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REENTRY AND DISPOSAL PHENOMENA
FOR NUCLEAR AUXILIARY POWER SYSTEMS

Final Report

November 1963

TECHNICAL DOCUMENTARY REPORT NO. RTD-TDR-63-3003

Research and Technology Division
Air Force Systems Command
AIR FORCE WEAPONS LABORATORY
Kirtland Air Force Base
New Mexico

Project No. 1831, Task No. 183101

DDC

DEC 5 1963

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Research and Technology Division
Air Force Systems Command
AIR FORCE WEAPONS LABORATORY
Kirtland Air Force Base
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ABSTRACT

This study examines the several phenomena associated with the reentry from orbit of a satellite-borne nuclear power source and the subsequent dispersal of radioactive debris. The obvious hazards to the civilian population in the dispersal areas motivated this study, one of several being conducted under contract with the United States Air Force Weapons Laboratory. The immediate purposes of the present study, differing somewhat from those of other contractors, was to establish the possibility that reentry through the atmosphere would cause the nuclear fuel elements to incinerate and thereby to be rendered innocuous. Several configurations have been included in the detailed hypotheses, including rods, cylinders, and random asymmetric shapes--all of them studied under the many parameters of reentry phenomena. Included are test data from the actual recovery of debris of a satellite booster which placed a Mercury capsule in orbit in 1962. While a large volume of data in many parameters has been derived from this study, no definitive conclusions can be made as yet. It is, however, generally indicated that incineration alone is not a sufficient guarantee that the hazards of radioactive fuel deposition on earth have been eliminated.

PUBLICATION REVIEW

This report has been reviewed and is approved.

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PREFACE

The format of this report has been tailored to permit a quick evaluation of the results and conclusions, unhindered by unnecessary detail, if such detail is not required by the reader.

A more detailed discussion of the areas considered during the course of this study is included in the Appendices for the reader who requires a deeper insight into the problem area examined.

Secondary report number for this report is AE 63-0143.

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1. INTRODUCTION

A. Objectives of the Study

This study was motivated by a concern for the safety of the civilian populace in the event that a satellite-borne nuclear reactor reenters the atmosphere. This particular study was one of eight, conducted by various contractors under the direction of the Air Force Weapons Laboratory, the purpose of which was to examine in greater detail, various questions raised by a previous and very comprehensive study of this problem area, conducted by the AVCO Corporation¹ under the sponsorship of the Air Force Special Weapons Center.

The purpose of this particular study was to investigate various problems associated with the removal of a nuclear auxiliary power reactor from an earth orbit without engendering a hazard to human life. Since the AVCO study examined the reentry characteristics of the reactor-vehicle systems in great detail, this study and five of the other associated studies were directed toward a more detailed examination of the reentry phenomena pertaining to the reactor fuel elements and materials.

Interest in the reentry characteristics of the reactor-vehicle system and fuel elements grew out of the hope that incineration due to reentry heating and subsequent dispersal of combustion products by atmospheric mechanisms would obviate deposit of harmful amounts of radioactive materials upon the surface of the earth. The AVCO study indicated that without modification, flight models of the particular reactor-vehicle systems considered at that time would not be reduced to a harmless residue by reentry through the atmospheric environment.

This study and the related seven studies were initiated to examine in greater detail key areas which had not received an exhaustive treatment in previous studies, and which could have indicated an unnecessarily pessimistic conclusion if not completely examined.

2. DISCUSSION AND CONCLUSIONS

A. Trajectory Analysis of Reentry Fuel Elements

In the early stages of the research program it was expected that the orbital trajectories and mission profiles for the Nuclear Auxiliary Power systems would be well enough defined to permit a detailed analysis of the reentry problem without recourse to a parametric study. However, neither the trajectories nor the system configuration were well enough known to permit such an approach at the time. Therefore, it was decided to make the best assumptions permitted by the available information and proceed with a parametric study (see Appendix A).

A reactor-vehicle system was assumed which had a ballistic coefficient ($W/C_D A$) equal to 300 lb/ft^2 for a reentry with the longitudinal axis of the reactor-vehicle aligned parallel with the air flow and a ballistic coefficient equal to 20 lb/ft^2 for a reentry with the axis aligned normal to the airflow. These assumed values bracket the value for a tumbling reentry body whose ballistic coefficient is approximately equal to 50 lb/ft^2 . No analysis, however, was made of the trajectory of a tumbling body. It was also assumed that the reentry body remained intact to earth impact.

The geometry of the individual fuel elements was such that their ballistic coefficients could be assumed to be 300 lb/ft^2 and 20 lb/ft^2 for the same reentry attitudes, i. e., axially aligned and axially normal. Hence, the number of trajectories analyzed was greatly reduced.

It was assumed that reentry was initiated at 400,000 feet altitude and two trajectory computations were run to earth impact, one for a $W/C_D A$ of 20 lb/ft^2 and the other for a $W/C_D A$ of 300 lb/ft^2 . These trajectories were considered to be the trajectories of the vehicle in its two permitted orientations, i. e., axially aligned and normally aligned. Fuel elements were assumed to be ejected at discrete altitudes of 300,000; 250,000; 200,000; 150,000 and 100,000 feet from each of the vehicles. The ejected elements were assumed to align themselves only in the axial and normal configurations. Therefore, an element ejected in the same orientation as the parent vehicle was assumed to follow the same

trajectory as the parent vehicle to earth impact and it was only necessary to compute the trajectory of the other element configuration from the ejection altitude to earth impact. For the calculation of this fuel-element trajectory the initial conditions of altitude, velocity, flight path angle, latitude, longitude and heading angle of the fuel element were assumed to be the conditions of the parent vehicle at the instant of ejection. Analysis of tumbling fuel elements was not performed.

Since the information available suggested that polar orbits would be operationally preferable, reentry was assumed to be initiated at Latitude 80°N , Longitude 0° with the parent vehicle on a heading of 050° ($\approx$ North-east) at a velocity of 25,680 ft/sec.

It should be noted that under these assumptions, fuel elements ejected in orbit (i.e., 400,000 ft) would follow the trajectories of the parent vehicles as discussed above.

This analysis, of course, assumes no ablation of the fuel elements since a constant ballistic coefficient was utilized throughout for each of the permitted reentry orientations ($W/C_D A = 20 \text{ lb/ft}^2$ and 300 lb/ft^2).

The trajectory program code which was used assumed a rotating, oblate earth and the 1959 ARDC model atmosphere.

The method and results of the analysis are discussed in detail in Appendix A of this report.

It is evident from Tables A-2, A-3 and A-4 contained in Appendix A, that in the event the ejected fuel elements are not consumed during reentry (and this is the result indicated by this study), the recovery of these elements after earth impact could pose great problems, if they are in fact ejected from the parent vehicle. This conclusion is based upon the fact that the ballistic coefficients of the fuel elements chosen for this analysis are limiting values. If the elements tumble, then their ballistic coefficients and hence trajectory history will be intermediate between those presented here. Hence, tumbling or trimmed elements will impact the earth randomly along a line whose end

points are the impact coordinates tabulated in Appendix A. The values of these coordinates will of course vary according to the particular trajectory chosen but some general conclusions may be drawn from the data presented for the trajectories analyzed. These conclusions are

- (1) If the reactor-vehicle system reenters in the axial orientation ($W/C_D A = 300 \text{ lb/ft}^2$), the unconsumed fuel elements will be dispersed over a minimum area if they are ejected at a minimum altitude; e.g. 100,000 feet or less in the present case.
- (2) If the reactor-vehicle system reenters in the normal orientation ($W/C_D A = 20 \text{ lb/ft}^2$) the unconsumed fuel elements will be dispersed over a minimum area, if they are ejected at a high altitude; e.g., approximately 300,000 feet for the present case.
- (3) A maximum dispersal of the unconsumed fuel elements will result from an orbital ejection; e.g., 400,000 feet for the present case.
- (4) If it is conclusively shown that the ejected fuel elements will not be consumed during reentry and the resultant combustion products dispersed at sufficiently high altitudes to obviate dangerous levels of radioactive fallout, consideration should be given to making provision for the reactor to reenter and impact the earth INTACT to enhance the possibility of early recovery of ALL contaminated fuel elements rather than hazard widespread dispersal of individual, contaminated fuel elements. Other possible means to prevent widespread dispersal of contaminated fuel elements are
 - (a) to cause the intact reactor to impact the earth in an uninhabited area such as an ocean or a desert,
 - (b) to prevent reentry until the radioactivity level of the contaminated fuel elements has been reduced to acceptable levels, or
 - (c) to send the reactor into a deep space orbit and prevent atmospheric reentry of any kind.

In light of the conclusions which must be drawn from the results of this study, i.e., that the consumption of the fuel elements by reentry heating is not to be expected, some considerations should be given to the alternatives suggested. At the present time, random earth impact of the intact reactor is probably the most feasible technique without a major redesign of the system being required. However, this raises the possibility of a contaminated reactor landing in an extremely sensitive area, e.g., a city water reservoir. If it is possible to redesign the parent vehicle to carry a rocket, fuel and command and control devices, than a retro-firing at the proper moment could cause the entire reactor to land in a pre-designated, uninhabited area; however, it is not suggested that such a redesign is by any means a minor modification. The redesign penalty associated with this technique is however not as great as that associated with the degree of redesign necessary to develop a system capable of being sent into a deep space orbit. The fuel required to accomplish such a disposal would be comparable to the amount of fuel required to place the parent vehicle in orbit, initially. Even if a low-thrust device, such as a plasma propulsion device, were utilized for this purpose a very sophisticated guidance system would be needed to orient the vehicle properly at the precise instant required. It is obvious that a modification of this magnitude is not to be lightly accepted. If the reactor is to be kept in a long-lived orbit; however, no modification to the reactor-vehicle need be considered, but this system immediately eliminates the possibility of low-altitude orbital missions. Depending upon the expected mission profiles, this technique may or may not be an acceptable solution to the problem.

B. Thermodynamic Analysis of the Atmospheric Reentry of Fuel Elements

Utilizing the data generated by the trajectory program code discussed previously and in Appendix A, a thermodynamic analysis of reentry fuel elements was carried out using an available, in-house,

aero-thermodynamic, IBM 7090 program code. For this particular analysis the reentry fuel elements were programmed to be heated aerodynamically, to conduct heat internally in two dimensions and to reradiate heat to the ambient atmosphere. The matrices constructed for the two element orientations utilized in the analysis, i.e., axially aligned ($W/C_D A = 300 \text{ lb/ft}^2$) and normally aligned ($W/C_D A = 20 \text{ lb/ft}^2$), are discussed in detail in Appendix B of this report, as are the mathematical formulations used to calculate the thermal mechanisms involved. Only one calculation simulating the heating of a tumbling reentry element was accomplished. This tumbling calculation was performed primarily to examine the feasibility of using the program matrices, which were designed to calculate trimmed, fuel element reentry heating, to calculate the thermodynamic history of tumbling reentry elements. This calculation is also discussed in detail in Appendix B.

The thermodynamic analysis assumed an unclad, unhydrided fuel element with a constant emissivity of 0.75 and an initial temperature of 460°R at the time of ejection from the reactor vessel. The elements were assumed to be ejected and to remain in the axial and the normal orientation throughout the reentry trajectory. No other orientation was permitted. The elements were assumed to be ejected at various discrete altitudes from an axially aligned reactor-vehicle system ($W/C_D A = 300 \text{ lb/ft}^2$) and from a normally aligned reactor-vehicle system ($W/C_D A = 20 \text{ lb/ft}^2$). The assumed ejection altitudes were 400,000 (orbital ejection), 300,000; 250,000; 200,000; 150,000; and 100,000 feet. The analysis did not account for ablative mechanisms of the material nor for phenomena such as phase changes, surface combustion of gases or combustion of eroded material.

Because the analysis assumed an unclad, unhydrided fuel element, the results of the analysis are optimistic in the sense that none of the available thermal energy was considered to be expended in removing the cladding material and in heating and evolving internal hydrogen. Of course, the thermal and mechanical properties of the hydrided material are not the same as those of the unhydrided material but due to the uncertainties associated with the reported values of the hydrided properties (see Appendices

B and C) and since the unhydrided case represented the most optimistic situation for reentry dispersal, it was decided that under the circumstances an analysis of the unhydrided reentry fuel elements would yield the most useful results.

Possible errors arise from the fact that the analysis did not take account of ablative mechanisms or adjust the heating and cooling equations for errors resulting from calculated wall temperatures greater than the assumed melting temperature of 3400°F (the approximate melting point of pure zirconium). Under the circumstances these possible errors are not considered to be great enough to cast doubts on the validity of the conclusions which must be drawn from the results of the present analysis for the reasons discussed below.

The failure to take account of ablation allowed the forward portions of the fuel element model to exceed the assumed melting temperature and hence the calculated convective heat transfer to the element was reduced due to an apparently high wall temperature (Appendix B, Equation 1) and also the reradiation rates from matrix elements with an excessive wall temperature were too high. But it will be observed that the wall temperature is incorporated in the heating equation as an enthalpy term contained in an enthalpy ratio and will not be a controlling factor during the major portions of the trajectories studied. Although an excessive wall temperature will result in a much greater calculated heat loss by the affected matrix elements due to reradiation (since it is utilized as a fourth power term in the reradiation equation), this result may be closer to actuality than if reradiation had been limited to the assumed melting temperature as a maximum. This conclusion is drawn from the fact that the experimental studies of the various associated contractors have indicated the formation of oxides on the portions of the fuel material samples exposed to heating in an atmosphere of oxygen. Since zirconium oxide is a refractory material with a higher melting point than pure zirconium, allowing the stagnation-area, matrix element wall temperatures to rise to $\sim 5000 - 6000^{\circ}\text{F}$, as was done for some of the analyzed cases, may be

a more realistic simulation of the actual case than arbitrarily limiting the wall temperature to 3400°F . This is especially true since none of the experimental results indicates that ablation or melting occurs for the trajectories considered. At least one experimental study has measured wall temperatures between $5000 - 6000^{\circ}\text{F}$ without noticeable degradation of the fuel element. Additionally, the results of this analytical study do not indicate that any appreciable mass of the fuel elements will rise above the arbitrary figure of 3400°F . For these reasons, the results of the thermodynamic analysis are considered to be valid in general and further refinements of the program and technique could not be expected to yield appreciably different results.

(1) Summary of Results

The results of the thermodynamic analysis of greatest interest are summarized in Table B-1-1. A more extensive discussion of these results and graphical presentations are contained in Appendix B. All referenced times in the tabulation are based upon ejection from the reactor vessel as time zero.

It can be seen from the tabulation that in general the fuel element aligned with its longitudinal axis normal to the air stream ($W/C_D A = 20 \text{ lb/ft}^2$) is more susceptible to reentry destruction than the axially aligned element regardless of the orientation of the parent vehicle. One reason for this result is that the convective heating rate to the reentry body (Appendix B, Equation 1) is directly proportional to the value $\frac{867.7}{\sqrt{R}}$ where R is the effective nose radius of the body and for the normal orientation R is equal to the geometric radius of the cylinder. For a cylinder in the axial orientation the effective nose radius is equal to four times the geometric nose radius.⁶ If we denote the term $\frac{867.7}{\sqrt{R}}$ as K then the values of K for the two configurations considered are

TABLE B-1-1
Summary of Results
Thermodynamic Analysis of Reentry Fuel Elements
AXIAL VEHICLE ($W/C_D A = 300 \text{ lb/ft}^2$)

Ejection Altitude (ft)	Element Orientation (Normal or Axial)	Max. Volume of Material Exceeding 3400° F (Value & Time of Occurrence) (BTU/sec)	Max. Convective Heat Input (Value & Time of Occurrence) (BTU/hr)	Heating Rate Exceeded By Cooling Rate (Value & Time of Occurrence) (BTU/hr)	***
300,000	N	~ 20 % at 130 sec*	3.5×10^4 at 140 sec*	~ 140 sec*	130*
	A	~ 3 % at 640 sec	4.1×10^4 at ~800 sec	2.5×10^4 at 925 sec	500
250,000	N	~ 18.5% at 105 sec	9.94×10^4 at ejection	7.5×10^4 at 80 sec	20
	A	N.D.	N.D.	N.D.	N.D.
200,000	N	~ 11 % at 35 sec	2.35×10^5 at ejection	5×10^4 at 45 sec	2.5
	A	~ 5.5% at 105 sec	4.1×10^4 at 95 sec	2.1×10^4 at 135 sec	2
150,000	N	~ 6% at 6 sec	3.6×10^5 at ejection	1.5×10^4 at 30 sec	1
	A	~ 5.5% at 45 sec	4.96×10^4 at ejection	1.1×10^4 at 70 sec	0.9
100,000	N	0%	3.1×10^5 at ejection	2.2×10^4 at 10 sec	**
	A	~ 1 % at 4 sec	4.3×10^4 at ejection	2.2×10^3 at 44 sec	1.5

* Best observed value due to difficulties in timing programed calculational period.

N.D. No reportable data obtained for this case.

** Minimum value not attained or data of no interest due to low values calculated.

*** Time at which stagnation point reached 3400° F (sec).

TABLE B-1-2
Summary of Results
Thermodynamic Analysis of Reentry Fuel Elements
NORMAL VEHICLE ($W/C_D A = 20 \text{ lb/ft}^2$)

300,000	N	~ 30 % at 230 sec	7.5×10^4 at 225 sec	7.4×10^4 at 235 sec	135
	A	~ 11 % at 430 sec	4.1×10^4 at 420 sec	2.8×10^4 at 450 sec	180
250,000	N	18.5% at 70 sec	8.57×10^4 at ejection	6.2×10^4 at 100 sec	25
	A	5.5% at 145 sec	4.22×10^4 at 135 sec	2.2×10^4 at 167 sec	25
200,000	N	0%	9.1×10^4 at ejection	$\sim 1 \times 10^4$ at 49 sec	**
	A	~ 3 % at 20 sec	3.2×10^4 at 55 sec	6×10^3 at 85 sec	16
150,000	N	0%	1.64×10^4 at ejection	~ 38 sec	**
	A	0%	3.6×10^3 at ejection	~ 60 sec	**
100,000	N	**	**	**	**
	A	**	**	**	**

ORBITAL EJECTION FROM EITHER VEHICLE

400,000	N	~ 6.5% at 1320 sec*	N.D.	N.D.	1300
	A	5.5% at 2040 sec	4.1×10^4 at 2100 sec	2.6×10^3 at 2125 sec	1770

$$K_{\text{normal}} = 2840.13$$

$$K_{\text{axial}} = 1901.02$$

These results indicate a modification to the fuel elements which might be considered to enhance the possibility of burn-up upon reentry; i.e., to round the blunt ends of the cylinders into a hemispherical shape. This modification would result in an increase in the stagnation point heat transfer rate to fuel elements reentering in the axial configuration by a factor of approximately 1.5.

It can be seen that a fuel element ejection delayed to low altitudes is essentially useless with respect to pure melting, especially if the reentry vehicle assumes a normal orientation or is tumbling upon reentry. For such a vehicle, ejection must take place above 200,000 feet to realize any appreciable heating (300,000 feet is to be preferred). However, even for ejection at 300,000 feet or higher, one cannot depend with any degree of confidence (e.g., a safety factor of 2) upon reentry heating to consume the fuel elements. For example, for ejections in orbit (400,000 feet) the volume of the fuel element material, exceeding the assumed melting temperature, is approximately 6 percent for both configurations but the entire volume is at a temperature in the vicinity of the melting point. But even for these cases one cannot establish a safety factor of even unity, much less a value of 2 or 3, which is desirable. We must then consider that the analysis presented here is essentially optimistic in its assumptions in that the fuel element was assumed to be unclad, unhydrided, and was not considered to be tumbling, which situation would cause a more uniform internal distribution of temperatures in the fuel element but would result in lower unit temperatures and hence a smaller volume exceeding the assumed melting temperature (See Appendix B).

One must conclude that destruction of the fuel elements by reentry heating and dispersal of the residue at high altitude is not to be

expected for the trajectories analyzed, if one depends solely on fuel element melting as the predominant mechanism.

However, as indicated in Appendix B, the thermodynamic analysis indicated the existence of severe thermal gradients within the fuel elements for many of the cases analyzed. Plots of these gradients are presented in that section of this report. These gradients are severe enough to indicate the possibility of fuel element shattering due to thermal stresses. Time did not permit conducting a stress analysis to verify this possibility but at least one of the associated contractors has experimentally induced a shattering phenomenon which lends weight to such speculation. It is believed that shattering was not observed experimentally with any frequency since most of the experimental studies conducted under this program did not truly simulate the rapid variations in heating rates and pressures to be expected during reentry but rather simulated these conditions by step-function techniques which allowed the samples to attain some modicum of thermal equilibrium between variations. No conclusion will be drawn here as to whether or not such shattering of the fuel elements into smaller fragments is advantageous to the hoped for solution of high altitude destruction and dispersal of the fuel element material since many parameters affect the ultimate result. But one may say a priori that reduction of the fuel element dimensions by any means is a step in the desired direction. A rough estimate of the possibility of fragments reentering intact was performed (Appendix D), assuming that the fragments were discs whose diameter was equal to the diameter of the cylinder. The results of this calculation indicate that a disc less than 1/2 inch thick would probably reenter through the atmosphere intact. Although the geometrical assumptions made for this analysis are far removed from those which could be expected as a result of thermal shattering, the analysis is useful if, as has been suggested, the fuel rod could be constructed of thin discs to enhance the possibility of reentry destruction.

Figures 1, 2 and 3 are the cold wall heat transfer rates calculated by means of Equation 1 in Appendix B and utilized by the associated contractors to experimentally simulate the thermal fluxes to be expected for these reentry trajectories.

FIGURE 1 Stagnation point convective heat transfer rate. Zero time equals point of ejection.

FIGURE 2 Stagnation point convective heat transfer rate. Zero time equals point of ejection.

FIGURE 3 Stagnation point convective heat transfer rate.
Zero time equal to initial re-entry point.

C. The Hazard Associated with the Dispersal of Small Particles from Reentering Reactors

It was decided early in the program to investigate the possibility of defining the dimensions of an ablated particle of the fuel element material, contaminated by the fission fragments resulting from orbital operation of the reactor, which particle size would result in a radiological hazard at the earth's surface. This was done to aid the efforts of two associated contractors, one of whom was investigating the size distribution of the combustion products of a reentry fuel element and the other, the possibility of increasing the dispersion of the fuel material by the addition of small amounts of chemical additives. The definition of the hazardous particle size was judged to be of extreme value to all concerned.

A very detailed fallout study was therefore undertaken (Appendix E). For purposes of this study a 30 kw, SNAP-8 type nuclear auxiliary power system, reentering after a year of orbital operation, was assumed. It was pessimistically assumed that reentry was initiated one hour after shutdown. A horizontal line source of spherical debris particles at an altitude of 30 km was also assumed, this altitude being a limit beyond which no combustion could reasonably be expected to occur for the reentry trajectories examined.

This analysis indicates that for the system and conditions assumed no radiological hazard to humans at the earth's surface will develop if the mean radius of the particles of fuel material released above 30 km is less than 35 to 60 microns. It was further found that if the power level of the reactor is raised a factor of 10 to 300 kw, thereby increasing the quantum of fission fragments per particle, the radius of the "safe" particle must be reduced to 14 microns. Additionally, the probability of a radiological hazard occurring is less than 10 percent if the average particle radius is less than 100 microns for the example of a 30 kw reactor (See Appendix E).

D. Reentry and Dispersion of the Beryllium Reflectors

As a corollary of the study of the reentry and dispersion of the fuel element materials, a number of topics relating to the reentry of the reactor's beryllium reflectors were examined. The reentry of beryllium is of interest since inhalation of beryllium particles can be dangerous to humans if the concentration of particles is great enough. An optimum situation, therefore, would be to have the reflectors totally melted and widely dispersed as small particles at high altitudes.

To this end the effect of interstitial helium nuclei produced in the beryllium by fast neutrons was examined as a possible mechanism of degradation and hence a contributing mechanism of dispersal (Appendix F). It was concluded that the total neutron flux (9.8×10^{18} nvt), to which the reflectors will be subjected by the reactor over an assumed operating period of one year, will generate an insufficient volume of gaseous helium ($0.02 \text{ cc He}^4 / \text{cc Be}^9$) to produce significant changes in the density, yield strength or brittleness of the material. Thus the irradiated material should not be expected to exhibit reentry characteristics radically different from those of the unirradiated material. It is estimated that the reflectors must be subjected to a total flux greater by two or three orders of magnitude before appreciable changes in the physical properties of the beryllium can be expected.

Since the total volume of helium produced within the reflector by fast neutrons was found to be insufficient to produce significant changes in the beryllium properties, the feasibility of using chemical additives which would decompose and produce volumes of internal gas by reentry heating was investigated. It was concluded that reflector materials cannot be dispersed by the evolution of an inert gas within the material through the thermal decomposition of chemical additives, no matter how much gas may be produced or how long the subsequent annealing process is carried out. This is because the maximum possible swelling of the material is largely independent of the mechanical properties of the metal

and the mode of evolution of the gas within the metal, being determined primarily by geometrical considerations. Also, available experimental data indicate no tendency of the metal to fragment. Thus it appears most unlikely that internally evolved gases will greatly aid in the dispersal of the beryllium reflectors. (See Appendix G).

A simplified calculation was then undertaken to determine the area over which the beryllium must be distributed to ensure a non-hazardous environment, assuming that the entire reflector had been reduced to particles of such a size that they would be hazardous (See Appendix B). This area was then compared with the minimum area of dispersion which could be expected to result from an ordinary reentry trajectory and atmospheric winds. This calculation assumed a maximum permissible exposure level for safe habitation* defined in units of the density of beryllium in the atmosphere and the time of exposure, viz., 6×10^{-3} gm sec/m³. It was assumed that the burn-up trajectory extended over 100 km and the particles were acted upon by a constant average wind greater than 10 m/sec. The result was in the form of a safety factor which is the ratio of the actual wind distribution area to the maximum area for which a hazard would exist. This safety factor, S , was found to be $> 10^3$. This simplified calculation was verified by a much more detailed analysis of the hazard associated with the dispersal of small particles from reentering reactors. This analysis was in essential agreement, i.e., a safety factor varying from 10^4 to 10^5 was found for the range of dangerous particle sizes (0.1 micron to 5 microns).

The conclusion drawn from these analyses must be that no beryllium hazard is to be associated with a high-altitude dispersal, but the possible hazard connected with the reentry, relatively intact, of large pieces of beryllium was not evaluated.

E. Vaporization of Hypersonic Particles

During the analysis of the fallout problem, it was realized that the experimental determination of the size of ablated fuel materials, by

those contractors assigned that particular task, would probably not result in a complete definition of the range of particle sizes to be expected. This conclusion follows from the fact that particles of a finite size, coming off a melting reentry body, will not retain their initial dimensions since they are separated from the parent body with a velocity almost equal to that of the fuel element at that point. If the particle velocity at release is appreciable, e.g., > 15000 fps for Zr-U particles, the particles themselves will experience reentry heating phenomena and may possibly be reduced in size by ablative mechanisms. Hence this size diminution would be of importance in calculating the fallout patterns resulting from reentry destruction of the fuel elements.

The analysis of particle size reduction due to reentry heating of ablated particles or droplets was done parametrically since no information was available at the time the analysis was initiated concerning the expected size distribution of ablated particles. The results of this analysis are presented in Appendix I and describe the vaporization of hypersonic particles in terms of velocity ratios, dimensionless ratios of the particle's initial and final radii, dimensionless distances and constants representing various possible reentry regimes.

Assuming exposure and ablation of a 30 kw reactor core above an altitude of 200,000 feet, the results of this analysis indicate that the initial diameter of the particles ablated from the fuel elements must be approximately 3000 microns or less if the diameters of particles decelerated to their terminal velocity are not to exceed the 90 micron diameter particle which may result in a radiological hazard on the ground (as discussed in Appendix E). This conclusion assumes an initial particle velocity of 26,000 ft/sec. As the initial velocity of the ablated particle decreases, the permissible initial diameter of the particle rapidly decreases until at an initial velocity of 15,000 fps it becomes ≈ 90 microns, the maximum size

*Private communication from A. L. Field (Astronuclear Laboratory, Westinghouse Electric Corporation, Pittsburgh, Pa.) to L. Haar, General Dynamics/Astronautics.

of a nonhazardous particle.

No definite experimental data are available at this time which defines the spectrum of particle sizes which may be expected from an ablating fuel element. However, the versatility of the vaporization analysis lends itself readily to the evaluation of size reduction phenomena in various reentry regimes of ablative products of various initial sizes and velocities, and therefore may be of value in the future. In light of the conclusion that fuel elements, following the trajectories assumed for purposes of this study, will experience negligible melting, the analysis is solely of academic importance at present.

F. Investigation of Recovered Fragments from Atlas 109-D Booster

During the course of this investigation, it was brought to our attention by Captain M. Rozner, who was then Project Officer, that a number of relatively large-sized fragments of the Atlas 109-D Booster had been recovered relatively intact. Since this was the booster which put the Mercury Capsule Friendship 7 into orbit in the NASA MA-6 Orbital Flight on 20 February 1962, the reasons for the intact reentry of large fragments were of great interest to the problem under study. Through the efforts of Captain Rozner, some of these fragments were obtained for examination and identification.

The details of the examination performed (which examination was funded in part by Contract AF 29(601)-4893) are outlined in Appendix J and were published in August, 1962, as AFSWC-TDR-62-78.

It was tentatively concluded that the fragments had remained fairly intact due to the fact that (1) they had relatively low ballistic coefficients due to their geometry, (2) the fragments had large radiative areas which would have rapidly dissipated heat, and (3) the fragment materials had rather high emissivity values which would also tend to enhance radiative cooling. These conclusions raised no startling circumstances with respect to the current problem and no extensive

theoretical analysis of the aerothermodynamic history of the fragments was attempted.

However, in the course of examination it was discovered that the fragments were pitted in a number of places by what appeared to be hyper-velocity impacts. The possibility that these craters might be due to micrometeorite impacts aroused considerable interest. The subsequent discovery of additional fragments free of such pitting suggests; however, that the craters may have been caused by material generated by the vehicle itself. More definite conclusions must await the completion of pending investigations.

G. Atmospheric Parameters of Interest

During the course of the present investigation, it was necessary to collect various data concerning the physical properties of the atmosphere as a function of altitude. The data obtained are based upon the 1956 and 1959 ARDC Model Atmospheres. These data are collected in Appendix H as well as information dealing with the prevailing winds which was also required in the analysis of the atmospheric diffusion of debris particles.

H. Scaling of Reentry Model Tests

Reentry phenomena are frequently difficult to analyze theoretically and it is necessary to resort to experiment for information concerning them. The experiments themselves require such rigorous conditions that they are often unreliable or impossible to conduct. Several techniques are usually brought into use, each of them designed to study reentry from a limited aspect in the hope that a fairly complete composite picture can be assembled from the composite results. It is necessary to consider model tests which often utilize different material properties and energy levels, proceeding in some cases at unreal time rates. Thus the scaling laws become important in that they may indicate the conditions which must be preserved in designing an interpretable test.

Accordingly, in May 1962, Captain M. Rozner requested that a "handbook" of reentry simulation requirements be compiled. Such a study

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has been made and the results are presented in Appendix K. The scope of the study was limited peculiarly to reentry phenomena and to subjects which have not been treated completely elsewhere in the literature so that there has been little, if any, duplication.

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3. APPENDICES

APPENDIX ATRAJECTORY ANALYSIS OF REENTRY FUEL ELEMENTS1. INTRODUCTION AND ANALYSIS

The trajectory program code utilized in this study was obtained from the Air Force Special Weapons Center² since this code was readily available and sufficiently detailed to efficiently perform the required analysis. This program, the Earth Coordinate Trajectory Program, utilizes a rotating, oblate earth and 1959 ARDC model atmosphere and permits an analysis of the motion of a mass point in three degrees of freedom.

Previous investigators^{10,11} have encountered discrepancies in the results obtained from analyses of reentry trajectories utilizing spherical, nonrotating earth models on the one hand and oblate, rotating earth models on the other. The discrepancies were greatest for very shallow reentry angles similar to those which are germane to this study and it was therefore decided to utilize the more complicated earth model to ensure the closest possible approximation of the actual reentry regime.

The first step in the trajectory analysis was to determine the reentry characteristics of the reactor-vehicle system and it was hoped that this information would be available from the manufacturer. However, at an April 1962 working group meeting of the associated contractors engaged in this program, it was pointed out that such information was not firmly established at that time but that best estimates indicated that the reactor-vehicle system's ballistic coefficients were $W/C_D A = 270 \text{ lb/ft}^2$ for a reentry attitude where the longitudinal axis was aligned parallel to the air flow and $W/C_D A = 27 \text{ lb/ft}^2$ for a reentry attitude with the axis aligned normal to the air flow.

