Abstract

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GAMMA-RAY SPECTROSCOPY ON THE $N = 82$ NUCLEAR ISOTONES: ^{139}Ba , ^{140}Ce , ^{141}Pr , AND ^{142}Nd

By Helmut W. Baer

Gamma radiation from the $N=82$ nuclei ^{139}La , ^{140}Ce , ^{141}Pr , and ^{142}Nd was investigated using the techniques of gamma-ray spectroscopy. Gamma-ray energies and intensities were measured with a 2-m curved-crystal spectrometer and Ge(Li) detectors. Gamma-ray decay sequences were determined by γ - γ coincidence measurements. The following results were obtained.

The energy values 165.854 ± 0.012 , 145.4498 ± 0.0049 , and 1575.42 ± 0.39 keV were determined for the excitation energies of the first excited states in ^{139}La , ^{141}Pr , and ^{142}Nd , respectively. A total of 26 gamma-ray energies and intensities were measured in the decay of ^{140}La to levels in ^{140}Ce . A decay scheme is presented which includes 15 excited levels and 28 γ -transitions. Twelve excited levels were found to be populated by β^- -decay; logft values were determined using partial β -transition probabilities and end-point energies. The multipole character of most of the γ -transitions could be determined when the present data was combined with recently published conversion electron data. The spins and parities of nearly all levels in ^{140}Ce populated via the decay of ^{140}La were determined. Of particular interest are the discovery of a 5- level at 2350.2 keV and an isomeric, 6+ level at 2108.2 keV; they are the first states with these J^π values identified in $N=82$ nuclei.

A compilation and discussion of the properties of $N=82$ nuclei is presented; evidence for special structure at $N=82$ is reviewed. The excitation spectra are discussed in the framework of the quasi-particle description of pairing correlations in spherical nuclei. Future experiments using the techniques of radioactive decay and nuclear reactions are proposed.

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^{139}Ba , ^{140}Ce , ^{141}Pr , AND ^{142}Nd

by

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Dedicated

to my parents

who gave years of encouragement

and

to my wife

who gave unremitting cheerful assistance.

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CHAPTER I

INTRODUCTION TO THE PROBLEM

This work was undertaken to further the understanding of nuclei with the magic neutron number $N=82$. An appreciation for the type of nuclear structure problem involved in these nuclei can be gained by examining the spectra of two typical nuclei of this group. Portions of the low-energy excitation spectra of Ce-140 and La-139 are shown below.

In the elementary shell model description of the La-139 nuclear ground state, the 82 neutrons and 50 of 57 protons fill up major shells. Of the 7 remaining protons, 6 couple pairwise to $J=0$ in the $1g_{7/2}$ orbital. The one remaining proton occupies a $1g_{7/2}$ orbital, thereby giving the entire nucleus in its lowest energy state an angular momentum and parity $7/2+$. The simplest excitation of this system corresponds to the promotion of the last proton to the next higher shell model orbital, viz., $2d_{5/2}$. This simple model therefore seems to contain elements of a proper description of the ground- and first excited state of La-139. If an 8th proton is added to this nucleus, the model predicts that the 7th and 8th proton will couple pairwise to $J=0$ and this

corresponds to the ground state of Ce-140. The simplest excitation of this system would be obtained by either promoting a pair of protons to the $2d_{5/2}$ orbital or by breaking a $(g_{7/2})^2$, $J=0$ pair and promoting one proton to the $2d_{5/2}$ orbital. We would expect, therefore, to find an excited state in Ce-140 somewhere between 0.17 and 0.34 MeV. However, the first excited state in Ce-140 occurs at 1.6 MeV. It is difficult to see why an energy of 1.6 MeV is required to produce the lowest possible excitation of this system. Furthermore, it is difficult to interpret the clustering of levels above 1.6 MeV in terms of other models which take into account the rotational or vibrational motion of the entire system.

The idea of a pairing force which favored the coupling of like particles pairwise to $J=0$ was already present in the early shell model of Mayer²⁾ and Haxel, Jensen and Suess³⁾. However, the full appreciation of the role of pairing in producing a high degree of correlation in nuclear motion, and the theoretical methods for dealing with this correlation, appear to have come only after the theory of superconductivity of metals was developed by Bardeen, Cooper, and Schrieffer⁵⁾ in 1957. The possible analogy between the energy gap in the electronic excitation of a superconducting metal and the energy gap found in even-even nuclei (such as the 1.6 MeV gap in ^{140}Ce) was first suggested by Bohr, Mottelson, and Pines⁶⁾ in 1958. The systematic exploitation of these ideas was started by Belyaev⁷⁾ in 1959. He found that the introduction of a great deal of correlation in the nucleon motion through the very short range pairing interaction (described mathematically by use of the quasiparticle transformation) leads, indeed, to an energy gap in the excitation spectra of even-even nuclei. Shortly after this, Kisslinger and Sorensen⁸⁾ performed extensive numerical calculations using the new theory and found remarkably good agreement with experiment for single-closed-shell (scs) nuclei throughout the mass table.

Both of these theoretical investigations made a distinction between two types of nuclear excitations, viz., in-

trinsic excitations of independent quasiparticles and collective quadrupole vibrations characterized by a "phenomenological" parameter. A conceptually pleasing unification was achieved by Baranger⁹⁾ in 1960 when he developed a theoretical description of heavy spherical nuclei (method of linearized equations of motion) in which single particle excitations and collective vibrations were described by the same equations, and further, the two types of excitations were orthogonal to each other.

The evaluation of this description and its various extensions by detailed comparisons of calculated spectra with observed spectra is currently of interest in nuclear physics. Single-closed-shell nuclei are especially suited for a comparison of this theory with experiment since the theory is more satisfactory when only the residual interaction between like nucleons must be included. In the N=82 isotones, it is expected that a reasonably good description of the low-energy spectra may be obtained by taking into account only the coordinates and two-body residual interactions of the protons outside the Z=50 core. Detailed calculations using the quasiparticle description were recently performed^{1,10)}.

The data on the N=82 nuclei have been sparse. It was therefore considered desirable to initiate an experimental investigation of these nuclei. The present work is a study of gamma rays corresponding to transitions between levels in N=82 nuclei which follow the β^- -decay of N=83 nuclei. We have performed gamma-ray energy and relative intensity measurements in the following four decays: $^{139}\text{Ba} \rightarrow ^{139}\text{La}$, $^{140}\text{La} \rightarrow ^{140}\text{Ce}$, $^{141}\text{Ce} \rightarrow ^{141}\text{Pr}$, $^{142}\text{Pr} \rightarrow ^{142}\text{Nd}$. Of these, the investigation of the ^{140}La decay was most fruitful. A total of 26 gamma-ray energies and intensities were determined and gamma-gamma coincidence studies were used to determine gamma-decay sequences. Using these results it was possible to construct a level scheme for ^{140}Ce which included 28 transitions and 15 excited levels. The multipole character of most of the transitions could be determined when the present data was combined with that of other

investigators. Moreover, spin and parity values could be uniquely assigned to most of the 15 excited levels. The level at 2108.2 keV (6+) was deduced to be isomeric. The presentation and interpretation of results is given in Chapter III.

Many of the energy and relative intensity measurements were performed using the University of Michigan's 2 meter curved-crystal spectrometer. The quartz crystal with diffraction planes, 310, and the Ge crystal with diffraction planes, 022, were used. Energy and intensity measurements were also performed using planar and coaxial Ge(Li) detectors having depleted volumes in the range 0.16 - 17.0 cm³. The gamma-gamma coincidence studies were performed using Ge(Li) - NaI and Ge(Li) - Ge(Li) detector arrangements. The experimental details are discussed in Chapter II and Appendices A and B.

Our studies have shed some light on the structure problem of N=82 nuclei. The discovery of the low-lying negative parity state 5- and the high J value, positive parity state 6+ in ¹⁴⁰Ce are the first of this type of state found in N=82 nuclei. Theoretical descriptions must therefore include nucleon configurations not considered in earlier calculations. On the other hand, some previously unobserved but predicted levels have now been found. Moreover, the observation of some anomalous gamma-decay branching ratios puts strict requirements on theoretical wave functions for certain states in ¹⁴⁰Ce. An extensive discussion of the structure problem for single-closed shell nuclei is given in Chapter IV. The nucleus ¹⁴⁰Ce is discussed at the end of Chapter III.

A compilation and discussion of the properties of N=82 nuclei is given in Chapter V. Evidence for special structure at N=82 is presented. The chapter ends with a discussion of possible future investigations which would further elucidate the structure of this interesting family of nuclei.

CHAPTER II

EXPERIMENTAL PROCEDURES

A. The Curved-Crystal Spectrometer

The determination of the wavelength (energy) of a gamma ray with a crystal diffraction spectrometer is based on the Bragg relation between the wavelength λ , inter-planar spacing of the crystal lamina d , and the Bragg angle θ . The Bragg equation is

$$n\lambda = 2d\sin\theta \quad , \quad (2.1)$$

where n denotes the order of diffraction. This equation relates the wavelength outside the crystal to the external diffraction angle θ (the angle θ corresponds to the peak intensity of the diffraction peak) to a precision of a few parts in 10^6 for the case of diffraction by transmission through a crystal lamina with diffraction planes perpendicular to the incident surface¹¹).

The energy measurements performed with the UM curved-crystal spectrometer involve a comparison of gamma-ray energies with the energy of the 411.800 keV gamma-ray following the decay of ^{198}Au . This obviates the necessity of determining the inter-planar spacing d . The relative relationships are

$$\frac{E_1}{E_2} = \frac{\lambda_2}{\lambda_1} = \frac{\sin\theta_2/n_2}{\sin\theta_1/n_1} = \frac{X_2/n_2}{X_1/n_1} \quad , \quad (2.2)$$

where E_1 and E_2 are the energies of two gamma rays having wavelengths λ_1 and λ_2 respectively, and n_1 , n_2 are the corresponding orders of diffraction. The quantities X_1 , X_2 (in screw-divisions (s.d.)) correspond to the positions of the two diffraction peaks along the linear screw which is part of a sine-screw mechanism. Through the use of a sine-screw, a simple, inverse relationship between gamma-ray energies and peak positions is obtained. By writing Eq.(2.2) in the form

$$E_1(X_1/n_1) = E_2(X_2/n_2) = k, \quad (2.3)$$

it is apparent that a diffraction spectrometer may be calibrated by determining the first-order position X_2/n_2 of the diffraction peak of a gamma-ray having a well determined energy. The calibration constant k is related to particular crystal planes and a sine-screw by $k = \alpha(L/d)$ where L is the arm length of the sine screw and α is a constant ($\alpha = hc/2$).

We may derive the relationship between the energy and position differentials from Eq.(2.3). One obtains the equation

$$\Delta E = \frac{1}{k} \cdot E^2 \cdot \frac{\Delta X}{n} = \frac{1}{\alpha} \cdot \frac{d}{L} E^2 \cdot \frac{\Delta X}{n} \quad (2.4)$$

We see that for a crystal diffraction spectrometer the fractional resolution $\Delta E/E$ increases linearly with gamma-ray energy, decreases with increasing order, and decreases with decreasing inter-planar spacing.

The UM 2 meter curved-crystal spectrometer was used for measuring gamma-ray energies and relative intensities. A complete description of the design, operation, and calibration of this instrument is given in ref. ¹²⁾. Also described is the unique manner of construction of the flat, ribbon-like sources.

Measurements were made with the two crystals, Ge(022) and Q(310), having effective areas of 4cm^2 and 20cm^2 , respectively. After the start of the measurements, the Ge(022) crystal was recalibrated using the fourth-through-eighth order reflection of the 411.800 keV transition in the decay of ^{198}Au . The resulting calibration constant $7297.989 \pm 0.025\text{ keV} - \text{s.d.}$ is slightly smaller than the one previously determined¹²⁾. The calibration constant for the Q(310) crystal is $12369.34 \pm 0.50\text{ keV} - \text{s.d.}$ (For this spectrometer, 1.0 s.d. $\approx 88\text{ sec of arc}$, for angles less than 3° .)

The sources were composed of BaO -, La_2O_3 -, CeO -, and PrO_2 - epoxy mixtures, and had a width (smallest dimension) of $76\text{ }\mu\text{m}$ or $130\text{ }\mu\text{m}$ (3 mil or 5 mil). The maximum total activity after thermal neutron irradiation to $\geq 90\%$ saturation in the UM Ford Reactor ($\approx 2 \times 10^{13}\text{ n/cm}^2 \cdot \text{sec}$) was estimated to be approximately 1 Cur. The sources were aligned by rotating them in the source holder until a minimum diffraction peak line width was

achieved. The resolution widths (fwhm) of the diffraction profiles obtained with the Ge(022) planes were approximately 10 sec and 13.5 sec of arc for the two different source widths. These correspond to an energy resolution of $\Delta E = 16 \times 10^{-6} \times E^2 / n$ keV and $\Delta E = 20 \times 10^{-6} \times E^2 / n$ keV, respectively, where E is in keV and n the order of diffraction. The resolution width obtained with the Q(310) planes is 25-30 sec of arc for both source widths. Data were taken using the first-through-fifth order reflection from the Ge(022) planes and the first-order reflection from the Q(310) planes.

In addition to the gamma-ray energy and intensity measurements, an investigation of the diffraction peak profiles and crystal reflectivity of the Ge(022) crystal was performed. An effort was made to understand the variations in diffraction peak line shapes with order of diffraction observed for low-energy gamma-rays. The results of this study are reported in Appendix A. Some curves of the peak reflectivity function for gamma-rays in the energy range 0.87 - 1.60 MeV are also presented.

B. The Ge(Li) Spectrometer

In the span of this thesis work the Ge(Li) detector evolved from the embryonic stages to its present central position in gamma-ray spectroscopy. The first detector used (1964) in this investigation was a RCA planar detector with an 0.16 cm^3 active volume, mounted on a home-made copper cold-finger, and insulated with styrofoam. The last detector used (1967) was an ORTEC, 17 cm^3 , coaxial detector mounted in a commercial high-vacuum cryostat. The resolution widths (fwhm) of the two detectors plus associated electronics were approximately 9 and 7 keV, respectively, for a 2.5 MeV gamma-ray line.

The Ge(Li) spectrometer with which nearly all of the present measurements were obtained consisted of the following components: an ORTEC detector-cryostat with a depleted volume $4 \text{ cm}^2 \times 0.5 \text{ cm}$, biased + 400 volts; a Tennelec TC - 130 (FET)

Fig. 2.1. The source-detector arrangement used in the present work.

pre-amp and TC-200 amplifier; a Victoreen (SCIPP) 1600 channel pulse-height analyzer. The best resolution of this system at low count rates was obtained with a unipolar pulse, and by using a 1.6μ sec time constant in the 1st differentiator and integrator of the TC-200 amplifier. Some typical spectra and resolution widths are shown in figs. 3.2 and 3.3.

The source-detector arrangement used to obtain all the gamma-ray spectra is shown in fig. 2.1. The dimensions shown in the figure relating to the detector and absorbing layers were provided by the manufacturer (ORTEC). The source holders are lucite discs with a small depression for the source at the center. The source material was in all cases dissolved or suspended in a liquid, and was deposited with an eyedropper into the depression. The liquid was allowed to evaporate and the source was covered with Scotch tape. The source disc was placed in a lucite source tower rigidly attached to the detector-cryostat. Concentric source and detector alignment was achieved by use of cross hair on the source tower, source-disc holder, and the front face of the cryostat. The source could be placed as close as 1 cm from the detector.

The La source consisted of several mg of $\text{La}(\text{NO}_3)_3$ dissolved in water and irradiated with thermal neutrons for 2-8 h in the UM Reactor. The activities were in the range 0.1 - 1.0 m Cur.

Considerable effort went into the measurement of relative gamma-ray emission rates in a manner that would make best use of the high resolution available. The relative full-energy peak detection efficiency in the energy range 0.08 - 3.2 MeV was determined for two detectors using a pair-point method. A full description of the calibration is given in Appendix B.

C. The Coincidence Circuit

The coincidence circuit used with both Ge(Li) - Ge(Li) and Ge(Li) - NaI detectors is shown in block diagram form in fig. 2.2. The dashed lines show the circuit connections for the Ge(Li) (display) - NaI (gate) arrangement.

The fast coincidence timing was performed in the fast coincidence (FC) unit (CI - 1441) which has a variable resolving time $2\tau = 5$ to 100 nsec. The two timing pulses into the FC were obtained from two timing single channel analyzers (TSCA). The TSCA - 2 in the fast gate side was set in differential mode such that pulses at the input in the voltage range 5.0 - 5.5 volts resulted in a timing pulse to the FC. Cross-over timing was used in the TSCA. In the display side the TSCA was set in integral mode and pulses at the input in the range 0.2 - 10.0 volts resulted in timing pulses to the FC. The coincidence efficiency, with a resolving time of 50 nsec, was constant for pulses in these voltage ranges. A double-delay line amplifier (CI - 1410) was used in the display side so as to obtain sharp cross-over timing for pulses in this large voltage range. In the fast gate side the gain of the double differentiating amplifier was always adjusted to bring the gate pulses into the voltage region 5.0 to 5.5 volts.

