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PROPAGATION OF DENSITY DISTURBANCES IN AIR-WATER FLOW

by

George P. Nassos

RELEASED FOR ANNOUNCEMENT
IN NUCLEAR SCIENCE ABSTRACTS

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IN NUCLEAR SCIENCE ABSTRACTS

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and
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FOREWORD

This report is one of a series that describes heat-transfer and fluid-flow studies performed at Argonne under a program sponsored jointly by the Associated Midwest Universities and the Argonne National Laboratory.

The earlier reports in this series are:

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NOMENCLATURE

Symbol	Description	Dimension	Symbol	Description
A	Cross-sectional area	L^2	$\Delta P_{F\ell}$	Frictional pressure drop, liquid system only
a	Characteristic quantity	a	Q	Volumetric flow rate
a_0, a_1, a_2	Constants defined for Eq. (C.11)	Dimensionless	Q_0	Strength of air source (ft^3/ft^2)
B	Number of cycles per run	Dimensionless	q	Characteristic flow
b	Value of i at maximum Y	Dimensionless	R	Radius
C	Capacitance		R	Resistance
C	Flow parameter	Dimensionless	R_0	Gas constant
C_0	Flow parameter defined by Eq. (2.1)	Dimensionless	r	Radius, variable
C_1	Constant defined by Eq. (3.8)		s	Number of bubbles
C_2	Constant defined by Eq. (C.8)	Dimensionless	T	Time interval
C_3	Constant defined by Eq. (C.9)	$1/L$	t	Time
D	Diameter	L	t_{avg}	Average dwell time of bubble
E	Voltage		t_0	Initial time
E_C	Constant voltage		t_s	Sampling time
e	Input voltage to integrating circuit		V	Average velocity
e_0	Output voltage of integrating circuit		V_d	Weighted average drift velocity
f	Fanning friction factor	Dimensionless	$V_{\ell\ell}$	Average liquid velocity, liquid system only
g	Gravitational acceleration	L/t^2	V_0	Steady-state value of V
H	Distance, as seen in Fig. 3.6	L	V_s	Superficial velocity
h	Manometer deflection of two-phase mixture	L	v_v	Local drift velocity
h_1	Distance, as seen in Fig. 3.7	L	v	Local velocity
h_2	Distance, as seen in Fig. 3.7	L	W	Mass flow rate
h_ℓ	Manometer deflection of liquid system	L	X	Quality
i_R	Current through resistor		x	Velocity-profile exponent
i_C	Current through capacitor		Y	Normalized value of probe output signal
K	Flow parameter defined by Bankoff	Dimensionless	$\bar{Y}$	Defined by Eq. (5.1)
k	Characteristic concentration	a/L	$\bar{\bar{Y}}$	Defined by Eq. (5.5)
L	Distance between pressure taps, characteristic length	L	y	Decayed fraction of perturbation
M	Characteristic mass	M	z	Distance
N	Number of samples per cycle	Dimensionless		
N_{Re}	Reynolds number, $DV\rho/\nu$	Dimensionless	Greek Symbols	
n	Concentration-profile exponent	Dimensionless	α	Local void fraction
P	Pressure	M/L^2	α_{avg}	Average local void fraction
P_0	Atmospheric pressure	M/L^2	α_{cor}	Corrected local void fraction
ΔP_F	Frictional pressure drop, two-phase mixture	M/L^2		

<u>Dimension</u>	<u>Greek Symbols</u>	<u>Description</u>	<u>Dimension</u>
M/L^2	$\langle a_y \rangle$	Void fraction defined by Eq. (6.29)	Dimensionless
L^3/t	$\langle a_z \rangle$	Void fraction defined by Eq. (C.6)	Dimensionless
L	β	Volumetric gas fraction	Dimensionless
a/t	Γ	Gas source strength	$1/t$
L	ϵ	Defined by Eqs. (5.2) and (5.3)	Dimensionless
	η	Number of whole wavelengths between two probes	Dimensionless
ML/θ	θ	Temperature	
L	κ	Diffusion coefficient	L^2/t
Dimensionless	λ	Wavelength	L
t	μ	Viscosity	M/Lt
t	ξ	Maximum value of Y	Dimensionless
t	ρ	Density; density of manometer fluid	M/L^3
t	σ	Standard deviation	Dimensionless
t	τ	Period	t
L/t	ϕ	Slip-velocity ratio	Dimensionless
L/t	<u>Indices</u>		
	i	Sample number	
L/t	j	Cycle number	
L/t	k	Signal number	
L/t	p	Probe number	
L/t	<u>Subscripts</u>		
L/t	1	Downstream	
M/t	2	Upstream	
Dimensionless	A } B } C }	Points in Fig. 3.6	
Dimensionless	c	Centerline	
Dimensionless	g	Refers to gas phase	
Dimensionless	l	Refers to liquid phase	
Dimensionless	m	Refers to mixture	
Dimensionless	o	Refers to steady-state conditions	
L	Sw	Refers to "shock" wave	
	W	Wall of circular channel	
	w	Refers to void wave	
Dimensionless	<u>Superscripts</u>		
Dimensionless	*	Dimensionless quantity	
Dimensionless	'	Time-fluctuating quantity	
	-	Time-average quantity	
	$\langle \rangle$	Cross-sectional average quantity	

PROPAGATION OF DENSITY DISTURBANCES IN AIR-WATER FLOW

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ABSTRACT

The principal objective of this study was to investigate the behavior of density waves propagating vertically upwards in an atmospheric-pressure air-water system. A square-wave pattern was imposed on the solenoid valve controlling the secondary air flow, resulting in a regular sequence of density disturbances. These disturbances were measured at five axial and five radial locations by an improved version of an electrical-resistivity probe developed earlier. When the radius of curvature of the probe tip was decreased to less than 0.001 in., and the mixture velocity was increased by a factor of three, the error in the void-fraction determination decreased from 35% to less than 7%.

Two air-water mixers were used for producing the primary air-water flow and the secondary perturbations. Twenty runs were made in which the steady-state air flow, the perturbation air flow, and the liquid velocity were varied. For each run, the void response at each axial and radial position was recorded on an FM tape recorder, and subsequently sampled by an analog-to-digital converter, the output of which was analyzed by a computer. All the computer results are presented in their original form; a few typical runs are presented in graphical form.

A plot of the computer results depicted qualitative agreement with the kinematic wave theory. Although analysis of the void-fraction response is subject to errors when the sampling rate and/or the number of samples are low, this sampling technique was used to calculate the propagation velocity and the decay of the disturbances.

A comparison of the experimental propagation velocity with predictions from the kinematic wave theory shows fairly good agreement, but there was a tendency, at low mixture velocities, for the experimental velocity to be greater than the theoretical velocity; for large mixture velocities, the reverse was true. Also, at low mixture velocities, the propagation velocity was greater than the gas velocity; at higher mixture velocities, the correlation was reversed. This does not agree with the assumptions that the flow parameter, C_0 , and the weighted average drift velocity, V_d , remain essentially constant for a given flow regime. A correction for the discrepancy of the experimental results with theory is suggested by modifying the basic continuity equations to take into account turbulent diffusion, which is believed to be very important.

A comparison of the experimental decay rate with that predicted by the kinematic wave theory shows, in general, a perturbation decay rate considerably higher than predicted. This was probably attributed to axial turbulent diffusion, which is not considered by the kinematic wave theory. A diffusion equation for two-phase flow was solved, and diffusion coefficients were calculated from the experimental results. These coefficients tended to increase with increasing downstream void fraction and increasing mixture velocity. Also, the radial diffusivity profile decreased strongly near the wall.

Steady-state data were plotted on a V_g -versus- V_m plane, and the points, in general, could be described by a straight line with a slope of $C_0 = 1.1$ and an intercept of $V_d = 0.7$ ft/sec. This value of C_0 would indicate flat velocity and concentration profiles, while experimental data showed concentration profiles described by a one-third or one-fourth power law distribution. The time-averaged transient data were also plotted on a V_g -versus- V_m plane resulting in a rather straight line with a slope of $C_0 = 1.2$ and $V_d = 1.0$ ft/sec. The difference between the steady-state data and time-averaged transient data may be attributed to the fact that $dC_0/dt \neq 0$, and that both C_0 and V_d may be functions of the mixture velocity.

I. INTRODUCTION

Instability of a boiling-water nuclear reactor can easily result from the presence of a feedback loop consisting of the reactivity of the reactor, the vapor fraction in the coolant channel, and the power level of the reactor. In the past, investigations of these instabilities could not be completed without making some assumptions as to the behavior of variations or changes in the vapor fraction (void fraction). Recently, theoretical models have been suggested for predicting, from steady-state characteristics, the propagation velocity and the decay (or dispersion) of variations (perturbations) in the void fraction. However, limited experimental data are available for verification of these theoretical predictions.

In this work, a forced-circulation air-water loop was employed for investigating the behavior of void perturbations. Upon attaining steady-state conditions, disturbances in the void fraction were superimposed at very low frequencies (~ 0.4 cps). Transient measurements of the void fraction required further development of an electrical-resistivity probe. This technique is capable of measuring locally the void fraction and also gives information on bubble frequency and bubble-size distribution. Five probes were inserted in the clear plastic test section at different axial positions to facilitate observance of the void perturbations as they propagate through the channel. The probes were also traversed in a radial direction. The electrical response of each probe was recorded on a magnetic tape recorder. These signals were sampled by an analog-to-digital converter, the results of

which were analyzed by a computer. The experimental results were then compared with the theoretical predictions, and on this basis, improvements in the theory were suggested.

A better understanding of hydrodynamic transient behavior will generate more confidence in the design of boiling-water reactor systems.

II. REVIEW OF LITERATURE

A. Void-fraction Measuring Techniques

In the study of two-phase gas-liquid flow, one of the most important parameters of interest is the void fraction or gas fraction. Consequently, the measurement of this parameter has been given considerable attention. Techniques have been developed for measuring the cross-sectional average void fraction, and more recently for measuring local values of void fraction.

One of the simplest methods for determining the average void fraction in small-diameter channels is from pressure-drop measurements. Corrections must be made for pressure drops due to friction and momentum losses. However, in bubbly flow at low liquid velocities these corrections are small and, in some cases, negligible. This technique is explained in more detail in Section III-C-1.

The pressure-drop technique was modified by Zuber and Hench¹ by fixing one tap near the bottom of the test section and placing a movable tap near the top. The manometer leg connected to the fixed tap was filled with water. The other leg was filled with air by bleeding a small amount of air into it so that the air bubbled out of the upper tap very slowly. By observing the manometer readings before and after the air flow was initiated, one could determine the average void fraction between the two taps.

In systems where the void fraction is a function of axial distance, the pressure-drop method does not determine the cross-sectional average void fraction but the volumetric average between the two taps.

Johnson and Abou-Sabe² placed special disk-type valves, of uniform tube diameter when open, at each end of the test section. The spring-loaded valves were closed rapidly and simultaneously to isolate the contents of the test section. This method also measures the volumetric average void fraction as compared to a cross-sectional average.

A dilatation technique was used by Schrock et al.³ to measure the voids in a heated, pressurized loop. A reference state was obtained with the loop completely filled with liquid. The liquid that was displaced from the system when voids were formed was measured in a calibrated burette.

A tracer technique was used by Dengler⁴ for measuring the cross-sectional, average void fraction. A radioactive salt, $\text{Mn}^{52}\text{Cl}_2$, was dissolved in the two-phase mixture. Assuming that the salt was not present in the gaseous phase and did not adhere to the channel walls, the radioactive count was proportional to the liquid fraction.

Probably the most popular method for void-fraction measurements is by gamma-ray attenuation. This technique, studied extensively by Hooker and Popper,⁵ employed a radioactive source, such as thulium, and a crystal detector housed in a photomultiplier tube. The strength of the attenuated beam passing through the two-phase mixture was a function of the mixture density and, hence, related to the void fraction.

Petrick⁶ improved the gamma-ray technique by traversing the source and detector across the channel width, and thus a chordal density profile was obtained. Petrick⁷ also introduced a polynomial-fitting technique for obtaining void-fraction profiles in circular channels.

Perkins *et al.*⁸ attenuated beta rays for measuring voids similar to the gamma-ray technique. Because the beta rays were more readily absorbed, this method was more sensitive; however, because of their low energy, only a limited channel depth and wall thickness could be used.

Orbeck⁹ designed an impedance void meter for measuring the transient void content inside the shroud of an instrumented fuel assembly of a boiling-water reactor. This void meter consisted of concentric shells insulated from each other so that the impedance between them could be measured. The impedance was composed of the vector sum of the resistive and reactive or, in this case, capacitive components. Introduction of a two-phase mixture increased the two components in the same proportion so that the phase angle remained unchanged, and the sum gave a measure of the void content.

Each of the previously-mentioned techniques was designed for measuring the average void fraction in a two-phase system. However, in many instances it is necessary to determine local values of the void fraction along with other parameters. A few methods capable of measuring local values of void fraction are reviewed in the following discussion.

Neal¹⁰ constructed an electrical-resistivity probe for measuring local parameters in mercury-nitrogen flow. This technique was based on the assumption that the electrical resistivity of each of the two phases differed substantially. The probe consisted of an insulated needle with an exposed tip pointing into the flow. In employing this technique, however, the continuous phase had to be electrically conductive.

This was modified by Nassos¹¹ to adapt it for an air-water system. A trigger circuit was introduced to eliminate irregularly shaped pulses

caused by wetting of the probe tip, which was not present in the mercury system. Void-fraction measurements were then taken at various radial positions, and the resulting void-fraction profiles were integrated to obtain cross-sectional averages. These average values were compared with values obtained from the gamma-ray attenuation and pressure-drop techniques. The results showed that the probe values were low, and improvement of this technique was needed. This is discussed in Chapter IV.

Solomon¹² used the same principle and constructed a similar probe to be used as a flow-regime transition detector in air-water flow. This probe was constructed primarily for qualitative studies, as compared to the previously-mentioned ones that were used for quantitative studies. Consequently, it was not necessary for this probe to be constructed for very sensitive measurements. Griffith¹³ employed this flow-regime detector to investigate the annular slug-flow regime in vertical upflow without heat addition and at elevated pressures.

Rather than employing an electrical-resistivity probe, Lackme¹⁴ inserted a wire perpendicular to the flow of a two-phase air-water system. The response of the bare tip was observed on a high-frequency recorder, and the results were used to make a statistical analysis of the local structure of two-phase flow.

Bencze and Orbeck¹⁵ more recently developed a probe which measures the difference in a.c. impedances between the gas and liquid phases as compared to the previously-mentioned probes that measure d.c. resistance. This probe was constructed from a coaxial cable encased in a stainless-steel tube. A 100-kc/sec signal was used as the input to a simple bridge circuit, which was balanced when the probe tip was completely in water. In a gas or vapor state, an unbalanced output voltage was obtained and then introduced to a triggering circuit, the ultimate output of which was analyzed.

An electronic method of recording the stratification of a steam-water mixture was used by Katarzhis, Kosterin, and Sheinin.¹⁶ The temperature and the ohmic resistance of an electrically-heated filament placed in a steam-water mixture at a known pressure varied, depending on whether steam or water surrounded the filament. This was due to the different rates of heat transfer in the two phases and made it possible to determine which phase was present.

Hsu, Simon, and Graham¹⁷ applied the same heat-transfer principle in constructing a hot-film anemometer capable of detecting the void distribution. Since the heat-transfer rate is also a function of the velocity of the phase in contact, this technique, with further improvement, could be used in determining local values of gas and liquid velocities. It appears, however, that further improvement in the electronic circuitry is needed before this technique can be used for void-fraction measurements because of the lack of a sharp demarcation in distinguishing the two phases.

A sampling probe was designed by Wicks and Dukler¹⁸ for measuring entrainment in air-water horizontal flow. This probe consisted of a stainless-steel tube that was open into the direction of flow and thus withdrew a sample of the two-phase mixture that passed into a separator. The liquid was directed to a calibrated burette, and the gas was vented to the atmosphere. Knowing the pipe and probe dimensions, the total entrainment was determined. The capture efficiency (the ratio of the actual droplet capture rate to that if there were no interference from the sampler) was found to be essentially 100%.

A similar sampling probe was constructed by Adorni *et al.*¹⁹ for determining phase and velocity distributions in two-phase adiabatic flow. The sampling was called isokinetic because the pre-existing conditions at the inlet of the probe were not changed. A different isokinetic sampling probe was developed for phase- and velocity-distribution measurements in two-phase flow near the wall of a conduit.²⁰

B. Steady-state, Two-phase, Flow Correlations

Several theoretical models for predicting two-phase flow quantities have been proposed, but only one will be discussed here.

Neal²¹ derived expressions for predicting the slip-velocity ratio and phase velocities in the bubble and slug-flow regimes. He noted that slip in two-phase (gas-liquid) flow is a result of the difference in the gravitational forces acting upon the phases and the existence of radial gradients of void fraction and stream velocity. The model introduced two flow parameters: (1) a measure of the resistance to gas motion with respect to the liquid, and (2) a correlation coefficient between the void fraction and the mixture (stream) velocity.

The rise velocity (with respect to the surrounding liquid) of slugs or bubbles large enough so that the tube walls influence their shape was given by

$$C \sqrt{gD \frac{\rho_l - \rho_g}{\rho_l}},$$

where C is the parameter that is a measure of the resistance to bubble motion with respect to the liquid, and D is the tube diameter. Since C is dependent upon the inertial, viscous, surface, and gravitational forces, the buoyant velocity of a group of bubbles depends upon the tube diameter rather than the individual bubble dimensions.

When the liquid flow rate is not zero, a second component for the gas velocity was introduced because of the gas being carried along the channel by the surrounding liquid. In fully-developed flow, this component

is generally greater than the cross-sectional average stream velocity, because the bubble concentration is a maximum at the center of the channel where the velocity is maximum. Thus a correlation coefficient was defined by

$$C_0 \equiv \frac{\int_0^A \alpha v \, dA}{\frac{1}{A} \left[\int_0^A \alpha \, dA \right] \left[\int_0^A v \, dA \right]} = \frac{\langle \alpha v \rangle}{\langle \alpha \rangle \langle v \rangle}, \quad (2.1)^\dagger$$

where $\langle \alpha \rangle$ is the cross-sectional average void fraction, and v_m is the local stream velocity defined by

$$v = \alpha v_g + (1 - \alpha) v_\ell. \quad (2.2)$$

In Eq. (2.2), v_g and v_ℓ are the local gas and liquid velocities, respectively. The average gas, liquid, and mixture velocities are then defined by

$$V_g \equiv \frac{Q_g}{A \langle \alpha \rangle} = \frac{\langle \alpha v \rangle}{\langle \alpha \rangle};$$

$$V_\ell \equiv \frac{Q_\ell}{A \langle 1 - \alpha \rangle} = \frac{\langle (1 - \alpha) v \rangle}{\langle 1 - \alpha \rangle};$$

and

$$V_m \equiv \frac{Q_g + Q_\ell}{A} = \langle v \rangle.$$

Addition of the two components results in a cross-sectional average gas velocity with respect to the tube wall given by

$$V_g = C_0 V_m + C \sqrt{gD \frac{\rho_\ell - \rho_g}{\rho_\ell}}. \quad (2.3)$$

Using the relation

$$V_m = \langle \alpha \rangle V_g + (1 - \langle \alpha \rangle) V_\ell$$

and Eq. (2.3), the following expression for the slip-velocity ratio was derived:

$$\Phi = \frac{V_g}{V_\ell} = \left[\frac{1 - \langle \alpha \rangle}{1/C_0 - \langle \alpha \rangle} \right] \left\{ 1 + \frac{C}{C_0} \left[\frac{(\rho_\ell - \rho_g) gD}{\rho_\ell V_{sl}^2} \right]^{\frac{1}{2}} \right\} \quad (2.4)$$

[†] Neal inadvertently omitted the factor $1/A$.

The second factor on the right-hand side of Eq. (2.4) is the contribution to slip due to the buoyant forces. If this is neglected, Eq. (2.4) is then identical to that derived by Bankoff²² with

$$C_0 = \frac{1}{K}.$$

More recently, a general expression for analyzing and interpreting experimental data was derived by Zuber and Findlay.²³ Their analysis took into account the effect of nonuniform flow and concentration profiles, as well as the effect of the local relative velocity between the phases.

A three-dimensional problem was considered, with the velocities expressed in terms of vectors. Employing the continuity equation and obtaining cross-sectional average values from the velocity vectors, an equation identical to Eq. (2.3) was derived. The first term on the right-hand side of Eq. (2.3) takes into account the effect of nonuniform flow and concentration profiles; the second term (defined as the weighted average drift velocity) accounts for the effect of the local relative velocity between the two phases. Expressions for this second term were given for each of the slug-flow and bubbly churn-turbulent regimes.

Values for C_0 can be determined from Eq. (2.1) by integrating the concentration and velocity profiles. Zuber and Findlay, however, postulated that by plotting the average gas velocity, V_g , along the ordinate, and the average mixture velocity, V_m , along the abscissa, C_0 as well as the weighted average drift velocity could be obtained from the slope and intercept, respectively. Furthermore, it was stated that a changing slope is indicative of changing profiles or developing flow.

An expression similar to Eq. (2.3) was obtained by Griffith and Wallis²⁴ in an analysis of two-phase slug flow. Since flat profiles were obtained, the value of C_0 in this case was equal to unity.

C. Transient Analysis; Kinematic Waves

A new class of wave motions, physically quite distinct from the classical wave motions encountered in dynamical systems, was discussed by Lighthill and Whitham in analyzing flood movement in long rivers²⁵ and traffic flow on long crowded roads,²⁶ and by Kynch²⁷ in analyzing the problem of sedimentation. These waves were considered only for one-dimensional flow systems where there is a functional relationship between the flow, q (quantity passing a given point per unit time), the concentration, k (quantity per unit distance), and the position, z . The wave property follows from the equation of continuity alone; therefore, these waves were described as "kinematic" in contrast to "dynamic" waves, which depend on Newton's second law of motion.

An important difference between the two types of waves is that kinematic waves possess only one wave velocity at each point, but dynamic waves possess at least two velocities, i.e. forwards and backwards relative to the medium. The lone wave velocity of kinematic waves results from the fact that the equation of continuity

$$\frac{\partial k}{\partial t} + \frac{\partial q}{\partial z} = 0 \quad (2.5)$$

is of the first order. If it is assumed that

$$q = q(k, z), \quad (2.6)$$

then, on multiplying Eq. (2.5) by

$$V_w = \left(\frac{\partial q}{\partial k} \right)_z, \quad (2.7)$$

the following relationship is obtained:

$$\frac{\partial q}{\partial t} + V_w(k, z) \frac{\partial q}{\partial z} = 0. \quad (2.8)$$

Equation (2.8) shows that q is constant on waves traveling past the point with velocity, V_w . This equation has but one system of characteristics ($dz/dt = V_w$) along each of which the flow q is constant. On the other hand, the characteristics of the wave equation are the solution of a quadratic equation.

The local[†] velocity at a point is given by

$$V = \frac{q}{k},$$

so that, from Eq. (2.7),

$$V_w = V + k \frac{\partial V}{\partial k}. \quad (2.9)$$

If $\partial V / \partial k$ is positive, i.e., the local velocity increases with concentration (as in rivers where concentration is taken as the depth), the wave velocity is greater than the mean velocity; whereas, if $\partial V / \partial k$ is negative (as in traffic flow), the wave velocity is less than the mean velocity.

Kinematic waves are not dispersive; however, they do change in form because of nonlinearity. This occurs when faster waves overtake slower

[†]Since this analysis is based on a one-dimensional problem, the local velocity is equal to the average velocity.

ones. Since it is impossible to have more than one value of concentration at any particular point (at the same time), a discontinuity results. Because their process of formation is similar to that of shock waves in a gas, these discontinuities were also described as shock waves. If q_1 and k_1 are the flow and concentration values on one side of the shock wave, and q_2 and k_2 are the values on the other side, then the quantity crossing the shock wave per unit time may be written as either $q_1 - V_{Sw}k_1$ or $q_2 - V_{Sw}k_2$, where V_{Sw} is the velocity of the shock wave. Equating the two above expressions gives

$$V_{Sw} = \frac{q_2 - q_1}{k_2 - k_1} \quad (2.10)$$

This is the slope of the chord joining the two points on a flow-concentration curve, for a particular value of z , which correspond to the states ahead of and behind the shock wave when it reaches z . In the limit, when the shock wave becomes a continuous wave, the slope of the chord becomes the slope of the tangent, and the velocity, V_{Sw} , becomes the wave velocity. Then

$$V_{Sw} = \left(\frac{\partial q}{\partial k} \right)_z$$

Lighthill and Whitham²⁵ also showed that the group-velocity phenomenon is a special case of the kinematic wave phenomenon. In Eqs. (2.5) and (2.8), if q (flow) is taken to be the frequency of a wave, and k (concentration) is taken to be the wave number, then the group velocity is given by Eq. (2.7).

Zuber and Hench²⁸ extended the kinematic wave theory to investigate transients in two-phase flow. The equation of continuity for the gas phase is

$$\frac{\partial \langle \alpha \rangle}{\partial t} + \frac{\partial V_{sg}}{\partial z} = 0, \quad (2.11)$$

where $\langle \alpha \rangle$ is the average gas fraction (void fraction), and V_{sg} is the superficial gas velocity. Equation (2.11) can also be expressed as

$$\frac{\partial \langle \alpha \rangle}{\partial t} + V_w(\langle \alpha \rangle) \frac{\partial \langle \alpha \rangle}{\partial z} = 0,$$

where $V_w(\langle \alpha \rangle)$,[†] the void wave velocity, is given by

$$V_w(\langle \alpha \rangle) = \left(\frac{\partial V_{sg}}{\partial \langle \alpha \rangle} \right)_{V_m} \quad (2.12)$$

[†]Note the implied assumption that V_{sg} does not depend on V_L .

In a space-time (z - t) plane, the slope of curves of constant $\langle \alpha \rangle$ is

$$\frac{dz}{dt} = V_w(\langle \alpha \rangle). \quad (2.13)$$

To determine when a change in $\langle \alpha \rangle$ will reach a given point, z , in the system, Eq. (2.13) is integrated to give

$$t = \int_0^z \frac{dz}{V_w(\langle \alpha \rangle)} + t_0,$$

where t_0 is the constant of integration or, in this case, the initial time.

The wave velocity can also be expressed in terms of the average gas velocity. Since

$$V_g = \frac{V_{sg}}{\langle \alpha \rangle}, \quad (2.14)^\dagger$$

this relation is substituted in Eq. (2.12) to obtain

$$V_w = V_g + \langle \alpha \rangle \frac{dV_g}{d\langle \alpha \rangle}. \quad (2.15)$$

Equation (2.15) shows that void waves will travel forwards or backwards, relative to the gas velocity, depending on whether $dV_g/d\langle \alpha \rangle$ is positive or negative.

The velocity with which a discontinuity in void fraction ("shock wave") propagates can now be written analogous to Eq. (2.10) as

$$V_{Sw} = \frac{V_{sg_2} - V_{sg_1}}{\langle \alpha_2 \rangle - \langle \alpha_1 \rangle}. \quad (2.16)$$

To investigate the behavior or decay of this shock wave, it is necessary, for convenience, to make a graphical analysis instead of an analytical one. This is explained in Section VI-D.

Wallis²⁹ also applied the kinematic wave theory to one- and two-component systems and referred to these waves as continuity waves since they are derived from the continuity equation. Wallis, however, went

[†]In general, two-phase flows in ducts of uniform cross section are not one-dimensional. An approximate application of this theory employs cross-sectional average quantities such as defined by Eq. (2.14).

further, as did Lighthill and Whitham in their analysis of traffic flow and flooded rivers, and compared continuity waves with dynamic waves. This comparison is not reviewed here, but a final result was a stability criterion showing that the continuity-wave velocity must be within the velocities of forward- and backward-facing dynamic waves. If the continuity waves overtake the dynamic waves, shock waves result, as was stated by Lighthill and Whitham.

III. EXPERIMENTAL APPARATUS

A. Flow System

1. Natural-circulation, Air-Water Loop

The natural-circulation, air-water loop used in these studies is shown schematically in Fig. 3.1. This system consisted of an air-injection

Fig. 3.1. Natural-circulation
Flow System

network, two air-water mixers, a test section, a separator, and the downcomer. The entire loop was filled with water, and air was then allowed to bubble through the air-water mixers, up the test section, and out of the loop by way of the separator. The water was returned to the test section via a downcomer by gravity. To allow for visual observation, the flow system was constructed almost entirely of Plexiglas.

The separator was a 33 x 24 x 18-in. Plexiglas tank, with an opening in the top. The velocity of the two-phase mixture entering the separator decreased sufficiently, because of the increase in cross-sectional area, for the air to be completely removed from the water.

In a natural-circulation system, the two-phase mixture flow rate is dependent on the gas flow rate. The buoyant forces between the two phases causes the gas bubbles to rise and carry the liquid phase along with them. In a forced-circulation system, a pump is used as the driving force for the flowing mixture.

2. Forced-circulation, Air-Water Loop

Upon completion of the studies on the natural-circulation loop, this project was continued on a forced-circulation, air-water loop. This loop is schematically illustrated and pictured in Fig. 3.2. In general, this system was similar to the natural-circulation system with the exception of introducing a 100-gpm pump. Because of the addition of the pump, water

coolers were used to retain the two-phase mixture at a constant temperature. The same perturbation air-water mixer and test section were used in both loops.

Fig. 3.2. Forced-circulation Flow System

3. Air-Water Mixers

Two air-water mixers were used in each of the two loops to inject the metered air streams. One was used for the steady-state air flow; the other was used for the perturbation air flow.

A cross-sectional view of the steady-state mixer in the natural-circulation system is depicted in Fig. 3.3. The air entered through two 1-in. copper tubes and merged with the vertically-flowing water stream through 168 holes (0.041-in. diam) spaced along the periphery of the $2\frac{3}{4}$ -in. pipe.

Fig. 3.3
Steady-state, Air-Water
Mixer

In the forced-circulation system, the steady-state mixer was similar to the one used in the natural-circulation system but was made of aluminum. In this mixer, however, the air entered through four $1\frac{1}{2}$ -in. copper tubes and then passed through 300 holes (0.0595-in. diam) to merge with the vertically-flowing water stream. The two-phase mixture flowed through a cone-shaped transition piece toward the test section. Two different, steady-state mixers were used only for convenience.

The perturbation mixer was located 32 in. above the steady-state one in the natural-circulation system and 40 in. above in the forced-circulation system. An exploded view of this mixer is presented in Fig. 3.4. The mixer consisted of seven vertical $5/8$ -in.-ID tubes, Fig. 3.4(b), which allowed the passage of the two-phase mixture formed upstream at the steady-state mixer. The air entered through two 1-in. copper tubes, Fig. 3.4(c), and merged with the mixture by passing through a porous disc, Fig. 3.4(d). The porous disc occupied the balance of the cross-sectional area not used by the seven tubes. The disc was a grade 2000 (super fine) Oilite bronze, $3\frac{11}{16} \times 1/16$ -in. A ring, Fig. 3.4(e), held the disc in place. With the exception of the seven tubes, the bottom face of the mixer was solid, Fig. 3.4(a), forcing the entering air up through the porous disc.

Fig. 3.4. Perturbation Air-Water Mixer

air-water mixer. The air flow rate to each mixer was regulated by a needle valve in each line. The flow rate was measured with an orifice meter. To minimize flow disturbances, 18 in. of straight 1-in. pipe (ASA Schedule 40) was provided upstream from the orifice union and 12 in. downstream. The knife-edged orifice was installed and sealed with O rings into 1-in. orifice unions provided with pressure taps. The pressure drop across the orifice was measured with a manometer. In the natural-circulation system, a 0.125-in. orifice and a 10-in. slant manometer filled with water were used; in the forced-circulation system, a 0.0625-in. orifice and a 100-in. manometer filled with mercury were used. These two different pressure-drop systems were used only for convenience. A constant pressure of 80 psig was maintained upstream from the orifice by two Norgren regulators.

The manifold arrangement is shown in Fig. 3.5. Downstream from the orifice was a shut-off valve ($1/4$ -in. Hoke), V_2 , operated by a solenoid. The air flow then separated into two lines, one going to the

4. Test Section

The test section was constructed from a $2\frac{3}{4}$ -in. Lucite pipe, $8\frac{1}{3}$ ft long. Probe taps were inserted in the test section at 12 different locations, the first one being only 4 in. downstream from the perturbation mixer. The remaining probe positions were located as shown in Fig. 3.2. The probe taps were more closely spaced at the entrance in order to observe the behavior at this point.

Pressure taps were also inserted in the test section. These were located at the same distances from the perturbation mixer as the probe taps, but 180° from them. These taps allowed for pressure-drop readings between any two points.

In the natural-circulation loop, the downcomer was somewhat longer, thus requiring the addition of another $2\frac{3}{4}$ -in. pipe to the test section, this one being $2\frac{1}{2}$ ft long.

B. Flow Measurement and Control

1. Air Flow

a. Physical Arrangement

The building air supply provided air at 100 psig for both the steady-state air-water mixer and the perturbation

steady-state mixer, and the other to the perturbation mixer. In the steady-state line, the flow was controlled by a needle valve (1/4-in. Hoke); in the perturbation line, the flow was controlled by a needle valve and a solenoid valve (1/4-in. Hoke), V_1 . The needle valve controlled the amplitude of the air pulses; the solenoid valve controlled the frequency and duration of the air pulses.

Fig. 3.5. Air-flow Manifold Arrangement

pulses was determined by the "on time" of a monostable multivibrator. The output of this network controlled a sensitive relay, which simultaneously energized a driver relay and triggered a digital counter. The counter recorded the total number of pulses. The driver relay, in turn, was used to energize the solenoid valve and to operate indicating lamps on the front panel.