In light of this uncertainty it was agreed to assume ballistic coefficients of 20 and 300 in order to encompass any variations in these

parameters resulting from future modifications of the reactor-vehicle systems.

These values also bracket the computed ballistic coefficients of the fuel elements. Utilizing information supplied by the manufacturer and assuming a drag coefficient of 1.62 for the axially oriented fuel element and 1.23 for the normally aligned fuel element these ballistic coefficients are:

- (1) $W/C_D A = 231.22 \text{ lb/ft}^2$ axial-hydrided
- (2) $W/C_D A = 24.92 \text{ lb/ft}^2$ normal-hydrided
- (3) $W/C_D A = 267.83 \text{ lb/ft}^2$ axial-unhydrided
- (4) $W/C_D A = 28.86 \text{ lb/ft}^2$ normal-unhydrided

The material densities utilized were 6 gm/cc hydrided and 6.95 gm/cc unhydrided.

The values used in the analysis also bracket the ballistic coefficient of a tumbling fuel element with the same dimensions, i.e., 1.25 inch diameter and 12 inches long, whose $W/C_D A \approx 50 \text{ lb/ft}^2$.

It was agreed that polar orbits would be utilized since there were strong indications that such orbits would be preferable from the operational viewpoint.

Since the trajectory program code exhibits anomalous behavior in the vicinity of the poles and since the exact proximity to the poles, giving rise to this behavior was difficult to assess, polar orbits designed to pass the poles by 5-15 degrees of latitude were selected. Table A-1 lists the positions assumed for the initiation of reentry of the reactor-vehicle and its heading at that position.

TABLE A-1

REACTOR-VEHICLE ASSUMED INITIAL POSITION AT REENTRY

<u>Trajectory</u>	<u>Position</u>	<u>Heading</u>
Polar Initiation	Latitude 80° N	050° (≈Northeast)
	Longitude 0°	
Equatorial Initiation	Latitude 0°	010° (≈North)
	Longitude 0°	

The polar initiation trajectories were those utilized in analyzing the behavior of the fuel elements ejected from the reentry body, i.e., the reactor-vehicle was assumed to begin reentry at 80° north latitude. The assumed initial conditions of the reactor-vehicle system were 400,000 feet altitude with a trajectory angle (with respect to the local horizontal) of zero degrees and with an inertial velocity slightly less than orbital velocity at 400,000 feet.

The decay of an elliptical orbit can be approximated by assuming that the body receives an impulse, due to increasing aerodynamic drag, in a direction opposite to its orbital motion. The point (perigee) at which the impulse is sensed by the body is always on the new orbit whose apogee is always less than the apogee of the old orbit. Thus, the decaying elliptical orbit approaches a circular orbit at the perigee altitude (assumed to be 400,000 feet in this case) and one can therefore utilize the circular orbit velocity at 400,000 feet, if reentry is assumed to begin at that altitude. The circular orbit velocity¹² is given by the equation

$$V_{co} = \sqrt{\frac{\gamma M_p}{r}} \quad (1)$$

where γ = constant of gravitation

M_p = planetary mass

r = radius of the orbit measured from the earth's center.

The computed velocity for a 400,000 foot orbit is 25 691.89 ft/sec.

Since, to initiate reentry the inertial velocity must be less than this value, a velocity of 25,680 ft/sec was assumed (99.954% of circular orbital velocity).

The first trajectories calculated were the 400,000 feet altitude polar initiation cases for both the axial and normal reactor-vehicle orientations. These trajectories were calculated to earth impact. These trajectories were also assumed to be the trajectories of fuel elements ejected from the reactor-vehicle system at 400,000 feet and which were assumed to orient themselves in either the axial or the normal attitude.

Using the reactor-vehicle trajectories as determinative of the initial conditions of the trajectories of fuel elements ejected at various altitudes, both axial and normal orientation fuel element trajectories were calculated to earth impact. At each ejection altitude it was assumed that the fuel elements assumed either an axial attitude ($W/C_D A = 300 \text{ lb/ft}^2$) or a normal attitude ($W/C_D A = 20 \text{ lb/ft}^2$) i.e., the reactor-vehicle system (either axial or normal) ejected its fuel elements in either an axial or a normal attitude and the elements maintained their initial orientation to impact. No analysis of tumbling bodies was attempted. Tables A-3 and A-4 list the initial and final conditions for the trajectories analyzed. Table A-2 lists the initial and final conditions for elements ejected in orbit; i.e., 400,000 feet. Note that these conditions are also valid for the reactor-vehicle assumed.

It was assumed that neither the elements nor the vehicles changed shape or orientation during reentry and that their drag coefficients remained constant to impact. Hence the analyses are of reentry bodies with constant ballistic coefficients.

The velocities listed in the tables are relative velocities with respect to the earth. The trajectory program converts the inertial velocity computed by equation (1) to relative velocity with respect to the earth by a solution of equation (2).

$$V_{rel} = (-V_{\omega\phi}) (\cos \gamma) (\sin \xi) + \sqrt{V_{inert.}^2 + (V_{\omega\phi}^2) (\cos^2 \gamma \sin^2 \psi - 1)} \quad (2)$$

where

V_{inert}	= inertial velocity of the body
V_{rel}	= relative velocity of the body
$V_{\omega\phi}$	= earth's rotational velocity component
$V_{\omega\phi}$	= $(r) (\omega) (\cos \xi)$
r	= distance of body from center of the earth
ω	= angular rotation of the earth
γ	= trajectory angle of V_{rel} vector with respect to the local horizontal
ξ	= latitude
ψ	= azimuth angle of V_{rel} vector

The trajectory program outputs showed an increase in the value of the relative velocity vector as the instantaneous position of the body moved northward toward the poles, a decrease as it moved southward to the equator and another increase as it continued to move south from the equator to the pole. This phenomenon is due to the changing value of the rotational velocity vector ($\vec{V}_{\omega\phi} = \vec{r}\omega \cos \xi$) and has been found to be correct by numerical verification.

In addition, apparently anomalous increases in altitude were evident in the trajectory data. These increases are due, however, to earth oblateness factors; i.e., the earth's polar radius is approximately 70,500 feet less than its equatorial radius and gives rise to differences in gravity terms in the trajectory equations as well as in terms involving geometric altitude fluctuations. Figure A-1 is a plot of altitude as a function of time and shows that these oblate earth effects are greatest for the positions of closest proximity to the poles. The equatorial initiation orbital trajectory (initiation at 0° Latitude and 0° Longitude) exhibits cyclic variations in altitude due to the fact that it traverses several complete earth orbits.

These trajectories were included as a comparison but were not utilized in the subsequent analysis.

Figure A-3 shows the velocity histories of the elements ejected from the reactor-vehicle system as a function of time while Figure A-4 shows the altitude histories of the ejected elements as a function of time.

The trajectories are further discussed in Appendix B in conjunction with the thermodynamic analysis of the reentry of fuel elements.

TABLE A-2

INITIAL AND FINAL TRAJECTORY CONDITIONSPARENT VEHICLES AND FUEL ELEMENTS EJECTED IN ORBIT

- I. Ejection Altitude = 400,000 ft
Initial Conditions - Elements and Vehicles - Both Orientations
 Velocity = 25,473.21 ft/sec
 Flight Path Angle = 0.0°
 Heading Angle = 050°
 Latitude = 80° North
 Longitude = 0°

Final Conditions at Earth Impact

<u>Attitude:</u>	<u>Normal Vehicle and Element</u>	<u>Axial Vehicle and Element</u>
Time to Impact (sec)	1775	2255.5
Velocity (ft/sec)	125.18	553.02
Altitude (ft)	8113	32
Range (ft)	35,517,377	53,518,584
Latitude	1.45° South	50.04° South
Longitude	124.32° East	130.48° East

TABLE A-3

INITIAL AND FINAL TRAJECTORY CONDITIONS
FUEL ELEMENTS EJECTED FROM AN AXIAL VEHICLE

- I. Ejection Altitude = 298,562 ft
Initial Conditions - Both Element Orientations
 Velocity = 25,570.6 ft/sec
 Flight Path Angle = -0.1405°
 Heading Angle = 175.57°
 Latitude = 62.39° North
 Longitude = 123.75° East

Final Conditions at Earth Impact

<u>Attitude</u>	<u>Normal Element</u>	<u>Axial Element</u>
Time to Impact (sec)	775	955.5
Velocity (ft/sec)	64.50	553.02
Altitude (ft)	1389.3	31.8
Range (ft)	9,242,938	53,118,584
Latitude	18.79° South	50.04° South
Longitude	125.71° East	130.48° East

- II. Ejection Altitude = 252,313 ft
Initial Conditions - Both Element Orientations
 Velocity = 25,250.6 ft/sec
 Flight Path Angle = -0.5332°
 Heading Angle = 174.09°
 Latitude = 29.80°
 Longitude = 126.83° East

Final Conditions at Earth Impact

<u>Attitude</u>	<u>Normal Element</u>	<u>Axial Element</u>
Time to Impact (sec)	520	430.5
Velocity (ft/sec)	195.98	same as I. above
Altitude (ft)	5717.75	"
Range (ft)	3,589,957	"
Latitude	39.43° South	"
Longitude	128.21° East	"

III. Ejection Altitude = 204,077 ft

Initial Conditions - Both Element Orientations

Velocity = 24,158.2 ft/sec

Flight Path Angle = -1.2626°

Heading Angle = 172.20°

Latitude = 41.52° South

Longitude = 128.56° East

Final Conditions at Earth Impact

<u>Attitude</u>	<u>Normal Element</u>	<u>Axial Element</u>
Time to Impact (sec)	440	255.5
Velocity (ft/sec)	183.28	same as I. above
Altitude (ft)	3980.75	"
Range (ft)	1,453,116	"
Latitude	45.38° South	"
Longitude	129.34° East	"

IV. Ejection Altitude = 158,325 ft

Initial Conditions - Both Element Orientations

Velocity = 21,702.4 ft/sec

Flight Path Angle = -2.3240°

Heading Angle = 171.09°

Latitude = 46.20° South

Longitude = 129.51° East

Final Conditions at Earth Impact

<u>Attitude</u>	<u>Normal Element</u>	<u>Axial Element</u>
Time to Impact (sec)	390	180.5
Velocity (ft/sec)	49.54	same as L. above
Altitude (ft)	3809.00	"
Range (ft)	594,669	"
Latitude	47.72° South	"
Longitude	129.87° East	"

V. Ejection Altitude = 109,380 ft

Initial Conditions - Both Element Orientations

Velocity = 14,877.5 ft/sec

Flight Path Angle = -4.5135°

Heading Angle = 170.31°

Latitude = 48.75° South

Longitude = 130.13° East

Final Conditions at Earth Impact

<u>Attitude</u>	<u>Normal Element</u>	<u>Axial Element</u>
Time to Impact (sec)	360	130.5
Velocity (ft/sec)	36.72	same as L. above
Altitude (ft)	1269.00	"
Range (ft)	148,841	"
Latitude	49.12° South	"
Longitude	130.22° East	"

TABLE A-4

INITIAL AND FINAL TRAJECTORY CONDITIONS
FUEL ELEMENTS EJECTED FROM A NORMAL VEHICLE

I. Ejection Altitude = 297,266 ftInitial Conditions - Both Element Orientations

Velocity = 25,324.65 ft/sec

Flight Path Angle = -0.3030° Heading Angle = 175.28° Latitude = 16.64° NorthLongitude = 122.91° EastFinal Conditions at Earth Impact

<u>Attitude</u>	<u>Normal Element</u>	<u>Axial Element</u>
Time to Impact (sec)	625	580.125
Velocity (ft/sec)	125.18	543.50
Altitude (ft)	8112.99	137.00
Range (ft)	35,517,377	11,249,746
Latitude	14.50° South	13.99° South
Longitude	124.32° East	125.30° East

II. Ejection Altitude = 248,793 ftInitial Conditions - Both Element Orientations

Velocity = 23,431.27 ft/sec

Flight Path Angle = -1.1204° Heading Angle = 175.71° Latitude = 4.96° NorthLongitude = 123.84° East

Final Conditions at Earth Impact

<u>Attitude</u>	<u>Normal Element</u>	<u>Axial Element</u>
Time to Impact (sec)	450	286.25
Velocity (ft/sec)	Same as I above	539.65
Altitude (ft)	"	86.25
Range (ft)	"	3,908,118
Latitude	"	5.70° North
Longitude	"	124.64° East

III. Ejection Altitude = 196,695 ft

Initial Conditions - Both Element Orientations

Velocity = 17,086.39 ft/sec

Flight Path Angle = -3.2648°

Heading Angle = 175.74°

Latitude = 0.70° North

Longitude = 124.16° East

Final Conditions at Earth Impact

<u>Attitude</u>	<u>Normal Element</u>	<u>Axial Element</u>
Time to Impact (sec)	375	178.38
Velocity (ft/sec)	same as I. above	536.65
Altitude (ft)	"	0
Range (ft)	"	1,350,098
Latitude	"	2.99° South
Longitude	"	124.44° East

IV. Ejection Altitude = 140,708 ft

Initial Conditions - Both Element Orientations

Velocity = 7086.67 ft/sec

Flight Path Angle = -9.3497°

Heading Angle = 175.69°

Latitude = 0.96° South

Longitude = 124.28° East

Final Conditions at Earth Impact

<u>Attitude</u>	<u>Normal Element</u>	<u>Axial Element</u>
Time to Impact (sec)	325	125.13
Velocity (ft/sec)	same as I. above	537.15
Altitude (ft)	"	55
Range (ft)	"	360,297
Latitude	"	1.95° South
Longitude	"	124.36° East

V. Ejection Altitude = 92,083 ft

Initial Conditions - Both Element Orientations

Velocity = 1251.79 ft/sec

Flight Path Angle = -39.6332°

Heading Angle = 175.52°

Latitude = 1.40° South

Longitude = 124.32° East

Final Conditions at Earth Impact

<u>Attitude</u>	<u>Normal Element</u>	<u>Axial Element</u>
Time to Impact (sec)	275	92.125
Velocity (ft/sec)	same as I. above	543.16
Altitude (ft)	"	25
Range (ft)	"	42,453
Latitude	"	1.52° South
Longitude	"	124.33° East

2. CONCLUSION

It is evident from Tables A-2, A-3 and A-4, that if the ejected fuel elements are not consumed during reentry, recovery of these elements after earth impact will pose great problems. This conclusion is based upon the fact that the ballistic parameters chosen for this analysis are limiting values. If the elements tumble, their ballistic coefficients and hence trajectory history will be intermediate between those presented here. Hence, tumbling or trimmed elements will impact the earth randomly along a line whose end points are the tabulated impact coordinates. The values of these coordinates will, of course, vary according to the trajectory chosen, but some approximate conclusions may be drawn from the data presented for the trajectories analyzed.

- (1) If the reactor-vehicle system reenters in the axial orientation, ($W/C_D A = 300 \text{ lb/ft}^2$) the unconsumed fuel elements will be dispersed over a minimum area, provided they are ejected at a minimum altitude; e.g., for the present case 100,000 feet or less.
- (2) If the reactor-vehicle system reenters in the normal orientation ($W/C_D A = 20 \text{ lb/ft}^2$) the unconsumed fuel elements will be dispersed over a minimum area, provided they are ejected at a high altitude; e.g., for the present case at approximately 300,000 feet.
- (3) Maximum dispersal of the unconsumed fuel elements will result from an orbital ejection; e.g., 400,000 feet for the present case.
- (4) If it is conclusively shown that the ejected fuel elements will not be consumed during reentry and the combustion products dispersed, consideration should be given to making provisions for the reactor to reenter and impact the earth intact to enhance the possibility of early recovery of all contaminated fuel elements rather than hazard widespread dispersal of dangerously contaminated fuel elements whose appearance would not put the casual finder on notice of the hazard of contact.

FIGURE A-1 Altitude Histories for Re-entry from Polar Orbits.
 (Initial conditions $h=400,000 \text{ ft.}$, $V_E=25,680 \text{ ft/sec.}$)

FIGURE A-2 Velocity Histories for Re-entry from Polar Orbits
 (Initial conditions $h = 400,000 \text{ ft.}$, $V_E = 25,680 \text{ ft/sec.}$,
 $\gamma_E = 0^\circ$.)

FIGURE A-3 Velocity histories for elements ejected at discrete altitudes

FIGURE A-4 Altitude histories for elements ejected at discrete altitudes

APPENDIX B

THERMODYNAMIC ANALYSIS OF THE ATMOSPHERIC REENTRY OF
FUEL ELEMENTS1. INTRODUCTION

Utilizing the trajectory data generated by the Earth Coordinate Trajectory Program² previously discussed, a thermodynamic analysis of the reentering fuel elements was conducted using an available aerothermodynamic IBM 7090 program³.

This program utilizes a "lumped parameter" procedure, and the first law of thermodynamics represented by a well-known numerical approximation of the diffusion equation. Problems involving either one-dimensional or two-dimensional transient heat conduction with nonlinear boundary conditions are readily solved. The problem media are divided into a finite number of individual volume elements, each of homogeneous material and temperature. A heat balance is performed simultaneously for all elements of the matrix. Variable thermal properties of the material are readily accommodated. Optional program subroutines are available to accommodate a broad variety of boundary conditions, including aerodynamic heating, free or forced convection, radiation to an external environment, radiative heat exchange between elements, internal heat exchange between elements, internal heat generation and any other arbitrary time dependent or constant heat flux.

2. PROBLEM GEOMETRY AND EQUATIONSA. Normal Orientation

Two matrices were designed to conform with the trajectory assumptions, previously discussed in Appendix A; a matrix for the normal attitude element (Figure B-1) and a matrix for the axial attitude element (Figure B-2). The normal matrix represents an element twelve inches

in length and oriented with its longitudinal axis normal to the airflow. The program assumes that for this orientation the stagnation point remains unchanged, i.e., the same point on the surface of the rod is subjected to the maximum heat flux throughout the entire reentry trajectory. It will be noticed that the normal orientation matrix is represented by a half-circle with the diameter through the stagnation point defined as an adiabatic wall. Since the thermodynamic mechanisms for this orientation are symmetrical about the stagnation point diameter, this technique was utilized to attain the smallest possible mesh size, there being a limit of 99 available segments. The dimensions of the segments of the normal matrix are contained in Table B-1. Segments 1, 19, 35, 49, 61, 71, 79, 85, and 89 were programed to have an aerodynamic heat input while these segments plus segments 18, 34, 48, 60, 70, 78, 84, 83, and 90 were programed to reradiate to the ambient environment. Internal heat conduction in the x- and y- directions was permitted but it was assumed that there was no internal heat conduction in the z- direction, along the length of the rod. The stagnation point heat transfer to the normal element was computed by means of the Kemp-Riddell⁴ expression for heat transfer to the stagnation point of a sphere in laminar flow:

$$q_c = \frac{867.7}{\sqrt{R}} \sqrt{\frac{\rho_\infty}{\rho_o}} \left(\frac{V}{10^4} \right)^{3.15} \left[\frac{H_s - H_w}{H_s - H_{w300}} \right] \text{ Btu/ft}^2 \text{ sec} \quad (1)$$

This value was converted to the stagnation point heat transfer to a normal cylinder by means of the expression

$$q_{cyl} = 0.747 q_c \quad (2)$$

The heat transfer to a point on the forward side of the body is a function of the heat transfer to the stagnation point and the angle between radii to the point in question and the stagnation point for laminar flow⁵ and was computed by the expression

$$\frac{q_\theta}{q_{cyl}} = (\cos \frac{\theta}{2})^{\frac{2}{5}} \quad (3)$$

Radiative heat input to the element and reradiation of the end faces of the cylinder were neglected as were ablation, phase changes, etc.

The analysis of the normal element assumes that the stagnation point remains unchanged throughout the reentry trajectory. This may not be the case during the actual reentry and hence the thermal gradients shown by the analysis may not be the most representative of the actual case. If one assumes random orientation of the fuel rods upon ejection, at least some of the rods will be subjected to dynamic motions which approximate those suggested by past studies (e.g., AVCO)¹. To examine the variation in results caused by such dynamic motions, possible methods of evaluating the total heat flux imparted to a fuel rod were studied.

Ideally, one should calculate the local flow conditions around the rod as it reenters. Knowing these, the local heat transfer may be computed. The literature was surveyed to determine available methods of calculating these local heat fluxes. The most accurate method for the anticipated trajectories appears to be that of Reference 13. Their results may be expressed as:

$$\frac{q}{q_s} = \frac{1}{2} \sqrt{\frac{(\rho\mu)_w}{(\rho\mu)_{ws}}} \left[\frac{\rho_s}{2(p_s - p_\infty)} \right]^{\frac{1}{4}} \frac{\sqrt{R} u_e \sin \theta}{\int_0^x u_e \sin^2 \theta dx} \frac{g_{\eta w}}{g_{\eta ws}} \quad (4)$$

for a constant temperature wall. The notation used herein is identical with Reference 13.

The stagnation point heat transfer, q_s , may then be computed using Reference 15. The main problem in using Equation 4 involves the calculation of the local velocity around the rod, μ_e . There is no accurate method known to the writer of calculating μ_e , when the rod is reentering at hypersonic velocities and in tumbling motion. In view of this difficulty, one approach is to calculate the ratio, q/q_s , for a rod in both axial and cross-axial attitude and then use a spin-correction factor.

Even then, the calculation of μ_e may not be made with great accuracy for the anticipated fuel rod trajectories. Preliminary calculations of μ_e were made for the rod in cross-axial flow at a Mach number of 20, assuming a perfect gas and a Newtonian pressure distribution. More refined computations were made taking real gas effects into account by using Reference 17 to evaluate the local conditions behind a normal shock at $M_\infty = 20$, $T_\infty = 225^\circ\text{K}$, $\gamma_2 = 1.16$, $Z_2 = 1.46$, where M_∞ , T_∞ , γ_2 and Z_2 signify Mach number and temperature at infinity, specific heat ratio and compressibility factor behind a normal shock, respectively. Comparison of the two calculations indicate that the ratio

$$\frac{u_e(\text{perfect gas})}{u_\infty} \times \frac{u_2(\text{real gas})}{u_2(\text{perfect gas})} \approx \frac{u_e(\text{real gas})}{u_\infty} \quad (5)$$

where u_2 is the local velocity behind a normal shock. This approximation is in error by about 15% at worst. However, this approximation greatly simplifies the use of Equation 4 to calculate the local heat flux distribution. Any errors in using Equation 5 to solve Equation 4 are greatly reduced since u_e enters Equation 4 as roughly the square root.

Pressure distributions over cylinders in axial flow have been measured in Reference 6 at Mach numbers 2.01 and 4.76. The results of Reference 6 indicate this pressure distribution to be insensitive to the higher Mach numbers. This approach should serve in computing u_e for the rod in axial flow.

No attempts were made to actually evaluate Equation 4 using the computed values of u_e . The only feasible method is to program Equation 4. The AVCO report¹ indicates that this was their method. However, only the crossaxial case was done and it is not clear what velocity distribution they used. Their results gave them values of

$$\frac{q_{ave}}{q_s} \approx 1/3$$

This is probably a reasonable value for the cross-flow case. Values of q/q_s were obtained from the experimental heat transfer coefficient measurements of Reference 14 at $M_\infty = 5.5$ for approximately 1/2 inch diameter cylinders. These results were integrated giving

$$\frac{q_{ave}}{q_s} = 0.356$$

which compares well with AVCO's 1/3 value.

However, some measurements on wires in cross-flow were briefly examined and some of these indicate higher values of q_{ave}/q_s .

Also, there are other theoretical analyses available, which are considerably easier to use. However, it is believed that the accuracy of these is not as good as that of Reference 13.

To examine the effect of total heating on a tumbling fuel element, the analysis was re-run for the case of a normal element ejected at an altitude of 109,380 feet from an axially aligned reactor-vehicle system, utilizing an average heating rate to the surface of the body equal to 1/3 of the stagnation point value. This approximates a rod in the cross-flow configuration, rotating around its longitudinal axis, at rates in excess of 1 cycle per second, as it reenters.

As might be expected, the thermal gradients calculated for this case are in a radial direction whereas in the non-rotating case the gradients were essentially parallel to the adiabatic wall through the stagnation points. The magnitude of the radial gradients are less than the chordwise gradients by approximately a factor of five if the most severe gradient for each case is considered. The rotating rod also reaches an equilibrium temperature more quickly than the nonrotating case and this temperature is approximately 400 ° higher than in the nonrotating case. The segment temperatures in the case of the rotating rod quite expectedly never reach the high values attained in the nonrotating case and, of course, do not approach the melting

temperature of the material. Some plots of the calculated isotherms for the rotating, cross-flow configuration are included in this report as an illustration of these results (Figures B-11, B-12).

The radial symmetry of the temperatures calculated in this case, which in fact can be calculated with a one-dimensional code, clearly demonstrate the adequacy of the calculational technique and the effective independence of the result with respect to mesh spacing.

B. Axial Orientation

The matrix utilized to represent a fuel element whose longitudinal axis is oriented parallel with the airflow is shown in Figure B-2 and the dimensions of the 95 segments of the matrix are tabulated in Table B-2. To permit an analysis of a three-dimensional problem with a two-dimensional program, it was assumed that there was no heat conduction in a direction parallel to circumferences of the cylinder. The matrix was then constructed by flattening out annular segments of the cylinder, each 0.125 inches thick and permitting heat conduction in the x- and y- direction but not in the z- direction. Segments 1 through 20 and segments 39, 58, and 77 were programed to reradiate to the atmosphere. The reradiation of end areas of segments 19, 38, 56, 76 and 95 was neglected since these segments were not expected to reach very high temperatures.

The frontal areas of the cylinder (frontal areas of segments 1, 20, 39, 58, and 77) were programed to receive the heat flux computed by Equation 1. The radius "R" in Equation 1 was made equal to four times the geometric radius of the cylinder to convert Equation 1 into an expression applicable to a blunt nosed body in conformance with the experimental results of Curtiss and Boison⁶.

The upper areas of segments 1 through 19 were programed to receive flat plate heating in addition to the heat flux applied to the frontal area of the rod. The program uses the following general expression³ in calculating aerodynamic heating on a flat plate.

$$q_i = h_{a_i} \frac{(az)_i}{144} (EN_R - EN_i) \quad (6)$$

in which

$$EN_R = EN_o + R \frac{U_o^2}{2gJ} \quad (7)$$

For the flat plate in laminar flow, the Blasius solution of the boundary layer momentum equation and the modified Reynolds analogy are used to obtain the Stanton number:

$$St = \frac{cf}{P} P_R^{-\frac{2}{3}} = \frac{0.332 P_R^{-\frac{2}{3}}}{\sqrt{Re}} \quad (8)$$

The local heat transfer coefficient can then be expressed as

$$h_{a_L} = 3.42 \times 10^{-2} \left(\frac{P_o U_o}{L} \right)^{0.5} \left(\frac{\rho^* \mu^*}{\rho_b \mu_b} \right)^{0.5} \quad (9)$$

For the flat plate in turbulent flow, the modified Reynolds analogy relating the Stanton number to the skin friction coefficient and the Blasius local skin friction coefficient,

$$\frac{C_f}{2} = 0.0296 Re^{-0.2} \quad (10)$$

TABLE B-1

NORMAL MATRIX
SEGMENT DIMENSIONS

<u>Segment Number</u>	<u>x dim. (in.)</u>	<u>y dim. (in.)</u>	<u>z dim. (in.)</u>
1 and 18	0.0125	0.1250	12.0
2 and 17	0.0188	0.1250	12.0
3 and 16	0.0313	0.1250	12.0
4 and 15	0.0375	0.1250	12.0
5 and 14	0.0438	0.1250	12.0
6 and 13	0.0625	0.1250	12.0
7 and 12	0.0906	0.1250	12.0
8 and 11	0.1438	0.1250	12.0
9 and 10	0.1813	0.1250	12.0
19 and 34	0.0188	0.0625	12.0
20, 33, 35 and 48	0.0313	0.0625	12.0
21, 32, 36, 47, 49 and 60	0.0375	0.0625	12.0
22, 31, 37, 46, 50, 59, 61 and 70	0.0438	0.0625	12.0
23, 30, 38, 45, 51 58, 62, 69, 71 and 78	0.0625	0.0625	12.0
24, 29, 39, 44, 52 57, 63, 68, 72, 77 79 and 84	0.0906	0.0625	12.0
25, 28, 40, 43, 53, 56, 64, 67, 73, 76 80, 83, 85 and 88	0.1438	0.0625	12.0
26, 27, 41, 42, 51, 55, 65, 66, 74, 75, 81, 82, 86, 87, 89 and 90	0.1813	0.0625	12.0

TABLE B-2
AXIAL MATRIX
SEGMENT DIMENSIONS

Segment Number	x dim. (in.)	y dim. (in.)	z dim. (in.)
1 and 2	0.125	0.0125	3.5340
20 and 21	0.125	0.0125	2.7490
39 and 40	0.125	0.0125	1.9640
58 and 59	0.125	0.0125	1.1780
77 and 78	0.125	0.0125	0.3930
3	0.125	0.0250	3.5340
22	0.125	0.0250	2.7490
41	0.125	0.0250	1.9640
60	0.125	0.0250	1.1780
79	0.125	0.0250	0.3930
4	0.125	0.0500	3.5340
23	0.125	0.0500	2.7490
42	0.125	0.0500	1.9640
61	0.125	0.0500	1.1780
80	0.125	0.0500	0.3930
5 and 6	0.125	0.2750	3.5340
24 and 25	0.125	0.2750	2.7490
43 and 44	0.125	0.2750	1.9640
62 and 63	0.125	0.2750	1.1780
81 and 82	0.125	0.2750	0.3930
7	0.125	0.5500	3.5340
26	0.125	0.5500	2.7490
45	0.125	0.5500	1.9640
64	0.125	0.5500	1.1780
83	0.125	0.5500	0.3930
8 through 19	0.125	0.9000	3.5340
26 through 38	0.125	0.9000	2.7490
45 through 57	0.125	0.9000	1.9640
64 through 76	0.125	0.9000	1.1780
83 through 95	0.125	0.9000	0.3950

are used to develop the heat transfer coefficient for use at Reynolds numbers of less than 10^7

$$h_{aT} = 4.83 \times 10^{-3} \frac{(P_o U_o)^{0.8}}{L^{0.2}} \left(\frac{\mu^* \mu_b}{\rho_b \mu_b} \right)^{0.8} \left(\frac{\mu^*}{\mu_b} \right)^{0.8} \quad (11)$$

At Reynolds numbers beyond 10^7 , the von Karman local skin friction coefficient,

$$(c_f)^{-\frac{1}{2}} = 1.7 + 4.15 \log_{10} (Re c_f) \quad (12)$$

results in

$$h_{aT} = 1.88 \times 10^{-3} \frac{(P_o U_o)^{0.8691}}{L^{0.1309}} \left(\frac{\mu^* \mu_b}{\rho_b \mu_b} \right)^{0.8691} \left(\frac{\mu^*}{\mu_b} \right)^{-0.7382} \quad (13)$$

All flat plate calculations are based upon the assumption that conditions outside the boundary layer are fully expanded, free stream conditions. A transition Reynolds number of 2×10^5 was used throughout.

The development of the heat transfer coefficients are set forth in detail in References 7, 8 and 9.

C. Thermal Properties

Throughout the thermodynamic analysis a SNAP type fuel element was assumed, whose geometry was that of a right circular cylinder 12 inches in length, 1.25 inches in diameter and composed of an unhydrided alloy of zirconium -10% uranium. The rod was assumed to be bare, i. e., without cladding material.

The thermal properties utilized in the program are tabulated in Tables B-3 and B-4. A constant emissivity of 0.75 and a constant density of 433.9 lb/ft³ was utilized throughout the analysis.

The thermal properties utilized were those measured by the Armour Research Foundation under AF 29(601)-5012 and reported during the course of the research program.

The decision to utilize only the unhydrided properties was based in part upon the fact that the measurements of the hydrided material had not been completed by the time it became necessary to begin the theoretical thermodynamic analysis.

Secondly, the unhydrided, unclad material would be more likely to be destroyed by reentry heating than the clad, hydrided material and hence if negative results were obtained for this material, it would then be fairly certain that the hydrided material would not be destroyed. This conclusion, of course, arises from the fact that in the reentry of an unclad, unhydrided rod all of the available thermal energy is devoted to heating the fuel element whereas in the clad, hydrided case, an appreciable portion of the available energy is applied to the destruction of the cladding and the heating and evolution of the internal hydrogen. Additionally the evolving hydrogen will probably reduce the heat transfer rate at the wall surface.