In the Ge(Li) - Ge(Li) arrangement the FC output gate a biased amplifier (TC-250). Thus the output pulses of the biased amplifier corresponded to the pulses accepted by the fast gate (TSCA - 2) and satisfying the coincidence requirements. Sharp, narrow gates could then be set with two more TSCA. Timing

pulses to the slow coincidence (SC) unit (CI 1445) from either of these two TSCA resulted in a gate pulse to the analyzer. The SC in this arrangement was merely used to generate a gate pulse when a pulse appeared at one of the two separate inputs. Routing pulses to the analyzer were obtained from TSCA - 3 and TSCA - 4. Two 800 channel sub-groups were used.

In the Ge(Li) (display) - NaI (gate) arrangement the FC output was connected directly to the slow coincidence unit. The logic requirements in the SC were set such that the FC pulse plus one pulse from TSCA - 3 or TSCA - 4 were needed to obtain the analyzer gate pulse. Again, routing pulses were obtained from these two TSCA.

CHAPTER III

EXPERIMENTAL RESULTS AND INTERPRETATION OF DATA

A. The Decay of ^{140}La to Levels in ^{140}Ce

1. Experimental Results

1a. Gamma-ray energy and relative intensity values

Gamma-ray energy and relative intensity values for 26 gamma rays were determined using the ccs and Ge(Li) spectrometers. The results are given in table 3.1. The method of data analysis and some measurements of special interest are discussed below.

Curved-Crystal Spectrometer

The energy values of gamma rays measured with the 2m ccs are listed in column 1, table 3.1. The data were taken using first-through-fifth order diffraction from the Ge(022) planes and first-order diffraction from the Q(310) planes. The peak positions in screw-divisions (s.d.) were determined by the method of overlapping-profiles. The uncertainty in any one determination of a peak position includes a monochrometer error of 0.0033 s.d. as discussed in ref.¹²⁾. All energies were measured at least twice and in most cases three or four times. The uncertainties correspond to the standard deviation of the mean.

The relative intensities given in column 3, table 3.1 were measured with the css using the Ge(022) planes in second order reflection. The reflection efficiency of this crystal in second order, $R(E_\gamma, 2)$, has been investigated¹²⁾ in the energy range from 130 to 2110 keV and is reasonably described by $R(E_\gamma, 2) \propto E^{-k}$ with $0.6 \leq k \leq 1.2$. A value of $k=1.0$ was used for the present work. The relative intensities were obtained

Table 3.1

Energy and relative intensity values for the gamma rays in the $^{140}\text{La} + ^{140}\text{Ca}$ decay

Photon energy (keV)		Relative gamma ray intensity ^{b)}		
Curved-crystal ^{a)}	Ge(Li)	Curved-crystal ^{a)}	Ge(Li)	Curved-cryst + Ge(Li)
(1)	(2)	(3)	(4)	(5)
64.135 ± 0.010			<2	<0.01
	69.0 ± 0.3		10.7 ± 1.8	0.065 ± 0.013
109.418 ± 0.007			44.1 ± 2.3	0.27 ± 0.04
131.121 ± 0.008			100	0.61 ± 0.09
173.536 ± 0.012			21.5 ± 6.4	0.13 ± 0.05
241.966 ± 0.012			73.8 ± 5.5	0.45 ± 0.06
266.551 ± 0.014			92.1 ± 6.6	0.56 ± 0.06
306.9 ± 0.2		0.044 ± 0.023		0.022 ± 0.011
328.768 ± 0.012		45 ± 10	43.2 ± 3.0	21.4 ± 1.1
397.79 ± 0.11		0.11 ± 0.050		0.054 ± 0.025
432.530 ± 0.029		6.1 ± 1.5	6.29 ± 0.31	3.11 ± 0.16
487.029 ± 0.019		100	100	49.4 ± 2.5
618.2 ± 0.7		0.090 ± 0.045		0.044 ± 0.022
751.827 ± 0.080		19 ± 3.8	18.3 ± 0.6	4.40 ± 0.26
815.801 ± 0.086		100	100	24.1 ± 1.2
867.82 ± 0.14		22.5 ± 4.5	23.4 ± 0.7	5.64 ± 0.28
919.64 ± 0.33	919.5 ± 0.2	11.0 ± 2.7	11.3 ± 0.6	2.73 ± 0.16
925.20 ± 0.17	925.1 ± 0.2	30.0 ± 6.0	30.0 ± 6.0	7.24 ± 0.43
950.88 ± 0.72	951.1 ± 0.4	2.6 ± 0.65	2.33 ± 0.14	0.56 ± 0.05
1596.58 ± 0.30	1596.6 ± 0.2			100
	2348.8 ± 0.6		25.5 ± 0.8	0.901 ± 0.045
2522.6 ± 2.5	2522.2 ± 0.4		100	3.52 ± 0.18
	2548.6 ± 0.8		3.46 ± 0.17	0.122 ± 0.009
	2899.7 ± 0.5		2.00 ± 0.14	0.070 ± 0.005
	3118.3 ± 0.7		0.775 ± 0.008	0.027 ± 0.003
	3319.7 ± 2.5		0.22 ± 0.11	0.008 ± 0.004

a) The values in these columns were obtained with a curved-crystal spectrometer and are reported in ref. 39).

b) The γ -ray intensities in columns (3) and (4) are given relative to the most intense member within each subgroup. In column (5), the data of both instruments are combined and the intensities are given relative to the 1596.6 keV γ -ray. When multiplied by 0.955 these numbers correspond to % of total decay.

from the relative peak counting rates corrected for crystal reflection efficiency and photopeak detection efficiency. The uncertainties in intensity values are derived from the uncertainty in crystal reflection efficiency and the statistical fluctuations in the peak counting rates. The agreement of these intensity values with those obtained with the Ge(Li) detector is excellent. This suggests that the stated uncertainties in the intensity values derived from the css data are too large and that the reflection efficiency in the energy range 300-1000 keV is well represented by the relation $R(E_\gamma, 2) \propto E^{-1.0}$.

The following energy and intensity measurements are of special interest:

919.64 and 925.20 keV. Previous to this investigation these two transitions were reported as a single transition with an energy of approximately 923 keV. The diffraction profiles for this doublet obtained with two different source widths and dispersions are shown in fig. 3.1. Fig. 3.1b shows that these two lines are well separated in fourth-order diffraction from the Ge(022) planes using a source of 76 μm (3 mil) width.

618.2 keV. The energy measurement on this weak transition had to be made in first-order diffraction since the second-order diffraction profile contains a contribution of the relatively strong third-order diffraction of the 920-925 keV lines. For the intensity determination of this γ -ray, relative to the 487.0 keV γ -ray, the peak reflectivity curves (Appendix A) were used to relate the first-order peak count rate to the second-order peak count rate.

306.9 keV. This transition has not been reported previously in the decay of ^{140}La although it has been observed in the $^{140}\text{Pr} \rightarrow ^{140}\text{Ce}$ decay¹⁴⁾.

Even though the 306.9 keV transition is very weak, the measurement on this line necessitated using the second-order diffraction from the Ge(0 $\bar{2}2$) planes for the following reason. The first-order diffraction of the 306.9 keV γ -ray occurs at nearly the same Bragg angle as the third-order diffraction of the

Fig. 3.1. Diffraction profiles of the 919.6 - 925.2 keV gamma-ray doublet occurring in the decay $^{140}\text{La} \rightarrow ^{140}\text{Ce}$ obtained with two different source widths and dispersions. The solid line merely indicates the trend of the data points.

920-925 keV doublet. Thus the diffraction peak consists of three different γ -rays in various proportions of intensity. By setting the amplifier-discriminator to accept the photopeak of the 306.9 keV γ -ray, a portion of the Compton distributions due to the 920-925 keV γ -rays is also accepted. In the region of the first-order diffraction of the 306.9 keV line (and so the third-order diffraction of the 920-925 keV doublet) the relative intensities of these two contributions to the count rate is large enough to produce serious distortions to the diffraction profile of the 306.9 keV line. By measuring the 306.9 keV gamma ray in second order, this distortion is greatly reduced since the intensity of the 920-925 keV lines (sixth-order reflection now) is reduced by about a factor of ten, whereas the 306.9 keV line is only reduced in intensity by ≈ 2 . In this manner a good determination of the energy and relative intensity of this transition was possible, though it occurs in only $2.1 \times 10^{-2} \%$ of the decays.

Ge(Li) Spectrometer

The gamma-ray spectrum in the energy range 0.05 - 3.45 MeV obtained with a 2 cm³, planar, Ge(Li) detector is shown in fig. 3.2. Of the 26 gamma rays listed in table 3.1, peaks corresponding to 22 of these can be seen clearly in the spectrum. The presence of peaks corresponding to the weak 64.1, 306.9, 397.8, 618.2 keV gamma rays observed with the curved-crystal spectrometer is made plausible by the appearance of a few high points in the spectrum. (In the singles spectrum obtained with the 17 cm³ coaxial detector the photo-peaks corresponding to the 306.9, 397.8 and 618.2 keV gamma rays can definitely be seen). Enlarged views of several regions of the gamma-ray spectrum are shown in fig. 3.3. It is interesting to note that the separation of the 919.5 - 925.1 keV doublet into two separated peaks is approximately the same as that obtained with the curved-crystal spectrometer in fourth-order diffraction from the Ge(0 $\bar{2}$ 2) planes. Also, it is observed that the peak of the weak 2548.6 keV gamma ray is well separated from the much stronger 2522.2 keV gamma ray.

Fig. 3.2. The gamma-ray spectrum following the decay of ^{140}La in the energy range 0.05 - 3.45 MeV obtained with a 2cm³, planar, Ge(Li) detector.

Fig. 3.3. Portions of the gamma-ray spectrum following the decay of ^{140}La obtained with a 2cm^3 planar, Ge(Li) detector. The dashed lines illustrate the manner in which the peak area A was determined.

The relative emission rates of 21 gamma rays were measured using the 2 cm³, planar, Ge(Li) detector. This detector was calibrated for detection efficiency in the energy range 0.08 - 3.5 MeV using sources with well measured relative gamma-ray emission rates. The calibration method is described in detail in Appendix B. The method uses as a measure of the gamma-ray intensity the peak area A defined by the sum $A = \sum_i (N_i - B_i)$, where N_i are the actual counts in a channel and B_i are the background counts in that channel as defined by the straight line separating the peak from the total distribution. Several examples of the manner in which the line was drawn for peaks in the ¹⁴⁰Ce spectrum are shown in fig. 3.3.

The uncertainties in the relative intensity values (table 3.1) arise from two independent contributions. First, there is the uncertainty with which the relative area of two peaks in the spectrum can be determined. This uncertainty depends on the shape of the spectral distribution adjacent to each peak and on the number of counts in each peak. The area ratios for each pair of peaks were determined from a minimum of 5 different spectra, and rms fluctuations in these ratios were used to assign an uncertainty to each ratio. Second, there is the uncertainty in the relative detection efficiency for each pair of gamma rays (discussed in Appendix B). The magnitude of this uncertainty depends on the energy separation of the two lines. For this reason the uncertainties given in column four, table 3.1 are less than those given in column 5.

For the purpose of energy determinations, the centroid of the peak area A was taken as the definition of the peak position. Gamma-ray energies between 919.5 and 2548.6 keV were determined using the 487.03 and 815.80 keV gamma-ray lines in the spectrum for the calibration. The energies of the 1596.6 and 2522.2 keV gamma rays were determined from the double escape peaks. The three gamma-ray energies 2899.7, 3118.3, and 3319.7 keV were measured using the calibration energy $E_\gamma = 2753.92 \pm 0.12 \text{ keV}^{15)}$ for the gamma ray following the decay of ²⁴Na. The uncertainties in the energies are

based on the uncertainties in the peak positions and the measured nonlinearities in the amplifier-analyzer system¹⁶⁾. All energies were measured at least twice.

1b. Gamma-gamma coincidence spectra.

Coincidence spectra were obtained using various NaI-Ge(Li) and Ge(Li) - Ge(Li) detector arrangements. The detectors and the coincidence circuit are described in Chapter II. The singles counting rates in both the display and gate side were generally in the range $(0.1-2.0) \times 10^4$ counts/sec. The resolving time 2τ was approximately 50 ns for all prompt coincidence runs. The spectra in coincidence with most of the stronger gamma rays were obtained and the results are summarized below.

Gate on 1596.6 keV gamma ray. The γ -ray spectrum in the energy range 0.07 - 2.02 MeV in coincidence with the 1596.6 keV gamma ray is shown in fig. 3.4. The running time was 18 hours and the true to accidental ratio was greater than 20 to one. It may be noted that the double- and single escape peaks (DEP, SEP) of the 1596.6 keV gamma ray can not be seen in the coincidence spectrum. Gamma rays with the following energies have approximately the same relative peak areas in the coincidence spectrum as in the singles spectrum: 109.4, 256.6, 328.8, 432.5, 487.0, 751.8, 815.8, 867.8, 919.6, 925.2, 950.9 keV. The 131.1 keV peak is reduced approximately 50% in the coincidence spectrum (relative to the above lines), and a peak corresponding to the 242.0 keV gamma ray can not be seen at all.

Gate on 487.0 keV gamma ray. The γ -ray spectrum in the energy range 0.06-1.18 MeV in coincidence with the 487.0 keV gamma ray is shown in fig. 3.5. The running time was 13 hours and the true to accidental ratio was greater than 20 to one. The events coincident with the Compton distributions in gate A were subtracted using the spectrum coincident with gate B. The

small 487.0 keV peak in the coincidence spectrum is due to the presence of annihilation radiation in gate A.

Gamma rays with energies 266.6, 328.8, 397.8, 432.5 keV and the 1596.6 DEP have approximately the same relative intensity in coincidences with the 487.0 keV gamma ray as in singles. The 109.4 and 131.1 keV peaks are reduced approximately 50% relative to 266.6 keV peak. There is no evidence for the presence of gamma rays in prompt coincidence having the following energies: 242.0, 751.8, 815.8, 867.8, 919.6, 925.2, 950.9 keV.

The low-energy portion of the γ -ray spectrum in delayed coincidence (delay = 200 ns, 2τ = 100 ns) with the 487 keV gamma-ray is shown in fig. 3.6. The running time was 42 hours. The 131.1 and 242.0 keV gamma rays are seen to be in delayed coincidence. The value of the peak area ratio $A(131.1)/A(242.0)$ in the delayed spectrum is approximately one half of the value of this ratio in the singles spectrum.

Gate on 328.8 keV gamma ray. The γ -ray spectrum in the energy range 0.07 - 1.02 MeV in coincidence with the 328.8 keV gamma ray is shown in fig. 3.7. The running time was 17 hours and the true to accidental ratio was greater than 20 to one. For the subtraction of the gate-B spectrum from the gate-A spectrum the relative count rates were adjusted so as to remove the 815.8 keV peak. With this normalization, lines in coincidence with the 487.0 keV gamma ray will appear in the subtracted spectrum since the 328.8 keV peak sits on the Compton knee of the 487.0 keV gamma ray.

The 109.0, 487.0, 1596.6 keV gamma rays are seen to be in coincidence with the 328.8 keV gamma ray. The presence of small peaks in the subtracted spectrum corresponding to gamma rays with energies 131.1, 328.8 and 432.5 keV may be attributed to the fact that they are in coincidence with the 487.0 keV gamma ray. There is no evidence for the presence of γ -ray lines in the coincidence spectrum with the energies: 242.0, 266.6, 751.8, 815.8, 867.8, 919.6, 925.2, 950.9 keV.

Gate on 2348.8 and 2522.2 keV gamma rays. The γ -ray spectra in the energy range 0.05 - 1.15 MeV in coincidence with the 2348.8 and 2522.2, 2548.6 keV gamma rays are shown in fig. 3.8. The running time was 44 hours. It can be seen from the inset that the gate for the 2522.2 keV gamma ray also accepts most of the photopeak of the weak 2548.6 keV gamma ray.

There appears to be no gamma ray in coincidence with either the 2522.2 or 2548.6 keV gamma ray. The coincidence spectrum deviates noticeably from the singles spectrum (solid line) below 300 keV. Since there is $\approx 11\%$ β^- -feeding to the 2522 keV level, this deviation is probably caused by the brehmstrahlung in coincidence with the 2522.2 keV gamma ray.

The two gamma rays with energies 64.1 and 173.5 keV are seen to be in coincidence with the 2348.8 keV gamma ray.

Gate on 751.8 keV gamma ray. The γ -ray spectrum in the energy range 0.07 - 1.00 MeV in coincidence with the 751.8 keV gamma ray was obtained using a Ge(Li) - NaI arrangement. The 173.5 and 1596.6 keV gamma rays appear in coincidence.

Gate on 867.8, 919.6 and 925.2 keV gamma rays. By using a Ge(Li) - Ge(Li) arrangement with a biased amplifier on the gate side, it was possible to gate on each of these peaks separately. The coincidence spectra in the range 0.07 - 1.65 MeV were obtained with running times of ≈ 35 hours. Only the 1596.6 keV gamma ray could definitely be seen in coincidence with each of these three gamma rays.

Fig. 3.4. The γ -ray spectrum in coinc. with the 1596.6 keV γ -ray.

Fig. 3.5. The gamma-ray spectrum in the energy range 0.07 - 1.18 MeV in coincidence with the 487.0 keV gamma ray.

Fig. 3.6. Low-energy portion of the gamma-ray spectrum following the decay of La-140 in prompt and delayed (delay = 200 ns, $2\tau = 100$ ns) coincidence with the 487.0 keV gamma ray. The 131.1 and 242.0 keV gamma rays are seen to be in delayed coincidence.

Fig. 3.7. The gamma-ray spectrum in the energy range 0.07 - 1.02 MeV in coincidence with the 328.8 keV gamma ray.

