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SwRI-1228-8-16

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FUNDAMENTAL INVESTIGATION OF LIQUID-METAL LUBRICATED JOURNAL BEARINGS

Quarterly Technical Report No. 5

Contract No. AT (II-I)-1228

Project Agreement No. 8

Modification No. 7

R. A. Burton

Y. C. Hsu

H. J. Carper

RELEASED FOR ANNOUNCEMENT
IN NUCLEAR SCIENCE ABSTRACTS

to

U.S. ATOMIC ENERGY COMMISSION

Washington 25, D.C.

November 1, 1965

SOUTHWEST RESEARCH INSTITUTE
SAN ANTONIO HOUSTON

Report No. RS-471

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Department of Aerospace Propulsion Research

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APPROVED:

P. M. Ku, Director
Department of Aerospace
Propulsion Research

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Report No. RS-471

FOREWORD

This report was prepared by Southwest Research Institute under Contract No. AT(11-1)-1228, Project Agreement No. 8, Modification No. 7. The work was administered by the Engineering Development Branch, Division of Reactor Development, U.S. Atomic Energy Commission, Washington 25, D. C., with Mr. Nicholas Grossman as Project Engineer.

This report covers work performed from August 1, 1965 through October 31, 1965.

ABSTRACT

An analytical approach to continuous turbulent films, described in previous reports, is exploited to provide estimates of performance of very short, full bearings operating in the turbulent regime. Similarly, the analysis is applied to tilted pads without end leakage, and shown to check with experimental measurements. Computer techniques are described whereby the analysis technique can be applied to finite bearings, of arbitrary film thickness variation.

Experimental data are reported for leading edge pressure rise under tilted pads, and for the pressure distribution under stepped pads. Further analysis is given for steady and fluctuational velocity profiles near a blockage in the bearing clearance.

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LIST OF SYMBOLS

a	Slider bearing slope
b	Local half-thickness of film
$\bar{b}$	Mean half-thickness of film
C_f	Friction coefficient
C_x	Friction coefficient in tangential direction
c	Radial bearing clearance
D	Journal diameter
e	Eccentricity
H	Dimensionless film thickness
h	Local film thickness
$\bar{h}$	Mean film thickness
h_c	Film thickness at point where only Couette flow prevails
k	Exponent in power-law friction-coefficient equation
L	Bearing length in axial direction
ℓ	Pad bearing length in tangential direction
m	Velocity profile exponent for axial flow
n	Velocity profile exponent for tangential-flow
P	Dimensionless pressure, $\frac{p}{\rho(u_s/2)^2}$
p	Gage pressure
p_a	Ambient pressure

LIST OF SYMBOLS (Cont'd)

R	Reynolds number based on local film thickness, bu_b/ν
$\bar{R}$	Reynolds number based on mean film thickness, $\bar{b}u_s/2\nu$, $\bar{b}u_{bc}/\nu$
r	Journal radius
U	u/u_{bc}
U^+	Dimensionless velocity
u	Velocity in tangential direction
V	Velocity vector, $\sqrt{u^2 + w^2}$
v	Velocity across film
W	Dimensionless velocity w/u_{bc}
w	Velocity in axial direction
X	Dimensionless distance
x	Coordinate in tangential direction
Y^+	Dimensionless wall distance
y	Coordinate across film
Z	Dimensionless axial coordinate
z	Axial coordinate
β	$\bar{h}/r$
δ^*	Displacement thickness
ϵ	Eccentricity ratio
ζ	Dimensionless coefficient in integrated turbulent film equation
μ	Absolute viscosity
ν	Kinematic viscosity

LIST OF SYMBOLS (Cont'd)

ξ	See Equation (39)
ρ	Fluid density
τ	Shear stress
ϕ	Angular coordinate on bearing
ψ	Attitude angle
χ	See Equation (40)
$()_b$	Refers to midchannel
$()_{bc}$	Refers to Couette flow at midchannel
$()_{bh}$	Refers to midchannel velocity relative to stationary wall
$()_{bo}$	Refers to midchannel velocity relative to moving wall
$()_c$	Refers to Couette flow
$()_h$	Refers to stationary wall
$()_o$	Refers to moving wall
$()_s$	Refers to surface velocity of journal

I. INTRODUCTION

This report summarizes progress in experimental measurements in a large-scale turbulent film apparatus, and in the analytical prediction of turbulent-film bearing behavior. Being a Progress Report, it must of necessity document the contributions made during the report period on a number of separate topics, all of which will be tied together more closely in the final report. Despite the somewhat broken-up nature of the subject, it has been found possible to include an introduction to each topic, briefly summarizing the past work, and setting down the accomplishments of the report period. Following this, detailed derivations, or analyses of experimental data are included for those who are interested in going more deeply into each topic. The topics are listed and discussed each individually in the following paragraphs.

A. Short Bearings

Experimental measurements were shown in Progress Report No. 5 to correlate well with analytical predictions for the short full bearing, as represented by the large-scale turbulent film apparatus. For the dimensions of the apparatus, inertial forces were sizeable relative to other forces (more so than would be expected in actual bearings); consequently, the experiments provided a check not only on ability to take account of turbulent film forces, but also to take account of inertia. The agreement between experiment and measurement was felt to be satisfactory. This suggested that the approach to analysis would be fruitful when carried over to bearing

dimensions, where any errors in inertial effects would be reduced considerably below those in the apparatus. In this report, a series of calculations is carried out for the very short bearing for variation of film thickness relative to diameter, length, and Reynolds number. These calculations are made with and without accounting of inertial effects, and comparison between the two types of calculations serves to show the importance of inertial effects in any given configuration.

Eccentricity plots suggest that the stability problem of the turbulent film bearing with large displacements may be somewhat different from that of laminar film bearings. This is particularly true when inertial effects are considered.