TABLE B-3
SPECIFIC HEAT

<u>T(°R)</u>	<u>C_p(Btu/lb °F)</u>
910	0.0605
1110	0.0664
1310	0.0724
1510	0.0783
1610	0.0865
2010	0.1140
2060	0.0706
2460	0.0759
2860	0.0812
3260	0.0865

TABLE B-4
THERMAL CONDUCTIVITY

<u>T(°R)</u>	<u>k(Btu/hr ft °F)</u>
640	8.5
772	8.7
865	8.8
974	9.0
1080	9.2
1163	9.3
1310	9.6
1410	9.8

Thirdly, because of the difficulty of measuring the physical properties of the hydrided material, the hydrided properties obtained by Armour were the result of testing in idealistic environments, which environments will not be encountered in an actual reentry situation. For example, some of the measurements were made in a hydrogen environment under one atmosphere of pressure and there were subsequent indications that the properties measured were pressure dependent. Since a reentering fuel element will experience a wide spectrum of pressure variations, adapting the data generated experimentally to the current analysis would be practically impossible. Appendix C presents a more detailed discussion of problems associated with experimental measurements of high temperature properties of hydrided zirconium-uranium alloy.

D. Summary of Results

The results of the thermodynamic analysis for the various assumed ejection altitudes are summarized below. The convective heating rates and radiative cooling rates discussed are total body rates (BTU/hr).

(1) 100,000 Feet Altitude

(A) Ejection From Axial Vehicle

Altitude = 109,380 feet

Velocity = 14,877 ft/sec

1 Normal Element (Figure B-3)

No portion of the fuel element attained the assumed melting temperature (3400°F), but rather severe thermal gradients were generated internally (Fig. B-4), the maximum gradient occurring 1.5 seconds after ejection and the maximum stagnation point temperature of 2514°F occurring at the same time. The maximum convective heat input of 3.1×10^5 BTU/hr was attained immediately after ejection. Within 10 seconds after ejection the radiative cooling rate exceeded the convective heating rate. Figures

B-5 through B-10 are plots of internal isotherms calculated for this case while Figures B-11 and B-12 are plots of the internal isotherms resulting from applying an average heat flux to the whole body equal to $1/3$ the calculated stagnation point convective heating rate. The average heat flux calculation was performed only for this case.

2 Axial Element (Fig. B-13)

For this case the fuel element experienced a maximum convective heating rate of 4.3×10^4 BTU/hr immediately after ejection. The cooling rate exceeded the heating rate at a value of 2.2×10^3 BTU/hr 44 seconds after ejection. A maximum volume of 1% of the fuel element material reached or exceeded the assumed melting temperature of 3400°F within 4 seconds of exposure. The stagnation point temperature rose to 3400°F within 1.5 seconds after exposure. Severe thermal gradients were observed affecting the forward $1/4$ inch of the fuel element but no graphs of these gradients were made.

(B) Ejection From Normal Vehicle

Altitude = 92,083 feet

Velocity = 1,252 ft/sec.

1 Normal Element

This element experienced negligible heating from ejection to impact due to the fact that the vehicle has decelerated to a relatively low velocity by the time it has reached this altitude. No burnup of the fuel element can be expected for these conditions. No graphs of these results were prepared since the temperatures attained were of little interest.

2 Axial Element

This element also experienced negligible heating due

to the relatively low velocity of the parent vehicle at this altitude. No burnup of the fuel element can be expected for these conditions. No graphs of these results were prepared since the temperatures attained were of little interest.

(2) 150,000 Feet Altitude

(A) Ejection From Axial Vehicle

Altitude = 158,325 feet

Velocity = 21,702 ft/sec

1 Normal Element (Figure B-14)

For these conditions of ejection the stagnation point reached the assumed melting temperature of 3400° F within 1 second after ejection and approximately 6 seconds after ejection a maximum volume of approximately 6 percent of the fuel element material had reached or exceeded that temperature. Thermal gradients were observed (Fig. B-15), being most severe during the first second of exposure. The fuel element attained its equilibrium temperature of 1700° F about 60 seconds after ejection. The maximum convective heating rate of 3.6×10^5 BTU/hr occurred immediately after ejection and the radiative cooling rate exceeded the convective heating rate approximately 30 seconds after ejection.

2 Axial Element (Figure B-16)

For these conditions of ejection the stagnation point reached 3400° F within 0.9 seconds of ejection and approximately 45 seconds after ejection a maximum volume of 5.5 percent of the fuel element material had reached or exceeded that temperature. Thermal gradients were again in evidence (Fig. B-17), the severe gradients

occurring within the first 35 seconds after ejection and affecting approximately the forward 2 inches of the fuel element until the internal temperatures began to equilibrate within 45 seconds. Internal equilibrium was attained after 115 seconds at a temperature of approximately 1200° F. The maximum convective heating rate of 4.96×10^4 BTU/hr was attained immediately after ejection and the radiative cooling rate exceeded the convective heating rate 70 seconds after ejection at a value of 1.08×10^4 BTU/hr.

(B) Ejection From Normal Vehicle

Altitude = 140,708 feet

Velocity = 7,087 ft/sec

1 Normal Element (Figure B-18)

For these ejection conditions the maximum stagnation temperature of 489° F was attained about 10 seconds after exposure and no portion of the fuel element reached the assumed melting point. Moderate thermal gradients were observed (Fig. B-19). An equilibrium temperature of approximately 250° F was attained about 40 seconds after exposure. The maximum convective heat rate of 1.64×10^4 BTU/hr was attained immediately after ejection and the radiative cooling rate exceeded the heating rate approximately 38 seconds after exposure of the fuel element.

2 Axial Element (Figure B-20)

For these ejection conditions the maximum stagnation point temperature of 1302° F was attained 35 seconds after ejection and no portion of the fuel element reached the assumed melting point. Mod-

erate thermal gradients were observed affecting the forward inch of the fuel element (Fig. B-21), the most severe being observed during the first 10 seconds after ejection. The equilibrium temperature of about 300°F was attained approximately 65 seconds after exposure. The maximum convective heating rate of 3.6×10^5 was attained immediately after ejection. The radiative cooling rate exceeded the heating rate approximately 60 seconds after ejection.

(3) 200,000 Feet Altitude

(A) Ejection From Axial Vehicle

Altitude = 204,076 feet

Velocity = 24,158 ft/sec

1 Normal Element (Figure B-22)

For these conditions of ejection the stagnation point temperature reached the 3400°F temperature within 2.5 seconds subsequent to ejection and within 35 seconds a maximum volume of 11 percent of the fuel element material had reached or exceeded the assumed melting temperature of 3400°F . A maximum convective heating rate of 2.35×10^5 BTU/hr was observed immediately after ejection and the heating rate was exceeded by the cooling rate 45 seconds after ejection at a value of 5×10^4 BTU/hr. Moderate internal thermal gradients (Fig. B-23) were observed, being most severe during the first 10 seconds after ejection and diminishing to the equilibrium temperature of about 1900°F after 110 seconds.

2 Axial Element (Figure B-24)

For these ejection conditions the stagnation point temperature increased to 3400°F within 2 seconds after ejection and after 105 seconds a maximum volume of approximately 5.5 percent of the fuel material had reached or exceeded that temperature. Several thermal gradients (Fig. B-25) were again observed, the most severe occurring within the first 3 seconds after exposure of the fuel element. The equilibrium temperature of approximately 1400°F was attained about 175 seconds after exposure. The maximum convective heating rate of $4.1 \times 10^4 \text{ BTU/hr}$ occurred 95 seconds after ejection and the radiative cooling rate exceeded the heating rate approximately 135 seconds after ejection at a value of $2.1 \times 10^4 \text{ BTU/hr}$.

(B) Ejection From Normal Vehicle

Altitude = 196,695 feet

Velocity = 17,086 ft/sec

1 Normal Element (Figure B-26)

For these conditions of ejection the stagnation point temperature reached a maximum value of 2650°F within 18 seconds after ejection and no portion of the fuel element attained the assumed melting temperature. Moderate thermal gradients were observed (Fig. B-27) and the equilibrium temperature of approximately 1500°F was attained within 90 seconds of ejection. The maximum convective heating rate of $5.1 \times 10^4 \text{ BTU/hr}$ was observed immediately after ejection. The cooling rate exceeded the heating rate within 49 seconds after exposure at a value of 10^4 BTU/hr .

2 Axial Element (Figure B-28)

For these conditions of ejection the stagnation point

temperature reached the assumed melting temperature 16 seconds after ejection and about 3 percent of the volume of the fuel material attained the melting temperature. Thermal gradients (Fig. B-29) were observed, being most severe during the first 15 seconds after ejection. The equilibrium temperature of approximately 800°F was attained 115 seconds after ejection. The maximum convective heating rate of 3.2×10^4 BTU/hr was attained about 55 seconds after ejection. The cooling rate exceeded the heating rate about 85 seconds after exposure at a value of approximately 6×10^3 BTU/hr.

(4) 250,000 Feet Altitude

(A) Ejection From Axial Vehicle

Altitude = 252,313 feet

Velocity = 25,251 ft/sec.

1 Normal Element (Figure B-30)

For these conditions the stagnation point temperature reached 3400°F some 20 seconds after ejection and a maximum volume of 18.5 percent of the fuel element material had reached or exceeded the assumed melting temperature within 105 seconds after ejection. Moderate thermal gradients (Fig. B-31) were observed. The maximum convective heating rate of 9.94×10^4 BTU/hr occurred immediately after ejection and the heating rate was exceeded by the cooling rate 80 seconds after ejection at a value of 7.5×10^4 BTU/hr. Thermal equilibrium was attained within 200 seconds at an approximate temperature of 1900°F .

2 Axial Element

The output data for this computation were in error and the available time did not permit a recalculation of the problem.

(B) Ejection From Normal Vehicle

Altitude = 248,793 feet

Velocity = 23,431 ft/sec

1 Normal Element (Figure B-32)

For these conditions the stagnation point temperature reached 3400°F approximately 25 seconds after ejection and a maximum volume of 18.5 percent of the fuel element material had reached or exceeded the assumed melting temperature within 70 seconds after exposure of the fuel element. The thermal gradients observed were moderate (Fig. B-33) and an equilibrium temperature of about 1800°F was reached after 160 seconds. Maximum convective heating occurred immediately after ejection at a value of $8.57 \times 10^4 \text{ BTU/hr}$. The cooling rate exceeded the heating rate 100 seconds after ejection at a value of $6.2 \times 10^4 \text{ BTU/hr}$.

2 Axial Element (Figure B-34)

For these conditions the stagnation point temperature reached 3400°F within 25 seconds after ejection and within 145 seconds after ejection a maximum volume of 5.5 percent of the fuel element material had reached or exceeded the assumed melting temperature. Severe thermal gradients (Fig. B-35) were observed, primarily affecting the forward one inch of the rod and were most severe within the first 15-20 seconds. Thermal equilibrium was attained after 220 seconds at a value

of 1200-1400° F. Maximum convective heating occurred 135 seconds after exposure at a value of 4.22×10^4 BTU/hr. The heating rate was exceeded by the cooling rate 167 seconds after ejection at a value of 2.2×10^4 BTU/hr.

(5) 300,000 Feet Altitude

(A) Ejection From Axial Vehicle

Altitude = 298,562 feet

Velocity = 25,571 ft/sec

1 Normal Element (Figure B-36)

The stagnation point temperature reached 3400° F within approximately 130 seconds after fuel element ejection. Unfortunately this computation was cut off prematurely at 130 seconds but at that time some 20 percent of the volume of the fuel element had neared 3400° F. By extrapolation it was found that the convective heating rate was exceeded by the cooling rate at 140 seconds after exposure so it is not likely that very much more material would have attained the melting temperature, if the calculation had run its full course. The interesting result, however, may be seen in Fig. B-37, i.e., the thermal gradients are shallow at this high temperature. For these conditions therefore, the fuel element reaches relatively high temperatures with some semblance of uniformity. The maximum value of convective heating rate which was obtained was 3.5×10^4 BTU/hr but this value may not be the highest rate to be expected for this trajectory. However, the data is of academic interest only since the total cooling rate exceeded the heating rate at this value.

2 Axial Element (Figure B-38)

For these ejection conditions the stagnation point temperature reached 3400°F approximately 500 seconds after ejection and a maximum volume of approximately 3 percent of the fuel element material reached or exceeded 3400°F about 640 seconds after exposure. Fairly severe thermal gradients were observed (Fig. B-39) acting over the forward two inches of the fuel element. The rod reached an equilibrium temperature of about 1400°F 840 seconds after ejection. The maximum convective heating rate occurred approximately 800 seconds after ejection at a value of $4.1 \times 10^4 \text{ BTU/hr}$ and the heating rate fell below the cooling rate 925 seconds after ejection at a value of approximately $2.5 \times 10^4 \text{ BTU/hr}$.

(B) Ejection From Normal Vehicle

Altitude = 297,266 feet

Velocity = 25,325 ft/sec

1 Normal Element (Figure B-40)

For these conditions the stagnation point temperature reached 3400°F within 135 seconds after ejection and a maximum volume of approximately 30 percent of the fuel element material reached or exceeded 3400°F about 230 seconds after exposure. The observed thermal gradients were relatively shallow (Fig. B-41) indicating that the entire volume of the rod attained rather high temperatures uniformly. The maximum convective heating rate occurred 225 seconds after ejection at a value of approximately $7.5 \times 10^4 \text{ BTU/hr}$ and the heating rate was exceeded by the cooling rate within 10 seconds more at a value of about $7.4 \times 10^4 \text{ BTU/hr}$.

2 Axial Element (Figure B-42)

For these exposure conditions the stagnation point temperature reached 3400°F within 180 seconds after ejection and a maximum volume of approximately 11 percent of the fuel element material reached or exceeded 3400°F within 430 seconds after exposure. Observed thermal gradients were appreciable (Fig. B-43) and affected the forward two inches of the rod. Thermal equilibrium was attained 500 seconds after exposure at approximately 1600°F . The maximum convective heating rate occurred about 420 seconds after exposure at a value of $4.1 \times 10^4 \text{ BTU/hr}$. The cooling rate exceeded the heating rate about 30 seconds later at a value of approximately $2.8 \times 10^4 \text{ BTU/hr}$.

(6) 400,000 Feet Altitude(A) Ejection From Either Vehicle

Altitude = 400,000 feet

Velocity = 25,473 ft/sec

1 Normal Element (Figure B-44)

The computation for this condition of ejection was prematurely ended at 1320 seconds at an altitude of 250,940 feet. At that time 6.5 percent of the volume of the fuel element material had reached or exceeded 3400°F and from the plot of the thermal gradient (Fig. B-45) it will be seen that the internal temperature of the fuel element was fairly uniform, indicating that with additional heating the major portion of the fuel element might have attained the melting temperature. However, cold wall calculations indicate that maximum heating for this trajectory occurs between 1325 and 1350 seconds after ejection. Hence, no significant additional amount of heating could be

expected after the 1320 second value especially since the heating rate and cooling rate were rising steeply (Fig. B-44) and closely parallel. Although at 1320 seconds the heating rate exceeded the cooling rate by some 2000 BTU/hr, the heating rate drops sharply after the maximum point has been attained while the cooling rate will continue to rise for a short time and tend to decrease more slowly. It should not then be expected that any significant additional volume of material will exceed the melting temperature. It is unfortunate that this calculation was not carried to completion since it is one of the more attractive ejection conditions examined.

2 Axial Element (Figure B-46)

For orbital ejection of the fuel element in the axial attitude, the stagnation point temperature reached 3400°F approximately 1770 seconds after exposure of the fuel rod and a maximum volume of 5.5 percent of the fuel element material reached or exceeded the assumed melting temperature 2040 seconds after ejection. Moderate thermal gradients (Fig. B-47) were observed affecting the forward two inches of the fuel element. Thermal equilibrium was reached 2190 seconds after ejection at a value of approximately 1400°F . The maximum convective heating rate occurred at a value of 4.1×10^4 BTU/hr 2100 seconds after ejection. The cooling rate exceeded the heating rate at a value of approximately 2.6×10^3 BTU/hr 2125 seconds after ejection.

E. Cold-Wall Parametric Analysis

It was possible to begin an analysis of the aerodynamic heating of the

fuel elements when the trajectory parameter values became available. Since thermal property data of the alloy were at the time unavailable, cold-wall analyses were performed to determine the altitude at which the stagnation point convective heat transfer rate becomes a maximum. The stagnation point convective heat transfer rate to a blunt body of revolution in hypersonic flow⁴ can be expressed as

$$\dot{q}_c = \frac{17600}{\sqrt{R}} \sqrt{\frac{\rho_\infty}{\rho_o}} \left(\frac{V}{V_c} \right)^{3.15} \left[\frac{H_s - H_w}{H_s - H_{w-300}} \right] \text{ BTU/ft}^2 \text{ sec} \quad (1)$$

where R = effective nose radius (ft)
 ρ_∞ = ambient density (slugs/ft³)
 ρ_o = sea level density (slugs/ft³)
 V = flight velocity (ft/sec)
 V_c = satellite velocity (ft/sec)
 H_s = stagnation enthalpy
 H_w = wall enthalpy
 H_{w-300} = wall enthalpy evaluated at 300° K.

If we take the satellite velocity as 26,000 ft/sec, Eq. (1) becomes

$$\dot{q}_c = \frac{867.7}{\sqrt{R}} \sqrt{\frac{\rho_\infty}{\rho_o}} \left(\frac{V}{10^4} \right)^{3.15} \left[\frac{H_s - H_w}{H_s - H_{w-300}} \right] \quad (2)$$

To perform a cold wall analysis of the convective heat transfer rate we assume that the wall temperature remains at 300° K and Eq. (2) becomes

$$\dot{q}_c = \frac{867.7}{\sqrt{R}} \sqrt{\frac{\rho_\infty}{\rho_o}} \left(\frac{V}{10^4} \right)^{3.15} \quad (3)$$

Taking the first derivative of Eq. (3) to evaluate the point at which $\dot{q}_c$ is a maximum we obtain

$$\dot{dq}_c = \frac{867.7}{\sqrt{R}} \left[\left(\frac{\rho_\infty}{\rho_o} \right)^{\frac{1}{2}} d \left(\frac{V}{10^4} \right)^{3.15} + \left(\frac{V}{10^4} \right)^{3.15} d \left(\frac{\rho_\infty}{\rho_o} \right)^{\frac{1}{2}} \right] \quad (4)$$

If we note that $\rho_o = \text{constant}$ Eq. (4) becomes

$$\dot{dq}_c = \frac{867.7}{\sqrt{R}} \left[\left(\frac{3.15}{10^4} \right) \left(\frac{\rho_\infty}{\rho_o} \right)^{\frac{1}{2}} \left(\frac{V}{10^4} \right)^{2.15} dv + \left(\frac{1}{2} \right) \left(\frac{V}{10^4} \right)^{3.15} \left(\frac{\rho_\infty}{\rho_o} \right)^{-\frac{1}{2}} \left(\frac{d\rho_\infty}{\rho_o} \right) \right] \quad (5)$$

Collecting terms we then have

$$\dot{dq}_c = \frac{867.7}{\sqrt{R}} \left(\frac{\rho_\infty}{\rho_o} \right)^{\frac{1}{2}} \left(\frac{V}{10^4} \right)^{3.15} \left[\left(\frac{3.15}{10^4} \right) \left(\frac{V}{10^4} \right)^{-1} dv + \left(\frac{1}{2} \right) \left(\frac{\rho_\infty}{\rho_o} \right)^{-1} \left(\frac{d\rho_\infty}{\rho_o} \right) \right] \quad (6)$$

We can therefore write Eq. (6) in the form

$$\dot{dq}_c = \dot{q}_c \left[3.15 \frac{dV}{V} + \frac{1}{2} \frac{d\rho_\infty}{\rho_\infty} \right] \quad (7)$$

The drag force can be written in the form

$$F = 1/2 \rho_\infty V^2 C_D A = -m \frac{dV}{dt} \quad (8)$$

where

- C_D = coefficient of drag
- A = characteristic area of the body
- m = mass of the body

$$\therefore \frac{dV}{dt} = -\frac{1}{2} \frac{\rho_\infty V^2 C_D A}{m} \quad (9)$$

We can rewrite Eq. (9) in the form

$$\frac{dV}{dh} = -\left(\frac{1}{2}\right) \frac{\rho_{\infty} V^2 C_D A}{m \left(\frac{dh}{dt}\right)} \quad (10)$$

where

h = altitude.

But

$$\frac{dh}{dt} = -V \sin \gamma \quad (11)$$

where γ = flight path angle.

Therefore, by substitution of Eq. (11) in Eq. (10) and rewriting, we have

$$\frac{dV}{V} = \left(\frac{1}{2}\right) \left(\frac{C_D A \rho_{\infty}}{m \sin \gamma} \right) dh \quad (12)$$

Eq. (7) can then be expressed in the form

$$\dot{dq}_c = \dot{q}_c \left[\left(\frac{3.15}{2} \right) \left(\frac{C_D A}{m} \right) \left(\frac{\rho_{\infty}}{\sin \gamma} \right) dh + \frac{1}{2} \frac{d\rho_{\infty}}{\rho_{\infty}} \right] \quad (13)$$

Setting Eq. (13) equal to zero we have

$$3.15 \left(\frac{C_D A}{m} \right) \left(\frac{\rho_{\infty}}{\sin \gamma} \right) dh + \frac{d\rho_{\infty}}{\rho_{\infty}} = 0 \quad (14)$$

$$\text{But } \rho_{\infty} = \rho_0 e^{-Bh} \quad (15)$$

$$\text{where } B = \frac{1}{22,000 \text{ ft}}$$

$$\rho_0 = 0.002378 \text{ slugs/ft}^3$$

Substituting Eq. (15) in Eq. (14) we have

$$3.15 \left(\frac{C_D A}{m} \right) \left(\frac{\rho_o e^{-Bh}}{\sin \gamma} \right) dh + \frac{d(\rho_o e^{-Bh})}{\rho_o e^{-Bh}} = 0 \quad (16)$$

Performing the indicated operations, canceling terms and solving Eq. (16) for the term involving h we have

$$e^{-Bh} = \left(\frac{B \sin \gamma}{3.15 \rho_o} \right) \left(\frac{m}{C_D A} \right) \quad (17)$$

Solving Eq. (17) for h we have finally

$$h = \frac{1}{B} \ln \left[\frac{(3.15) (\rho_o)}{(B) (\sin \gamma)} \left(\frac{C_D A}{m} \right) \right] \quad (18)$$

This is in good agreement with the results of Allen and Eggers¹⁶ for the altitude at which the maximum value of the heat transfer rate per unit area at the stagnation point occurs. Figure B is a plot of the altitude at which the maximum occurs as a function of initial trajectory angle at the time of fuel element exposure for the two ballistic coefficients of 20 and 300 lb/ft². Also shown in Figure B is a plot of the variation of trajectory angle as a function of altitude of the reference vehicle. These values were taken from the trajectory program outputs and as discussed previously these values may serve for the trajectory of the fuel elements ejected in orbit as well as the reactor vehicle combination.

Figure B clearly shows the advantages of fuel element ejection at a point as early in reentry as possible. For the case of the vehicle reentering in the cross-flow configuration ($W/C_D A = 20$) the elements should be ejected prior to about 175,000 feet or those elements which assume a cross-flow configuration ($W/C_D A = 20$) upon ejection will have been shielded during the time of maximum heat transfer rate. For the case where the vehicle is reentering in the expected axial flow configuration ($W/C_D A =$

300) the elements should be ejected prior to about 220,000 feet or those elements which assume a cross-flow configuration ($W/C_D A = 20$) upon ejection will again have been shielded during the time of maximum heat transfer rate.

It should be noted that for fuel elements which assume the cross-flow configuration upon ejection from the reactor the maximum stagnation point heating occurs at an altitude higher than that at which the maximum stagnation point heating is sensed by elements which are trimmed axially ($W/C_D A = 300$). Since the attitudes of the fuel elements upon ejection can be assumed to have a random distribution, ejection should be accomplished at as great an altitude as possible (at least above approximately 220,000 feet) to ensure that all the elements, regardless of their orientation, are exposed during peak stagnation point heating.

Figure B also shows a plot of $(\frac{dQ}{dy})_{\max}$ which is the maximum rate of total heat transfer with altitude. These maxima occur¹⁶ at an altitude given by the expression

$$h = \frac{1}{B} \ln \frac{p_o}{(B)(\sin \gamma)} \quad (19)$$

It is again apparent that early ejection of the elements is desirable, especially if the vehicle-reactor combination assumes an axial flow configuration upon reentry. If such is the case, then the elements should be ejected prior to about 190,000 feet to insure that the elements that assume a cross-flow configuration ($W/C_D A = 20$) upon ejection are exposed to the maximum heat pulse.

FIGURE B COLD WALL VALUES OF $\dot{q}_{\max}$ AND $(\frac{dQ}{dy})_{\max}$ AS A FUNCTION OF TRAJECTORY ANGLE AT EJECTION

FIG.B-1 PROGRAMMED SEGMENTS
NORMAL CONFIGURATION
($W/C_0A = 20 \text{ LBS/FT}^2$)

FIG.B-2 : PROGRAMMED SEGMENTS
AXIAL CONFIGURATION
($W/C_p A = 300 \text{ lb/ft}^2$)

1	2	3	4	
20	21	22	23	
39	40	41	42	
58	59	60	61	
77	78	79	80	

ENLARGED VIEW OF
DETAIL "A"
NOT TO SCALE

SEE ENLARGED VIEW

DETAIL "A"	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19
	24	25	26	27	28	29	30	31	32	33	34	35	36	37	38
	43	44	45	46	47	48	49	50	51	52	53	54	54	55	56
	62	63	64	65	66	67	68	69	70	71	72	73	74	75	76
	81	82	83	84	85	86	87	88	89	90	91	92	93	94	95

SIDE VIEW (NOT TO SCALE)

FIG. B-3 NORMAL ELEMENT-AXIAL VEHICLE
 100,000 FEET EJECTION

FIG.B-4 INTERNAL TEMPERATURE ALONG A DIAMETER (X) THROUGH THE STAGNATION POINT

TIME = 0.1 SEC. FROM EJECT
 STAG. PT. TEMP. $\approx$ 1085 °F
 VEL. $\approx$ 14,706 FT./SEC.
 ALT. $\approx$ 109,300 FT.
 DIAM. OF ROD = 1.25 IN.

FIG. B-5

TIME = 0.2 SEC. FROM EJECT
 STAG. PT. TEMP = 1436 °F
 VEL. = 14,536 FT./SEC.
 ALT. = 109,230 FT.
 DIAM. OF ROD = 1.25 IN

FIG. B-6

SCALE: 4/1

TIME = 0.5 SEC. FROM EJECT
STAG. PT. TEMP. $\approx$ 2000 °F
VEL. = 14,044 FT./SEC
ALT. = 109,010 FT.
DIAM. OF ROD = 1.25 IN.

FIG. B-7

SCALE : 4 / 1

TIME = 1.0 SEC. FROM EJECT
 STAG. PT. TEMP. $\approx$ 2384 °F
 VEL. = 13,167 FT./SEC.
 ALT. = 108,610 FT.
 DIAM. OF ROD = 1.25 IN.

FIG. B-8

TIME = 6.5 SEC. FROM EJECT
 STAG. PT. TEMP. $\approx$ 1962 °F
 VEL. = 7,131 FT./ SEC.
 ALT. = 105,120 FT.
 DIAM. OF ROD = 1.25 IN.

FIG. B - 9

SCALE : 4/1

TIME = 11.5 SEC. FROM EJECT
STAG. PT. TEMP. $\approx 1439^{\circ}\text{F}$
VEL. = 4,474 FT./ SEC.
ALT. = 102,660 FT.
DIAM. OF ROD = 1.25 IN.

FIG. B-10

SCALE : 4/1

TIME = 0.1 SEC. FROM EJECT
SURFACE TEMP. $\approx$ 410°F
VEL. = 14,706 FT./SEC.
ALT. = 109,300 FT.
DIAM. OF ROD = 1.25 IN.

FIG. B-II

TIME = 0.2 SEC. FROM EJECT
 SURFACE TEMP. $\approx$ 565 °F
 VEL. = 14,536 FT./SEC.
 ALT. = 109,230 FT.
 DIAM. OF ROD $\approx$ 1.25 IN.

FIG. B - 12

FIG. B-13 AXIAL ELEMENT-AXIAL VEHICLE
 100,000 FEET EJECTION

FIG. B-14 NORMAL ELEMENT-AXIAL VEHICLE 150,000 FEET EJECTION

FIG. B-15 : THERMAL GRADIENT ALONG STAG. POINT DIAMETER - AXIAL VEHICLE 150,000 FEET EJECTION.

FIG B-16 AXIAL ELEMENT - AXIAL VEHICLE
 150,000 FEET EJECTION

FIG B-17 THERMAL GRADIENT ALONG ROD SURFACE
AXIAL ELEMENT-AXIAL VEHICLE 150,000
FEET

FIG. B-18 NORMAL ELEMENT-NORMAL VEHICLE 150,000 FEET EJECTION

FIG. B-19. THERMAL GRADIENT ALONG THE
STAGNATION POINT DIAMETER
NORMAL VEHICLE-NORMAL ELEMENT
150,000 FEET EJECTION.

FIG. B-20 AXIAL ELEMENT - NORMAL VEHICLE 150,000 FEET EJECTION.

FIG. B-21: THERMAL GRADIENT ALONG THE ROD SURFACE
NORMAL VEHICLE AXIAL ELEMENT
150,00 FEET EJECTION.

FIG B-22 NORMAL ELEMENT-AXIAL VEHICLE 200,000 FEET EJECTION

FIG. B - 23 THERMAL GRADIENT ALONG STAGNATION POINT DIAMETER AXIAL VEHICLE - NORMAL ELEMENT 200,000 FEET EJECTION.

FIG. B-24 AXIAL ELEMENT-AXIAL VEHICLE
 200,000 FEET EJECTION

FIG B-25. THERMAL GRADIENT ALONG THE ROD SURFACE
AXIAL VEHICLE -AXIAL ELEMENT 200,000
FEET EJECTION.

FIG. B-26 NORMAL ELEMENT - NORMAL VEHICLE
200,000 FEET EJECTION

FIG. B-27 THERMAL GRADIENT ALONG THE
STAGNATION POINT DIAMETER
AXIAL VEHICLE - NORMAL ELEMENT
EJECTION = 200,000

FIG. B-28 AXIAL ELEMENT - NORMAL VEHICLE 200,000 FEET EJECTION

FIG B-29: THERMAL GRADIENT ALONG THE ROD
SURFACE - AXIAL VEHICLE - AXIAL
ELEMENT 200,000 FEET EJECTION

FIG. B-30 NORMAL ELEMENT-AXIAL VEHICLE 250,000 FT. EJECTION

FIG. B-31: THERMAL GRADIENT ALONG THE
STAGNATION POINT DIAMETER
AXIAL VEHICLE - NORMAL ELEMENT
250,000 FEET EJECTION.

FIG. B-32 NORMAL ELEMENT - NORMAL VEHICLE
 250,000 FEET EJECTION

FIG. B-33 . THERMAL GRADIENT ALONG THE
STAGNATION POINT DIAMETER
NORMAL VEHICLE - NORMAL ELEMENT
250,000 FEET EJECTION

FIG. B-34 AXIAL ELEMENT-NORMAL VEHICLE 250,000 FEET EJECTION

FIG. B-35: THERMAL GRADIENT ALONG THE ROD SURFACE
NORMAL VEHICLE - AXIAL ELEMENT 250,000
FEET EJECTION

FIG. B-36 NORMAL ELEMENT - AXIAL VEHICLE 300,000 FEET EJECTION

FIG B-37 THERMAL GRADIENT ALONG STAGNATION POINT DIAMETER AXIAL VEHICLE- NORMAL ELEMENT 300,000 FEET EJECTION

FIG. B-38 AXIAL ELEMENT-AXIAL VEHICLE
 300,000 FEET EJECTION

FIG. B - 39 : THERMAL GRADIENT ALONG THE ROD
SURFACE -AXIAL VEHICLE -AXIAL ELEMENT
300,000 FEET EJECTION.

FIG. B-40 NORMAL ELEMENT-NORMAL VEHICLE
 300,000 FEET EJECTION

FIG. B- 41 : THERMAL GRADIENT ALONG STAG.
POINT DIAMETER - NORMAL VEHICLE
NORMAL ELEMENT 300,000 FEET
EJECTION .

FIG. B-42 AXIAL ELEMENT-NORMAL VEHICLE 300,000 FEET EJECTION

FIG. B - 43. THERMAL GRADIENT ALONG ROD SURFACE
NORMAL VEHICLE - AXIAL ELEMENT
300 000 FEET EJECTION

FIG. B-44 NORMAL ELEMENT- ORBITAL 400,000 FEET EJECTION

FIG. B-45: THERMAL GRADIENT ALONG STAG. DIAMETER - NORMAL ELEMENT - EITHER VEHICLE - 400,000 FEET EJECTION.