Fig. 3.8. The gamma-ray spectra in the energy range 0.05 - 1.15 MeV in coincidence with the 2348.8 and 2522.2, 2548.6 keV gamma rays.

2. Construction of the $^{140}\text{La} \rightarrow ^{140}\text{Ce}$ Decay Scheme

2a. Transition energies, intensities, and K-shell conversion coefficients.

Energy and intensity values for 28 transitions in ^{140}Ce following the β^- -decay of ^{140}La are shown in table 3.2. The gamma-ray and K-shell conversion transition intensities were obtained from the relative intensities of the present work and that of Karlsson et al.¹⁷⁾. The theoretical value¹⁸⁾ $\alpha_K(1596.6, E2) = 0.69 \times 10^{-3}$ and the fact that the ground state β -feeding²⁹⁾ is $\approx 4 \times 10^{-4}\%$ were used in determining the normalizations. For the 1903.5 keV transition, the theoretical branching ratio¹⁹⁾ $\lambda_{\pi}/\lambda_{Ke} = 0.21$ was used to estimate the internal pair transition intensity.

2b. Transition multipolarities.

The data available for the determination of transition multipolarities consist of γ - $\gamma(\theta)$ correlation data^{20,21,36)}, K-shell and L-subshell conversion coefficient data^{17,22,37)}, and ratios of internal pair positons to K-shell conversion electrons corresponding to four ground state transitions²⁸⁾. All these data were considered in ascertaining the transition multipolarities.

A comparison of theoretical and experimental K-shell conversion coefficients is given in table 3.3. The possible transition multipolarities, based on this comparison, are shown in column 2, table 3.4. Also shown in this table are the possible multipolarities consistent with the data on L-subshell ratios¹⁷⁾, K/L ratios¹⁷⁾ and the γ - $\gamma(\theta)$ correlation data^{20,21)}. For several transitions where an appreciable admixture of a component with $L \geq 3$ was not excluded by the conversion coefficient data, the fact that the corresponding gamma ray was seen in a coincidence spectrum was used to exclude these higher multipolarities. In column 5, table 3.4, the possible multipolarities consistent with all the data are indicated. The transition multipolarities consistent with the

Table 3.2

Energy, intensity, and K-shell conversion coefficient values for transitions
in ^{140}Ce following β^- -decay of ^{140}La

Transition energy ^{a)} (keV)	I ^{d)} (% decays)	I _e (K) ^{e)} (% decays)	I _{Total} ^{d,e)} (% decays)	$\alpha_K \times 10^3$
24.595 ^{b)}			0.32 $\pm$ 0.04	
64.135		(2.4 $\pm$ 0.4) $\cdot 10^{-2}$	0.04 $\pm$ 0.02	
68.916 ^{b)}	0.062 $\pm$ 0.012	(1.5 $\pm$ 0.3) $\cdot 10^{-1}$	0.24 $\pm$ 0.06	2420 $\pm$ 674
109.418	0.26 $\pm$ 0.04	(1.48 $\pm$ 0.05) $\cdot 10^{-1}$	0.43 $\pm$ 0.04	569 $\pm$ 90
131.121	0.58 $\pm$ 0.09	(2.10 $\pm$ 0.08) $\cdot 10^{-1}$	0.83 $\pm$ 0.09	362 $\pm$ 58
173.536	0.12 $\pm$ 0.05	(2.2 $\pm$ 0.3) $\cdot 10^{-2}$	0.15 $\pm$ 0.05	183 $\pm$ 80
241.966	0.43 $\pm$ 0.06	(3.19 $\pm$ 0.22) $\cdot 10^{-2}$	0.47 $\pm$ 0.06	74.2 $\pm$ 11.5
266.551	0.53 $\pm$ 0.06	(3.0 $\pm$ 0.3) $\cdot 10^{-2}$	0.56 $\pm$ 0.06	56.6 $\pm$ 8.5
306.9	0.021 $\pm$ 0.011		0.022 $\pm$ 0.011	
328.768	20.4 $\pm$ 1.1	(7.24 $\pm$ 0.10) $\cdot 10^{-1}$	21.2 $\pm$ 1.1	35.5 $\pm$ 2
397.79	0.052 $\pm$ 0.024	(2.2 $\pm$ 1.4) $\cdot 10^{-3}$	0.055 $\pm$ 0.024	42 $\pm$ 33
432.530	2.97 $\pm$ 0.15	(5.5 $\pm$ 0.3) $\cdot 10^{-2}$	3.03 $\pm$ 0.15	18.5 $\pm$ 1.4
487.029	47.2 $\pm$ 2.4	(4.20 $\pm$ 0.22) $\cdot 10^{-1}$	47.7 $\pm$ 2.4	8.90 $\pm$ 0.65
618.2	0.042 $\pm$ 0.021	$< 2.1 \times 10^{-3}$	0.042 $\pm$ 0.021	
751.827	4.20 $\pm$ 0.25	(1.88 $\pm$ 0.22) $\cdot 10^{-2}$	4.22 $\pm$ 0.25	4.48 $\pm$ 0.59
815.801	23.0 $\pm$ 1.1	(9.1 $\pm$ 0.6) $\cdot 10^{-2}$	23.1 $\pm$ 1.1	3.96 $\pm$ 0.32
867.82	5.39 $\pm$ 0.27	(5.8 $\pm$ 1.4) $\cdot 10^{-3}$	5.40 $\pm$ 0.27	1.08 $\pm$ 0.27
919.64	2.61 $\pm$ 0.15	(5.8 $\pm$ 1.4) $\cdot 10^{-3}$	2.62 $\pm$ 0.15	2.22 $\pm$ 0.55
925.20	6.91 $\pm$ 0.41	(2.17 $\pm$ 0.22) $\cdot 10^{-2}$	6.93 $\pm$ 0.41	3.14 $\pm$ 0.37
950.88	0.53 $\pm$ 0.05	(1.3 $\pm$ 0.4) $\cdot 10^{-3}$	0.53 $\pm$ 1.3	2.45 $\pm$ 0.79
1596.6	95.5 $\pm$ 0.2	6.59 $\times 10^{-2}$	95.6 $\pm$ 0.2	0.69
1903.5 ^{c)}		(1.45 $\pm$ 0.14) $\cdot 10^{-2}$	(1.93 $\pm$ 0.16) $\cdot 10^{-2}$ f)	
2348.4 ^{c)}	0.860 $\pm$ 0.043	(3.2 $\pm$ 0.5) $\cdot 10^{-4}$	0.860 $\pm$ 0.043	0.37 $\pm$ 0.06
2521.8 ^{c)}	3.36 $\pm$ 0.17	(1.16 $\pm$ 0.14) $\cdot 10^{-3}$	3.36 $\pm$ 0.17	0.345 $\pm$ 0.045
2547.5 ^{c)}	0.117 $\pm$ 0.009	(3.3 $\pm$ 0.8) $\cdot 10^{-5}$	0.117 $\pm$ 0.009	0.28 $\pm$ 0.07
2899.7	0.067 $\pm$ 0.005	(1.7 $\pm$ 0.3) $\cdot 10^{-5}$	0.067 $\pm$ 0.005	0.25 $\pm$ 0.05
3118.3	0.026 $\pm$ 0.003	(6.5 $\pm$ 2.2) $\cdot 10^{-6}$	0.026 $\pm$ 0.003	0.25 $\pm$ 0.09
3319.7	0.008 $\pm$ 0.004	(9 $\pm$ 5) $\cdot 10^{-7}$	0.008 $\pm$ 0.004	0.11 $\pm$ 0.08

- a) Values taken from table 1 of present work except where indicated otherwise.
b) Value taken from ref. 17).
c) Value adopted on basis of energy sums of lower energy transitions.
d) Values based on the relative γ -ray intensities given in table 1 and on the assumption that the 1596.6 keV transition is pure E2 with $\alpha_K(1596.6) = 0.69 \times 10^{-3}$.
e) Values based on the relative conversion electron intensities of ref. 17).
f) The theoretical branching ratio, $\lambda_K/\lambda_{KE} = 0.21$, has been used to estimate the internal pair transition intensity.

Table 3.3

Comparison of experimental and theoretical K-shell conversion coefficients

Transition energy (keV)	Theoretical values ^{a)} of $\alpha_K \times 10^3$						Experimental value of $\alpha_K \times 10^3$	
	E1	E2	E3	M1	M2	M3		
68.9	570	3350		2970	36000		2420	+ 674
109.4	165	930		790	6700		569	+ 90
131.1	100	530		470	3450		362	+ 58
173.5	45.3	220		215	1275		183	+ 80
242.0	18.7	76	275	86	420	1700	74.2	+ 11.5
266.6	14.5	56.5	195	67	305	1150	56.6	+ 8.5
306.9		36						
328.8	8.5	29.3	92	38.0	155	520	35.5	+ 2.0
397.8	5.3	16.8	49	23.5	84	260	42	+ 33
432.5	4.32	13.4	38.2	18.5	65	195	18.5	+ 1.4
487.0	3.28	9.7	24.3	13.3	43	120	8.90	+ 0.65
618.2		5.3						
751.8	1.30	3.30	7.60	5.00	13.5	30.5	4.48	+ 0.59
815.8	1.08	2.67	6.02	4.05	10.6	23.0	3.96	+ 0.32
867.8	0.96	2.35	5.15	3.52	9.0	19.2	1.08	+ 0.27
919.6	0.86	2.05	4.45	3.08	7.70	16.4	2.22	+ 0.55
925.2	0.855	2.03	4.40	3.05	7.65	16.0	3.14	+ 0.37
950.9	0.810	1.93	4.10	2.86	7.15	14.8	2.45	+ 0.79
1596.6		0.690						
2348.4	0.175	0.325	0.570	0.380	0.780	1.28	0.37	+ 0.06
2521.8	0.155	0.290	0.485	0.320	0.650	1.05	0.345	+ 0.045
2547.5	0.153	0.288	0.481	0.317	0.643	1.04	0.28	+ 0.07
2899.7	0.127	0.232	0.382	0.242	0.480	0.77	0.25	+ 0.05
3118.3	0.114	0.205	0.33	0.210	0.41	0.65	0.25	+ 0.09
3319.7	0.105	0.185	0.29	0.185	0.35	0.55	0.11	+ 0.08

a) Theoretical values taken from Sliv and Band 18).

Table 3.4

Multipolarity determinations for transitions in ^{140}Ce

Transition energy (keV)	Possible multipolarities			Multipolarity assignment	
	K-shell conv. coefficients ^{a)}	L-subshell ratios ^{c)} $K/\sum L$ -ratios	γ - $\gamma(\theta)$ ^{d)}	Data	Data+internal consistency ^{e)} with level scheme
24.6		M1+E2,E2		M1+E2,E2	E2 (< 42% M1)
64.1		M1+E2,M1		M1+E2,M1	M1 (< 3.7% E2)
68.9	E1+M2,M1,E2,M1+E2	M1+E2,M1		M1+E2,M1	M1 (< 0.7% E2)
109.4	E1+M2,M1	M1+E2		M1+E2	M1+E2
131.1	M1+M2,M1	E1+M2		E1+M2	E1 + (2-19%) M2
173.5	E1+M2,M1,E2,M1+E2	E1+M2,E1,M1,M1+E2		E1+M2,M1,M1+E2	M1+E2,M1
242.0	E1+M2,M1,E2,M1+E2	E1+M2,E1,M1,M1+E2		E1+M2,M1,M1+E2	E1 + (11-17%) M2
266.6	E1+M2,M1,E2,M1+E2	E1+M2,M1,M1+E2,M2		E1+M2,M1,M1+E2	E1 + (12-17%) M2
328.8	E1+M2,M1,M1+E2	E1+M2,E1,M1,M1+E2	L=1; L=2 $\leq$ 0.005	M1	M1
397.8	E1+M2,M1,E2,M1+E2			E1+M2,M1,E2,M1+E2	M1+E2,M1
432.5	E1+M2,M1,M1+E2	E1+M2,M1,M1+E2,E2,M2		E1+M2,M1,M1+E2	E1+M2,M1,M1+E2
487.0	E1+M2,E2	M1+E2,E2	L=2; L=3 $\leq$ 0.005	E2	E2
751.8	E1+M2,M1,M1+E2			E1+M2,M1,M1+E2	M1 + (0-65%) E2
815.8	E1+M2,M1,M1+E2	E1+M2,E1,M1,M1+E2,E2,M2	L=1; L=2 $\leq$ 0.002	M1	M1
867.8	E1+M2,E1			E1+M2,E1	E1 + (0-5%) M2
919.6	E1+M2,E2,M1+E2			E1+M2,E2,M1+E2	E1+M2,E2,M1+E2
925.2	E1+M2,M1,M1+E2	E1+M2,E1,M1,M1+E2,E2,M2		E1+M2,M1,M1+E2	M1 + (0-27%) E2
950.9	E1+M2,M1,E2,M1+E2			E1+M2,M1,E2,M1+E2	M1+E2,M1
1596.6	E2 ^{b)}		L=2	E2	E2
1903.5	E0	E0		E0	E0
2348.4	M1,E2			M1,E2	E2
2521.8	M1,E2			M1,E2	E2
2597.5	M1,E2			M1,E2	M1,E2
2899.7	M1,E2			M1,E2	M1,E2
3118.3	M1,E2			M1,E2	M1,E2
3319.7	E1,M1,E2			E1,M1,E2	M1,E2

a) Based on values given in table 3.

b) Adopted transition multipolarity.

c) Based on work of ref. (7).

d) Values of L consistent with the γ - $\gamma(\theta)$ measurements of ref. 20,21,36).

e) Admixtures for transitions in the energy range 24.6 - 131.1 keV correspond to those reported in ref. 17). Admixtures for all other transitions are based on the K-shell conversion coefficients of the present work.

data and with the presently proposed level scheme are shown in column 6.

There appears to be a discrepancy in the multipolarity assignment for the 109.4 keV transition. The L-subshell ratios are consistent with an M1 + E2 admixture and exclude an E1 + M2 admixture. The K-shell conversion coefficient, on the other hand, is consistent with an E1 + M2 admixture and appears to exclude an M1 + M2 admixture. Since the measurements involved in determining α_K are subject to systematic errors, the L-subshell ratios were considered to be more reliable. Also, since the parities of the 2521.8 and 2412.4 keV levels can be established independently to have a + value, the 109.4 keV transition between these levels cannot have an E1 + M2 character.

2c. The Decay: $^{140}_{57}\text{La}_{83} \rightarrow ^{140}_{58}\text{Ce}_{82}$

In fig. 3.9 is presented a scheme for the decay of ^{140}La to levels of ^{140}Ce which incorporates in a consistent manner all the data of the present work as well as that of earlier investigators. To the most recent level scheme of Karlsson et al.¹⁷⁾, two new levels at 2108.2 and 2464.4 keV have been added, and further evidence for the proposed levels at 2350.2 and 2481.4 keV is presented. The existence of the two proposed levels at 2325.5 and 3216.1 keV is not supported.

The β -ray end-point energies were obtained using the measured¹⁷⁾ value $Q_\beta = 3.763 \pm 0.008$ MeV, and the level excitation energies from the present work. The β -feeding percentages are based on the γ -transition intensity balance for each level. The value, $(4 \pm 1) \times 10^{-4}\%$, of the ground state β -feeding is reduced by a factor of two from that reported by Dzhelepov et al.²⁹⁾ since they assumed that the β -feeding to the 1596.6 keV level is 10%. From the present work it is seen that this feeding is approximately 5%. The indicated log ft's are calculated values³⁰⁾, obtained by making the allowed approximation and using partial β -transition probabilities and end-point energies. The total β -decay life-time, $T_{1/2} = 40.2 \pm 0.2$ h, is taken from ref.¹⁷⁾.

The mean lifetime values of several of the low-lying states in ^{140}Ce are taken from refs. 23-27). On the basis of the present coincidence studies, an upper limit of 30 ns can be placed on the mean lifetime of each level in the range 2348.4 - 2547.5 keV, with the exception of the upper limit on the 2464.4 keV level; this value is discussed later. The magnetic moment of the 2083.6 keV level has been measured by various investigators using the perturbed angular correlation technique. The value $\mu = 4.4$ nm is taken from ref.³¹⁾.

A value of 3 has been determined³²⁾ for the ground state spin of ^{140}La using the atomic beam magnetic resonance method. The negative parity is expected from shell model considerations. The 57th proton of $^{140}\text{La}_{83}$ is expected to occupy the positive parity orbital $1g_{7/2}$ and the 83rd neutron the negative parity orbital $2f_{5/2}$. Since the parity of one component of the ground state wave function of ^{140}La can reasonably be expected to be determined solely by the 57th proton and 83rd neutron, the negative parity assignment is strongly favored. Further, the $J\pi=3^-$ assignment is not inconsistent with the conversion coefficient data and $\gamma\text{-}\gamma(\theta)$ correlation data³³⁾ on γ -ray transitions in ^{140}La , nor with the $\log ft$ values of β -transitions to levels in ^{140}Ce . The value, $\log ft = 14.5$, for the ground state transition is not inconsistent with a 3rd-forbidden β -transition, although it is somewhat smaller than that observed in several other 3rd-forbidden transitions³⁴⁾.

The spin and parity assignments shown in fig. 3.9 for the 15 excited states of ^{140}Ce are based on the interpretation of all the data presently available. Values of J and/or π enclosed by parenthesis are regarded as tentative. A discussion of how spin and parity assignments were made and of other data pertinent to the level structure follows.