B. Pads without End Leakage

Experiments on tilted pads without end leakage have been carried out, and these have been compared with theoretical calculations. The experimental data give information on the initial "impact" pressure rise at the leading edge as well as the pressure distribution under the pad. Again this impact pressure and other inertial effects are exaggerated in the experiment, but appear to be accounted for well in the analysis.

C. Computer Program

A unique iterative technique has been devised for obtaining solutions for bearings of arbitrary film-thickness distribution and length-to-width ratio. This technique has been subjected to numerous tests of its accuracy and convergence, and some preliminary results have been produced for

simple tilting pads. As set up, the program is adaptable to the case where inertial effects are present, yet is of relatively secondary importance.

D. Stepped Pad

Pressure distributions for stepped pads have been measured experimentally, and found to be considerably different from what would have been expected on the basis of laminar flow theory, without accounting for inertia. Analyses for such pads are not complete, but are expected to be included in a general program of analysis which is presently under way. The data are offered at present because of their provocative nature.

E. Blocked Flow

In previous reports, severely distorted flows were produced by placing a blockage in the annulus of the turbulent film apparatus. These provide both tangential and axial pressure-flows of corresponding magnitude to the Couette flow in the channel. Calculations of wall shear stress, and comparisons of the velocity profile with the universal velocity profile have proved instructive.

II. STEADY STATE PERFORMANCE ESTIMATES FOR A SHORT, FULL BEARING

A. Introduction

A solution has been obtained for pressure variation in the large scale turbulence apparatus, and this has been shown to conform substantially to the experimental measurements. In the following, this solution has been adapted to more realistic bearing clearances and has been used to compute attitude angle, load carrying capacity and friction coefficient. The effects of inertia have been carried along in such a way that they may be removed from the remainder of the pressure solution, thus permitting a comparison of the consequences of including or neglecting these.

Nomenclature is illustrated in Figure 1.

In Reference (1) the pressure equation has been written as:

$$P(\phi, z) = \left[1 - \left(\frac{2z}{L} \right)^2 \right] \left[P_f + P_i \right] \quad (1)$$

where

$$\begin{aligned} P_f &\equiv P_{\text{friction}} \\ &= \frac{\zeta_6 C_{xc} \epsilon (L/D)}{4\zeta_7 (\bar{h}/L)} : \frac{\sin \phi}{(1 + \epsilon \cos \phi)^2} \end{aligned} \quad (2)$$

and

$$\begin{aligned} P_i &\equiv P_{\text{inertia}} \\ &= \frac{\zeta_2 \epsilon (L/D)^2 \cos \phi}{2\zeta_7 (1 + \epsilon \cos \phi)} + \frac{\zeta_3 (\epsilon L/D)^2 \sin^2 \phi}{(\zeta_7)^2 (1 + \epsilon \cos \phi)^2} \end{aligned} \quad (3)$$

FIGURE 1. NOTATION FOR SHORT FULL JOURNAL BEARING

In the right-hand side of Equation (1), the first and second terms denote the pressure components due to friction and inertia, respectively.

If the inertia effect upon the pressure is neglected, P is reduced to

$$P = P_f \quad (4)$$

where P_f is in the same form as in Equation (2).

From the pressure solution, bearing performance estimates for total load, attitude angle, frictional force and frictional coefficient may be obtained for the following cases:

- (1) Pressure distributions for $0 \leq \phi \leq 360^\circ$, both with and without accounting of inertial effects.
- (2) Positive pressure distribution only, without inertia effect, for $0 \leq \phi \leq 80^\circ$.

The general equations of the components of the total load and the circumferential frictional force at shaft are

$$W_\ell \sin \psi = \frac{D}{2} \int_{-L/2}^{L/2} \int_{\phi=\phi_1}^{\phi=\phi_2} p(\phi, z) \sin \phi \, d\phi \, dz \quad (5)$$

$$W_\ell \cos \psi = \frac{D}{2} \int_{-L/2}^{L/2} \int_{\phi=\phi_1}^{\phi=\phi_2} p(\phi, z) \cos \phi \, d\phi \, dz \quad (6)$$

where

ψ = attitude angle, degree

ϕ = angular coordinates measured from lines of centers in direction of journal motion, degree

L = bearing length

D = journal diameter

W_ℓ = bearing load

and

$p = \rho U_{bc}^2 P$, where P is the dimensionless pressure.

$$F = \frac{LD}{2} \int_{\phi=\phi_1}^{\phi=\phi_2} T_{xo} d\phi \quad (7)$$

where

$$T_{xo} = \frac{\rho C_{xc}}{2} (U_{bc})^2 \quad (8)$$

F = circumferential frictional force at shaft

B. Performance Estimates for Full 360° Short Journal Bearing, without Inertial Effects

Using $\phi_1 = 0^\circ$, $\phi_2 = 360^\circ$ and substituting Equation (4) into Equations (5) and (6), the components of the total dimensionless load, obtained by integration, are

$$\bar{W}_\ell \sin \psi = \frac{\pi \zeta_6 C_{xc} (L/D)}{6 \zeta_7 (\bar{h}/L) \epsilon} [(1 - \epsilon^2)^{-1/2} - 1] \quad (9)$$

$$\bar{W}_\ell \cos \psi = 0 \quad (10)$$

where

$$\bar{W}_\ell = \frac{W_\ell}{\rho (U_{bc})^2 DL} \quad (11)$$

From Equations (9) and (10), the quantities of $\bar{W}_\ell$ and ψ are

$$\bar{W}_\ell = \frac{\pi \zeta_6 C_{xc} (L/D)}{6 \zeta_7 (\bar{h}/L) \epsilon} [(1 - \epsilon^2)^{-1/2} - 1] \quad (12)$$

and

$$\psi = 90^\circ \quad (13)$$

Using $\phi_1 = 0^\circ$, and $\phi_2 = 360^\circ$ and Equations (7) and (8), the frictional force, F , is given by

$$F = \frac{\pi}{2} \rho C_{xc} L D (u_{bc})^2 \quad (14)$$

Using the definition of f and Equations (12) and (14), the frictional coefficient at the shaft is found to be

$$\begin{aligned} f &= F/W_\ell \\ &= \frac{3\zeta_7(\bar{h}/L)\epsilon}{\zeta_6(L/D)[(1 - \epsilon^2)^{-1/2} - 1]} \end{aligned} \quad (15)$$