FIG. B-46 AXIAL ELEMENT-ORBITAL
 400.000 FEET EJECTION

FIG. B-47: THERMAL GRADIENT ALONG ROD SURFACE
AXIAL ELEMENT - EITHER VEHICLE
40000 FEET EJECTION.

APPENDIX CTHERMAL PROPERTIES OF THE HYDRIDED FUEL MATERIAL

A trip was taken on 15 November 1962 to Armour Research Foundation to review techniques for measuring high temperature properties of materials which were conducted under the current research program. The trip was prompted by our concern as to whether the enthalpy and specific heat data for the hydrided zirconium-uranium alloy as measured by ARF could be used in the theoretical analysis of a re-entering fuel rod. This concern could not be satisfied by the Armour Research Foundation reports on these properties since they lacked adequate discussion about the experiments and their results.

A discussion was held with J. C. Hedge, Project Engineer, and J. Lang, experimentalist on the project, concerning the enthalpy data. It was agreed that the experiments did not simulate actual flight conditions and that this was not called for in the contract.

The chief reasons that the experiments did not simulate flight conditions are: first, the samples were heated for long periods of time (approximately 30 minutes) to obtain equilibrium conditions; and secondly, the hydrided samples were in hydrogen gas at one atmosphere pressure. The first technique obviously does not simulate reentry because a re-entering fuel rod will experience heating times of approximately 400 seconds or less and not 1800 seconds as used in the experiments. The second technique does not simulate reentry for the apparently obvious reason that the fuel rod is traveling through air and therefore will experience a very large variation of hydrogen gas pressure at its surface, but the more important reason is more subtle. The enthalpy data reported for the hydrided alloy shows a rise and then a decrease with increasing temp-

erature such that above 2000° F the data, as shown, would indicate that the sample would give off energy if the temperature is raised above 2000° F, thus the negative specific heat as reported. This discrepancy results because the sample is in hydrogen gas at one atmosphere, and, since the vapor pressure of the sample is greater than one atmosphere at these higher temperatures, the system loses hydrogen gas and thus the energy that was absorbed by the sample. Therefore, the enthalpy data reported does not give the total energy absorbed by the sample when heated to these higher temperatures. In addition, if the hydrogen pressure is changed to a value different from one atmosphere, there would be a different enthalpy curve for the hydrided alloy from that reported. The initial temperature at which the enthalpy curve begins to drop will increase with increasing hydrogen pressure. Finally, when the hydrogen pressure is equal to or greater than the vapor pressure of the hydrided alloy at the highest experimental temperature, the enthalpy curve will be a continuously increasing curve which will include all of the energy absorbed by the alloy. The enthalpy data reported by ARF is just part of that required for a theoretical analysis of a reentering hydrided fuel rod.

The energy lost by the hydrided alloy through the loss of hydrogen gas may be accounted for if the heat of formation of the hydrided zirconium-uranium alloy is known. The total enthalpy for the alloy and the evolved hydrogen gas is

$$H_T = m_c [h_T]_c + m_{H_2} [h_T]_{H_2} \quad (1)$$

where

m_c = the mass fraction of the alloy
 m_{H_2} = the mass fraction of the evolved hydrogen gas
 H_T = total enthalpy for the constant mass of alloy and hydrogen gas at temperature T.

$[h_T]_c$ = enthalpy of the alloy at temperature T

$[h_T]_{H_2}$ = enthalpy of the evolved hydrogen gas at temperature T.

The enthalpy of the alloy $[h_T]_c$ is reported by ARF and that for the evolved gas has to be calculated as follows. The sensitive enthalpy of hydrogen gas versus temperature may be found in Reference 18. It is given as

$$[H_T - H_{298^\circ K}]_{H_2} \quad (2)$$

in cal/mole H_2 .

Therefore, to calculate $[h_T]_{H_2}$ we must know $H_{298^\circ K}$ or

$$h_T = \frac{1}{M_{H_2}} [H_T - H_{298^\circ K}]_{H_2} + \frac{1}{M_{H_2}} [H_{298^\circ K}]_{H_2} \quad (3)$$

where M_{H_2} = molecular weight of H_2 .

ARF chooses the following enthalpy base

$$[H_{298^\circ K}]_{U_{0.0434} ZrH_{1.8}} = 0 \quad (4)$$

$$[H_{298^\circ K}]_{U_{0.0434} Zr} = 0 \quad (5)$$

where $U_{0.0434} ZrH_{1.8}$ is the hydrided alloy given in molar proportions and $U_{0.0434} Zr$ is the unhydrided alloy also in molar proportions. The alloy may be formed at $298^\circ K$ by the reaction

where $\Delta H_{298^\circ K}$ is the energy released and is defined as

$$\Delta H_{298^\circ K} = [H_{298^\circ K}]_{U_{0.0434} ZrH_{1.8}} - 0.9 [H_{298^\circ K}]_{H_2} - [H_{298^\circ K}]_{U_{0.0434} Zr} \quad (7)$$

Substituting equations 4 and 5 into equation 7, we have

$$[H_{298^\circ K}]_{H_2} = -1.111 \Delta H_{298^\circ K} \quad (8)$$

Substituting equation 8 and 3 into equation 1, we have

$$H_T = m_c [h_T]_c + m_{H_2} \left\{ \frac{1}{M_{H_2}} [H_T - H_{298^\circ K}]_{H_2} - \frac{1.111}{M_{H_2}} \Delta H_{298^\circ K} \right\} \quad (9)$$

Equation 9 may be used to calculate the total enthalpy of the alloy-hydrogen gas mixture if the heat of reaction $\Delta H_{298^\circ K}$ is known.

The heat of reaction $\Delta H_{298^\circ K}$ for reaction 6 has not been directly measured. However, $\Delta H_{298^\circ K}$ may be evaluated by combining other known chemical reactions which involve the three chemical compounds in reaction 6. The following reactions are such a combination at a temperature of $298^\circ K$.

Reaction 6 may be had from reactions 10 through 14 by combining the above reactions in the following proportions

$$0.9[11] + 0.0434[12] + [13] - [10] - [14]$$

The heat of reaction $\Delta H_{298^\circ \text{K}}$ is then given as

$$\Delta H_{298^\circ \text{K}} = 0.9\Delta H_{11} + 0.0434\Delta H_{12} + \Delta H_{13} - \Delta H_{10} - \Delta H_{14} \quad (15)$$

The Armour Research Foundation measured the heat of combustion of the hydrided alloy; however, they did not report the products of combustion for the experiment nor the standard temperature. It will be assumed that their combustion reaction is given by reaction 10. They reported the following value for the heat of combustion

$$\Delta H_{10} = -2868 \text{ Btu/lb} = -164,650 \text{ cal/mole} \quad (16)$$

The heats of reaction for reactions 11, 12 and 13 are given in Reference 19,

$$\Delta H_{11} = -68,317 \text{ cal/mole } \text{H}_2\text{O} \quad (17)$$

$$\Delta H_{12} = -270,000 \text{ cal/mole} \quad (18)$$

$$\Delta H_{13} = -258,200 \text{ cal/mole } \text{ZrO}_2 \quad (19)$$

The heat of reaction for reaction 14 is not known. However, the atom concentration of uranium in zirconium is so small that it may be taken as approximately zero.

$$\Delta H_{14} = 0 \quad (20)$$

Substituting the heat of reactions for reactions 10 through 14 into equation 15 gives

$$\Delta H_{298^{\circ}\text{K}} = 0.9[-68,317] + 0.0434[-270,000] - 258,000 + 164,650$$

or

$$\Delta H_{298^{\circ}\text{K}} = 166,753 \text{ cal/mole} \quad (21)$$

Substituting $\Delta H_{298^{\circ}\text{K}}$ into equation 9, we have for the total enthalpy of the mixture in BTU/lb

$$H_T = m_c [h_T]_c + m_{H_2} \left\{ 0.893 [H_T - H_{298^{\circ}\text{K}}] + 165,427 \right\} \quad (22)$$

The mass fractions m_c and m_{H_2} have not been reported and must be obtained from the Armour Research Foundation.

The heat of reaction $\Delta H_{298^{\circ}\text{K}}$, as given by (21), may be too large. Its absolute value would be reduced if the combustion of the hydrided alloy was incomplete, that is, if not all of the Zr, U and H_2 were converted to ZrO_2 , UO_2 and H_2O . This cannot be evaluated, however, since the compositions of the combustion products were not reported.

APPENDIX D

THERMODYNAMIC ANALYSIS OF FUEL "DISKS"

It has been suggested that fuel rods made up of a number of disks would enhance burn-up during atmosphere reentry. However, Reference 20 has shown that some specific sized disks may reenter successfully. It is therefore necessary to determine if a fuel rod "disk" follows the general trend found in the cited reference.

Following the procedure outlined in Reference 21, the radiation heat flux is set equal to the maximum convective heat flux (laminar flow).

$$\epsilon \sigma T_w^4 = 590 K_1 \sqrt{\frac{w}{C_D A}} R g c \bar{q}_{\max} \quad (1)$$

$\bar{q}_{\max} = 0.218$; reentry from decaying earth orbit (Ref. 21)

$K_1 = 1$; ratio of local heat flux to stagnation value (Ref. 22)

$\sigma = 0.48 \times 10^{-12} \text{ Btu/ft}^2 \text{ sec}^\circ \text{R}^4$

The assumed shape under investigation is a disk of fixed diameter and variable thickness t . The disk is oriented perpendicular to the flight path and therefore the ballistic coefficient is defined as:

$$\frac{w}{C_D A} = \frac{\rho A t}{C_D A} = 18t \text{ lb/ft}^2 \quad (2)$$

for t in inches, $C_D \equiv 2$, and $\rho = 434 \text{ lb/ft}^3$. Since heat is radiated from both sides of the disk (for small values of t) and $R = 4r$ (Reference 6), Equation 1 becomes

$$2t\sigma T_w^4 = 590 \sqrt{\frac{18t}{4rgc}} \bar{q} \quad (3)$$

It was assumed that the material has the following characteristics and properties:

$$r = 0.625 \text{ inch}$$

$$t = 0.75$$

$$\rho = 434 \text{ lb/ft}^3$$

Solving Equation 3 for T_w gives

$$T_w = 4140 t^{\frac{1}{8}} \quad (4)$$

and is plotted as Figure D-1 up to a thickness of $t = 1/2$ inch since it is felt that the simple assumptions are valid only for a thin disk. Based on this simple analysis one can say that disks of thickness $t < 1/2$ inch will probably reenter intact.

FIG. D-1 : RE-ENTERING FUEL MATERIAL DISKS MAXIMUM TEMPERATURES ATTAINABLE AS A FUNCTION OF THICKNESS

APPENDIX E

THE HAZARD ASSOCIATED WITH THE DISPERSAL OF SMALL PARTICLES FROM REENTERING REACTORS AS A FUNCTION OF PARTICLE SIZE

1. INTRODUCTION

The problem to be considered is a reentering vehicle which "burns" (i.e., ablates, melts, etc.) in the atmosphere. The burn up produces a distribution of particles, approximately a line source, which settles to earth under the influence of gravity and the earth's atmosphere.

A general procedure for determination of an upper limit to the areal concentration of mass and number of particles at ground level will be developed in Sections 2. and 3. This procedure is applied to the U-Zr-H fuel rods of a 30 kilowatt nuclear auxiliary power system and to its beryllium shield.

In order to decide the question of safety, one must know the maximum allowable areal concentration of material. This question is attacked in Section 4. for the fuel rods and 5. for the beryllium shield.

There are several mechanisms which act to spread a line source of particles over a large area before they reach the ground.

- (1) Molecular diffusion
- (2) Turbulent eddy diffusion
- (3) Turbulence in the wake of the vehicle
- (4) The finite length of burn path, L , giving a sizeable horizontal spread to the particles initially
- (5) The finite difference in altitude Δh of the burn path which allows horizontal winds to displace the upper particles relative to the lower during the time required to fall through the difference Δh . This causes the upper particles to see a wind pattern displaced in space and time from that seen by the lower particles.

- (6) Particles of differing radii have differing terminal fall velocities. Thus the particles which fall more slowly spend more time in a layer with given horizontal wind velocity, so are carried a greater horizontal distance before reaching the ground.
- (7) A given element of interest may, after the burning process, be distributed among several constituents. Each constituent has a different density, size distribution, etc., resulting in a total spread at the ground greater than that for a single material.

Of these effects number (1) is negligible for particles of micron size or larger. This is shown in Section 7.

It is likely that all of the other 6 processes cause significant spreading of particles. However, numbers (2), (4), and (6) taken together are sufficient to spread particles from SNAP-8 to an acceptable radiological or chemical level if the "burn" process is efficient. We will therefore ignore effects (3) and (5), by assuming a horizontal line source, and effect (7) by assuming all of the element of interest is concentrated into its most dense constituent. We will also assume particles to be spherical. By so doing we will estimate an upper limit to the areal concentration on the ground of the element of interest.

Effects (4) and (6) will be considered in Section 2., effect (2) in Section 3.

In line with the objective of calculating an upper limit to the concentration of products, it will be assumed that the release altitude is 30 km (92,000 ft), a lower limit on the likely burning altitude.

2. AREAL CONCENTRATION WIND PERPENDICULAR TO THE TRAJECTORY

A. Formula for the Areal Concentration

Consider a point release of particles of constituent α at altitude h_r having a continuous distribution in radius. The number of

particles with radii between p and $p + dp$ is $N_\alpha(p) dp$. We assume the particles to fall with their terminal velocity and to be carried horizontally with the local wind. We assume further that the terminal velocity, U , is a function of particle radius, p . At successively later times $t_0, t_1, \dots$ the locus of the position of these particles is a line as shown in Figure E-1.

In Figure E-1, t_0 is the time of release. At t_3 the faster falling particles have reached the ground, while at t_4 all particles have reached the ground. The horizontal separation of the slow and fast falling limits of the particle distribution is Δr .

If the position on the ground of a particle of radius p is $r(p)$ the number of particles in an element dr is

$$N_\alpha(p) \frac{dp}{dr} dr. \quad (1)$$

Fig. E-1. Pattern of particle fall from a point source due to horizontal wind.

Therefore, if we are given $N_\alpha(p)$ and can calculate

$$\frac{dp}{dr} = \left(\frac{dr}{dp} \right)^{-1} \quad (2)$$

then we know the number of particles per unit length on the ground.

Suppose now that instead of a point release at h_r , we are dealing with a horizontal line release at h_r . We can imagine the particles which originate at given positions $l_0, l_1, l_2, \dots$ on the original line as tracing their corresponding lines on the ground as in Figure E-2.

The number of particles with radii between p and $p + dp$ released between positions l and $l + dl$ is

$$N_\alpha(p, l) dp dl \quad (3)$$

These fall on an element of area

$$\frac{\partial \vec{r}}{\partial p} \cdot \frac{\partial \vec{r}}{\partial l} dp dl \quad (4)$$

so the number of particles per unit area is given by

$$N = \frac{N_\alpha(p, l)}{\frac{\partial \vec{r}}{\partial p} \cdot \frac{\partial \vec{r}}{\partial l}} \quad (5)$$

Fig. E-2. Pattern of particle fall from a line source due to horizontal wind.

As illustrated in Figure E-2, it is possible for a curve from a given point ℓ_1 to turn back on itself. It is also possible for particles originating at different points on ℓ to overlap. In the following estimate these possibilities will be treated as minor perturbations (causing an error much less than a factor of 10) and ignored.

For the present assume

$$\frac{\partial \vec{r}}{\partial p} \cdot \frac{\partial \vec{r}}{\partial \ell} = \left| \frac{\partial \vec{r}}{\partial p} \right| \left| \frac{\partial \vec{r}}{\partial \ell} \right| \quad (6)$$

This in effect assumes the mean wind velocity to be perpendicular to the line of release. This assumption will be removed in Section 3.

On the average the displacement at the ground caused by release at different positions above the ground is just $|\Delta \vec{r}| = |\Delta \ell|$ so take

$$\frac{\partial \vec{r}}{\partial \ell} = 1. \quad (7)$$

We have then the number of particles per unit area on the ground as

$$N \approx \frac{N_\alpha(p, \ell)}{\frac{\partial \vec{r}}{\partial p}} \quad (8)$$

B. Evaluation of $\frac{\partial \vec{r}}{\partial p}$

The equation relating horizontal position to release altitude

is:

$$\vec{r}(p) = \int_0^t \vec{V} dt = \int_{h_r}^0 \vec{V} \frac{dt}{dz} dz = \int_0^{h_r} \frac{\vec{V}(z)}{U(z, p)} dz \quad (9)$$

where $\vec{r}$ is the position on the ground measured relative to the point directly beneath the release point, $\vec{V}$ is horizontal wind velocity (a function of altitude, z), t is time of fall, and U is terminal fall velocity (which may be

a function of both z and p).

Differentiating equation (9) with respect to p

$$\begin{aligned}\frac{\partial \vec{r}}{\partial p} &= \frac{\partial}{\partial p} \int_0^{h_r} \frac{\vec{V}(z)}{U(z, p)} dz = \int_0^{h_r} \vec{V}(z) \frac{\partial}{\partial p} \left(\frac{1}{U} \right) dz \\ &= - \int_0^{h_r} \frac{\vec{V}}{U^2} \left(\frac{\partial U}{\partial p} \right) dz .\end{aligned}\quad (10)$$

We will see that U can always be written in the form

$$U = U_0 p^{\gamma} f(z) \quad (11)$$

so (10) becomes

$$\frac{\partial \vec{r}}{\partial p} = -\frac{\gamma}{p} \int_0^{h_r} \frac{\vec{V}}{U} dz \quad (12)$$

The behavior of U as a function of p and z requires consideration of several different velocity regions. For the accuracy required here it is sufficient to consider three regions. These, in order of decreasing altitude (or increasing p), are :

- I. The molecular flow region, defined as that region in which the particle radius p is less than the mean free path, λ , of an ambient atmospheric molecule. In this region

$$U = U_s \frac{\lambda}{p} \quad (13)$$

to 10% accuracy. In (13) U_s is the velocity of fall in the Stokes region.

- II. The Stokes region, defined as that region in which $\lambda < p$ and the Reynolds number is less than 1. In this region

$$U = U_s = \frac{2g\rho_\alpha p^2}{9\eta} \quad (14)$$

where g is gravitational acceleration, ρ_α is the particle density, (assumed much greater than air density) and η is the coefficient of viscosity of air.

III. The turbulent flow region, defined by the Reynold's number being greater than 1. In this region

$$U = \left(\frac{8}{3} \left(\frac{\rho_\alpha}{\rho_a} \right) \frac{pg}{C_D} \right)^{\frac{1}{2}} \quad (15)$$

where ρ_a is ambient density and C_D is the aerodynamic drag coefficient.

(1) Region I

In Region I, U depends on altitude only through the mean free path of an air molecule. Since λ varies inversely as the air density and the air density varies approximately exponentially one may write

$$U = U_s \frac{\lambda}{p} = \frac{2g\rho_\alpha}{9\eta} p\lambda_o e^{Z/h_o} \equiv U_o \lambda_o p e^{Z/h_o} \quad (16)$$

where λ_o is the mean free path at some reference altitude which will be taken as sea level and h_o is the e-folding height of the atmosphere.

Substituting (16) into (12) gives (since $\gamma = 1$ in this region)

$$\frac{\partial \bar{r}}{\partial p} = \frac{1}{U_o \lambda_o p^2} \int_0^{h_r} V e^{-Z/h_o} dz \quad (17)$$

A glance at a mean wind profile (Fig. E-3) shows that the average horizontal wind velocity increases approximately as e^{Z/h_o} for about 2 scale heights, then drops off rapidly, never to become sufficiently strong at high altitudes to overcome the exponential factor. A good

FIG. E-3 WIND SPEED VERSUS ALTITUDE

- (1) Exceeded 50% of the Time in Calmest Area of North America
- (2) Exceeded 50% of the Time in Windiest Area of North America

approximation to (17) for particles which are in Region I all the way to the ground then is

$$\left| \frac{\partial \vec{r}}{\partial p} \right| = \frac{1}{U_o \lambda_o p^2} \int_0^{2h_o} V_o dz = \frac{2h_o V_o}{U_o \lambda_o p^2} \quad (18)$$

where V_o is the mean wind velocity at ground level. Substituting equation (18) into equation (8) gives the number per unit area for particles which are in Region I throughout their fall.

$$N \approx \frac{U_o \lambda_o p^2}{2V_o h_o} N_\alpha(p, t) \quad [\text{Region I}] \quad (19)$$

To obtain the mass per unit area, one needs to multiply (19) by $\frac{4}{3} \pi \rho_\alpha p^3$ to obtain

$$M \approx \frac{2\pi}{3} \rho_\alpha \frac{U_o \lambda_o p^5}{V_o h_o} N_\alpha(p, t) \quad [\text{Region I}] \quad (20)$$

where λ_o is the mean free path of an air molecule at sea level.

(2) Region II

In Region II, from (14) and (16)

$$U = \frac{2g\rho_\alpha p^2}{9\eta} = U_o p^2 \quad (21)$$

and is independent of altitude, so $\gamma = 2$ and substituting into equation (12) gives

$$\frac{\partial \vec{r}}{\partial p} = - \frac{2}{U_o p^3} \int_0^h \vec{r} V dz \quad (22)$$

For an order of magnitude estimate over the range 0 to 30 km take V as half the maximum value, i. e., about 5 times the surface value. Since the integral covers about 4 scale heights one has

$$\left| \frac{\partial r}{\partial p} \right| \approx \frac{40 V_o h_o}{U_o p^3} \quad (23)$$

Substituting equation (23) into equation (8) gives

$$N \approx \frac{U_o p^3}{40 V_o h_o} N_\alpha(p, l) \quad [\text{Region II}] \quad (24)$$

and

$$M \approx \frac{\pi}{30} \frac{U_o p^6 \rho_\alpha}{V_o h_o} N_\alpha(p, l) \quad [\text{Region II}]. \quad (25)$$

(3) Region III

In Region III, from (15)

$$U = \left[\frac{8}{3} \left(\frac{\rho_\alpha}{\rho_o} \right) \frac{g}{C_D} \right]^{\frac{1}{2}} p^{\frac{1}{2}} e^{\frac{Z}{2h_o}} \equiv U_{III} p^{\frac{1}{2}} e^{\frac{Z}{2h_o}} \quad (26)$$

so that $\gamma = \frac{1}{2}$ and equation (12) becomes

$$\left| \frac{\partial r}{\partial p} \right| = \frac{1}{2 p^{3/2} U_{III}} \int_0^h V e^{\frac{Z}{-2h_o}} dz. \quad (27)$$

Equation (27) has assumed the drag coefficient to be constant; this assumption is sufficiently accurate for our purpose. One may now proceed as in Region I except that due to the weaker exponential in equation (27) one needs to take about 4 scale heights times the surface wind velocity.

$$\left| \frac{\partial \mathbf{r}}{\partial p} \right| \approx \frac{2h_o V_o}{p^{3/2} U_{III}} \quad (28)$$

Substituting in equation (8) yields

$$N \approx \frac{p^{3/2} U_{III}}{2h_o V_o} N_{\alpha}(p, l) \quad [\text{Region III}] \quad (29)$$

and

$$M \approx \frac{2\pi}{3} \frac{p^{9/2} U_{III}^{\rho_{\alpha}}}{h_o V_o} N_{\alpha}(p, l) \quad [\text{Region III}] \quad (30)$$

C. Numerical Evaluation

As is apparent from any of the pairs of equations (19), (20); (24), (25); or (29), (30); the highest concentration of particles or mass occurs where $N_{\alpha}(p, l)$ is peaked. In order to obtain an evaluation of the radiation or chemical danger one must make some assumption as to the shape of $N_{\alpha}(p, l)$.

It seems unreasonable to expect a natural burning process to produce a number distribution with less than a width of, say, 10 percent. Therefore we assume that $N_{\alpha}(p)$ is peaked at p with a 10 percent spread in p (see Figure E-4).

Fig. E-4. Assumed radius distribution of burned particles.

It will also be assumed that burning occurs uniformly over a horizontal distance L . So that

$$N_{\alpha}(p, \ell) = \frac{10N_{\alpha}}{pL} \quad (31)$$

Where

$$N_{\alpha} = \int dp \int d\ell N_{\alpha}(p, \ell) \cong \frac{M_{\alpha}}{\frac{4}{3}\pi\rho_{\alpha}p^3} \quad (32)$$

where M_{α} is the total mass of the α th constituent. Substituting (32) and (31) into (19), (20), (24), (25), (29), (30) gives

$$\left. \begin{aligned} N &= \frac{15}{4\pi} \frac{U_o \lambda_o}{V_o h_o p^2 L} \frac{M_{\alpha}}{\rho_{\alpha}} \\ M &= 5 \frac{U_o \lambda_o}{V_o h_o} \frac{p}{L} M_{\alpha} \end{aligned} \right\} \text{Region I} \quad (33)$$

$$\left. \begin{aligned} N &= \frac{3}{16\pi} \frac{U_o}{V_o h_o p L} \frac{M_{\alpha}}{\rho_{\alpha}} \\ M &= \frac{U_o p^2}{4V_o h_o L} M_{\alpha} \end{aligned} \right\} \text{Region II} \quad (34)$$

$$\left. \begin{aligned} N &= \frac{15}{4\pi} \frac{U_{III}}{h_o V_o L p^{5/2}} \frac{M_{\alpha}}{\rho_{\alpha}} \\ M &= 5 \frac{U_{III} p}{h_o V_o L} M_{\alpha} \end{aligned} \right\} \text{Region III} \quad (35)$$

Figure E-5 is a graph of altitude-radius space showing the three fall-velocity regions. To obtain the accuracy required, calculations were carried out between extreme ranges of particle radius for all three regions. The true curve lies intermediate in the regions of overlap.

Numerical factors used are:

$$\begin{aligned}
 \lambda_o &= 8 \times 10^{-6} \text{ cm at sea level} \\
 V_o &= 6 \times 10^2 \text{ cm/sec at sea level} \\
 h_o &= 7 \text{ km} \\
 \eta &= 10^{-4} \text{ gm/cm sec} \\
 \rho &= 6.17 \text{ gm/cm}^3 \text{ for U-Zr-H} \\
 \rho &= 2 \text{ gm/cm}^3 \text{ for Be} \\
 C_D &= 1.0 \\
 L &= 900 \text{ km*}
 \end{aligned}$$

If one divides the mass per unit area by M_α the fraction of material deposited per unit area, f , is obtained. The quantity f depends on the material constituent but not the particular re-entering system, and is therefore a convenient measure of spread of material. Figure E-6 shows a plot of f , denoted f_L , as calculated from equations (33), (34), and (35) for U-Zr-H and Figure E-7 shows f_L calculated for Be. Since the mean burning radius, p , is not known at this time, the calculations cover the range of meaningful p .

These curves represent an upper limit on f for mean horizontal wind velocity perpendicular to the trajectory of the re-entry body. The case of mean horizontal wind parallel to the trajectory is treated in Section 3.

3. MAXIMUM VALUE OF f CORRESPONDING TO WIND PARALLEL TO THE TRAJECTORY

If the mean wind is parallel to the re-entry body trajectory, the most effective mechanism for dispersal of debris is likely to be turbulent eddy diffusion, effect (2) of the introduction.

* See Appendix A

FIG. 1 - FALL REGIONS FOR SMALL PARTICLES

FIG. 5-7 FRACTIONAL MASS PER UNIT AREA FOR D-

Experiment ³⁰ has shown that near sea level the radius of a point source cloud of smoke increases according to

$$\overline{y^2} = \overline{V^2} t^2 \quad (36)$$

to radii $\overline{y^2}^{\frac{1}{2}} \geq 4$ meters, where $\overline{V^2}^{\frac{1}{2}} \approx 10$ cm/sec.

Kellogg³¹ has found a similar result for altitudes between 8 km and 20 km. His values of $\overline{V^2}^{\frac{1}{2}}$ range from 4 cm/sec to 10 cm/sec to values of $\overline{y^2}^{\frac{1}{2}} \approx 100$ meters.

Sutton³² computes that for diffusion to hundreds of kilometers

$$\overline{y^2} \propto (\overline{vt})^{1.75} \quad (37)$$

The applicability of equation (37) to the current problem is uncertain, however, it will be used since it yields smaller values of $\overline{y^2}$ and thus larger values of f than equation (36).

Using these results the value of f may be computed as follows:

If the terminal velocity, U , of particles in question is greater than 10 cm/sec, take the time of fall equal to

$$t = \int_0^{h_r} \frac{dz}{U} \quad (38)$$

where h_r is release altitude.

Now assume equation (36) holds until $\overline{y^2}^{\frac{1}{2}} = 100$ meters, and equation (37) holds thereafter, with $\overline{V^2}^{\frac{1}{2}} = 5$ cm/sec, yielding a lower limit on y of

$$y^2 = 10^4 \left[\frac{t}{2 \times 10^3} \right]^{1.75} \text{ square meters.} \quad (39)$$

Then the area over which the material spreads is given approximately by

$$A = \begin{cases} \overline{Ly^2}^{\frac{1}{2}} & \text{if } L^2 > y^2 \\ \overline{y^2} & \text{if } L^2 < \overline{y^2} \end{cases} \quad (40)$$

and

$$f_{II} = A^{-1} \quad (41)$$

If the terminal velocity, U , of particles is less than the turbulent diffusion velocity, $\overline{V^2}^{\frac{1}{2}}$, it seems at first glance that a better value of t than equation (38) would be

$$t' = h_r / \overline{V^2}^{\frac{1}{2}} \quad (42)$$

In fact, t' is the approximate time at which the first particles appear at the surface. Their average fall velocity, however, is still given by the terminal velocity U , as is apparent when one considers the fact that the turbulent eddy motion is random as was found numerically by Banister and Davis.³³ Equation (38) then holds for all terminal velocities U , at least to a first approximation.

Substituting expressions (13), (14), and (15), from Section 2, for terminal velocity into equation (38), one finds for the three regions:

$$t_I = \frac{h_o}{U}, \quad t_{II} = \frac{h_r}{U}, \quad t_{III} = \frac{2h_o}{U} \quad (43)$$

U being the terminal velocity at the surface for the fall region in question. Equations (39), (40), and (41) yield the curves marked f_{11} in Figures E-6 and E-7 for U-Zr-H and B_e respectively. They represent an upper limit on f for re-entry trajectory parallel to mean wind velocity.

4. CALCULATION OF ACCEPTABLE RADIATION EXPOSURE LEVEL

A. Introduction

Dale³⁴ has made estimates of the acceptable levels of radiation exposure resulting from nuclear weapons tests. He finds that the threshold for economic loss from irradiation of plants, cattle, etc., is higher than that for danger to humans. Therefore, the human threshold is the limiting factor. He calculates two thresholds, level A at which no observable effects arise, and level B at which small observable effects could arise such as slight temporary sickness among individuals with a low threshold sensitivity to radiation.

A direct comparison between debris from a nuclear explosion and a reactor is not possible except for exposure times large compared to the period of operation of the reactor. This is due to the difference in early decay rates.

In the case of a nuclear explosion, the activity approximately follows the law

$$N = N_0 \left(\frac{t_0}{t} \right)^{1.2} \quad (44)$$

where N is the activity at time t , N_0 the activity at time t_0 .

However, in the case of a reactor operating for a time T , the activity is the sum of all the activities of fission debris produced throughout the time T . If the time after shutdown, τ , is short compared to T a mixture of fission products exists whose ages vary from $T + \tau$ to τ where the activity from fission products produced at t' is given by

$$dN = \frac{dN}{dt'} \left(\frac{t_0}{t - t'} \right)^{1.2} dt' \quad (45)$$

where $\frac{dN}{dt}$ is the rate of generation of fission activity at t' and t is the time of observation. It is convenient to take the time t_0 as 1 hour. Integrating equation (45) over the operating time yields, for constant $\frac{dN}{dt}$,

$$N = 5 \frac{dN}{dt} (t_0)^{1.2} \left(\frac{1}{\tau^{1/5}} - \frac{1}{(T + \tau)^{1/5}} \right) \quad (46)$$

For times τ small compared to T , (46) reduces to

$$N = 5 t_0 \frac{dN}{dt} \left(\frac{t_0}{\tau} \right)^{1/5} \quad (\tau < T). \quad (47)$$

Comparison of (47) with (44) illustrates the difference in decay rates of reactor products as compared to nuclear explosion debris.

If $\tau \gg T$ equation (46) reduces to

$$N = T \frac{dN}{dt} \left(\frac{t_0}{\tau} \right)^{1.2} \quad (48)$$

which is to say it finally acts as an instantaneous source of activity $N_0 = T \frac{dN}{dt}$.

The SNAP system is expected to operate for a time T of about a year. This means that N begins to behave like equation (48) only for times τ near a year. The $t^{-1.2}$ law is not accurate for times this long, however it will be used since it overestimates the dose rate and great accuracy at large τ is not required.