1903.5 keV. The only observed transition ($2^+ \rightarrow 0^+$, 618.2 keV) populating this 0^+ level in the decay of ^{140}La has an intensity of 0.042 ± 0.021 % decay. (The stated uncertainty may be too large, as discussed section 1.a.) Depopulating this level are the 306.9 keV γ -transition, 1903.5 keV electron con-

version transition, and the 1903.5 keV internal-pair transition, having respective intensities of 0.022 ± 0.011 , 0.0162 ± 0.0015 , and 0.003 % decay. The last number is based on the K-shell conversion electron intensity¹⁷⁾ $N_{K\epsilon} = (1.45 \pm 0.14) \times 10^{-2}$ % decay and the theoretical branching ratio¹⁹⁾ $\lambda_{\pi}/\lambda_{K\epsilon} = 0.21$. The total depopulating intensity is 0.041 ± 0.012 % decay, which indicates that an intensity balance with negligibly small β -feeding is achieved. This is consistent with the fact that a 3rd-forbidden β -transition is required to feed this level.

2108.2 keV. An isomeric state at this energy is postulated for the following reasons. The energy 266.551 ± 0.014 keV of the cross-over transition compares well with the sum $24.595 + 241.966 = 266.561 \pm 0.013$ keV of the proposed cascade transitions. The 50 % reduction in intensity of the 131.1 keV γ -ray peak relative to the 266.6, 432.5 and 328.8 keV γ -ray peaks in the coincidence spectrum gated by the 487.0 keV gamma ray indicates that the 2350.2 keV level is being depopulated by a gamma ray with an intensity nearly equal to that of the 266.6 keV gamma ray. The 242.0 and 266.6 keV gamma rays have nearly the same intensity. The decay sequence of the 242.0 - 24.6 keV cascade is most clearly established by the photonuclear reaction data of Krehbiel and Meyer-Berkhout²⁷⁾. They found two gamma rays with energies 480 ± 10 keV and 1600 ± 40 keV having nearly equal intensity in delayed coincidence with the primary excitation pulse in the reactions $^{140}\text{Ce}(\gamma, \gamma')^{140\text{m}}\text{Ce}$ and $^{142}\text{Ce}(\gamma, 2n)^{140\text{m}}\text{Ce}$. If the sequence of decay were 24.6 - 242.0 keV, as suggested by Karlsson et al.¹⁷⁾, the 242.0 keV gamma ray would also have been observed in delayed coincidence with the excitation pulse. Since this gamma ray seems not to have been observed, the decay sequence must be as shown in fig. 3.9. The highly L-converted (theoretical $\alpha_L(24.6) \approx 465$) 24.6 keV transition would have escaped detection in the work of Krehbiel and Meyer-Berkhout. These investigators determined the decay half life of the 480 and 1600 gamma rays relative to the excitation pulse to be 6.5 ± 1.5 μs . This lifetime must now be attributed to the 2108.2 keV level. All the coincidence data of the pre-

sent investigation, including the delayed coincidence spectrum shown in fig. 3.6, support this interpretation.

Only the 242.0 and 24.6 keV transition have been observed to feed and depopulate the 2108.2 keV level; their respective intensity values are 0.47 ± 0.06 and 0.32 ± 0.04 % decay. These numbers indicate that there is very little β -feeding to the 2108.2 keV level and that any other depopulating γ -transitions must be weak. The upper limits on the intensities of γ -ray transitions to the ground state, 1596.6, and 1903.5 keV levels as determined from the Ge(Li) singles spectrum are: $I_\gamma(2108.2) < 0.03$ %, $I_\gamma(512) < 0.2$ %, $I_\gamma(200.7) < 0.1$ % decay. The intensity value $I_\gamma(24.6) = 0.0007$ % decay is obtained using the theoretical value $\alpha_L(24.6, E2) = 4.65 \times 10^2$ and the experimental value $I_L(24.6) = 0.32$ % decay.

The possible J^π values of this state consistent with the E2(+M1) character of the 24.6 keV transition and the fact that the 2083.6 keV level is 4+ are 2+, 3+, 4+, 5+, and 6+. The mean lifetime of this state is 9.4 μ s, which corresponds to a total decay probability $\lambda_t = 1.1 \times 10^5 \text{ sec}^{-1}$. A comparison of this transition probability with single proton transition estimates (Appendix D) for various unobserved, depopulating, γ -ray transitions favors a 6+ assignment. The hinderance factors for the unobserved 2108.2 and 511.6 keV transitions between the 2108.2 keV level and the ground- and 1st-excited state which would exist if the 2108.2 keV level had the various J^π values in the above mentioned range are given below. Also given is the hinderance factor for the 24.6 keV, M1 transition if the 2108.2 keV level were assumed to be 5+.

Assumed J^π for 2108.2 keV level	Single proton transition estimate $\lambda_{sp} \text{ sec}^{-1}$	Maximum γ -ray transition prob. from data	Hinderance factor $\lambda_{sp}/\lambda_\gamma$
2+	$\lambda(2108, E2) = 2.2 \times 10^{12}$	$\lambda_\gamma < 0.1 \lambda_t$	$> 10^8$
3+	$\lambda(511.6, M1) = 3.5 \times 10^{12}$	$\lambda_\gamma < \lambda_t$	$> 10^7$
4+	$\lambda(511.6, E2) = 1.5 \times 10^9$	$\lambda_\gamma < \lambda_t$	$> 10^4$
5+	$\lambda(24.6, M1) = 4.4 \times 10^8$	$\lambda_\gamma < 0.2 \lambda_t$	$> 10^4$
6+	$\lambda(24.6, E2) = 4.8 \times 10^2$	$\lambda_\gamma \approx 2.3 \times 10^2 \text{ sec}^{-1}$	≈ 2

The maximum γ -ray transition probabilities for the 2+, 3+, and 4+ cases derive from the intensity limits stated above for the unobserved γ -ray transitions and the fact that the 24.6 keV transition has an intensity 0.32% decay. For the 5+ case, λ_γ is obtained using the theoretical conversion coefficient¹⁹⁾ $\alpha_L(24.6, M1) \approx 7$. For the 6+ case, it has been assumed that $\lambda_{Le} \approx \lambda_t$, so that the γ -ray transition probability is given by $\lambda_\gamma \approx \lambda_t/\alpha_L(24.6, E2)$. When the above lower limits on hinderance factors are compared to the range of factors generally observed in nuclear transitions with the same multipolarity³⁵⁾ it is seen that they fall above this range. At present one does not expect M1 and E2 γ -ray transition rates between low-lying states in single-closed-shell even-even nuclei to deviate more than four orders of magnitude from single-proton estimates. Thus, since any value of J^π other than 6+ would require the existence of an extremely large hinderance factor, the assignment 6+ to the 2108.2 keV level is favored.

Furthermore, a J^π value of 2+, 3+, 4+, or 5+ would permit a 1st- forbidden β -transition to populate this state, whereas a 6+ value requires a 3rd- forbidden transition. In this decay, β - transitions proceed to all the observed levels which can be populated via a 1st- forbidden transition. The absence of β - feeding to the 2108.2 keV level therefore adds weak support to the 6+ assignment.

2412.4 keV. The spin and parity of this level have generally been adopted to be 3+ without regard to conflicting evidence. It is important that the J^π value be firmly established since the transition multipolarities of the 5 γ -transitions populating and depopulating this level are used to establish the spins and parities of other levels. The γ - $\gamma(\theta)$ correlation measurements^{20,21,36)} on the 328.8 - 487.0 keV cascade are consistent with a pure E2, 487.0 keV transition and an M1, 328.8 keV transition with 0.1 to 0.5 % quadrupole admixture. The K-shell conversion coefficient data has not been consistent with this. Experimental determinations^{17,22)} of the K-shell

conversion coefficient of the 487.0 keV transition have given a value very nearly equal to the theoretical value $\alpha_K(487.0, E2) = 9.7 \times 10^{-3}$. The experimental K-shell conversion coefficients for the 328.8 keV transition determined by Bashilov et al.²²⁾ is 29×10^{-3} and that determined by Karlsson et al.¹⁷⁾ is $(29.8 \pm 2.4) \times 10^{-3}$. Comparison with theoretical values, $\alpha_K(328.8, M1) = 38.0 \times 10^{-3}$ and $\alpha_K(328.8, E2) = 29.3 \times 10^{-3}$, would suggest a nearly pure E2 character for the 328.8 keV transition (in contradiction to the γ - $\gamma(\theta)$ data).

Since the 487.0 keV transition is very nearly pure E2, the theoretical value $\alpha_K(487.0, E2) = 9.7 \times 10^{-3}$ should be used to determine the K-shell conversion coefficient of the 328.8 keV transition so as to reduce the effect of systematic errors in the relative γ -ray and K-shell conversion electron intensities. By doing this, and by using the relative K-shell conversion electron intensities¹⁷⁾ $I_e(328.8)/I_e(487.0) = 1.72 \pm 0.09$ and the relative gamma-ray intensities (column 4, table 3.1) $I_\gamma(487.0)/I_\gamma(328.8) = 0.232 \pm 0.015$ the value $\alpha_K(328.8) = 38.6 \pm 2.8$ is obtained. This is in good agreement with the theoretical value for an M1 transition and thereby resolves the ambiguity.

The data on the 815.8 keV transition agree only with the assumption that this transition is of nearly pure M1 character.

The M1 multipolarities determinations of the 328.8 and 815.8 keV transitions and the assumption that the 1596.6 and 2083.6 keV levels are 2+ and 4+ respectively, uniquely affix the value $J\pi = 3+$ to the 2412.4 keV level.

2348.4 keV. The 2+ assignment to this level is based on the following two arguments. The multipolarity of the 2348.4 keV ground state transition is M1 or E2. The multipolarity of the 64.1 keV transition between the 2412.4 keV, 3+ level and the 2348.4 keV level is M1(<3.7 % E2). Only a 2+ assignment is compatible with both these data.

The total intensity of the 751.8 and 2348.4 keV transitions depopulating this level is 5.08 ± 0.25 % decay, whereas

the total intensity of the 64.1 and 173.5 keV transitions depopulating this level is 0.19 ± 0.06 % decay. The intensity balance therefore requires a β -feeding of 4.89 ± 0.26 % decay. The corresponding logft value, 8.7, falls into the range generally observed^{34,35)} for 1st- forbidden β - transitions.

2350.2 and 2481.3 keV. Levels at these energies were suggested by Karlsson et al. on the basis of energy sum relationships. The coincidence data of the present work give strong support. In the coincidence spectrum gated by the 487.0 keV gamma ray the 266.6, 328.8, 397.8 and 432.5 keV lines have approximately the same relative intensity as in the singles spectrum. Within the framework of the present level scheme, this indicates that the 266.6 and 397.8 keV transitions must feed the 2083.4 keV level, thereby establishing levels at 2350.2 and 2481.3 keV.

The positive parity of the 2481.3 keV level is established by the fact that the 68.9 keV transition between the 2481.4 and 2412.4 keV, $3+$ levels is of character $M1 + E2$ or pure $M1$. The negative parity of the 2350.2 keV level is then established by the fact that the 131.1 keV transition between the 2481.3 and 2350.2 keV levels is of character $E1 + M2$. The 266.6 and 242.0 keV transitions depopulating the 2350.2 keV, $+$ level to positive parity states therefore must have $E1 + M2$ character. The multipolarities of these two transitions determine the J of the 2350.2 keV level to be 5 if the 2083.6 and 2108.2 keV levels are assumed to have $J=4$ and $J=6$ respectively. For the 2481.4 keV level, only the value $J=4$ is now consistent with the $E1 + M2$ character of the 131.1 keV transition and the $M1 + E2$ character of the 68.9 keV transition. The multipolarity of the 397.8 keV transition between the 2481.3 ($4+$) and 2083.6 ($4+$) keV levels must therefore be $M1(+E2)$, which is consistent with the data. For these reasons the assignments $J^\pi = 5^-$ and $J^\pi = 4+$ for the 2350.2 and 2481.4 keV levels, respectively, are strongly favored.

The total intensity of the 242.0 and 266.6 keV transitions depopulating the 2350.2 keV level is 1.04 ± 0.08 % decay,

whereas the intensity of the populating 131.1 keV transition is 0.83 ± 0.12 % decay. To within uncertainties, an intensity balance is therefore achieved with negligibly small β - feeding. The absence of β - feeding is consistent with the fact that a 2nd- forbidden transition is required to feed this 5- level. For the 2481.4 keV level, the total depopulating intensity of the 68.9, 131.1 and 397.8 keV gamma-decays is 1.12 ± 0.10 % decay. Since no populating gamma-transitions have been observed, β - feeding of approximately 1.1 % must be assumed. The corresponding logft value, 9.1, falls on the high side of the range of values generally observed³⁴⁾ for 1st- forbidden β - transitions in this mass region.

Gamma rays corresponding to the possible E2 transitions between the 2481.3, 4+ and 2108.3 (6+), 1596.6, 2+ keV levels were not observed. The upper limits, $I_\gamma(373 \text{ keV}) < 0.04$ % and $I_\gamma(885 \text{ keV}) < 0.1$ % decay were determined.

2464.4 keV. A level at this excitation energy was first suggested by Hansen and Nathan³⁸⁾ on the basis of inelastic α -particle scattering experiments. In the reaction $^{140}\text{Ce}(\alpha, \alpha' \gamma)^{140}\text{Ce}$ they detected an 0.87 MeV gamma ray which they attributed to the transition between levels at 2.47 and 1.60 MeV. The coincidence data of the present investigation is in good agreement with this interpretation if the 0.87 MeV gamma ray is identified with the 868.8 keV gamma ray and the 2.47 and 1.60 MeV levels with the 2464.4 and 1596.6 keV levels. Hansen and Nathan also assigned the value $J^\pi=3^-$ to this level and interpreted it as an octupole vibrational state. The E1 character of the 867.8 keV transition (table 3.4) is consistent with the 3- assignment.

An upper limit of 0.3 ps on the mean lifetime of the 2464.4 keV level is obtained using the measured value³⁸⁾ $B(E3, 0+ \rightarrow 3-) = 76 \text{ e}^2 10^{-74} \text{ cm}^6$ and the intensity limit $I_\gamma(2464.4 \text{ keV}) < 100 I_\gamma(867.8 \text{ keV})$ determined from the Ge(Li) singles spectrum.

The intensity of the 867.8 keV transition depopulating the 2464.4 keV level is 5.40 ± 0.27 % decay. Since no gamma-transitions has been observed to feed this level, it must be assumed that this level is primarily populated through β -decay.

The corresponding $\log ft$ value, 8.5, falls on the high side of the observed range³⁴⁾ for allowed β -transitions in this mass region.

Gamma rays corresponding to the possible E1 transitions between the 2464.4, 3- and 2412, 3+, 2348.4, 2+, 2083.6, 4+ keV levels were not observed. The upper limits, $I_\gamma(116 \text{ keV}) < 0.08 \%$ and $I_\gamma(381 \text{ keV}) < 0.04 \%$ decay were determined.

2516.1 keV. A level at this energy has been suggested^{17,39)} primarily on the basis of energy sum relationships. The coincidence data of the present work add strong support. The 432.5 and 328.8 keV gamma rays have approximately the same relative intensity in the spectrum coincident with the 487.0 keV gamma ray as in the singles spectrum. One must therefore assume that the 432.5 keV transition is in cascade with the 487.0 keV gamma ray, thereby establishing a level at 2516.1 keV. Also, the fact that the 919.6 keV gamma ray appears only in coincidence with the 1596.6 keV gamma ray is evidence for the existence of the 2516.1 keV level.

The possible multipolarities of the 919.6 and 432.5 keV transitions (table 3.4) and the fact that 1596.6 and 2083.6 keV levels have J^π values of 2+ and 4+ respectively limit the J^π values of the 2516.1 keV level to 3-, 3+, or 4+. The experimental values of the K-shell conversion coefficients of the 919.6 and 432.5 keV transitions have nearly the same values as the theoretical conversion coefficients for pure E2 and M1 transitions, respectively. This tends to favor a 4+ assignment. Other weak arguments based on the absence of γ -ray transitions also favor the 4+ assignment.

The total intensity of the depopulating, 919.6 and 432.5 keV, transitions is $5.65 \pm 0.21 \%$ decay. Since no populating γ -transitions have been observed, this level must be fed primarily through β -decay. The corresponding value, $\log ft = 8.7$, falls within the observed range^{34,35)} of allowed and 1st-forbidden β -transitions.

Several upper limits on γ -ray intensities for unobserved, depopulating γ -ray transitions are reported in ref.39). Upper

limits for transitions between the 2516.1 and 2350.2(-5) and 2108.2(6+) keV levels are: $I_{\gamma}(166\text{keV}) < 0.1 \%$ and $I_{\gamma}(408\text{ keV}) < 0.1 \%$ decay.