The air shutoff solenoid valve was controlled by a switch on the air-flow regulator.

A more detailed explanation of this electronic system is contained in Appendix A.

2. Water Flow

The water supply was obtained from the main tap-water line. Tap, rather than demineralized water, was used because of its higher electrical conductivity, a necessity for proper functioning of the probe.

In the natural-circulation system, the water flow rate was regulated by an inverse-percentage-flow plug valve. The flow was measured

b. Electronic Control

The operation of the perturbation solenoid valve was controlled by the air-flow regulator, an electronic system designed specifically for this application. Because the generation of the air perturbations was an integral part of this study, the principles of this control will be presented here.

The frequency of the void transients was determined by the frequency of an astable multivibrator. The duration of these

at a 1.375-in. orifice at the downstream end of the downcomer. A standard 60-in. manometer, which could be read to ± 0.05 in., measured the pressure drop across the orifice. The manometer was filled with a fluid of specific gravity 1.25.

In the forced-circulation system, the water entered the test section by way of one of two lines. For low water rates ($V_{sl} < 2$ ft/sec), a 1-in. line containing a 0.499-in. orifice was used. For high water rates ($V_{sl} \geq 2$ ft/sec), a 5-in. line containing a 1.75-in. orifice was used. The pressure drop across the orifices was measured with a 100-in. manometer containing mercury and a 30-in. manometer containing 2.75 specific-gravity fluid. All of the air and water orifices met specifications listed in Grace and Lapple³⁰ and in Fluid Meters.³¹

The water temperature was measured by a dial thermometer, with accuracy of $\pm 1^\circ\text{C}$, inserted in the separator.

C. Void-fraction Measurements

1. Pressure-drop Technique

The total pressure drop in a column of a flowing air-water mixture is equal to the sum of three separate pressure-drop terms: one due to the density or head, one due to the momentum of the fluid, and one due to the fluid frictional losses. At low liquid velocities, the momentum and frictional terms are negligible, leaving the pressure drop due to the void fraction as the main contribution. The negligible pressure-drop terms are in the opposite direction of the void fraction term. This means that void-fraction values calculated from the pressure drop will be lower than the true values.

Fig. 3.6. Pressure-drop Technique

From Fig. 3.6, it can be seen that

$$P_A = \rho g h + \rho_l g [H - (h + L)] + \rho_m g L + P_C, \quad (3.1)$$

and

$$P_B = \rho_l g H + P_C.$$

Since the relation $P_A = P_B$ must hold,

$$(\rho - \rho_l) h - (\rho_l' - \rho_m) L = 0. \quad (3.2)$$

The density of the two-phase mixture is given by

$$\rho_m = \langle \alpha \rangle \rho_g + [1 - \langle \alpha \rangle] \rho_\ell,$$

but it is safe to assume that

$$\rho_m = [1 - \langle \alpha \rangle] \rho_\ell, \quad (3.3)$$

because

$$\rho_\ell \gg \rho_g.$$

Using Eq. (3.3), one can rearrange Eq. (3.2) to give

$$\langle \alpha \rangle = (\rho - \rho_\ell) h / \rho_\ell L. \quad (3.4)$$

A correction for the frictional and momentum pressure drop can be made by noting the pressure drop when the liquid phase is flowing alone. Once again, it must be assumed that the frictional and momentum pressure drop in a two-phase mixture is due to the liquid phase only. This correction term, however, cannot be determined in a natural-circulation system because the gas phase must be present for a flowing mixture. It can, on the other hand, be determined for a forced-circulation system.

If the frictional pressure drop is not neglected, the term ΔP_F is added to the right side of Eq. (3.1). This alters Eq. (3.4) to

$$\langle \alpha \rangle = [(\rho - \rho_\ell) gh + \Delta P_F] / \rho_\ell gL. \quad (3.5)$$

The frictional pressure drop is related to the liquid velocity by the Fanning equation,

$$\Delta P_F = 4fLV_\ell^2 / 2gD, \quad (3.6)$$

where f is the Fanning friction factor. For smooth tubes, it is given as a function of Reynolds number in the following equation:

$$f = 0.046(N_{Re})^{-0.2}. \quad (3.7)$$

Substituting Eq. (3.7) into Eq. (3.6), one obtains

$$\Delta P_F = C_1 V_\ell^{1.8}$$

where

$$C_1 = 0.092 \frac{L}{D^{1.2}} \left(\frac{\mu}{\rho_\ell} \right)^{0.2}. \quad (3.8)$$

In a system where the liquid phase is flowing alone, the frictional pressure drop is similarly given by

$$\Delta P_{F\ell} = C_1 V_{\ell\ell}^{1.8}{}^\dagger$$

For constant liquid-mass flow rate

$$V_\ell = \frac{V_{\ell\ell}}{1 - \langle \alpha \rangle},$$

and using this relationship, one obtains

$$\Delta P_F = \frac{\Delta P_{F\ell}}{(1 - \langle \alpha \rangle)^{1.8}}.$$

Since $\Delta P_{F\ell}$ can be calculated by using

$$\Delta P_{F\ell} = \left(\frac{\rho - \rho_\ell}{\rho_\ell} \right) g h_\ell,$$

Eq. (3.5) is simplified to

$$\langle \alpha \rangle = \frac{\rho - \rho_\ell}{\rho_\ell L} \left[h + \frac{h_\ell}{(1 - \langle \alpha \rangle)^{1.8}} \right]. \quad (3.9)$$

For the calculation of the cross-sectional average void fraction, standard 60-in. manometers filled with a fluid of 1.25 specific gravity were used. Four pressure taps and two manometers were used to assure the reliability of the data taken. If the data from the two manometer systems agreed, this was a good indication that no air was trapped in either system.

2. Fluid-level Height Technique

Another method for determining the cross-sectional average void fraction in any channel is called the "fluid-level height technique." This method, however, can only be used in a stagnant system because the fluid height must be distinct.

In a U-shaped apparatus (Fig. 3.7), the gas is bubbled up through one leg of the liquid-filled tube. The other leg is filled entirely with the liquid phase. To maintain equilibrium, the weights of the fluids in each leg must be equal. The increased height of the fluid level of one leg is due to the volume occupied by the gas phase. The void fraction is equal to the

[†]The additional subscript ℓ is used to distinguish a solid-liquid system from a two-phase system.

Fig. 3.7. Fluid-level Height Technique

difference in height between the two columns, divided by the height of the two-phase mixture; or,

$$\langle \alpha \rangle = \frac{h_2 - h_1}{h_2}.$$

3. Electrical-resistivity Probe

In the present studies, the electrical-resistivity probe that was utilized was a slight modification of the one used in Reference 11. The two probes differed in that the present one was somewhat sharper than the original one. For convenience and durability, the size of the probe and the insulation were also changed.

An actual probe and its schematic diagram are shown in Fig. 3.8. This probe consisted of a 6-in. length of 0.024-in. spring-steel wire, bent at right angles 1 in. from one end. The tip of the shorter segment was sharpened to a fine point (radius of curvature of tip ≈ 0.001 in.), and this segment was dip-coated with a silicone varnish and cured at 450°F for 2 hr. Since the probe tip had an insufficient surface area, the varnish did not adhere and the tip was left exposed as desired. The longer segment was then encased in a 4-in. piece of 1/8-in. stainless-steel tubing but was insulated from it by Teflon spaghetti. To ensure complete insulation and no leakage, both ends of the tubing were dipped in liquid paraffin and allowed to cool. After the probe was successfully insulated, it was inserted in the channel with the use of a modified Swagelock tube fitting. The first ferrule of the fitting was replaced by a 1/8-in. O ring, and the second ferrule was reversed. This allowed the probe to be traversed radially across the test section and remain leak-proof. This was also enhanced by using Teflon tape on the Swagelock fitting threads. In the channel, the probe was oriented with the exposed tip pointing into the direction of flow. The steps taken to modify the probe are described in Chapter IV.

In addition to the electrical-resistivity probe, three other electronic systems necessary for the investigation of void perturbations were (a) the void-probe trigger, (b) a magnetic tape recorder, and (c) an analog-to-digital converter.

1. Tip not Insulated	6. 1/8-in. "O" Ring
2. Silicone Varnish Insulation	7. Ferrule
3. Parafin Insulation	8. Parafin Insulation
4. 1/8-in. diam SST Tube	9. Teflon Insulation
5. 1/8-in. Fitting	10. 0.024-in. diam Steel Wire

Fig. 3.8. Electrical-resistivity Probe

a. Void-probe Trigger

Because of a probe-wetting problem (as explained in Reference 11), an electronic system was introduced to facilitate the handling of the probe response. This electronic circuit, called the void-probe trigger, converts an electrical signal of variable amplitude to a signal having only two values: zero and a prespecified constant, E_C . The voltage level of the input signal at which the output changes is predetermined.

When this circuit was applied to the probe, the irregularly-shaped probe response was converted to a pseudorandom square wave with amplitude E_C . The pulse widths were identical to the dwell times of the air bubbles in contact with the probe tip. This meant that the triggering level

of the input probe signal was set slightly above zero (or that voltage corresponding to all liquid in contact with the probe). The output of the void-probe trigger could then be electronically integrated.

With the exception of an additional stage, the circuit was identical to one used in Reference 11. A variable time-delay stage was incorporated to allow the output signal to have a longer pulse width than the input signal. When the variable delay was calibrated (explained in Section IV-G), this stage could be used as a correction factor for any known errors that would give low output results. If a known error caused high output results, the polarity of the input to the delay could be reversed and the delay stage would subtract a correction factor. A third possibility would be the complete elimination of this stage by use of a bypass switch.

The principles involved in the operation of the void-probe trigger are explained completely in Appendix B. Since multiple probes were used in this project, six individual channels of the void-probe trigger were constructed. A remote readout panel was incorporated to facilitate the observation of the bubble pulses from any of the six channels on an oscilloscope.

b. Magnetic Tape Recorder

Analysis of the probe response data required the use of an analog-to-digital converter located in a different nearby laboratory on the site. This meant using a tape recorder to transfer the experimental data to the analog-to-digital converter.

A Minneapolis Honeywell 8100 portable tape recorder, equipped with seven data channels, was utilized for this purpose. This FM tape recorder had the capability of recording and playing back electrical signals on all seven channels at any of four different tape speeds ($1\frac{7}{8}$, $3\frac{3}{4}$, 15, or 30 in./sec). The frequency responses associated with each of these tape speeds were 0 to 0.625, 1.250, 5, and 10 kc/sec, respectively. The recorder was also equipped with a reel-rotation counter for locating a particular position on the tape, an oscilloscope for observing the data recorded or played back on any channel, and a voice channel for describing or labeling the data.

c. Analog-to-digital Converter

An analog-to-digital converter is an electronic system which accepts a continuous signal, samples it at a predetermined rate, and presents the output in discrete numerical values. The system used was a Packard Bell Converter Model 247, with a modified IBM format. It was capable of up to six channels of input data and sampling rates of 0.5 to 768 msec for one channel, or 3 to 768 msec for all six channels simultaneously. The digital output was recorded on magnetic tape and could therefore be analyzed by a computer.

IV. MODIFICATION OF ELECTRICAL-RESISTIVITY PROBE

Neal¹⁰ constructed an electrical-resistivity probe for measuring local parameters in mercury-nitrogen flow. The probe consisted of an insulated needle with an exposed tip pointing into the flow. When a battery and a resistor in series with the probe were connected to ground, a change in voltage was observed, as mercury and then nitrogen came into contact with the probe. This resistivity probe was developed in a system, with the continuous phase being a liquid metal.

Nassos¹¹ extended this void-fraction measuring technique to an air-water system. Utilizing the probe, he measured the local void fraction at various radial positions and constructed a void profile. Under identical operating conditions, a similar void profile was obtained from the gamma-ray attenuation technique. In each case, the cross-sectional, average void fraction was determined. These results were compared for several different values of quality ranging from 1.25×10^{-4} to 1.045×10^{-3} . This comparison concluded that the probe gave low void-fraction values relative to the gamma-ray method. The error varied from 22 to 57% low, with an average error of 35%.

Before the electrical-resistivity probe could be employed in procuring useful data, its results had to be more reliable. An investigation was conducted to determine the cause of the probe's invalidity. This investigation consisted of a series of experiments, each of which were designed to answer one or more questions on the characteristics of the probe.

A. Probe Response

Probably the first question asked in reference to the probe is whether a lag exists between the physical response and the electrical response. In other words, is there a delay between the time the probe first pierces a bubble and the time when this is observed on the output of the void-probe trigger? The answering of this question required the use of high-speed photography.

A 5-ft length of 1-in.-ID Lucite tubing was employed as the test section for this experiment. A probe was inserted $1\frac{1}{2}$ ft from the top, with the tip pointing downwards along the axis of the tube. The tube was filled with water, and individual bubbles were allowed to flow past the probe. The resulting signal from the probe was fed into the void-probe trigger, and both the input and output of the void-probe trigger were displayed on a dual-trace oscilloscope (Tektronix Type 531A with plug-in unit Type CA). A high-speed, rotating-prism camera (Fastex WF17)

was focused on the probe tip to observe the probe-bubble response. This camera was equipped with an auxiliary lens for superimposing, simultaneously, an oscilloscope trace on the main image.

The camera film speed was set at about 5000 frames/sec, and the probe-bubble interaction was filmed. One-hundred-foot rolls of film were used, which, at this speed, required less than 1 sec to expose. This meant that several rolls were needed to insure that the necessary observations were "captured."

The processed film was viewed at 24 frames/sec, or approximately 1/167 of the filmed speed. The observations showed clearly that the electrical response of the probe was almost instantaneous with the physical contact of the probe and an air bubble.

The instantaneous response of the probe and bubble was also verified by viewing this phenomenon under a microscope having a magnification factor of 125. A probe and a water drop were placed on a specimen glass, and the probe was moved slowly toward the droplet until physical contact was made. The probe was then moved away slowly until contact was broken with the droplet. This making and breaking of contact were observed, simultaneously, on an ohmmeter.

B. Bubble Deflection and Deformation

The probe used in the original studies was not sharpened symmetrically with respect to its axis but was shaped similarly to a knife; i.e., the probe was sharp in only one of two dimensions. It was believed that, because of its construction, the probe may have caused deflection and/or deformation of the air bubble. In the present investigation, however, the probe tip was sharpened symmetrically, and a much-improved probe resulted.

The high-speed camera observations indicated no noticeable deformation of an air bubble as it was pierced by the sharp probe. Deflection of the bubble by the probe could not be determined from the films, because a small-diameter tube was used in this system and the air bubble occupied almost the entire cross-sectional area, preventing any deflection at all. If a larger-diameter tube were used, the probe tip would have to have been situated near the wall for observation. This, again, would have partially prevented deflection. Therefore, it could not be deduced from these films if any deflection is caused by the probe.

C. Probe Reproducibility

In future studies, multiple electrical-resistivity probes were to be used simultaneously for void-fraction measurements. For this reason,

deviation between void measurements from different probes under identical operating conditions had to be negligible. A test was performed to establish the reliability of the reproduction of probes. The results, shown in Table I, prove to be favorable.

TABLE I. Reproducibility of Probes

Probe	α	$ \alpha - \alpha_{avg} $	$\frac{ \alpha - \alpha_{avg} }{\alpha} (\%)$
1	0.221	0.001	0.44
2	0.229	0.009	3.93
3	0.221	0.001	0.44
4	0.218	0.002	0.87
5	0.210	0.010	4.40
$\alpha_{avg} = 0.220$			

D. Probe Angle

In the construction of the probes, a spring-steel wire was bent at a 90° angle so that when the probe was inserted perpendicular to the test section, the probe tip was parallel to its axis. However, it was possible for the probe not to be parallel to the axis, even with a 90° angle bend. This would have occurred if the probe had rotated about its longer segment, resulting in two possible errors.

One error would be due to the misalignment of the probe tip. This means that the void measurements would be for a radial position further away from the center line than the realized position. The magnitude of this error would depend upon the degree of misalignment and $\partial\alpha/\partial r^*$ at the radial position in question.

Another error which could ensue from the rotated probe was the deflection (or increase in deflection) of the bubbles. A test was conducted to determine the size of this error. Two probes with angles of 95° and 100° were used, along with a standard type (90° angle). The tips were located at identical radial and axial positions. Data were taken for two sets of conditions, and the results indicate a negligible error, as indicated in Table II.

TABLE II. Effect of Probe Angle

Condition	Probe Angle	α	% Error [†]
1	90°	0.1577	
	95°	0.1517	-3.81
	100°	0.1570	-0.45
2	90°	0.2110	
	95°	0.2131	+1.00
	100°	0.2145	+1.65

[†]Relative to 90° angle probe.

E. Probe Stability

When being used as a void-fraction measuring device (can also be used to determine bubble frequency), the probe may have to be used over a long period of time. In that case, the response of the probe must not deviate during the course of the experiment. To verify this condition a probe-stability analysis was performed. Under constant operating conditions, the void fraction was measured at approximately 20-min intervals for $3\frac{1}{2}$ hr. The results are shown in Table III.

TABLE III. Probe Stability

Reading	Elapsed Time (min)	α	% Deviation
1	0	0.219	+2.62
2	25	0.210	-1.59
3	45	0.213	-0.19
4	65	0.207	-3.00
5	85	0.215	+0.75
6	105	0.214	+0.28
7	125	0.214	+0.28
8	165	0.215	+0.75
9	190	0.214	+0.28
10	210	0.213	-0.19
		$\alpha_{avg} = 0.2134$	

These results indicate that there is no consistency in the void-fraction measurements to depict instability in the probe response.

F. Improvement of Probe Technique

The initial step taken in the hopeful improvement of the electrical-resistivity probe was to make the tip sharper. This was accomplished by polishing the probe tip with emery paper. The result was a probe tip having a radius of curvature of less than 0.001-in. After the completion of this step, the previous investigations were undertaken. Since the probe was constructed as sharp as reasonably possible, no other improvement was attempted on the probe itself, with the exception of the insulation. Dow Corning 991 silicone varnish was used for electrical insulation, and, at the same time, the hard nonwetting surface of the probe served as a protective coating.

In the earlier development program, sodium ethyl xanthate was added to the water system in an effort to decrease the surface tension and improve the probe response. The results were negative. In a second effort, the probe was treated with this same xanthate for a 2-hr period, and another

probe was soaked in water during this period. The two probes were tested, and, again, the results were negative.

The investigation of the probe characteristics was completed, and the probe was prepared for testing. To check the probe void-fraction readings against another method, three different types of systems were employed: (1) stagnant, (2) natural-circulation, and (3) forced-circulation.

1. Stagnant System

To produce a stagnant air-water system, the natural-circulation loop was filled with water to a level just below the entry to the separator. This assured no liquid flow. The probe was inserted in the test section approximately midway between the air-water mixer and the two-phase fluid level. Void-fraction readings were obtained at five different radial positions for various operating conditions. A void profile was constructed from these data, and a value for the cross-sectional, average void fraction was obtained.

A value for the average void fraction was also obtained from the fluid-level height technique as described in Section III-C-2 above. The results from the two techniques were compared, and the findings indicated that the probe measurements were 9 to 18% lower than the fluid-level measurements.

The void-fraction values obtained from the fluid-level height technique could not be termed as a cross-sectional average, because the cross-sectional average is not a constant but a function of the axial distance from the air-water mixer. As the air propagates up the channel, it expands because of the change in head. Consequently, the cross-sectional average void fraction increases with increasing axial length. Since the change in pressure is linearly related with the test-section length, the void fraction is also, to a first approximation, linearly related with length. The values obtained from this technique should be referred to as the volumetric-average void fraction. It was essential, therefore, that the probe be situated so that its readings be representative of the volumetric average. Within a negligible error, this axial position was found to be the midpoint between the air-water mixer and the two-phase fluid level. (See Appendix C.) Actually, the two values being compared were a cross-sectional average and an axially-integrated average of the cross-sectional average.

2. Natural-circulation System

A probe was inserted in the natural-circulation, air-water loop, and void-fraction measurements were obtained at five different radial positions for 16 different values of quality. From the data, void profiles were computed, and cross-sectional averages were evaluated by graphical integration.

Average values for void fraction were also obtained from pressure-drop data. Pressure taps were incorporated 1 and 2 ft upstream and downstream from the probe. The taps 1 ft from the probe were connected to a 60-in. manometer; the taps 2 ft from the probe were connected to another 60-in. manometer. Equation (3.4) was used to determine the void fraction, the value of L being taken as 24 and 48 in., respectively, for the two manometer systems.

To check for the presence or absence of radial pressure gradients, i.e., to check the reliability of pressure readings at the wall, measurements were made at both the channel axis and the channel wall. The results of these measurements indicated that the wall-pressure readings were less than 2% lower than the axial readings. Since the true cross-sectional average pressure is somewhere between the pressure readings at the limits, the assumption that either of the limiting pressure readings is equal to the average pressure introduces a negligible error.

Just as in the previous void-measuring technique, the pressure-drop method gives the volumetric average void fraction for the length of the test section between the two pressure taps, rather than a cross-sectional average. Therefore the probe again had to be situated in the test section at an axial point from which the cross-sectional average value of void fraction obtained would be representative of the volumetric average. (See Appendix C.)

The probe and pressure-drop void-measuring techniques are compared in Fig. 4.1. As seen in the graph, the probe void measurements are, in general, lower than the pressure-drop voids. The range of error in the probe measurements is from 2 to 27%.

Fig. 4.1. Comparison of Void-fraction Measuring Techniques

Upon completion of the stagnant-system and natural-circulation-system tests, no definite conclusions could be reached on the cause of error in the probe measurements. The average error in each of the two tests was about 15%, but the range of errors differed. This was because more data were taken for the natural-circulation tests.

A comparison of these results with those of Nassos¹¹ (Fig. 4.1) in his earlier studies showed a remarkable improvement in the response of the probe. In identical systems, the average error decreased from 35 to 15%. Since the only change was in the construction of the probe, this indicates the effect of the probe sharpness on the response. In view of this fact, it is surprising that Bencze and Orbeck¹⁵ were able to obtain accurate (reported 2.05% deviation) void-fraction measurements with their a.c. impedance probe, the tip of which had a radius of curvature on the same order of magnitude as that of Nassos's original probe. Furthermore, a graphical integration of their reported void-fraction profiles shows an average error of about 7% higher than reported.

Although considerable improvement was achieved in the present program, further work was indicated. The probe was then tested in the forced-circulation system.

3. Forced-circulation System

Upon completion of the tests in the natural-circulation system, the test section was removed and positioned in the forced-circulation loop. Pressure taps and a probe were inserted in the same locations as in the previous tests. Void-fraction measurements were then obtained at five different radial positions for ten different values of quality. Average values of void fraction were determined by both the probe method and the pressure-drop technique. In the pressure-drop technique, Eq. (3.9) was employed to take into account the frictional losses.

As seen in Fig. 4.1, a better agreement between the two void-fraction measuring techniques (probe and pressure-drop) is obtained in the forced-circulation system. In this system, the maximum error was about 9%, the average error being about 7%.

A possible explanation for the difference in the reliability of the probe in the two systems can be attributed to the liquid velocities attained. In the natural-circulation system, the superficial liquid velocities ranged from 0 to 1 ft/sec; in the forced-circulation system, the velocities ranged from 0 to 4 ft/sec. Because of the increased liquid velocities in the latter system, it is believed that there was a smaller possibility of the air bubbles being deflected by the probe. As stated in Section IV-B, the deflection of the bubbles was not verified by the high-speed moving pictures. The a.c. impedance probe developed in Norway showed good agreement with the

quick-closing-valve technique, but the liquid velocities ranged from 2 to 6.5 ft/sec, causing bubble deflection to be a minimum.

The only other difference between the natural- and forced-circulation systems was that the frictional losses were not taken into account in the former. Since the frictional pressure drop is in the opposite direction of the pressure drop due to voids, a correction factor would have raised the void-fraction value, and a larger variance between the two measuring techniques would have been incurred. However, because of the low liquid velocities, this correction factor was negligible.

4. Conclusions

Two steps in improving the electrical-resistivity probe response were attained in this investigation. Improving on the sharpness of the probe decreased the average error in void-fraction measurements from 35 to 15%. Additional improvement was achieved when the void measurements were obtained at higher liquid velocities (>1.0 ft/sec), and the average error then decreased to 7%.

G. Calibration of Void-probe Trigger Delay

To correct for any uncontrollable errors, such as deflection of the air bubbles by the probe, a delay circuit was added to the void-probe trigger, as mentioned in Section III-C-3-a above. This circuit adds or subtracts a small Δt to each bubble pulse. If s is the number of bubbles pierced by a probe during a time interval T , then the void fraction added due to the delay circuit can be expressed as

$$\Delta\alpha = st/T.$$

Without employing the delay, the measured void fraction can be mathematically stated by

$$\alpha = st_{\text{avg}}/T,$$

where t_{avg} is the average dwell time of a bubble. Then the corrected void fraction is the sum of the two above equations, or

$$\alpha_{\text{cor}} = s(t_{\text{avg}} + t)/T.$$

Determination of t requires the knowledge of s/T , t_{avg} , and the error in the probe-measured void fraction. Employing Δt then requires the calibration of the delay circuit with time. This lengthy and involved process was replaced by the following procedure.

With the delay circuit inoperative, a particular value of void fraction was measured with the probe. The actual void fraction was held constant, the delay mode was incorporated, and probe readings were obtained for several values of Δt .

The actual value of Δt was never determined, but the value corresponding to the setting of the potentiometer (R_{18} in Fig. B.1) was noted. This was repeated for several void-fraction conditions over the range of interest. Calibration of the delay circuit (Fig. 4.2) in the system where it was used was advantageous in that correction of other existing errors would have been taken into account.

Fig. 4.2. Calibration of Delay Circuit

Although the individual electronic components of each delay circuit came within their specified tolerances, variances in their inherent characteristics resulted in slight deviations in the outputs. Thus each channel had to be calibrated individually. Coincidentally, however, two sets of three channels each had similar outputs. Therefore, only two families of calibration curves had to be plotted.

V. PROCEDURE

A. Verification of Electrical-resistivity Probe

In each experiment in the series performed for analyzing the probe characteristics, a basic procedure was employed. The void fraction of the two-phase mixture was measured with the probe, and then compared with other probe measurements or with other void-fraction measurements, depending on the particular test.

Void-fraction measurements were conducted by electronically integrating the output signal of the trigger circuit for a time interval T . The simple integrating circuit used (shown in Fig. 5.1) yields the relationship

$$e_0 = -\frac{1}{RC} \int_0^T e(t) dt,$$

from which is obtained the working equation of

$$\alpha = 1 - \frac{1}{T} \int_0^T \frac{e(t)}{E} dt.$$

The voltage $e(t)$ took on the value E when liquid was in contact with the probe, and zero when air was in contact with the probe. If the values of $e(t)$ were reversed, then the working equation would be

Fig. 5.1. Integrating Circuit

$$\alpha = \frac{1}{T} \int_0^T \frac{e(t)}{E} dt,$$

which was used in all of the previously-mentioned tests.

B. Experimental Operation

The five probes were inserted in the proper axial positions and adjusted to the proper radial positions. The loop was filled with tap water, and all the manometer systems were bled to remove any air bubbles present. The five channels of the trigger circuit were nulled to give a zero output with the probe in the liquid phase only.

With the water pump in operation, the flow rate was set and the frictional pressure drop was noted. The air flow was set to give the desired steady-state void fraction. The perturbation valve was then energized

long enough for the transient void fraction to be noted. After the potentiometers of the trigger circuit were adjusted for the proper triggering level, delay, and output amplitude, and the potentiometers of the air-flow regulator were adjusted for the proper frequency and duration, data were ready to be taken.

A test run was initiated by starting the precalibrated tape recorder, resetting the cycle counter, and starting a stop watch. During the run, the flow rates were kept constant, the probe responses were observed on an oscilloscope, and the number of cycles elapsed was periodically announced on the voice channel of the tape recorder. Upon completion of a run, the probes were adjusted to a different radial position, and their responses were again recorded on tape without changing any of the other conditions. This was repeated for each radial position. All the variables were adjusted for the next set of conditions, and the data-taking procedure was repeated until two-thirds of the analog tapes were filled with data.

The tapes, along with the recorder, were transported to the analog-to-digital converter. The analog data was sampled by the converter, and the digital output was recorded on magnetic tape. These digital data were then used for the computer input, thus providing the final results.

C. Computer Analysis of Data

Six signals were fed into the analog-to-digital converter for sampling. Five of the signals were obtained from the probes and were composed of a random number of pulses of random length and in a random sequence. The amplitude of these pulses was constant, taking on a value of one for the presence of a bubble, and a value of zero for the absence of a bubble. The sixth signal was obtained from the output of the air-flow regulator. This signal consisted of an unsymmetrical square wave, having the value of one when the perturbation valve was energized, and a value of zero when it was de-energized. Each cycle of this square wave represented a repetition of the experiment from which the first five signals were obtained.

A computer program was written for obtaining the average response of the probe signals. This program was based on the following mathematical statements.

The inputs to the converter were sampled for B cycles at a sampling time of t_s . If the period of each cycle were τ , the number of samples per cycle would be given by

$$N = \frac{\tau}{t_s}.$$

Let Y_{ij}^k be the value of the i th sample in the j th cycle of the k th signal, where

$$i = 1, 2, \dots, N;$$

$$j = 1, 2, \dots, B;$$

and

$$k = 1, \dots, 5.$$

The function Y_{ij}^k has a value of either zero or one, as mentioned above. The computer was then asked to find

$$\bar{Y}_i^k = \frac{1}{B} \sum_{j=1}^B Y_{ij}^k, \quad (5.1)$$

and these values be printed out for each i and each k . By plotting these values, the average response was obtained.

The propagation velocity of the perturbation was found by considering the maximum value of $\bar{Y}_i^k$ for each k . Let

$$\max \bar{Y}_i^k = \xi^k(b^k),$$

where the maximum occurred at $i = b^k$. Find $b^{p+1} - b^p$ for $p = 1, \dots, 4$.

If

$$b^{p+1} - b^p > 0, \text{ let } \epsilon^p = \frac{b^{p+1} - b^p}{N}. \quad (5.2)$$

If

$$b^{p+1} - b^p < 0, \text{ let } \epsilon^p = 1 - \left| \frac{b^{p+1} - b^p}{N} \right|. \quad (5.3)$$

Then

$$\bar{\lambda}^{p,p+1} = \frac{z^{p+1} - z^p}{\eta + \epsilon^p},$$

where $z^{p+1} - z^p$ was the distance between probes $p + 1$ and p , and η was the number of whole wavelengths between the same two probes. The propagation velocity was then given by

$$v_{Sw}^{p,p+1} = \frac{\bar{\lambda}^{p,p+1}}{\tau}. \quad (5.4)$$

D. Determination of Experimental Parameters

1. Operating Conditions

Since the goal of this experiment was to investigate the propagation of void waves in an air-water bubbly system, the range of void-fraction values was restricted between 0 and 20%. Data were therefore taken with the steady-state void fraction at 0, 5, 10, and 15%, and the perturbation void fraction was set at 5, 10, and 15%. Because the superficial liquid velocity has a definite effect on the transients, this parameter was varied, with values of 0.5, 1.5, and 3.0 ft/sec. Varying the three parameters resulted in a maximum of 36 different operating conditions.

2. Probe Positions

Figure 3.2 indicates that nine different axial-probe positions were possible; however, because of the input-channel limitation of the analog-to-digital converter (maximum of six), only five probes could be used simultaneously. Five probes were inserted over the entire length of the test section, but more closely spaced near the entrance, where the changing characteristics of the transients were more pronounced. The distances at which the probes were located were $3/4$, $1\frac{1}{4}$, $2\frac{1}{4}$, $4\frac{1}{4}$, and $7\frac{1}{4}$ ft from the entrance.

To note any effects in the radial direction, data were procured at the following distances from the wall of the $2\frac{3}{4}$ -in. pipe: $1/4$, $1/2$, $3/4$, 1, and $1\frac{3}{8}$ in. These values are equivalent to dimensionless radii of $r/R = 0.81$, 0.63, 0.45, 0.27, and 0.0, respectively. All five probes were positioned at identical radial values during any one run.

3. Analog Tape Speed

The analog tape recorder was equipped with tape speeds of 30, 15, $3\frac{3}{4}$, and $1\frac{7}{8}$ in./sec, which corresponded to frequency responses of 10, 5, 1.25, and 0.625 kc/sec, respectively. For economy, it was necessary to determine the slowest tape speed that could be used reliably. A test run was made, and the probe response was recorded at all four tape speeds. The signals were then played back and analyzed. On the basis of this test, it was found that only at $1\frac{7}{8}$ in./sec was the frequency response inadequate. All succeeding tests were therefore recorded at $3\frac{3}{4}$ in./sec.

4. Digital Sampling Time

The analog-to-digital converter was capable of sampling analog signals in intervals varying from 0.5 to 768 msec. The selection of the proper sampling rate was dependent on several factors.

First of all, a representative number of samples was required between any two successive perturbations so that a smooth curve could be drawn

depicting the wave shape. Experience showed that this required a minimum of 20 samples per cycle.

Another factor which was necessary for consideration was the change in the digital output format between various sampling times. At a sampling time of 96 msec or higher, an end-of-record gap was incorporated after each set of six samples (or whatever the number of channels used at the time). This meant that the computer used for analyzing the data could accept one record at a time into its memory core and then perform the specified operation before accepting another record. Consequently, an infinite number of records could be analyzed. If, however, the sampling time is less than 96 msec, no end-of-record gap is present, and all the data for each file must be accepted by the computer memory core before any operation can be executed. This limits the number of samples to the size of memory core which is 32,000 words for the IBM-704. Allowing 2,000 words for the program and a minimum of 20 samples per cycle per channel, the maximum number of cycles was 250. This was later found to be insufficient. Therefore, the sampling time was chosen from the variables 96, 192, 384, and 768 msec, all of which possessed the desired format. Since the time period of the perturbations was of the order 2 sec, the only sampling time that could be used was 96 msec.