In order to make clear the similarities and differences between effects of nuclear weapons fallout and effects of reactor debris, these will be discussed in some detail.

B. Dose From a Nuclear Explosion

A somewhat simplified method for treating fallout from nuclear explosions is the following: We pick a critical radiation damage level corresponding to Dale's level A or B. Since Dale has already accounted for the physical effects produced by a uniform layer of radiating material, we may search for a critical damage level, R_c , in units of microcurie hours per square meter.

Approximately 20 percent of radiation damage in humans is permanent while the remaining 80 percent is eliminated (that is, recovered from) at a rate of about 8.33×10^{-3} per hour (20 percent per day).

The radiation damage level at time t is then the sum of the irrecoverable part and the recoverable part.

$$R = R_1 + R_2. \quad (49)$$

The irrecoverable part, R_1 , using equation (44) is given by

$$R_1(t) = .2 \int_{t_1}^t N dt = .2 N_o t_o^5 \left[\left(\frac{t_o}{t_1} \right)^{\frac{1}{5}} - \left(\frac{t_o}{t} \right)^{\frac{1}{5}} \right] \quad (50)$$

where t_1 is the time at which exposure begins.

The recoverable damage level $R_2(t)$ is the solution to the equation

$$R_2 = .8 N_o \left(\frac{t_o}{t} \right)^{1.2} - a R_2 \quad (51)$$

where $a = 8.33 \times 10^{-3}$. The solution to equation (51) is given by

$$R_2 = 4 N_o t_o (at_o)^{\frac{1}{5}} \left\{ \frac{e^{a(t_1-t)}}{(at_1)^{\frac{1}{5}}} \left[1 - S(at_1) \right] - \frac{1}{(at_2)^{\frac{1}{5}}} \left[1 - S(at_2) \right] \right\} \quad (52)$$

where

$$S(x) = \sum_{n=1}^{\infty} \frac{(-)^{(n+1)} x^n}{(n - \frac{1}{5}) (n - 1 - \frac{1}{5}) \dots \dots \dots (\frac{4}{5})} \quad (53)$$

For times when $t \gg 1$, the solution to (51) may be approximated by setting R_2 to zero to obtain

$$R_2(t_1) \approx \frac{.8N_o}{a} \left(\frac{t_o}{t_1} \right)^{1.2} = .8N_o t_o (at_o)^{\frac{1}{5}} (at_1)^{-1.2} \quad (54)$$

Figure E-8 shows a graph of R vs t for Dale's level A with exposure beginning at 1 hr, 12 hr, and 60 hr after detonation. As may be seen, all damage levels reach a maximum value of about $R_c = 2.3 \times 10^4 \mu c hr/m^2$ within the 10 week period covered by Dale's calculations.

It may be noted that the highest curve (60 hr initial exposure) does not cross R_c until the exposure time reaches a year. Thus Dale's conclusions would have been unchanged if he had assumed a year was required for the debris to leach out of the soil.

C. Dose From Dispersed Reactor Material

To be consistent with the policy of calculating the most dangerous case, we will assume a residence time for the active debris of one year. To simplify the calculation we will assume exposure begins one hour after shutdown. This is the most pessimistic case.

Assuming the reactor continuously generates power at a constant rate for a time T , the activity at time t later is given by equation (46)

$$N(t) = 5 \frac{dN_o}{dt} t_o \left[\left(\frac{t_o}{t} \right)^{\frac{1}{5}} - \left(\frac{t_o}{T+t} \right)^{\frac{1}{5}} \right] \quad (46)$$

The quantity $5 \frac{dN_o}{dt} t_o$ in curies may be calculated from the known power level of a reactor or, for better accuracy, it may be calculated by setting $N(t)$ equal to the activity rate of a SNAP reactor one hour after shutdown. This rate is available from N. A. Hart at Atomics International, Canoga Park, California.

The exposure levels R_1 and R_2 may be calculated in a manner similar to that of Dale if the fraction of the reactor's fission debris deposited per square meter, f , is known. Since 20 percent of the radiation is irrecoverable

$$R_1 = .2f \int_{t_1}^{t_2} \dot{N} dt$$

$$= f \frac{5}{4} \frac{dN_o}{dt} t_o \cdot 1.2 \left[(t_2)^{\frac{4}{5}} - (t_1)^{\frac{4}{5}} - (T + t_2)^{\frac{4}{5}} + (T + t_1)^{\frac{4}{5}} \right] \quad (55)$$

Because the decay rate given by equation (46) falls only a factor of 2.6 between one hour and the recovery period ($a^{-1} = 5$ days), it is sufficiently accurate for our purposes to approximate the recoverable damage, R_2 , by

$$R_2 = .8fN \Big|_{1 \text{ hr}} (t_2 - t_1) \text{ if } t_2 \ll 5 \text{ days} \quad (56)$$

and

$$R_2 = .8f \frac{\dot{N}(t_2)}{a} \text{ if } t_2 \gg 5 \text{ days} \quad (57)$$

using equation (46) for $N(t_2)$ and interpolating across the maximum.

Taking as an example the 30 kw system operating for one year, one obtains from Hart:

$$f 5 \frac{dN_o}{dt} t_o = 7.8 \times 10^{10} \text{ f } \mu \text{ c/m}^2 \quad (58)$$

Using equations (55), (56), (57) for calculation of R and taking

$$f_D = 2.7 \times 10^{-9} \text{ m}^{-2} \quad (59)$$

yields curve IV of Figure E-8. As can be seen, the dose level does not cross R_c until the leach-out time, 1 year, has passed, provided $f < f_D$.

The problem of calculating the danger level for a re-entering reactor is thus reduced to that of calculating the fraction of active material deposited per square meter at the surface of the earth.

For comparison, an acceptable value of f may be determined from an independent criterion. The dosage rate from nuclear weapon fallout one hour after detonation is currently taken as 1800 Roentgen/hour per kiloton/mile².³⁵ Equation (46) shows that the activity rate one hour after shutdown of a reactor is approximately the rate corresponding to an instantaneous yield equal to the energy released in 5 hours. The dose rate in Roentgen/hr for the 30 kw system is then about

$$\begin{aligned} R \left(\frac{\text{Rtgn}}{\text{hr}} \right) &= 1800 \frac{\text{Rtgn/hr}}{\text{kt/mile}^2} \times \frac{\text{reactors}}{\text{mile}^2} \times 5 \text{ hrs} \times \frac{\text{ergs}}{\text{kw hr}} \times \frac{\text{kw}}{\text{reactor}} \times \frac{\text{kt}}{\text{erg}} \\ &= 1.8 \times 10^3 \times f(\text{m}^{-2}) \times 2.59 \times 10^6 \frac{\text{m}^2}{\text{mile}^2} \times 5 \text{ hrs} \times 3.6 \times 10^{13} \frac{\text{ergs}}{\text{kw hr}} \\ &\times 30 \frac{\text{kw}}{\text{reactor}} \times 2.3 \times 10^{-20} \frac{\text{kt}}{\text{erg}} = 5.79 \times 10^5 f \text{ Roentgen/hr} \end{aligned} \quad (60)$$

The accepted population dosage³⁶ is 0.5 rem per year. Since γ radiation is most important, this is about 0.5 Roentgen per year.

We can use curve IV of Figure E-1 to calculate the upper limit of $f(f_{\text{NBS}})$ which yields 0.5 rem per year.

We found that a one hour activity level $f_A 7.8 \times 10^{10} \mu\text{c/m}^2$ yields a dose level R_1 of $2.3 \times 10^4 \mu\text{c hr/m}^2$ at the end of a year. But the

irrecoverable portion R_1 represents only 20% of the total dose. Therefore an activity level R in arbitrary units will yield a dose

$$D = R \cdot 5 \frac{2.3 \times 10^4}{f_A \cdot 1.60 \times 10^{11}} \text{ in one year.} \quad (61)$$

Setting $R = 5.79 \times 10^5 f_{\text{NBS}}$ Roentgen/hr from equation (60) and $D = 5 \times 10^{-1}$ Roentgen yields

$$f_{\text{NBS}} = 1.2 f_D \quad (62)$$

which is clearly consistent with Dale's level A.

5. THE PERMISSIBLE BERYLLIUM CONCENTRATION

Particles of beryllium in the proper size range will cause cumulative damage to the respiratory system.

A. L. Field of Westinghouse in a survey of the literature and conversations with workers in the field (in connection with contract AF 29(601)-5078) has arrived at a human tolerance of about 6×10^{-3} gm sec/m³ for Be particles resident in the atmosphere in the dangerous size region of $0.1 \text{ micron} < p < 5 \text{ micron}$.

In order to relate the tolerance level to the fraction of material deposited per unit area, f , consider the descending cloud of vertical thickness x and density ρ . If the time of exposure is T , the human tolerance requirement can be written

$$\rho T \leq (\rho T)_c = 6 \times 10^{-3} \text{ gm sec/m}^3 \quad (63)$$

The time required by the cloud of material to pass a given point is

$$T = \frac{x}{U} \quad (64)$$

FIG. E-8 RADIATION DOSAGE VERSUS TIME

where U is terminal velocity of the particulate material. The density is

$$\rho = \frac{Mf}{x} . \quad (65)$$

The product is then given by

$$\rho T = \frac{Mf}{U} . \quad (66)$$

The safety requirement on f is therefore

$$f \leq f_c = \frac{U}{M} (\rho T)_c = 6 \times 10^{-3} \frac{U}{M} . \quad (67)$$

This argument is in essentials identical to the simpler one of Haar given in Section 8. The line f_c in Figure E-7 is derived from (67), assuming a mass of 100 kg of B_e reflector which is greater than currently planned for the SNAP series of nuclear auxiliary power systems.

Comparison of f_c with the curves for f_{II} and $f_{I\Delta}$ in Figure E-7 shows a safety factor which varies from 10^4 to 10^5 over the dangerous range of particle size, in substantial agreement with Haar's evaluation.

6. CONCLUSIONS

A. The Radiation Hazard

Conclusions which may be drawn from this study are all embodied in Figure E-6.

- (1) Reference to the curve for f_{II} in Figure E-6 shows that in no case does a radiological hazard develop if the mean radius of U-Zr-H particles is less than ~ 45 microns. Because of the steep slope of the f_{II} vs p curve below the crossover point, the 45 micron number is relatively

insensitive to increases in expected values of f or decreases in acceptable values of f .

For example, if the power level of the reactor is raised a factor of ten to 300 kw, lowering the acceptable f values a factor of ten, the safe radius drops from 45u to 14u.

- (2) The value of p at which the f curve crosses the acceptable levels, $\sim 10^3 \mu$, has a more ambiguous meaning for three reasons:
 - (A) Literal interpretation of the curves indicates this to be the value of p above which measurable radiation damage is certain, but since the f curve is an upper limit, this value of radius becomes the lower limit on the particle size certain to produce measurable radiological hazard.
 - (B) Since p is proportional to f^2 in this range of particle sizes, a small shift in acceptable f (or error in f) produces a large change in the value of p at the crossing point.
 - (C) When the average particle radius becomes larger than a millimeter ($10^3 \mu$), the radiological hazard cannot be evaluated by the statistical arguments used here. The danger from millimeter or greater sized particles is largely dependent on proximity of a single particle rather than average distribution of the particles.
- (3) A curve of probability of occurrence of measurable radiation damage may be computed, assuming the relative orientation of the mean wind and trajectory is random. Figure E-9 is such a curve based on the formula for probability of measurable damage:

$$P = \begin{cases} \frac{2}{\pi} \sin^{-1} \left[f_1 / \frac{1}{2} (f_D + f_{NBS}) \right] & \text{if } f_{11} > \frac{1}{2} (f_D + f_{NBS}) \\ 0 & \text{if } f_{11} < \frac{1}{2} (f_D + f_{NBS}) \end{cases} \quad (68)$$

Because the region of interest lies in the transition region between fall Regions II and III, three curves are calculated corresponding to f_1 evaluated for Regions II, III, and the dotted interpolation shown in Figure E-7. It is felt that the principal effect of improvements in the calculation (i.e., variations in mean wind strength, inclusion of other effects, etc.) will be to move the $P = 0$ probability radius slightly as described in paragraph (1), and the $P = 1$ radius considerably. With the understanding that the radius for $P = 0$ may be in error by a factor of 2 or so and the $P = 1$ radius by a factor of 10, the shaded area of Figure E-9 represents the range of probability for measurable radiation damage to occur from reentry of the 30 kw nuclear auxiliary power system.

B. The Beryllium Hazard

A glance at Figure E-7 clearly demonstrates that no beryllium hazard will be associated with high altitude burn up. Such a large safety factor obviates the necessity of more detailed consideration.

The possibility of hazard connected with the successful reentry of large pieces of Be is not considered in this report.

7. THE EFFECT OF MOLECULAR DIFFUSION

One of several processes which influence the deposition of small particles released at high altitudes is molecular diffusion (as distinguished from turbulent eddy diffusion). It will be shown that for particles of micron or larger size this effect is generally negligible.

The assumption that particles diffuse to distances small compared to the scale height will be made in all of the following work. This assumption considerably simplifies the problem, allowing one to treat the vertical

FIG. E-9 PROBABILITY OF OCCURANCE OF MEASURABLE RADIATION DAMAGE VS MEAN PARTICLE RADIUS FOR SNAP - 8

fall velocity as a unique function of particle size and altitude when calculating the horizontal diffusion. This assumption will turn out to be true for particle sizes down to the molecular range. For the sake of simplicity the assumption will also be made that the particles of interest are no larger than about 50μ in radius. This assumption guarantees that the Reynolds number be less than 1 at all altitudes. Therefore, in the region of continuum flow one is dealing with Stokes flow. It will turn out that a particle of radius 50μ diffuses through a distance of only 30μ so that the theory, as developed, applies to all particles which are affected by diffusion.

Under the two assumptions made in the preceeding paragraph, there are two altitude regimes for any given particle radius. These are the (higher) molecular flow region and the (lower) Stoke's flow region. The regions are differentiated by the behavior of the fall velocity of the particle. For the accuracy required here these velocities are given by

$$U_s = \frac{2g\rho p^2}{9\eta} \text{ (Stokes)} \quad (69)$$

and

$$U_m = \frac{\lambda}{p} U_s \text{ (Molecular Flow)} \quad (70)$$

where U_s is the Stokes law free fall velocity, U_m the molecular flow fall velocity, g is gravitational acceleration, ρ is particle density, p is particle radius, λ the mean free path of an air molecule, and η is atmospheric viscosity, given by

$$\eta = \frac{.0448}{b^2} \sqrt{kmT} \quad (71)$$

k is Boltzman's constant, T temperature, m the mass of an air molecule, and b the radius of an air molecule.

The value of U_m given by equation (70) is 10% too low. This is not enough to seriously affect results.

The essential difference between the velocities given in expressions (69) and (70) is that in the case of Stokes flow the fall velocity is independent of altitude, while in the case of molecular flow the fall velocity depends on altitude through λ , the mean free path of an air molecule. There are therefore two different diffusion equations to be solved corresponding to the two regions of altitude, the dividing line being approximately the point in altitude at which $\lambda = p$, the radius of the particle.

As long as the particle is large compared to an air molecule the diffusion coefficient is given by

$$D = \frac{3}{8np^2} \sqrt{\frac{kT}{2\pi m}} \quad (72)$$

where n is the number density of air molecules.

A. Solution for the Molecular Flow Region

Assuming an exponential atmosphere and taking as a reference altitude the point at which $\lambda = p$ one can write (70) and (72) as

$$U_m = U_x e^{z/h_o} \quad (73)$$

and

$$D = D_o e^{z/h_o} \quad (74)$$

where D_o is the value of the diffusion coefficient at the reference altitude and z is the distance above the reference altitude.

If w is the probability of the particle being between horizontal

distances r and $r + dr$ then

$$\frac{\partial \omega}{\partial t} = D \nabla_r^2 \omega \quad (75)$$

Equation (75) may be reduced to the standard form of a diffusion equation if one writes

$$\frac{\partial}{\partial t} = U_m \frac{\partial}{\partial z} \quad (76)$$

and substitutes (73) and (74) for U_m and D to obtain

$$\frac{\partial \omega}{\partial z} = \frac{D_o}{U_s} \nabla_r^2 \omega \quad (77)$$

where now D_o/U_s is a constant.

The line source solution to (77) is

$$\omega = \frac{U_s e^{-\frac{r^2 U_s}{4 D_o z}}}{2 D_o z} \quad (78)$$

where r is the horizontal distance from the center and z is the distance of fall.

Equation (78) will be regarded as accurate until the altitude is reached at which $\lambda = p$.

B. Solution for Stokes Region

As soon as the particle reaches the altitude at which $\lambda = p$ we assume equation (69) to hold so that U is no longer a function of altitude.

Now the equation to be solved is

$$\frac{\partial \omega}{\partial t} = D_o e^{z/h_o} \nabla_r^2 \omega \quad (79)$$

but since U is now a constant $Z = -Ut$ and (79) can be written

$$\frac{\partial \omega}{\partial t} = D_o e^{-U_s t/h_o} \nabla_r^2 \omega \quad (80)$$

Define a new variable

$$\tau \equiv \frac{h_o}{U_s} (1 - e^{-Ut/h_o}) \quad (81)$$

so that

$$\frac{\partial}{\partial t} = \frac{d\tau}{dt} \frac{\partial}{\partial \tau} = e^{-Ut/h_o} \frac{\partial}{\partial \tau} \quad (82)$$

Substituting (82) into (80) yields the standard form of the diffusion equation

$$\frac{\partial \omega}{\partial \tau} = D_o \nabla_r^2 \omega \quad (83)$$

The line source solution to (83) is

$$\omega = \frac{e^{-\frac{r^2}{4D_o\tau}}}{2D_o\tau} \quad (84)$$

Converting back to time for a variable one has

$$w = \frac{U_s}{2D_o h_o (1 - e^{-Ut/h_o})} \exp \frac{-U_s r^2}{4D_o h_o (1 - e^{-Ut/h_o})} \quad (85)$$

as t gets large, or after the particle has fallen through more than one scale height, equation (85) reduces to

$$w = \frac{U_s e^{-\frac{U_s r^2}{4D_o h_o}}}{2D_o h_o} \quad (86)$$

C. Conclusion

The only difference between equations (86) and (78) is the substitution of h_o for z . To calculate the diffusion at the ground for particles of a given radius one therefore has two prescriptions (a) if the altitude of release is in the Stokes region, set D_o equal to the diffusion coefficient at release altitude and apply equation (86), (b) if the altitude of release is in the molecular flow region apply equation (78) with z equal to the distance from altitude of release to one scale height below the point at which $\lambda = p$, and set D_o equal to the diffusion coefficient at the altitude for which $\lambda = p$.

(1) Numerical Evaluation

A convenient measure of the effect of diffusion is the value of r at which w has fallen to $\frac{1}{e}$ of its central value when the particle has reached the ground. If a value of r is chosen to represent significant diffusion then one may calculate the largest particle radius which is diffused significantly as a function of altitude of release.

Choosing r equal to one meter as representing significant diffusion, the exponent in (86) or (78) may be set equal to 1 to yield the required relation.

$$\frac{10^4 U_s}{4D_o d} = 1 \quad (87)$$

where d is either h_0 or z as determined by prescriptions (a) or (b).

Substituting (69) for U_s , (71) for η , and (72) for D_0 in (87) yields

$$p = \left(\frac{4.20 \times 10^{-20} TD}{\rho g} \right)^{1/3} \quad (88)$$

evaluating D_0 and U_0 at the point where $\lambda = p$, that is, assuming release to be in the molecular flow region. Assuming $T = 280^\circ K$, $\rho = 6 \text{ gm/cm}^3$, and $g = 950 \text{ cm/sec}^2$ gives

$$p = 1.27 \times 10^{-7} (d)^{1/3} \text{ cm} \quad (89)$$

Equation (89) may be solved iteratively, one guesses a value of d for a given altitude of release, calculates p , then looks at a graph of the line of altitude for which $\lambda = p$ versus p to determine a second guess on d , etc. By this iterative procedure one arrives at Table E-1.

TABLE E-1

The maximum value of particle radius for which molecular diffusion is significant versus altitude of release. Diffusion through 1 meter deemed significant.

$h(\text{km})$	30	40	50	60	70	80	90	100
$p(\mu)$	.165	.185	.202	.217	.231	.241	.252	.263

As can be seen from the table, diffusion is unimportant for particle radii greater than .25 microns at all altitudes of release to 100 km.

8. BERYLLIUM FALLOUT HAZARD

In this note we calculate the area over which the beryllium reflector must be distributed, if the entire reflector were broken up into particles of sizes that would be hazardous if ingested. This area is then compared with the minimum area of dispersion resulting from an ordinary re-entry trajectory and atmospheric wind. The results are obtained in terms of a safety factor, which is the ratio of the actual wind distribution area, to the maximum of the area for which a hazard would exist.

The maximum permissible exposure for humans inhabiting a Be contaminated atmosphere is given in units of the density of the Be in the atmosphere and the time of exposure. The Westinghouse study indicates that this quantity is

$$6 \times 10^{-3} \text{ gm sec/m}^3. \quad (90)$$

The Be reflector of the reactor weighs on the order of 100 lbs, or approximately $\frac{10^5}{2}$ gm. For the "no-hazard" case the time of exposure t and the volume V over which this amount of Be is distributed must be such that the quantity in Eq. (90) would not be exceeded, that is for safety we must satisfy the relation,

$$\frac{10^5}{2} \text{ gm} \frac{t}{V} < 6 \times 10^{-3} \text{ gm sec/m}^3 \quad (91)$$

For particles of a given size, V may be obtained from the product of the dispersal area A , (corresponding to a particular trajectory and ordinary atmospheric winds) and a thickness L which is the altitude to which the atmosphere is contaminated with Be when significant amounts of Be reach ground level (the quantity L is therefore essentially a diffusion length). The exposure time t is the time it takes for a Be particle to diffuse through the distance L . Thus we have

$$V = AL, \quad t = \frac{L}{v},$$

so that

$$\frac{t}{V} = \frac{1}{Av}, \quad (92)$$

where v is the velocity of fall at the surface. Using the relations of Eq. (92) it is seen that Eq. (91) may be written in terms of a requirement on the area of dispersal,

$$A > \frac{10^7}{V}. \quad (93)$$

Rewriting Eq. (93) we obtain the condition on the safety factor S .

$$S = \frac{A}{10^7/v} > 1. \quad (94)$$

The dispersal area of course will depend on reentry angle. For ordinary reentry configuration, if burnup occurs, it would presumably occur over a trajectory T of at least several hundred kilometers of travel in the upper atmosphere; $T > 10^5$ m. The wind dispersal distance r may be estimated to be

$$r = \int W dt = \int_H^0 \frac{W}{v} dz \quad (95)$$

where W is the wind velocity, v the fall velocity for the Be particle and H the height at release. The particle sizes that could be dangerous vary between 0.1μ to 5μ . For these particles the fall velocity near ground level is approximately the Stokes value. As a lower limit we neglect wind effects above one scale height. Using a constant average

wind velocity $w > 10$ m/sec and approximately 10 km for the scale height, and a burnup trajectory $T > 100$ km, the area of dispersal becomes

$$A = r \times T > \frac{10 \times 10^4 \times 10^5}{v}, \text{ or} \quad (96)$$

$$A > \frac{10^{10}}{v}$$

The safety factor defined by Eq. (94) then becomes

$$S > 10^3 \quad (97)$$

Note that this result is for the worst possible case.

Some question might be raised regarding the validity of Eq. (95). In Eq. (95) the dispersal mechanism is assumed to be only the particle rate of fall: the large particles on the average fall faster than the small particles. That is, for a particle of cross sectional area a we could define a distance δr which on the average is the additional distance the wind will carry a particle of size $a + \delta a$,

$$\delta r = \left| \delta a \frac{\partial}{\partial a} r \right| \approx \frac{\delta a}{a} r,$$

for Stokes fall.

Now δa is of the order of $(5\mu)^2$. That is $\delta a \leq a$. Thus we obtain

$$\delta r \leq r,$$

and our estimate is still on the conservative side when we approximate the dispersal length by the quantity r as in Eq. (95).

APPENDIX F

REENTRY OF NAP REACTOR BERYLLIUM REFLECTORS; EFFECT OF
FAST NEUTRON PRODUCED INTERSTITIAL HELIUM NUCLEI1. INTRODUCTION

The reentry of beryllium neutron reflectors may engender a hazard to health caused by the toxicity of this metal if the beryllium reaches the surface of the earth in sufficient concentrations. An optimum situation, therefore, would be one in which the beryllium would be totally melted and widely dispersed as small particles at high altitudes. In this event, concentrations reaching the earth's surface would (a priori) be biologically nonhazardous. This report discusses the effect of interstitial helium nuclei produced in the beryllium by fast neutrons as a possible mechanism of beryllium degradation and hence a contributing mechanism of dispersal.

2. PROPERTIES OF BERYLLIUMA. Mechanical Properties (Vacuum Hot-Pressed Powder)

(1)	Tensile strength						
	Temperature(° C)	20	100	200	400	600	800
	Strength(lb/in ²)	50000	48000	43000	28000	20000	6000
(2)	Young's Modulus						
	Temperature(° C)	20	425	540	650	730	875
	Modulus(lb/in x 10 ⁶)	43	41	37	28	14	11
(3)	Density: 1.8477 gm/cm ³						

B. Thermal Properties

(1)	Melting point:	1284° C (2343° F)
(2)	Boiling point:	2970° C (5378° F)
(3)	Vapor Pressure:	
		$\log P = \left[6.494 - \frac{11.710}{T(^{\circ}K)} \right] \text{ mmHg}$
(4)	Latent heat of fusion:	2.8K cal/gm-atom

(5)	Specific heat: (cal/gm-°C)						
	Temperature(°C)	20	100	200	400	600	800 900
	Specific heat:	0.43	0.51	0.57	0.64	0.69	0.75 0.77
(6)	Thermal conductivity (cal/cm ² - sec - °C[cm])						
	Temperature (°C)	50	200	400	600		
	Conductivity	0.46	0.34	0.27	0.24		
(7)	Mean Thermal Expansion						
	Temperature Range (°C)		25-200	25-400	25-700	25-1000	
	Linear Coefficient x 10 ⁻⁶		13.5	15.2	17.2	18.8	

C. Nuclear Properties

- (1) Mass number: Be⁹ - 100 percent abundance

Under neutron bombardment the following reactions occur:

$\sigma = 600 \text{ mb}$, neutron energies $> 2.70 \text{ Mev}$

$\sigma = 80 \text{ mb}$, effective neutron energies $> 2.41 \text{ Mev}$

3. EFFECT OF NEUTRON IRRADIATION OF BERYLLIUM

The sizable cross sections for production of He⁴ under fast neutron bombardment of beryllium indicate that interstitial helium nuclei will be produced within the beryllium. For a total neutron flux of 10²¹ nvt, 2 cm³ (at STP) of helium is produced per cm³ of Be⁹. If the irradiated beryllium is subsequently heated, the helium will migrate to edges of crystal planes and will form bubbles by agglomeration, causing embrittlement of the beryllium and an accompanying change in volume. This process of agglomeration does not occur to any significant extent below a temperature of 500°C. At temperatures greater than this value, the rate of agglomeration (and hence expansion and embrittlement) is a function of the temperature and the volume of interstitially entrapped gas. At a

temperature of 800° C gross changes in density can occur because of the growth of He⁴ bubbles and the resultant internal gas pressure. For a total flux of 10²² nvt an average of 23 cm³ He⁴ per cm³ Be⁹ was obtained. Raising the temperature of the specimen to 800° C permits migration of the interstitial gas and a subsequent increase of 30 percent in the volume of the specimen because of the pressure of the entrapped gas (expansion proceeds until the pressure of the trapped gas bubbles is equal to the yield strength of the specimen). The entrapped gas has only a minor effect on the yield strength but it does cause embrittlement.

The total weight of beryllium utilized as a neutron reflector in the SNAP reactor is approximately 126 pounds. The reflector is about 2.3 inches thick and surrounds the 16 inch by 14 inch diameter reactor core. This mass of beryllium will be subjected to a neutron flux of 3.1×10^{11} n/cm² -sec at a temperature somewhat above 500° C.

After a year's operation of the reactor a total neutron flux of (365 x 24 x 3600s) (3.1×10^{11} n/cm² -sec) = 9.8×10^{18} nvt will have been incident upon the beryllium reflector. If we assume that these are all high energy neutrons and the entire reflector receives this dose, the resultant total gas volume will be

$$\left[\frac{2 \text{ cm}^3 \text{ He}^4 / \text{cm}^3 \text{ Be}^9}{10^{21} \text{ nvt}} \right] \left[9.8 \times 10^{18} \text{ nvt} \right] \approx 0.02 \text{ cm}^3 \text{ He}^4 / \text{cm}^3 \text{ Be}^9$$

This volume of gas (2×10^{-2} cm³ He⁴ / cm³ Be⁹) can slowly migrate from its initial interstitial location to edges of crystal planes if the temperature is great enough. At the temperature to be expected this migration will be slow but significant. However, the total gas volume involved is too small to produce significant changes in density, yield strength, or brittleness of the beryllium. The irradiated reflector metal, therefore, should upon reentry react in the same manner as unirradiated beryllium. The beryllium reflectors must be subjected to a total flux greater by two or three orders of magnitude before appreciable changes in the physical properties can be expected.

APPENDIX G

DISPERSAL OF BERYLLIUM SHIELDS

1. INTRODUCTION

The object of this analysis is to inquire into the feasibility of causing the dispersal, on reentry, of the reactor's beryllium shields by the evolution of quantities of gas within the beryllium. The gases evolved may come from the action of the fast neutron bombardment of the shield during the operating lifetime of the reactor or from a suitable additive placed in the beryllium for this purpose which, on heating during reentry, would decompose.

2. NEUTRON INDUCED GASES

The production of helium within the beryllium shield of a NAP reactor has been considered in Appendix F, the conclusion being that the total volume of helium produced during a twelve month operating period of the reactor is insufficient to produce significant changes in the density, yield strength or brittleness of the beryllium.

3. BERYLLIUM CONTAINING ADDITIVES

When the beryllium shields are subjected to reentry heating, the additive must decompose forming a gas. This gas, depending on the initial distribution of the additive, will be either uniformly dispersed throughout the lattice of the metal or will be concentrated in small pockets within the metal.

The effect of dissolved gases on the annealing of beryllium appears to have received little attention; however, the effect of neutron induced helium on beryllium has been the subject of several studies. Rich, Walten and Bames^{2,3} found that beryllium subjected to fast neutron bombardment contained helium dispersed as fine bubbles within the metal. On heating, the number of bubbles decreased while the average size of the bubbles

increased. A hardening of the beryllium takes place when it is irradiated below the temperature at which thermally produced vacancies diffuse. On heating the irradiated metal, Rich, et. al., found that the beryllium begins to recover its original ductility only after annealing at temperatures above 950°C , although its hardness is reduced after annealing for one hour at 800°C . Both Rich, Walten and Rich, Ridding and Bames²⁴ note that the lowest temperature at which the hardness of the metal is appreciably reduced is near that at which the first measurable increase in volume of the beryllium takes place. In addition to the hardening caused by the radiation damage, the helium in solution also hardens the beryllium. Such hardening has also been observed (Rich, Ridding and Bames) in beryllium containing, in solution, injected helium. Since the beryllium recovers its initial mechanical properties only after annealing at elevated temperatures, when most of the helium is contained in bubbles within the metal rather than in solution, a considerable amount of the hardening of the beryllium is due to the presence of dissolved helium. Therefore the behavior of irradiated beryllium is largely determined by the presence of helium within the metal and thus one may expect beryllium containing gases, evolved from additives to react in much the same way on annealing as irradiated beryllium containing helium.

Ells and Perryman²⁵ have found that after annealing the irradiated beryllium for 2 1/2 hours at 700°C the helium has diffused to the grain boundaries. Detectable swelling of the metal was not observed below a threshold temperature of greater than 400°C ; the temperature depending on the gas content. At low annealing temperatures, the degree of swelling at a particular annealing temperature increases with the gas content of the metal, which is consistent with the assumption that swelling is due to gas pressure in the voids. The ratio of swelling of high-flux material to low-flux material decreases with increasing temperature indicating that there is some upper limit of expansion at which interconnected porosity

will permit escape of the entrapped gas. In this connection, Ells and Perryman observed that a detectable amount of helium was released from the beryllium on annealing for two hours at 800°C ; the density change being 10%.