2521.8 keV. The parity of this level has generally been considered to be +, and the spin has variously been considered 17,28,30) to be 1 or 2. The uncertainty in the spin value can be attributed to the fact that the theoretical values $\alpha_K(2521.8, M1) = 0.320 \times 10^{-3}$ and $\alpha_K(2521.8, E2) = 0.290 \times 10^{-3}$ differ by only 10 %. The experimental value $\alpha_K(2521.8) = (0.345 \pm 0.045) \times 10^{-3}$ given in table 3.2 is consistent with (and only with) an M1 or E2 multiplicity assignment. It is again reasonable to re-determine the experimental conversion coefficient so as to reduce the effect of systematic errors in the relative γ -ray and K-shell conversion electron intensities. Using the relative K-shell conversion electron intensity¹⁷⁾ $I_e(2521.8)/I_e(2348.4) = 3.50 \pm 0.35$, the relative γ -ray intensity (table 3.1) $I_{\gamma}(2348.4)/I_{\gamma}(2521.8) = 0.255 \pm 0.008$, and the theoretical value¹⁸⁾ $\alpha_K(2348.4, E2) = 0.325$, the conversion coefficient $\alpha_K(2521.8) = 0.29 \pm 0.03$ is determined. This agrees very well with the theoretical value for an E2 transition although M1 cannot be excluded. However, the multipolarity of the 109.4 keV transition between the 2521.8 and 2412.4 keV, 3+ level has M1 + E2 character (but not pure E2). Hence, only a 2+ assignment to the 2521.8 keV level is compatible with both data. The 925.2 keV transition between the 2521.8, 2+ and 1596.6, 2+ keV levels must be M1(+E2) which is in good agreement with the data.

It should be mentioned that the 2+ assignment contradicts the suggestion of Antonova et al.²⁸⁾ who proposed an M1 multipolarity for a 2527 keV transition by measuring the ratio of internal-pair positons to K-shell conversion electrons and comparing this to theoretical estimates. Although the uncertainties in this measurement were not stated, an E2 multipolarity for the 2557 keV transition does not appear to be definitely excluded.

The total intensity of the five γ -transitions depopulating the 2521.8 keV level is $10.92 \pm 0.45 \%$ decay. No populating γ -transitions have been observed. The logft value, 8.2, based on

the intensity balance, falls within the observed range^{34,35)} of 1st-forbidden non-unique transitions.

A gamma ray corresponding to the possible E1 transition between the 2521.8, 2+ and 2464.4, 3- keV levels was not observed. The intensity limit $I_\gamma(57 \text{ keV}) < 0.05 \%$ decay was determined.

2547.5 keV. The possible $J\pi$ values of this level can be restricted to 1+ or 2+ by the multipolarity determination of the ground state transition. The relative K-shell conversion electron intensity¹⁷⁾ $I_e(2547.5)/I_e(2348.4) = 0.106 \pm 0.022$, the relative gamma ray intensity (table 3.1) $I_\gamma(2348.4)/I_\gamma(2547.5) = 7.35 \pm 0.75$, and the theoretical conversion coefficient $\alpha_K(2348.4, E2) = 0.325$ combine to give $\alpha_K(2547.5) = 0.25 \pm 0.07$. This agrees only with the theoretical values for an M1 or E2 transition, thereby establishing the spin and parity to be either 1+ or 2+. The 950.9 keV transition between the 2547.5 and 1596.6 keV, 2+ level must therefore have M1 character. This is consistent with the data.

The total intensity of the 950.9 and 2547.5 keV transitions depopulating this level is $0.65 \pm 0.05 \%$ decay. No populating γ -transitions have been observed. If a β -feeding of 0.65 % is assumed, the corresponding logft value is 9.3. This falls on the high side of the range generally observed^{34,35)} for 1st-forbidden transitions.

2899.7, 3118.3, and 3319.7. The presence of these levels is established by the fact that the gamma rays and K-shell conversion electrons^{17,37)} corresponding to the ground state transitions have been observed. On the basis of the measured K-shell conversion coefficients, the multipolarities of the 2899.7 and 3118.3 keV transitions are restricted to M1 or E2. For the 3319.7 keV transition, the possible multipolarities are E1, M1, or E2. However, the E1 multipolarity seems unlikely. If a $J\pi = 1-$ value is adopted for the 3319.7 keV level, the β -transition feeding this level would be 2nd-forbidden. The logft value, 9.5, based on the intensity balance, is smaller than the values gen-

erally observed³⁴⁾ for 2nd-forbidden transitions. Thus the spin and parity of all three levels is either $1+$ or $2+$, and the β -transitions feeding these levels are 1st-forbidden.

3. Comparison With Theory and Discussion

There have been several theoretical calculations^{1,40,41)} of the low-lying excitation spectra of the $N=82$, single-closed-shell nuclei. All these calculations were performed in the framework of the quasi-particle (qp) treatment of pairing correlations in spherical nuclei (discussed in Chapter IV). Plastino, Arvieu and Moszkowski⁴¹⁾ and M. Rho¹⁾, described the low-lying positive parity states as combinations of 2 qp configurations (Tamm-Dancoff approximation). Plastino et al. took as the two-body residual interaction the surface delta interaction. Rho used a spin dependent interaction with a Gaussian radial dependence. His calculations were the most detailed and included the evaluation of several gamma-transition rates and the magnetic moment of one state in ^{140}Ce . A comparison of this calculation^{1,42)} with the experimental spectrum follows.

The single proton (shell model) levels were taken to be, in MeV, as $1g_{7/2}(0)$, $2d_{5/2}(1.00)$, $1h_{11/2}(2.18)$, $2d_{3/2}(2.88)$, and $3s_{1/2}(3.20)$. The 2 qp configuration used in the computation were $(g_{7/2})^2$, $(g_{7/2}d_{5/2})$, $(d_{5/2})^2$, $(d_{3/2})^2$, $(h_{11/2})^2$, $(g_{7/2}d_{3/2})$, and $(d_{5/2}d_{3/2})$, with J values from 1 to 4. All the configurations have positive parity. The 2 qp amplitudes were determined by a perturbation treatment of the 2 qp interaction hamiltonian H_{22} arising from the BCS transformation.

A comparison of the theoretical and experimental spectrum of ^{140}Ce is shown in fig. 3.10. The dominant 2 qp configurations of each state are indicated. (The complete wave function is a linear combination of 2 qp configurations.) It is seen that in this calculation the lowest $2+$ and $4+$ states are

Fig. 3.10. Comparison of the observed excitation spectrum of $^{140}_{58}\text{Ce}_{82}$ with the theoretical spectrum obtained by M. Rho.

described as primarily $(g7/2)^2$, $J=2,4$ configurations and the next highest $2+$ and $4+$ states as primarily $(d5/2)^2$, $J=2,4$ configurations. These four states can probably be identified with the observed levels at 1596.6, $2+$, 2083.6, $4+$ and 2348.5, $2+$, 2481.3 ($4+$) keV respectively. The nearly pure⁴²⁾ 2 qp states with configurations $(g7/2d5/2)$, $J\pi = 1+, 2+, 3+, 4+$ occur at approximately 2.5 MeV. One can see in the experimental spectrum that levels with these $J\pi$ values occur at approximately 2.5 MeV.

In addition to energy spectra, it is possible to compare several electromagnetic transition rates. The observed enhancement factor, 16, of the E2 transition between the first $2+$ and ground state is well reproduced by the calculated value, 13. Moreover, the observed retardation factor, 17, for the E2 transition between the first $4+$ and $2+$ levels is reasonably well reproduced by the calculated value, 28. In Rho's calculation the effective proton charge was taken to be $2e$.

The magnetic moment of the 2083.6 keV, $4+$ level was calculated to be 3.6 nm which compares well with the measured³¹⁾ value 4.4 nm. This supports the proposition that the dominant configuration of this level is $(g7/2)^2$. For the computation, the g -factors of the $g7/2$ and $d5/2$ orbits were taken from neighboring odd-mass nuclei, and Schmidt values were used for $d3/2$ and $h11/2$ orbits.

The observed $6+$ level at 2108.2 keV is the first of this high J value found in $N=82$ nuclei. Since none of the calculations include configurations with $J=6$ it is difficult to determine the nature of this level. It would be of interest to see if it can be adequately described by 2 qp configurations, and if so, which configurations are dominant. A $6+$ determination has not been made for any level in the scs, Sn nuclei³³⁾, although a $6-$ level has been found at 2.76 MeV excitation in ^{116}Sn .

A discussion of the $0+$, $3-$, $5-$, and several positive parity, higher-lying states follows.

1903.5 keV, 0+. The E2 gamma-transition rate between this 0+ state and the first excited 2+ state is approximately 6 single particle units²⁴⁾. There has been no published theoretical estimate of this transition rate. A calculation of the excitation energies of 0+ states in the N=82 nuclei was recently made by Bès and Veje⁴³⁾ on the basis of the quasi-boson treatment⁴⁵⁾ of the pairing force. According to this calculation there are four low-lying 0+ states in each even-even nucleus and each is a combination of five 2 qp states. So far only one 0+ state has been found in $^{140}_{58}\text{Ce}_{82}$, one in $^{142}_{60}\text{Nd}_{82}$ at 2.21 MeV and two in $^{144}_{62}\text{Sm}_{82}$ at 1.67 and 2.45 MeV. No such state has yet been identified in $^{138}_{56}\text{Ba}_{82}$. In view of this paucity of data, it is difficult to assess the adequacy of the theoretical description.

2464.4 keV, 3-. Low-lying 3- levels have been observed in a large number of even-even nuclei throughout the mass table. Lane and Pendlebury⁴⁴⁾ have shown that the 3- levels can be described as collective octupole oscillations about a spherical equilibrium shape. The arguments supporting this interpretation have been summarized by Perdrisat⁴⁶⁾ as follows:

- (a) The electric-octupole excitation probabilities of the 3- levels are enhanced relative to single-particle transitions.
- (b) The observation of these levels in almost all even-A spherical nuclei suggests that they are due to excitations of the core rather than the cloud nucleons.
- (c) The consistently high $\log ft$ values observed for the allowed beta transitions populating these levels suggest that they have a structure very different from that of the odd-odd nuclear ground states.

For the 3- state in ^{140}Ce , the measured electric-octupole excitation probability, relative to the single-particle transition rate, has the value³⁸⁾, $B(E3)/B(E3)_{sp} = 93$. The $\log ft$ for the allowed β -transition between the 3- ground state of ^{140}La and the 3- state in ^{140}Ce is 8.5. The average $\log ft$ value ob-

served for allowed β -transitions (based on 480 cases) is³⁴⁾ 5.7 ± 1.1 . We see that according to the above criteria the 3- level in ^{140}Ce qualifies as an octupole vibration.

The 867.8 keV, E1 transition between the 2464.4 and 1596.6, 2+ keV levels is the only observed depopulating transition. To obtain a non-vanishing E1 transition matrix element the following selection rules (discussed in Chapter IV B) on the single particle quantum numbers of the initial (i) and final (f) states of the "jumping" proton must be obeyed: $\pi_i \pi_f = -1$, $|j_i - j_f| \leq 1$, and $|\ell_i - \ell_f| \leq 1$. The ℓ values of the shell model orbitals between the Z=50 and Z=82 major shells are 4, 2, 5, 2, 0. The parity and ℓ value selection rules limit the possible E1 transitions to between states having $\ell=4$ and $\ell=5$ proton configurations. However, since the corresponding j values are 7/2 and 11/2, one sees that E1 transitions are j-forbidden. From this consideration one must conclude that the 3- state has some components involving core excitations. If the 1596.6 keV, 2+ state has a dominant $(1g7/2)^2$ configuration (as suggested by Rho's calculations) it seems likely that the 3- state has components involving the $1f5/2$ proton orbital from below the Z=50 major shell. This would result in non-vanishing E1 transition matrix elements between the 3- and 2+ levels.

2350.2 keV, 5-. All the transitions either populating or depopulating this 5- level have mostly E1 character. We have already noted that E1 transitions can not take place between states having configurations involving only the shell model orbitals between the Z=50 and Z=82 major shells. Therefore a more elaborate treatment of this 5- level is required. For the Sn isotopes, where several 5- levels have been observed, it has been suggested⁴⁵⁾ that the 5- levels contain 4 qp excitations. No calculations involving this type excitation in N=82 nuclei have been published.

2412.4 keV, 3+. The lifetime of this state has a measured value²⁶⁾, $(7 \pm 3) \times 10^{-13}$ sec. The relative intensities of the 815.8, 328.8, 64.1 keV, depopulating, M1 gamma-ray transitions

are 100, 98, 0.05, respectively. Using these data, the absolute γ -transition rates in terms of single particle units have values 0.05, 0.6, 0.04 respectively. Thus the 328.8 keV M1 transition proceeds at nearly the rate expected from the single-particle estimate, whereas the 815.8 and 64.1 keV transitions are retarded by a factor of ≈ 20 .

2481.3 keV, 4+. We may compare the intensity ratios $I_{\gamma}(397.8)/I_{\gamma}(131.1)/I_{\gamma}(68.9) = 9/100/11$ of the depopulating γ -ray transitions to the corresponding single particle transition ratios $\lambda(397.8, M1)/\lambda(131.1, E1)/\lambda(68.9, M1) = 30/100/0.0015$. It is apparent that the 68.9 keV transition between the 2481.3 keV, (4+) and 2412.4 keV, 3+ levels is preferred to the other transitions.

On the basis of single-particle transition estimates it is expected that the possible 884.7 keV, E2 transition between the 2481.3 keV, 4+ and 1596.6 keV, 2+ levels should compete favorably with the 68.9 keV transition. However, this transition has not been observed.

2516.1, keV, (4+). If this level has $J\pi=4+$, its character must be quite different from the 2481.2 keV, (4+) level. This is suggested by the fact that these two levels depopulate to different levels. For example, the 2516.1 keV level depopulates to the 1596.6 keV, 2+ level via an E2 transition while the corresponding transition is not observed for the 2481.2 keV level.

2521.8 keV, 2+. This level has an interesting property in that only those transitions expected on the basis of relative single-particle transition rates are observed. We may compare the intensity ratios of the five depopulating γ -ray transitions, $I_{\gamma}(2521.8, E2)/I_{\gamma}(925.2)/I_{\gamma}(618.2)/I_{\gamma}(173.5)/I_{\gamma}(109.4) = 100/206/1.3/3.6/7.8$ with the corresponding single particle transition ratios $\lambda(2521.8, E2)/\lambda(925.2, M1)/\lambda(618.2, E2)/\lambda(173.5, M1)/\lambda(109.4, M1) = 100/403/0.09/2.9/0.73$. The 109.4 and 618.2 keV transitions are enhanced by a factor of ≈ 10 relative to the other three transitions. It would be interesting to know the lifetime of this

level so that the above transition ratios could be evaluated on an absolute scale.

B. Gamma-ray Transition Energies in $^{139}_{57}\text{La}_{82}$, $^{141}_{59}\text{Pr}_{82}$, $^{142}_{60}\text{Nd}_{82}$

1. Experimental Results and Decay Schemes

The following gamma-ray energies have been measured using the 2m curved-crystal spectrometer with the Ge(022) crystal. All three gamma-rays are emitted from nuclei having a neutron number $N=82$.

<u>Decay</u>	<u>Half-life</u>	<u>E (keV)</u>	<u>ΔE (keV)</u>
$^{142}\text{Pr} \rightarrow ^{142}\text{Nd}$	19.2 h	1575.42	± 0.39
$^{141}\text{Ce} \rightarrow ^{141}\text{Pr}$	32.5 d	145.4498	± 0.0049
$^{139}\text{Ba} \rightarrow ^{139}\text{La}$	83 min	165.854	± 0.012

At least five measurements were made on each gamma ray. The indicated uncertainties correspond to the standard deviation of the mean.

The decay schemes for the above nuclei are shown in fig. 3.11. The transitions for which energy values were determined are identified in the figure. In the decay of ^{139}Ba , several other weak gamma-ray transitions have recently been observed⁴⁷⁾, and a more complete decay scheme will be available soon.

A discussion of these three nuclei in the general context of the structure problem of $N=82$ nuclei is given in Chapter V.

In addition to the 0.166 keV γ -ray, at least 12 other weak γ -rays have been observed⁴⁷⁾.

Fig. 3.11. Decay schemes for three nuclei that β^- decay to states in $N=82$ nuclei. Decay schemes are taken from ref.33) and γ -ray transition energies are from present work.

CHAPTER IV

THEORETICAL CONSIDERATIONS

A. The Quasi-Particle Description of Pairing Correlations in Spherical Nuclei

1. Introduction

Many descriptions of nuclear properties begin with the picture (model) of point nucleons moving nearly independently in an average nuclear potential. The motion is only nearly independent because of the presence of residual two-body forces which introduce correlations. The hamiltonian operator for nucleon motion in this model is^{48,49)}

$$H = \sum_{i=1}^A (T_i + V_i) + \sum_{i < j}^A V(X_i, X_j) \quad (4.1)$$

where $X_i = (r_i, s_i, t_i) = (\text{space, spin, isospin})$, T_i denotes the kinetic energy of the i -th nucleon and V_i denotes the Hartree-Fock self-consistent nuclear potential (spherical or non-spherical) felt by the i -th nucleon. The residual two-nucleon interaction is denoted by $V(X_i, X_j)$.

One ambition of theoretical nuclear physics has been to find the A -particle eigenstates of this hamiltonian and thereby to see whether such a model hamiltonian is indeed a good description of nuclear dynamics. In this part of Chapter IV we will discuss some theoretical concepts and approximation techniques developed in recent years for the solution of this complex problem. The purpose will be simply to present the current theoretical ideas relevant to the interpretation of the spectra of single-closed-shell (scs) nuclei such as the $N=82$ isotones.