C. Performance Estimates for Full 360° Short Journal Bearing, with Inertial Effects

Using $\phi_1 = 0^\circ$, $\phi_2 = 360^\circ$ and substituting Equation (4) into Equations (5) and (6), the components of the total dimensionless load, obtained by integration, are

$$\bar{W}_\ell \sin \psi = \frac{\pi \zeta_6 C_{xc} (L/D) [(1 - \epsilon^2)^{-1/2} - 1]}{6 \zeta_7 (\bar{h}/L) \epsilon} \quad (16)$$

$$\begin{aligned} W_\ell \cos \psi &= - \frac{\pi \zeta_2 (L/D)^2 [1 - (1 - \epsilon^2)^{-1/2}]}{3 \epsilon \zeta_7} \\ &\quad + \frac{2\pi \zeta_3 (L/D)^2}{3 \epsilon (\zeta_7)^2} \left\{ \epsilon^2 (1 - \epsilon^2)^{-1/2} [1 - (1 - \epsilon^2)^{-1}] \right. \\ &\quad \left. + [2 - 3(1 - \epsilon^2)^{-1/2} + (1 - \epsilon^2)^{-3/2}] \right\} \end{aligned} \quad (17)$$

From Equations (16) and (17), $\bar{W}_\ell$ is

$$\begin{aligned}\bar{W}_\ell &= [(\bar{W}_\ell \sin \psi)^2 + (\bar{W}_\ell \cos \psi)^2]^{1/2} \\ &= \frac{2\pi(L/D)^2}{3\epsilon(1 - \epsilon^2)^{1/2}} \cdot G\end{aligned}\quad (18)$$

where

$$\begin{aligned}G &= \frac{1}{2\xi_7} \left\{ \left[\left(\frac{\xi_6 C_{xc}}{2 \cdot \frac{L}{D} \cdot \frac{\bar{h}}{L}} \right)^2 + (\xi_2)^2 \right] [(2 - \epsilon^2) - 2(1 - \epsilon^2)^{1/2}] \right. \\ &\quad + 4(\xi_3/\xi_7)^2 [8(1 - 3\epsilon^2)(1 - \epsilon^2)^{-2} - (8 - 12\epsilon^2 + 4\epsilon^4)(1 - \epsilon^2)^{-1/2} \\ &\quad + \epsilon^4(5 - \epsilon^2)^2(1 - \epsilon^2)^{-2}] - 4(\xi_2 \xi_3/\xi_7) [(4 - 3\epsilon^2) \\ &\quad \left. - (4 - \epsilon^2)(1 - \epsilon^2)^{1/2}] \right\}^{1/2}\end{aligned}\quad (19)$$

Dividing Equation (16) by (17), ψ is

$$\psi = \tan^{-1} \left[\frac{\xi_6 C_{xc} (1 - \epsilon^2) [1 - (1 - \epsilon^2)^{1/2}]}{2 \left(\frac{L}{D} \right) \left(\frac{\bar{h}}{L} \right) \left\{ \xi_2 (1 - \epsilon^2) [1 - (1 - \epsilon^2)^{1/2}] + \frac{2\xi_3}{\xi_7} [-(2 - 3\epsilon^2 + \epsilon^4) + 2(1 - \epsilon^2)^{3/2}] \right\}} \right]\quad (20)$$

When $\epsilon \rightarrow 0$, the limit of ψ in Equation (20) is

$$\psi = \lim_{\epsilon \rightarrow 0} \psi = \tan^{-1} \left[\frac{\xi_6 C_{xc}}{2\xi_2 (\bar{h}/L)(L/D)} \right]\quad (21)$$

When $\epsilon \rightarrow 1$, the limit of ψ in Equation (20) is

$$\psi = \lim_{\epsilon \rightarrow 1} \psi = \tan^{-1} \left\{ \frac{(\xi_6 C_{xc}) / [(2L/D)(\bar{h}/L)]}{\xi_2 - 2\xi_3/\xi_7} \right\}\quad (22)$$

The frictional force, F , is in the same general form as Equation (14).

Using Equations (14), (18), and (19),

$$f = F/W_\ell$$

$$= \frac{3C_{xc}\epsilon(1 - \epsilon^2)^{1/2}}{4(L/D)^2G} \quad (23)$$

where G is identical in form with Equation (19).

When $\zeta_2 = \zeta_3/\zeta_7 = 0$, the quantities of $\bar{W}_\ell$, ψ , and f in Equations (18), (20), and (23) can be reduced to those without inertia effect in Equations (12), (13), and (15), respectively.

D. Discussion of Performance Estimates for Short Full Journal Bearings

The parameters corresponding to $\bar{R} = 1200$ and $\bar{R} = 3400$ are listed in Table 1.