Rich, Ridding and Barnes²⁴, in a study of the behavior during annealing of beryllium containing neutron induced helium, found that there was a negligible volume increase until the beryllium had been heated to about 600°C for one hour and that the volume increase, although initially differing widely between samples, eventually reaches about 30% in all samples (after heating to about $1,000^{\circ}\text{C}$ for one hour) with little further increase. The space occupied per helium atom increases when bubbles are formed in the metal, the formation of bubbles is governed by the diffusion of vacancies, mainly from the grain boundaries while the rate of volume increase will depend upon the temperature, the helium content, and the grain size. If the beryllium contains voids, as is suggested by the wide range of initial density, then this will also influence the final density change. However, constancy of the maximum volume increase (30%) for samples having widely different gas contents, suggests that in this condition the sample is so permeable that helium can leak from it and there is no further volume increase. The only sample analyzed by Rich, Ridding and Barnes contained very little helium after a 25% volume increase. Similar maximum volume increases have been noted in annealing uranium containing fission gases by Barnes, et. al.²⁶, who observed a 30% maximum volume increase and by Churchman, Barnes and Gottrel²⁷ who observed a 23% swelling of the uranium. The similarity of the maxima for different metals containing very different amounts of dissolved gases suggests that this is the value at which the majority of the internal bubbles touch and release their gas. This value would be geometrically determined and would be independent of the material and the amount of gas which it contains.

The above results have all been obtained experimentally by studying the effects of helium induced by fast neutron bombardment. However, the conclusion of Rich, et. al.²⁴ that the maximum swelling of the material is largely independent of the mechanical properties of the metal, a conclusion which is supported by the results of studies on uranium where the induced gases were obtained by nuclear fission, indicates that the existence of the upper limit to the swelling is independent of any changes induced in the mechanical properties of the metal by the fast neutron bombardment and hence is independent of the mode of evolution of the gas within the metal.

Thus, one cannot expect to effect the dispersal of the beryllium by the evolution of an inert gas within the metal and subsequent heating, no matter how much gas is evolved or for how long the subsequent annealing process is carried out. In the actual reentry case, there are limits to the amount and type of gas which can be evolved, (since the addition of the additive should be kept as small as possible in order to prevent its adversely affecting the nuclear properties of the shield) and to the time of annealing, which would be determined by reentry heating. If a chemically reactive gas is released within the beryllium, then its mechanical properties may be changed; however, since the maximum swelling of the material is determined by geometrical considerations, it could not be dispersed by the build-up of gas pressure within the shield.

4. CONCLUSIONS

The complete mechanism of the swelling, on annealing, of metal containing dissolved gases is not well understood. However, there is sufficiently good agreement in the experimental results to indicate that an upper limit to the swelling exists. None of the experimental work in this field indicates that there is any tendency for the metal to fragment. Thus it appears to be most unlikely that gases released within the beryllium during reentry heating will greatly aid in the dispersal of the reactor reflectors.

APPENDIX H

ATMOSPHERIC PARAMETERS OF INTEREST

1. INTRODUCTION

Information pertinent to the studies conducted by the various contractors is assembled here. Additional data are available in the included references.

2. ATMOSPHERIC PROPERTIES FROM THE ARDC MODEL ATMOSPHERE, 1959

The ARDC Model Atmosphere is a representation of atmospheric physical properties prepared by the Geophysics Research Directorate, Air Force Cambridge Research Center. The 1959 model is a revised edition of the 1956 model. The atmospheric properties of greatest interest are presented graphically as function of altitude in metric units. These and additional properties are presented in both graphical and tabular form for metric and English units in the ARDC report.³⁷

The kinetic temperature is shown in Figure H-1. The pressure is shown in Figure H-2 for two common units, kilograms of force per square meter and millimeters of mercury. The density is shown in Figure H-3. The mean particle speed is defined as the arithmetic average of the Maxwellian distribution of all air molecules within a given elemental volume, and is shown in Figure H-4. The mean free path is shown in Figure H-5.

The velocity of sound, coefficient of thermal conductivity, and coefficient of viscosity are shown in Figures H-6, H-7 and H-8. These values are terminated by ARDC at 90 km because at higher altitudes the mean free path is larger than the dimensions of likely vehicles and the

values will thus also be dependent upon vehicle dimensions.

The mean molecular weight is relatively constant in the altitude regime being considered. It has a value of 28.966 at sea level and is constant to 75 km. It decreases to 28.90 at 100 km, 28.71 at 120 km, 28.45 at 140 km, and 28.04 at 160 km.

3. WINDS

Winds below 100,000 feet are considered first. The prevailing wind in the middle latitudes is westerly, increasing in velocity from the surface to the tropopause (30,000 to 40,000 feet) and then decreasing with altitude to about 80,000 feet. Above 80,000 feet, the wind speed slowly increases again, generally remaining westerly in direction in the winter and becoming easterly in summer. Synthetic wind speed profiles and maximum shears which are exceeded 1, 5, 10, 20, and 50% of the time during winter are shown in Figure H-9 for the windiest area in the United States and in Figure H-10 for the calmest area in the United States.²

Only a limited number of wind observations are available for altitudes above 100,000 feet. It is evident that there is a seasonal reversal from easterly winds in the summer (April through September) to westerly winds during the winter (October through March) between 100,000 and 230,000 feet. Above 230,000 feet, westerly winds occur in the summer over middle latitudes and there is a strong indication of easterlies over low and high latitudes. In the winter, westerlies are also present over the middle latitudes at these levels. Estimated mean seasonal zonal wind components for altitudes between 30 and 100 km are shown in Figures H-11 and H-12.³ Rocket-grenade experiments have indicated a range of 35 to 104 m/sec in winter and 5 to 9 m/sec in summer in the altitude regime from 30 to 70 km with maximum speeds occurring between 50 and 60 km.⁴

4. ATMOSPHERIC COMPOSITION

The relative composition of the permanent atmospheric gases is considered to be invariant up to 90 km. The composition is shown in Table H-1.⁴¹ The distribution of these gases with altitude is shown in Figure H-13.

TABLE H-1

COMPOSITION OF DRY ATMOSPHERE UP TO ABOUT 90 KM

<u>Constituent</u>	<u>% Volume</u>	<u>% Weight</u>	<u>Reduced Thickness (atm-cm. NTP)</u>
Nitrogen	78.088	75.527	624,343.
Oxygen	20.949	23.143	167,495.
Argon	0.930	1.282	7,435.69
Carbon Dioxide	0.030	0.0456	239.861
Neon	1.8×10^{-3}	1.25×10^{-3}	14.3817
Helium	5.24×10^{-4}	7.24×10^{-5}	4.18957
Methane	1.4×10^{-4}	7.75×10^{-5}	1.11935
Krypton	1.14×10^{-4}	3.30×10^{-4}	0.911472
Nitrous Oxide	5×10^{-5}	7.6×10^{-5}	0.399768
Xenon	8.6×10^{-6}	3.90×10^{-5}	0.068760
Hydrogen	5×10^{-5}	3.48×10^{-6}	0.399768

FIG. H-1 KINETIC TEMPERATURE VERSUS ALTITUDE.

FIG.H-2 PRESSURE VERSUS ALTITUDE

FIG.H-3 MASS DENSITY VERSUS ALTITUDE.

FIG.H-4 MEAN PARTICLE SPEED VERSUS ALTITUDE.

FIG.H-5 MEAN FREE PATH (METERS) VERSUS ALTITUDE.

FIG.H-6 : SPEED OF SOUND VERSUS ALTITUDE .

FIG:H-7 : COEFFICIENT OF THERMAL CONDUCTIVITY
VERSUS ALTITUDE.

FIG.H-8: COEFFICIENT OF VISCOSITY VERSUS ALTITUDE.

FIG. H-9 : SYNTHETIC WIND SPEED PROFILES EXCEEDED 1, 5, 10, 20, AND 50 % OF THE WINTER FOR THE WINDIEST AREA (NORTHEASTERN PART) OF THE UNITED STATES.

FIG.H-10: SYNTHETIC WIND PROFILES EXCEEDED 1,5,10, 20,AND 50% OF THE WINTER FOR THE CALMEST AREA (NORTHWESTERN PART) OF THE UNITED STATES.

+ WESTERLY }
 - EASTERLY } METERS/SEC

FIG.H-II NORTHERN HEMISPHERE WINTER ZONAL WIND DISTRIBUTION.

+ WESTERLY } METERS/SEC
- EASTERLY }

FIG.H-12 NORTHERN HEMISPHERE SUMMER ZONAL
WIND DISTRIBUTION.

FIG.H-13 VERTICAL DISTRIBUTION OF ATMOSPHERIC CONSTITUENTS FROM SURFACE TO 250 KM.

APPENDIX I

VAPORIZATION OF HYPERSONIC PARTICLES1. INTRODUCTION

Bodies passing through the upper atmosphere at hypersonic speeds vaporize rapidly as they slow-down. Some bodies vaporize or ablate completely before reaching terminal velocity. This is the case for most meteorites. Others reach terminal velocity or impact on the earth before vaporizing, which is true for nose cones and large meteorites. The present analysis employs a simple estimate of the heat transfer to determine the velocity and size of vaporizing particles. This information is useful when studying several reentry problems. The rate of disintegration and trajectory of meteorites can be estimated. This analysis was originally derived to investigate the hazards associated with the destruction of nuclear reactors through aerodynamic heating when reentering the earth's atmosphere from satellite orbits. In this instance it is desirable for all the droplets of nuclear material to vaporize completely.

2. THEORY

Initially several assumptions are made in order to obtain an analytic result. Later, these assumptions will be explored quantitatively. Radiative heat transfer is assumed small compared to the convective heat transfer; therefore, the mass loss of a particle can be equated to the heat transfer.

$$-\frac{dm}{dt} = \frac{q_c}{\Delta H} \quad (1)$$

where m = mass of particle

ΔH = heat of ablation

q_c = convective heat transfer

t = time

The convective heat transfer rate to a particle is expressed by

$$q_c = \frac{\alpha \rho v^3 A}{2} \quad (2)$$

where ρ = ambient density

v = velocity of particle

A = frontal area of particle

α = heat transfer coefficient

The term α is not the usual heat transfer coefficient; but is the fraction of the kinetic energy of the air in the tube swept by the particle, which is absorbed by the particle. Under free molecule flow conditions, α will approach one.

The usual aerodynamic drag equation can be written as

$$m \frac{dv}{dt} = - \frac{\rho v^2 C_D A}{2} \quad (3)$$

When the particle is a sphere of radius r and density ρ_p

$$\frac{dv}{dt} = - \frac{3 C_D \rho v^2}{8 \rho_p r} \quad (4)$$

Combining equations (1), (2), (4) and integrating gives

$$a(V^2 - 1) = \log_e R \quad (5)$$

where v_o and r_o are initial values, $V = \frac{v}{v_o}$, $R = \frac{r}{r_o}$, and $a = \frac{\alpha v_o^2}{6 C_D \Delta H}$.

The particle radius as computed by equation (5) is plotted versus velocity in Figure I-1 for values of "a" between 0.1 and 10. For values of "a" less than 0.1 the particle slows down without appreciable mass loss. For values of "a" greater than 10 the final particle radius is less than 10^{-3} times the initial radius. The final radius cannot be computed precisely

because at low velocities the particle radiates much of the energy transferred to it by convection. The error introduced by radiation will be estimated in a later section.

The distance travelled by the particle as a function of its final velocity can be obtained by combining equations (4) and (5).

$$bX = \int_1^V \frac{e^{a(V^2-1)}}{V} dV \quad (6)$$

where $X = \frac{x}{r_o}$, $b = \frac{3C_D \rho}{8\rho_p}$, and x is the distance from the particle's starting point in the atmosphere. The integral in equation (6) was evaluated by representing e^{aV^2} by the series $1 + \sum_{n=1}^n \frac{(aV^2)^n}{n!}$, which gives

$$bX = e^{-a} \left\{ \log_e \frac{1}{V} + \sum_{n=1}^n \frac{a^n (1-V^{2n})}{2n : n!} \right\} \quad (7)$$

The dimensionless distance, bX is presented as a function of V in Figure I-2. The distance travelled by the particle, x , is measured from the point at which the particle is first exposed to the undisturbed atmosphere. Often it is desired to know how far the particle is behind a heavier parent body, which doesn't slow down appreciably. The distance behind the parent body ℓ , is the difference in the distance travelled by the body and the particle.

$$d\ell = (v_o - v)dt \quad (8)$$

The dimensionless length, $L = \frac{\ell}{r_o}$, is obtained by solving equations (4),

(5) and (8) simultaneously, which gives

$$bL = - \int_1^V \frac{(1-v)e^{a(v^2-1)}}{V^2} dV \quad (9)$$

FIG.I-1 Vaporization of Hypersonic Particles

FIG.I-2 Distance Travelled by Particles

The approximation $e^{aV^2} = 1 + aV^2$ was used to evaluate the integral in equation (9). The results are given in Figure I-3. One point on each curve was checked by graphical integration of equation (9). The maximum difference was 20%, which is small compared to errors caused by uncertainties in α and q_r .

3. EXAMPLES

The example cited previously will be used to illustrate a practical case. It is desired to know the ultimate disposition of molten droplets of radioactive Zr-U from a fuel rod. A satellite-borne nuclear reactor, hopefully, breaks up at altitudes above 200,000 feet. It is presumed here that the separated fuel rods are melted by aerodynamic heating and deposit droplets of about 100 microns diameter in their wakes. Will the droplets burn-up and how far will they travel in doing so? The following data is applicable.

r_o	= 50 microns
v_o	= 26,000 feet per second
ΔH	= 10^3 Btu per lb
ρ	= 10^{-7} g/cm ³
ρ_p	= 8 g/cm ³
C_D	= 1
α	= 1

The values of the dimensionless parameters are $a = 4.5$ and $b = 4.7 \times 10^{-9}$. Zirconium, the major constituent in the rod, will melt at 1900°C and vaporize at 2900°C under standard conditions. The particle velocity producing a stagnation temperature equal to the vaporization temperature is 13,000 fps. To be conservative it will be assumed that vaporization ceases when the particle velocity reaches 15,000 feet per second. Figure I-1 shows the particle radius will be reduced to 0.05 of its original value, or 2.5 microns. Figure I-2 shows the particle will have travelled about 3km. This means the particles are small, and still at a high altitude; and will, therefore,

FIG.I-3 Velocity of Particles Behind a Heavy Hypersonic Body

take considerable time to settle to the earth. This analysis assumes the vaporization rates are rapid to such a degree that zirconium does not oxidize to the refractory ZrO_2 .

For a second example we ask the question, can particles of ablated material exist at large distances behind reentering bodies?

$$\begin{aligned} v_o &= 23,000 \text{ ft/sec} \\ \Delta H &= 2 \times 10^3 \text{ Btu/lb} \\ \rho &= 10^{-5} \text{ g/cm}^3 \text{ (100,000 feet altitude)} \\ \rho_p &= 10 \text{ g/cm}^3 \\ C_D &= 1 \\ \alpha &= 10^{-1} \end{aligned}$$

The value of the dimensionless parameters are $a = 0.37$ and $b = 0.38 \times 10^{-6}$. The final velocity is assumed to be 15,000 feet per second as in the first example. The initial particle size required to survive to a point 1000 feet behind the reentry body is approximately 10^{-2} inches. The particles lose 20% of their diameter in passing through the wake. There are larger uncertainties in the value of α and radiative heat transfer than in the values of the other data used. If α is larger than the assumed value of 10^{-1} , the particles will ablate faster, thereby, requiring larger particles. Radiation, which was neglected, however, will tend to reduce the ablation rate.

4. ANALYSIS OF ERRORS

Assuming α and the emissivity, σ , are unity, a minimum ratio of the convective to radiative heat transfer can be estimated as follows:

$$\frac{q_c}{q_r} = \frac{\rho v^3}{8\sigma T^4}$$

$v = 7 \text{ km/sec}$ (minimum reentry velocity expected)

$T = 3800^\circ \text{K}$ (sublimation temperature of carbon, which is highest surface temperature expected)

$\rho = 10^{-7} \text{ g/cm}^3$ (density at 230,000 feet altitude)

The ratio $\frac{q_c}{q_r}$ is 3.3 which means the vaporization rate is over-estimated by approximately 30% for the above conditions. This indicates that equation (1) predicts transfer rates adequately at altitudes below 200,000 feet for particles travelling at ICBM reentry velocities or greater. This altitude is obviously not unique, but changes with velocity, surface temperature and particle size.

If heat transfer rates computed by means of relations developed for hypersonic continuum flow, such as that by Detra⁴, are compared with that of equation (2), it is found that the results agree only at an altitude near which, the body radius is equal to the ambient mean free path. In one example, the particle size was 10^{-2} inches, and the altitude was 100,000 feet. The mean free path at 100,000 feet is 10^{-4} inches; therefore, α would equal 1 for a 10^{-4} inch body. Since the heat transfer varies as $r^{-\frac{1}{2}}$ in continuum flow, one might reasonably expect α to be 10^{-1} for a 10^{-2} inch body.

A third factor not previously mentioned is the initial condition of the particle. If the particle is cool when ejected into the atmosphere its ΔH will be increased. However, particles created by ablation or mechanical disintegration of a reentry object will, in general, not be ejected cleanly into the atmosphere. Most particles will be carried along by the hot wake following the parent body before entering the undisturbed atmosphere. The wake gases are hot and will vaporize materials with low vaporization temperature even in the absence of a hypersonic gas stream impinging on the particle. The particle will not be slowed while in the wake as much as it would have been if it had been in undisturbed air; therefore, the total vaporization may be greater than predicted by the present analysis. However, refractory particles may be decelerated to some degree by the wake without being vaporized. In this case, the total vaporization will be less. No attempt was made to assess the enhanced or decreased vaporization caused by the wake of the parent body.

Note Bene

This analysis was initiated by Dr. W. W. Short under AF 29(601)-4893 and was completed by him under other funding at the Defense Research Corporation, Santa Barbara, California. We wish to express our appreciation to Dr. Short for his extra effort in assisting the completion of this report.

APPENDIX J

INVESTIGATION OF RECOVERED FRAGMENTS FROM ATLAS 109-D
BOOSTER

1. PREFACE

This document serves a dual purpose: (1) It reports some of the results to date from a continuing investigation of Atlas 109-D fragments which were recovered in the Republic of South Africa. In this sense it constitutes a progress report, the detailed contents of which are subject to possible change on completion of the investigation. (2) It includes all of the work relative to the 109-D fragments which has been performed under support of Contract AF 29(601)-4893 with Air Force Special Weapons Center, Kirtland AFB, New Mexico. From the standpoint of subject contract, the original interest in the 109-D fragments was to obtain information relative to the mode of reentry whereby relatively large portions of the unprotected vehicle successfully reached earth impact from a naturally decaying orbit. In this sense, a plausible explanation of the successful reentry of portions of Atlas 109-D would have direct applicability to the subject contract. However, the preliminary investigation of the fragments indicated that a detailed study program would be required to obtain the maximum information of interest to space technology, and such a program falls outside the scope of the work statement and funding of the present contract.

In view of the above, this document contains data and test results relative to Atlas vehicle 109-D available at time of writing with a minimum of discussion and/or explanations for the events reported and conclusions drawn on the basis of the data reported herein should be considered tentative, pending completion of all investigations on all recovered 109-D fragments.

2. INTRODUCTION

This report presents some of the preliminary results of a current investigation of four recovered fragments of Atlas Booster 109-D which put the Mercury Capsule Friendship 7 into orbit in the NASA MA-6 Orbital Flight on 20 February 1962.

All four fragments have manufacturer's markings which permit their positive identification as part of Atlas 109-D, and further permit exact location of their original position on the booster, as shown in Figure J-1. Fragments P-1, P-2 and P-3 are pieces of Type 301 stainless steel skin, and fragment S-1 is a Type 6-4 titanium alloy spherical helium bottle.

The fragments were recovered in the Republic of South Africa and forwarded by the USAF Air Attache, Col. D. G. Gaylor, for examination. The three pieces of skin were found near Aliwal North (Figure J-2) at coordinates $30^{\circ} 28'S - 26^{\circ} 49'E$, and the sphere was recovered near Noupoort, coordinates $31^{\circ} 08'S - 24^{\circ} 49'E$. The letter accompanying P-1 stated: "South African Government officials advise that the fragment was found subsequent to a reported series of explosions in that area at 0100 hours on 21 February 1962." If these explosions were the final reentry of the 109-D, then the booster reentered on the sixth orbit, since the time of the reported explosions corresponds to 1800 hours EST 20 February 1962 which is slightly over eight hours after launch. An interview with the finder of this first fragment, P-1, via Col. Gaylor, indicated that P-1 was located at 1800 hours local time on 22 February 1962 which corresponds to 1100 hours EST 22 February. This is about $49 \frac{1}{4}$ hours after launch, and with a period of roughly $1 \frac{1}{2}$ hours the upper limit on the number of orbits the 109-D could have made is $33 \frac{1}{3}$; i.e., the maximum orbit for reentry is the beginning of the 34th orbit and the finder would virtually have had to catch the fragment as it fell.

Information released by NASA on the MA-6 Orbital Flight parameters is given in Table J-1. NASA made no provision for tracking the 109-D after separation from the Friendship 7 capsule. However, on learning of the fragment recovery, NASA Manned Spacecraft Center, Langley AFB, used the separation conditions of Table J-1 as input for a calculation of orbital decay and reentry of the 109-D. The program uses ballistic coefficients ($W/C_D A$) calculated as a function of trajectory Mach number, and NASA used the tumbling coefficients partially tabulated in Table J-2. With these input conditions, and using the composite model atmosphere (NASA Working Paper 205), NASA calculated ten orbits without reentry. The case was terminated here since no reentry favorable to impact in South Africa would occur in the next several orbits. The calculation was performed again using the same input except that the composite model atmosphere was increased by 92.5 percent. This calculation gave the orbital projections shown as heavy lines on Figure J-3, and resulted in impact near the fragment recovery sites on the beginning of the sixth orbit.

The only tracking data available on the 109-D orbits were provided by SPADATS, Ent AFB, Colorado Springs. The NORAD-ADC network tracked the 109-D continuously on all passes over North America, the first observation initiating at 1117 hours EST and the last terminating at 1608 EST, both 20 February 1962. Their data consist of 130 observations, 75 percent of which are radar (principally Laredo, Milstone, Trinidad), but also include beacon tracking (Patrick) and a few visual observations. SPADATS performed an orbit calculation using a smoothing technique with all tracking data as input; their calculated 109-D orbit parameters are given in Table J-3. Also shown in Table J-3 is a comparison of a SPADATS Epoch, the South to North Equatorial pass near the end of the third orbit, and several SPADATS observations with the corresponding values from the NASA calculation of Figure J-3; the event comparisons are rather good but the altitude discrepancies are severe. The accuracy of the SPADATS altitudes is not known at this time.

TABLE J-1

MA-6 Orbital Flight Parameters

Launch, 20 February 1962	09:47:39 EST
SECO: Elapsed Time, hr:min:sec	00:05:01.4
Surface Altitude, ft	528,381
Inertial Velocity, ft/sec	25,730
Inertial Flight path angle, deg	- 0.047
Separation: Elapsed time, hr:min:sec	00:05:03.6
Surface Altitude, ft	528,373
Inertial Velocity, ft/sec	25,704
Inertial Flight path angle, deg	- 0.0462
Friendship 7 Orbit:	
Perigee, nm	86.92
Apogee, nm	140.92
Period, hr:min:sec	00:88:29
Inclination, deg	32.54

TABLE J-2

W/C_DA (lb/ft²) For 109-D Spent Booster, From NASA

0°		90°		Tumbling	
M	W/C _D A	M	W/C _D A	M	W/C _D A
0.0	35.33	0.0	283	0.0	214
0.5	36.11	0.4	314	0.25	217
0.6	37.68	0.7	1115	0.95	1022
1.2	76.93	0.85	1068	1.7	682
1.3	77.7	0.9	1421	2.2	643
1.6	76.93	1.1	1217	2.5	637
4.0	69.87	1.5	989	3.0	631
7.0	65.16	2.0	903	4.5	631
30.	65.16	3.0	871		

TABLE J-3

109-D Orbital Data, From SPADATS - ADC

Orbit: Period, min 88.100
 Semi-major axis, Earth 1.02888
 Eccentricity 0.00583
 Arc perigee, deg 73.02
 Right Ascension, deg 233.23
 Inclination, deg 32.53

Epoch: S-N Equatorial pass, 3rd orbit, 20 February 1962
 SPADATS NASA
 Event hr:min:sec 13:59:59.4 EST 13:59:11.9 EST

Track:

<u>Time</u> Sec	<u>Latitude</u> °N		<u>Longitude</u> °W		<u>Altitude</u> Ft	
	<u>ADC</u>	<u>NASA</u>	<u>ADC</u>	<u>NASA</u>	<u>ADC</u>	<u>NASA</u>
5,408	27.24	26.98	107.44	108.13	548,048	520,698
5,655	32.09	31.31	91.32	90.37	613,059	517,280
5,883	32.91	32.70	75.01	72.94	785,455	520,392
5,929	32.57	32.67	69.37	69.45	494,092	521,736
11,007	32.46	32.04	103.93	107.45	553,937	504,280
11,103	33.06	32.57	100.14	100.15	559,399	505,355

The ADC network observed the 109-D only during the latter portions of the first three orbits; the 109-D was not observed on the fourth or subsequent orbits. This may have been due solely to the fact that the fourth and fifth orbits were too far south in their passes over the western hemisphere; however it does present the possibility that the 109-D reentered as early as the beginning of the fourth orbit. Figure J-3 shows that orbits four, five, and six all pass over the general impact area so

that reentry on any of these three is possible under the present information. An extrapolation of the NASA orbit calculations on Figure J-3, shown as dashed lines, indicates that after the 6th orbit, impact in the recovery area is not possible until the 19th and 20th orbits. The recovery time for the first fragment, P-1, discussed above, put an upper limit of 34 orbits on the history of 109-D, and Figure J-3 indeed indicates that orbit 34 would pass near the southern tip of Africa. However, Figure J-3 also shows that extrapolated orbits 16 through 20 pass over North America and hence should have been tracked by the NORAD-ADC network if they occurred. SPADATS states that the 109-D was not tracked after the end of the third orbit; confirmation of this statement would indicate strongly that the 109-D reentered on the 4th, 5th or 6th orbits. A careful analysis of the SPADATS data together with more detailed information from recovery personnel interviews must precede additional discussion of the orbital decay and reentry history of the 109-D.

3. FRAGMENT TEST PROGRAM

Each of the four fragments is being subjected to a series of tests to provide as much data as possible for determining an envelope of reasonable aero-thermodynamic histories for the fragments. In addition particular care has been taken to note any possible hyper-velocity impact craters. The test program is proceeding along the general outline given in Table J-4.

TABLE J-4

TEST PROGRAM ON 109-D FRAGMENTS

- (1) Identification markings, location on missile, geometry, weight, initial composition and physical properties
- (2) Visual examination (with eyepiece/microscope) noting and logging the following: (a) possible hypervelocity impact craters; (b) dents with apparent foreign inclusions; (c) holes of regular shape; (d) typical slag for analysis;

- (e) typical streaks for analysis; (f) local melting or flow for possible microstructure; (g) "snap-back" due to rupture under tension as indicated by welded doublet sections
- (3) Photographs of overall fragment and closeups of important areas noted in Item (2)
- (4) Chemical tests for composition of chosen samples of flakes, slag, streaks, etc.
- (5) Spectrographic tests for composition of chosen samples
- (6) Mechanical properties of fragment sample (yield, ultimate, % elongation, hardness)
- (7) Microstructure of selected burned or melted areas
- (8) Measurement of geometry of impact craters
- (9) Microstructure and microprobe chemical analysis of selected craters

The logging code for the visual examination is given in Figure J-4. The test program is in its initial stages; results to date are summarized in the following sections on the individual fragments. In connection with the measurement and description of the apparent hypervelocity impact craters, the noted diameters are measured with a calibrated eyepiece to the inside edge of the crater lip and are accurate to about ± 0.002 inch for craters above 0.010 inch diameter; for counting and logging purposes craters about 0.010 inch diameter and less were grouped as 0.010 inch nominal or 0.005 inch nominal. Craters of the order of 0.001 inch diameter were found, but the counting of these and smaller sizes cannot be considered accurate at this time.

4. TEST RESULTS FOR FRAGMENT P-1

Fragment P-1 is a portion of the LO₂ tank skin of Atlas 109-D and was found north of Aliwal North, Republic of South Africa, coordinates 30° 28'S - 26° 49'E at 1800 hours local time on 22 February 1962. General data on P-1 is given in Table J-5.

The general configuration of P-1 is sketched in Figures J-5 and J-6 which also show the location and logging code of the specimens taken for testing purposes. Photographs presented as Figures J-7, J-8, J-9, show the over-all appearance of inner and outer surfaces of P-1 and close-ups of typical outer surface features. Both surfaces were uniformly oxidized; though not shown clearly in the photographs, the inside surface had a high iridescence whereas the outside surface was dull and gray. This indicates that the outside surface suffered more extensive oxidation.

Both surfaces were streaked with molten metal, with a much larger quantity striking the outside. The impingement pattern on the outside surface indicated that the great majority of the metal drops were moving toward the rear of the 109-D parallel to the axis (see Figure J-9 and

TABLE J-5

General Data, Fragment P-1

Material:	Type 301 XH Stainless Steel Sheet			
Area:	8.8 ft ² (one side)			
Dimensions:	38" x 45" x 0.014" and 0.015" thick			
Weight:	6.9 lbs.			
Properties:	<u>Gage, in.</u>	<u>F_{tu}, Ksi</u>	<u>F_{ty}, Ksi</u>	<u>% e</u>
P-1	0.014	145.6	43.1	44.0
Original	0.014	211.4	--	14.0
Fully				
Annealed 301	--	110.0	40.0	60.0
Hardness:	(Laboratory tests on sheets of 301XH heated to indicated temperature for 30 seconds)			

<u>°F</u>	<u>Superficial Hardness, 15T</u>
1200	94.0, 94.5
1400	94.6, 95.0
1600	89.1, 89.0
1800	87.0, 87.0
2000	85.0, 85.2, 85.2
2200	84.0, 85.0, 84.2
P-1	85.0, 85.5, 86.2, 86.2

TABLE J-6

RESULTS OF SPECTROGRAPHIC ANALYSIS
OF SELECTED SPECIMENS FROM FRAGMENT P-1

Location No.	Description	Main Constituent Identified Spectrographically as
<u>Outside Surface</u>		
L1	purple green streak	Titanium
L2	yellow splash	not identified
L3	metal splash, blue green	not identified
L4	gold colored fleck	aluminum
L5	molten metal, large area	aluminum
L6	molten metal	magnesium
L7	molten metal	Titanium
L8	metal splash, yellowish	Titanium
L9	cavity (dent)	(see text)
L10	molten metal streak	aluminum
P1	molten metal streak	Titanium
P2	molten metal streak	Titanium
P3	molten metal streak	
P4	burnt surface	
P5	-	
P6	molten metal streak	Titanium
<u>Inside Surface</u>		
M1	rusty area	not identified
M2	metal splash, silvery	Titanium
M3	molten metal	not identified
M4	molten metal	aluminum
M5	whitish metal splash	Titanium
M6	molten metal	Titanium
M7	bright metal splash	Titanium
M8	bright metal splash	Titanium
M9	blue green area	iron
M10	metal splash	Titanium
N1	-	not identified
N2	dark iridescent area	Titanium
N3	cavity (dent)	(see text)
N4	molten metal	Titanium
N5	metal splatter	Titanium
N6	molten metal	Titanium
N7	molten metal	not identified

the arrows on Figure J-5). On the other hand, numerous small droplets impinged on the inner surface moving in a more random fashion (see arrows on Figure J-6).

The results of the spectrographic analysis on the selected surface specimens identification code and location shown in Figures J-5 and J-6, are presented in Table J-6. Impinging materials foreign to the stainless steel skin were titanium alloy and aluminum alloy. These were available on the upper portion of the 109-D only in the Mercury Capsule adapter, Figure J-1. This adapter is made of titanium alloy and contains several aluminum brackets on the inside at the top of the bulkhead. Both titanium and aluminum droplets impinged on the outer surface of P-1, but very few aluminum droplets were found on the inner surface. All of the above streaking and oxidation evidence strongly indicates that at least the LO_2 tank of the 109-D suffered severe reentry heating while still intact with a nose-first (0° angle of attack) trajectory. It should be noted that for insulation purposes the upper stainless steel bulkhead of the 109-D was covered with 0.050 inch thick fiberglass with the top of the bulkhead surmounted by a 0.250 inch thick 30 inch diameter phenolic ring, Figure J-1. These materials may have acted as heat shields during early reentry.

Two dents, apparently made by low-velocity impact, were analyzed using X-ray emission techniques. Specimen L9 on the outside, Figure J-5, contained splattered oxidized material which showed roughly $1/3$ the Fe content of the Type 301 Stainless Steel skin. Specimen N3 on the inside surface, Figure J-6, contained a flowed copper colored material which showed roughly $3/4$ the Fe content of the skin. Both specimens showed Cr in roughly the same proportion as Type 301, and no elements foreign to Type 301 were found in either. This suggests that these dents were caused by impacts of hot stainless steel, the low Fe content being explained by boil-off, the latter being greater on the hotter outer surface.