The first major approximation for scs nuclei is to take into account only the degrees of freedom of nucleons outside the closed shell. For $N=82$ nuclei, the neutron coordinates

are ignored. The neutrons are considered to form an inert, spherical core having a relatively high excitation energy. Further, 50 of the 51 to 65 protons of these nuclei form a similar core. Hence only the coordinates of the protons outside this core are explicitly taken into account. The observed low-energy states are then described simply in terms of the various correlations of these extra-core protons as effected by the residual two-nucleon forces and the interaction of each extra-core proton with the core. Obviously this picture constitutes a rather drastic approximation, but this is our starting point. We shall concern ourselves with a discussion of the problems confronted after this tremendous truncation has been made, viz., the description of the correlated motion of the few extra-core nucleons.

At present a unique expression for the residual two body force can not be given. The approach has been rather, to assume some force and to calculate nuclear properties with it which can be compared with data. From efforts of this type over the past 10 years, has emerged an appreciation for the crucial role played by the short range, attractive part of the residual interaction in determining the low-energy spectra of spherical nuclei. In order to understand this role we consider in the next section a simple idealized system before discussing the general situation as applicable to the $N=82$ nuclei.

Throughout our discussion we conform to present-day usage and write operators and wave functions in the second quantisation representation. For a discussion of this formalism refs.⁴⁸⁻⁵²⁾ may be consulted. Since we will be concerned with heavy spherical nuclei, we use $j-j$ coupling, in which single particle states are characterized by the quantum numbers $(n \ l \ j \ m) = \alpha$. The creation and annihilation operators will be denoted by $a_{\alpha}^{\dagger}$ and a_{α} , and they obey the anticommutation rules for fermions.

2. Simple Spectra of a Short Range Residual Interaction.

In order to gain an appreciation for the correlation in nucleon motion that a short range attractive interaction tends to produce, we discuss the spectra of two and four identical nucleons in a degenerate shell model orbital⁴⁸⁻⁵¹). The spectrum of two nucleons interacting with the very short range δ -force is shown in fig. 4.1. It is seen that this force is most fully exploited, when the nucleons move so as to couple to $J=0$. The other states for which $J \neq 0$ cluster together at a much higher energy. These are the seniority two ($V=2$) states. (The seniority quantum number V gives the number of particles not coupled to $J=0$.) The $2+$ state is slightly favored over other $V=2$ states. Also shown in the figure are the two and four particle spectra of the pairing interaction. This interaction preserves the essential features of the δ -force but is simpler to treat. Again the $J=0$ states are lowest. Next lowest are the $J \neq 0$ states permitted by the exclusion principle in which only two particles are not coupled to $J=0$, i.e., $V=2$ states. Above these are the $V=4$ states.

The picture that emerges from a study of these simple cases is that nucleons can take fullest advantage of a short range residual force by moving pairwise in $J=0$ states. When the requirement that two nucleons do not couple to $J=0$ is imposed, the nucleons can not make as good use of the short range interaction and therefore move in a state of higher energy. These $J \neq 0$ states have the common quantum number $V=2$. When the requirement that four particles do not couple to $J=0$ is imposed, the nucleons can take even less advantage of a short range interaction. These states are at a higher energy and have the quantum number $V=4$.

We are thus led to the idea that when constructing n -particle states for real nuclei, we should couple pairs to $J=0$ to make best use of any short range residual force that might exist. However the restriction of pairs to degenerate orbitals does not in general correspond to the situation in real nuclei.

Fig. 4.1. Spectra for an even number of identical particles in a degenerate ($\Omega=j+1/2$) shell model orbital under the influence of a δ -force and a pairing force. The seniority quantum number V gives the number of particles not coupled to $J=0$. For an even- n nucleon system, $V = 0, 2, 4, \dots, n$.

For example, the protons outside the $Z=50$ core of the $N=82$ isotones have available for occupation the orbitals $1g7/2$, $2d5/2$, $1h11/2$, $2d3/2$, $3s1/2$, and maybe others. Therefore we are faced with constructing a wave function which will correlate particles pairwise to $J=0$ when these particles can occupy a set of non-degenerate orbits. Such a wave function for an even- n system has the form⁴⁸⁻⁵⁰⁾

$$\Psi_{V=0} = N \left(\sum_{\alpha} \sum_{m_{\alpha}} C_{\alpha m_{\alpha}} a_{\alpha m_{\alpha}}^{\dagger} a_{\alpha - m_{\alpha}}^{\dagger} \right)^{\frac{n}{2}} |0\rangle, \quad 4.2$$

where $\alpha=(nlj)$, N is a normalization factor, and the amplitudes $C_{\alpha m_{\alpha}}$ depend on the details of the nuclear force. Unfortunately, general methods for dealing with this type of wave function have not yet become available. A somewhat different wave function which contains the desired type of correlation but which is simpler was constructed by Bardeen, Cooper, and Schrieffer (1957)⁵⁾ to describe the pair correlated motion of electrons in a superconductor. The same type of wave function was introduced to nuclear physics by Bohr, Mottleson, and Pines (1958)⁶⁾. In the next section we will discuss this wave function and its most elementary excitations, viz., quasi-particle excitations.

3. The BCS Treatment of Pairing Correlations.

3a. The BCS wave function and quasi-particles.

As discussed in the last section, the single most important effect of a short range two body force in scs nuclei is to produce pairing correlations. A wave function which contains the desired type of correlation and which is mathematically tractable⁴⁸⁻⁵²⁾ is

$$\Psi = \prod_{\alpha} \prod_{m_{\alpha}=0}^{j_{\alpha}+1/2} (u_{\alpha} + v_{\alpha} s_{\alpha m_{\alpha}} a_{\alpha m_{\alpha}}^{\dagger} a_{\alpha - m_{\alpha}}^{\dagger}) |0\rangle, \quad 4.3$$

where the normalization of Ψ is assured if we have

$$u_{\alpha}^2 + v_{\alpha}^2 = 1. \quad 4.4$$

(In the following discussion Greek indices denote a complete set of single particle quantum numbers, $\alpha=(nljm)$, and corresponding Roman indices denote all except the magnetic quantum number $a=(nlj)$; also, $s_{\alpha}=(-1)^{j_{\alpha}-m_{\alpha}}$. This notation has been used by Baranger⁵³⁾ and Rho^{1,54)}.) The product extends over a range of levels, a , in the vicinity of the Fermi level of the nuclear system. The BCS function Ψ is strikingly different from usual wave functions. It contains components with different number of particles, i.e., if we expand the product (4.3) there is a component with no particles present, components with two particles, etc., all the way to components with 2Ω particles, ($\Omega = \sum_{\alpha} (j_{\alpha}+1/2) =$ pair degeneracy). The products $a_{a m_a}^{\dagger} a_{a -m_a}^{\dagger}$ assure that only $J=0$ pairs occur in all components. Thus Ψ is referred to as a state of generalized seniority $V=0$ and is sometimes written $\Psi_{V=0}$. The component of Ψ with a given even number of particles can be projected out. It is given by⁴⁹⁾

$$P_n |\Psi_{V=0}\rangle = \left(\prod_{\alpha} u_{\alpha}^{2j_{\alpha}+1} \frac{1}{(n/2)!} \left[\sum_{\alpha} \sum_{m_{\alpha}} \left(\frac{v_{\alpha}}{u_{\alpha}} \right) s_{\alpha m_{\alpha}} a_{a m_a}^{\dagger} a_{a -m_a}^{\dagger} \right]^{n/2} |0\rangle \right). \quad (4.5)$$

This component has the desired form (4.2) for an even- n system.

It is also of interest to know the average number of particles in the state Ψ . This can be shown to be^{48,49)}

$$\langle \Psi_{V=0} | N_{op} | \Psi_{V=0} \rangle = \langle \Psi_{V=0} | \sum_{\alpha} \sum_{m_{\alpha}} a_{a m_a}^{\dagger} a_{a m_a} | \Psi_{V=0} \rangle = \sum_{\alpha} (2j_{\alpha}+1) v_{\alpha}^2. \quad (4.6)$$

Hence the average number of particles in level a is $(2j_{\alpha}+1)v_{\alpha}^2$.

The numbers u_{α}^2 and v_{α}^2 are determined by the dynamics of the problem, i.e., the residual interaction, the number of particles present, and the orbitals available for occupancy. The problem of determining the u 's and v 's and the description of excitation spectra is greatly facilitated by introducing the quasi-particle (qp) transformation.

Quasi-particle creation and annihilation operators are defined by⁴⁸⁾

$$b_{\alpha}^{\dagger} = u_{\alpha} a_{\alpha}^{\dagger} + s_{\alpha} v_{\alpha} a_{\bar{\alpha}}$$

$$b_{\alpha} = u_{\alpha} a_{\alpha} + s_{\alpha} v_{\alpha} a_{\bar{\alpha}}^{\dagger}$$

where $\bar{a} = (a, -m_a)$. By virtue of the relation (4.4), the qp operators also satisfy the anticommutation rules for fermions:

$$b_\alpha b_\beta + b_\beta b_\alpha = b_\alpha^\dagger b_\beta^\dagger + b_\beta^\dagger b_\alpha^\dagger = 0$$

$$b_\alpha^\dagger b_\beta + b_\beta b_\alpha^\dagger = \delta_{\alpha\beta}.$$

The great usefulness of qp operators is exhibited by the following properties^{48,49)}

$$b_\alpha |\Psi_{v=0}\rangle = 0 \quad b_{\bar{\alpha}} |\Psi_{v=0}\rangle = 0$$

Thus the BCS wave function is the qp vacuum, and one writes $|\Psi_{v=0}\rangle = |\tilde{0}\rangle$. One qp states are given by

$$b_{am_a}^\dagger |\Psi_{v=0}\rangle = a_{am_a}^\dagger \prod_{\alpha \neq am_a} (u_\alpha + v_\alpha s_\alpha a_\alpha^\dagger a_{\bar{\alpha}}^\dagger) |0\rangle \equiv |v=1, am_a\rangle.$$

We see that a one qp state has the odd(real) particle in a definite orbital (n l j m) just as in the shell model, whereas other particles are paired off. One qp states form an orthonormal set of states and are also orthogonal to $|\tilde{0}\rangle$.

Two qp states are given by

$$b_{am_a}^\dagger b_{bm_b}^\dagger |\Psi_{v=0}\rangle = a_{am_a}^\dagger a_{bm_b}^\dagger \prod_{\substack{\alpha \neq am_a \\ \alpha \neq bm_b}} (u_\alpha + v_\alpha s_\alpha a_\alpha^\dagger a_{\bar{\alpha}}^\dagger) |0\rangle \equiv |v=2; am_a, bm_b\rangle$$

We see that two qp states have two real particles in definite orbitals whereas other particles are paired off. Two qp states form an orthonormal set of states and are also orthogonal to $|\tilde{0}\rangle$ and to one qp states.

With this discussion of the qp vacuum and of one and two qp states we are in a position to consider an approximation technique for finding eigenstates of the shell model hamiltonian (4.1). In this we follow the discussion of discussion of Baranger⁵³⁾ and Rho^{1,54)}.

3b. Dynamics; solving the gap equations.

The u's and v's of the BCS function are the parameters containing the nuclear dynamics. In this section we discuss the equations to be solved for the u's and v's corre-

sponding to low energy states in scs nuclei. These equations derive from the diagonalization of the shell model Hamiltonian in the BCS representation. In the second quantisation representation the shell model hamiltonian (4.1) is written

$$H = \sum_{\alpha} \epsilon_{\alpha} a_{\alpha}^{\dagger} a_{\alpha} + \frac{1}{4} \sum_{\alpha\beta\gamma\delta} (\alpha\beta|V|\gamma\delta) a_{\alpha}^{\dagger} a_{\beta}^{\dagger} a_{\gamma} a_{\delta} , \quad (4.8)$$

Where ϵ_{α} are the single particle energies. The round bracketed matrix elements denote a direct minus an exchanged term, i.e.,

$$(\alpha\beta|V|\gamma\delta) = \langle \alpha\beta|V|\gamma\delta \rangle - \langle \alpha\beta|V|\delta\gamma \rangle .$$

By using the inverse relations between particle and quasi-particle

$$\begin{aligned} a_{\alpha}^{\dagger} &= u_{\alpha} b_{\alpha}^{\dagger} + s_{\alpha} v_{\alpha} b_{\bar{\alpha}} \\ a_{\alpha} &= u_{\alpha} b_{\alpha} + s_{\alpha} v_{\alpha} b_{\bar{\alpha}}^{\dagger} , \end{aligned} \quad (4.9)$$

the hamiltonian (4.8) can be expressed in terms of qp operators.

A difficulty arises because of an approximation that is made. Certain terms in the transformed hamiltonian will be dropped, so the eigenstates of the reduced H are mixtures of states of different number of particles. In order to pick out the state with the closest relation to the correct state for a system of a given number n of nucleons, we insist that the condition

$$\langle \tilde{0} | N_{op} | \tilde{0} \rangle = \sum_{\alpha} (2j_{\alpha} + 1) v_{\alpha}^2 = n \quad (4.10)$$

be satisfied. This extra condition can be taken into account by using a Lagrange multiplier λ , and by working with a new hamiltonian $\mathcal{H}$, related to H by

$$\mathcal{H} = H - \lambda N_{op} . \quad (4.11)$$

When the transformation (4.9) is inserted into $\mathcal{H}$, it becomes (with algebra using Wick's theorem):

$$\mathcal{H} = U + \mathcal{H}_{11} + \mathcal{H}_{20} + \mathcal{H}_{int}$$

$$\begin{aligned}
 U &= \sum_a (2j_a + 1) [v_a^2 (\epsilon_a - \lambda - \frac{1}{2} \mu_a) - \frac{1}{2} u_a v_a \Delta_a] \\
 \mathcal{H}_{11} &= \sum_a [(u_a^2 - v_a^2) \eta + u_a v_a \Delta_a] b_a^\dagger b_a \\
 \mathcal{H}_{20} &= \sum_a s_a [u_a v_a \eta_a + \frac{1}{2} (v_a^2 - u_a^2) \Delta_a] (b_a^\dagger b_a^\dagger + b_a b_a) ,
 \end{aligned} \tag{4.12}$$

where

$$\begin{aligned}
 \Delta_a &= -\frac{1}{2} \sum_b \left(\frac{2j_b + 1}{2j_a + 1} \right)^{\frac{1}{2}} u_b v_b (a a 0 | V | b b 0) \\
 \mu_a &= - \sum_b \sum_J \left(\frac{2J + 1}{2j_a + 1} \right) v_b^2 (a b J | V | a b J) \\
 \eta_a &= \epsilon_a - \lambda - \mu_a
 \end{aligned} \tag{4.13}$$

The subscripts on the $\mathcal{H}$'s refer to the number of qp creation and annihilation operators appearing in the expression. The term $\mathcal{H}_{int}$ is a complicated expression containing four qp operators. It can be written as a sum of three terms:

$$\mathcal{H}_{int} = \mathcal{H}_{22} + \mathcal{H}_{13} + \mathcal{H}_{40}$$

We give an explicit expression only for $\mathcal{H}_{22}$ since this is the term used by Rho in calculating the spectra of the N=82 nuclei. It is:

$$\begin{aligned}
 \mathcal{H}_{22} &= \frac{1}{4} \sum_{\alpha \beta \gamma \delta} \{ (\alpha \beta | V | \gamma \delta) (u_\alpha u_\beta u_\gamma u_\delta + v_\alpha v_\beta v_\gamma v_\delta) \\
 &\quad - 4 (\alpha - \gamma | V | \beta \delta) s_\beta s_\delta u_\alpha u_\delta v_\beta v_\gamma \} b_\alpha^\dagger b_\beta^\dagger b_\gamma b_\delta
 \end{aligned} \tag{4.14}$$

An approximate diagonalization of $\mathcal{H}$ is achieved if the u_α , v_α are chosen to eliminate $\mathcal{H}_{20}$, and the $\mathcal{H}_{int}$ term dropped completely. (For a discussion on the justification for dropping $\mathcal{H}_{int}$ see ref.⁴⁸.) Requiring that $\mathcal{H}_{20} = 0$ results in the following equations:

$$2 u_\alpha v_\alpha = \frac{\Delta_\alpha}{E_\alpha} \quad u_\alpha^2 - v_\alpha^2 = \frac{\eta_\alpha}{E_\alpha} \tag{4.15}$$

where $E_\alpha = (\eta_\alpha^2 + \Delta_\alpha^2)^{1/2}$. These two equations together with Eq.(4.10) are called the gap equations and must be solved for the u 's, and v 's, and λ . Before we discuss what properties

of nuclei can be deduced from these solutions, we briefly list the sequence of steps followed in obtaining the solution. This will clarify what information must actually be put in.

To perform the calculation one must:

(1) determine the set of single particle levels ($n_a l_a j_a m_a$) and their relative energy spacing ϵ_a to be included in the calculation. The states in the vicinity of the Fermi level clearly must be included, however, often it is not clear just how many other levels to include^{1,55)}. The problem of choosing the levels and level spacings is one of the most serious in the calculation on N=82 nuclei¹⁾.

(2) Calculate all the necessary two body matrix elements $\langle abJM|V|cdJM\rangle$. (Only J=0 matrix elements are needed for the present considerations, i.e., determining ground state correlations). This entails choosing single particle wave functions and a two-body residual interaction V. Several choices have been made for V. Among these are the pairing force⁹⁾, δ -force⁹⁾, surface δ -force¹⁰⁾, and the so-called realistic interaction⁵⁵⁾ (Tabakin) which to some extent fits the two nucleon scattering data. Rho¹⁾ used a simple spin dependent force with a Gaussian radial dependence. He also used the radial wave functions of the harmonic oscillator.