5. Number of Cycles

It was very important to determine the optimum number of cycles to be sampled for each run. If this number were too small, the resulting wave shapes would contain a high percentage of scatter. If an exceedingly large number of cycles were sampled, the time and tape consumed might not outweigh the decrease in scatter.

To determine the optimum number of cycles, a probe signal was recorded for 1000 cycles. The probe response was that of steady-state conditions. Therefore, the period of each cycle was hypothetical, since no perturbations existed. The recorded signal was then played back and sampled several times in lengths of 100, 250, 500, and 1000 cycles. Each of these four files was analyzed, and the standard deviation was calculated. Let

$$\bar{\bar{Y}}_i = \frac{1}{N} \sum_{i=1}^N \bar{Y}_i. \quad (5.5)$$

Then the standard deviation is given by

$$\sigma = \left[\frac{\sum_{i=1}^N (\bar{Y}_i - \bar{\bar{Y}}_i)^2}{N - 1} \right]^{\frac{1}{2}}.$$

The results of this analysis are given in Table IV.

TABLE IV. Scatter in Steady-state Data

Number of Cycles	$\sigma/\bar{Y}_i$ (%)
100	13.15
250	10.72
500	7.99
1000	6.92

On the basis of these results, it was decided that 500 was the optimum number of cycles for each run since the improvement at 1000 cycles was not worthy of the consumption of twice as much time and tape.

VI. DISCUSSION OF RESULTS

A. Steady-state Analysis

Although the prime purpose of this research problem was an investigation of void disturbances in two-phase flow, limited data were obtained from steady-state air-water flows. Part of the steady-state data was taken during the comparison of the electrical resistivity probe with pressure-drop techniques in a forced-circulation system; additional information was obtained from the verification of the delay-circuit calibration. Since the latter data were from a natural-circulation system, this afforded a comparison of the two types of circulation systems.

Zuber and Findlay²³ derived the expression

$$\frac{V_{sg}}{\langle \alpha \rangle} = V_g = C_0 V_m + \frac{\langle \alpha v_{vg} \rangle}{\langle \alpha \rangle} \quad (6.1)$$

for analyzing and interpreting steady-state data. Equation (6.1) is identical to, but somewhat more general than, that derived by Neal,²¹ given in Eq. (2.3). On the basis of Eq. (6.1), Zuber and Findlay theorized that a plot of the average gas velocity, V_g , versus the average mixture velocity, V_m , lent itself to a simple and expeditious method of interpreting data. If the "weighted average drift velocity," $V_d = \langle \alpha v_{vg} \rangle / \langle \alpha \rangle$, does not depend on the concentration, then the average gas velocity is linearly dependent on the average mixture velocity. Therefore, if the distribution parameter, C_0 , defined by Eq. (2.1), is constant, this type of plot will result in a straight line with the slope equal to C_0 , and with an intercept equal to the "weighted average drift velocity."

By assuming exponential velocity and concentration profiles of the form

$$\frac{V_m}{V_{mc}} = 1 - r * x^{\frac{1}{x}} \quad (6.2)$$

and

$$\frac{\alpha - \alpha_W}{\alpha_c - \alpha_W} = 1 - r * n^{\frac{1}{n}}, \quad (6.3)$$

where x and n are positive constants less than one, and the subscripts c and W refer to values evaluated at the centerline and at the wall of circular channels, C_0 can be expressed in terms of x , n , and α_c by substituting Eqs. (6.2) and (6.3) into Eq. (2.1). Then,

$$C_0 = \frac{n(1+2x)}{x+n+2xn} \left[1 + \left(\frac{\alpha_c}{\langle \alpha \rangle} \right) \frac{x}{n(1+2x)} \right]. \quad (6.4)$$

In evaluating the "weighted average drift velocity," which takes into account local slip, both Neal and Zuber and Findlay suggested

$$V_d = C \left[\frac{g(\rho_l - \rho_g) D}{\rho_l} \right]^{\frac{1}{2}} \quad (6.5)$$

for the slug-flow regime; however, a value of 0.35 was set for C by Zuber and Findlay, while Neal used $C = 0.60$ to fit data from both the slug-flow and bubbly-flow regimes. A value of $C = 0.35$ was also used by Griffith³² for the bubble-rise velocity. For the churn-turbulent bubble regime, Zuber and Findlay used the following expression:

$$V_d = 1.53 \left[\frac{\sigma g(\rho_l - \rho_g)}{\rho_l^2} \right]^{\frac{1}{4}}. \quad (6.6)$$

Figure 6.1a indicates that the steady-state data, when plotted according to Eq. (6.1), show a linear relationship with respect to the average mixture velocity. The relation is independent of the type of circulation system, which is expected since the bubbles are unaware of the presence of a pump in the system. The slope of the line is equal to $C_0 = 1.1$, which would indicate flat profiles ($x = n = 1/7$) for both the velocity and concentration distributions. A check for agreement between the actual void profiles and the profiles indicated by $C_0 = 1.1$ can easily be made, since the average void fraction corresponding to the plotted data was obtained from local values of void fraction.

Fig. 6.1. Gas Velocity as a Function of Mixture Velocity,
a) Steady-state Data and b) Time-averaged Data

The criterion used to fit Eq. (6.3) to the local-void-fraction data was that the average void fraction calculated from Eq. (6.3) be equal to the actual average void fraction. This was done by finding the centerline void fraction in terms of the average void fraction. Solving Eq. (6.3) for $\langle \alpha \rangle$ and then rearranging, one obtains

$$\alpha_c = \langle \alpha \rangle (2n+1)^{-1} \quad (6.7)$$

when α_w is taken as zero (experimental data verify this).

Using Eq. (6.7), values for n were obtained from the concentration data, as seen in Fig. 6.2. The smallest value of n used to fit the data was $1/4$, which shows that the profiles were not as flat as indicated by a value of $C_0 = 1.1$. A value for x can then be found by rearranging Eq. (6.4) to give

$$x = n \frac{C_0 - 1}{\frac{\alpha_c}{\langle \alpha \rangle} - C_0} \quad (6.8)$$

Substituting $n = 1/4$ into Eq. (6.8) results in a value of $x = 1/16$, which is indicative of a very flat velocity profile.

Before one can discuss any disagreement between experimental data and theory, it should be noted that the values of x and n are very sensitive to C_0 . Had the value of C_0 been 1.2 with $n = 1/4$, then x would have increased to $1/6$. However, in this case, it does not seem possible for a line with slope 1.2 to fit the data in Fig. 6.1a. It seems unlikely that simultaneous values of $n = 1/4$ and $x = 1/16$ can exist in the same tube, but this possibility should not be ruled out, since velocity-profile measurements are not available in this type of flow. A more likely explanation is that C_0 and $\langle \alpha v_{vg} \rangle / \langle \alpha \rangle$ are not independent of $\langle \alpha \rangle$.

The intercept in Fig. 6.1a indicates a value of 0.7 ft/sec for V_d . Good agreement is shown by evaluating V_d from Eq. (6.6), which predicts a value of 0.8 ft/sec. If one used Eq. (6.5) as the expression for the slug-flow regime (which is indicated by $C_0 = 1.1$), a value of 0.95 ft/sec would be obtained, if C were taken as 0.35. With a pipe size of 2 in., Eqs. (6.5) and (6.6) predict the same value for V_d . Consequently, with circular channels between 1 and 3 in., agreement can be found using either equation.

In addition to plotting the steady-state data, the time-averaged transient data were used to note the effect of void disturbances on C_0 and V_d .

Before looking at some of these results, consider some relationships concerning gas velocities. Assume that the transient, cross-sectional, average gas velocity, $V_g(t)$, can be expressed in terms of a steady-state component, V_{g0} , and a time-varying component, $V'_g(t)$. Then

Fig. 6.2. Void-fraction Profiles

$$V_g(t) = V_{g0} + V'_g(t), \quad (6.9)$$

and

$$\overline{V_g(t)} = \overline{V_{g0} + V'_g(t)}. \quad (6.10)$$

The gas velocity can be expressed in terms of the superficial-gas velocity, V_{sg} , and the cross-sectional average void fraction, $\langle \alpha \rangle$, so that

$$V_{sg0} = V_{g0} \langle \alpha_0 \rangle, \quad (6.11)$$

and

$$V'_{sg} = (V_g \langle \alpha \rangle)'. \quad (6.12)$$

Since Eq. (6.12) can be expressed as

$$(V_g \langle \alpha \rangle)' = [(V_{g0} + V'_g)(\langle \alpha_0 \rangle + \langle \alpha' \rangle)]' - V_{g0} \langle \alpha_0 \rangle,$$

the time-varying gas velocity can then be found to be

$$V'_g(t) = \frac{V'_{sg} - V_{g0} \langle \alpha' \rangle}{\langle \alpha_0 \rangle + \langle \alpha' \rangle}. \quad (6.13)$$

Substitution of Eqs. (6.11) and (6.13) into Eq. (6.10) results in

$$\overline{V_g(t)} = \frac{V_{sg0}}{\langle \alpha_0 \rangle} + \frac{V'_{sg} - V_{g0} \langle \alpha' \rangle}{\langle \alpha_0 \rangle + \langle \alpha' \rangle},$$

which, upon expansion, can be written as

$$\begin{aligned} \overline{V_g(t)} &= \frac{V_{sg0}}{\langle \alpha_0 \rangle} + \frac{V'_{sg} - V_{g0} \langle \alpha' \rangle}{\langle \alpha_0 \rangle} \left[1 - \frac{\langle \alpha' \rangle}{\langle \alpha_0 \rangle} + \left(\frac{\langle \alpha' \rangle}{\langle \alpha_0 \rangle} \right)^2 - \dots \right] \\ &= \frac{V_{sg0} + V'_{sg}}{\langle \alpha_0 \rangle} - \frac{V_{g0} \langle \alpha' \rangle}{\langle \alpha_0 \rangle} - \frac{\langle \alpha' \rangle}{\langle \alpha_0 \rangle} \left(\frac{V'_{sg} - V_{g0} \langle \alpha' \rangle}{\langle \alpha_0 \rangle} \right) + \dots \quad (6.14) \end{aligned}$$

In evaluating $\overline{V_g(t)}$ from the experimental data, however, a time-averaged superficial-gas velocity and void fraction were used so that

$$\overline{V_g(t)} = \frac{\overline{V_{sg0} + V'_{sg}}}{\langle \alpha_0 \rangle + \langle \alpha' \rangle},$$

which, upon expansion, can be written as

$$\begin{aligned}
 V_g(t) &= \frac{\overline{V_{sg0} + V'_{sg}}}{\langle \alpha_0 \rangle} \left[1 - \frac{\langle \alpha' \rangle}{\langle \alpha_0 \rangle} + \left(\frac{\langle \alpha' \rangle}{\langle \alpha_0 \rangle} \right)^2 - \dots \right] \\
 &= \frac{\overline{V_{sg0} + V'_{sg}}}{\langle \alpha_0 \rangle} - \overline{V_{g0}} \frac{\langle \alpha' \rangle}{\langle \alpha_0 \rangle} - \frac{\langle \alpha' \rangle}{\langle \alpha_0 \rangle} \frac{\overline{V'_{sg}}}{\langle \alpha_0 \rangle} + \dots
 \end{aligned} \tag{6.15}$$

A comparison of Eqs. (6.14) and (6.15) indicates that an error in the second term could arise if the time-average of the products differs from the product of the time-averaged quantities. In general, this is the case unless the distributions of the two quantities in question are identical. In the third term of each equation, a similar error arises, plus the addition of $V_{g0} \langle \alpha' \rangle^2 / \langle \alpha_0 \rangle^2$ in Eq. (6.14). These errors, however, are of second order and can be considered negligible.

For evaluating the time-averaged mixture velocity, a similar analysis can be made. Here

$$\overline{V_m(t)} = \overline{V_{sg}(t)} + \overline{V_{sl}(t)},$$

or, if expressed in terms of components,

$$\overline{V_m(t)} = \overline{V_{sg0} + V'_{sg}(t) + V_{sl0} + V'_{sl}(t)}. \tag{6.16}$$

The time-averaged mixture velocity was calculated from the experimental data by using

$$\overline{V_m(t)} = \overline{V_{sg0} + V'_{sg}(t) + V_{sl0} + V'_{sl}(t)}. \tag{6.17}$$

In this case, however, it can easily be seen that Eqs. (6.16) and (6.17) are identical.

Using Eqs. (6.15) and (6.17), these time-averaged data were plotted as shown in Fig. 6.1b. A comparison of Figs. 6.1a and 6.1b shows that in the latter, a higher slope, $C_0 = 1.2$, and intercept, $V_d = 1.0$ ft/sec, resulted. This difference might be attributed to the error arising from the use of Eq. (6.15), since $dC_0/dt \neq 0$ in perturbed flow. However, if the use of Eq. (6.15) is valid, then a value of $C_0 = 1.2$ could indicate that the void perturbations tend to be more concentrated near the center of the channel, which would also justify a higher value for V_d . Also, it seems possible that both C_0 and V_d may be functions of the mixture velocity; since, if V_m is increased, more mixing could occur, thus causing flatter concentration profiles (decrease in C_0) and decreasing the value of V_d .

B. Presentation of Computer Results

As mentioned in Section V-C, a program was written to analyze the probe transient-response data on a computer. Runs were made for 20 different operating conditions. For each of these 20 runs, the average void response was determined at each of five axial positions and at each of five radial positions. The two-phase system was repeatedly perturbed with a pulse of air at a rate of about 0.4 cps for a total of approximately 500 cycles. In other words, a particular experiment was repeated 500 times, and the average void response of these 500 experiments was obtained by a computer. This was accomplished by sampling the void signal at constant phase angle with respect to a reference signal. The sampling rate was 0.096 sec, which indicated that for a period of $\tau = 2.5$ sec, each cycle was divided into approximately 26 equal segments. However, if the period were not an even multiple of 0.096 sec, the last segment would not be equal to other segments and would therefore be neglected.

The computer results for all 20 runs are tabulated in Appendix E, and eight representative runs are graphically illustrated in Figs. 6.3-6.10. The values of $\langle \alpha_1 \rangle$ and $\langle \alpha_2 \rangle$ represent the void fraction at the inlet corresponding to steady-state operation without and with the secondary air flow. Ideally, therefore, the subscripts 1 and 2 represent conditions downstream of, and in the middle of, the perturbation, respectively, immediately after the secondary air-injection position, $z^* = 0$. The values for $r^* = r/R$

Fig. 6.3. Average Void Response, Run 3: $V_{sl} = 1.0$ ft/sec,
 $\langle \alpha_1 \rangle = 0$, $\langle \alpha_2 \rangle = 0.102$

Fig. 6.4. Average Void Response, Run 5: $V_{sl} = 0.5$ ft/sec,
 $\langle \alpha_1 \rangle = 0$, $\langle \alpha_2 \rangle = 0.05$

Fig. 6.5. Average Void Response, Run 7: $V_{sl} = 0.5$ ft/sec,
 $\langle \alpha_1 \rangle = 0.042$, $\langle \alpha_2 \rangle = 0.171$

Fig. 6.6. Average Void Response, Run 15: $V_{sl} = 1.5$ ft/sec,
 $\langle \alpha_1 \rangle = 0.091$, $\langle \alpha_2 \rangle = 0.175$

Fig. 6.7. Average Void Response, Run 9: $V_{sl} = 1.5$ ft/sec,
 $\langle \alpha_1 \rangle = 0.048$, $\langle \alpha_2 \rangle = 0.176$

Fig. 6.8. Average Void Response, Run 11: $V_{sl} = 3.0$ ft/sec,
 $\langle \alpha_1 \rangle = 0.045$, $\langle \alpha_2 \rangle = 0.144$

Fig. 6.9. Average Void Response, Run 12: $V_{sl} = 0.5$ ft/sec,
 $\langle \alpha_1 \rangle = 0.085$, $\langle \alpha_2 \rangle = 0.133$

Fig. 6.10. Average Void Response, Run 17: $V_{sl} = 3.0 \text{ ft/sec}$,
 $\langle \alpha_1 \rangle = 0.087$, $\langle \alpha_2 \rangle = 0.170$

and $z^* = L/D$ represent the radial and axial positions, respectively, of the probes during the void-fraction measurements. Values of 0 and 1 for the REF. curves indicate when the secondary air-flow (perturbation) valve was closed or open, respectively.

As indicated by the kinematic wave theory, void waves travel forwards or backwards with respect to the gas velocity, depending on the flow regime in existence. This means that a perturbation in void will either become more concentrated (increase in void amplitude) or will spread out and eventually disappear. Runs 3 and 5 (Figs. 6.3 and 6.4) are examples in which the downstream (of the perturbation) void fraction is zero, or $\langle \alpha_1 \rangle = 0$. However, in Run 3, $\langle \alpha_2 \rangle = 0.12$ at the inlet, and the cross-sectional average void amplitude increased with axial distance due to the formation of a spherical cap bubble. In Run 5, on the other hand, $\langle \alpha_2 \rangle = 0.06$ at the inlet, but in this case, the perturbation had the tendency to spread out.

A look at the relative position of corresponding void maxima gives an indication of the propagation velocity of these maxima. A comparison of Runs 3 and 5 shows that the propagation velocity in Run 3 is greater than in Run 5, but this is expected since the superficial liquid velocity is greater in Run 3.

Run 7 (Fig. 6.5) for which the void-fraction perturbation was relatively large ($\langle \alpha_2 \rangle = 0.20$; $\langle \alpha_1 \rangle = 0.05$) shows the formation of a double maximum at the inlet. In addition to the first perturbation spreading out, the relative time position of the two peaks also increased with increasing axial distance. The spreading out of the hump is either caused by the leading edge of the hump traveling faster than the gas velocity, or by the trailing edge traveling slower than the gas velocity.

A double maximum was also formed at the inlet in Run 15 (Fig. 6.6). However, in this case, the relative time position of the two humps decreased with increasing axial distance. Also, the void amplitude of the major (first) perturbation increased with axial distance. This indicated that the trailing void waves traveled faster than the leading edge of the perturbation and eventually caught up, forming a discontinuity or "shock wave" as described by Zuber and Hench.²⁸

Figure 6.7 (Run 9) illustrates another example of the perturbation void amplitude increasing with axial distance. Here void waves upstream from the leading edge of the perturbation traveled faster than the leading edge and eventually caught up to form a shock wave.

C. Propagation Velocity of Void Waves

The void-disturbance propagation velocity was computed from the displacement on the time axis of the void peak between successive probes, as in Figs. 6.3-6.10. Since the time between successive samples was 0.096 sec, the maximum error ranged from ± 3 to $\pm 20\%$, depending upon the probe spacing. The actual error was considerably less, because of interpolation to locate the void peak position. The maximum error was further reduced to $\pm 1.5\%$ by computing the propagation velocity, according to Eq. (5.1), between the first ($z^* = 3.6$) and last ($z^* = 32.0$) probes. Using this technique, local values of the axial-average propagation velocity were found for five different radial positions from which velocity profiles were constructed. The cross-sectional average-wave velocity was then found by graphically integrating these profiles.

Figure 6.11 indicates that the centerline (local) wave velocity is relatively independent with axial position. The open symbols represent values of the wave velocity midway between two successive probes; the dotted symbols represent the average values midway between the first and last probes. If expansion of the gaseous phase can be assumed to have negligible effect, the errors are probably due to the low sampling rate and/or the small (500) number of samples.

A plot of the local wave velocity as a function of radius (Fig. 6.12) shows flat profiles, the velocity decreasing only quite close to the wall. Hence, the use of a cross-sectional average wave velocity is justified.

Fig. 6.11. Wave Velocity
as a Function of
Axial Position

Fig. 6.12. Wave Velocity
as a Function of
Radial Position

The kinematic wave theory relates the transient response of a two-phase flow to steady-state information. For this reason, steady-state measurements were made in the forced-circulation air-water loop. These data are reported in Appendix D, with the run numbers prefixed by an S, and are illustrated in a reduced form in Fig. 6.13. Since, from theoretical considerations, the wave velocity is given by Eq. (2.12) or

$$V_w = \left(\frac{\partial V_{sg}}{\partial \langle \alpha \rangle} \right)_{V_m}, \quad (6.18)$$

the data were first plotted as taken in the form V_{sg} versus $\langle \alpha \rangle$, with the superficial liquid velocity, V_{sl} , as a parameter (a), and then cross-plotted with the mixture velocity, V_m , as the parameter (b). Zuber and Hench²⁸ assumed that V_{sg} is a function of $\langle \alpha \rangle$ only, since their experimental work was limited to bubblers ($V_{sl} = 0$). This assumption does not apply to all cases where $V_{sl} \neq 0$, as shown by Fig. 6.13. More generally, $V_{sg} = V_{sg}(\langle \alpha \rangle, V_m)$, and the continuity equation requires that

$$\frac{\partial \langle \alpha \rangle}{\partial t} + \left(\frac{\partial V_{sg}}{\partial \langle \alpha \rangle} \right)_{V_m} \frac{\partial \langle \alpha \rangle}{\partial z} = \Gamma - \left(\frac{\partial V_{sg}}{\partial V_m} \right)_{\langle \alpha \rangle} \frac{\partial V_m}{\partial z}, \quad (6.19)$$

where Γ represents an equivalent, instantaneous, cross-sectional-average, gas-source strength. For adiabatic ducts of constant cross section, with no internal sources of gas, $\partial V_m / \partial z = 0$, neglecting compressibility effects. This may be compared to a derivation given recently by Bijwaard, Staub, and Zuber,³³ beginning with the continuity equation

$$\frac{\partial \langle \alpha \rangle}{\partial t} + \frac{\partial V_{sg}}{\partial z} = \Gamma, \quad (6.20)$$

where

$$V_{sg} = \langle \alpha \rangle (C_0 V_m + V_d). \quad (6.21)$$

Fig. 6.13. Steady-state Correlations, (a) V_{sg} as Parameter and (b) V_m as Parameter

From a correlation of existing data, it is concluded that C_0 and V_d remain essentially constant for a given flow regime, so that Eqs. (6.20) and (6.21) become

$$\frac{\partial \langle \alpha \rangle}{\partial t} + (C_0 V_m + V_d) \frac{\partial \langle \alpha \rangle}{\partial z} = \Gamma - \langle \alpha \rangle C_0 \frac{\partial V_m}{\partial z}. \quad (6.22)$$

Equation (6.19) is of the same form as Eq. (6.22) under these assumptions. This states that $V_w = (\partial V_{sg} / \partial \langle \alpha \rangle) V_m = V_g$, which does not agree with the present transient data (Fig. 6.15). Furthermore, the slopes of the steady-state curves of V_{sg} versus $\langle \alpha \rangle$ depend both upon V_m and $\langle \alpha \rangle$, and hence are not, in general, equal to V_g .

To determine the void propagation velocity theoretically, slopes of the V_{sg} -versus- $\langle \alpha \rangle$ curves must be calculated at the $\langle \alpha \rangle$ in question and at constant V_m . Since there is only slight curvature in the V_{sg} -versus- $\langle \alpha \rangle$ relationship, the wave velocity was approximated by a chord joining the points corresponding to the conditions downstream and at the peak of the perturbation. Thus the propagation velocity of the leading edge of the perturbation was calculated by using

$$V_w = \frac{V_{sg2} - V_{sg1}}{\langle \alpha_2 \rangle - \langle \alpha_1 \rangle}. \quad (6.23)$$

Note that Eq. (6.23) is identical to that used for determining the shock-wave velocity (Eq. 2.16). This is reasonable since a shock wave forms immediately at the leading edge of the perturbation (lagging edge if $\partial V_g / \partial \alpha < 0$). The formation of the shock wave could be seen (graphically) by constructing the characteristics ($dz/dt = V_w$) and noting when they intersect with a changing slope of dz/dt .

Approximating V_w by Eq. (6.23) is justified when compared with V_w determined from the tangent at the linear average void fraction of the perturbation. Table V indicates fairly good agreement between the two procedures.

TABLE V. Graphical Approximation of the Wave Velocity

Run	V_w (Chord Method)	V_w (Tangent Method)
2	1.67	1.67
7	1.68	1.73
14	2.71	3.00
17	3.99	4.20
19	3.77	3.70

In Fig. 6.14, the experimentally-obtained propagation velocities are compared with those predicted by Eq. (6.23). [A time-averaged mixture velocity, $\bar{V}_m$, was used for evaluation of Eq. (6.23).] The agreement is fairly good, but there is a tendency at low mixture velocities for the experimental propagation velocity to be greater than the theoretical velocity, while for large V_m , the reverse is true.

Fig. 6.14. Comparison of Experimental and Theoretical Wave Velocities

When expressing the wave velocity in terms of the gas velocity,

$$V_w = V_g + \langle \alpha \rangle \left(\frac{\partial V_g}{\partial \langle \alpha \rangle} \right)_{V_m}, \quad (6.24)$$

it can be seen that void waves will travel either forwards or backwards relative to the gas velocity, depending on whether $(\partial V_g / \partial \langle \alpha \rangle)_{V_m}$ is positive or negative. Figure 6.13 indicates that in the present experimental system, the flow regime was such that $(\partial V_g / \partial \langle \alpha \rangle)_{V_m}$ was positive. This indicates that the void waves should propagate faster than the gas velocity by a term equal to $\langle \alpha \rangle (\partial V_g / \partial \langle \alpha \rangle)_{V_m}$.

A plot of the wave velocity versus the time-averaged gas velocity (Fig. 6.15) illustrates a contradiction to Eq. (6.24). At low mixture velocities, $V_m = 0.5-2.0$ ft/sec, qualitative agreement is depicted; at higher liquid velocities, $V_{sl} = 3.0-3.5$ ft/sec, the void propagation velocity is less than the gas velocity, thus contradicting Eq. (6.22). The experimental wave velocities are listed in Table VI.

Fig. 6.15. Comparison of Wave Velocity with Gas Velocity

TABLE VI. Void Propagation Velocities

Run	V_m	V_g	V_w at $r^* =$					V_w	
			0.0	0.27	0.45	0.63	0.81	Exp. [†]	Theo.
1	1.00	-	2.12	2.05	1.99	1.83	1.99	1.88	1.60
2	1.00	-	1.99	1.99	2.05	1.78	1.78	1.79	1.67
3	1.03	-	2.08	2.08	2.05	2.05	1.69	1.74	1.73
4	2.52	-	2.94	-	2.94	3.38	3.08	2.92	2.88
5	0.51	-	1.54	1.50	1.50	1.54	1.44	1.37	1.24
6	0.52	-	1.59	1.54	1.54	-	1.57	1.43	1.25
7	0.69	2.22	2.60	2.60	2.51	2.51	2.60	2.54	1.68
8	1.70	2.89	3.31	3.22	3.22	3.22	2.95	2.97	2.44
9	1.78	3.38	3.56	3.56	3.56	3.56	3.56	3.56	2.89
10	3.50	5.95	3.38	3.91	4.17	3.68	4.67	4.04	3.96
11	3.40	5.58	3.47	4.23	3.76	3.98	3.76	3.63	4.19
12	0.67	1.72	2.60	2.60	2.71	2.60	2.71	2.59	1.63
13	0.71	1.88	2.35	2.30	2.71	2.71	2.71	2.56	1.87
14	1.79	2.79	3.65	3.91	3.65	4.05	3.65	3.47	2.71
15	1.90	3.41	3.98	4.23	3.98	3.98	4.23	3.99	3.29
16	3.50	4.80	3.56	3.22	-	3.56	3.56	3.41	4.49
17	3.56	5.02	3.76	3.38	3.22	3.56	3.22	3.14	3.99
18	0.80	2.11	2.82	2.82	2.82	2.82	-	2.82	2.18
19	2.00	3.51	3.98	3.98	3.98	3.76	3.76	3.63	3.77
20	3.85	5.83	3.98	3.98	3.98	3.98	3.76	3.67	4.78

[†] $V_w(\text{Experimental}) = \langle v_w \rangle$.

The discrepancy between the theory and the experimental results may be attributed to three possible reasons. The kinematic wave theory is applicable to one-dimensional phenomena only. Two-phase flow is not, in general, one-dimensional; however, the data show (Fig. 6.12) that this error is probably small here.

A second possibility is the existence of a shock wave across which the derivatives required in Eq. (6.24) do not exist. From continuity, however, Eq. (6.23) describes the shock-wave velocity in terms of the conditions on either side of the shock, and therefore represents the correct one-dimensional equation for this system. For weak shocks, or if V_{sg} is nearly a linear function of $\langle \alpha \rangle$ under steady conditions, the slope of the chord approximates that of the tangent, as shown above. This error should therefore also be small, and another explanation for the low propagation velocity, as compared to the theoretical prediction, must be found.

Fig. 6.16. Path of Void Waves in Forming a Shock

Eventually, the leading edge of each hump should exhibit a discontinuity in gas concentration, as would be observed in a hypothetical flow consisting entirely of flat-nosed slugs. This is not observed, probably on account of turbulent mixing effects.

To take these into account, it is desirable to reexamine the basic continuity equations. Assuming the shock region is of finite thickness, Δz , continuity for the gas requires that

The formation of a shock wave can be described by constructing characteristics and plotting the locus of the intersections. Zuber and Hench²⁸ constructed characteristics as straight lines, since the wave velocity was assumed to be a function of $\langle \alpha \rangle$ only. However, since $V_w = dz/dt$ is a function of V_m also, the characteristics will change slope as V_m changes. This is the case when a steady flow is perturbed repeatedly, as seen in Fig. 6.16 for Run 7. This illustration depicts the path of two void waves 5 ft apart, propagating through the two-phase mixture and intersecting in a shock wave. With a series of "hump-shaped" perturbations, a continuous change in the slope of the characteristics results, and a shock wave is formed immediately at each leading edge.

$$(V_{g2} - V_w) \langle \alpha_2 \rangle - \kappa \left. \frac{\partial \langle \alpha \rangle}{\partial z} \right|_2 = (V_{g1} - V_w) \langle \alpha_1 \rangle - \kappa \left. \frac{\partial \langle \alpha \rangle}{\partial z} \right|_1, \quad (6.25)$$

where κ is a turbulent diffusion coefficient for axial gas transport, and the downstream and upstream conditions outside the shock zone are denoted by the subscripts 1 and 2. Assuming that the shock-zone thickness is small,

$$\left. \frac{\partial \langle \alpha \rangle}{\partial z} \right|_2 - \left. \frac{\partial \langle \alpha \rangle}{\partial z} \right|_1 \approx \frac{\partial^2 \langle \alpha \rangle}{\partial z^2} \Delta z,$$

where the curvature is to be evaluated at some interior point of the shock zone. Then Eq. (6.25) can be rewritten as

$$V_w = \frac{V_{sg2} - V_{sg1}}{\langle \alpha_2 \rangle - \langle \alpha_1 \rangle} - \kappa \frac{\partial^2 \langle \alpha \rangle}{\partial z^2} \frac{\Delta z}{\Delta \langle \alpha \rangle}. \quad (6.26)$$

If $\partial \langle \alpha \rangle / \partial z^2 > 0$, Eq. (6.26) would show the correct trend, in that the low propagation velocities are observed at high V_m , where the turbulent diffusivity, κ , would be expected to be higher.

D. Growth or Decay of Void Perturbations

Turbulent diffusion of bubbles, which, in general, will have the effect of causing gas-concentration peaks to spread out, will modify the foregoing predictions.

Calculation of the cross-sectional average amplitude of the void perturbation was generated from the computer results (Figs. 6.3-6.10) by noting the maximum value of the void fraction (α_2) and also the steady-state value (α_1). The value α_1 actually represents the time-averaged void fraction neglecting the perturbation. The quantity of interest, therefore, is $\langle \alpha' \rangle$ where

$$\alpha' = \alpha_2 - \alpha_1. \quad (6.27)$$

Radial profiles of α' were constructed for each axial position, and values of $\langle \alpha' \rangle$ were determined by taking the cross-sectional average. Each local value of α' was obtained by noting the maximum for each axial position. Within the accuracy of the sampling technique, the phase lag of the maximum was found to be independent of the radial friction.

A theoretical prediction of the rate of decay of a void perturbation or "hump" was given by Zuber and Hench,²⁸ following the method of Lighthill and Whitham.²⁵ Since the variation with time t of the superficial gas velocity at the inlet is known, a graph is plotted with the corresponding wave velocity, V_w , as the ordinate, and its product with the time, $V_w t$, as the abscissa. Three plots in this form are shown in Fig. 6.17.