TABLE 1. VALUES OF PARAMETERS FOR
 $\bar{R} = 1200$ AND $\bar{R} = 3400$

$\bar{R}$	C_{xc}	U	W	ζ_2	ζ_3	ζ_6	ζ_7
1200	0.011	0	$W \doteq 0$	0.8571	0.75	1	0.85714
3400	0.0068	0	$W \doteq 0$	0.9	0.81818	1	0.9

The quantities of W_ℓ in Equations (12) and (18), ψ in Equations (13) and (20), and f in Equations (15) and (23) are plotted in Figures 2 to 11 for various values of L/D , $\bar{h}/L$ and $\bar{R}$ for the cases without and with inertia effect, respectively. Plots of load versus eccentricity are shown in

FIGURE 2. VARIATION OF LOAD SUPPORT WITH ECCENTRICITY FOR A FULL BEARING WITH $L/D = 1/7$

FIGURE 3. VARIATION OF LOAD SUPPORT WITH ECCENTRICITY FOR A FULL BEARING WITH $L/D = 1/4$

FIGURE 4. VARIATION OF LOAD SUPPORT WITH ECCENTRICITY FOR A FULL BEARING WITH $L/D = 1/3$

FIGURE 5. ATTITUDE ANGLE VERSUS ECCENTRICITY FOR $L/D = 1/7$

FIGURE 6. ATTITUDE ANGLE VERSUS ECCENTRICITY FOR $L/D = 1/4$

FIGURE 7. ATTITUDE ANGLE VERSUS ECCENTRICITY FOR $L/D = 1/3$

FIGURE 8. FRICTION COEFFICIENT VERSUS ECCENTRICITY FOR $L/D = 1/7$

FIGURE 9. FRICTION COEFFICIENT VERSUS ECCENTRICITY FOR $L/D = 1/4$

FIGURE 10. FRICTION COEFFICIENT VERSUS ECCENTRICITY FOR $L/D = 1/3$

FIGURE 11. ATTITUDE PLOTS FOR 180°, SHORT, JOURNAL-BEARINGS

Figures 2 to 4 are for different values of L/D and $\bar{h}/L$ for two Reynolds numbers $\bar{R} = 1200$ and $\bar{R} = 3400$. The lower the Reynolds number, the higher the dimensionless load capacity, with or without accounting of inertia effect. For a corresponding film-thickness/journal-diameter ratio, $\bar{h}/D$ and $\bar{R}$, the load capacity can be increased while eccentricity ratio increases. The inertia effect on the load capacity becomes, indeed, significant for the relatively large values of $\bar{h}/D$ and $\bar{R}$. In fact, the load capacity can be improved and increased by taking account of the inertial effect. The smaller the value of $\bar{h}/D$, the higher the load capacity.

The eccentricity versus attitude angle locus are shown in Figures 5 to 7 for different values of L/D and $\bar{h}/L$ for two Reynolds number $\bar{R} = 1200$ and $\bar{R} = 2400$. For the relatively large values of $\bar{h}/D$ and $\bar{R}$, the contribution to the attitude angle from the inertial effect is dominant. Considering inertia effect, the attitude angle becomes sensitive to the eccentricity ratio when ϵ is in the range such that $0.6 \leq \epsilon \leq 1$. For the case without inertia effect, the attitude angle must be 90° . In other words, the load, W_ℓ , acts perpendicular to the centerline. This means that the journal center will (under load and without considering inertia effect) move over along a line at right angles to the load line. For the very thin film, i. e., smaller values of $\bar{h}/D$, the attitude angle for different value of eccentricity with inertia effect is in the neighborhood of $\psi = 90^\circ$.

Plots of frictional coefficient versus eccentricity ratio are shown in Figures 8 to 10 for different values of L/D and $\bar{h}/L$ for two Reynolds numbers $\bar{R} = 1200$ and $\bar{R} = 3400$. For the relatively large values of $\bar{h}/D$ and $\bar{R}$, the inertial effect on the frictional coefficient becomes important in the range of ϵ such that $0.4 \leq \epsilon \leq 0.6$. For a corresponding value of $\bar{h}/D$ and $\bar{R}$, the lower the frictional coefficient the higher the eccentricity ratio. The smaller the value of $\bar{h}/D$, the smaller the frictional coefficient.

It is concluded from these data that a thin turbulent film behavior causes a reduction in frictional coefficient and an increase in load capacity as compared with a thick turbulent film, for the same values of $\bar{R}$ and ϵ .

E. Computed Performance of a 180° Sector, Short Journal Bearing, Neglecting Inertia

In Equation (4), the value of $P(\phi, z)$ is positive in the converging film such that $0 \leq \phi \leq 180^\circ$. Substituting Equation (4) into Equations (5) and (6), the components of the total dimensionless load are:

$$\bar{W}_\ell \sin \psi = \frac{\pi \zeta_6 C_{xc}(L/D)[(1 - \epsilon^2)^{-1/2} - 1]}{12 \zeta_7 \bar{\epsilon}(\bar{h}/L)} \quad (24)$$

$$\bar{W}_\ell \cos \psi = \frac{\zeta_6 C_{xc}(L/D)}{12 \zeta_7 \bar{\epsilon}(\bar{h}/L)} \left[\frac{2}{(1 - \epsilon^2)} - \frac{1}{\epsilon} \ln \frac{(1 + \epsilon)}{(1 - \epsilon)} \right] \quad (25)$$

where

$$\bar{W}_\ell = \frac{\bar{W}_\ell}{\rho(u_{bc})^2 DL} \quad (26)$$

From Equations (24) and (25), $\bar{W}_\ell$ is

$$\bar{W}_\ell = \frac{\zeta_6 C_{xc}(L/D) M}{6\epsilon(\bar{h}/L)} \quad (27)$$

where

$$M = \frac{1}{2\zeta_7} \left\{ \frac{\pi^2}{(1-\epsilon^2)} [(2-\epsilon^2) - 2(1-\epsilon^2)^{1/2}] + \left[\left(\ln \frac{1+\epsilon}{1-\epsilon} \right)^2 - \frac{4\epsilon}{(1-\epsilon^2)} \ln \frac{(1+\epsilon)}{(1-\epsilon)} + \frac{4\epsilon^2}{(1-\epsilon^2)^2} \right] \right\}^{1/2} \quad (28)$$

Dividing Equation (24) over (25), ψ is

$$\psi = \tan^{-1} \left\{ \frac{\pi(1-\epsilon^2)^{1/2} [1 - (1-\epsilon^2)^{1/2}]}{2\epsilon - (1-\epsilon^2) \ln [(1+\epsilon)/(1-\epsilon)]} \right\} \quad (29)$$