(3) Choose a particular nucleus by choosing n. This n is not the mass number A of the nucleus, rather it refers to the nucleons of one type which are considered to be spectroscopically active. In Rho's calculation on N=82 nuclei, n referred to the number of nucleons outside the Z=50 core. It should be mentioned that the calculation is slightly different for even- and odd- n nuclei since the BCS function contains only components with an even number of particles. Frequently in practice it turns out that the u's and v's of the pairwise occupied states don't vary much in going from an even-n to the neighboring odd- (n + 1) nucleus. Then the u's and v's calculated for an even-n nucleus are used to determine the properties of the adjacent odd-(n + 1) nucleus.

(4) The complicated set of coupled Eqs. (4.10) and (4.15),

called the gap equations, are solved for u_a , v_a , Δ_a , μ_a , and λ . (For the case of a pairing force Δ_a is independent of a). With these solutions we obtain the quantities $E_a = (\eta_a^2 + \Delta_a^2)^{1/2}$ which are the single qp excitation energies (to be discussed in the next section).

To get a feel for the magnitude of the various quantities we show a typical solution of the gap equations in table 4.1. The numbers were obtained by Rho¹⁾ for the ground state of $^{140}_{58}\text{Ce}_{82}$. We observe that the values of the energy gap Δ_a do not fluctuate very much and have an average value 1.07 MeV. Hence the notion of an energy gap without reference to a single particle level is still meaningful. Also, we point out that the range of single particle energies ϵ_a is 3.20 MeV while that of qp energies E_a is 1.76 MeV. This corresponds to the observed "compression" effect of low-lying levels observed for odd-A nuclei⁴⁸⁾. The quantity $1/2(2j_a + 1)v_a^2$ corresponds to the probability for finding a pair in level a in the ground state of Ce-140.

Table 4.1
Solution of the gap equations corresponding to $^{140}_{58}\text{Ce}_{82}$.

	$1g_{7/2}$	$2d_{5/2}$	$1h_{11/2}$	$2d_{3/2}$	$3s_{1/2}$
ϵ_a	0	1.00	2.18	2.88	3.20
Δ_a	1.116	1.145	0.980	1.144	0.972
v_a	0.820	0.521	0.250	0.215	0.167
$\frac{1}{2}(2j_a+1)v_a^2$	0.67	0.20	0.094	0.023	0.007
E_a	1.189	1.288	2.022	2.721	2.954

The quantities E_a , Δ_a , and E_a are given in MeV. The parameters of the calculation were $n=8$, $V=V_P e^{-\beta r_{12}}$, $\beta^{-1/2} = 1.732$ fermis. The value of λ is 0.411 MeV.

3c. Energy spectra.

By dropping $\mathcal{H}_{int}$ and determining the u 's and v 's such that $\mathcal{H}_2 = 0$, the truncated hamiltonian has the very simple form

$$\mathcal{H} = U + \mathcal{H}_{II} = U + \sum_{\alpha} E_{\alpha} b_{\alpha} b_{\alpha} . \quad (4.16)$$

The energy spectra of this hamiltonian can be obtained directly in the qp representation.

Ground States

The ground state of an even- n nucleus corresponds to the BCS vacuum. The theoretical value of the energy of this n -nucleon system is

$$\begin{aligned} E(n, v=0) &= \langle \tilde{0} | \mathcal{H} | \tilde{0} \rangle = \langle \tilde{0} | \mathcal{H} + \lambda N_{op} | \tilde{0} \rangle = U + \lambda n \quad (4.17) \\ &= \sum_{\alpha} (2f_{\alpha} + 1) \left[v_{\alpha}^2 (\epsilon_{\alpha} - \lambda - \frac{1}{2} \mu_{\alpha}) - \frac{1}{2} u_{\alpha} v_{\alpha} \Delta_{\alpha} \right] + \lambda n . \end{aligned}$$

The ground state of the odd- $(n+1)$ nucleus corresponds to the one qp state of lowest energy. The theoretical value of the energy of this $(n+1)$ -nucleon system is

$$\begin{aligned} E(n+1, v=1, \alpha) &= \langle v=1, \alpha | U + \mathcal{H}_{II} + \lambda N_{op} | v=1, \alpha \rangle = U + E_{\alpha} + \lambda(n+1) \quad (4.18) \\ &= E(n, v=0) + E_{\alpha} + \lambda \end{aligned}$$

where E_{α} is the smallest single qp energy. The last relation is only approximately true since the u 's and v 's of the pairwise occupied states of a one qp state are slightly different than the u 's and v 's of the qp vacuum.

To relate these theoretical energies to the mass data we must give the theoretical expression for the total nuclear binding energy $B.E.(Z, N)$. In terms of the physical picture we have adopted for the $N=82$ nuclei (neutron core, proton core, n extra-core protons) the binding energy may be regarded as a sum of three terms⁸⁾, i.e.,

$$B.E.(Z, N) = W_0(Z-n, N) + \sum_i^n [\epsilon_0 - (\epsilon_i - \epsilon_1)] - \sum_{i,j} v_{ij} \quad (4.19)$$

where v_{ij} is positive (negative) for a repulsive (attractive) residual interaction. The total binding energy of the neutron ($N=82$) and proton ($Z=50$) core is denoted by W_0 . The additional binding of one extra core proton is ϵ_0 . As more protons are added the additional binding energy is determined by the single

particle energies ϵ_i and the residual interaction V_{ij} . If we assume that W_0 and ϵ_0 are independent of n , Eq. (4.19) can be written

$$\begin{aligned} B.E.(Z,N) &= W_0 + n\epsilon_0 - \left\{ \sum_{i=1}^n (\epsilon_i - \epsilon_0) + \sum_{i,j}^n v_{ij} \right\} \\ &= W_0 + n\epsilon_0 - \langle \tilde{O} | H | \tilde{O} \rangle = W_0 + n\epsilon_0 - E(n, v=0), \end{aligned} \quad (4.20)$$

where Z and n are even. The binding energy for the odd- $(Z+1)$ nucleus is ($n=Z-50$):

$$B.E.(Z+1,N) = W_0 + (n+1)\epsilon_0 - E(n+1, v=1). \quad (4.21)$$

We are interested in comparing theoretical energies with the experimental odd-even mass differences and single-proton separation energies of $N=82$ nuclei. The definition of the proton separation energy S_p in terms of total nuclear binding energies B.E. is

$$S_p \equiv B.E.(Z,N) - B.E.(Z-1,N). \quad (4.22)$$

The experimental binding energies of many nuclei can be obtained from mass tables⁵⁶⁾. The theoretical proton separation energy is (using the definition (4.22) and Eq. (4.20):

$$S_p(Z+1,N) = \epsilon_0 + E(n, v=0) - E(n+1, v=1) \cong \epsilon_0 - E_\pi - \lambda. \quad (4.23)$$

Since the absolute value of the lowest single particle binding energy ϵ_0 is undertermined, a comparison between theory and experiment on any one nucleus is not possible. However the trends of the S_p vs. Z curves can be compared. The data on the $N=82$ nuclei shows a decrease in S_p with increasing Z (see fig. 5.6b). For example, the single proton separation energy of $^{137}_{55}\text{Cs}_{82}$ is 7.7 MeV while that of $^{143}_{61}\text{Pm}_{82}$ is 4.2 MeV. Theoretical values of ΔS_p for the $N=82$ nuclei have not been published.

The odd-even proton mass difference P_p (pairing energy) may be defined for the odd- $(Z+1)$ nucleus in terms of the proton separation energies by the equation

$$P_p(Z+1,N) \equiv \frac{1}{2} [S_p(Z+2,N) - S_p(Z+1,N)]. \quad (4.24)$$

The theoretical value of $P_p(Z+1,N)$ is closely approximated by the smallest qp energy of the neighboring even- (Z,N) nucleus¹⁾. As an example we observe that the experimental odd-

-even mass difference for $^{141}_{59}\text{Pr}_{82}$ is 1.185 MeV while $E_{\alpha}(\text{min})$ for $^{140}_{58}\text{Ce}_{82}$ is 1.189 MeV (table 4.1). A graph of experimental and theoretical pairing energies for the $N=82$ isotones is shown in fig. 5.7.

Excited States

For odd-A nuclei, there is no essential difference in the theoretical description of the ground state and low-lying excited states. They all correspond to 1 qp states with the ground state being the lowest-lying. The excitation energy between the ground state and other 1 qp states is given by

$$E(n+1, v=1, \alpha) - E(n+1, v=1, \beta) \cong E_{\alpha} - E_{\beta} = \sqrt{\gamma_{\alpha}^2 + \Delta_{\alpha}^2} - \sqrt{\gamma_{\beta}^2 + \Delta_{\beta}^2}.$$

The qp energy differences ($E_{\alpha} - E_{\beta}$) are in many cases much smaller than the single qp energies themselves. They are also smaller than the shell model energy differences ($\epsilon_{\alpha} - \epsilon_{\beta}$). This is in agreement with the observation that excitation energies of low-lying states in odd-A nuclei are small compared with odd-even mass differences, and with the observation that energy spacings between low-lying states of many odd-A nuclei are smaller than the corresponding shell model spacings. We can also understand now why the simple shell model of Mayer and Jensen is so successful for odd-A nuclei in accounting for the spin and parity of the ground state and first few excited states in terms of the degrees of freedom of the odd particle alone. Due to the pairing correlation the three and five qp states are pushed an energy $2E_{\alpha}$ and $4E_{\alpha}$ above the low-lying states thereby simplifying considerably the low energy part of the spectrum.

In even-A nuclei, the description of the ground state and low-lying excited states is quite unsymmetrical. Excited states correspond to 2 qp states while the ground state is the qp vacuum. The 2 qp excitation energy is given by

$$\begin{aligned} E(n, v=2; \alpha\beta) &= \langle v=2, \alpha\beta | U + \mathcal{H}_{11} + \lambda N_{\alpha\beta} | v=2, \alpha\beta \rangle = U + \lambda n + E_{\alpha} + E_{\beta} \\ &= E(n, v=0) + E_{\alpha} + E_{\beta}. \end{aligned}$$

We see that to excite an even-A nucleus, two pairs must be broken with the consequent cost in energy $E_\alpha + E_\beta$. From table 4.1 we see that for $^{140}_{58}\text{Ce}_{82}$ this energy is in excess of 2 MeV which is in qualitative agreement with the observation that there is no excited level below 1.5 MeV and that there are many levels in the vicinity of 2.4 MeV.

The general features of the low energy spectra of even- and odd-A nuclei are described quite well in terms of independent qp motion. However, a more detailed comparison of this theory with the data reveals the need for an extension of the theory. The presence of at least one level in many even-even nuclei below the two qp energy must be explained. The need for such an explanation motivated Rho to extend the simple independent qp theory described here to include to some extent the effects of the neglected $\mathcal{H}_{int}$ term. This term contains the effects of qp interactions and could lead to a lowering of certain levels. A brief statement of Rho's work is given in the next section.

4. Treatment of Quasi-Particle Interactions; Tamm Dancoff Approximation.

There are two notable features of the low-energy spectra of even-n spherical nuclei that the independent qp description fails to predict: the existence of a 2+ state below the 2 qp energy $E_\alpha + E_\beta$, and the enhanced E2 transition rate between this state and the ground state. In an attempt to understand these features, Rho¹⁾ investigated the role of the 4 qp operator $\mathcal{H}_{int}$, which so far has been neglected. Rho considered the $\mathcal{H}_{22}$ term to be most important for even-n nuclei. The diagonalization of this term results in eigenstates which are linear combinations of two qp states (TDA). We will briefly review the manner in which the two qp amplitudes and energy eigenvalues are obtained.

For the consideration of excited states, Rho takes

the simplified hamiltonian

$$\mathcal{H} = \mathcal{H}_{11} + \mathcal{H}_{22} .$$

The eigenstates of this $\mathcal{H}$ and of total angular momentum may be written in terms of 2 qp wave functions

$$\Phi_J^{(i)} = \sum_{ab} C_{abJ}^{(i)} \Phi_{abJ} . \quad (4.26)$$

The states Φ and Φ have the following properties:

$$\mathcal{H}_{11} \Phi_{abJ} = (E_a + E_b) \Phi_{abJ}$$

$$\mathcal{H} \Phi_J^{(i)} = \omega_J^{(i)} \Phi_J^{(i)}$$

$$\vec{J}^2 \begin{Bmatrix} \Phi_{abJ} \\ \Phi_{abJ} \end{Bmatrix} = J(J+1) \begin{Bmatrix} \Phi_{abJ} \\ \Phi_{abJ} \end{Bmatrix}$$

$$\langle \Phi_J^{(i)} | \Phi_J^{(j)} \rangle = \delta_{ii'} \delta_{JJ'} ,$$

The superscript i distinguishes the different eigenstates of $\mathcal{H}$ with the same J value. An explicit expression for a 2 qp wave function of definite J is

$$\Phi_{abJ} = \frac{1}{\sqrt{1 + \delta_{abJ}}} \sum_{m_a m_b} \langle j_a m_a j_b m_b | JM \rangle b_{j_a m_a}^+ b_{j_b m_b}^+ | \tilde{0} \rangle , \quad (4.27)$$

where $\langle j_a m_a j_b m_b | JM \rangle$ are Clebsch-Gordon coefficients. The problem is to obtain the energies $\omega_J^{(i)}$ and the 2 qp amplitudes $C_{abJ}^{(i)}$. Rho shows that these are obtained by solving the matrix equation

$$C_{abJ}^{(i)} = \frac{\sum_{cd} \langle \Phi_{abJ} | \mathcal{H}_{22} | \Phi_{cdJ} \rangle C_{cdJ}^{(i)}}{(\omega_J^{(i)} - E_a - E_b)} ,$$

where $\mathcal{H}_{22}$ is given by Eq. (4.14). He obtains sets of solutions $(C_{abJ}^{(i)}, \omega_J^{(i)})$ for states $(J\pi = 1+, 2+, 3+, 4+)$ of even- A nuclei in the range $A=136 - 142$.

An example of a solution for $^{140}_{58}\text{Ce}_{82}$ is shown in table 4.2. This table shows the actual 2 qp configurations taken into account for all calculations. The numbers correspond to the lowest $J=2$ state in Ce-140. For this state the $(g7/2)^2$ and $(d5/2)^2$ components are seen to be dominant. Un-

Fortunately one cannot say that the calculation comes out very well on the excitation energy $\omega_{J=2} = 1.558$ MeV vs experimental $E=1.596$ MeV, since this energy was used to fix a parameter in the two-body interaction (spin-triplet-to-singlet ratio). The $B(E2)$ γ -ray transition rate between this $J=2$ state and the ground state was calculated to be 13 times larger than the single particle estimate. The experimental value is ≈ 16 . Further comparisons of experiment with theory are given in Chapter III.

Table 4.2

Two qp amplitudes and excitation energy of the first $2+$ state in $^{140}_{58}\text{Ce}_{82}$

	$(g_{7/2})^2$	$(d_{5/2})^2$	$(d_{3/2})^2$	$(h_{1/2})^2$	$(d_{5/2}g_{7/2})$	$(d_{5/2}d_{3/2})$	$(g_{7/2}d_{3/2})$
C_{abJ}	0.87	0.42	0.06	0.15	-0.11	-0.10	0.13

$$\omega_{J=2} = 1.558 \text{ MeV}$$

Without going into a deeper analysis, we simply observe that it is difficult to assess the adequacy of this theory (BCS + TDA) for the $N=82$ nuclei since many parameters are varied. Also, the single particle level spacings are not well known. The role of 4 qp configurations should be further investigated. Nevertheless, it is eminently clear that the model does describe qualitatively some general features of these nuclei.

5. Effects of Pairing Correlations on Electromagnetic Moments and Transition Rates

We have seen that the short range residual interaction produces a high degree of pairing correlations in nucleon motion. Thus nucleons do not fill completely one shell model level before starting to fill the next highest, but have a non-zero occupation probability for several levels near the Fermi level. On physical grounds one expects this correlation to affect the electromagnetic matrix elements. To determine this effect quantitatively one expresses the electromagnetic multipole operators (discussed in Chapter IV B) in terms of qp operators and evaluates matrix elements between $0, 1, 2, \dots$ qp states. It is found ^{1,48,52,60} that these matrix elements are related to ordinary particle motion matrix elements by simple factors involving the u 's and v 's. We give explicit expressions for these factors for electromagnetic moments and transition rates.

5a. Magnetic dipole moment.

Odd-A Nucleus, $V=1$ States⁸⁾

$$\mu = \langle V=1, \alpha | \mu_z | V=1, \alpha \rangle_{m_\alpha = j_\alpha} = j_\alpha g_\alpha = j_\alpha \left\{ g_p \pm \frac{1}{2j_\alpha + 1} (g_s - g_l) \right\}, \quad j_\alpha = l_\alpha \pm \frac{1}{2} \quad (4.28)$$

$$g_s = \begin{Bmatrix} 5.9 \\ -3.8 \end{Bmatrix} \quad \text{and} \quad g_l = \begin{Bmatrix} 1 \\ 0 \end{Bmatrix} \quad \text{for} \quad \begin{Bmatrix} \text{protons} \\ \text{neutrons} \end{Bmatrix}$$

The magnetic moment of a qp is seen to be the same as that of a particle. Thus odd-A nuclei should have magnetic moments close to Schmidt values.