(a) Based on Square-wave Input

(b) Based on Local Probe Response

(c) Based on Average Probe Response

Fig. 6.17. Graphical Analysis of Perturbation Decay

(Additional results for the construction of these plots appear in Appendix F.) The time when a shock wave first appears is given by the reciprocal of the slope of the tangent to the curve at its left-hand point of inflexion. For the case illustrated by Fig. 6.17a, the shock wave is formed at $t = 0$. To determine further progress of a shock wave, chords are drawn so that the areas above and below (between the chord and the curve) are equal. Then the reciprocal of the slope of a chord is the time at which the shock wave absorbs the two waves on which V_w and t have the values corresponding to the end points of the chord. Therefore, at the time represented by this reciprocal of the slope, the void fraction upstream of the shock wave will be given by that value corresponding to the wave velocity at the right-hand end point of the chord (e.g., A, B, or C). Hence, this technique can be used to compare with the experimental rate of decay of a void perturbation. A mathematical analysis of this graphical technique is given by Lighthill and Whitham.²⁵

Construction of a plot of V_w versus $V_w t$ could be based on three different forms, each of which will be discussed in detail. The first form considered was based on the assumption that the perturbation existed at the inlet as a perfect square wave, with values of $\langle \alpha_1 \rangle$ and $\langle \alpha_2 \rangle$. In other words, downstream of the disturbance, the void fraction was taken as $\langle \alpha_1 \rangle$, while the void fraction in the disturbance was assumed to be constant throughout with a value of $\langle \alpha_2 \rangle$. The duration of the disturbance was noted from the reference signal shown in Figs. 6.3-6.10 as the bottom curve. The value of $\langle \alpha_1 \rangle$ was determined from pressure-drop readings with no perturbations. The value of $\langle \alpha_2 \rangle$ was then determined by holding the perturbation valve open (along with the steady-state valve) long enough so that a new value for the steady-state void fraction is achieved and measured. From Fig. 6.13, values of the wave velocity corresponding to $\langle \alpha_1 \rangle$ and $\langle \alpha_2 \rangle$ at the mixture velocity under consideration were calculated from the slopes at these conditions. These values of V_w were then plotted as shown in Fig. 6.17a. At $t = 0$, and consequently $V_w t = 0$, a discontinuity arises from the presence of both $\langle \alpha_1 \rangle$ and $\langle \alpha_2 \rangle$ at the leading edge of the perturbation. This discontinuity is indicated by a vertical line superimposed on the ordinate axis. A second discontinuity arises at the trailing edge of the perturbation at some finite time. This discontinuity, however, has a finite slope since for different values of V_w at constant t the product $V_w t$ varies.

To determine when the void amplitude decays to a particular value, the previously discussed argument, due to Lighthill and Whitham, was used. Choosing a value of $\langle \alpha_y \rangle$ given by

$$\langle \alpha_y \rangle = (1 - y) \langle \alpha_2 - \alpha_1 \rangle + \langle \alpha_1 \rangle \quad (6.28)$$

or

$$\langle \alpha_y \rangle = \langle 1 - y \rangle \langle \alpha' \rangle + \langle \alpha_1 \rangle, \quad (6.29)$$

where y is the fraction of the void amplitude that has decayed, a chord is constructed from a value of V_w corresponding to $\langle \alpha_y \rangle$; to the value of V_w corresponding to $\langle \alpha_1 \rangle$, so that lobes of equal area are formed. In Fig. 6.17a, the chord AA' was constructed for a value of $y = 0.1$, and chords BB' and CC' relate to values of $y = 0.25$ and $y = 0.5$, respectively. The reciprocal of the slope of these chords indicates the time when the void amplitude decays to a value of $\langle \alpha_y \rangle$. Since the propagation velocity of the perturbation is known, the axial position when the void amplitude is given by $\langle \alpha_y \rangle$ can be evaluated. It was assumed that no interference occurred from successive perturbations in constructing the chords AA', etc. This is verified by considering, for example, Run 7, where the perturbation wave traveled at a rate of 0.23 ft/sec faster than the steady-state void waves. Since the wave velocity of the disturbance was 1.61 ft/sec and the distance between the first and last probes was $6\frac{1}{2}$ ft, in this distance the perturbation traveled 0.93 ft with respect to the steady-state waves, and the distance between successive perturbations was 1.7 ft.

The continuity equation for the cross-sectional average void fraction is

$$\frac{\partial \langle \alpha \rangle}{\partial t} + \frac{\partial v_{sg}}{\partial z} = 0; \quad (6.30)$$

the equation for the local void fraction is

$$\frac{\partial \alpha}{\partial t} + \frac{\partial v_{sg}}{\partial z} = 0, \quad (6.31)$$

where v_{sg} is a local superficial gas velocity and $\langle v_{sg} \rangle = V_{sg}$. It follows from Eqs. (6.30) and (6.31) that $\langle \partial v_{sg} / \partial z \rangle = \partial V_{sg} / \partial z$; however,

$$\frac{\partial v_{sg}}{\partial z} = \frac{\partial V_{sg}}{\partial \langle \alpha \rangle} \frac{\partial \langle \alpha \rangle}{\partial z},$$

and

$$\left\langle \frac{\partial v_{sg}}{\partial z} \right\rangle = \left\langle \frac{\partial v_{sg}}{\partial \alpha} \frac{\partial \alpha}{\partial z} \right\rangle.$$

Since

$$\left\langle \frac{\partial v_{sg}}{\partial \alpha} \right\rangle \neq \frac{\partial \langle v_{sg} \rangle}{\partial \langle \alpha \rangle},$$

it can be seen that

$$\left\langle \frac{\partial v_{sg}}{\partial \alpha} \frac{\partial \alpha}{\partial z} \right\rangle \neq \left\langle \frac{\partial v_{sg}}{\partial \alpha} \right\rangle \frac{\partial \langle \alpha \rangle}{\partial z}.$$

Consequently, in general,

$$V_w \neq \left\langle \frac{\partial v_{sg}}{\partial \alpha} \right\rangle.$$

For flat profiles, the error in equating these quantities will be small. A second form was suggested by constructing a plot of V_w versus V_{wt} (Fig. 6.17b) using local values of void fraction. From a void response at the lowest probe position ($z^* = 3.6$)[†] and at the centerline ($r^* = 0$), Fig. 6.17b was constructed by assuming $V_w = \langle \alpha v_{sg} / \partial \alpha \rangle V_m$ for each value of α and t . The chords AA', BB', and CC' were then constructed in the same manner described previously for values of $y = 0.1, 0.25$, and 0.5 , respectively.

The reliability of the previous assumption is questionable; consequently, a third alternative criterion for constructing a plot of V_w versus V_{wt} was considered. From the void responses at the initial probe position and each radial position, the cross-sectional average void fraction was calculated graphically for each value of time, t . An average void response ($\langle \alpha \rangle$ versus t) was then plotted, and by employing Eq. (6.18) and Fig. 6.13, Fig. 6.17c was constructed. This variation of V_w with time is based on an actual (cross-sectional average) void perturbation. The chords represent the same values of y as in the other two alternate forms.

In Fig. 6.18, the theoretical decay rates based on each of the three criteria just mentioned are compared with the experimental decay rates. As expected, the decay rate based on the square-wave perturbation does not agree with the other theoretical and experimental results, but fairly good agreement is shown between the two theoretical rates based on actual void responses of a perturbation. The reason for this agreement is that the centerline void response has a similar shape to that of the cross-sectional average. However, the experimental results indicate, in general, a perturbation decay rate considerably higher than any of the theoretical predictions. Further, the experimental curves frequently exhibit local maxima and minima not predicted by the theory. In part, these effects may be attributed to an insufficient number of samples (500), shown earlier to correspond to a standard deviation of about 8%. More probably, however, axial turbulent diffusion, which is not considered by the present theory, contributed to the rapid experimental decay.

[†]In some cases, entrance effects made it necessary to choose the second probe position ($z^* = 5.8$).

Fig. 6.18. Comparison of Void Decay

From the previous discussion, it does not appear that the kinematic wave theory adequately predicts the decay of a void perturbation. In the kinematic wave theory, the decay of a perturbation is assumed to be due to different propagation velocities for different void waves. As discussed in Section VI-C, axial diffusion plays an important role in the behavior of void perturbations. Assume that the flow of gas is given by

$$V_{sg} = \kappa \frac{\partial \langle \alpha \rangle}{\partial z},$$

and when substituted into the continuity equation, one obtains

$$\frac{\partial \langle \alpha \rangle}{\partial t} + \frac{\partial V_{sg}}{\partial \langle \alpha \rangle} \frac{\partial \langle \alpha \rangle}{\partial z} - \kappa \frac{\partial^2 \langle \alpha \rangle}{\partial z^2} = 0, \quad (6.32)$$

which is nonlinear and cannot be solved by ordinary means. Using the transformation

$$z' = z - V_w t,$$

where $V_w = \partial V_{sg} / \partial \langle \alpha \rangle$, results in

$$\left(\frac{\partial \langle \alpha \rangle}{\partial t} \right)_z = \left(\frac{\partial \langle \alpha \rangle}{\partial t} \right)_{z'} + \left(\frac{\partial \langle \alpha \rangle}{\partial z'} \right)_t \left(\frac{\partial z'}{\partial t} \right)_z$$

and

$$\left(\frac{\partial \langle \alpha \rangle}{\partial z} \right)_t = \left(\frac{\partial \langle \alpha \rangle}{\partial z'} \right)_t$$

Then

$$\frac{\partial \langle \alpha \rangle}{\partial t} - \kappa \frac{\partial^2 \langle \alpha \rangle}{\partial z'^2} = 0 \quad (6.33)$$

which is the diffusion equation in a stationary medium; Eq. (6.32) is the diffusion equation in a slab traveling with velocity, V_w .

If both V_w and κ are assumed constant, one solution to Eq. (6.33) is given by

$$\Delta \alpha = \frac{Q_0}{2\sqrt{\pi \kappa t}} e^{-z'^2/4\kappa t}, \quad (6.34)$$

where Q_0 is the strength of the initial plane source of voids in $\text{ft}^3/\text{unit area}$, and $\Delta \alpha$ is the amplitude of the perturbation. This solution assumes that the system is perturbed by a source in the form of an impulse at some negative z' and at $t = 0$, and the subsequent shape of the perturbation is approximated by the error function.

We now look at the diffusion of a void perturbation (Fig. 6.19). If the subscripts 1 and 2 are used in referring to conditions at $t = t_0$ and

Fig. 6.19. Diffusion of Void Perturbation

$t = t_0 + t'$ for the same perturbation, Eq. (6.34) becomes

$$\Delta\alpha_1 = \frac{Q_0}{2\sqrt{\pi\kappa t_0}}, \quad (6.35)$$

since $z' = 0$. If z_1' is the pulse length for an amplitude of $(1/2)\Delta\alpha$, we obtain similarly

$$\frac{1}{2} \Delta\alpha_1 = \frac{Q_0}{2\sqrt{\pi\kappa t_0}} e^{-z_1'^2/4\kappa t_0}, \quad (6.36)$$

so that, from Eq. (6.35),

$$\ln 2 = \frac{z_1'^2}{4\kappa t_0}. \quad (6.37)$$

In the same manner, at $t = t_0 + t'$,

$$\ln 2 = \frac{z_2'^2}{4\kappa(t_0 + t')}. \quad (6.38)$$

By rearranging Eq. (6.37), one obtains

$$\kappa = \frac{z_1'^2}{4t_0 \ln 2}. \quad (6.39)$$

When Eq. (6.38) is substituted in Eq. (6.37), the result is

$$t_0 = \frac{z_1'^2 t'}{z_2'^2 - z_1'^2}. \quad (6.40)$$

The value for t' can easily be calculated, since the distance between the probes (at which the perturbation is observed) and the wave velocity, V_w , are known.

Void perturbation curves similar to Fig. 6.19 were constructed from the experimental data by finding the cross-sectional average void fraction at each value of time. Then $\Delta\alpha$ versus t was plotted for each of two probe positions for a particular run. Values for z_1' and z_2' were measured, and values of κ were calculated from Eq. (6.39), the results of which are shown in Table VII. In calculating κ , one assumes that the entire decay of a void perturbation is due to diffusional effects only.

TABLE VII. Average Diffusion Coefficients

Run	$\langle \alpha_1 \rangle$	$V_m(\text{ft/sec})$	$\kappa(\text{ft}^2/\text{sec})$	$\kappa_w(\text{ft}^2/\text{sec})$
5	0	0.51	0.0219	0.177
1	0	1.00	0.0332	0.346
12	0.10	0.67	0.309	0.282
13	0.10	0.71	0.273	0.83
17	0.10	3.56	2.43	3.38
3	0	1.03	-0.0268	0.288
9	0.05	1.78	-5.9	4.72
15	0.10	1.90	-1.9	3.82

The calculated values of κ appear to follow several trends. Positive values of κ indicate that the variation in void fraction spread out; negative values indicate that the kinematic wave phenomenon dominated and the variation became more concentrated. If the order of magnitude of the diffusion coefficient is noted, dependence on the downstream void fraction, $\langle \alpha_1 \rangle$, and the mixture velocity, V_m , is suggested. An increase in the void fraction causes more two-phase mixing (consequently, a higher diffusion coefficient), as does an increase in mixture velocity. A void perturbation may therefore become more concentrated due to the formation of a kinematic shock wave and then spread when diffusional effects become dominant. Examples of this change in the controlling decay process were illustrated in Figs. 6.4 and 6.5.

This approach can be applied to local disturbances so that the radial dependency on the diffusion coefficient can be determined. Local values of κ were calculated for each of four different runs, the results of which are tabulated in Table VIII.

TABLE VIII. Radial Diffusion Coefficients

Run	r^*	κ	Run	r^*	κ
7	0.0	0.0049	12	0	0.0041
	0.27	0.0043		0.27	0.0020
	0.45	0.0045		0.45	0.0027
	0.63	0.0041		0.63	0.0012
	0.81	0.0014			
11	0.0	0.0252	17	0	0.1591
	0.27	0.0686		0.27	0.3228
	0.45	0.1125		0.45	0.1183
	0.63	0.046		0.63	0.1375
	0.81	0.0067		0.81	0.1279

Table VIII indicates a general trend of the diffusion coefficients to decrease remarkably near the wall. This shows that two-phase mixing is more predominant in the center of a channel than near the wall.

In Lighthill and Whitham's analysis of the progression of a shock wave based on the kinematic wave theory, a diffusion coefficient of the order of $(V_{w_2} - V_{w_1})(V_{w_1} \Delta t)$ was suggested, where Δt is the dwell time of the disturbance. Values for these diffusion coefficients were calculated from the experimental data by

$$\kappa_w = (V_{w_2} - V_{w_1})(V_{w_1} \Delta t) \quad (6.41)$$

and compared with κ in Table VII. In most of the runs, the absolute values of the calculated diffusion coefficients exhibit values of the same order of magnitude.

VII. CONCLUSIONS

The propagation of density disturbances in an air-water system was investigated experimentally by varying the primary air flow, the secondary air flow, and the liquid velocity. These disturbances were measured at each of five axial and five radial positions by an electrical-resistivity probe.

Steady-state void-fraction measurements by the electrical-resistivity probe showed better agreement with pressure-drop measurements upon modification of the original probe. It was shown that decreasing the radius of curvature of the probe tip, and increasing the liquid velocity of the system, decreased the error in the probe measurements from 35% to less than 7%. These two factors are believed to have played an important role in decreasing the deflection of bubbles by the probe and thus improving the accuracy of the measurements.

The steady-state data were plotted on a V_g -versus- V_m plane, and the points, in general, could be described by a straight line with a slope of $C_0 = 1.1$ and an intercept of $V_d = 0.7$ ft/sec. From theory, a value of $C_0 = 1.1$ would indicate flat velocity and concentration profiles; experimental data showed concentration profiles readily described by a one-third or one-fourth power-law distribution. For this value of C_0 to hold, the velocity profile would be required to obey a power-law distribution with an exponent of $m = 1/16$. Although it is unlikely that these distributions exist simultaneously, this possibility cannot be ruled out, since velocity-profile measurements are not available in this type of flow. A more likely explanation is that C_0 and V_d are not independent of $\langle \alpha \rangle$.

The time-averaged transient data were also plotted on the V_g -versus- V_m plane, resulting in a rather straight line with a slope of $C_0 = 1.2$ and an intercept of $V_d = 1.0$ ft/sec. The difference between the steady-state data and time-averaged transient data might be attributed to the fact that $dC_0/dt \neq 0$ in perturbed flow. Also, that both C_0 and V_d may be functions of the mixture velocity; if V_m is increased, more mixing could occur, thus causing flatter concentration profiles (decrease in C_0) and decreasing the value of V_d .

From the statistical average void response of the transient data, qualitative agreement is depicted when the data are compared with the kinematic wave theory. Analysis of the void-fraction response is subject to errors when the sampling rate and/or the number of samples are low; however, it is shown that the sampling technique can readily be used for determining the propagation velocity and decay of a void perturbation.

Local void data indicate that the local wave velocity is relatively independent of axial position, but dependency on the radial position is only seen quite close to the wall.

A comparison of the experimental propagation velocity of the density disturbances with predictions from the kinematic wave theory as applied by Zuber and Hench shows fairly good agreement, but there is a tendency, at low mixture velocities, for the experimental velocity to be greater than the theoretical velocity; for large V_m , the reverse is true. Also, at low mixture velocities, the propagation velocity is greater than the time-averaged gas velocity, but at higher mixture velocities, the correlation is reversed. This does not agree with the assumptions of Zuber and Findlay that C_0 and V_d remain essentially constant for a given flow regime.

The kinematic wave theory was originally applied to two-phase flow by assuming $V_{sg} = V_{sg}(\alpha)$, whereas the experimental data indicate that V_{sg} is probably a function of the mixture velocity also. A correction for the discrepancy of the experimental results with theory is suggested by modifying the basic continuity equations to take into account turbulent diffusion, which is believed to be important.

A comparison of the experimental decay rate with that predicted by the kinematic wave theory shows, in general, a perturbation decay rate considerably higher than predicted. This is probably attributed to axial turbulent diffusion, which is not considered by the kinematic wave theory.

Assuming that turbulent diffusion cannot be neglected provides a solution to the diffusion equation, and values for the diffusion coefficient were calculated from the experimental data. A general trend of the diffusion coefficient to increase with increasing downstream void fraction and increasing mixture velocity is depicted. Also, the coefficients showed a trend to decrease remarkably, near the wall. Since the solution of the diffusion equation assumes that the decay is entirely due to diffusion, a few calculated coefficients were negative in value. This indicated that the variation in void fraction was becoming more concentrated because the kinematic-wave phenomenon had a greater contribution than the diffusion phenomenon.

APPENDIX A

Air-flow Regulator

The operation of the perturbation solenoid valve (V_1) was controlled by the air-flow regulator, an electronic system designed for this specific application. This system was described very briefly in Section III-B-1, but a more detailed explanation of its operation is presented here. The main objective of this system was to operate a solenoid valve with the ability to vary continuously (as opposed to discretely) the frequency and the duration of the perturbations.

A schematic diagram of this air flow regulator circuit is shown in Fig. A.1, and the values of the circuit components are listed in Table IX. Electronic pulses, whose repetition rate was determined by an RC network (R_5C_1 and R_6C_2), were generated in an astable multivibrator consisting of Q_1 , Q_2 , and associated components. A multivibrator is a two-stage network in which only one stage conducts at any instant. In an astable multivibrator, both stages are unstable. The frequency of the air perturbations was determined by the frequency of this network. For the conducting time of one stage to equal that of the other stage, which was instituted for a more efficient system, the following relation must hold: $R_5C_1 = R_6C_2$. With $C_1 = C_2$, the resistances, R_5 and R_6 , were ganged potentiometers of equal value so that the frequency could be a continuous variable.

The pulses, which ranged from 0.15 to 0.55 cps, were transmitted through proper wave-shaping circuitry (C_3 and R_8) and then applied to a two-stage complimentary emitter follower. Components C_3 and R_8 were incorporated to alter the pulses to a more desirable shape. The purpose of the emitter follower was to match impedances between two successive stages. The output of the emitter follower was applied to another waveshape-maintaining stage (R_{10} and C_4) and then through a diode (CR_1), which clipped any negative-going, leading or lagging, edges of pulses. These pulses were applied to the base of Q_5 , which was a portion of a monostable multivibrator. This type of multivibrator consists of one stable and one unstable stage. To send Q_5 , which was in the stable stage, into the conducting state, a pulse was needed. The length of time this transistor remained on was determined by $R_{14}C_6$, which was in the unstable stage. After Q_5 turned off, another pulse was needed before entering the conducting state. The duration of the air perturbations was determined by the output of Q_5 . This output went through another waveshape-maintaining stage ($R_{17}C_7$), and then through another emitter follower, across the load of which was recorded the signal on magnetic tape.

Pulses from the collector of Q_5 were transmitted through proper steering diodes (CR_2 and CR_3) to another load consisting of a relay (K_1) coil, whose sensitivity was determined by a trimpot (R_{21}). One set of contacts on this relay operated a digital counter, which tabulated the total number of

Fig. A.1. Air-flow Regulator, Schematic Diagram

TABLE IX. Air-flow Regulator Components

<u>Resistors and Potentiometers</u>		<u>Resistors and Potentiometers</u>	
R ₁	2 k	R ₁₅	2 k
R ₂ , R ₃	100 k	R ₁₆	330 k
R ₄	2 k	R ₁₇	220 k
R ₅ , R ₆	500 k, 5 W	R ₁₈ , R ₁₉	10 k
R ₇	330 k	R ₂₀	2 k
R ₈	220 k	R ₂₁	1 k, 1/4 W
R ₉	10 k	R ₂₂ , R ₂₃	10, 5 W
R ₁₀	1.5 k	R ₂₄ , R ₂₅	500, 5 W
R ₁₁	1 k	R ₂₆ , R ₂₇	200, 5 W
R ₁₂	26 k	R ₂₈ , R ₂₉	510, 5 W
R ₁₃	5 k	R ₃₀	12, 10 W
R ₁₄	100 k, 5 W		

All resistors are 1/2-watt, $\pm 1\%$, unless otherwise specified. The values are in ohms.

<u>Capacitors</u>		<u>Semiconductors</u>	
C ₁ , C ₂	10	CR ₁	F2
C ₃	0.5	CR ₂ -CR ₅	1N270
C ₄	10	CR ₆ , CR ₇	F6
C ₅	560 pf	CR ₈	M500
C ₆	100	Q ₁ -Q ₈	2N652
C ₇	50	CRZ ₁	1N1355A
C ₈	250	CRZ ₂	1N1355RA
All values are in microfarads, unless otherwise specified.		CRZ ₃ , CRZ ₄	6 V

Miscellaneous

K ₁	Stevens Arnold A-22
K ₂	KRP 14DG
L ₁ , L ₂	44
S ₁ , S ₂	SPST, Fuse, and Lamp Combination
T ₁	115-V Isolation Transformer
T ₂ , T ₃	6.3-V Filament Transformer
V ₁ , V ₂	Solenoid Valve

pulses. Another set of contacts on the sensitive relay operated a driver relay (K₂) which, in turn, operated the solenoid valve. Contacts on this relay were also used for indicating lamps on the front panel. The operation of this system was controlled by switch S₁. The on-off solenoid valve (V₂) was controlled by switch S₂.

APPENDIX B

Void-probe Trigger

The electrical-resistivity probe was originally designed so that a simple series circuit of a battery, a resistor, and the probe would result in an output that could be used directly in measuring void fractions. However, as explained in Reference 11, this was not feasible. The probe was used as a switch of variable impedance and as the input to a trigger circuit. A schematic diagram of this circuit is shown in Fig. B.1, and the values of the circuit components are listed in Table X.

TABLE X. Void-probe Trigger Components

<u>Resistors and Potentiometers</u>		<u>Resistors and Potentiometers</u>	
R ₁	50 k	R ₁₂	130
R ₂	10 M	R ₁₃	1 M
R ₃	50 k	R ₁₄	2 k
R ₄	5 k, 5 W	R ₁₅	10 k
R ₅	2 k, 2 W	R ₁₆	5.6 k
R ₆	3 k, 2 W	R ₁₇	47
R ₇	20 k	R ₁₈	100 k, $\pm 0.05\%$, 5 W
R ₈	5 k, 5 W	R ₁₉	910
R ₉	2 k, 2 W	R ₂₀	50 k, 5 W
R ₁₀	33 k	R ₂₁	20 k, 5 W
R ₁₁	22 k	R ₂₂	10 k

All resistors are 1/2 watt, $\pm 1\%$, unless otherwise specified. The values are in ohms.

<u>Capacitors</u>		<u>Semiconductors</u>	
C ₁	100 pf	CR ₁ , CR ₂	F ₂
C ₂	0.1 μ f	Q ₁ -Q ₅	2N652
		CRZ ₁	1N3024B
		CRZ ₂	1N746

<u>Miscellaneous</u>	
L ₁	NE51
L ₂ -L ₇	47
S ₁ , S ₂	DPDT
T ₁	6.3-V Filament Transformer

When the probe tip was contained in an air bubble, an open circuit was created at the input. When the probe tip was in the liquid phase, its low impedance (relative to an open circuit) resulted in a voltage drop across R₂. Resistor R₂ was across the base and emitter of the input transistor of the solid-state amplifier, P-2. The base was biased so that a low-impedance

Fig. B.1. Void-probe Trigger, Schematic Diagram

input would result in current flow. This pulse went through the voltage amplifier, P-2, which was an operational amplifier with high gain. The gain stability and amplitude were achieved by the feedback resistor, R_1 , and the current driver (or output amplifier), P-5. A current driver has current gain but no voltage gain and is able to drive a low-impedance stage.

The current drive was followed by a Schmitt trigger circuit or level detector. This standard circuit performs as a switch to turn on full conduction whenever a preset voltage level is exceeded by any small amount. This circuit is also known as a zero-level detector. The triggering level was set slightly above zero so as not to turn on with undesirable pulses or low-level noise. This level was controlled simultaneously by potentiometers R_4 and R_8 , which were known as the sensitivity and level "pots," respectively.

Pulses from the Schmitt trigger, which was represented by Q_1 and Q_2 , were directed to one of two output circuits. The pulses were then either directed to a d.c. bucking-voltage circuit, 20 V supply and R_{21} , or were switched (S_1) to a variable-delay-mode type of operation. The time-delay circuit consisted of a basic monostable multivibrator: Q_4 , Q_5 , and associated components. Pulses from the Schmitt trigger output, at the collector of Q_2 , caused a voltage drop across R_7 , which was applied to the base of Q_3 . Transistor Q_3 was the primary component of an emitter follower or buffer stage. It maintained the wave shape of a pulse going from high-impedance to low-impedance stages. The variable delay was determined by the network consisting of R_{18} and C_2 . The delay was incorporated to elongate, with respect to time, the bubble pulses. Diodes CR_1 and CR_2 prevented undesirable triggering of the multivibrator and insured quick recovery of the quasi-conditioned multivibrator. The output of this time-delay circuit was, in turn, fed to the same d.c. bucking-voltage circuit.

The bucking-voltage circuit was needed so that a zero output would be maintained as noted for a meter with a low-impedance input. In this manner, the output could be electronically integrated to measure the void fraction. Potentiometer R_{20} could be adjusted to give any desired amplitude from 0 to 10 volts in the output signal. Three different points in the void-probe trigger were tapped to BNC connectors on the front panel so that the pulses at these points could be observed on an oscilloscope. These three points were designated as: A, the primary signal or input to the Schmitt trigger; B, the secondary signal or output of the Schmitt trigger and input to the time-delay circuit; and C, the delayed signal or output of the time-delay circuit.

Because five probes were employed simultaneously, at least five void-probe-trigger systems were necessary and were consequently mounted on two separate panels. A third panel was constructed to facilitate the observance of points A, B, and C from any of the six systems. Panel four consisted of the Dressen Barnes power supplies.

APPENDIX C

Determination of Void-fraction Relationship with Axial Length

To make a true comparison between void-fraction measurements of the resistivity probe and the pressure-drop techniques, it was necessary to locate the probe in the proper axial position so that the cross-sectional average void fraction at this point was representative of the volumetric average void fraction between the two pressure taps in a constant cross-sectional area conduit.

In an air-water system open to the atmosphere, a first approximation would indicate a linear relationship between the cross-sectional average void fraction, $\langle \alpha \rangle$, and the axial length or head, z . This means that the location of the resistivity probe would be midway between the pressure taps. However, due to the presence of slip (ratio of the average gas velocity to the average liquid velocity) a second-order-relation equation would better describe the relation between $\langle \alpha \rangle$ and z .

Bankoff²² showed that

$$\langle \alpha \rangle = K\beta, \quad (C.1)$$

where K is the flow parameter, having a value of 0.71 to a first approximation, and β is the volumetric gas fraction, which can be expressed as

$$\beta = \frac{W_g/\rho_g}{W_g/\rho_g + W_l/\rho_l} \quad (C.2)$$

Using Eq. (C.2) and rearranging, one can write Eq. (C.1) as

$$\langle \alpha \rangle = K \left(1 + \frac{W_l \rho_g}{W_g \rho_l} \right)^{-1} \quad (C.3)$$

Under steady-state conditions, ρ_g is the only term of Eq. (C.3) that is a function of axial distance. If the ideal gas law is assumed,

$$\rho_g = \frac{P}{R_0 \theta} \quad (C.4)$$

where P is the pressure, θ is the temperature, and R_0 is the gas-law constant. Since

$$P = z\rho_m + P_0,$$

and

$$\rho_m = \langle \alpha_z \rangle \rho_g + (1 - \langle \alpha_z \rangle) \rho_\ell,$$

Eq. (C.4) can be rewritten as

$$\rho_g = \frac{1}{R_0 \theta} [z(1 - \langle \alpha_z \rangle) \rho_\ell + P_0], \quad (C.5)$$

where P_0 is atmospheric pressure and

$$\langle \alpha_z \rangle = \frac{1}{z} \int_z^0 \langle \alpha \rangle(z) dz. \quad (C.6)$$

The additional assumption of $(1 - \langle \alpha_z \rangle) \rho_\ell \gg \langle \alpha_z \rangle \rho_g$ was also considered. Substitution of Eq. (C.5) into Eq. (C.3) results in

$$\langle \alpha \rangle(z) = K \left\{ 1 + C_2 [z C_3 (1 - \langle \alpha_z \rangle) + 1] \right\}^{-1}, \quad (C.7)$$

where

$$C_2 = \frac{W_\ell P_0}{W_g \rho_\ell R_0 \theta}, \quad (C.8)$$

and

$$C_3 = \rho_\ell / P_0. \quad (C.9)$$

Equation (C.7) can be expanded by long division to give

$$\begin{aligned} \langle \alpha \rangle(z) = \frac{K}{C_2 + 1} \left\{ 1 - \left[\frac{C_2 C_3 (1 - \langle \alpha_z \rangle) z}{C_2 + 1} \right] + \left[\frac{C_2 C_3 (1 - \langle \alpha_z \rangle) z}{C_2 + 1} \right]^2 \right. \\ \left. - \left[\frac{C_2 C_3 (1 - \langle \alpha_z \rangle) z}{C_2 + 1} \right]^3 + \dots \right\} \end{aligned} \quad (C.10)$$

Assume that $\langle \alpha \rangle(z)$ is a second-order equation of the form

$$\langle \alpha \rangle(z) = a_0 + a_1 z + a_2 z^2, \quad (C.11)$$

where $\langle \alpha \rangle(0)$ is taken to be at the exit.

Then Eq. (C.6) becomes

$$\langle \alpha_z \rangle = -[a_0 + (1/2)a_1z + (1/3)a_2z^2]. \quad (C.12)$$

Substitution of Eq. (C.12) into Eq. (C.10) gives

$$\langle \alpha \rangle(z) = \frac{K}{C_2 + 1} \left\{ 1 - \frac{C_2 C_3}{C_2 + 1} (1 + a_0) z + \frac{C_2 C_3}{C_2 + 1} \left[\frac{C_2 C_3}{C_2 + 1} (1 + a_0)^2 - (1/2)a_1 \right] z^2 \right\}$$

from which the constants of Eq. (C.11) can be evaluated to give

$$a_0 = \frac{K}{C_2 + 1},$$

$$a_1 = -\frac{K}{(C_2 + 1)^3} [C_2 C_3 (K + C_2 + 1)],$$

and

$$a_2 = \frac{K(C_2 C_3)^2}{(C_2 + 1)^5} [(3/2)K^2 + (5/2)K(C_2 + 1) + 2(C_2 + 1/2) + C_2^2].$$

At the exit, the void fraction is then given by

$$\langle \alpha \rangle(0) = \frac{K}{C_2 + 1}; \quad (C.13)$$

at $z = 1$, it is given by

$$\begin{aligned} \langle \alpha \rangle(1) = & \frac{K}{C_2 + 1} - \frac{K}{(C_2 + 1)^3} [C_2 C_3 (K + C_2 + 1)] \\ & + \frac{K(C_2 C_3)^2}{(C_2 + 1)^5} [(3/2)K^2 + (5/2)K(C_2 + 1) + 2(C_2 + 1/2) + C_2^2]; \end{aligned} \quad (C.14)$$

at $z = 1/2$, it is given by

$$\begin{aligned} \langle \alpha \rangle(1/2) = & \frac{K}{C_2 + 1} - \frac{K}{2(C_2 + 1)^3} [C_2 C_3 (K + C_2 + 1)] \\ & + \frac{K(C_2 C_3)^2}{4(C_2 + 1)^5} [(3/2)K^2 + (5/2)K(C_2 + 1) + 2(C_2 + 1/2) + C_2^2]. \end{aligned} \quad (C.15)$$

To determine the degree of deviation from linearity, the average value of Eqs. (C.13) and (C.14) must be compared with Eq. (C.15). This was found to be equal to the last term of Eq. (C.15), or

$$\frac{\langle \alpha \rangle(0) + \langle \alpha \rangle(1)}{2} - \langle \alpha \rangle(1/2) = \frac{K(C_2 C_3)^2}{4(C_2 + 1)^5} [(3/2)K^2 + (5/2)K(C_2 + 1) + 2(C_2 + 1/2) + C_2^2]. \quad (C.16)$$

The following typical values were used to evaluate Eqs. (C.15) and (C.16):

$$W_\ell = 4 \text{ lb/sec},$$

$$W_g = 2 \times 10^{-3} \text{ lb/sec},$$

and

$$\theta = 520^\circ \text{F}.$$

The results were

$$\langle \alpha \rangle(1/2) = 0.2033,$$

and

$$\frac{\langle \alpha \rangle(0) + \langle \alpha \rangle(1)}{2} - \langle \alpha \rangle(1/2) = 0.0004.$$

On the basis of this analysis, the assumption of a linear relationship between $\langle \alpha \rangle$ and z introduces an error of 0.197%.