When $\epsilon \rightarrow 0$, the limit of ψ in Equation (29) is

$$\psi = \lim_{\epsilon \rightarrow 0} \psi = 90^\circ \quad (30)$$

When $\epsilon \rightarrow 1$, the limit of ψ in Equation (29) is

$$\begin{aligned} \psi &= \lim_{\epsilon \rightarrow 1} \psi \\ &= \lim_{\epsilon \rightarrow 1} \left\{ \tan^{-1} \left[\frac{\pi(1-\epsilon^2)^{1/2}}{8\epsilon} \right] \right\} \\ &= 0^\circ \end{aligned} \quad (31)$$

Using $\phi_1 = 0^\circ$, $\phi_2 = 180^\circ$, and Equations (7) and (8), F is

$$F = \pi \rho C_{xc}(LD) (u_{bc})^2 \quad (32)$$

Using the quantities of F and W_ℓ from Equations (32) and (27), f is found to be:

$$\begin{aligned}
 f &= F/W_\ell \\
 &= \frac{6\pi\epsilon(\bar{h}/L)}{\zeta_6 M(L/D)}
 \end{aligned}
 \tag{33}$$

where M is identical to one in Equation (28).

For the convergent laminar film without inertial effect⁽²⁾

($0^\circ \leq \phi \leq 180^\circ$), ψ is

$$\psi = \frac{\pi(1 - \epsilon^2)^{1/2}}{4\epsilon}
 \tag{34}$$

The values of ψ for the laminar and turbulent films in Equations (34) and (29) are plotted in Figure 11.

At values of ϵ near zero, the line of centers becomes nearly perpendicular to the load line, and it is parallel to the load line at $\epsilon = 1$ for the laminar and turbulent flows, respectively. The locus of the journal for the case of laminar flow lies within a semicircle; but, for the case of turbulent flow, it lies outside a semicircle.

III. COMPARISON BETWEEN EXPERIMENTAL AND ANALYTICAL RESULTS FOR TILTED PADS WITHOUT END LEAKAGE

A. Experimental Results

In the large-scale turbulence apparatus, a tilted pad was simulated by removing all but one 3-foot section of the external cylindrical surface, and tilting this at selected angles to the rotating drum. End leakage was restricted by stiff plastic curtains which spanned the gap between the stationary pad surface and the rotating drum surface, rubbing at the latter contact. Pressure was so small that it did not tend to force these curtains away from the drum. For such a pad, velocity profile traverses and pressure data were obtained at different angles of pivoting, holding the film thickness constant at a distance, x , back from the leading edge, equal to $x/L = 0.7$. The thickness at this point was held to 0.54 inch. The film thickness at leading and trailing edges was measured, and, in the analytical approach to the problem, a best parabola was fitted to these dimensions for each operating condition to give an approximation of the actual thickness variation.

Pressure was measured at midspan in the axial direction, at the leading edge, and at $x/L = 0.7$. Table 2 summarizes the dimensional data and pressure data. The results of these measurements are shown on the data plots of Figure 12, compared with theoretical curves for the same dimensional variation of the film thickness, and Reynolds number for which the data were taken. In viewing these data, it is of interest

FIGURE 12. THEORETICAL AND EXPERIMENTAL PRESSURE DISTRIBUTIONS FOR A TILTED PAD

that a significant amount of the pressure rise occurred at the leading edge, due to what has been termed an "impact pressure" effect. It must be remembered that this effect is somewhat exaggerated due to the emphasis of inertial contributions, as a consequence of the relative dimensions chosen for the large-scale turbulent-film apparatus. Such inertial contributions would be expected to be minimized somewhat in actual bearing clearances, just as inertial effects were minimized in the thinner clearances in the previous section on short bearings. Nevertheless, the leading edge pressure rise (which is not calculable from film considerations) forms a significant part of the problem of estimating tilting pad performance.

B. Analytical Approach to the Problem

It is of interest to determine how closely the simplified film equations which have been developed in previous progress reports fit the measured data for the tilted pad without end leakage. As has been pointed out, the analytical approach to the short bearing fit experimental data very well. The opposite extreme test would be the case of no end leakage where the x-velocity components rather than the z-velocity components dominate. Consequently, application of the simplified analysis to the no-end leakage problem provides a second test of this analysis under a limiting extreme case. As will be shown, satisfactory agreement was obtained insofar as it could be checked, and confidence in the analytical approach is heightened.

From Equation (73) in Reference 3, a solution for dimensionless pressure without end leakage was shown to be

$$-\frac{dP}{dX} = \frac{-\zeta}{\chi^2} \frac{dH/dX}{H^3} - \frac{\zeta(\chi - 1)}{H^2 \chi^2} \frac{dH}{dX} + \left(\frac{1 - H}{H^2} \right) \quad (35)$$

where

$$\frac{x}{h_c} = X, \quad \frac{\Delta u_b}{u_{bc}} = U, \quad \frac{p}{\rho u_{bc}^2} = P \quad (36)$$

$$H = \frac{h}{h_c} = \frac{a}{h_c} + \frac{b}{(r + h)} x + \frac{c}{(r + h)^2} x^2 \quad (37)$$

$$\zeta = \frac{(1 + U^2)/(1 + 2n) + 2(n + U)/(1 + n)}{(1 + U)^2} \quad (38)$$

$$\xi = \frac{(1 + U)^{2-k} - (1 - U)^{2-k}}{4U} \quad (39)$$

$$\chi = \frac{1}{1 + n} \quad (40)$$

Equations (38) to (40) are identical to Equations (40), (35), and (46) in Reference 3. The general magnitudes of ζ , ξ , and χ in Equations (38) to (40) are valid for any kind of bearing films without end leakage. The geometric constants a , b , and c of Equation (37), measured film thickness, and measured pressures are listed in Table 2.