On the basis of the tensile and hardness test results shown in Table J-5, Fragment P-1 was apparently heated to about 1900°F during reentry.

The test specimen indicated that the fragment was not fully annealed, whereas the fully annealed properties are obtained with Type 301 after holding 1 to 3 minutes at about 2000° F. Also, examination of the spot welded doublet section on P-1 indicated evidence of "snap-back" suggesting that when the 109-D LO₂ tank ruptured and P-1 tore loose there was still considerable elastic energy in the skin.

All of the preceding chemical and physical information of P-1 was abstracted from General Dynamics/Astronautics Materials Research Report MRG-313 prepared by L. Girton, May, 1962.

The first examination of fragment P-1 for possible hyper-velocity impact craters revealed no such craters. A part of the fragment was intercepted before being returned to the Republic of South Africa and close examination revealed hyper-velocity cratering. Figures J-10, J-11 show the part of P-1 returned to the Republic of South Africa, the part under examination at this time, and the portion lost due to testing performed on P-1. The examination is in progress; therefore, a crater size distribution for P-1 is not possible at this time. The cratering shown on the upper portion of P-1 was determined by a rapid examination and it is felt that a closer examination would yield a larger number of small diameter cratering (2 - 7 mil diameter).

5. TEST RESULTS FOR FRAGMENT P-2

Fragment P-2 is also a portion of the LO₂ tank skin of the 109-D. Present knowledge of the recovery site of P-2 is limited to "the same general area" where P-1 was found, and the recovery date and time are unknown. Specific information on these matters has been requested via the U.S. Air Attache in Pretoria, South Africa. General data on P-2 are given in Table J-7. The materials tests on P-2 are still in progress and results are not yet available. This section will be limited to a summary of the results of the visual examination of P-2, in particular the apparent hyper-velocity impact craters found thereon.

TABLE J-7

General Data, Fragment P-2

Material:	Type 301 XH Stainless Steel Sheet
Area:	8.2 ft ² (one side)
Dimensions:	40" x 45" x 0.014" and 0.015" thick
Weight:	5.9 lbs

A photograph of the outside surface of P-2 is presented as Figure J-12, and the inside surface is shown in Figure J-13. Both surfaces show numerous hyper-velocity impact craters of diameters 0.020 inch and larger. In addition, the inside surface of P-2 has a very large number (over 1200) of craters less than 0.020 inch in diameter. Figures J-14 and J-15 are sketches of P-2 showing the locations and approximate sizes of these craters with respect to diameter and surface appearance are shown in Figures J-16, J-17. The in situ appearance of these craters is typical in all obvious respects of those found on Fragment P-3, discussed in the next section. Cross sections of two craters on P-2, taken by microtome section of rubber impressions, are shown in Figure J-18.

6. TEST RESULTS FOR FRAGMENT P-3

Fragment P-3 is a strip of 0.022 inch thick stainless steel approximately 9 inches wide and 13 feet long. It constitutes almost half of the transition skin between the cylindrical and conical sections of the 109-D, Figure J-1. This fragment was also found in "the same general area" as P-1; more exact information on recovery site and time for P-3 has also been requested. The general data on P-3 is listed in Table J-8. Materials tests on this fragment are also in progress, so discussion is limited here to the hyper-velocity impact craters found on P-3.

An overall view of Fragment P-3 is shown as Figure J-19. Numerous craters were found on both surfaces of P-3 and the locations, diameters and appearances of these craters are indicated in the sketches of Figure J-20a, b, c and d, which use the Y-Y axis as the origin for measurement,

Figure J-1. Photographs of three typical impact craters from P-3 are shown in Figures J-21 and J-22. These were all taken from the outside surface of P-3; their appearance, however, is representative of all craters found on both P-2 and P-3. Crater C-11 on Figure J-21 represents the large diameter craters in size and the dull rusty red craters in appearance. Crater C-10, Figure J-22, is typical of the medium size and the shiny bright appearance. The small crater C-12 of Figure J-21 barely shows detail in the photograph, but under the microscope is representative of craters less than 0.010 inch diameter. The distribution of craters with respect to diameter and surface appearance for both surfaces of P-3 and for the total impacts is shown in Figures J-23, J-24 and J-25. Craters of approximately 5 mil diameter have been found on P-3 but have not been logged in Figures J-23, J-24 and J-25.

TABLE J-8

General Data, Fragment P-3

Material:	Type 301 XH Stainless Steel Sheet (welded doublet)
Area:	9.8 ft ²
Dimensions:	8" x 160" x 0.044" thick (double 0.022" sheets)
Weight:	12.3 lbs.

7. TEST RESULTS FOR FRAGMENT S-1

Fragment S-1 is a titanium alloy spherical helium bottle originally located near the tail of the 109-D, Figure J-1. This fragment was recovered on 24 February 1962 near Noupoot, South Africa, coordinates 31° 08' S - 24° 49' E, Figure J-2. The sphere suffered very limited damage as shown in the photographs of Figures J-26 and J-27. The stainless steel pressure fitting and titanium alloy boss at one end melted almost completely and flowed back over the sphere, Figure J-26. The identical fitting opposite this is virtually undamaged, and may be screwed in and out by hand. The sphere is about 21 inches in diameter and weighs 47.8 lbs; the wall thickness

is 0.18 inch, Figure J-28 shows a close-up of the surface near the melted boss.

A chemical analysis of a chip from the molten slag near the melted boss indicated a 9:1 ratio of titanium alloy to stainless steel. A spectrographic analysis of the dull coppery coating on the sphere indicated 90-95% rutile (TiO_2). Additional tests, including mechanical properties of three skin segments, are in progress. No hyper-velocity impact craters have been found on S-1.

8. SUMMARY

Since the foregoing data are purely preliminary, no conclusions are in order here. Information available from direct observation of the impact craters on the recovered fragments is rather limited. Of much greater value would be measurements of the exact geometry of the craters, permitting statistical analysis of the variation of geometrical parameters such as those shown in Figure J-29 with crater size for the various types of craters found.

LOCATION OF RECOVERED FRAGMENTS (SHADED) ON MA-6
BOOSTER ATLAS 109-D

Figure J-2 Recovery Sites of Atlas 109-D Fragments

Figure J-3 NASA Calculated Orbital Decay and Re-Entry of Atlas 109-D Booster (From data furnished by NASA Manned Spacecraft Center, Langley AFB; calculated impact $27^{\circ}44'56''\text{E}$ $30^{\circ}39'11''\text{S}$ at 18:12:23 EST 20 Feb. 62.)

VISUAL EXAMINATION OF 109-D FRAGMENTS -- LOGGING CODE

<u>CODE</u>	<u>DESCRIPTION</u>	<u>TYP. X-SECTION</u>	<u>COLOR</u>	
C	crater		<u>Code</u>	<u>Description</u>
D	dent		B	black
H	hole		C	copper
G	slag		G	gold
S	streak		P	purple-blue
			R	red-brown
			S	silver
			Y	yellow-green
			<u>REFLECTANCE</u>	
			<u>Code</u>	<u>Description</u>
			B	bright
			D	dull

Example:

Type Dia-thou in. Color Refl

C - 095 - S - B

The example is thus a crater 0.095 inch diameter with a silver - bright appearance.

Location:

Results of visual observations are to be logged on coordinate paper showing location of feature noted on fragment.

Figure J-4

Figure J-5 Sketch of Outer Surface of Fragment P-1 Showing Location of Test Specimens. (Arrows indicate direction molten metal was traveling at impact.)

Figure J-6 Sketch of Inner Surface of Fragment P-1 Showing Location of Test Specimens. (Arrows indicate direction molten metal was traveling at impact.)

Figure J-7 View of outside surface of Fragment P-1, Atlas 109-D. Note axial streak pattern. Analysis of circled streaks given in Table J-6.

Figure J-8 View of inside surface of Fragment P-1, Atlas 109-D. Note absence of axial streaking. Analysis of circled streaks given in Table J-6.

Figure J-9. Closeup of forward edge, outer surface, Fragment P-1. Streak P6 is titanium; L10 is aluminum.

Figure J-10. Outside View of P-1 as of 1 July 1962.

Figure J-11. Inner View of P-1 as of 1 July 1962.

Figure J-12. View of Outside Surface of Fragment P-2, Atlas 109-D.
Note Axial Streak Pattern.

Figure J-13. View of Inside Surface of Fragment P-2, Atlas 109-D
Note Absence of Axial Streaking.

Figure J+14.

Figure J-15.

Figure J-16.

Figure J-18. Cross-sections of craters from Fragment P-2

P-2, Inside Surface
Crater # 2032
Picture # 3023
Diameter 0.040 inch
Magnification 25X
Type: Silver Dull

P-2, Outside Surface
Crater # 2015
Picture # 3024
Diameter 0.030 inch
Magnification 25X
Type: Red (Rusty) Dull

Figure J-19. View of Fragment P-3, Atlas 109-D

Y-Y P-3 OUTSIDE

Y-Y P-3 INSIDE

Figure J-20a. Location of craters on P-3. See Fig. J-15 for symbol notation.

P-3 OUTSIDE

H-075-B-D

3'

4'

5'

6'

P-3 INSIDE

H-075

Figure J-20b.

Figure J-20c.

P-3 OUTSIDE

-236-

6'

5'

4'

P-3 INSIDE

Figure J-20d.

Figure J-21. Closeup of Two Craters on Outside Surface of Fragment P-3

Figure J-22. Closeup of Typical Shiny Crater on Outside Surface of Fragment P-3

Figure J-23.

DISTRIBUTION OF CRATERS ON OUTSIDE OF P-3

Figure J-24.
DISTRIBUTION OF CRATERS ON INSIDE OF P-3

Figure J-25.

TOTAL DISTRIBUTION OF CRATERS ON P-3

Figure J-26. View of hemisphere of Fragment S-1 which suffered the most severe heating. Note streak patterns away from melted boss.

Figure J-27. View of hemisphere of Fragment S-1 showing secondary but still severe heating. Features indicate this hemisphere was forward during early re-entry.

Figure J-28. Closeup of region near melted boss on Fragment S-1. Melt pit near center was only one of kind on sphere and was deepest penetration of wall.

Figure J-29. Some geometrical parameters for a typical crater

APPENDIX K

ON THE SCALING OF PROCESSES OF SIGNIFICANCE IN THE REENTRY
OF NUCLEAR FUEL ELEMENTS1. INTRODUCTION

Scaling laws may be determined for a physical phenomenon in the manner indicated here by a general procedure and example.

First, it is presumed that the phenomenon is understood sufficiently well to permit its mathematical expression in a set of differential equations and an appropriate set of boundary conditions. These expressions are written down.

Second, each factor in each term of these expressions is rendered dimensionless by dividing by a reference quantity having the same physical dimensions as the original dimensional factor. It is attempted to minimize the number of the basic factors of which the reference quantities are composed. Not only must the resulting factors be dimensionless but they must depend, strictly speaking, only upon scaled quantities. If there exists weak dependence upon an unscaled quantity, it may sometimes be ignored.

Third, the dimensionless dependent variables, the dimensionless independent variables, the dimensionless parameters, and the dimensionless boundary conditions are distinguished from one another. The dimensionless parameters may be implicitly dependent upon the dimensionless dependent variables, but the dimensionless boundary conditions are constants.

Fourth, presuming that the problem could be solved analytically, each dimensionless dependent variable could be expressed in terms of all of the other types of dimensionless quantities. These expressions which make up the solution of the problem will be universal, that is, applicable to all similar problems, if they are independent of the various scale factors which occur.

Fifth, the appropriate scaling laws which must be observed to ensure universality of the solution are those which preserve, respectively, the identical values of all the dimensionless factors appearing in the solution. The exact scaling laws are compiled.

Sixth, any special cases which can be treated with the aid of a simplifying restriction upon the scope of the problem are discussed.

These six steps are employed in the following example:

- (1) Given the differential equations

$$y_t + a(x)y + b(x)x^2 = 0$$

$$x_{tt} + c(y)x^3 = 0$$

and the boundary (initial) conditions

$$y(0) = Y$$

$$x(0) = X$$

$$x_t(0) = X_t$$

where x and y are to be determined as functions of t . (The scripts denote differentiation with respect to t .)

- (2) Let $y^* = y/L$, $x^* = x/L$
 $\tau = t/t_0$, $a^* = a/a_0$
 $b^* = b/b_0$, $c^* = c/c_0$

and transform the problem to

$$y_\tau^* + a^*y^* + b^*x^{*2} = 0,$$

$$x_{\tau\tau}^* + c^*x^{*3} = 0,$$

$$y^*(0) = Y^*, \quad x^*(0) = X^*,$$

$$x_{\tau}^*(0) = X_{\tau}^*$$

where use has been made of the freedom to choose

$$a_o = 1/t_o,$$

$$b_o = 1/Lt_o \text{ and } c_o = 1/L^2 t_o^2.$$

- (3) The dimensionless problem has as its dependent variables x^* and y^* , as its independent variable τ , as its parameters $a^*(x^*)$, $b^*(x^*)$ and $c^*(y^*)$, and as its boundary conditions Y^* , X^* and X_{τ}^* .
- (4) If solved, the result could be stated as

$$x^* = f_1 (X^*, X_{\tau}^*, Y^*; a^*, b^*, c^*; \tau)$$

$$y^* = f_2 (X^*, X_{\tau}^*, Y^*; a^*, b^*, c^*; \tau)$$

and these results are universal if no scale factors appear in them.

- (5) To ensure universality of the results we must preserve, for example, the value of a^* as between model and prototype: $a^*_{\text{model}} = a^*_{\text{prototype}}$. That is,

$$\left(\frac{a}{a_o} \right)_m = \left(\frac{a}{a_o} \right)_p,$$

$$(a t_o)_m = (a t_o)_p,$$

and

$$\frac{a_m}{a_p} = \frac{t_{op}}{t_{om}} ,$$

or by definition of scale factor

$$N_a = N_{t_o}^{-1} .$$

The scaling laws may now be compiled as:

$$N_x = N_X = N_y = N_Y = N_L$$

$$N_{x_t} = N_L / N_{t_o} = N_L / N_a$$

$$N_b = 1 / N_L N_{t_o} = N_a / N_L$$

$$N_c = N_a^2 / N_L^2$$

$$N_t = 1 / N_a .$$

- (6) It may happen that a^* , b^* and c^* depend upon other dimensionless quantities such as the Mach number, the Reynolds number, etc. If so, these quantities must be taken into account in compiling the scaling laws.

If, as we have tacitly assumed above, the quantities a^* , b^* and c^* are either independent of such quantities or only weakly dependent upon them over the range of interest, we may ignore those additional quantities provided both model and prototype

problems remain within the range of weak dependence.

It may now be of interest to consider the case $N_c \equiv 1$.

Then it follows that $N_a = N_L$ and the compilation of scaling laws may be rewritten as:

$$N_x = N_X = N_y = N_Y = N_a = N_L$$

$$N_{x_t} = N_b = N_c = 1$$

$$N_t = 1/N_L.$$

2. SUMMARY

A. Model and Prototype Similarity in Selected Reentry Processes

(1) Nonlifting free fall

(A) Spheres

Identical trajectories for falling spheres of various weights and sizes can be achieved, assuming constant gravitation and an isothermal atmosphere, and providing the following points are observed:

- 1 Vertical coordinate of the trajectory is measured downward from initial (ejection) point;
- 2 initial trajectory angles are identical;
- 3 initial velocities are identical; and
- 4 initial points are chosen so that the atmospheric densities are in the same proportion to one another as the ballistic coefficients, $W/C_D A$.

(B) Tumbling rods

Even with four simplifying assumptions (see Section 3.A.(2)) it is impossible to simulate the

trajectories of rapidly rotating free falling rods (except for extremely rapid rotations in the plane of fall which are included in case 2.A.(1)(A)).

Under the additional assumption of slow rotation in the plane of fall, tumbling rod trajectories can be made identical under the conditions of A.(1)(A) plus the following requirements:

- 1 The initial rates of rotation are identical;
- 2 the fractional displacements of the center of gravity from the aerodynamic center are scaled with rod length; and
- 3 the moments of inertia are scaled as the products of mass times length squared.

(C) Steep-angle, high-velocity fall with zero lift

The only useful simulation here is to achieve identical velocity and atmospheric density values at all corresponding points of model and prototype trajectories. The only scaling involved, then, is that by which increases of ballistic coefficient may be compensated by initial trajectory angle reductions according to the scale law:

$$N_{(W/C_D A)} = 1/N_{\sin \gamma_1} \quad (1)$$

(2) Aerodynamic heating

(A) Hypervelocity flight

In Section 3.B.(1) it is shown on the basis of semi-empirical theory that tests for heating intensity and

distribution, for example, may be conducted with consideration of the general scaling law

$$N_q = N_\rho^m N_V^n N_L^p \quad (2)$$

where m , n and p have one set of values for laminar flow and another set for turbulent flow. Here the model and prototype surfaces are assumed to be much colder than the effective stagnation temperature.

To obtain full-scale heating rates, as in certain materials tests if those involving transient heating of solids (see also Section 2.A. (3)), one must specify $N_q = 1$.

At high velocities gas reactions may occur which cannot be simulated but instead must be faithfully reproduced. This generally will require both $N_V = 1$ (to ensure that the same temperatures are reproduced), and $N_\tau = N_L/V = N_L/N_V = 1$ (to allow for the same time intervals for the various reactions to occur). Needless to say, the same gas mixtures must be used as well.

However, if this latter restriction is not important because in both model and prototype cases sufficient time is allowed (relatively slow flows), then for $N_q = 1$ we can vary N_L to accommodate N_ρ according to

$$N_\rho^m = N_L^{-p} \quad (3)$$

It is shown that the full scale Nusselt number will be observed in a model test of a laminar flow at the full-

scale Reynolds number. Since for a reactive flow it is necessary to use the same gas and hold $N_V = 1$, this comes to simply maintaining

$$N_p = 1/N_L. \quad (4)$$

Binary reactions may be reproduced by observing this latter scaling law.

(B) Not-so-cold-model tests

The more complex heat flux relationship which applies in this case makes general similarity more difficult to attain.

Overall heating rates per unit mass of material may be simulated by observing the law

$$N_{\dot{q}} = N_L \quad (5)$$

where $N_{\dot{q}}$ is determined by the $\dot{q}$ relationship of Equation (53).

In tests of melting ablation in which the shape is preserved the applicable scaling law is

$$(L/L_0)^{1-p} = 1 - C \dot{q}_0 t, \quad (6)$$

which is derived in Section 3.B.(2)(B). To put this relationship into a form involving the usual scale factors we may proceed as follows:

Take the natural logarithm of both sides to obtain

$$(1-p) \ln (L/L_0) = \ln (1 - C \dot{q}_0 t). \quad (7)$$

Now consider the usual case in which $C \dot{q}_0 t \ll 1$ and write the approximation

$$(1 - p) \ln(L/L_0) = -C \dot{q}_0 t + 0 (C \dot{q}_0 t)^2 \quad (8)$$

Thus to first order we may write an expression for the variation of N_{q_0} , the scale of initial rates, in terms of initial prototype values, the variation of N_L with time and the time itself:

$$N_{q_0} N_C = 1 - (1 - p) \ln N_L(t) / (C \dot{q}_0)_p t \quad (9)$$

The last term will usually be small compared to one, so

$$N_{q_0} = (N_C)^{-1} = N_{L_0} / N_{\rho_s} N_{h_{s-l}} \quad (10)$$

2.A.(3), 2.A.(4), 2.A.(5), 2.A.(6) Transient temperatures, thermal stresses, and sorption expansion in homogeneous solids

Complete similarity requires such a number of parameters to be properly scaled that they will be stated only once. See Sections 3.C., 3.D., 3.E. and 3.F.

3. SIMILARITY ANALYSIS

A. Nonlifting free fall

(1) Trajectory similarity for falling spheres

Fig. K-1. FORCE DIAGRAM FOR FALLING SPHERE

The governing equations for $V(t)$, $\gamma(t)$ and $y(t)$ in dimensional form are:

$$(W/g_c) \dot{V} + D + (W/g_c) g \sin \gamma = 0 \quad (11)$$

$$V \dot{\gamma} + g \cos \gamma = 0 \quad (12)$$

$$\dot{y} = V \sin \gamma \quad (13)$$

where

g_c is the acceleration due to gravity at sea level, $[LT^{-2}]$

g is the acceleration due to gravity at altitude y , $[LT^{-2}]$

W is the weight of the body at sea level, $[MLT^{-2}]$

ρ is the atmospheric density at altitude y , $[ML^{-3}]$

C_D is the drag coefficient, $D/(\rho V^2/2)A$, $[1]$

A is the frontal area of the body, $[L^2]$

$(\dot{})$ indicates differentiation with respect to time, $[T^{-1}]$

the fundamental dimensions being indicated in brackets on the right.

Initial conditions are:

$$V(0) = V_E \quad [LT^{-1}]$$

$$\gamma(0) = \gamma_E \quad [1]$$

$$y_E - y(0) = z_E = 0. \quad [L] \quad (14)$$

Note that the altitude of interest is that measured positively downward from the ejection altitude y_E which is of adjustable magnitude.

In dimensionless form this problem may be stated as follows.

$$\frac{dV^*}{dt^*} + \Pi_1 \rho^* V^{*2} + g^* \sin \gamma = 0 \quad (15)$$

$$V^* \frac{d\gamma}{dt^*} + g^* \cos \gamma = 0 \quad (16)$$

$$\frac{dz^*}{dt^*} + V^* \sin \gamma = 0 \quad (17)$$

$$V^*(0) = 1 \quad (18)$$

$$\gamma(0) = \gamma_E$$

$$z^*(0) = z_E^* = 0$$

in which the basic reference quantities have been chosen as

$$V_R = V_E, \rho_R = \rho_E \text{ and } g_R = g_c \quad (19)$$

leading to secondary reference quantities such as

$$t_R = V_E / g_c \text{ and } L = V_E^2 / g_c, \quad (20)$$

and y has been replaced by $y_E - z$. The dimensionless coefficient which appears in the first differential equation is defined as:

$$\Pi_1 = (\rho_E V_E^2 / 2) / (W / C_D A) \quad (21)$$

The solution of this problem may be stated in functional form as

$$V^*(t^*) = f_1 [\Pi_1 \rho^*(z^*), g^*(z^*) \gamma_E | t^*],$$

$$\gamma(t^*) = f_2 [\text{same variables}],$$

$$z^*(t^*) = f_3 [\text{same variables}]. \quad (22)$$

Two different trajectories will be similar if each variable upon which the functions f_1 , f_2 and f_3 depend have the same values, respectively, in the two cases. These functions are universal; in other words, they may be plotted once and for all as V^* , γ and z^* versus t^* with Π_1 and γ_E as parameters.

These universal curves will apply only to those trajectories having identical functions $g^*(z^*)$ and $\rho^*(z^*)$, and, strictly speaking, these are geophysical functions of y rather than of z^* so γ_E would necessarily be the same in all cases. However, since g^* is practically unity from the "edge" of the atmosphere down to sea level, this variable may be set equal to one. Also, a satisfactory approximation for ρ^* is to assume an isothermal atmosphere which obeys the relationship

$$\rho = \rho_0 e^{-\beta y} \quad (23)$$

where β is a physical constant. Then we have that

$$\Pi_1 \rho^* = \exp(\beta z^* V_E^2 / g_c) (\rho_E V_E^2 / 2) / (W / C_D A) \quad (24)$$

which will depend purely upon z^* only if V_E is identical in all cases. Variations of $(W / C_D A)$

may be compensated, however, by properly selecting ρ_E , i.e., by measuring z downward from a suitable initial altitude y_E .

The resulting scale factors which apply are:

$$N_V = N_\gamma = N_z = N_t = N_{V_E} = N_{\gamma_E} = 1,$$

$$N_{\rho_E} = N_{(W/C_D A)} \quad (25)$$

(2) Trajectory similarity for falling rods which are tumbling end over end. ⁴⁷

Five assumptions are made:

- (A) Effects of the earth's (and its atmosphere's) rotation are negligible;
- (B) effects of the earth's curvature are negligible;
- (C) the acceleration due to gravity is constant;
- (D) the motion takes place essentially in a plane with the axis of rotation normal to this plane (this restriction will later be removed); and
- (E) the relative tumbling rate is small (this restriction will not be used at first but will be seen to be necessary later).

The equations of motion in dimensional form are:

$$m\dot{V} + D + mg \sin \gamma = 0, \quad (26)$$

$$mV\dot{\gamma} - L + mg \cos \gamma = 0, \quad (27)$$

$$I\ddot{\theta} = M, \quad (28)$$

$$\dot{y} = V \sin \gamma, \quad (29)$$

Fig. K-2. FORCE DIAGRAM FOR TUMBLING ROD IN FREE FALL

where, in addition to quantities defined in Section 3.A. (1)

L is the aerodynamic lift force, $[L]$
 I is the moment of inertia of the rod, $[ML^2]$
 M is the applied aerodynamic moment, $[ML^2 T^{-2}]$

the fundamental dimensions being given on the right..

Initial conditions to be specified are:

$$\begin{aligned}
 y(0) &= y_E, & [L] \\
 V(0) &= V_E, & [LT^{-1}] \\
 \gamma(0) &= \gamma_E, & [1] \\
 \dot{\theta}(0) &= \dot{\theta}_E, & [1] \\
 \theta(0) &= \theta_E, & [T^{-1}]
 \end{aligned} \quad (30)$$

In dimensionless form the problem may be restated:

$$\frac{dV^*}{dt^*} + \frac{1}{2} \rho^* V^{*2} + g^* \sin \gamma = 0 \quad (31)$$

$$V^* \frac{d\gamma}{dt^*} - \Pi_2 \rho^* V^{*2} + g^* \cos \gamma = 0 \quad (32)$$

$$I^* \frac{d^2 \theta}{dt^{*2}} = \Pi_3 \rho^* V^{*2} \quad (33)$$

$$\frac{dz^*}{dt^*} + V^* \sin \gamma = 0 \quad (35)$$

$$z^*(0) = z_E^* = 0 ,$$

$$V^*(0) = 1 ,$$

$$\gamma(0) = \gamma_E ,$$

$$\theta(0) = \theta_E ,$$

$$\frac{d\theta}{dt^*}(0) = \left(\frac{d\theta}{dt^*} \right)_E . \quad (36)$$

In addition to the definitions of Section 3.A.(1) we have

$$\Pi_2 = (C_L/C_D) \Pi_1$$

$$\Pi_3 = (C_M/C_D) \Pi_1 \Pi_4$$

where

$$\Pi_4 = (V_E^2 / g_c l) , \quad (37)$$

and l is the length of the rod. The coefficients C_L , C_D and C_M depend upon α and $\theta l/V$ in the Newtonian approximation. The latter product may be written

$$\dot{\theta} l / V = (1 / V^*) (d\theta / dt^*) / \Pi_4 \quad . \quad (38)$$

The solution of this problem may be expressed in functional form:

$$\begin{aligned} V^*(t^*) &= f_1 \left[\Pi_1, \rho^*(z^*), C_M / C_D, \Pi_4, I^*; \gamma_E, \theta_E, \left(\frac{d\theta}{dt^*} \right)_E | t^* \right] , \\ \gamma(t^*) &= f_2 \left[\text{same variables} \right] , \\ z^*(t^*) &= f_3 \left[\text{same variables} \right] , \\ \theta(t^*) &= f_4 \left[\text{same variables} \right] , \quad (34) \end{aligned}$$

The quantity g^* has been omitted by Assumption (C) above. Also C_L / C_D is omitted because in the Newtonian approximation⁴⁷ it equals $\cot \alpha$ which is determined by the solution (34).

If in the cases compared the rods are of geometrically similar shape (they are not necessarily circular cylinders) and if Π_4 is the same, C_L , C_D and C_M will be the same. However, if Π_4 is maintained, the arguments introduced in Section 3.A.(1) for Π_1, ρ^* being held constant will again require V_E to be the same and hence, in this case, l to be the same. Therefore, strictly speaking, no modeling can be done of a tumbling rod in free fall, and only full scale tests will yield useful results. It will therefore be necessary to restrict the generality of the analysis by imposing Assumption (E).

If the relative tumbling rate $\dot{\theta} l / V$ is sufficiently

small, ⁴⁷ $C_M/C_D = \bar{r} \sin \alpha$, where $\bar{r} = r/l$ is the fractional displacement of the center of gravity from the aerodynamic center of the rod, and this ratio is also determined by the similarity of the models and by the solution (34). However, in this case Π_4 appears only in Equation (33) (not in the coefficients C_L and C_D) and therefore in the results (34) it appears with C_M/C_D as the product

$$(C_M/C_D) \Pi_4 = \bar{r} \sin \alpha V_E^2 / g_c l. \quad (39)$$

Thus, we can adjust $\bar{r}$ and l while holding $\bar{r}/l$ constant and still maintain the same value of the product (39). The fact that it is necessary to maintain Π_4 constant is no handicap since this is always possible by a redistribution of the mass within the rod unless material homogeneity is required for other reasons not considered in this section.

The resulting scale factors which apply to a slowly tumbling free fall are:

$$N_V = N_Y = N_Z = N_\theta = N_t = N_{V_E} = N_{Y_E} = N_{\theta_E} = N_{\theta_E} = 1$$

$$N_{\rho_E} = N(W/C_D A), \quad N_r = N_l, \quad N_I = N_m l^2. \quad (40)$$

With regard to more general tumbling motions than the simple end-over-end motion considered above, it would be considerably more difficult to

deal with the more complex equations involved. However, it may be anticipated that for sufficiently slow rotations the coupling of modes (gyroscopic effects) will not be a large factor, and no new types of parameter will be introduced. If the scale factors N_Y , $N_{\dot{Y}}$, N_{Y_E} and $N_{\dot{Y}_E}$ are now regarded as applying to each of the axes involved, we may take the above results to be applicable to the more general problem.

(3) Steep angle, high velocity free fall of nonlifting body

For $r - \gamma$ and V both large, the angle is slowly varying compared to ρ/ρ and may be taken as constant. The equation of motion reduces to

$$\dot{V}/g_c + (\rho V^2/2)/(W/C_D A) = 0 \quad (41)$$

For an isothermal atmosphere,

$$\dot{\rho} = -\beta \rho V \sin \gamma, \quad (42)$$

so that we may write (41) as

$$2\beta \sin \gamma_1 d(\ln V) = g_c (C_D A/W) d\rho \quad (43)$$

and the solution is

$$\ln(V/V_1) = \Pi(\rho - \rho_1)/\rho_1, \quad (44)$$

where

$$\Pi = \rho_1 (g_c/2\beta)/(W/C_D A) \sin \gamma_1. \quad (45)$$

The subscript 1 rather than E is used here to emphasize that similarity may be effected in some lower altitude range without faithfully simulating upper portions of the trajectory. The use of downward accelerated rockets to simulate peak heating conditions is a particular case. In this case subscript 1 may indicate an initial exposure point for the test model.

According to (44) and (45) the model and prototype velocities will be in constant ratio to one another at a fixed atmospheric density difference provided Π , ρ_1 and V_1 in the two cases are the same. This is a useful conclusion only if exact simulation is obtained, i.e., $V_m = V_p$ and $\rho_m = \rho_p$ at all points. The parameter Π/ρ_1 shows that ballistic coefficient increases may be compensated by γ_1 reductions. In terms of scale factors

$$N_{(W/C_D A)} = N^{-1} \sin \gamma_1 \quad (46)$$

This scaling remains valid only so long as the approximation is reasonable.

B. Aerodynamic Heating

(1) Hypervelocity flight

The results of detailed analyses and correlations of experimental data for hypersonic, aerodynamic heat transfer rates $\dot{q}$ to bodies at constant, relatively low, surface temperature may be expressed conveniently in the following form: ^{13, 49}

$$\dot{q} = K_\rho^m V^n L^p e^{-k_1 V^2} \quad (47)$$

Here K is a function of body geometry, location upon the body, and the specific heats of the gases involved if there is mass transfer; and k_1 is inversely proportional to the heat of ablation. The exponents have the particular values shown in Table K-1.

Table K-1

<u>Boundary Layer State</u>	<u>Laminar</u>	<u>Turbulent</u>
M	0.5	0.8
n	3.15	3.5
p	-0.5	-0.2

If the material's ablation temperature is not exceeded, k_1 will have the value zero. On the other hand, for most cases the exponential factor will not be less than one-tenth.