APPENDIX D
Steady-state Data

Run	X (10^{-3})	V_{sl} (ft/sec)	$\langle \alpha \rangle$	Run	X (10^{-3})	V_{sl} (ft/sec)	$\langle \alpha \rangle$
C-1	0.322	4.1	0.139	S-31	0.218	1.5	0.093
C-2	0.161	4.1	0.079	S-32	0.273	1.5	0.110
C-3	0.0396	4.1	0.019	S-33	0.346	1.5	0.128
C-4	0.592	2.5	0.211	S-34	0.403	1.5	0.142
C-5	0.485	2.5	0.179	S-35	0.482	1.5	0.160
C-6	0.250	2.5	0.120	S-36	0.559	1.5	0.178
C-7	0.0840	2.5	0.037	S-37	0.041	2.0	0.020
C-8	0.739	1.5	0.203	S-38	0.086	2.0	0.040
C-9	0.140	1.5	0.058	S-39	0.115	2.0	0.056
				S-40	0.153	2.0	0.072
V-1	0.0833	0.56	0.017	S-41	0.196	2.0	0.092
V-2	0.173	0.97	0.052	S-42	0.265	2.0	0.121
V-3	0.276	1.25	0.099	S-43	0.343	2.0	0.139
V-4	0.385	1.21	0.135	S-44	0.393	2.0	0.152
V-5	0.63	0.74	0.154	S-45	0.431	2.0	0.163
V-6	0.91	0.51	0.191	S-46	0.027	2.5	0.013
				S-47	0.047	2.5	0.022
S-1	0.134	0.5	0.023	S-48	0.071	2.5	0.034
S-2	0.236	0.5	0.051	S-49	0.101	2.5	0.047
S-3	0.354	0.5	0.086	S-50	0.118	2.5	0.058
S-4	0.425	0.5	0.097	S-51	0.165	2.5	0.078
S-5	0.495	0.5	0.108	S-52	0.213	2.5	0.102
S-6	0.551	0.5	0.116	S-53	0.244	2.5	0.118
S-7	0.613	0.5	0.127	S-54	0.291	2.5	0.136
S-8	0.708	0.5	0.136	S-55	0.323	2.5	0.146
S-9	0.778	0.5	0.148	S-56	0.363	2.5	0.158
S-10	0.867	0.5	0.158	S-57	0.022	3.0	0.012
S-11	0.983	0.5	0.172	S-58	0.045	3.0	0.020
S-12	1.14	0.5	0.189	S-59	0.078	3.0	0.034
S-13	1.30	0.5	0.208	S-60	0.108	3.0	0.048
S-14	0.066	1.0	0.013	S-61	0.138	3.0	0.062
S-15	0.116	1.0	0.040	S-62	0.180	3.0	0.082
S-16	0.174	1.0	0.062	S-63	0.220	3.0	0.102
S-17	0.209	1.0	0.073	S-64	0.254	3.0	0.116
S-18	0.240	1.0	0.082	S-65	0.276	3.0	0.126
S-19	0.271	1.0	0.092	S-66	0.313	3.0	0.140
S-20	0.306	1.0	0.101	S-67	0.024	4.0	0.011
S-21	0.349	1.0	0.112	S-68	0.034	4.0	0.016
S-22	0.407	1.0	0.124	S-69	0.045	4.0	0.022
S-23	0.473	1.0	0.137	S-70	0.072	4.0	0.036
S-24	0.523	1.0	0.146	S-71	0.105	4.0	0.050
S-25	0.600	1.0	0.164	S-72	0.135	4.0	0.063
S-26	0.697	1.0	0.173	S-73	0.163	4.0	0.078
S-27	0.044	1.5	0.018	S-74	0.190	4.0	0.088
S-28	0.078	1.5	0.033	S-75	0.215	4.0	0.102
S-29	0.102	1.5	0.046	S-76	0.248	4.0	0.117
S-30	0.174	1.5	0.076				

APPENDIX E

Transient Data

The transient data were obtained as described in Chapter V. The operating conditions for obtaining the transient data are summarized in Table XI.

The actual data resulting from the computer are shown in Table XII for each of the 20 runs. The data were taken at each of the five radial positions (r^*) and five axial positions (z^*) listed. The column headed by REF. indicates when the perturbation valve was closed or open by values of 0.0 or 1.0, respectively. (A value approaching 1.0 indicated an open valve.) The numbers listed for each r^* and z^* are the values of void fraction calculated by Eq. (5.1) at constant phase angle. The void-fraction values were obtained from a statistical average for each time interval of 0.096 sec.

TABLE XI. Summary of Transient Data

Run	V_{sl} (ft/sec)	$\langle \alpha_1 \rangle$	$\langle \alpha_2 \rangle$	X_1 (10^{-3})	X_2 (10^{-3})
1	1.0	0	0.025	0	0.065
2	1.0	0	0.057	0	0.096
3	1.0	0	0.102	0	0.35
4	2.5	0	0.052	0	0.15
5	0.5	0	0.050	0	0.29
6	0.5	0	0.107	0	0.59
7	0.5	0.042	0.171	0.22	1.38
8	1.5	0.045	0.131	0.14	0.41
9	1.5	0.048	0.176	0.14	0.72
10	3.0	0.043	0.091	0.13	0.27
11	3.0	0.045	0.144	0.13	0.39
12	0.5	0.085	0.133	0.42	0.74
13	0.5	0.089	0.174	0.43	1.22
14	1.5	0.090	0.135	0.26	0.40
15	1.5	0.091	0.175	0.29	0.71
16	3.0	0.087	0.136	0.23	0.33
17	3.0	0.087	0.170	0.22	0.46
18	0.5	0.128	0.177	0.80	1.31
19	1.5	0.124	0.177	0.44	0.72
20	3.0	0.132	0.178	0.39	0.55

TABLE XII. Transient Data (Contd.)

RUN 7

Z*
r*

0.00

0.27

0.45

0.63

0.81

3.6 5.8 10.2 18.2 32.0 REF.

0.18763E-00	0.24733E-00	0.66098E-01	0.85288E-01	0.29424E-00	0.
0.10043E-00	0.16205E-00	0.99494E-01	0.74627E-01	0.24307E-00	0.
0.18977E-00	0.22601E-00	0.63156E-01	0.63156E-01	0.14929E-00	0.
0.15991E-00	0.23241E-00	0.42644E-00	0.89552E-01	0.89552E-01	0.
0.19081E-01	0.17010E-00	0.30277E-00	0.89552E-01	0.74627E-01	0.
0.78891E-01	0.14072E-00	0.16418E-00	0.51173E-01	0.74759E-01	0.
0.35437E-01	0.03817E-01	0.15352E-00	0.57569E-01	0.11087E-00	0.
0.49041E-01	0.11544E-00	0.20043E-00	0.53305E-01	0.13433E-00	0.
0.83156E-01	0.70362E-01	0.20256E-00	0.78891E-01	0.89552E-01	0.
0.46908E-01	0.66098E-01	0.10021E-00	0.68230E-01	0.61834E-01	0.
0.12495E-01	0.07602E-01	0.11087E-00	0.13433E-00	0.28308E-01	0.
0.49041E-01	0.03817E-01	0.01684E-01	0.29211E-00	0.40041E-01	0.
0.34115E-01	0.09701E-01	0.78891E-01	0.34115E-00	0.21983E-01	0.
0.44776E-01	0.49041E-01	0.72495E-01	0.29211E-00	0.29951E-01	0.
0.44776E-01	0.53437E-01	0.63156E-01	0.16205E-00	0.42644E-01	0.
0.53305E-01	0.61834E-01	0.10448E-00	0.13859E-00	0.38380E-01	0.
0.29851E-01	0.55437E-01	0.78891E-01	0.15565E-00	0.36247E-01	0.30704E-00
0.40041E-01	0.40041E-01	0.10021E-00	0.15565E-00	0.44776E-01	0.09360E-00
0.42644E-01	0.46908E-01	0.10021E-00	0.10448E-00	0.68230E-01	0.98934E-00
0.13305E-01	0.36247E-01	0.78891E-01	0.63156E-01	0.57569E-01	0.98934E-00
0.14929E-00	0.13305E-01	0.78891E-01	0.55437E-01	0.40041E-01	0.98934E-00
0.31770E-00	0.61834E-01	0.78891E-01	0.59701E-01	0.91684E-01	0.98934E-00
0.17058E-00	0.53305E-01	0.53305E-01	0.51173E-01	0.21535E-00	0.98721E-00
0.14286E-00	0.40041E-01	0.10704E-00	0.	0.74627E-01	0.87755E-00

0.21702E-00	0.23830E-00	0.61702E-01	0.72340E-01	0.27234E-00	0.
0.17600E-00	0.14255E-00	0.88038E-01	0.10851E-00	0.21702E-00	0.
0.11702E-00	0.20436E-00	0.13830E-00	0.63830E-01	0.14464E-01	0.
0.12766E-00	0.17600E-00	0.34255E-00	0.63830E-01	0.10213E-00	0.
0.85106E-01	0.20426E-00	0.29362E-00	0.70213E-01	0.74468E-01	0.
0.23957E-01	0.10438E-00	0.17324E-00	0.60851E-01	0.72340E-01	0.
0.42553E-01	0.03817E-01	0.18511E-00	0.78723E-01	0.55745E-01	0.
0.76594E-01	0.68085E-01	0.20000E-00	0.76594E-01	0.89362E-01	0.
0.37447E-01	0.88085E-01	0.16000E-00	0.76594E-01	0.65957E-01	0.
0.46809E-01	0.57447E-01	0.11277E-00	0.82979E-01	0.45957E-01	0.
0.25532E-01	0.48936E-01	0.65106E-01	0.11913E-00	0.34043E-01	0.
0.53191E-01	0.51064E-01	0.70213E-01	0.30851E-00	0.27660E-01	0.
0.42553E-01	0.48936E-01	0.78723E-01	0.35319E-00	0.27660E-01	0.
0.27660E-01	0.44681E-01	0.63830E-01	0.28085E-00	0.48936E-01	0.
0.53191E-01	0.53191E-01	0.65106E-01	0.15319E-00	0.34043E-01	0.
0.48936E-01	0.61702E-01	0.76594E-01	0.13611E-00	0.36170E-01	0.
0.38298E-01	0.44681E-01	0.78723E-01	0.18298E-00	0.44681E-01	0.25532E-00
0.34043E-01	0.26170E-01	0.67234E-01	0.12979E-00	0.46809E-01	0.90787E-00
0.53191E-01	0.44681E-01	0.63830E-01	0.11702E-00	0.44681E-01	0.90787E-00
0.55319E-01	0.36170E-01	0.89362E-01	0.60851E-01	0.38298E-01	0.90787E-00
0.10851E-00	0.46809E-01	0.68085E-01	0.53191E-01	0.29787E-01	0.90787E-00
0.11044E-00	0.60851E-01	0.76594E-01	0.68085E-01	0.76594E-01	0.90787E-00
0.15745E-00	0.21702E-00	0.68085E-01	0.51064E-01	0.17021E-00	0.90787E-00
0.18841E-00	0.43478E-00	0.57971E-01	0.78084E-01	0.78084E-01	0.95652E-00

0.15171E-00	0.20513E-00	0.55556E-01	0.61966E-01	0.23504E-00	0.
0.16739E-00	0.12393E-00	0.57692E-01	0.72650E-01	0.17735E-00	0.
0.14530E-00	0.20726E-00	0.69744E-01	0.81197E-01	0.69744E-01	0.
0.10484E-00	0.10426E-00	0.27250E-00	0.65470E-01	0.76923E-01	0.
0.83333E-01	0.15598E-00	0.22209E-00	0.68376E-01	0.42735E-01	0.
0.61966E-01	0.11752E-00	0.11111E-00	0.59829E-01	0.40598E-01	0.
0.53419E-01	0.83333E-01	0.13171E-00	0.59829E-01	0.58991E-01	0.
0.29915E-01	0.71805E-01	0.13748E-00	0.52564E-01	0.72650E-01	0.
0.55556E-01	0.76923E-01	0.12607E-00	0.68376E-01	0.87607E-01	0.
0.38462E-01	0.72650E-01	0.10897E-00	0.64103E-01	0.51282E-01	0.
0.27353E-01	0.44672E-01	0.72650E-01	0.11752E-00	0.21368E-01	0.
0.25641E-01	0.57692E-01	0.57692E-01	0.76709E-01	0.27778E-01	0.
0.23504E-01	0.42735E-01	0.72650E-01	0.21838E-00	0.27778E-01	0.
0.13504E-01	0.53419E-01	0.15956E-00	0.22505E-00	0.27778E-01	0.
0.36325E-01	0.47029E-01	0.87607E-01	0.11538E-00	0.34188E-01	0.
0.19231E-01	0.44672E-01	0.70513E-01	0.12607E-00	0.38462E-01	0.
0.19915E-01	0.25641E-01	0.68176E-01	0.68176E-01	0.44672E-01	0.28419E-00
0.23504E-01	0.38462E-01	0.70513E-01	0.11111E-00	0.32051E-01	0.90787E-00
0.25641E-01	0.47029E-01	0.79660E-01	0.64017E-01	0.42735E-01	0.90787E-00
0.13504E-01	0.27787E-01	0.72784E-01	0.12654E-01	0.28419E-01	0.90787E-00
0.01880E-01	0.34188E-01	0.55556E-01	0.55556E-01	0.21368E-01	0.90787E-00
0.21368E-01	0.44672E-01	0.44672E-01	0.57692E-01	0.55556E-01	0.90787E-00
0.10470E-00	0.20000E-00	0.28462E-01	0.57692E-01	0.11038E-00	0.90787E-00
0.95238E-01	0.21746E-00	0.15873E-01	0.79365E-01	0.17460E-00	0.98413E-00

0.15106E-00	0.20426E-00	0.55319E-01	0.63830E-01	0.23404E-00	0.
0.16170E-00	0.12140E-00	0.59574E-01	0.72340E-01	0.17660E-00	0.
0.14681E-00	0.20638E-00	0.89362E-01	0.80851E-01	0.91498E-01	0.
0.10638E-00	0.15957E-00	0.27447E-00	0.85106E-01	0.76594E-01	0.
0.82979E-01	0.15932E-00	0.21913E-00	0.46085E-01	0.42735E-01	0.
0.63830E-01	0.11702E-00	0.11277E-00	0.59574E-01	0.40426E-01	0.
0.53191E-01	0.82979E-01	0.15319E-00	0.59574E-01	0.97872E-01	0.
0.29787E-01	0.63830E-01	0.13404E-00	0.55319E-01	0.72340E-01	0.
0.55319E-01	0.78723E-01	0.12766E-00	0.68085E-01	0.87234E-01	0.
0.38298E-01	0.72340E-01	0.10851E-00	0.63830E-01	0.51064E-01	0.
0.42553E-01	0.44681E-01	0.72340E-01	0.11915E-00	0.21277E-01	0.
0.25532E-01	0.57447E-01	0.57447E-01	0.26596E-00	0.27660E-01	0.
0.25532E-01	0.42553E-01	0.72340E-01	0.31915E-00	0.27660E-01	0.
0.23404E-01	0.55319E-01	0.55319E-01	0.22553E-00	0.27660E-01	0.
0.36170E-01	0.46809E-01	0.87234E-01	0.11489E-00	0.34043E-01	0.
0.19149E-01	0.44681E-01	0.70213E-01	0.12766E-00	0.38298E-01	0.
0.29787E-01	0.25532E-01	0.68085E-01	0.13404E-00	0.44681E-01	0.28511E-00
0.23404E-01	0.38298E-01	0.70213E-01	0.11064E-00	0.31915E-01	0.90787E-00
0.25532E-01	0.46809E-01	0.80851E-01	0.93617E-01	0.42553E-01	0.90787E-00
0.38298E-01	0.27660E-01	0.89362E-01	0.72340E-01	0.29787E-01	0.90787E-00
0.71489E-01	0.36170E-01	0.55319E-01	0.55319E-01	0.21277E-01	0.90787E-00
0.21277E-00	0.44681E-01	0.44681E-01	0.57447E-01	0.55319E-01	0.90787E-00
0.10426E-00	0.22128E-00	0.38298E-01	0.57447E-01	0.14043E-00	0.90787E-00
0.55238E-01	0.21746E-00	0.15873E-01	0.79365E-01	0.17460E-00	0.98413E-00

0.38380E-01	0.57569E-01	0.17058E-01	0.38380E-01	0.29891E-01	0.
0.68230E-01	0.59701E-01	0.21322E-01	0.51173E-01	0.10661E-01	0.
0.59701E-01	0.87420E-01	0.21322E-01	0.70512E-01	0.19190E-01	0.
0.55437E-01	0.78891E-01	0.40512E-01	0.31983E-01	0.23454E-01	0.
0.51173E-01	0.81023E-01	0.46908E-01	0.29851E-01	0.21322E-01	0.
0.46908E-01	0.55437E-01	0.34115E-00	0.70512E-01	0.17058E-01	0.
0.29851E-01	0.61834E-01	0.83156E-01	0.38380E-01	0.14929E-01	0.
0.36247E-01	0.27719E-01	0.72495E-01	0.44776E-01	0.17719E-01	0.
0.27719E-01	0.57569E-01	0.49041E-01	0.44776E-01	0.14929E-01	0.
0.19190E-01	0.63966E-01	0.63966E-01	0.40512E-01	0.23454E-01	0.
0.23454E-01	0.36247E-01	0.51173E-01	0.68230E-01	0.10661E-01	0.
0.10661E-01	0.29851E-01	0.42644E-01	0.76759E-01	0.42644E-02	0.
0.14929E-01	0.25586E-01	0.51173E-01	0.53305E-01	0.14929E-01	0.
0.25586E-01	0.21322E-01	0.46908E-01	0.68230E-01	0.12793E-01	0.
0.42644E-02	0.36247E-01	0.46908E-01	0.36115E-01	0.14929E-01	0.26222E-00
0.53955E-02	0.17058E-01	0.46908E-01	0.59701E-01	0.23454E-01	0.09545E-00
0.42644E-02	0.10661E-01	0.53305E-01	0.42644E-01	0.12793E-01	0.09360E-00
0.23454E-01	0.21322E-01	0.42644E-01	0.42644E-01	0.14929E-01	0.09360E-00
0.19190E-01	0.17058E-01	0.25586E-01	0.34115E-01	0.14929E-01	0.09360E-00
0.57569E-01	0.49041E-01	0.27719E-01	0.27719E-01	0.21322E-01	0.09360E-00
0.55437E-01	0.12122E-00	0.21322E-01	0.44444E-01	0.33333E-01	0.06667E-00

TABLE XII. Transient Data (Contd.)

RUN 8

 $\frac{Z^*}{r^*}$

0.00

0.27

0.45

0.63

0.81

3.6

5.8

10.2

18.2

32.0

REF.

0.89744E-01	0.91880E-01	0.65470E-01	0.40598E-01	0.70513E-01	0.
0.70513E-01	0.10228E-02	0.16239E-00	0.49145E-01	0.57692E-01	0.
0.91880E-01	0.11966E-00	0.11325E-00	0.32051E-01	0.59829E-01	0.
0.81197E-01	0.10684E-00	0.10043E-00	0.49145E-01	0.47009E-01	0.
0.57692E-01	0.10570E-00	0.13034E-00	0.17094E-01	0.72650E-01	0.
0.38462E-01	0.10684E-00	0.11966E-00	0.34188E-01	0.51282E-01	0.
0.42735E-01	0.57692E-01	0.12821E-00	0.42735E-01	0.49145E-01	0.
0.29015E-01	0.57692E-01	0.13248E-00	0.10256E-00	0.42735E-01	0.
0.44872E-01	0.55556E-01	0.12393E-00	0.70513E-01	0.61966E-01	0.
0.36325E-01	0.38462E-01	0.98291E-01	0.61966E-01	0.55556E-01	0.
0.44872E-01	0.40598E-01	0.14786E-01	0.63333E-01	0.53419E-01	0.
0.51282E-01	0.50462E-01	0.10746E-00	0.87607E-01	0.59829E-01	0.
0.42735E-01	0.49145E-01	0.47009E-01	0.72650E-01	0.59829E-01	0.
0.25006E-01	0.42735E-01	0.61197E-01	0.17060E-01	0.34188E-01	0.
0.49145E-01	0.44872E-01	0.49145E-01	0.59829E-01	0.55556E-01	0.
0.29015E-01	0.51282E-01	0.51282E-01	0.57692E-01	0.53419E-01	0.
0.42735E-01	0.49145E-01	0.44103E-01	0.14786E-01	0.61197E-01	0.
0.42735E-01	0.53419E-01	0.83746E-01	0.61966E-01	0.96154E-01	0.24359E-00
0.27778E-01	0.51282E-01	0.41966E-01	0.62399E-01	0.59829E-01	0.99786E-00
0.70513E-01	0.21368E-01	0.40598E-01	0.40598E-01	0.53333E-01	0.99786E-00
0.91880E-01	0.32051E-01	0.55556E-01	0.51282E-01	0.72650E-01	0.99786E-00
0.91880E-01	0.13675E-00	0.46239E-01	0.61966E-01	0.70513E-01	0.99786E-00
0.79040E-01	0.91880E-01	0.44872E-01	0.44872E-01	0.46239E-01	0.99786E-00
0.58252E-01	0.11650E-00	0.48544E-01	0.29176E-01	0.58252E-01	0.94029E-00
0.87234E-01	0.74468E-01	0.57447E-01	0.51064E-01	0.46809E-01	0.
0.28773E-01	0.55745E-01	0.14295E-00	0.44681E-01	0.26170E-01	0.
0.68085E-01	0.25106E-01	0.93617E-01	0.44681E-01	0.55319E-01	0.
0.80851E-01	0.11277E-00	0.62979E-01	0.36170E-01	0.38298E-01	0.
0.38298E-01	0.13675E-00	0.11277E-00	0.24043E-01	0.40426E-01	0.
0.51064E-01	0.11702E-00	0.13404E-00	0.19149E-01	0.37447E-01	0.
0.68085E-01	0.76596E-01	0.12553E-00	0.46809E-01	0.38298E-01	0.
0.31915E-01	0.57447E-01	0.12746E-00	0.69362E-01	0.29787E-01	0.
0.42553E-01	0.40426E-01	0.95745E-01	0.85106E-01	0.31915E-01	0.
0.17021E-01	0.34043E-01	0.10213E-00	0.55319E-01	0.23404E-01	0.
0.31915E-01	0.25532E-01	0.10426E-00	0.11702E-00	0.11064E-01	0.
0.57447E-01	0.34043E-01	0.87234E-01	0.87234E-01	0.48936E-01	0.
0.42553E-01	0.40426E-01	0.74468E-01	0.76596E-01	0.55319E-01	0.
0.61702E-01	0.40426E-01	0.40598E-01	0.40598E-01	0.53419E-01	0.
0.40426E-01	0.46809E-01	0.55319E-01	0.68085E-01	0.53191E-01	0.
0.17021E-01	0.31915E-01	0.61702E-01	0.10213E-01	0.26170E-01	0.
0.27778E-01	0.21277E-00	0.61702E-01	0.40598E-01	0.70213E-01	0.31064E-00
0.25532E-01	0.31915E-01	0.31915E-01	0.51064E-01	1.00000E-01	0.99786E-00
0.27660E-01	0.48936E-01	0.46809E-01	0.68085E-01	0.42553E-01	0.99574E-00
0.40426E-01	0.40426E-01	0.42553E-01	0.57447E-01	0.91489E-01	0.99574E-00
0.91489E-01	0.11915E-01	0.51064E-01	0.46809E-01	0.72340E-01	0.99574E-00
0.72340E-01	0.11489E-00	0.59574E-01	0.53191E-01	0.45957E-01	0.99574E-00
0.74468E-01	1.00000E-01	0.51064E-01	0.44681E-01	0.44681E-01	0.99574E-00
0.25006E-01	0.10256E-00	0.	0.76873E-01	0.51282E-01	0.94029E-00
0.68085E-01	0.43830E-01	0.53191E-01	0.42553E-01	0.46809E-01	0.
0.95745E-01	0.60851E-01	0.72340E-01	0.44681E-01	0.40426E-01	0.
0.68085E-01	0.55319E-01	0.72340E-01	0.68085E-01	0.24043E-01	0.
0.61702E-01	0.62979E-01	0.65106E-01	0.46809E-01	0.36170E-01	0.
0.51064E-01	0.87234E-01	0.97872E-01	0.24043E-01	0.46809E-01	0.
0.25532E-01	0.14468E-01	0.11915E-00	0.38298E-01	0.29787E-01	0.
0.25532E-01	0.57447E-01	0.60851E-01	0.53191E-01	0.31915E-01	0.
0.21277E-01	0.53191E-01	0.12979E-00	0.76596E-01	0.44681E-01	0.
0.25532E-01	0.46809E-01	0.10851E-00	0.93617E-01	0.27660E-01	0.
0.14295E-01	0.36170E-01	0.68085E-01	0.53191E-01	0.21277E-01	0.
0.10638E-01	0.31915E-01	0.97872E-01	0.62979E-01	0.31915E-01	0.
0.36170E-01	0.19149E-01	0.91489E-01	0.10851E-00	0.25322E-01	0.
0.36170E-01	0.31915E-01	0.25322E-01	0.46809E-01	0.72340E-01	0.
0.36170E-01	0.31915E-01	0.62979E-01	0.76596E-01	0.38298E-01	0.
0.27778E-01	0.21915E-01	0.42553E-01	0.61702E-01	0.25322E-01	0.
0.31915E-01	0.31915E-01	0.34043E-01	0.51064E-01	0.51064E-01	0.
0.12766E-01	0.23404E-01	0.28298E-01	0.82979E-01	0.51064E-01	0.40851E-00
0.27660E-01	0.44681E-01	0.34043E-01	0.68085E-01	0.45957E-01	0.10000E-01
0.46809E-01	0.55319E-01	0.61702E-01	0.76596E-01	0.42553E-01	0.10000E-01
0.85106E-01	0.12766E-01	0.36170E-01	0.45957E-01	0.44681E-01	0.10000E-01
0.62979E-01	0.44681E-01	0.53191E-01	0.53191E-01	0.26170E-01	0.06778E-00
0.76596E-01	0.62979E-01	0.40426E-01	0.51064E-01	0.42553E-01	0.95332E-00
0.60000E-01	0.	0.60000E-01	0.60000E-01	0.	0.12000E-00
0.89099E-01	0.81425E-01	0.22901E-01	0.43257E-01	0.22901E-01	0.
0.50891E-01	0.63613E-01	0.38168E-01	0.28168E-01	0.20534E-01	0.
0.63613E-01	0.73791E-01	0.43257E-01	0.29445E-01	0.45802E-01	0.
0.71247E-01	0.61425E-01	0.43257E-01	0.33079E-01	0.43257E-01	0.
0.50891E-01	0.66124E-01	0.12477E-01	0.33079E-01	0.35823E-01	0.
0.33079E-01	0.45802E-01	0.61425E-01	0.58524E-01	0.25445E-01	0.
0.30534E-01	0.42802E-01	0.61425E-01	0.27990E-01	0.25445E-01	0.
0.20534E-01	0.43257E-01	0.16336E-01	0.59800E-01	0.38168E-01	0.
0.17812E-01	0.27990E-01	0.78880E-01	0.63613E-01	0.27990E-01	0.
0.10170E-01	0.25623E-01	0.13669E-01	0.50891E-01	0.23079E-01	0.
0.17812E-01	0.17812E-01	0.16336E-01	0.18168E-01	0.22901E-01	0.
0.30534E-01	0.35623E-01	0.16369E-01	0.61425E-01	0.10534E-01	0.
0.20534E-01	0.12723E-01	0.69059E-01	0.50891E-01	0.27990E-01	0.
0.15267E-01	0.30534E-01	0.16369E-01	0.48702E-01	0.15267E-01	0.
0.15267E-01	0.15267E-01	0.43257E-01	0.10891E-01	0.20534E-01	0.
0.20534E-01	0.35623E-01	0.38168E-01	0.48346E-01	0.17812E-01	0.
0.12723E-01	0.25445E-01	0.15267E-01	0.15267E-01	0.35623E-01	0.44855E-00
0.30534E-01	0.40712E-01	0.17812E-01	0.43613E-01	0.35623E-01	0.10000E-01
0.53435E-01	0.17812E-01	0.40712E-01	0.58524E-01	0.45802E-01	0.10000E-01
0.53435E-01	0.17812E-01	0.43257E-01	0.45802E-01	0.20534E-01	0.10000E-01
0.50891E-01	0.53435E-01	0.15267E-01	0.53435E-01	0.58902E-01	0.10000E-01
0.15267E-01	0.53435E-01	0.25445E-01	0.58524E-01	0.28168E-01	0.10000E-01
0.15267E-01	0.53435E-01	0.18168E-01	0.58524E-01	0.27990E-01	0.78117E-00
0.15267E-01	0.53435E-01	0.24043E-01	0.44681E-01	0.11622E-01	0.
0.44748E-01	0.43165E-01	0.21175E-01	0.28777E-01	0.16787E-01	0.
0.67146E-01	0.79137E-01	0.28777E-01	0.31175E-01	0.16787E-01	0.
0.50300E-01	0.91127E-01	0.55156E-01	0.21533E-01	0.19185E-01	0.
0.50300E-01	0.69446E-01	0.19717E-01	0.33731E-01	0.14388E-01	0.
0.23981E-01	0.10701E-00	0.33573E-01	0.28777E-01	0.14388E-01	0.
0.28777E-01	0.75923E-01	0.71942E-01	0.11983E-01	0.11990E-01	0.
0.49823E-01	0.61536E-01	0.16739E-01	0.45846E-01	0.26777E-01	0.
0.11990E-01	0.55156E-01	0.47146E-01	0.33981E-01	0.33573E-01	0.
0.14388E-01	0.31175E-01	0.74361E-01	0.33971E-01	0.23981E-01	0.
0.47942E-02	0.17812E-01	0.11177E-01	0.13733E-01	0.23981E-01	0.
0.14388E-01	0.23981E-01	0.98212E-01	0.45846E-01	0.28777E-01	0.
0.47942E-02	0.19185E-01	0.71942E-01	0.52758E-01	0.23981E-01	0.
0.14388E-01	0.14388E-01	0.33533E-01	0.43165E-01	0.43165E-01	0.
0.71942E-02	0.26379E-01	0.57554E-01	0.27146E-01	0.14388E-01	0.
0.14388E-01	0.19185E-01	0.38369E-01	0.31175E-01	0.19185E-01	0.
0.14388E-01	0.26379E-01	0.16787E-01	0.43165E-01	0.16787E-01	0.
0.11990E-01	0.26379E-01	0.19185E-01	0.45846E-01	0.11990E-01	0.56355E-00
0.23981E-01	0.52758E-01	0.35971E-01	0.50360E-01	0.14388E-01	0.99760E-00
0.71942E-02	0.21263E-01	0.31175E-01	0.57554E-01	0.11990E-01	0.99760E-00
0.26379E-01	0.26379E-01	0.35971E-01	0.50360E-01	0.21533E-01	0.99760E-00
0.43165E-01	0.28777E-01	0.35733E-01	0.57554E-01	0.26379E-01	0.99760E-00
0.45846E-01	0.35971E-01	0.16787E-01	0.64748E-01	0.21533E-01	0.99760E-00
0.52758E-01	0.55156E-01	0.31175E-01	0.50360E-01	0.33981E-01	0.99760E-00
0.60445E-01	0.64510E-01	0.	0.16129E-01	0.67774E-01	0.

TABLE XII. Transient Data (Contd.)