Equation (35) may be solved directly for pressure gradient when the variation of H is inserted and the remaining parameters are assigned numerical values. Given values of U , n , and k , the quantities ζ , ξ , and χ

TABLE 2. EXPERIMENTAL DATA FOR TILTING PADS

Case	Film Thickness, h, inches			Parameters, Equation (37)			Pressure, P	
	$x/l = 0$	$x/l = 0.7$	$x/l = 1$	a	b	c	$x/l = 0$	$x/l = 0.7$
1	1.0	0.54	0.265	0.28	0.428	1.0	0.28	0.428
2	0.85	0.54	0.357	0.257	0.264	0.85	0.257	0.264
3	0.7	0.54	0.449	0.149	0.099	0.7	0.149	0.099

were evaluated by means of Equations (38) to (40). The required quantities so calculated, or set by the experimental conditions of Figure 12, are listed in Table 3.

TABLE 3. REQUIRED QUANTITIES FOR EQUATION (35)

$\bar{R}$	U	k	n	ζ	ξ	χ	C_{xc}
2400	-0.31	0.25	0.125	1.155	0.621	0.889	0.0075

Once the gradient was calculated by Equation (35) for x-points along the pad, numerical integration provided magnitudes of P. To evaluate the constants of integration, initial pressure was taken as the experimental value, the trailing edge pressure was taken as zero.

Referring back to Figure 12, it is seen that for the two extreme cases the fit between data and predictions is exceptional. For the intermediate case, the fit is less satisfactory. The error in this one data point may possibly be due to errors in measurement of pressure or film thickness.

C. Leading Edge Pressure Rise

Since little information exists on leading edge pressure rise, additional analysis has been carried out to set the measurements reported in Figure 12 into perspective. In particular, it is of interest to compare cases on the basis of stagnation pressure, $p/\rho u_b^2 = P + (\zeta/2)(1 + U)^2 = P_o$. The velocity U was calculated from the computed curves in Figure 12, and results are summarized in Table 4.

TABLE 4. LEADING EDGE PRESSURE DATA

$h_{L.E.},$ inches	U	ζ	$(\zeta/2)(1 + U)^2$	P	P_o
1	-0.9665	1.155	0.000649	0.28	0.280
0.85	-0.5939	1.155	0.095244	0.257	0.352
0.7	-0.528	1.155	0.12829	0.149	0.277

From this, again considering only the extreme cases, it appears that $P_o = \text{const.} = 0.28$. Until more data become available, this is recommended as a design figure.

IV. ITERATIVE PROCEDURE FOR FINITE-LENGTH TURBULENT-FILM BEARING CALCULATIONS

The simplified turbulent-film equations are

$$\begin{aligned} \frac{\zeta_1}{\zeta_2} \left[2(1+U) \frac{\partial U}{\partial X} + \frac{(dH/dX)}{H} (1+U)^2 \right] + W \frac{\partial U}{\partial Z} + (1+U) \frac{\partial W}{\partial Z} \\ = - \frac{1}{\zeta_2} \frac{\partial P}{\partial X} - \frac{2\zeta_5}{H\zeta_2} C_{xc} U \end{aligned} \quad (41)$$

$$\begin{aligned} \frac{2\zeta_3}{\zeta_2} W \frac{\partial W}{\partial Z} + (1+U) \frac{\partial W}{\partial X} + W \frac{\partial U}{\partial X} + \frac{(dH/dX)}{H} W(1+U) \\ = - \frac{1}{\zeta_2} \frac{\partial P}{\partial Z} - \frac{\zeta_6}{\zeta_2} C_{xc} \frac{W}{H} \end{aligned} \quad (42)$$

$$\frac{(dH/dX)}{H} (1+U) + \frac{\partial U}{\partial X} + \zeta_7 \frac{\partial W}{\partial Z} = 0 \quad (43)$$

The boundary conditions for the present analysis are

$$\begin{aligned} \text{at } X = 0, \quad & P = P_0, \quad U = U_0 \\ \text{at } X = L, \quad & P = 0 \\ \text{at } Z = Z_{\max}, \quad & P = 0 \\ \text{at } Z = 0, \quad & W = 0 \end{aligned}$$

This means: that the shearing motion is in the x-direction; that Z-flow is leakage flow; that the line $Z = 0$ corresponds to the midline of a symmetrical bearing, where there is no flow across this line. It is assumed that there is some leading-edge pressure in the X-direction, P_0 , and some inlet velocity, U_0 . In the analysis which follows, the contour (variation of H) will be

assumed to be a function of X alone, though the general approach is not necessarily subject to this restriction.

To produce a first approximation, which may be improved by iteration, the inertial terms in Equations (41) and (42) may be neglected and the quantities ζ_i and C_{xc} may be treated as constants in the (X, Z) space. After pressures and velocities are computed on this trial basis, the velocity magnitude may be used to compute approximate magnitudes for the inertial term; and, with tables given in prior reports, may be used to compute improved values of ζ_i , as functions of X and Z . This is possible because in cases of real interest, the inertial terms are of secondary importance, and the variation of ζ_i is not great. Even with this simplification, there remains a significant problem in computing the pressures for a given set of edge conditions and a given variation of H .