If a test is conducted for the purpose of an empirical determination of K as a function of, say, location on the body surface, and if mass transfer is not involved, it would appear to be reasonable to observe the following scale relationships:

$$N_q^* = N_\rho^m N_V^n N_L^p \quad (48)$$

Thus, if full-scale heating rates are desired ($N_q = 1$),

$$N_\rho^m N_V^n = N_L^{-p} \quad (49)$$

If enthalpy levels are high enough to stimulate

chemical reactions in the flow, it will generally not be desirable to scale either ρ or V . However, if the density is sufficiently high in both model and prototype flows to ensure that chemical reaction times are short relative to appropriate flow times,⁴² the density may then be scaled as

$$N_{\rho}^m = N_L^{-p}, \quad (50)$$

again for $N_q = 1$.

Similarity of reactive flows may also be stated in terms of dimensionless coefficients which should have identical values for model and prototype tests. Thus for hypersonic flow and a cold surface, for which the free stream total (stagnation) enthalpy is well-represented by $V^2/2$ and compared to which the enthalpy at the surface is negligible,

$$\frac{N_{u_i}}{\sqrt{Re_L}} = \frac{2 \dot{q}_p C_p L}{k V^2 \sqrt{Re_L}} = K' \rho^{m-\frac{1}{2}} V^{n-\frac{5}{2}} L^{p+\frac{1}{2}} e^{-k_1 V^2} \quad (51)$$

where K' is a modified function of geometry and gas properties. Here also V should not be scaled. For laminar flow the expression on the right is independent of ρ and L . The Nusselt number will then be unscaled if the Reynolds number is maintained, that is, since $N_V = 1$, if

$$N_{\rho} = N_L^{-1} \quad (52)$$

This latter scaling law may also be applied even for

slow reverse reactions at low pressures where binary recombination is the dominant type of slow reaction.⁴³

It has been estimated for the reentry of an ICBM of one meter characteristic length that both radiative heat transfer and tertiary reactions may be neglected above about 180,000 feet or for a similar body of one centimeter characteristic length above 130,000 feet. Consequently, a ballistic range test of a 1/100-scale model of this ICBM must be conducted above the density altitude of 130,000 feet which, therefore, will simulate full-scale at altitudes above 250,000 feet (the density at 130,000 feet is ten times that at about 250,000 feet).

At the stagnation point it is probable that equilibrium prevails in both model and prototype cases below altitudes where aerodynamic heating becomes significant. The above scaling then applies.

(2) High enthalpy flow tests

Hypervelocity conditions cannot generally be maintained for long periods in ground test facilities. The models are also seldom of negligible enthalpy level, compared to those of available hot gas streams, i_s . Formula (47) should then be written

$$\dot{q} = K \rho^m i_s V^{n-2} L^p \left[\frac{i_s - i_w}{i_s - i_w(300)} \right] e^{-k_2(i_s - i_w)} \quad (53)$$

where $i_w(300)$ means "specific wall enthalpy i_w evaluated at 300° Kelvin" and the exponents have their former values (Table K-1).

If the material of the model does not vaporize or outgas, k_2 will be zero.

- (A) One use of Equation (53) is to compare ground tests of materials with free fall flight tests. In such a case direct scaling is not the question, but rather a simulation of resulting heat transfer rates to the material. To achieve comparable heating rates per unit mass would require that we observe the relationship

$$\frac{\dot{Q}'_m}{\dot{Q}'_p} = \frac{[\int \dot{q} dA / \int \rho_s dV]_m}{[\int \dot{q} dA / \int \rho_s dV]_p}$$

$$= \frac{\dot{q}_m}{\dot{q}_p} \cdot \frac{L_p}{L_m}, \quad (54)$$

where dA is the incremental surface area and dV the incremental volume. Thus we may write (for either two- or three-dimensional shapes)

$$\frac{\dot{Q}'}{\dot{q}} = \frac{N_q}{N_L}. \quad (55)$$

If the test is to determine melting rates of material having high thermal conductivity, for example, in a model of one-fifth scale, the heating rates per unit area should also be reduced to one-fifth. Alternatively, the time-scale may be considered to be changed. Then it is seen that for a one-fifth scale model tested at a full-scale unit heating rate, a comparable heating

effect should be expected in one-fifth of the time. That is, for the model, time elapses five times as fast as for the prototype. These relationships are invalid if the material has significant thermal resistance.

- (B) Another use of Equation (53) is in comparing one ground test with another of different scale and of different resulting heat transfer rate.

For example, if material melts and is removed immediately in such fashion as to preserve the shape of the model, it may easily be shown that the reduction of size versus time follows the law

$$(L/L_0)^{1-p} = 1 - C \frac{t}{q_0}, \quad (56)$$

where C is an ablation rate constant, p is taken from Table K-1, and the zero subscripts pertain to zero time.* It is seen that, if L represents a

* Equation (56) is derived as follows: Under the prescribed conditions $\dot{q}$ is simply proportional to L^p , or $\dot{q} = \dot{q}_0 (L/L_0)^p$. Then the rate of recession of the surface is equal to n which is proportional to L , $n = k L$, say, and therefore, if all incoming heat is consumed in change of state,

$$\dot{q} = -\rho_s h_{s-l} \dot{n}, \quad (58)$$

where ρ_s is the density of the solid and h_{s-l} the latent heat of fusion. Therefore, we may combine the above expressions and integrate from zero time to time t to obtain Equation (56) with

$$C = (1-p) / \rho_s h_{s-l} k L_0 \quad (59)$$

body radius, it will reduce more rapidly with time for a given $\dot{q}_0$ and more rapidly at all times for larger values of $\dot{q}_0$ than some reference value (say that of the prototype).

- (C) If material melts and is immediately removed in such fashion as to preserve $\dot{q}$, a different mass removal law applies. Then simply

$$\dot{l} \propto \dot{q} \quad (57)$$

where $\dot{l}$ is the time rate of growth of rod length. Such a process might reasonably describe the end-on heating of a long cylinder, for example.

C. Temperature distribution in homogeneous, solid bodies without heat sources

Finite conductivity will prevent immediate equilibration of internal temperatures. Surface temperatures will also vary with location with a secondary effect upon local heat transfer. This secondary effect will generally be unimportant in free fall, but may be quite significant in laboratory tests wherein the enthalpy difference causing heat transfer is not large compared to surface variations of enthalpy.

- (1) The transient heat conduction equation in dimensionless form is^{45, 50}

$$\frac{\partial \theta^*}{\partial t^*} = \nabla^2 \theta^* \quad (60)$$

and the boundary conditions are

$$\Theta^*(X_1^*, 0) = \Theta^*(X_1^*) \quad , \quad (\text{Initial condition})$$

$$\frac{\partial \Theta^*}{\partial n^*}(X_1^*, t^*) = q_{t^*}^*(X_1^*, t^*)/K^* - \left(\frac{\epsilon \sigma}{K}\right)^* [\Theta^*(X^*, t^*)]^4 \quad (61)$$

The dimensionless variables which appear in (60) and (61) are

$$x_1^* = x_1/L, \quad X_1^* = X_1/L, \quad t^* = t/t_0,$$

$$\Theta^* = \Theta/\theta_0, \quad \theta^* = \theta/\theta_0, \quad (\epsilon \sigma/K)^* = (\epsilon \sigma/K)L\theta_0^3,$$

$$q_{t^*}^* = \dot{q} L/K_0 \theta_0,$$

$$K^* = K/K_0 \text{ and } \dot{h}^* = \dot{h}t_0/L^2. \quad (62)$$

The small x_1 represents the coordinates of an interior point, the X_1 a surface point, the t_0 an arbitrary reference time, θ is an interior temperature, Θ is a surface temperature, θ_0 an arbitrary reference temperature, ϵ is the surface emissivity, K is the thermal conductivity of the solid material, and K_0 is an arbitrary reference value for the thermal conductivity of the solid material.

The similarity equation in this case is

$$\Theta^* = f[X^*, \Theta^*, \dot{h}^*, q_{t^*}^*, K^*, (\epsilon \sigma/K)^* | x_1^*, t^*] \quad (63)$$

The similarity conditions, therefore, are

$$N_{Y_i} = N_{X_i} = N_L, \quad N_\theta = N_\Theta = N_{\theta_o}, \quad N_t = N_{t_o},$$

$$N_q^* = N_{K_o} N_{\theta_o} / N_L, \quad N_k = N_L^2 / N_{t_o},$$

$$N_e = N_K / N_L N_{\theta_o}^3. \quad (64)$$

If model and prototype materials are the same and if all temperatures are unscaled,

$$N_k = N_K = N_e = 1, \quad N_\theta = N_\Theta = N_{\theta_o} = 1,$$

and

$$N_t = N_{t_o} = N_L^2, \quad N_q = N_L^{-1}, \quad (65)$$

This verifies the importance of considering finite thermal conductivity, for here it has been shown that temperature at an interior point increases at a rate inversely proportional to L^2 whereas for infinite thermal conductivity the temperature (heat per unit mass) increased inversely with L as shown in Section 3.B.(2) (see discussion following Equation [55]). Of course, similarity requires also in this instance that $\dot{q}$ vary with L^{-1} whereas in section 3.B.(2) $\dot{q}$ was unscaled. If $\dot{q}$ were unscaled, similarity could be maintained in theory by scaling K with L and then the rates would go as L^{-1} as in Section 3.B.(2).

Comparison of (3.C.(1) (E)) with the last statement of

3. C.(1)(F) reveals an incompatibility in the case of scaling ϵ , however. Therefore, the loss of heat by radiation cannot be scaled properly unless we allow the possibility of exerting separate control over the surface condition. Fortunately, for reentry heating problems this is not an important factor. In certain test facilities such as furnaces or radiant heaters it clearly is an important factor.

Since for identical properties and temperatures $\dot{q}$ must be scaled inversely to L , it is apparent that the stream enthalpy must be varied to accomplish this. Thus, for example, if in Equation (53) only L and ρ are scaled, then necessarily $N_\rho = N^{-1} N^P$. Other adjustments in (53) may accomplish the desired result also.

If a model test is performed in which a scale model ($N_L < 1$) of prototype material ($N_\eta = N_K = N_\epsilon = 1$) and for which $N_{\dot{q}} = 1$, it will be necessary to observe the time scale $N_t = N_L^2$ and the temperature scale $N_\theta = N_L$. For example, if $N_L = 1/5$ the temperature at a point will increase 1/5 of the prototype value increase at a corresponding point in the model but will do so in 1/25 of prototype time. The rate of temperature rise is, therefore, five times as fast in the model as in the prototype. If material melts it will do so five times as fast in the model.

D. Thermal Stresses

For homogeneous, isotropic solids within their elastic limits the thermal stress field is given by⁵¹

$$S_{ij}^* = \frac{1}{2(1+\nu)} \frac{\partial e_j^*}{\partial x_j^*} + \frac{\partial e_1^*}{\partial x_1^*} + \frac{1}{(1-2\nu)} \frac{\nu}{1+\nu} \frac{\partial e_k^*}{\partial x_h^*} - \alpha^* \theta^* \delta_{ij}$$

(66)

$$x_1^* = x_1/L, \quad e_1^* = e_1/L, \quad \theta^* = \theta/\theta_0, \quad \alpha^* = \alpha\theta_0,$$

and

$$S_{ij}^* = S_{ij}/E. \quad (67)$$

Here e_1 represents the total deformation made up of superimposed thermal, sorption, and mechanical strains, in general.

The thermal deformation is governed by

$$\frac{\partial^2 e_1^*}{\partial x_1^{*2}} + \frac{1}{1-2\nu} \frac{\partial^2 e_1^*}{\partial x_1^* \partial x_j^*} = \frac{2\alpha^*(1+\nu)}{1-2\nu} \frac{\partial \theta^*}{\partial x_1^*} \quad (68)$$

subject to the boundary condition

$$\frac{\partial e_1^*}{\partial x_j^*} + \frac{\partial e_1^*}{\partial x_1^*} + \frac{2\nu}{1-2\nu} \frac{\partial e_1^*}{\partial x_1^*} \delta_{ij} = \frac{2\alpha^*(1+\nu)}{1-2\nu} \theta^* \delta_{ij} \quad (69)$$

If the thermal deformation is evaluated by means of the second problem (Equations [68] and [69]) and substituted into (66), that equation will yield a functional expression for S_{ij}^* which may be written

$$S_{ij}^* = f[X^*, \theta^*(x^*, t^*), \alpha^* | x_1^*] . \quad (70)$$

If, in addition, $\theta^*(x^*, t^*)$ as determined by Equation (63) is put into the above function, the complete transient functional dependence of S_{ij}^* upon x^* and t^* may be given expression. It

is sufficient here to note, however, that, as shown by (53), the thermal stresses will be simulated under the same conditions as will θ plus the condition that $N_\alpha = N_\theta^{-1}$ which is satisfied if $N_\alpha = N_\theta^{-1} = 1$.

For rapid heating it is probable that creep and plastic flow of material will produce negligible effects and that the simulation of actual fracture of the material due to thermal stresses will be possible. Brittle materials will be especially amenable to such model tests.

E. Transient gas desorption

(1) Introduction

Structural strains and consequently significant stresses may occur in solid materials which are subjected to rapid heating. The usual thermal stresses discussed in Section 3.D. are an example of this, but if the solid contains large amounts of dissolved gas, an additional stress may be induced as this gas comes out of solution.⁴⁴

There is a class of metals, including most notably titanium, zirconium, and thorium, in which hydrogen is soluble in large amounts (over 200 cc. at S.T.P. per gm.). The sorption process is exothermic which implies that desorption occurs as heat is supplied and as the temperature is raised. Since the solubility is proportional to the square root of the pressure of the external hydrogen gas, it is evident that the gas dissociates upon entering the crystal lattice and presumably migrates as hydrogen ions. Thus the "heat of formation" of the "hydride" as measured is probably actually the net energy evolved in a process wherein dissociation and ionization absorb energy

and the evidently very strong lattice energy is reduced. As gas is absorbed the metal expands to such an extent that its density may be reduced as much as 12 percent.

The measurements of heat of solution, Q_s , are made indirectly, under conditions of thermodynamic equilibrium, by using the relationship

$$s = s_0 P^{\frac{1}{2}} \exp (Q_s / 2R_0 \theta) \quad (71)$$

and measuring s , cc(STP) gas per 100 g metal, at a series of temperatures while holding the pressure constant. $Q_s / 2R_0$ is then the slope of the curve $\ln s$ versus $1/\theta$, where R_0 is the universal gas constant, and θ the (isothermal) metal temperature.

If large samples of metal are "hydrided," the gas in solution must penetrate into the interior by diffusion. A separation of the processes of endothermic dissociation and ionization and from the process of exothermic lattice bonding may then occur. If it is postulated that energy is absorbed near the surface and released in the interior, heat conduction would play a limiting role since hydrogen ions could not be accepted into the interior crystals until energy thus released had been conducted away to the surface. If this is the process, it should be possible to demonstrate it in a steady-state diffusion experiment.

Diffusion rates have been measured for metals which do not behave like zirconium in that they absorb hydrogen endothermically or with little energy release and in

relatively small amounts. The metal samples used were generally thin sheets or slender wires. Thus, although these experiments involve the processes in which we are interested, it is virtually impossible to detect a thermal conductivity effect in the results.

Alternatively, if diffusion is not accompanied by a strong counter-flow of heat, conduction may be only a secondary process. This model postulates that hydrogen ions may move from one site to the next as vacancies occur and with a probability which increases with temperature because of the need to overcome a certain interstitial potential barrier. The principal exchange of heat then occurs at the surface.

In either case there is a possible fluid dynamic factor to be noted. As hydrogen ions emerge they would attract free electrons very readily but might not always be paired at the surface to form hydrogen molecules. This circumstance would be the more probable if the outer gas environment consisted of a high-speed, low density convective flow. In this case the molecular dissociation energy would not be entirely recovered at the surface and, for a given rate of energy supply from the environment, the rate of gas effusion would be reduced.

Another phenomenon which has a bearing upon the structural integrity of a rapidly heated homogeneous specimen is that of crystal expansion due to gas sorption. The density of metals of the zirconium type in the present context may be reduced by as much as twelve percent in the hydriding process. This will be referred to as "sorption expansion" in the following.

(2) Analytical model adopted herein

It is postulated that all hydrogen energy level changes occur at the metal surface except for clearing energy barriers between sites, which is reflected in the variation of the diffusion coefficient with temperature. A certain net heat flux will be required at the surface to allow the hydrogen to effuse. Excess thermal energy brought to the surface from the environment will penetrate into the interior of the metallic solid by conduction.

The metal will be postulated to have a known coefficient of linear thermal expansion, α , which in general may be a function of both temperature and dissolved gas content.

The metal will be postulated also to have a known coefficient of linear sorption expansion, β , which also may vary with temperature and dissolved gas content.

(3) Transient distribution of dissolved gas in homogeneous, solid bodies without interior sources

The mass diffusion equation in dimensionless form is ⁴⁴

$$\frac{\partial s^*}{\partial t^*} = \frac{\partial}{\partial x_1^*} \left(D^* \frac{\partial s^*}{\partial x_1^*} \right) \quad (72)$$

and the boundary conditions are

$$s^*(X_1^*, 0) = s_1^*(X_1^*) \quad (\text{initial condition})$$

$$\frac{\partial s^*}{\partial n^*} (X_1^*, t) = j^*(X_1^*) / \rho_n^* D^*(X_1^*) \quad (73)$$

The dimensionless variables which appear in (72) and (73) are

$$x^* = x_i / L, \quad X_i^* = X_i / L, \quad t^* = t / t_o,$$

$$j_o^* = j_o / s_o, \quad D^* = Dt_o / L^2,$$

$$\rho_m^* = j_o t_o / L s_o \rho_{mo}, \quad \rho_m^* = \rho_m / \rho_{mo}, \quad (74)$$

where s is the "solubility" in cc. (STP) per 100 gm. metal and $s_o \rho_{mo}$ is the "initial" solubility (STP) times the density of the metal at STP of $2s_o \rho_{mo} / 22,400 =$ number of grams of H_2 per 100 cc. of metal in its initially hydrided state at STP.

In exact analogy to the discussion of Section 3. C. (1) we may write down the similarity conditions for this case:

$$N_{x_i} = N_{X_i} = N_L, \quad N_s = N_{s_o}, \quad N_t = N_{t_o}, \quad N_{\rho_m} = N_{\rho_{mo}},$$

$$N_D = N_L^2 / N_{t_o}, \quad N_{j_o} = N_L N_{s_o} N_{\rho_{mo}} / N_{t_o}. \quad (75)$$

Although the scaling of j_o (the mass removal rate of hydrogen from unit surface area) is in principle controllable by the partial pressure of hydrogen in the environment, in practical experiments the range of available pressures will usually be inadequate to overcome the large dissociation pressures produced in the desorption process. Thus with little loss in generality we shall set $N_{j_o} = 1$.

For a given metal under isothermal conditions

$$N_D = N_{\rho_{mo}} = 1, \quad (76)$$

and, consequently, the scaling laws which result are

$$N_{s_o} = N_L$$

$$N_{t_o} = N_L^{\frac{1}{2}} \quad (77)$$

The analogy between gas diffusion and heat conduction is evident when (77) and (65) are compared.

However, in actuality the diffusion coefficient depends strongly upon temperature according to the empirical law

$$D = D_o \exp (- E_o / 2R_o \theta) , \quad (78)$$

where E_o is called the "heat of diffusion." Therefore, in a model experiment of gas desorption it is necessary to have simulated the transient temperature distribution as well. This then requires the simultaneous observation of conditions (77) and (65), which are quite compatible.

F. Superimposed thermal and sorption stresses

- (1) Since it is known that gas sorption results in a structural deformation, we may define a linear "sorption expansion" coefficient β in analogy to the linear thermal expansion coefficient α , and write a generalized volume expansion⁵¹

$$\epsilon = \epsilon_x + \epsilon_y + \epsilon_z = \epsilon(\theta, s, t) \quad (79)$$

is the local equation of state of the solid material and in which

$$\epsilon_x - \alpha T + \beta s = \frac{1}{E} [\sigma_x - \nu(\sigma_y + \sigma_z)] \quad (80)$$

where E is Young's modulus, ν is Poisson's ratio and the σ_i are unit tensile stresses. The ϵ_i are unit strains related to the deformation ϕ_i by

$$\epsilon_i = \frac{\partial \phi_i}{\partial x_i} \quad (81)$$

The equation for total strain is then

$$\nabla^2 \phi_i^* + \frac{1}{(1 - 2\nu^*)} \frac{\partial^2 \phi_j^*}{\partial x_i^* \partial x_j^*} = \frac{2(1 + \nu^*)}{(1 - 2\nu^*)} \alpha^* \frac{\partial \theta^*}{\partial x_i^*} + \beta^* \frac{\partial s^*}{\partial x_i^*} \quad (82)$$

subject to the boundary condition at $x_i \equiv X_i$

$$\frac{\partial \phi_i^*}{\partial x_j^*} + \frac{\partial \phi_j^*}{\partial x_i^*} + \frac{2\nu^*}{1 - 2\nu^*} \frac{\partial \phi_k^*}{\partial x_k^*} \delta_{ij} = \frac{2(1 + \nu^*)}{(1 - 2\nu^*)} [\alpha^* \theta^* + \beta^* s^*] \delta_{ij} \quad (83)$$

The new dimensionless terms are

$$\beta^* = \beta/s_o, \quad s^* = s/s_o \quad (84)$$

Similarity requirements for modeling this state of total strain and total stress are analogous to those for thermal stresses. That is, the conditions noted above in (65), (70), and (77) must all be observed and in addition

$$N_\beta = N_{s_o}^{-1} = 1 \quad (85)$$

This implies the same material in model and prototype. In addition this implies $N_L = 1$ (see [77]) so scale modeling of this phenomenon is out of the question.

4. CONCLUSIONS AND RECOMMENDATIONS

This similarity analysis has determined a number of scaling parameters which must be maintained at their prototype values in any valid scale model test. In some cases of significance similarity is found to require a full-scale test.

It is to be recommended that such analyses guide all experimentalists who are attempting to determine empirical laws. If the phenomena under scouting are not described by well-formulated equations, it is probably inadvisable to attempt scale testing. The cruder, but more flexible, guidance of dimensional analysis has been found to be useful in such instances, however.

Further studies of scaling parameters may well extend our knowledge of tests capable of being conducted at model scale.

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Nomenclature

<u>Symbol</u>	<u>Definition</u>
A	Area
a, b, c	Coefficients
C_D, C_L, C_M	$D/qA, L/qA, M/qA\ell$, force and moment coefficients
C	See Equation (59)
C_p	Specific heat at constant pressure
D	Drag force, diffusion coefficient
E	Young's modulus
E	Heat of diffusion
e_i	Strain in i-direction
f	Function
g	Dynamic pressure
g	Acceleration due to gravity at altitude
g_c	Acceleration due to gravity at sea level
$h_{s-\ell}$	Latent "heat" of fusion
I	Moment of inertia
i	Enthalpy, thermal energy per pound
j	Mass flux, mass flow per unit area per unit time
$K, \bar{k}_1, K', \bar{k}, \bar{k}_2$	Constants in Equations (47), (51) and (53), thermal conductivity of solid material in Equation (61)
L	Lift force, characteristic length
ℓ	Characteristic length, rod length
ℓ, L	Characteristic length
ℓ_n	Natural logarithm
M	Pitching moment
m	Mass

<u>Symbol</u>	<u>Definition</u>
N_a	$(a)_m / (a)_p$, scale factor
$o()$	Terms of order of () or higher
$\dot{Q}$	See Equation (54)
Q_s	"Heat of formation" of hydride, see Equation (71)
$\dot{q}$	Heat transfer rate, thermal energy per unit
R_u	Universal gas constant
Re_L	Reynolds number based upon free stream conditions and the length L
r	Distance from center of gravity to center of aerodynamic pressure
S_{ij}	Stress tensor
s_o	Solubility, see Equation (71)ff
t	Independent variable, time
V	Velocity
W	Weight at sea level
X, Y, Z	Initial values, boundary values
x, y, z	Dependent variables, space coordinates (y vertical), also see Equation (14)
α	Linear thermal expansion coefficient
α	Angle of attack, Figure 2
β	See Equation (23), also linear sorption expansion coefficient, see Section K, E. (3)
γ	See Figure 1 or 2
δ_{ij}	Kronecker's delta
ϵ	Thermal emissivity
θ	$\alpha + \gamma$, see Figure 2, temperature
Θ	Surface temperature

<u>Symbol</u>	<u>Definition</u>
K	Thermal diffusivity
ν	Poisson's ratio
Π	Dimensionless product
ρ	Mass density of gas
ρ_m	Mass density of metal
τ	t/t_0
ϕ_i	Deformation in i-direction
∇^2	The Laplacian operator
Subscripts	
E	Ejection value
i	Initial value
m, p	Model, prototype values
R, o	Reference value, in Equation (23) sea level value
s	Stream value
t	Differentiation of quantity () with respect to t
w	Wall value
Superscripts	
$(^o)$	Differentiation with respect to time
m, n, p	Constant exponents
*	Dimensionless quantity

REFERENCES

1. Research and Advanced Development Division, AVCO Corp., "Investigation of Reentry Destruction of Nuclear Auxiliary Powerplant," AFSWC-TR-61-69, Oct. 1961.
2. Pugh, H. F. L., "Description of Minimum IBM-704 Mass Point Trajectory Program, Rotating Oblate Earth and 1959 ARDC Model," AFSWC-TN-60-1, Jan. 1961.
3. O'Neill, R. F. and Reendaal, D. C., "The Variable Boundary Transient Heat Conduction Program," AE62-0401, GD/Astronautics, 1 May 1962.
4. Detra, Kemp and Riddell, "Addendum to "Heat Transfer to Satellite Vehicles Reentering the Atmosphere," Jet Propulsion 27, 12, p. 1256, Dec. 1957.
5. Eggers, Hansen and Cunningham, "Stagnation Point Heat Transfer to Blunt Shapes in Hypersonic Flow, Including the Effects of Yaw," NACA TN 4229.
6. Boison, J. C. and Curtiss, H. A., "An Experimental Investigation of Blunt Body Stagnation Point Velocity Gradient," ARS Journal, 29, 2, Feb. 1959.
7. Eckert, E. R., "Survey on Heat Transfer at High Speeds," ADC Technical Report 54-70.
8. Romig, M., "The Reference Temperature Method for Skin Friction and Heat Transfer in Compressible Flow," A-Atlas-151, Dec. 1954.
9. Fisher, L. and Smith, D. J., "Analytical Methods and Equations for Use in an IBM-704 Program to Determine the Effects of Aerodynamic Heating by the Reference Enthalpy Method," Convair Report T-C-15, 28 Oct. 1958.
10. Lockheed Aircraft Corp., Missiles and Space Division, "Thermodynamic Analyses of SNAP-1 Reentry," LMSD-445651-A, 9 Sept. 1960 (Secret).
11. Joint Technical Note, Lockheed Corp. and Martin Corp., "Sensitivity of Ballistic Satellite Reentry Trajectory Predictions to Differences in Geophysical Properties."

12. Corliss, W. R., Propulsion Systems for Space Flight, McGraw-Hill, 1960.
13. Kemp, N. H., Rose, P. H. and Detra, R. W., "Laminar Heat Transfer Around Blunt Bodies in Dissociated Air," AVCO Research Lab., AVCO Mfg. Co. Research Report 15, May 1958.
14. Tewfic, O. K. and Giedt, W. H., "Local Heat Transfer, Recovery Factor, Pressure Distribution Around a Circular Cylinder Normal to a Rarefied Supersonic Airstream," Heat Transfer and Fluid Mechanics Institute, 1959.
15. Fay, J. A. and Riddell, F. R., "Theory of Stagnation Point Heat Transfer in Dissociated Air," J.A.S., Vol. 25, No. 2, 1958.
16. Allen, H. and Eggers, A., "A Study of the Motion and Aerodynamic Heating of Missiles Entering the Earth's Atmosphere at High Supersonic Speeds," NACA TN 4047, October 1959.
17. Hochstim, A. R., Convair Report APH 9GP-002.
18. JANAF Thermochemical Tables, Vol. 1 and 2, ARPA, USAF Contract No. AF 33(616)-6149, Dow Chemical Company.
19. Selected Values of Chemical Thermo Dynamic Properties, N. B. S. Circular 500, 1952.
20. Gallaher, W. H. and Sibulkin, M. S., "On the Successful Re-entry of Space Fragments From a Decaying Earth Orbit," submitted to AIAA Journal.
21. Chapman, D. R., "An Approximate Analytical Method for Studying Entry Into Planetary Atmospheres," NACA TN4276, May, 1958.
22. Kemp, N. H., Rose, P. H., and Detra, R. W., "Laminar Heat Transfer Around Blunt Bodies in Dissociated Air," Journal Aero. Sci., Vol. 26, No. 7, pp. 421-430, July, 1959.
23. Rich, J. B., Walten, G. P. and Bames, R. S., J. Nuclear Materials 4, 287 (1961).
24. Rich, J. B., Ridding, G. B., and Bames, R. S., J. Nuclear Materials 1, 96 (1959).

25. Ells, C. E. and Perryman, E. C. W., J. Nuclear Materials 1, 73 (1959).
26. Bames, R. S., Churchman, A. T., Curtis, G. C., Eldred, V. W., Enderly, J. A., Foreman, A. J. E., Plail, O. S., Pugh, S. F., Walten, G. N. and Wyatt, L. M., Second Geneva Conference (1958).
27. Churchman, A. T., Bames, R. S. and Gotrell, A. M., J. Nuclear Energy 7, 88 (1958).
28. "Worldwide Effects of Atomic Weapons," Project Sunshine R-251-AEC, (1953) p. 34.
29. H. Lamb, Hydrodynamics, Dover Publications, P. 299, 1945.
30. F. N. Frankiel and J. Katz, "Studies of Small-Scale Turbulent Diffusion in the Atmosphere," Journal of Meteorology, 13, 388, (1956).
31. W. W. Kellogg, "Diffusion of Smoke in the Stratosphere," Journal of Meteorology, 13, 241, (1956).
32. O. G. Sutton, Micrometeorology, McGraw-Hill Book Co. (1953), p. 287.
33. J. R. Banister and H. L. Davis, "Descent of Particles and Molecules Through the Upper Atmosphere," Physics of Fluids, 5 #2, 136, (1962).
34. G. C. Dale, "Safety Levels for Contamination from Fall Out From Atomic Weapons Trials," FWE-107(AEC) Oct. 1955.
35. R. R. Rapp, private communication.
36. Addendum to NBS Handbook 59, "Permissible Dose from External Sources of Ionizing Radiation," April 15, (1958).
37. Minzner, R. A., Champion, K. W., and Pond, H., The 1959 ARDC Model Atmosphere, Air Force Surveys in Geophysics, No. 115, ARDC, 1959.
38. Sissenwine, N., Wind Speed Profiles, Wind Shear, and Gusts for Design of Guidance Systems for Vertical Rising Vehicles, Air Force Surveys in Geophysics, No. 57, ARDC, 1954.
39. Pant, P. S., Circulation in the Upper Atmosphere, Scientific Report No. 1: Contract No. AF 19(604)-1006, New York University, May 1955.

40. Stroud, W. G. Nordberg, W., and Walsh, J. R. Atmospheric Temperature and Winds Between 30 and 80 km, J. of Geo. Res., 61, 45, 1956.
41. Handbook of Geophysics, USAF ARDC, MacMillan, New York, 1960.
42. v. Karman, Theodore, "Fundamental Equations in Aerothermochemistry," Agard Combusion Colloquium, Liege, Belgium, Dec. 1955.
43. Acton, L. L. and J. A. L. Thomson, "Scaling Methods," (unpublished).
44. Dushman, S., Scientific Foundations of Vacuum Technique, Second Edition, John Wiley and Sons, 1962.
45. Scipio, L. Albert II and Lee C. Teng, "Analytical Study of Similarity Parameters for Aerodynamic Model Testing at High Temperatures," Rosemount Aeronautical Laboratory Research Report No. 104, University of Minnesota, August 1954.
46. Leadon, Bernard M., "Similarity Parameters for Aerodynamic Model Testing at High Temperatures," Aerodynamics Panel Meeting, Project Bumblebee, June 1955.
47. Remmler, Karl L., "Tumbling Bodies Entering the Atmosphere," ARS Journal, Vol. 32, No. 1, 92-95, Jan. 1962.
48. Markowitz, J. M., "Hydrogen Redistribution in Thin Plates of Zirconium under Large Thermal Gradients," AEC Research and Development Report WAPD-TM-104, Jan. 15, 1958.
49. Hanley, G. M., "Scaling Effects on the NAP Burnup," North American Aviation First Quarterly Progress Report, Contract AF29(601)-5104, Project 1831, Task 79512, 11 May 1962.
50. Carslaw, H. S., and J. C. Jaeger, Conduction of Heat in Solids, London, Oxford University Press, 1947.
51. Timoshenko, S. and J. N. Goodier, Theory of Elasticity, Second Edition, McGraw-Hill, 1951.

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