		RUN 10					REF.
Z^*	r^*	3.6	5.8	10.2	18.2	32.0	
0.00		0.19272E-01	0.42827E-01	0.21413E-01	0.64240E-02	0.42827E-02	0.
		0.23555E-01	0.23555E-01	0.27637E-01	0.85633E-02	0.21413E-02	0.
		0.10707E-01	0.79222E-01	0.38555E-01	0.	0.42827E-02	0.
		0.64240E-02	0.66301E-01	0.66301E-01	0.21413E-02	0.21413E-02	0.
		0.14989E-01	0.40689E-01	0.62099E-01	0.64240E-02	0.12848E-01	0.
		0.17131E-01	0.20979E-01	0.64240E-01	0.64240E-02	0.	0.
		0.12848E-01	0.23496E-01	0.34261E-01	0.17131E-01	0.64240E-02	0.
		0.10707E-01	0.38544E-01	0.44968E-01	0.42827E-02	0.42827E-02	0.
		0.21413E-02	0.14989E-01	0.23555E-01	0.14989E-01	0.23555E-02	0.
		0.64240E-02	0.19272E-01	0.36403E-01	0.21413E-01	0.21413E-02	0.
		0.14989E-01	0.32120E-01	0.32120E-01	0.23555E-01	0.64240E-02	0.
		0.10707E-01	0.38544E-01	0.23555E-01	0.85633E-02	0.21413E-02	0.
		0.64240E-02	0.20979E-01	0.20979E-01	0.85633E-02	0.21413E-02	0.
		0.65653E-02	0.21413E-01	0.40689E-01	0.64240E-02	0.10707E-01	0.
		0.10707E-01	0.34261E-01	0.32120E-01	0.85633E-02	0.21413E-02	0.
		0.10707E-01	0.14989E-01	0.23555E-01	0.64240E-02	0.17131E-01	0.
		0.85653E-02	0.19272E-01	0.14989E-01	0.42827E-02	0.14989E-01	0.
		0.42827E-02	0.19272E-01	0.14989E-01	0.64240E-02	0.32120E-01	0.
		0.85653E-02	0.21413E-01	0.20979E-01	0.10707E-01	0.12848E-01	0.
		0.64240E-02	0.34261E-01	0.34261E-01	0.10707E-01	0.12848E-01	0.
		0.42827E-02	0.21413E-01	0.21413E-01	0.42827E-02	0.85633E-02	0.98715E 00
		0.14989E-01	0.20979E-01	0.23555E-01	0.21413E-02	0.85633E-02	0.99358E 00
		0.85653E-02	0.34261E-01	0.23555E-01	0.21413E-02	0.42827E-02	0.99358E 00
		0.	0.21739E-01	0.14939E-01	0.21739E-01	0.72464E-02	0.97626E 00
0.27		0.40512E-01	0.74627E-01	0.59701E-01	0.10661E-01	0.63966E-02	0.
		0.36247E-01	0.11727E-00	0.49041E-01	0.63966E-02	0.	0.
		0.23555E-01	0.74627E-01	0.38380E-01	0.12793E-01	0.42644E-02	0.
		0.40512E-01	0.10874E-00	0.95949E-01	0.12793E-01	0.42644E-02	0.
		0.16848E-01	0.15235E-00	0.10102E-01	0.10661E-01	0.63966E-02	0.
		0.20851E-01	0.63966E-01	0.53305E-01	0.17058E-01	0.21322E-02	0.
		0.21322E-01	0.61846E-01	0.49041E-01	0.10661E-01	0.21322E-02	0.
		0.19100E-01	0.23496E-01	0.36300E-01	0.14925E-01	0.21322E-02	0.
		0.17058E-01	0.44776E-01	0.23496E-01	0.25586E-01	0.	0.
		U.23496E-01	0.37609E-01	0.36247E-01	0.30300E-01	0.	0.
		0.10661E-01	0.21771E-01	0.23496E-01	0.47767E-01	0.63966E-02	0.
		0.12793E-01	0.20851E-01	0.20851E-01	0.17058E-01	0.63966E-02	0.
		0.36247E-01	0.59701E-01	0.38380E-01	0.14925E-01	0.21322E-02	0.
		0.23496E-01	0.42644E-01	0.36300E-01	0.14925E-01	0.25586E-02	0.
		0.10661E-01	0.36247E-01	0.21322E-01	0.19100E-01	0.10661E-01	0.
		0.10661E-01	0.36247E-01	0.19100E-01	0.85288E-02	0.21322E-01	0.
		0.14925E-01	0.42644E-01	0.21322E-01	0.10661E-01	0.10661E-01	0.
		0.14925E-01	0.44776E-01	0.44776E-01	0.10661E-01	0.23496E-01	0.
		0.17058E-01	0.46908E-01	0.20851E-01	0.23496E-01	0.17058E-01	0.
		0.14925E-01	0.51173E-01	0.27778E-01	0.12793E-01	0.14925E-01	0.16205E-00
		0.21322E-01	0.44776E-01	0.47767E-01	0.17058E-01	0.10661E-01	0.99147E 00
		0.20851E-01	0.51173E-01	0.31983E-01	0.42644E-02	0.85288E-02	0.99147E 00
		0.27778E-01	0.46908E-01	0.36247E-01	0.21322E-02	0.85288E-02	0.99147E 00
		0.20851E-01	0.20851E-01	0.14925E-01	0.14925E-01	0.	0.94030E 00
0.45		0.27778E-01	0.83333E-01	0.22222E-01	0.11111E-01	0.13889E-01	0.
		0.47222E-01	0.97222E-01	0.36111E-01	0.83333E-02	0.83333E-02	0.
		0.27778E-01	0.86111E-01	0.47222E-01	0.83333E-02	0.16667E-01	0.
		0.38889E-01	0.88889E-01	0.50000E-01	0.11111E-01	0.16667E-01	0.
		0.47222E-01	0.83333E-01	0.75000E-01	0.55556E-02	0.83333E-02	0.
		0.36111E-01	0.83333E-01	0.44444E-01	0.27778E-02	0.83333E-02	0.
		0.41667E-01	0.46667E-01	0.69444E-01	0.16667E-01	0.25556E-02	0.
		0.33333E-01	0.47222E-01	0.36111E-01	0.16667E-01	0.83333E-02	0.
		0.19444E-01	0.41667E-01	0.38889E-01	0.30556E-01	0.16667E-01	0.
		0.38889E-01	0.60606E-01	0.33333E-01	0.47222E-01	0.83333E-02	0.
		0.33333E-01	0.61111E-01	0.19444E-01	0.13889E-01	0.19444E-01	0.
		0.33333E-01	0.52778E-01	0.25000E-01	0.22222E-01	0.13889E-01	0.
		0.22222E-01	0.44444E-01	0.22222E-01	0.34111E-01	0.25556E-02	0.
		0.19444E-01	0.44444E-01	0.22222E-01	0.22222E-01	0.16667E-01	0.
		0.36111E-01	0.50333E-01	0.30556E-01	0.83333E-02	0.25000E-01	0.
		0.22222E-01	0.50333E-01	0.30556E-01	0.27778E-02	0.16667E-01	0.
		0.36111E-01	0.61111E-01	0.44444E-01	0.13889E-01	0.36111E-01	0.
		0.13889E-01	0.47222E-01	0.13889E-01	0.83333E-02	0.25000E-01	0.
		0.27778E-01	0.63889E-01	0.36111E-01	0.55556E-02	0.10556E-01	0.
		0.25000E-01	0.41667E-01	0.25000E-01	0.27778E-02	0.11111E-01	0.14444E-00
		0.22222E-01	0.52778E-01	0.33333E-01	0.55556E-02	0.19444E-01	0.10000E 01
		0.19444E-01	0.30556E-01	0.75000E-01	0.55556E-02	0.55556E-02	0.97722E 00
		0.27778E-01	0.44444E-01	0.38889E-01	0.83333E-02	0.83333E-02	0.96667E 00
		0.	0.76923E-01	0.	0.	0.	0.
0.63		0.14085E-01	0.10563E-00	0.39906E-01	0.14085E-01	0.14085E-01	0.
		0.14085E-01	0.10094E-00	0.42294E-01	0.93897E-02	0.21127E-01	0.
		0.46948E-02	0.11268E-00	0.35211E-01	0.93897E-02	0.25822E-01	0.
		0.18779E-01	0.18207E-00	0.61035E-01	0.93897E-02	0.16432E-01	0.
		0.14085E-01	0.72770E-01	0.79812E-01	0.23474E-01	0.25822E-01	0.
		0.11737E-01	0.70423E-01	0.79812E-01	0.18779E-01	0.14085E-01	0.
		0.46948E-02	0.53991E-01	0.39906E-01	0.11737E-01	0.25822E-01	0.
		0.93897E-02	0.85202E-01	0.51643E-01	0.30516E-01	0.16432E-01	0.
		0.46948E-02	0.46948E-01	0.30516E-01	0.56338E-01	0.11737E-01	0.
		0.93897E-02	0.65728E-01	0.35211E-01	0.51643E-01	0.16432E-01	0.
		0.46948E-02	0.61035E-01	0.30516E-01	0.42294E-01	0.70423E-02	0.
		0.93897E-02	0.72770E-01	0.25822E-01	0.37559E-01	0.28169E-01	0.
		0.11737E-01	0.31643E-01	0.42294E-01	0.14085E-01	0.16432E-01	0.
		0.23474E-02	0.72770E-01	0.26169E-01	0.21127E-01	0.46948E-02	0.
		0.70423E-02	0.53991E-01	0.23474E-01	0.21127E-01	0.23474E-01	0.
		0.11737E-01	0.61035E-01	0.42294E-01	0.93897E-02	0.46601E-01	0.
		0.70423E-02	0.56338E-01	0.25822E-01	0.23474E-01	0.35211E-01	0.
		0.11737E-01	0.65728E-01	0.44601E-01	0.18779E-01	0.35211E-01	0.
		0.11737E-01	0.65728E-01	0.35211E-01	0.16432E-01	0.30516E-01	0.32254E-00
		0.16432E-01	0.86854E-01	0.56338E-01	0.14085E-01	0.30516E-01	0.99765E 00
		0.93897E-02	0.58885E-01	0.28169E-01	0.14085E-01	0.11737E-01	0.99765E 00
		0.11737E-01	0.72770E-01	0.35211E-01	0.11737E-01	0.16432E-01	0.87659E 00
		0.	0.37037E-01	0.	0.	0.18519E-01	0.
0.81		0.36496E-01	0.77859E-01	0.65693E-01	0.24331E-01	0.24331E-01	0.
		0.24331E-01	0.90024E-01	0.48602E-01	0.14959E-01	0.36496E-01	0.
		0.31630E-01	0.97324E-01	0.75426E-01	0.17032E-01	0.48602E-01	0.
		0.36496E-01	0.72770E-01	0.82729E-01	0.36496E-01	0.31630E-01	0.
		0.31630E-01	0.87591E-01	0.11192E-00	0.31630E-01	0.29197E-01	0.
		0.29197E-01	0.80242E-01	0.10949E-00	0.31630E-01	0.38929E-01	0.
		0.24764E-01	0.51095E-01	0.61858E-01	0.43796E-01	0.37919E-01	0.
		0.21828E-01	0.51363E-01	0.60827E-01	0.38929E-01	0.31630E-01	0.
		0.26764E-01	0.48602E-01	0.55961E-01	0.77859E-01	0.36496E-01	0.
		0.38929E-01	0.82729E-01	0.80242E-01	0.48602E-01	0.36496E-01	0.
		0.97324E-02	0.51363E-01	0.34063E-01	0.55961E-01	0.43796E-01	0.
		0.14959E-01	0.34063E-01	0.51095E-01	0.41363E-01	0.31630E-01	0.
		0.21899E-01	0.51528E-01	0.41363E-01	0.36496E-01	0.18988E-01	0.
		0.24331E-01	0.53528E-01	0.46229E-01	0.38929E-01	0.29197E-01	0.
		0.21899E-01	0.55961E-01	0.60827E-01	0.12165E-01	0.24331E-01	0.
		0.21899E-01	0.58394E-01	0.53528E-01	0.19459E-01	0.41363E-01	0.

TABLE XII. Transient Data (Contd.)

RUN II

Z*
r*

0.00

0.27

0.45

0.63

0.81

3.6 5.8 10.2 18.2 32.0 REF.

0.69474E-01	0.65263E-01	0.82105E-01	0.18947E-01	0.13684E-01	0.
0.68421E-01	0.67368E-01	0.69474E-01	0.23158E-01	0.25263E-01	0.
0.10737E-00	0.10526E-00	0.86316E-01	0.18947E-01	0.31579E-01	0.
0.92632E-01	0.94737E-01	0.65263E-01	0.37895E-01	0.25263E-01	0.
0.61053E-01	0.84737E-01	0.10105E-00	0.33684E-01	0.35789E-01	0.
0.71579E-01	0.75789E-01	0.11579E-00	0.27368E-01	0.33684E-01	0.
0.65263E-01	0.63158E-01	0.84211E-01	0.25263E-01	0.29474E-01	0.
0.44211E-01	0.67368E-01	0.63158E-01	0.33684E-01	0.31579E-01	0.
0.46316E-01	0.61053E-01	0.86316E-01	0.16842E-01	0.27368E-01	0.
0.52632E-01	0.75789E-01	0.65263E-01	0.31579E-01	0.23158E-01	0.
0.65263E-01	0.58947E-01	0.44211E-01	0.65263E-01	0.25263E-01	0.
0.69474E-01	0.82105E-01	0.75789E-01	0.52632E-01	0.27368E-01	0.
0.86316E-01	0.56842E-01	0.54737E-01	0.46316E-01	0.31579E-01	0.
0.48421E-01	0.54737E-01	0.90526E-01	0.58947E-01	0.33684E-01	0.
0.61053E-01	0.61053E-01	0.88421E-01	0.16842E-01	0.40000E-01	0.
0.61053E-01	0.71579E-01	0.65263E-01	0.25263E-01	0.37895E-01	0.
0.88421E-01	0.58947E-01	0.90526E-01	0.14737E-01	0.31579E-01	0.
0.63158E-01	0.61053E-01	0.54737E-01	0.10526E-01	0.67368E-01	0.
0.61053E-01	0.67368E-01	0.77895E-01	0.31579E-01	0.50526E-01	0.
0.71579E-01	0.73684E-01	0.69474E-01	0.16842E-01	0.58947E-01	0.
0.67368E-01	0.71579E-01	0.67368E-01	0.29474E-01	0.67368E-01	0.
0.73684E-01	0.61053E-01	0.61053E-01	0.23158E-01	0.58947E-01	0.89263E 00
0.76220E-01	0.36488E-01	0.54678E-01	0.27439E-01	0.48780E-01	0.99392E 00
0.54737E-01	0.46316E-01	0.56842E-01	0.16842E-01	0.25263E-01	0.
0.88421E-01	0.54737E-01	0.65263E-01	0.29474E-01	0.12632E-01	0.
0.11158E-00	0.88421E-01	0.88421E-01	0.21053E-01	0.2158E-01	0.
0.88421E-01	0.13263E-00	0.75789E-01	0.21053E-01	0.21053E-01	0.
0.90526E-01	0.84211E-01	0.96842E-01	0.29474E-01	0.23158E-01	0.
0.45263E-01	0.73684E-01	0.12211E-00	0.31579E-01	0.23158E-01	0.
0.58947E-01	0.73684E-01	0.10947E-01	0.14737E-01	0.18947E-01	0.
0.54737E-01	0.46316E-01	0.98947E-01	0.27368E-01	0.10526E-01	0.
0.69474E-01	0.52632E-01	0.75789E-01	0.21053E-01	0.14737E-01	0.
0.52632E-01	0.42105E-01	0.56842E-01	0.58947E-01	0.14737E-01	0.
0.77895E-01	0.52632E-01	0.56842E-01	0.67368E-01	0.16842E-01	0.
0.61053E-01	0.61053E-01	0.50526E-01	0.50526E-01	0.29474E-01	0.
0.71579E-01	0.65263E-01	0.42105E-01	0.61053E-01	0.23158E-01	0.
0.58947E-01	0.44211E-01	0.56842E-01	0.44211E-01	0.27368E-01	0.
0.56842E-01	0.40000E-01	0.50526E-01	0.37895E-01	0.27368E-01	0.
0.63158E-01	0.46316E-01	0.61053E-01	0.25263E-01	0.25263E-01	0.
0.61053E-01	0.44211E-01	0.65263E-01	0.21053E-01	0.35789E-01	0.
0.75789E-01	0.40000E-01	0.63158E-01	0.14737E-01	0.51579E-01	0.
0.73684E-01	0.54737E-01	0.69474E-01	0.37895E-01	0.52632E-01	0.
0.50526E-01	0.48421E-01	0.65263E-01	0.35789E-01	0.50526E-01	0.
0.63158E-01	0.46316E-01	0.73684E-01	0.31579E-01	0.46316E-01	0.
0.48421E-01	0.61053E-01	0.69474E-01	0.42105E-01	0.29474E-01	0.85895E 00
0.63830E-01	0.42553E-01	0.79027E-01	0.48421E-01	0.14108E-01	0.99392E 00
0.58947E-01	0.40000E-01	0.48421E-01	0.84211E-02	0.23158E-01	0.
0.11579E-00	0.67368E-01	0.73684E-01	0.16842E-01	0.12632E-01	0.
0.10105E-00	0.86316E-01	0.63158E-01	0.14737E-01	0.18947E-01	0.
0.10316E-00	0.96842E-01	0.88421E-01	0.25263E-01	0.21053E-01	0.
0.92632E-01	0.71579E-01	0.61053E-01	0.42105E-02	0.14737E-01	0.
0.63158E-01	0.46316E-01	0.77895E-01	0.18947E-01	0.21053E-01	0.
0.48421E-01	0.50526E-01	0.63158E-01	0.23158E-01	0.12632E-01	0.
0.69474E-01	0.44211E-01	0.86316E-01	0.12632E-01	0.10526E-01	0.
0.58947E-01	0.44211E-01	0.58947E-01	0.23158E-01	0.21053E-01	0.
0.10526E-01	0.46316E-01	0.50526E-01	0.16842E-01	0.25263E-01	0.
0.77895E-01	0.54737E-01	0.73684E-01	0.29474E-01	0.23158E-01	0.
0.75789E-01	0.40000E-01	0.46316E-01	0.52632E-01	0.18947E-01	0.
0.52632E-01	0.44211E-01	0.35789E-01	0.23158E-01	0.10526E-01	0.
0.50526E-01	0.65263E-01	0.67368E-01	0.25263E-01	0.10526E-01	0.
0.67368E-01	0.48421E-01	0.56842E-01	0.21053E-01	0.14737E-01	0.
0.52632E-01	0.29474E-01	0.69474E-01	0.18947E-01	0.18947E-01	0.
0.71579E-01	0.48421E-01	0.69474E-01	0.84211E-02	0.37895E-01	0.
0.61053E-01	0.42105E-01	0.52632E-01	0.84211E-02	0.40000E-01	0.
0.61053E-01	0.44211E-01	0.65263E-01	0.65263E-01	0.16842E-01	0.
0.35789E-01	0.33684E-01	0.44211E-01	0.27368E-01	0.46316E-01	0.
0.80000E-01	0.44211E-01	0.61053E-01	0.27368E-01	0.31579E-01	0.
0.75789E-01	0.40000E-01	0.54737E-01	0.21053E-01	0.40000E-01	0.85895E 00
0.72674E-01	0.66842E-01	0.58140E-01	0.17442E-01	0.23256E-01	0.10000E 01
0.58947E-01	0.50526E-01	0.61053E-01	0.14737E-01	0.21053E-01	0.
0.13474E-00	0.88421E-01	0.40000E-01	0.84211E-02	0.16842E-01	0.
0.12211E-00	0.88421E-01	0.63158E-01	0.27368E-01	0.18947E-01	0.
0.96842E-01	0.96842E-01	0.82105E-01	0.21053E-01	0.63158E-02	0.
0.77895E-01	0.65263E-01	0.86316E-01	0.21053E-01	0.14737E-01	0.
0.37895E-01	0.48421E-01	0.96842E-01	0.27368E-01	0.18947E-01	0.
0.40000E-01	0.40000E-01	0.80000E-01	0.10526E-01	0.21053E-01	0.
0.46316E-01	0.50526E-01	0.90526E-01	0.10526E-01	0.18947E-01	0.
0.46316E-01	0.50526E-01	0.67368E-01	0.16842E-01	0.16842E-01	0.
0.48421E-01	0.44211E-01	0.69474E-01	0.48421E-01	0.25263E-01	0.
0.58947E-01	0.35789E-01	0.42105E-01	0.48421E-01	0.25263E-01	0.
0.56842E-01	0.50526E-01	0.58947E-01	0.61053E-01	0.18947E-01	0.
0.58947E-01	0.61053E-01	0.50526E-01	0.44211E-01	0.84211E-02	0.
0.56842E-01	0.37895E-01	0.48421E-01	0.42105E-01	0.21053E-01	0.
0.33684E-01	0.48421E-01	0.48421E-01	0.33684E-01	0.27368E-01	0.
0.58947E-01	0.25263E-01	0.54737E-01	0.23158E-01	0.27368E-01	0.
0.46316E-01	0.44211E-01	0.52632E-01	0.14737E-01	0.44211E-01	0.
0.44211E-01	0.37895E-01	0.48421E-01	0.16842E-01	0.63158E-01	0.
0.56842E-01	0.40000E-01	0.42105E-01	0.25263E-01	0.40000E-01	0.
0.50526E-01	0.35789E-01	0.40000E-01	0.35789E-01	0.35789E-01	0.
0.40000E-01	0.52632E-01	0.50526E-01	0.23158E-01	0.44211E-01	0.80211E 00
0.44643E-01	0.41667E-01	0.59524E-01	0.59524E-02	0.14881E-01	0.10000E 01
0.10526E-00	0.69474E-01	0.65263E-01	0.35789E-01	0.29474E-01	0.
0.18105E-00	0.84211E-01	0.94737E-01	0.40000E-01	0.29474E-01	0.
0.17474E-00	0.10105E-00	0.88421E-01	0.42105E-01	0.21053E-01	0.
0.12000E-00	0.13895E-00	0.90526E-01	0.54737E-01	0.16842E-01	0.
0.11579E-00	0.10737E-00	0.11789E-00	0.58947E-01	0.35789E-01	0.
0.56842E-01	0.58947E-01	0.40000E-01	0.46316E-01	0.44211E-01	0.
0.86316E-01	0.82105E-01	0.14947E-00	0.54737E-01	0.44211E-01	0.
0.82105E-01	0.56842E-01	0.11158E-00	0.65263E-01	0.21053E-01	0.
0.73684E-01	0.58947E-01	0.80000E-01	0.65263E-01	0.18947E-01	0.
0.61053E-01	0.50526E-01	0.90526E-01	0.73684E-01	0.23158E-01	0.
0.73684E-01	0.54737E-01	0.77895E-01	0.11368E-00	0.31579E-01	0.
0.86316E-01	0.46316E-01	0.77895E-01	0.90526E-01	0.40000E-01	0.
0.77895E-01	0.63158E-01	0.46316E-01	0.77895E-01	0.35789E-01	0.
0.73684E-01	0.52632E-01	0.63158E-01	0.46316E-01	0.25263E-01	0.
0.80000E-01	0.50526E-01	0.77895E-01	0.44211E-01	0.31579E-01	0.
0.65263E-01	0.40000E-01	0.67368E-01	0.44211E-01	0.29474E-01	0.
0.65263E-01	0.48421E-01	0.84211E-01	0.44211E-01	0.23158E-01	0.
0.73684E-01	0.67368E-01	0.54737E-01	0.54737E-01	0.58947E-01	0.
0.69474E-01	0.50526E-01	0.46316E-01	0.50526E-01	0.75789E-01	0.
0.84211E-01	0.54737E-01	0.50947E-01	0.61053E-01	0.54737E-01	0.
0.65263E-01	0.52632E-01	0.52632E-01	0.71579E-01	0.67368E-01	0.3474E 00
0.71425E-01	0.40373E-01	0.68323E-01	0.49689E-01	0.49689E-01	0.99689E 00

TABLE XII. Transient Data (Contd.)

RUN 12							
Z*	3.6	5.8	10.2	18.2	32.0	REF.	
0.00	0.14792E-00	0.16042E-00	0.13125E-00	0.11250E-00	0.22917E-00	0.	
	0.13542E-00	0.97917E-01	0.12300E-00	0.13353E-00	0.21042E-00	0.	
	0.13958E-00	0.12500E-00	0.16875E-00	0.14375E-00	0.13958E-00	0.	
	0.12083E-00	0.12917E-00	0.29375E-00	0.13542E-00	0.95833E-01	0.	
	0.16042E-00	0.11250E-00	0.15208E-00	0.12500E-00	0.12083E-00	0.	
	0.15417E-00	0.11667E-00	0.87500E-01	0.14167E-00	0.12292E-00	0.	
	0.10208E-00	0.12708E-00	0.12708E-00	0.12500E-00	0.13125E-00	0.	
	0.10417E-00	0.13750E-00	0.12917E-00	0.13125E-00	0.14375E-00	0.	
	0.12083E-00	0.11042E-00	0.15000E-00	0.11875E-00	0.16875E-00	0.	
	0.95833E-01	0.11496E-00	0.19375E-00	0.15000E-00	0.12917E-00	0.	
	0.11250E-00	0.10000E-01	0.13958E-00	0.13750E-00	0.14167E-00	0.	
	0.93750E-01	1.00000E-01	0.14375E-00	0.25625E-00	0.12500E-00	0.	
	0.13958E-00	0.11042E-00	0.13542E-00	0.21250E-00	0.12292E-00	0.	
	0.14167E-00	0.10417E-00	0.14167E-00	0.11458E-00	0.14375E-00	0.	
	0.12292E-00	0.10000E-01	0.13542E-00	0.93750E-01	0.13542E-00	0.	
	0.12708E-00	0.13333E-00	1.00000E-01	0.12500E-00	0.16042E-00	0.	
	0.91667E-01	0.13333E-00	0.13333E-00	0.10625E-00	0.17083E-00	0.81875E 00	
	0.11042E-00	1.00000E-01	0.13833E-00	0.12500E-00	0.15833E-00	0.99375E 00	
	0.10625E-00	0.10208E-00	0.14375E-00	0.12292E-00	0.17292E-00	0.99833E 00	
	0.13750E-00	0.10208E-00	0.16458E-00	0.15417E-00	0.15208E-00	0.99833E 00	
	0.25417E-00	0.11042E-00	0.14583E-00	0.11875E-00	0.18542E-00	0.99833E 00	
	0.12292E-00	0.16042E-00	0.13958E-00	0.14375E-00	0.21458E-00	0.99833E 00	
	0.16114E-00	0.27014E-00	0.15640E-00	0.12796E-00	0.26066E-00	0.99052E 00	
0.27	0.13750E-00	0.16875E-00	0.12083E-00	0.10417E-00	0.25625E-00	0.	
	0.15417E-00	1.00000E-01	0.11875E-00	0.11875E-00	0.19792E-00	0.	
	0.14042E-00	0.11667E-00	0.16042E-00	0.16042E-00	0.12917E-00	0.	
	0.19204E-00	0.11250E-00	0.26250E-00	0.13750E-00	0.14583E-00	0.	
	0.14792E-00	0.13542E-00	0.16667E-00	0.14583E-00	0.12292E-00	0.	
	0.11250E-00	0.13125E-00	1.00000E-01	0.14167E-00	0.13542E-00	0.	
	0.12083E-00	1.00000E-01	0.11675E-00	0.12708E-00	0.12500E-00	0.	
	0.10208E-00	0.11250E-00	0.12708E-00	0.97917E-01	0.17170E-00	0.	
	0.14042E-00	0.11458E-00	0.16250E-00	0.11458E-00	0.14583E-00	0.	
	1.00000E-01	0.10208E-00	0.15408E-00	0.16667E-00	0.12292E-00	0.	
	0.89583E-01	0.85417E-01	0.14375E-00	0.22708E-00	0.15208E-00	0.	
	0.10833E-00	0.97917E-01	0.14375E-00	0.23333E-00	0.14583E-00	0.	
	0.13750E-00	0.10208E-00	0.16250E-00	0.17708E-00	0.14583E-00	0.	
	0.11875E-00	0.95833E-01	0.16458E-00	0.11042E-00	0.15208E-00	0.	
	0.10208E-00	0.11675E-00	0.13750E-00	0.77083E-01	0.15433E-00	0.	
	0.10833E-00	0.81250E-01	0.13958E-00	0.13542E-00	0.16042E-00	0.	
	0.95833E-01	0.85417E-01	0.15000E-00	0.13125E-00	0.17292E-00	0.69583E 00	
	0.11458E-00	0.70833E-01	0.14167E-00	0.12917E-00	0.17292E-00	0.10000E 01	
	0.12708E-00	0.81250E-01	0.12708E-00	0.10208E-00	0.10208E-00	0.10000E 01	
	0.11250E-00	0.87500E-01	0.14583E-00	0.85417E-01	0.13750E-00	0.99792E 00	
	0.25000E-00	0.10208E-00	0.15833E-00	0.11667E-00	0.17170E-00	0.10000E 01	
	0.15833E-00	0.13750E-00	0.13333E-00	0.13333E-00	0.17929E-00	0.10000E 01	
	0.12625E-00	0.24917E-00	0.79734E-01	0.11953E-00	0.23588E-00	0.10000E 01	
0.45	0.16458E-00	0.13958E-00	0.12292E-00	0.12500E-00	0.18542E-00	0.	
	0.13542E-00	1.00000E-01	0.13750E-00	0.10833E-00	0.18542E-00	0.	
	0.15833E-00	0.11250E-00	0.18542E-00	0.10833E-00	0.18125E-00	0.	
	0.13542E-00	0.11875E-00	0.26042E-00	1.00000E-01	0.14542E-00	0.	
	0.12917E-00	0.11458E-00	0.16042E-00	0.16250E-00	0.89583E-01	0.	
	0.10625E-00	0.13125E-00	0.13125E-00	0.12500E-00	0.87500E-01	0.	
	0.11458E-00	0.15000E-00	0.97917E-01	0.12292E-00	0.13542E-00	0.	
	0.12292E-00	0.12500E-00	0.12708E-00	1.00000E-01	0.11667E-00	0.	
	0.11042E-00	0.11250E-00	0.12292E-00	1.00000E-01	0.13750E-00	0.	
	0.89583E-01	0.10208E-00	0.20208E-00	0.13750E-00	0.11875E-00	0.	
	0.10417E-00	0.10417E-00	0.12292E-00	0.12500E-00	0.12500E-00	0.	
	0.10833E-00	0.10625E-00	0.13958E-00	0.21458E-00	0.11458E-00	0.	
	0.13125E-00	0.10208E-00	0.13125E-00	0.16875E-00	0.12292E-00	0.	
	0.89583E-01	0.97917E-01	1.00000E-01	0.13750E-00	0.13333E-00	0.	
	0.83333E-01	0.12708E-00	0.15000E-00	0.10208E-00	0.12083E-00	0.	
	0.83333E-01	0.95833E-01	0.12083E-00	0.10417E-00	0.12292E-00	0.	
	0.87500E-01	0.91667E-01	0.15000E-00	0.10208E-00	0.15417E-00	0.6025E 00	
	0.83333E-01	0.97917E-01	0.13542E-00	0.12500E-00	0.12708E-00	0.99792E 00	
	0.83333E-01	0.11042E-00	0.15833E-00	0.13750E-00	0.11667E-00	0.99792E 00	
	0.95833E-01	1.00000E-01	0.13125E-00	0.13125E-00	0.17083E-00	0.99792E 00	
	0.17500E-00	0.91667E-01	0.12917E-00	0.13125E-00	0.16875E-00	0.99792E 00	
	0.13958E-00	0.11875E-00	0.16458E-00	0.12917E-00	0.17292E-00	0.99792E 00	
	0.10758E-00	0.16023E-00	0.11047E-00	0.14535E-00	0.11802E-00	0.99709E 00	
0.63	0.92243E-01	0.13417E-00	0.10273E-00	0.11530E-00	0.19706E-00	0.	
	0.10273E-00	0.94340E-01	0.92243E-01	0.10642E-00	0.15933E-00	0.	
	0.11111E-00	0.94340E-01	0.10482E-00	0.92243E-01	0.14542E-00	0.	
	0.12159E-00	0.12579E-00	0.17610E-00	0.11950E-00	0.10273E-00	0.	
	0.81761E-01	0.86050E-01	0.13627E-00	0.10642E-00	0.96436E-01	0.	
	0.11371E-00	0.11153E-00	0.10901E-00	0.90147E-01	0.92243E-01	0.	
	0.73375E-01	0.8594E-01	0.8594E-01	0.11950E-00	0.11111E-00	0.	
	0.67086E-01	0.11530E-00	0.11321E-00	0.96436E-01	0.11740E-00	0.	
	0.90147E-01	0.10000E-01	0.10273E-00	0.10901E-00	0.12369E-00	0.	
	0.94340E-01	0.10901E-00	0.12369E-00	0.11321E-00	0.96436E-01	0.	
	0.92243E-01	0.10208E-00	0.96436E-01	0.15514E-00	0.88050E-01	0.	
	0.86050E-01	0.8594E-01	0.11950E-00	0.16352E-00	0.10482E-00	0.	
	0.79665E-01	0.11111E-00	0.98532E-01	0.15514E-00	0.10692E-00	0.	
	0.8594E-01	0.88050E-01	0.11308E-00	0.11950E-00	0.11111E-00	0.	
	0.82893E-01	0.73375E-01	0.11111E-00	0.8594E-01	0.11740E-00	0.	
	0.86050E-01	0.94340E-01	0.11950E-00	0.8594E-01	0.10273E-00	0.	
	0.67086E-01	0.71279E-01	0.10901E-00	0.10482E-00	0.12369E-00	0.54507E 00	
	0.79665E-01	0.10273E-00	0.12369E-00	0.96436E-01	0.11530E-00	0.9981E 00	
	0.83875E-01	0.90147E-01	0.13836E-00	0.86050E-01	0.11530E-00	0.9981E 00	
	0.8594E-01	0.79665E-01	0.11321E-00	0.10273E-00	0.12159E-00	0.9981E 00	
	0.12708E-00	0.77566E-01	0.11111E-00	0.13417E-00	0.14675E-00	0.99792E 00	
	0.86050E-01	0.10901E-00	0.8594E-01	0.11950E-00	0.14256E-00	0.99792E 00	
	0.84469E-01	0.10627E-00	0.12262E-00	0.95368E-01	0.14986E-00	0.99183E 00	
0.81	0.71579E-01	0.56842E-01	0.82105E-01	0.61053E-01	0.94737E-01	0.	
	0.69474E-01	0.52632E-01	0.86316E-01	0.71579E-01	0.77895E-01	0.	
	0.77895E-01	0.56842E-01	0.77895E-01	0.54737E-01	0.88421E-01	0.	
	0.61053E-01	0.77895E-01	0.77895E-01	0.96842E-01	0.69474E-01	0.	
	0.86316E-01	0.82105E-01	0.82105E-01	0.80000E-01	0.84211E-01	0.	
	0.56842E-01	0.66316E-01	0.69474E-01	0.77895E-01	0.96842E-01	0.	
	0.73684E-01	0.96842E-01	0.73684E-01	0.73684E-01	0.56842E-01	0.	
	0.54737E-01	0.96842E-01	0.84211E-01	0.71579E-01	0.88421E-01	0.	
	0.73684E-01	0.82105E-01	0.10737E-00	0.90526E-01	0.90526E-01	0.	
	0.54737E-01	0.61053E-01	0.10526E-00	0.96842E-01	0.84211E-01	0.	
	0.48421E-01	0.73684E-01	0.13686E-00	0.11158E-00	0.90947E-01	0.	
	0.54737E-01	0.61053E-01	0.10526E-00	0.88421E-01	0.96737E-01	0.	
	0.54737E-01	0.61053E-01	0.10526E-00	0.87368E-01	0.98947E-01	0.	
	0.54737E-01	0.61053E-01	0.10526E-00	0.80000E-01	0.69474E-01	0.	
	0.54737E-01	0.61053E-01	0.10526E-00	0.82105E-01	0.77895E-01	0.54737E 00	
	0.61053E-01	0.52632E-01	0.86316E-01	0.71579E-01	0.96842E-01	0.99792E 00	
	0.37895E-01	0.44211E-01	0.96842E-01	0.69474E-01	0.96842E-01	0.99792E 00	
	0.52632E-01	0.52632E-01	0.96842E-01	0.69474E-01	0.96842E-01	0.99792E 00	
	0.59621E-01	0.73171E-01	0.70461E-01	0.70461E-01	0.12737E-00	0.99792E 00	

TABLE XII. Transient Data (Contd.)