Without the inertia terms, Equations (41), (42), and (43) become respectively

$$F + \frac{\partial U}{\partial X} + \zeta_7 \frac{\partial W}{\partial Z} = 0 \quad (44)$$

$$\frac{\partial P}{\partial X} = - 2\zeta_5 C_{xc} \frac{U}{H} = A \frac{U}{H} \quad (45)$$

$$\frac{\partial P}{\partial Z} = - \zeta_6 C_{xc} \frac{W}{H} = B \frac{W}{H} \quad (46)$$

Since a number of terms have been relegated to the second iteration, linearity of (46) is achieved by similarly dropping U in this equation in the first iteration, and then supplying the approximate value of this variable

at the same time as the inertial terms and ζ_i terms for the second iteration. Letting $2\zeta_5 C_{xc}$ be defined as A, $\zeta_6 C_{xc}$ be defined as B, and letting $(1 + U) \frac{(dH/dX)}{H}$ be defined as F, the equations may be written in reverse order to become

$$F = \frac{\partial U}{\partial X} + \zeta_1 \frac{\partial W}{\partial Z} \quad (47)$$

$$\frac{\partial P}{\partial X} = - \frac{AU}{H} \quad (48)$$

$$\frac{\partial P}{\partial Z} = - \frac{BW}{H} \quad (49)$$

Solutions may be obtained that satisfy (47) and that may or may not satisfy (48) and (49); however, if a systematic group of solutions can be obtained, it is permissible to sum these to achieve a combined solution which satisfies all three equations. The problem is to devise a system of generation and summing such solutions, which is economical of computer time. The method chosen here is to begin with a primitive solution of Equation (47) and allow the discrepancy between the pressures computed from (48) and (49) from this solution to serve as a guide in generation of successive solutions. For example, the following represents a valid solution for (47)

$$F/2 = \frac{\partial U}{\partial X} \quad (50)$$

with the complementary statement

$$F/2 = \zeta_1 \frac{\partial W}{\partial Z} \quad (51)$$

It follows from (50) that

$$U = U_0 + \int_0^X -\frac{FdX}{2} \quad (52)$$

where U_0 is an initial value and $X = 0$. It also follows from (52) that

$$P_x = P_0 + \int_0^X \frac{A}{H} U dX \quad (53)$$

where P_0 is an initial value, and P_x is P as computed by the X integration.

Combining (52) and (53)

$$P_x = P_0 + \int_0^X \frac{A}{H} \left[U_0 + \int_0^X -\frac{FdX}{2} \right] dX \quad (54)$$

but U_0 is determined by the requirement that the trailing edge pressure be zero; hence

$$-U_0 = P_0 + \int_0^L \left[\frac{A}{H} \int_0^X -\frac{F}{2} dX \right] dX \quad (55)$$

Thus performing the integration of Equation (55) numerically, the solution for P_x can be found corresponding to the primitive solution given by (51) and (50).

By a corresponding process

$$\xi_7 F/2 = -\partial W/\partial Z \quad (56)$$

or

$$W = \int_0^Z -\frac{\xi_1 F}{2} \quad (57)$$

Since $P = 0$ at $Z_{\max}$ and P_z is P computed by integration in the Z -direction

$$P_z = \int_{Z_{\max}}^Z \frac{B}{H} \int_0^Z - \frac{\xi_1 F}{2} dZ dZ \quad (58)$$

it would be only by chance that P_z equal P_x throughout the entire field.

A solution can be obtained, however, which, when added to the prior solution, tends to produce values of P_z closer to those of P_x , than in the primitive solution. This is obtained by letting at each point

$$\frac{\partial U}{\partial X} = a(P_z - P_x) \quad (59)$$

and noting that to satisfy Equation (46) there must be a complementary solution

$$\frac{\xi_7 \partial W}{\partial Z} = - a(P_z - P_x) \quad (60)$$

In (59) and (60) $a \ll 1$, say 0.001, and is called the relaxation coefficient.

For the next approximate solution the boundary conditions become

$$W = 0 \text{ at } Z = 0$$

$$P = 0 \text{ at } Z = Z_{\max}$$

$$P = 0 \text{ at } X = 0$$

$$P = 0 \text{ at } X = L$$

(61)

A solution which satisfies these boundary conditions may be added to the previous solution, and thus the sum will satisfy all of the initial boundary conditions as stated. For the incremental solution this is equivalent to

letting $F = a(P_z - P_x)$, in Equations (53), (54), and (57), and letting $P_o = 0$. By this means, the new P_z and P_x magnitudes may be produced for all (X, Z) points and may be added to the previously obtained matrices of P_x and P_z . The new matrices of P_x and P_z may be used in Equations (59) and (60) for additional repeats of the process of successive approximation.

There is no question but what this iterative procedure produces successive solutions which tend to correct the errors in prior solutions. There may be questions as to how well the solution converges to the correct one, and these can best be dealt with by testing the behavior in actual calculation.

The illustrations which follow show the results of such calculations. In Figure 13, the solid curve shows the convergence toward a single solution for P_x and P_z . This plot represents the pressure and the center of the bearing. Leading edge pressure $P_{L.E.} = P_o = 0$, and the length was 250 times the film thickness, and the bearing length to width ratio was 0.54. Initially, the two solutions differ by 0.6. By ten iterations, they differ by 0.1, and, by twenty iterations, they are very close together. This is for a relaxation factor of $a = 0.001$.

The broken curve shows accelerated convergence caused by increasing the relaxation factor by 1.1 for each iteration. The number of iterations for a given degree of convergence is roughly halved. Figure 14 shows a comparison between the solutions for a finite length long-bearing, and the analytical solution for an infinite length bearing. Here it is seen that convergence is

FIGURE 13. ILLUSTRATION OF CONVERGENCE OF ITERATIVE CALCULATIONS

FIGURE 14. COMPARISON BETWEEN INFINITE-LENGTH-BEARING ANALYTICAL SOLUTION, AND LONG-BEARING COMPUTATION

rapid, but the computer solution is high. Since the integrations were carried out numerically, grid size rather than the iterative process was thought to be the factor in this discrepancy. This was checked for different mesh sizes and, in Figure 15, is shown to approach the analytical solution for $G = 40$ where G is the number of grid segments in the X-direction.

To illustrate the kind of results which can ultimately be obtained, Figure 16 has been prepared showing pressure variations for different length/width ratios for tilted pads. To account for inertial effects the solution was iterated twenty times as discussed previously, and the final W and U were used to estimate the quantities omitted in the previous solution. Then the iteration procedure was repeated with these new terms in the equations. In Figure 16, one can see how the effects of inertia raise somewhat the pressures throughout, particularly toward the trailing edge.