RUN 14

Z*

3.6

5.8

10.2

18.2

32.0

REF.

0.00

0.27

0.45

0.63

0.81

0.10438E-00	0.13151E-00	0.18789E-00	0.93046E-01	0.13152E-00	0.
0.15031E-00	0.16493E-00	0.21503E-00	0.81420E-01	0.13361E-00	0.
0.14196E-00	0.18789E-00	0.21503E-00	0.93046E-01	0.15658E-00	0.
0.12526E-00	0.14196E-00	0.18580E-00	0.10647E-00	0.19833E-00	0.
0.11055E-00	0.17537E-00	0.22347E-00	0.96031E-01	0.20251E-00	0.
0.10647E-00	0.13779E-00	0.25261E-00	0.10230E-00	0.16701E-00	0.
0.10230E-00	0.11273E-00	0.21921E-00	0.15658E-00	0.19207E-00	0.
0.11691E-00	0.11591E-00	0.21921E-00	0.17745E-00	0.16493E-00	0.
0.11055E-00	0.10954E-00	0.18998E-00	0.14405E-00	0.16284E-00	0.
0.79332E-01	0.12526E-00	0.20251E-00	0.11900E-00	0.16701E-00	0.
0.75157E-01	0.12109E-00	0.17111E-00	0.12944E-00	0.17111E-00	0.
0.79332E-01	0.85595E-01	0.19207E-00	0.16284E-00	0.18163E-00	0.
0.64718E-01	0.81420E-01	0.17328E-00	0.12944E-00	0.19833E-00	0.
0.11055E-00	0.12735E-00	0.18163E-00	0.12944E-00	0.15240E-00	0.
0.71658E-01	0.11591E-00	0.15447E-00	0.11273E-00	0.16910E-00	0.
0.79332E-01	0.10021E-00	0.15658E-00	0.10555E-00	0.19624E-00	0.
0.87683E-01	0.93944E-01	0.12944E-00	0.10021E-00	0.18998E-00	0.46731E 00
0.96033E-01	0.10021E-00	0.14910E-00	0.12317E-00	0.18998E-00	0.10000E 01
0.87683E-01	0.10230E-00	0.15658E-00	0.13361E-00	0.18789E-00	0.10000E 01
0.12526E-00	0.13779E-00	0.15658E-00	0.17570E-00	0.18998E-00	0.10000E 01
0.12944E-00	0.12944E-00	0.19833E-00	0.11273E-00	0.15240E-00	0.10000E 01
0.10647E-00	0.18789E-00	0.18789E-00	0.11273E-00	0.16701E-00	0.10000E 01
0.11168E-00	0.15136E-00	0.17370E-00	0.11414E-00	0.13151E-00	0.10000E 01
0.13296E-00	0.17674E-00	0.17674E-00	0.81399E-01	0.15116E-00	0.
0.16512E-00	0.15116E-00	0.15116E-00	0.13023E-00	0.17209E-00	0.
0.14651E-00	0.18605E-00	0.18605E-00	0.10233E-00	0.13953E-00	0.
0.14619E-00	0.12093E-00	0.12093E-00	0.11850E-00	0.15116E-00	0.
0.12093E-00	0.12093E-00	0.12093E-00	0.10233E-00	0.16512E-00	0.
0.12226E-00	0.12226E-00	0.12226E-00	0.10000E-01	0.16512E-00	0.
0.13256E-00	0.13256E-00	0.13256E-00	0.13468E-00	0.16910E-00	0.
0.97674E-01	0.10698E-00	0.10698E-00	0.12558E-00	0.15349E-00	0.
0.12093E-00	0.12093E-00	0.12093E-00	0.12093E-00	0.21860E-00	0.
0.12958E-00	0.12958E-00	0.12958E-00	0.15116E-00	0.16910E-00	0.
0.10698E-00	0.10698E-00	0.10698E-00	0.12326E-00	0.17442E-00	0.
0.88372E-01	0.88372E-01	0.88372E-01	0.16744E-00	0.16744E-00	0.
0.10698E-00	0.10698E-00	0.10698E-00	0.11620E-00	0.16021E-00	0.
0.11163E-00	0.11163E-00	0.11163E-00	0.10698E-00	0.17442E-00	0.
0.11860E-00	0.11860E-00	0.11860E-00	0.11860E-00	0.17674E-00	0.
0.10233E-00	0.10233E-00	0.10233E-00	0.12093E-00	0.21256E-00	0.
1.00000E-01	1.00000E-01	1.00000E-01	0.12791E-00	0.18605E-00	0.56512E 00
0.97674E-01	0.97674E-01	0.97674E-01	0.12791E-00	0.23023E-00	0.99335E 00
0.11395E-00	0.11395E-00	0.11395E-00	0.11395E-00	0.11395E-00	0.99335E 00
0.14651E-00	0.14651E-00	0.14651E-00	0.10698E-00	0.19535E-00	0.99302E 00
0.12326E-00	0.12326E-00	0.12326E-00	0.13468E-00	0.17907E-00	0.99302E 00
0.15581E-00	0.15581E-00	0.15581E-00	1.00000E-01	0.14651E-00	0.99302E 00
0.18605E-00	0.18605E-00	0.18605E-00	0.90341E-01	0.16566E-00	0.99096E 00
0.14375E-00	0.11458E-00	0.81250E-01	0.56250E-01	0.15208E-00	0.
0.12083E-00	0.15625E-00	0.14375E-00	0.70833E-01	0.13750E-00	0.
0.12292E-00	0.16250E-00	0.12917E-00	0.66667E-01	0.16625E-00	0.
0.12292E-00	0.17292E-00	0.14617E-00	0.66667E-01	0.16875E-00	0.
0.83333E-01	0.13958E-00	0.15000E-00	0.56333E-01	0.17500E-00	0.
0.87500E-01	0.11667E-00	0.15833E-00	0.60417E-01	0.15625E-00	0.
0.85417E-01	0.11458E-00	0.17292E-00	0.81250E-01	0.14167E-00	0.
0.58333E-01	0.11667E-00	0.15417E-00	0.11458E-00	0.14792E-00	0.
0.77083E-01	0.81250E-01	0.11458E-00	0.77083E-01	0.17292E-00	0.
0.70833E-01	0.10417E-00	0.12917E-00	0.97017E-01	0.18125E-00	0.
0.87500E-01	0.85417E-01	0.97017E-01	0.97017E-01	0.14375E-00	0.
0.70833E-01	0.85417E-01	0.12500E-00	0.10833E-01	0.11417E-00	0.
0.87500E-01	0.93750E-01	0.95833E-01	0.95833E-01	0.13958E-00	0.
0.83333E-01	0.10208E-00	0.13333E-00	0.79167E-01	0.1708E-00	0.
0.68750E-01	0.97017E-01	0.81250E-01	0.45417E-01	0.15625E-00	0.
0.47917E-01	0.10417E-00	0.97017E-01	0.49583E-01	0.17083E-00	0.
0.50000E-01	0.91667E-01	0.70833E-01	0.43333E-01	0.20000E-00	0.64792E 00
0.56250E-01	0.81250E-01	0.95833E-01	0.85417E-01	0.20000E-00	0.10000E 01
0.81250E-01	0.79167E-01	0.77083E-01	0.85417E-01	0.20000E-00	0.10000E 01
0.89583E-01	0.10417E-00	0.97017E-01	0.83333E-01	0.20000E-00	0.10000E 01
0.12083E-00	0.10833E-00	0.11458E-00	0.79167E-01	0.16512E-00	0.10000E 01
0.95833E-01	0.10625E-00	0.77083E-01	0.75000E-01	0.16458E-00	0.10000E 01
0.13720E-00	0.11890E-00	0.82317E-01	0.57927E-01	0.12805E-00	0.10000E 01
0.13542E-00	0.13125E-00	0.12500E-00	0.47917E-01	0.10625E-00	0.
0.11042E-00	0.13958E-00	0.79167E-01	0.79167E-01	0.13475E-00	0.
0.10625E-00	0.12917E-00	0.11250E-00	1.00000E-01	0.13333E-00	0.
0.87500E-01	0.14167E-00	0.15625E-00	0.87500E-01	0.13542E-00	0.
0.81250E-01	0.15000E-00	0.11250E-00	0.60417E-01	0.15833E-00	0.
0.56250E-01	0.11458E-00	0.15417E-00	0.64583E-01	0.13333E-00	0.
0.81250E-01	0.10417E-00	0.12708E-00	0.61250E-01	0.1042E-00	0.
0.68750E-01	0.96333E-01	0.13333E-00	0.93750E-01	0.14583E-00	0.
0.54167E-01	0.86667E-01	0.11042E-00	0.10625E-00	0.13958E-00	0.
0.72917E-01	0.81250E-01	1.00000E-01	0.10625E-00	0.1708E-00	0.
0.70833E-01	0.88750E-01	0.10208E-00	0.81250E-01	0.12917E-00	0.
0.54167E-01	0.70833E-01	0.10625E-00	0.97017E-01	0.1042E-00	0.
0.68750E-01	0.64583E-01	0.93750E-01	0.85417E-01	0.12708E-00	0.
0.47917E-01	0.81250E-01	0.83333E-01	0.66667E-01	0.15000E-00	0.
0.66667E-01	0.75000E-01	0.85417E-01	0.87500E-01	0.15000E-00	0.
0.81250E-01	0.89583E-01	0.93750E-01	0.66667E-01	0.13750E-00	0.66042E 00
0.81250E-01	0.98333E-01	0.88750E-01	0.89583E-01	0.14625E-00	0.99792E 00
0.45833E-01	0.98333E-01	0.93333E-01	0.64583E-01	0.17083E-00	0.99792E 00
0.95833E-01	0.77083E-01	0.10625E-00	0.83333E-01	0.13958E-00	0.99792E 00
0.11042E-00	0.87500E-01	0.95833E-01	0.87500E-01	0.12500E-00	0.99792E 00
0.15000E-00	0.13333E-00	0.44916E-01	0.56250E-01	0.11667E-00	0.99792E 00
0.12117E-00	0.13042E-00	0.96273E-01	0.9166E-01	0.11801E-00	0.99696E 00
0.10833E-00	0.81250E-01	0.95633E-01	0.45833E-01	0.10833E-00	0.
0.111250E-00	0.10417E-00	0.10833E-00	0.37500E-01	0.10625E-00	0.
0.10625E-00	0.11667E-00	0.13125E-00	0.25000E-01	0.11250E-00	0.
0.89583E-01	0.11250E-00	0.87500E-01	0.33333E-01	0.12500E-00	0.
0.72917E-01	0.10625E-00	0.12917E-00	0.50000E-01	0.11667E-00	0.
0.54167E-01	0.81250E-01	0.12292E-00	0.35417E-01	0.95833E-01	0.
0.58333E-01	0.81250E-01	0.97017E-01	0.39583E-01	0.10208E-00	0.
0.56250E-01	0.81250E-01	0.10625E-00	0.48583E-01	0.12708E-00	0.
0.60417E-01	0.83333E-01	0.13542E-00	0.35417E-01	0.11042E-00	0.
0.52083E-01	0.56250E-01	0.13333E-00	0.56250E-01	0.12500E-00	0.
0.56250E-01	0.83750E-01	0.12292E-00	0.50000E-01	0.14583E-00	0.
0.47917E-01	0.64583E-01	0.95833E-01	0.77083E-01	1.00000E-01	0.
0.47917E-01	0.68750E-01	0.12083E-00	0.45833E-01	0.10625E-00	0.
0.56250E-01	0.66667E-01	0.10417E-00	0.64583E-01	0.12708E-00	0.
0.70833E-01	0.53333E-01	0.10208E-00	0.52083E-01	0.87500E-01	0.
0.58333E-01	0.77083E-01	0.81250E-01	0.60417E-01	0.11875E-00	0.
0.66667E-01	0.68750E-01	0.89583E-01	0.41667E-01	0.10625E-00	0.63958E 00
0.41667E-01	0.68750E-01	0.77083E-01	0.77083E-01	0.13333E-01	0.99792E 00
0.54167E-01	0.70833E-01	0.64583E-01	0.54167E-01	0.11250E-00	0.99792E 00
0.77083E-01	0.41667E-01	0.58333E-01	0.50000E-01	0.13125E-00	0.99792E 00
0.80750E-01	0.70833E-01	0.89583E-01	0.56250E-01	0.13750E-00	0.99792E 00
0.89583E-01	0.64583E-01	0.77083E-01	0.35417E-01	0.77083E-01	0.95833E 00
0.76023E-01	0.84795E-01	0.61871E-01	0.52632E-01	0.10819E-00	0.99415E 00

TABLE XII. Transient Data (Contd.)

RUN 15

Z*
r*

0.00

0.27

0.45

0.63

0.81

3.6	5.8	10.2	18.2	32.0	REF.
0.15789E-00	0.12211E-00	0.26737E-00	0.10526E-00	0.12211E-00	0.
0.14947E-00	0.08947E-01	0.10158E-00	0.10105E-00	0.13053E-00	0.
0.12632E-00	0.13263E-00	0.1580E-00	0.77895E-01	0.15368E-00	0.
0.11789E-00	0.12642E-00	0.22737E-00	0.71579E-01	0.15368E-00	0.
0.08947E-01	0.77895E-01	0.21474E-00	0.86316E-01	0.13895E-00	0.
0.04211E-01	0.50526E-01	0.17053E-00	0.25053E-00	0.14947E-00	0.
0.10316E-00	0.53158E-01	0.13684E-00	0.29684E-00	0.16000E-00	0.
0.14316E-00	0.73684E-01	0.13263E-00	0.17053E-00	0.15579E-00	0.
0.12421E-00	0.65263E-01	0.14105E-00	0.18316E-00	0.14526E-00	0.
0.12211E-00	0.48421E-01	0.13474E-00	0.19158E-00	0.12211E-00	0.
0.11158E-00	0.67368E-01	0.11368E-00	0.12632E-00	0.13684E-00	0.
0.10526E-00	0.73684E-01	0.11368E-00	0.17263E-00	0.13474E-00	0.
0.06316E-01	0.63158E-01	0.15795E-00	0.12421E-00	0.13474E-00	0.
0.02632E-01	0.71579E-01	0.15789E-00	0.11579E-00	0.21263E-00	0.
0.12000E-00	0.54737E-01	0.12421E-00	0.10737E-00	0.39789E-00	0.
0.02632E-01	0.56842E-01	0.14105E-00	0.14105E-00	0.23579E-00	0.
0.07368E-01	0.40000E-01	0.94737E-01	0.10526E-00	0.18737E-00	0.58316E 00
0.02632E-01	0.48421E-01	0.11789E-00	0.98947E-01	0.15579E-00	0.10000E 01
0.10737E-00	0.44211E-01	0.82105E-01	0.17053E-00	0.15579E-00	0.10000E 01
0.16421E-00	0.33684E-01	0.82105E-01	0.13684E-00	0.16632E-00	0.10000E 01
0.18526E-00	0.77895E-01	0.90526E-01	0.14316E-00	0.11789E-00	0.10000E 01
0.15789E-00	0.19789E-00	0.98947E-01	0.20579E-01	0.11579E-00	0.10000E 01
0.16992E-00	0.14763E-00	0.10028E-00	0.10028E-00	0.14206E-00	0.10000E 01
0.16000E-00	0.12632E-00	0.21263E-00	0.10947E-00	0.11579E-00	0.
0.17895E-00	0.16421E-00	0.19789E-00	0.92632E-01	0.14526E-00	0.
0.11158E-00	0.13474E-00	0.16211E-00	0.88421E-01	0.16842E-00	0.
0.11158E-00	0.10532E-00	0.17895E-00	0.87368E-01	0.17368E-00	0.
0.90526E-01	0.84211E-01	0.16632E-00	0.88421E-01	0.16632E-00	0.
0.82105E-01	0.54737E-01	0.21684E-00	0.25053E-00	0.14737E-00	0.
0.02632E-01	0.77895E-01	0.17684E-00	0.30105E-00	0.15158E-00	0.
0.06842E-01	0.71579E-01	0.13895E-00	0.16632E-00	0.12000E-00	0.
0.88421E-01	0.84211E-01	0.12421E-00	0.18947E-00	0.13474E-00	0.
0.10316E-00	0.84211E-01	0.12826E-00	0.18947E-00	0.13474E-00	0.
0.94737E-01	0.80000E-01	0.12211E-00	0.18947E-00	0.12632E-00	0.
0.65263E-01	0.37895E-01	0.14105E-00	0.13895E-00	0.12421E-00	0.
0.10316E-00	0.80000E-01	0.16211E-00	0.10947E-00	0.13684E-00	0.
0.73684E-01	0.50526E-01	0.12421E-00	0.94737E-01	0.24211E-00	0.
0.46316E-01	0.52632E-01	0.12421E-00	0.12211E-00	0.34947E-00	0.
0.69474E-01	0.46316E-01	0.10316E-00	0.82105E-01	0.27368E-00	0.
0.02632E-01	0.56842E-01	0.11158E-00	0.10526E-00	0.18737E-00	0.53053E 00
0.08421E-01	0.44211E-01	0.11158E-00	0.11789E-00	0.17263E-00	0.10000E 01
0.71579E-01	0.46316E-01	0.80000E-01	0.15158E-00	0.15158E-00	0.10000E 01
0.13263E-00	0.44211E-01	0.92632E-01	0.14316E-00	0.11368E-00	0.10000E 01
0.19158E-00	0.10526E-00	0.88421E-01	0.11789E-00	0.10105E-00	0.10000E 01
0.21474E-00	0.18526E-00	0.69474E-01	0.69474E-01	0.82105E-00	0.10000E 01
0.16710E-00	0.18277E-00	0.99217E-01	0.13577E-00	0.10183E-00	0.10000E 01
0.11747E-00	0.13263E-00	0.18105E-00	0.86316E-01	0.12632E-00	0.
0.19158E-00	0.10316E-00	0.20421E-00	0.75789E-01	0.10526E-00	0.
0.15579E-00	0.13263E-00	0.17263E-00	0.52632E-01	0.14947E-00	0.
0.90526E-01	0.10947E-00	0.19368E-00	0.87368E-01	0.17368E-00	0.
0.84211E-01	0.52632E-01	0.20211E-00	0.94421E-01	0.16000E-00	0.
0.77895E-01	0.10105E-00	0.14737E-00	0.19789E-00	0.17263E-00	0.
0.71579E-01	0.90526E-01	0.14737E-00	0.26526E-00	0.13895E-00	0.
0.02632E-01	0.63158E-01	0.13895E-00	0.15158E-00	0.10737E-00	0.
0.80000E-01	0.73684E-01	0.10947E-00	0.14105E-00	0.13053E-00	0.
0.80000E-01	0.73684E-01	0.92632E-01	0.12826E-00	0.10526E-00	0.
0.96842E-01	0.42632E-01	0.12000E-00	0.12000E-00	0.94737E-01	0.
0.75789E-01	0.77895E-01	0.13053E-00	0.10737E-00	0.13053E-00	0.
0.58947E-01	0.80000E-01	0.12000E-00	0.10947E-00	0.14526E-00	0.
0.69474E-01	0.41032E-01	0.10947E-00	0.10105E-00	0.19368E-00	0.
0.71579E-01	0.52632E-01	0.13263E-00	0.92632E-01	0.34947E-00	0.
0.84211E-01	0.33684E-01	0.86316E-01	0.86316E-01	0.24211E-00	0.
0.67368E-01	0.15789E-00	0.11158E-00	0.11158E-00	0.15158E-00	0.52632E 00
0.65263E-01	0.48421E-01	0.77895E-01	0.11368E-00	0.15789E-00	0.99368E 00
0.58947E-01	0.77368E-01	0.75789E-01	0.15579E-00	0.17474E-00	0.99368E 00
0.15789E-00	0.54737E-01	0.56842E-01	0.98947E-01	0.98947E-01	0.99368E 00
0.20000E-00	0.10316E-00	0.10105E-00	0.98947E-01	0.84211E-01	0.99368E 00
0.18526E-00	0.77895E-00	0.73684E-01	0.98947E-01	0.10737E-00	0.99368E 00
0.15584E-00	0.18893E-00	0.62338E-01	0.10989E-00	0.98701E-01	0.99221E 00
0.13895E-00	0.11368E-00	0.16421E-00	0.52632E-01	0.10105E-00	0.
0.14947E-00	0.13474E-00	0.16421E-00	0.46316E-01	0.90526E-01	0.
0.11579E-00	0.10737E-00	0.16632E-00	0.44211E-01	0.10737E-00	0.
0.11158E-00	0.11368E-00	0.13895E-00	0.52632E-01	0.98947E-01	0.
0.73684E-01	0.77895E-01	0.15579E-00	0.68421E-01	0.14105E-00	0.
0.73684E-01	0.54737E-01	0.12842E-00	0.15789E-00	0.16421E-00	0.
0.61053E-01	0.50526E-01	0.16842E-00	0.19368E-00	0.16000E-00	0.
0.80000E-01	0.10526E-01	0.11368E-00	0.10737E-00	0.14737E-00	0.
0.69474E-01	0.58947E-01	0.11368E-00	0.11579E-00	0.14737E-00	0.
0.54737E-01	0.52632E-01	0.88421E-01	0.10105E-00	0.13684E-00	0.
0.73684E-01	0.41032E-01	0.10737E-00	0.12421E-00	0.13053E-00	0.
0.77895E-01	0.48421E-01	0.11368E-00	0.80000E-01	0.10526E-00	0.
0.52632E-01	0.37895E-01	0.10316E-00	0.98947E-01	0.12211E-00	0.
0.65263E-01	0.52632E-01	0.14526E-00	0.80000E-01	0.18947E-00	0.
0.67368E-01	0.42105E-01	0.10947E-00	0.69474E-01	0.28989E-00	0.
0.61053E-01	0.44211E-01	0.86316E-01	0.52632E-01	0.23158E-00	0.
0.89474E-01	0.35789E-01	0.82105E-01	0.92632E-01	0.15789E-00	0.
0.50526E-01	0.35789E-01	0.75789E-01	0.75789E-01	0.13053E-00	0.49053E-00
0.46316E-01	0.42105E-01	0.71579E-01	0.84211E-01	0.11368E-00	0.10000E 01
0.11368E-00	0.43158E-01	0.71579E-01	0.67368E-01	0.11789E-00	0.10000E 01
0.17684E-00	0.63158E-01	0.63158E-01	0.10105E-00	0.98947E-01	0.10000E 01
0.17895E-00	0.12842E-00	0.88421E-01	0.96842E-01	0.82105E-01	0.10000E 01
0.14079E-00	0.12136E-00	0.10473E-00	0.58292E-01	0.92233E-01	0.10000E 01
0.95137E-01	0.31712E-01	0.10782E-00	0.40169E-01	0.02452E-01	0.
0.05171E-00	0.42263E-01	0.12474E-00	0.12474E-00	0.97252E-01	0.
0.93023E-01	0.31712E-01	0.12051E-00	0.31712E-01	0.78224E-01	0.
0.73996E-01	0.31712E-01	0.10782E-00	0.25370E-01	0.12896E-00	0.
0.44397E-01	0.77895E-01	0.93023E-01	0.93023E-01	0.10168E-00	0.
0.50740E-01	0.33827E-01	0.71882E-01	0.84567E-01	0.84567E-01	0.
0.54968E-01	0.21142E-01	0.93023E-01	0.88795E-01	0.14168E-00	0.
0.59197E-01	0.40512E-01	0.10782E-00	0.48263E-01	0.12916E-00	0.
0.40512E-01	0.12685E-01	0.99368E-01	0.99368E-01	0.99368E-01	0.
0.40512E-01	0.12685E-01	0.10782E-00	0.61311E-01	0.97252E-01	0.
0.38055E-01	0.14789E-01	0.14789E-01	0.54968E-01	0.35368E-01	0.
0.33827E-01	0.42429E-02	0.12685E-01	0.38055E-01	0.80338E-01	0.
0.25370E-01	0.42283E-02	0.10359E-00	0.40169E-01	0.11893E-00	0.
0.27484E-01	0.63429E-02	0.10359E-00	0.40169E-01	0.12826E-00	0.
0.20598E-01	0.19027E-01	0.97125E-01	0.38055E-01	0.21353E-00	0.
0.40169E-01	0.12685E-01	0.76110E-01	0.48421E-01	0.17548E-00	0.
0.35941E-01	0.12685E-01	0.69767E-01	0.33827E-01	0.90795E-00	0.92369E-00
0.35941E-01	0.84567E-02	0.76110E-01	0.31712E-01	0.11628E-00	0.99368E 00
0.31712E-01	0.42283E-02	0.42283E-02	0.61311E-01	0.10571E-00	0.99368E 00
0.65939E-01	0.10571E-01	0.52895E-01	0.33827E-01	0.13006E-00	0.99368E 00
0.10782E-00	0.10571E-01	0.43539E-01	0.67368E-01	0.80795E-01	0.99154E 00
0.11416E-00	0.33827E-01	0.54968E-01	0.38055E-01	0.95137E-01	0.99154E 00
0.85293E-01	0.29945E-01	0.71429E-01	0.64516E-01	0.52903E-01	0.99078E 00

TABLE XII. Transient Data (Contd.)

RUN 16						
$\frac{Z^*}{r^*}$	3.6	5.8	10.2	18.2	32.0	REF.
0.00	0.98326E-01	0.11925E-00	0.12762E-00	0.35565E-01	0.83682E-01	0.
	0.85774E-01	0.11297E-00	0.12134E-00	0.46028E-01	0.92050E-01	0.
	0.62762E-01	0.11297E-00	0.89958E-01	0.75314E-01	0.94142E-01	0.
	0.71130E-01	0.1380E-00	0.11080E-00	0.66946E-01	0.89958E-01	0.
	0.66946E-01	0.10042E-00	0.10042E-00	0.66946E-01	0.85774E-01	0.
	0.71130E-01	0.87866E-01	0.10079E-00	0.64854E-01	0.98326E-01	0.
	0.71130E-01	0.10042E-00	0.79498E-01	0.71130E-01	0.96234E-01	0.
	0.50209E-01	0.75314E-01	0.69038E-01	0.85774E-01	0.13389E-00	0.
	0.56485E-01	0.85774E-01	0.73222E-01	0.12134E-00	0.98326E-01	0.
	0.71130E-01	0.75314E-01	0.94142E-01	0.10460E-00	0.10460E-00	0.
	0.66946E-01	0.73222E-01	0.69038E-01	0.94142E-01	0.79498E-01	0.
	0.66946E-01	0.79498E-01	0.66946E-01	0.73222E-01	0.79498E-01	0.
	0.60309E-01	0.73222E-01	0.71130E-01	0.98326E-01	0.11925E-00	0.
	0.66946E-01	0.73222E-01	0.69038E-01	0.79498E-01	0.10251E-00	0.
	0.66946E-01	0.85774E-01	0.77406E-01	0.66946E-01	0.14644E-00	0.
	0.77406E-01	0.92050E-01	0.85774E-01	0.66946E-01	0.14644E-00	0.
	0.58577E-01	0.77406E-01	0.93682E-01	0.3682E-01	0.16527E-00	0.
	0.60696E-01	0.71130E-01	0.54393E-01	0.62762E-01	0.16736E-00	0.51893E 00
	0.58577E-01	0.79498E-01	0.73222E-01	0.60696E-01	0.17155E-00	0.99791E 00
	0.58577E-01	0.73222E-01	0.64854E-01	0.62762E-01	0.17573E-00	0.99791E 00
	0.96234E-01	0.10079E-00	0.79498E-01	0.75314E-01	0.14226E-00	0.99791E 00
0.94142E-01	0.10669E-00	0.98326E-01	0.75314E-01	0.94142E-01	0.99582E 00	
0.94142E-01	0.12971E-00	0.11297E-00	0.52301E-01	0.85774E-01	0.99582E 00	
0.85774E-01	0.11875E-00	0.10914E-00	0.63592E-01	0.10152E-00	0.99492E 00	
0.27	0.10042E-00	0.89958E-01	0.15063E-00	0.77406E-01	0.10879E-00	0.
	0.94142E-01	0.11715E-00	0.18619E-00	0.56234E-01	0.11297E-00	0.
	0.85774E-01	0.96234E-01	0.17782E-00	0.85774E-01	0.10251E-00	0.
	0.66946E-01	0.96234E-01	0.10879E-00	0.10879E-00	0.96234E-01	0.
	0.62762E-01	0.81500E-01	0.13309E-00	0.89958E-01	0.11080E-00	0.
	0.54393E-01	0.66946E-01	0.13809E-00	0.12134E-00	0.13180E-00	0.
	0.62762E-01	0.71222E-01	0.11080E-00	0.13389E-00	0.12552E-00	0.
	0.64854E-01	0.85682E-01	0.10440E-00	0.14854E-00	0.14854E-00	0.
	0.41841E-01	0.54393E-01	0.10460E-00	0.15481E-00	0.10879E-00	0.
	0.60309E-01	0.64854E-01	0.11505E-00	0.15481E-00	0.13389E-00	0.
	0.54393E-01	0.85774E-01	0.10251E-00	0.13180E-00	0.13389E-00	0.
	0.62762E-01	0.71130E-01	0.93682E-01	0.14854E-00	0.17155E-00	0.
	0.50209E-01	0.54393E-01	0.89958E-01	0.12343E-00	0.15063E-00	0.
	0.60696E-01	0.66946E-01	0.98326E-01	0.71130E-01	0.14435E-00	0.
	0.54393E-01	0.66946E-01	0.92050E-01	0.89958E-01	0.15900E-00	0.
	0.81500E-01	0.10042E-00	0.12762E-00	0.87866E-01	0.16946E-00	0.
	0.54393E-01	0.69038E-01	0.11080E-00	0.89958E-01	0.16946E-00	0.
	0.66946E-01	0.62762E-01	0.92050E-01	0.11297E-00	0.21548E-00	0.50209E 00
	0.64583E-01	0.31250E-01	0.79498E-01	0.10042E-00	0.17573E-00	0.99582E 00
	0.92050E-01	0.85682E-01	0.11297E-00	0.85682E-01	0.15063E-00	0.99582E 00
	0.83682E-01	0.81500E-01	0.85774E-01	0.10879E-00	0.14226E-00	0.99582E 00
0.89958E-01	0.12134E-00	0.14226E-00	0.79498E-01	0.10879E-00	0.99582E 00	
0.98485E-01	0.11111E-00	0.14141E-00	0.95960E-01	0.15152E-00	0.99492E 00	
0.45	0.89958E-01	0.50000E-01	0.15208E-00	0.52083E-01	0.11042E-00	0.
	0.10417E-00	0.62500E-01	0.12708E-00	0.68750E-01	0.11250E-00	0.
	0.77083E-01	0.56250E-01	0.11075E-00	0.39583E-01	0.10000E-01	0.
	0.56250E-01	0.31250E-01	0.14900E-00	0.62500E-01	0.77017E-01	0.
	0.54167E-01	0.22917E-01	0.14167E-00	0.68750E-01	0.93750E-01	0.
	0.60667E-01	0.41667E-01	0.13958E-00	0.68750E-01	0.89958E-01	0.
	0.56250E-01	0.18750E-01	0.11042E-00	0.89958E-01	0.10417E-00	0.
	0.64583E-01	0.31250E-01	0.12272E-00	0.93750E-01	0.10417E-00	0.
	0.58333E-01	0.33333E-01	0.11667E-00	0.10208E-00	0.93750E-01	0.
	0.66667E-01	0.31250E-01	0.79167E-01	0.11042E-00	0.93750E-01	0.
	0.66667E-01	0.29167E-01	0.85417E-01	0.11250E-00	0.85417E-01	0.
	0.56250E-01	0.43750E-01	0.10208E-00	0.87500E-01	0.11667E-00	0.
	0.68750E-01	0.27083E-01	0.99167E-01	0.72917E-01	0.14583E-00	0.
	0.64583E-01	0.43750E-01	0.91667E-01	0.89958E-01	0.17917E-00	0.
	0.60417E-01	0.27083E-01	0.11667E-00	0.81250E-01	0.19167E-00	0.
	0.47917E-01	0.22917E-01	0.79167E-01	0.35417E-01	0.15417E-00	0.
	0.58333E-01	0.27083E-01	0.10833E-00	0.54167E-01	0.16842E-00	0.99375E 00
	0.50000E-01	0.22917E-01	0.89583E-01	0.91667E-01	0.15000E-00	0.99792E 00
	0.60417E-01	0.33333E-01	0.11042E-00	0.68750E-01	0.20417E-00	0.99792E 00
	0.94583E-01	0.29167E-01	0.91667E-01	0.56250E-01	0.16667E-00	0.99792E 00
	0.89583E-01	0.52083E-01	0.11667E-00	0.52083E-01	0.15625E-00	0.99792E 00
0.81250E-01	0.39583E-01	0.85417E-01	0.47917E-01	0.11667E-00	0.99792E 00	
0.13611E-00	0.56333E-01	0.13056E-00	0.63889E-01	0.94444E-01	0.99722E 00	
0.63	0.10625E-00	0.12917E-00	0.10208E-00	0.10417E-00	0.13333E-00	0.
	0.12917E-00	0.13333E-00	0.15625E-00	0.10833E-00	0.14583E-00	0.
	0.68750E-01	0.10208E-00	0.10208E-00	0.10208E-00	0.13333E-00	0.
	0.58333E-01	0.12500E-00	0.10625E-00	0.10625E-00	0.15417E-00	0.
	0.56250E-01	0.15208E-00	0.12292E-00	0.12292E-00	0.14167E-00	0.
	0.64583E-01	0.11667E-00	0.17292E-00	0.17292E-00	0.16042E-00	0.
	0.81250E-01	0.11667E-00	0.11667E-00	0.11667E-00	0.13958E-00	0.
	0.60417E-01	0.87500E-01	0.15208E-00	0.15208E-00	0.95833E-01	0.
	0.79167E-01	1.00000E-01	0.15208E-00	0.15208E-00	0.16250E-00	0.
	0.56250E-01	0.70833E-01	0.12500E-00	0.12500E-00	0.13542E-00	0.
	0.62500E-01	0.72917E-01	0.14583E-00	0.14583E-00	0.13750E-00	0.
	0.41667E-01	0.70833E-01	0.10417E-00	0.10417E-00	0.11458E-00	0.
	0.66667E-01	0.10208E-00	1.00000E-01	1.00000E-01	0.13750E-00	0.
	0.31250E-01	0.54167E-01	0.54167E-01	0.11042E-00	0.15833E-00	0.
	0.62500E-01	0.79167E-01	0.79167E-01	0.11458E-00	0.18125E-00	0.
	0.79167E-01	0.11875E-00	0.11875E-00	0.10417E-00	0.20417E-00	0.
	0.56333E-01	0.12917E-01	0.12917E-01	0.10833E-00	0.23958E-00	0.64000E 00
	0.41667E-01	0.87500E-01	0.87500E-01	0.95833E-01	0.21458E-00	0.10000E 01
	0.81250E-01	0.11042E-00	0.12708E-00	0.12708E-00	0.19375E-00	0.10000E 01
	0.10417E-00	0.75000E-01	0.12292E-00	0.12292E-00	0.17500E-00	0.10000E 01
	0.10208E-00	0.81250E-01	0.10417E-00	0.10417E-00	0.12292E-00	0.10000E 01
0.10615E-00	0.12849E-00	0.11173E-00	0.11173E-00	0.16480E-00	0.10000E 01	
0.81	0.10625E-00	0.12917E-00	0.10208E-00	0.10417E-00	0.23958E-00	0.
	0.12917E-00	0.13333E-00	0.15625E-00	0.10833E-00	0.14583E-00	0.
	0.68750E-01	0.10208E-00	0.10208E-00	0.10208E-00	0.13333E-00	0.
	0.58333E-01	0.12500E-00	0.10625E-00	0.10625E-00	0.15417E-00	0.
	0.56250E-01	0.15208E-00	0.12292E-00	0.12292E-00	0.14167E-00	0.
	0.64583E-01	0.11667E-00	0.17292E-00	0.17292E-00	0.16042E-00	0.
	0.81250E-01	0.11667E-00	0.11667E-00	0.11667E-00	0.13958E-00	0.
	0.60417E-01	0.87500E-01	0.15208E-00	0.15208E-00	0.95833E-01	0.
	0.79167E-01	1.00000E-01	0.15208E-00	0.15208E-00	0.16250E-00	0.
	0.56250E-01	0.70833E-01	0.12500E-00	0.12500E-00	0.13542E-00	0.
	0.62500E-01	0.72917E-01	0.14583E-00	0.14583E-00	0.13750E-00	0.
	0.41667E-01	0.70833E-01	0.10417E-00	0.10417E-00	0.11458E-00	0.
	0.66667E-01	0.10208E-00	1.00000E-01	1.00000E-01	0.13750E-00	0.
	0.31250E-01	0.54167E-01	0.54167E-01	0.11042E-00	0.15833E-00	0.
	0.62500E-01	0.79167E-01	0.79167E-01	0.11458E-00	0.18125E-00	0.
	0.79167E-01	0.11875E-00	0.11875E-00	0.10417E-00	0.20417E-00	0.
	0.56333E-01	0.12917E-01	0.12917E-01	0.10833E-00	0.23958E-00	0.64000E 00
	0.41667E-01	0.87500E-01	0.87500E-01	0.95833E-01	0.21458E-00	0.10000E 01
	0.81250E-01	0.11042E-00	0.12708E-00	0.12708E-00	0.19375E-00	0.10000E 01
	0.10417E-00	0.75000E-01	0.12292E-00	0.12292E-00	0.17500E-00	0.10000E 01
	0.10208E-00	0.81250E-01	0.10417E-00	0.10417E-00	0.12292E-00	0.10000E 01
0.10615E-00	0.12849E-00	0.11173E-00	0.11173E-00	0.16480E-00	0.10000E 01	

TABLE XII. Transient Data (Contd.)

RUN 18

 $\frac{Z^*}{r^*}$

0.00

0.27

0.45

0.63

0.81

3.6 5.8 10.2 18.2 32.0 REF.

0.26809E-00	0.22766E-00	0.20426E-00	0.22079E-00	0.26596E-00	0.
0.30851E-00	0.23191E-00	0.24809E-00	0.20000E-00	0.24681E-00	0.
0.25745E-00	0.24596E-00	0.31702E-00	0.19362E-00	0.20426E-00	0.
0.24255E-00	0.24894E-00	0.29362E-00	0.20213E-00	0.25106E-00	0.
0.23617E-00	0.27447E-00	0.21915E-00	0.21915E-00	0.24681E-00	0.
0.25951E-00	0.25745E-00	0.21489E-00	0.16298E-00	0.23830E-00	0.
0.20638E-00	0.19574E-00	0.24043E-00	0.19787E-00	0.23830E-00	0.
0.23191E-00	0.25106E-00	0.25319E-00	0.23830E-00	0.19787E-00	0.
0.20213E-00	0.24043E-00	0.23404E-00	0.25106E-00	0.19149E-00	0.
0.19787E-00	0.22340E-00	0.20213E-00	0.28085E-00	0.25319E-00	0.
0.21277E-00	0.24043E-00	0.20000E-00	0.31064E-00	0.20638E-00	0.
0.24408E-00	0.20426E-00	0.18936E-00	0.25745E-00	0.23830E-00	0.
0.19574E-00	0.25106E-00	0.17600E-00	0.21702E-00	0.22553E-00	0.
0.20213E-00	0.21915E-00	0.26383E-00	0.21277E-00	0.22979E-00	0.
0.18085E-00	0.21277E-00	0.21204E-00	0.22979E-00	0.24681E-00	0.
0.20213E-00	0.21915E-00	0.24408E-00	0.21489E-00	0.20213E-00	0.
0.20213E-00	0.18723E-00	0.24408E-00	0.22128E-00	0.20851E-00	0.58936E 00
0.21277E-00	0.23404E-00	0.21489E-00	0.15106E-00	0.24043E-00	0.58936E 00
0.21277E-00	0.19149E-00	0.19574E-00	0.15106E-00	0.24043E-00	0.58936E 00
0.27872E-00	0.22340E-00	0.21489E-00	0.16596E-00	0.26809E-00	0.99149E 00
0.32979E-00	0.24681E-00	0.16170E-00	0.17872E-00	0.27872E-00	0.98723E 00
0.25319E-00	0.33404E-00	0.20851E-00	0.40213E-00	0.35319E-00	0.98936E 00
0.21918E-00	0.29315E-00	0.18004E-00	0.27567E-00	0.31781E-00	0.98630E 00
0.24681E-00	0.24255E-00	0.22766E-00	0.20638E-00	0.30851E-00	0.
0.28723E-00	0.24043E-00	0.31489E-00	0.22979E-00	0.25957E-00	0.
0.22340E-00	0.23617E-00	0.38936E-00	0.20638E-00	0.23617E-00	0.
0.23617E-00	0.27600E-00	0.31212E-00	0.18085E-00	0.20638E-00	0.
0.20213E-00	0.27600E-00	0.25517E-00	0.21277E-00	0.25957E-00	0.
0.20638E-00	0.25319E-00	0.22766E-00	0.23617E-00	0.21064E-00	0.
0.24255E-00	0.21489E-00	0.25957E-00	0.21277E-00	0.23617E-00	0.
0.19574E-00	0.25957E-00	0.27872E-00	0.21277E-00	0.20638E-00	0.
0.21915E-00	0.25106E-00	0.26809E-00	0.24043E-00	0.21702E-00	0.
0.17447E-00	0.21277E-00	0.21702E-00	0.29149E-00	0.21489E-00	0.
0.19574E-00	0.18723E-00	0.24255E-00	0.34043E-00	0.25532E-00	0.
0.20851E-00	0.21064E-00	0.22553E-00	0.24894E-00	0.22553E-00	0.
0.21915E-00	0.24255E-00	0.22128E-00	0.22340E-00	0.24681E-00	0.
0.20426E-00	0.22340E-00	0.20213E-00	0.23404E-00	0.21915E-00	0.
0.21489E-00	0.24043E-00	0.25319E-00	0.22979E-00	0.21064E-00	0.
0.21702E-00	0.22128E-00	0.29362E-00	0.19362E-00	0.13191E-00	0.
0.18511E-00	0.16596E-00	0.29362E-00	0.22553E-00	0.20638E-00	0.44468E-00
0.19149E-00	0.21277E-00	0.24043E-00	0.17872E-00	0.23191E-00	0.99362E 00
0.18936E-00	0.21277E-00	0.25745E-00	0.21277E-00	0.23617E-00	0.99362E 00
0.21064E-00	0.22340E-00	0.22128E-00	0.20638E-00	0.25319E-00	0.99362E 00
0.34681E-00	0.21915E-00	0.24408E-00	0.20638E-00	0.30000E-00	0.99362E 00
0.23404E-00	0.28299E-00	0.23617E-00	0.22979E-00	0.33617E-00	0.99362E 00
0.216367E-00	0.40455E-00	0.23404E-00	0.27055E-00	0.27272E-00	0.99362E 00
0.23404E-00	0.24255E-00	0.20638E-00	0.19149E-00	0.24681E-00	0.
0.19149E-00	0.25532E-00	0.25106E-00	0.22766E-00	0.20213E-00	0.
0.20426E-00	0.25532E-00	0.26596E-00	0.15106E-00	0.22553E-00	0.
0.17021E-00	0.24255E-00	0.17872E-00	0.17447E-00	0.21915E-00	0.
0.20000E-00	0.22340E-00	0.21702E-00	0.15957E-00	0.20638E-00	0.
0.14489E-00	0.22340E-00	0.25319E-00	0.17872E-00	0.18511E-00	0.
0.19362E-00	0.25745E-00	0.22979E-00	0.18723E-00	0.22340E-00	0.
0.17660E-00	0.19149E-00	0.22553E-00	0.17362E-00	0.18936E-00	0.
0.14681E-00	0.22766E-00	0.19574E-00	0.28511E-00	0.20638E-00	0.
0.16596E-00	0.22766E-00	0.21064E-00	0.25745E-00	0.18723E-00	0.
0.16809E-00	0.20426E-00	0.20638E-00	0.22766E-00	0.20638E-00	0.
0.17447E-00	0.23191E-00	0.19149E-00	0.20000E-00	0.21702E-00	0.
0.17021E-00	0.21702E-00	0.19574E-00	0.20638E-00	0.22766E-00	0.
0.14681E-00	0.21277E-00	0.20638E-00	0.20000E-00	0.18299E-00	0.
0.14255E-00	0.19787E-00	0.21277E-00	0.18511E-00	0.19149E-00	0.
0.14681E-00	0.21915E-00	0.22340E-00	0.19362E-00	0.18511E-00	0.32979E-00
0.14408E-00	0.18723E-00	0.18299E-00	0.18299E-00	0.20638E-00	0.99574E 00
0.14255E-00	0.18936E-00	0.21489E-00	0.15957E-00	0.20851E-00	0.99574E 00
0.17447E-00	0.21702E-00	0.20000E-00	0.20213E-00	0.22766E-00	0.99574E 00
0.22340E-00	0.30426E-00	0.19149E-00	0.20851E-00	0.28085E-00	0.99574E 00
0.23427E-00	0.28633E-00	0.18655E-00	0.22560E-00	0.28850E-00	0.99566E 00
0.14894E-00	0.19362E-00	0.16596E-00	0.16596E-00	0.21064E-00	0.
0.16809E-00	0.18511E-00	0.17231E-00	0.14894E-00	0.17234E-00	0.
0.15745E-00	0.18170E-00	0.21489E-00	0.21489E-00	0.17234E-00	0.
0.14681E-00	0.17872E-00	0.18511E-00	0.16383E-00	0.17234E-00	0.
0.16596E-00	0.18085E-00	0.14894E-00	0.14681E-00	0.16170E-00	0.
0.12553E-00	0.17872E-00	0.14043E-00	0.17085E-00	0.16809E-00	0.
0.15532E-00	0.19574E-00	0.17600E-00	0.11915E-00	0.18511E-00	0.
0.10213E-00	0.18511E-00	0.16383E-00	0.15532E-00	0.15319E-00	0.
0.14255E-00	0.18511E-00	0.17447E-00	0.16085E-00	0.15957E-00	0.
0.11915E-00	0.17872E-00	0.18299E-00	0.21702E-00	0.14894E-00	0.
0.12979E-00	0.15957E-00	0.15106E-00	0.21702E-00	0.17447E-00	0.
0.14681E-00	0.16809E-00	0.14255E-00	0.14085E-00	0.20638E-00	0.
0.12979E-00	0.17872E-00	0.11489E-00	0.13617E-00	0.14894E-00	0.
0.14408E-00	0.18511E-00	0.15106E-00	0.16383E-00	0.18085E-00	0.
0.14043E-00	0.16596E-00	0.15319E-00	0.13404E-00	0.15957E-00	0.
0.10213E-00	0.15745E-00	0.16383E-00	0.14468E-00	0.14681E-00	0.
0.13617E-00	0.13617E-00	0.15745E-00	0.14255E-00	0.17600E-00	0.44468E-00
0.12553E-00	0.17872E-00	0.16170E-00	0.10426E-00	0.17447E-00	0.99936E 00
0.10638E-00	0.14408E-00	0.14681E-00	0.12340E-00	0.15532E-00	0.98936E 00
0.15957E-00	0.14894E-00	0.17872E-00	0.12128E-00	0.17600E-00	0.98936E 00
0.19787E-00	0.17872E-00	0.15745E-00	0.15319E-00	0.23404E-00	0.98936E 00
0.18085E-00	0.20638E-00	0.15319E-00	0.14255E-00	0.27600E-00	0.98936E 00
0.14932E-00	0.20814E-00	0.16299E-00	0.13575E-00	0.26018E-00	0.9869E 00
0.92979E-01	0.11915E-00	0.11277E-00	0.76596E-01	0.10638E-00	0.
0.76596E-01	0.11277E-00	0.00000E-01	0.16596E-01	0.10638E-00	0.
0.95745E-01	0.11489E-00	0.11915E-00	0.76596E-01	0.80951E-01	0.
0.11277E-00	0.19149E-00	0.10851E-00	0.89362E-01	0.11489E-00	0.
0.74408E-01	0.14043E-00	0.93617E-01	0.91489E-01	0.10638E-00	0.
0.10426E-00	0.13191E-00	0.11277E-00	0.82979E-01	0.10213E-00	0.
0.11064E-00	0.13191E-00	0.11489E-00	0.89362E-01	0.10213E-00	0.
0.10426E-00	0.15106E-00	0.13404E-00	0.83830E-01	0.11064E-00	0.
0.82979E-01	0.11915E-00	0.11702E-00	0.80851E-01	0.78723E-01	0.
0.00000E-01	0.10851E-00	0.85106E-01	0.11064E-00	0.85106E-01	0.
0.89362E-01	0.11277E-00	0.00000E-01	0.10426E-00	0.77872E-01	0.
0.93617E-01	0.12979E-00	0.10213E-00	0.63830E-01	0.13191E-00	0.
0.85106E-01	0.14043E-00	0.10851E-00	0.85106E-01	0.97872E-01	0.
0.11277E-00	0.14894E-00	0.12766E-00	0.86936E-01	0.12766E-00	0.
0.74408E-01	0.14894E-00	0.12553E-00	0.77872E-01	0.00000E-01	0.
0.97872E-01	0.00000E-01	0.14255E-00	0.63830E-01	0.89362E-01	0.
0.82979E-01	0.11277E-00	0.11702E-00	0.76596E-01	0.95745E-01	0.34468E-00
0.80851E-01	0.93617E-01	0.12979E-00	0.82979E-01	0.87234E-01	0.99362E 00
0.10426E-00	0.12340E-00	0.11489E-00	0.72340E-01	0.10851E-00	0.99362E 00
0.74408E-01	0.89362E-01	0.12128E-00	0.63830E-01	0.13617E-00	0.99362E 00
0.93617E-01	0.11915E-00	0.10426E-00	0.14255E-00	0.14255E-00	0.99362E 00
0.87234E-01	0.10213E-00	0.95745E-01	0.57447E-01	0.12128E-00	0.99149E 00
0.97614E-01	0.15618E-00	0.10412E-00	0.71584E-01	0.14317E-00	0.99349E 00

TABLE XII. Transient Data (Contd.)

RUN 20						
Z*	3.6	5.8	10.2	18.2	32.0	REF.
0.00	0.24043E-00	0.35104E-00	0.34681E-00	0.12120E-00	0.12760E-00	U.
	0.26383E-00	0.34043E-00	0.37234E-00	0.14255E-00	0.13617E-00	0.
	0.22766E-00	0.31702E-00	0.34681E-00	0.13404E-00	0.12553E-00	0.
	0.22979E-00	0.28936E-00	0.34681E-00	0.12034E-00	0.16809E-00	0.
	0.22340E-00	0.28936E-00	0.34043E-00	0.16596E-00	0.12128E-00	0.
	0.21702E-00	0.23617E-00	0.24448E-00	0.14043E-00	0.14468E-00	0.
	0.20426E-00	0.26596E-00	0.25532E-00	0.16809E-00	0.14468E-00	0.
	0.20426E-00	0.26170E-00	0.30213E-00	0.15957E-00	0.11277E-00	0.
	0.19574E-00	0.27234E-00	0.28298E-00	0.15106E-00	0.15319E-00	0.
	0.19362E-00	0.27660E-00	0.29574E-00	0.13617E-00	0.17447E-00	0.
	0.23191E-00	0.20005E-00	0.29787E-00	0.13191E-00	0.18085E-00	U.
	0.16170E-00	0.20213E-00	0.27660E-00	0.13404E-00	0.18298E-00	0.
	0.16170E-00	0.21064E-00	0.20426E-00	0.15532E-00	0.15106E-00	0.
	0.17872E-00	0.22766E-00	0.21702E-00	0.16809E-00	0.14681E-00	0.
	0.17234E-00	0.22340E-00	0.22766E-00	0.15745E-00	0.18298E-00	0.
	0.20000E-00	0.23830E-00	0.23404E-00	0.12760E-00	0.15745E-00	0.
	0.19936E-00	0.24681E-00	0.24448E-00	0.15319E-00	0.18936E-00	0.34255E-00
	0.21489E-00	0.28298E-00	0.27660E-00	0.65957E-01	0.20851E-00	0.10000E 01
	0.22128E-00	0.24255E-00	0.27021E-00	0.12340E-00	0.20851E-00	0.10000E 01
	0.20638E-00	0.24894E-00	0.27872E-00	0.10426E-00	0.16170E-00	0.10000E 01
	0.22979E-00	0.24255E-00	0.28085E-00	0.11489E-00	0.17234E-00	0.10000E 01
	0.26809E-00	0.29787E-00	0.29362E-00	0.11915E-00	0.16170E-00	0.10000E 01
	0.25957E-00	0.32553E-00	0.31064E-00	0.10213E-00	0.17234E-00	0.10000E 01
0.27	0.22553E-00	0.31277E-00	0.27660E-00	0.70213E-01	0.13617E-00	0.
	0.25319E-00	0.33617E-00	0.33830E-00	0.65957E-01	0.17447E-00	0.
	0.21064E-00	0.31702E-00	0.27660E-00	0.55106E-01	0.18936E-00	0.
	0.18723E-00	0.25106E-00	0.27234E-00	0.78723E-01	0.16389E-00	0.
	0.17872E-00	0.25957E-00	0.30851E-00	0.10851E-00	0.14681E-00	0.
	0.19574E-00	0.28723E-00	0.26809E-00	0.10000E-01	0.11915E-00	0.
	0.21702E-00	0.26383E-00	0.22766E-00	0.11489E-00	0.14894E-00	0.
	0.17660E-00	0.24448E-00	0.23191E-00	0.11915E-00	0.20638E-00	0.
	0.20213E-00	0.24894E-00	0.24681E-00	0.11489E-00	0.21064E-00	0.
	0.15319E-00	0.25106E-00	0.25532E-00	0.10851E-00	0.18511E-00	0.
	0.17660E-00	0.21915E-00	0.27234E-00	0.78723E-01	0.19149E-00	0.
	0.12979E-00	0.21277E-00	0.21277E-00	0.97872E-01	0.21489E-00	0.
	0.17234E-00	0.24255E-00	0.23191E-00	0.93617E-01	0.23404E-00	0.
	0.15745E-00	0.23617E-00	0.23830E-00	0.11277E-00	0.24255E-00	0.
	0.17234E-00	0.23404E-00	0.20426E-00	0.60851E-01	0.20426E-00	0.
	0.16170E-00	0.21915E-00	0.21489E-00	0.76596E-01	0.19149E-00	0.
	0.16809E-00	0.24681E-00	0.21277E-00	0.63830E-01	0.22128E-00	0.29574E-00
	0.15957E-00	0.21277E-00	0.19787E-00	0.68085E-01	0.22340E-00	0.10000E 01
	0.17660E-00	0.24894E-00	0.20426E-00	0.68085E-01	0.20638E-00	0.99787E 00
	0.21064E-00	0.28298E-00	0.22128E-00	0.82979E-01	0.18936E-00	0.99787E 00
	0.23617E-00	0.27447E-00	0.21915E-00	0.87234E-01	0.20638E-00	0.99787E 00
	0.25532E-00	0.31064E-00	0.31021E-00	0.80851E-01	0.18936E-00	0.99787E 00
	0.24468E-00	0.33830E-00	0.32128E-00	0.97872E-01	0.19574E-00	0.99787E 00
0.45	0.23191E-00	0.30851E-00	0.24681E-00	0.63830E-01	0.13404E-00	0.
	0.21702E-00	0.30851E-00	0.24681E-00	0.59574E-01	0.17447E-00	0.
	0.18936E-00	0.26383E-00	0.22128E-00	0.53191E-01	0.17660E-00	U.
	0.14681E-00	0.25745E-00	0.22553E-00	0.78723E-01	0.14468E-00	0.
	0.14043E-00	0.30213E-00	0.23404E-00	0.72340E-01	0.20000E-01	0.
	0.12760E-00	0.19149E-00	0.22553E-00	0.97872E-01	0.14468E-00	0.
	0.14043E-00	0.20213E-00	0.21915E-00	0.10000E-01	0.12340E-00	0.
	0.15106E-00	0.20213E-00	0.18298E-00	0.11915E-00	0.13630E-00	0.
	0.14894E-00	0.20851E-00	0.21489E-00	0.12340E-00	0.14681E-00	0.
	0.13930E-00	0.20426E-00	0.18936E-00	0.11489E-00	0.15106E-00	0.
	0.14255E-00	0.21489E-00	0.18085E-00	0.11277E-00	0.15745E-00	0.
	0.12553E-00	0.19362E-00	0.17447E-00	0.97872E-01	0.14468E-00	0.
	0.13191E-00	0.19149E-00	0.19149E-00	0.10426E-00	0.17021E-00	0.
	0.13404E-00	0.19574E-00	0.18298E-00	0.89362E-01	0.20426E-00	0.
	0.12979E-00	0.18936E-00	0.14894E-00	0.53191E-01	0.19574E-00	0.
	0.13191E-00	0.19149E-00	0.19149E-00	0.55319E-01	0.17660E-00	0.28511E-00
	0.11064E-00	0.14255E-00	0.12760E-00	0.48936E-01	0.21277E-00	0.10000E 01
	0.14468E-00	0.17447E-00	0.16170E-00	0.68085E-01	0.24043E-00	0.10000E 01
	0.17447E-00	0.22340E-00	0.15745E-00	0.57447E-01	0.16596E-00	0.10000E 01
	0.21277E-00	0.23404E-00	0.20426E-00	0.55319E-01	0.14043E-00	0.10000E 01
	0.21489E-00	0.30213E-00	0.19362E-00	0.57447E-01	0.10085E-00	0.10000E 01
	0.21277E-00	0.27872E-00	0.22340E-00	0.65957E-01	0.12979E-00	0.10000E 01
	0.63	0.18337E-00	0.26652E-00	0.24307E-00	0.83156E-01	0.11940E-00
0.16844E-00		0.23241E-00	0.23028E-00	0.76759E-01	0.12367E-00	0.
0.16205E-00		0.27505E-00	0.23241E-00	0.95949E-01	0.14925E-00	0.
0.15564E-00		0.19520E-00	0.22034E-00	0.11277E-00	0.20235E-00	0.
0.13646E-00		0.19829E-00	0.22175E-00	0.14072E-00	0.13006E-00	0.
0.13066E-00		0.20043E-00	0.24094E-00	0.17697E-00	0.12580E-00	0.
0.10874E-00		0.17213E-00	0.23028E-00	0.17494E-00	0.12367E-00	0.
0.13433E-00		0.17058E-00	0.21109E-00	0.17910E-00	0.14499E-00	0.
0.10874E-00		0.17058E-00	0.16631E-00	0.16631E-00	0.12793E-00	0.
0.10641E-00		0.17697E-00	0.17213E-00	0.23454E-00	0.14713E-00	0.
0.85288E-01		0.13433E-00	0.12793E-00	0.10337E-00	0.11727E-00	0.
0.78891E-01		0.17697E-00	0.13859E-00	0.13646E-00	0.14286E-00	0.
0.10021E-00		0.13646E-00	0.15091E-00	0.12580E-00	0.18124E-00	0.
0.13433E-00		0.18530E-00	0.13220E-00	0.12154E-00	0.18124E-00	0.
0.14072E-00		0.17697E-00	0.13646E-00	0.10661E-00	0.25373E-00	0.
0.10021E-00		0.13646E-00	0.14499E-00	0.11940E-00	0.21748E-00	0.
0.15332E-00		0.18530E-00	0.16631E-00	0.10448E-00	0.25160E-00	0.30917E-00
0.18550E-00		0.19616E-00	0.16205E-00	0.98081E-01	0.22175E-00	0.99574E 00
0.18550E-00		0.25160E-00	0.22388E-00	0.91684E-01	0.25373E-00	0.99574E 00
0.22222E-00		0.25641E-00	0.19072E-00	0.15212E-00	0.98574E-01	0.99574E 00
				0.11514E-01	0.16844E-00	0.99574E 00
				0.98081E-01	0.15139E-00	0.99360E 00
				0.11966E-00	0.13679E-00	0.99360E 00
0.81	0.18337E-00	0.26652E-00	0.24307E-00	0.83156E-01	0.11940E-00	0.
	0.16844E-00	0.23241E-00	0.23028E-00	0.76759E-01	0.12367E-00	0.
	0.16205E-00	0.27505E-00	0.23241E-00	0.95949E-01	0.14925E-00	0.
	0.15564E-00	0.19520E-00	0.22034E-00	0.11277E-00	0.20235E-00	0.
	0.13646E-00	0.19829E-00	0.22175E-00	0.14072E-00	0.13006E-00	0.
	0.13066E-00	0.20043E-00	0.24094E-00	0.17697E-00	0.12580E-00	0.
	0.10874E-00	0.17213E-00	0.23028E-00	0.17494E-00	0.12367E-00	0.
	0.13433E-00	0.17058E-00	0.21109E-00	0.17910E-00	0.14499E-00	0.
	0.10874E-00	0.17058E-00	0.16631E-00	0.16631E-00	0.12793E-00	0.
	0.10641E-00	0.17697E-00	0.17213E-00	0.23454E-00	0.14713E-00	0.
	0.85288E-01	0.13433E-00	0.12793E-00	0.10337E-00	0.11727E-00	0.
	0.78891E-01	0.17697E-00	0.13859E-00	0.13646E-00	0.14286E-00	0.
	0.10021E-00	0.13646E-00	0.15091E-00	0.12580E-00	0.18124E-00	0.
	0.13433E-00	0.18530E-00	0.13220E-00	0.12154E-00	0.18124E-00	0.
	0.14072E-00	0.17697E-00	0.13646E-00	0.10661E-00	0.25373E-00	0.
	0.10021E-00	0.13646E-00	0.14499E-00	0.11940E-00	0.21748E-00	0.
	0.15332E-00	0.18530E-00	0.16631E-00	0.10448E-00	0.25160E-00	0.30917E-00
	0.18550E-00	0.19616E-00	0.16205E-00	0.98081E-01	0.22175E-00	0.99574E 00
	0.18550E-00	0.25160E-00	0.22388E-00	0.91684E-01	0.25373E-00	0.99574E 00
0.22222E-00	0.25641E-00	0.19072E-00	0.15212E-00	0.98574E-01	0.99574E 00	

APPENDIX F

Cross-sectional Average Perturbations

Run	z^*	$t^* = t/t_s$	$\langle \alpha \rangle$	Run	z^*	$t^* = t/t_s$	$\langle \alpha \rangle$
1	5.8	0	0.0058	5	3.6	0	0.0089
		1	0.0045			1	0.0508
		2	0.0158			2	0.0265
		3	0.0361			3	0.0273
		4	0.0186			4	0.0220
		5	0.0138			5	0.0084
		6	0.0084				
		7	0.0041				
	32.0	0	0.0038		32.0	0	0.0192
		1	0.0147			1	0.0416
		2	0.0103			2	0.0267
		3	0.0072			3	0.0031
		4	0.0107			4	0.0142
		5	0.0045			5	0.0157
		6	0.0046			6	0.0114
		7	0.0056			7	0.0136
2	5.8	8	0.0038	6	5.8	8	0.0150
		0	0.0178			0	0
		1	0.0311			1	0.134
		2	0.0333			2	0.0357
		3	0.0316			3	0.0247
		4	0.0177			4	0.0225
		5	0.0087			5	0.0158
						6	0.0058
	32.0	0	0.0089	7	5.8	0	0.0113
		1	0.0372			1	0.0158
		2	0.0552			2	0.0595
		3	0.0398			3	0.160
		4	0.0321			4	0.0553
		5	0.0206			5	0.0393
		6	0.0084			6	0.0613
3	5.8	0	0.0151	8	3.6	0	0.0153
		1	0.0702			1	0.0301
		2	0.0297			2	0.0442
						3	0.0513
						4	0.0547
						5	0.0680
						6	0.0689
						7	0.0561
	32.0	0	0.0056				
		1	0.0156				
		2	0.0104				
		3	0.0057				
		4	0.0100				
4	5.8	0	0.0056				
		1	0.0156				
		2	0.0104				
		3	0.0057				
		4	0.0100				

Run	z^*	$t^* = t/t_s$	$\langle \alpha \rangle$	Run	z^*	$t^* = t/t_s$	$\langle \alpha \rangle$
9	5.8	0	0.0361	12	18.9	3	0.142
		1	0.111			4	0.109
		2	0.103			5	0.0826
		3	0.117				
		4	0.121			0	0.0688
		5	0.129			1	0.138
		6	0.110			2	0.111
		7	0.107			3	0.124
		8	0.111			4	0.130
		9	0.0833			5	0.132
		10	0.0929			6	0.137
		11	0.0752			7	0.0898
		12	0.0479			8	0.0849
		13	0.0337				
	32.0	0	0.0643		18.9	0	0.099
		1	0.0835			1	0.112
		2	0.149			2	0.162
		3	0.0839			3	0.181
		4	0.0796			4	0.146
		5	0.0888			5	0.104
		6	0.0496			6	0.116
						7	0.125
10	5.8	0	0.371	14	5.8	0	0.0793
		1	0.0707			1	0.0885
		2	0.0812			2	0.0953
		3	0.0843			3	0.0938
		4	0.0791			4	0.113
		5	0.0703			5	0.122
		6	0.0620			6	0.113
11	3.6	0	0.0680	15	3.6	0	0.0404
		1	0.129			1	0.0910
		2	0.118			2	0.130
		3	0.0940			3	0.125
		4	0.0845			4	0.106
12	3.6	0	0.0621			5	0.123
		1	0.0660			6	0.129
		2	0.110			7	0.102
		3	0.0843			8	0.0786
	18.9	0	0.0870		32.0	0	0.112
		1	0.106			1	0.156
		2	0.127			2	0.246
						3	0.193

Run	z^*	$t^* = t/t_s$	$\langle \alpha \rangle$	Run	z^*	$t^* = t/t_s$	$\langle \alpha \rangle$
15	32.0	4	0.112	17	32.0	5	0.211
		5	0.118			6	0.181
16	3.6	0	0.0375			7	0.190
		1	0.0688			8	0.204
		2	0.0835			9	0.147
		3	0.0781	18	5.8	0	0.126
		4	0.0960			1	0.153
		5	0.0870			2	0.184
		6	0.0950			3	0.184
17	3.6	0	0.0730			4	0.154
		1	0.121			5	0.151
		2	0.127	19	5.8	0	0.150
		3	0.135			1	0.182
		4	0.158			2	0.193
		5	0.159			3	0.214
		6	0.158			4	0.192
		7	0.124			5	0.189
		8	0.0950	20	5.8	0	0.200
		9	0.0827			1	0.250
		10	0.0765			2	0.258
	32.0	0	0.128			3	0.266
		1	0.157			4	0.247
		2	0.184			5	0.249
		3	0.178			6	0.218
		4	0.188				

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