FIGURE 15. ILLUSTRATION OF CONVERGENCE OF LONG BEARING MID POINT PRESSURE TOWARD EXACT VALUE OF 1.4 AS FUNCTION OF GRID SIZE

FIGURE 16. PRESSURE SOLUTIONS FOR SELECTED LENGTH WIDTH RATIOS FOR TILTED PADS

V. EXPERIMENTAL MEASUREMENTS ON STEPPED PADS

A stepped pad was created in the large-scale turbulent film apparatus by cementing a 1/4-inch thick sheet of rubber to the after half of the surface of the previously used tilting pad. When this was placed concentric with the rotating drum, this made a stepped pad for which the first half of the circumferential length had a film thickness almost double that of the second half. Pressure measurements were made at three points along the larger film thickness and five points along the thinner film, these measurements being along the centerline in the axial direction. Shields were not applied to the ends of the film, so this represents a short stepped-pad with end leakage.

Results are plotted in Figures 17 and 18, showing a rise of pressure from the leading edge up to the step, a drop to negative magnitudes at the step, and a rise in pressure back toward zero in the thin film portion under the pad. This was not at all anticipated and is believed to be due almost entirely to the exaggerated inertial effects in the relatively thick film. One may ask (since inertial effects are so important) why there was not a corresponding leading edge pressure rise as encountered in the tilted pad. Looking at the scale for the dimensionless pressure, P , however, one sees that such a leading edge pressure rise would be very small compared to the overall pressures measured in this bearing.

FIGURE 17. PRESSURE DISTRIBUTION FOR
STEPPED PAD AT $R = 1200$

FIGURE 18. PRESSURE DISTRIBUTION FOR
STEPPED PAD AT $R = 2400$

Though inertial effects have shown slight significance in other bearing situations, they are believed to be more exaggerated here. It is believed that because of their predominant effect here, they will remain significant in thinner films. It is hoped, that in the forthcoming period, a check can be made where it will be determined if the iterative procedure outlined in Section IV will predict the measurements found here for the thick films. If they predict at all well the type of phenomena encountered, they will then be applied to thinner films to determine what inertial effects remain.

VI. DECELERATED FLOWS

As discussed in previous reports, operating with a full external cylinder and with internal cylinder concentric, it was possible to produce extremely distorted flows by attaching to the outer cylinder a blockage (parallel to the axis of the cylinders), so that this blockage completely spanned the film except for a very thin clearance. In so doing, the shear flow changed from a Couette flow (at approximately 180° away from the blockage), to an extremely decelerated flow ($U \rightarrow -1$) near the blockage. These extreme flows are of interest in determining to what extent the basic profile parameters differ from those velocity profiles reported for simple Couette and pressure flows. A decelerated flow profile is plotted in Figure 19 where y is measured from the stationary surface. In this case, the full channel profile is used, and one question might be as to whether this resembles, in this particular case, a half channel profile of pure pressure flow, since shear stress varies from a maximum at the moving surface to zero at the stationary wall. A typical pressure flow would vary from maximum at the moving surface to zero at midstream.

Since the profiles themselves are relatively insensitive to these details, their shape alone does not give one the desired evidence. However, measurement of the fluctuation velocity component u and its distribution across the channel does provide such an indication. In Figure 20, the data for this component measured across the channel are plotted and compared

(a)

(b)

FIGURE 19. NORMALIZED FULL-CHANNEL VELOCITY PROFILES IN FIELD WITH LARGE PRESSURE GRADIENT FOR CONCENTRIC OPERATION
(a) $\bar{R} = 2400$ AND (b) $\bar{R} = 250$

NOTE: X REPRESENTS DISTANCE FROM BLOCKAGE IN FILM

FIGURE 20. VARIATION OF VELOCITY FLUCTUATION ACROSS FILM IN BLOCKED FLOW

with measurements for pressure flow by Reichart where, for purposes of comparison, midstream was allowed to correspond to $h = 1$. In other words, h was used as the full channel width for the measurements reported here and was used as a distance from zero to midstream for Reichart's pressure flow. In either case, the point $y/h = 1$ corresponds to a point of zero shear stress. The close correspondence of the two variations indicates that the case of zero shear stress at a wall corresponds to the case zero shear stress at midstream.

A second test has been to fit the data for velocity and wall distance, to the universal velocity profile, and thus compare the relative fit at different wall distances. By this we mean that the values of density and viscosity for the fluid were taken, along with a given measurement of actual velocity and wall distance. Then a value of friction coefficient was found which would set the given measurements on the universal velocity profile. At each wall distance this calculation was repeated.

If the actual profile corresponded to the universal velocity profile, the same value of friction coefficient would have served in each case. If it did not correspond, then one would find a variation of apparent friction coefficient with wall distance. Figure 21 represents an illustrative case where this was done, and it was found that friction coefficient was remarkably constant. Thus the velocity profile in this extremely distorted flow still corresponded very closely to the universal velocity profile, when measured out from the moving wall. This is not intended to

FIGURE 21. VARIATION OF APPARENT FRICTION COEFFICIENT ACROSS FILM IN BLOCKED FLOW

illustrate any mysterious relationship, but only that the velocity profile of turbulent flow is not greatly sensitive to detailed conditions of the flow. This was illustrated earlier in Reference 2 by showing that the velocity profiles for pressure flow and Couette flow were very close to each other and that a single curve would fit either one of them fairly well. Here again, it is illustrated that details of shear stress variation and other unknown variables simply do not change the velocity profile itself greatly.

VII. CONCLUSIONS

We conclude that the chosen analytical approach lends itself easily to computation, and that the computed results agree with experimental measurements to the extent that they can be checked.