Even-A Nucleus, $V=2$ States¹⁾

$$\mu_J = \langle \Phi_{abJM=J} | \mu_z | \Phi_{abJM=J} \rangle = g_J J \quad (4.29)$$

$$= \frac{1}{2J+1} \left[(g_a + g_b) J(J+1) + (g_a - g_b) \{ j_a(j_a+1) - j_b(j_b+1) \} \right]$$

where g_a is the g-factor for the j_a state. Again the magnetic moment is not influenced by the pairing correlation (in the approximation of non-interacting qp).

5b. Electric quadrupole moments

Odd-A Nucleus, $V=1$ States⁸⁾

$$\begin{aligned} Q_{sqp} &= \langle v=1, j_a m_a = j_a | Q_{op} | v=1, j_a m_a = j_a \rangle = (u_a^2 - v_a^2) \left\{ -\frac{2j_a - 1}{2j_a + 2} \langle j_a | r^2 | j_a \rangle \right\} e_{eff} \\ &= (u_a^2 - v_a^2) \cdot Q_{sp} \end{aligned} \quad (4.30)$$

The single qp (sqp) electric quadrupole moment is $(u_a^2 - v_a^2)$ times the single particle moment. For a state, a, far above the Fermi level, i.e., $u_a^2 \approx 1$, $v_a^2 \approx 0$, the sqp has nearly the same moment as the real particle. For a state far below the Fermi level, i.e., $u_a^2 \approx 0$, $v_a^2 \approx 1$, the qp has nearly the magnetic moment of a hole. For a state near the Fermi level, i.e., $u_a^2 \approx v_a^2 \approx 0.5$ the sqp electric quadrupole moment is much less than the particle moment. It is seen that the effect of pair correlated motion is to reduce the quadrupole moments for odd-A nuclei.

Even-A Nucleus, $V=2$ States

The ground states of even-even nuclei have $J=0$ and therefore can have no measurable static deformation. The quadrupole moments of excited $J \neq 0$ states have not yet been measured for any $N=82$ nuclei so we will not discuss the corresponding theoretical quantity.

In this regard an important recent development should be mentioned. A large static electric quadrupole moment (≈ 0.5 barn) has been measured⁵⁷⁾ for ^{114}Cd in its first excited $2+$ state. In a theoretical description which regards this state as one phonon quadrupole vibration, the predicted quadrupole moment is zero⁵⁸⁾. Hence a modification of a traditional theoretical description is required. Such a description, in terms of qp motion, has been set forth within the last several months by

G. Do Dang and A. Klein⁵⁹⁾, but detailed calculations are not yet available. The measurement of the quadrupole moment of an excited state in a scs nucleus, such as ¹⁴⁰Ce, might have very interesting consequences.

5c. Electromagnetic transition rates in even- and odd-A nuclei.

The effect on γ -ray transition rates caused by the pairing correlation may be expressed by a factor $D(i \rightarrow f; \sigma L, \Delta v)$ which multiplies the ordinary particle transition rates. We may write^{1,8,60)}

$$T_{sp}(i \rightarrow f; \sigma L, \Delta v) = [D(i \rightarrow f; \sigma L, \Delta v)]^2 T_{sp}(i \rightarrow f; \sigma L)$$

where L denotes the transition multipolarity, σ denotes the type of transition (electric or magnetic), and Δv is the change in seniority between the initial (i) and final (f) qp states. The single particle transition probabilities T_{sp} are discussed in part B of this chapter. Expressions for the D -factors^{52, 60)} are given in the table below.

Table 4.3

The D -factors relating particle to quasi-particle gamma-transition rates

Radiation type	Change in seniority	
	$\Delta v = 0$	$\Delta v = 2$
EL	$(u_i v_f - v_i u_f)$	$(u_i v_f + v_i u_f)$
ML	$(u_i v_f + v_i u_f)$	$(u_i v_f - v_i u_f)$

It can be seen that electric transitions having $\Delta v=0$, can be affected rather drastically. For states i and f near the Fermi level, the factor $D(i \rightarrow f; EL, \Delta v = 0)$ has small values⁵²⁾. Similarly, in $\Delta v=2$ transitions, such as excited-to-ground-state transitions in odd-A nuclei, magnetic transition rates can be reduced significantly. In a transition between an ex-

cited 2 qp state ($V=2$) and the ground state ($V=0$) the u's and v's refer to the two quasi-particles of the initial state¹).

In Rho's description of the even, $N=82$ nuclei, excited states are linear combinations of 2 qp states. The transition rates are therefore given by the absolute square of a sum of matrix elements. In this case each term in the sum contains its own D-factor. This can lead to further enhancement or retardation in the transition rate. The observed enhancements for $2_1+ \rightarrow 0+$, E2, transitions and the observed retardations for $4_1+ \rightarrow 2_1+$, E2 transitions in $N=82$ nuclei are reasonably well reproduced by Rho's calculation. A comparison with data on $^{140}_{58}\text{Ce}_{82}$ is given in Chapter III.

B. GAMMA-DECAY TRANSITION RATES

1. Introduction

The electromagnetic interaction with the nuclear system has played an important role in our understanding of the nucleus. This stems in part to the fact that the basic interactions are probably rather well understood so that any measurement pertaining to these quantities starts, at least, with something that is more or less known. Further, the fact that these interactions are weak allows the use of perturbation theory in its lowest non-vanishing order, thus an essentially "unperturbed" nuclear system is measured. It is for these reasons, according to Professor de-Shalit⁶¹⁾, "that electromagnetic tests of nuclear-model wave-functions have proved very useful and inspiring,...".

A nuclear model wave function is tested, by comparing calculated matrix elements with measured ones. This assumes that we know the operators which go with the particular quantity being measured and with the particular set of wave functions. These quantities must in fact be postulated and therefore they constitute a part of the nuclear model⁶¹⁾. For our discussion we will be concerned only with the wave functions obtained with the assumption of a nuclear shell model. The interaction operators correspond to point nucleons interacting with the electromagnetic field simply through their charges and magnetic moments (meson coordinates have been eliminated⁶²⁾). Also, the non-relativistic motion of nucleons is assumed.

The calculation and discussion of electromagnetic matrix elements within this framework can be found in many places in the literature^{63,64)}. We will confine ourselves to a statement of pertinent results as needed for the interpretation of data on nuclei dealt with in this thesis.

Since relative γ -ray transition rates were measured in the present experiments we are interested in comparing the theory

with these measurements. Before giving the theoretical formulae, we state the quantum numbers used to characterize a γ -ray transition. Since nuclear states are considered to be eigenstates of energy, parity, and angular momentum we may specify the initial and final states by $|E_i, \pi_i, J_i, M_i\rangle$ and $|E_f, \pi_f, J_f, M_f\rangle$ respectively. The emitted photon is specified by its total angular momentum, LM , its type σ (electric or magnetic), and its energy $\hbar\omega = E_i - E_f$ or, equivalently, by its wave number $k = \omega/c$. The quantities σL also specify the parity of the radiation field.

2. The Multipole Operators

The transition probability/sec for a nucleus to make a transition between states of angular momenta (J_i, J_f) and parity (π_i, π_f) with emission of a photon of a multipole order L and energy $\hbar k = E_i - E_f$, is given (in first order perturbation theory) by the general expression⁶⁴⁾

$$T(\sigma L; J_i \pi_i \rightarrow J_f \pi_f) = \frac{8\pi(L+1)}{L[(2L+1)!!]^2} \frac{1}{\hbar} k^{2L+1} B(\sigma L; J_i \pi_i \rightarrow J_f \pi_f). \quad (4.31)$$

The long wavelength approximation $ka \ll 1$, where a denotes the nuclear radius, has been assumed. When written in this form all the nuclear structure effects are contained in the reduced matrix element⁶⁴⁾

$$B(\sigma L; J_i \pi_i \rightarrow J_f \pi_f) = \frac{1}{2J_i+1} \sum_{M_i, M_f} |\langle \pi_f J_f M_f | \mathcal{M}_{LM}^\sigma | \pi_i J_i M_i \rangle|^2. \quad (4.32)$$

The multipole operators $\mathcal{M}_{LM}^\sigma$ for electric and magnetic transitions respectively are given by the following sums over all A nucleons^{64,65)}:

$$\mathcal{M}_{LM}^E = \sum_{i=1}^A \left\{ e_i r_i^L Y_{LM}^*(\theta_i, \varphi_i) - i g_{s_i} \mu_0 \hbar \frac{1}{L+1} \vec{S}_i \cdot \vec{r}_i [\text{grad}_i r_i^L Y_{LM}(\theta_i, \varphi_i)]^* \right\} \quad (4.33)$$

$$\mathcal{M}_{LM}^M = \mu_0 \sum_{i=1}^A \left\{ \frac{2}{L+1} (g_{l_i} \vec{L}_i + g_{s_i} \vec{S}_i) \cdot [\text{grad}_i r_i^L Y_{LM}(\theta_i, \varphi_i)]^* \right\} \quad (4.34)$$

The g 's are the orbital- and intrinsic g -factors for nucleons, and $\mu_0 = e\hbar/2m_p c = 0.505 \times 10^{-13}$ erg/gauss is the nuclear magneton (nm). For protons $e_i = e$ and for neutrons $e_i = 0$.

3. Selection Rules

The conservation of angular momentum and parity for the system of nucleus plus gamma-radiation and the assumption that nuclear states are eigenstates of angular momentum and parity impose general selection rules on the possible multipole orders L of a gamma-radiation between two nuclear states. The selection rules which are satisfied rigorously for transitions between nuclear states of angular momenta (J_i, J_f) and parities (π_i, π_f) are^{64,65}:

- (1) $|J_i - J_f| \leq L \leq |J_i + J_f|$
- (2) $\pi_i \pi_f = (-1)^L$ for EL radiation
 $= -(-1)^L$ for ML radiation
- (3) $J_i = 0 \rightarrow J_f = 0$ single photon emission forbidden

Possible multipolarities for γ -ray transitions consistent with these selection rules are given in table 4.4. For a given $\Delta J = |J_i - J_f|$ and parity change, the table gives the transition multipolarity to be expected⁶⁴). Only the lowest two multipole orders are given; the second is in parenthesis to denote that it is generally much weaker (except for the case M1 (E2)) and is forbidden when either J_i or J_f is zero.

Table 4.4

Possible Multipolarities for Gamma-Radiation

Parity change	Change in angular momentum, $\Delta J = J_i - J_f $							
	$\Delta J = 0$			$\Delta J = 1$	$\Delta J = 2$	$\Delta J = 3$	$\Delta J = 4$	$\Delta J = 5$
	$0 \rightarrow 0$	$\frac{1}{2} \rightarrow \frac{1}{2}$	$J_i = J_f \geq 1$					
No	forbidden	M1	M1(E2)	M1(E2)	E2(M3)	M3(E4)	E4(M5)	M5(E6)
Yes	forbidden	E1	E1(ME)	E1(E2)	M2(E3)	E3(M4)	M4(E5)	E5(M6)

Isobaric Spin Selection Rules

To the extent that nuclear states are eigenstates of isobaric spin, certain selection rules should be observed in electromagnetic transitions. A recent discussion of the foundation and implications of this hypothesis was given by E. K. Warburton⁶⁶⁾. Gamma-ray transitions between pure states of isobaric spin (T_i, T_f) in a nucleus with the z-component of isobaric spin $T_z = 1/2(N - Z)$ must obey the selection rules:

(4) $\Delta T = 0, \pm 1$ for all multipolarities

(5) Corresponding transitions in conjugate nuclei (opposite signs of T_z) with $\Delta T = 1$ proceed at the same rate.

Rule (4) can be seen physically as a consequence of the fact that only one nucleon may change its quantum number in an electromagnetic transition and the nucleon has an isobaric spin of $1/2$.

For the special case of E1 transitions in self-conjugate nuclei ($N=Z$) a further selection rule is predicted, viz., $\Delta T = 0$ transitions are forbidden. Data on this class of transition indicate⁶⁶⁾ that the inhibition due to this selection rule is not complete, but is about a factor of 30. (This has been interpreted to show the presence of approximately 3 % of isobaric spin impurities⁶⁶⁾.)

The role of the isobaric spin quantum number in heavy nuclei has been of considerable interest in recent years but is as yet not well understood⁶⁷⁾. Its effect on γ -ray transitions in the $N=82$ nuclei is therefore not considered further.

4. Transition rates between single neutron and single proton states

In this section we give the theoretical γ -ray transition probability /sec for a nucleon to make a transition between the single particle shell model states identified by (n_i

l_i s j_i) and $n_f l_f$ s j_f). This model is a simple approximation to odd-A nuclei. In this model, the A-1 non-jumping nucleons form an inert core which has no spin or charge and is fixed to a point in space. The coordinates $\vec{r}$ of the jumping nucleon are relative to this fictitious point. If this nucleon is a proton, it is considered to have a charge e and intrinsic magnetic moment $\mu_p = 2.79 \text{ nm}$, if a neutron the net charge is zero and $\mu_n = -1.91 \text{ nm}$. Since the spin of the core is zero, the observed spins of the nucleus are $J_i = j_i$ and $J_f = j_f$.

This is of course an oversimplified picture of the nucleus. In cases where the transition rates predicted by this model disagree with the observations, one cannot always assess very well the roles played in the discrepancy by the above assumptions. In cases where agreement is found one cannot conclude that the model is completely correct. On the other hand, this simple model is quite useful since transition rates for more realistic models can be expressed in terms of the results of this model. In part A we saw, for example, that the quasi-particle transition rates are related to the particle transition rates by simple D-factors.

Electric Transitions

The gamma-decay transition probability/sec for electric transitions (obtained when the general expression (4.31) is evaluated in the strict single particle shell model) is given by⁶⁸⁾

$$T_{sp}(\text{electric}, L) = \frac{2(L+1)}{L[(2L+1)!!]^2} \frac{k^{2L+1}}{\hbar} a^{2L} I_L^2 S(j_i, j_f, L) \left[\epsilon + \frac{\eta}{L+1} k\mu \right]^2 \quad (4.35)$$

where for a proton transition $\epsilon=e$, $\mu=\mu_p$; for a neutron transition $\epsilon=0$, $\mu=\mu_n$. The nuclear radius is denoted by a . The other quantities are defined as follows:

$$I_L = \int_0^\infty R_{n_f, l_f} \left(\frac{r}{a} \right)^L R_{n_i, l_i} r^2 dr$$

$$\eta = l_i(l_i+1) - j_i(j_i+1) - l_f(l_f+1) + j_f(j_f+1)$$

$$S(j_i j_f L) = S(l_i j_i l_f j_f; \frac{1}{2} L) = \frac{Z_o^2(l_i j_i l_f j_f; \frac{1}{2} L)}{2 j_f + 1}$$

$$= (2 l_i + 1)(2 l_f + 1)(2 j_i + 1)(2 L + 1) \begin{Bmatrix} l_i j_i \frac{1}{2} \\ j_f l_f L \end{Bmatrix}^2 \begin{pmatrix} l_i & l_f & L \\ 0 & 0 & 0 \end{pmatrix}^2$$

Values of 6-j symbols, $\left\{ \begin{matrix} & & \\ & & \end{matrix} \right\}$, and 3-j symbols, $\begin{pmatrix} & & \\ & & \end{pmatrix}$, have been tabulated for convenient use by Rotenberg et al.⁶⁹⁾.

Within the framework of this model, further selection rules are predicted for electric transition: $\pi_{l_i} \pi_{l_f} = (-1)^L$ and $|l_i - l_f| \leq L \leq |l_i + l_f|$. Also, it should be noted⁶⁸⁾ that for the special case $L = |j_i - j_f|$, $j_f = 1/2$ the statistical factor reduces to 1.

The following additional approximations are frequently made⁶⁴⁾:

- (a) $I_L = \frac{3}{L+3}$ - this corresponds to assigning to the radial functions a constant value $R_{ne} = 3 a^{-3/2}$ inside the nucleus and $R_{ne} = 0$ outside the nucleus.
- (b) The second term in the brackets is neglected for proton transitions since it is small compared to the first term for transition energies > 10 MeV.

With these approximations the transition probability/sec for protons reduces to

$$T_{sp}(EL) = \frac{4.38(L+1)}{L[(2L+1)!!]^2} \frac{3}{L+3} \left(\frac{E_p(\text{MeV})}{197.3} \right)^{2L+1} (a \cdot 10^{-13} \text{ cm})^{2L} S(j_i j_f L) 10^{21} \text{ sec}^{-1} \quad (4.36)$$

For neutron transitions, only the second term in the brackets (Eq. (4.35)) is present. Thus neutron transitions are predicted to proceed at much slower rates than proton transitions. Since this disagrees with the observations, the need for modifications of this model is already apparent.

The unit for specifying electric transition rates, commonly referred to as the Weisskopf unit⁶³⁾, is given by Eq. (4.36) with S set equal to 1. The Moszkowski unit⁶⁴⁾ is given by Eq. (4.36).

Magnetic Transitions

The γ -ray transition probability/sec for a magnetic transition (obtained when the general expression (4.31) is evaluated for the strict single particle shell model) is given by⁶⁸⁾

$$T_{sp}(\text{magnetic}, L) = \frac{8(L+1)}{L[(2L+1)!!]^2} \frac{1}{\hbar} \alpha^{2L+1} I_{L-1}^2 S(j_i, j_f, L) \left[F\mu + \frac{FG\delta}{L+1} \right]^2 \mu_o^2, \quad (4.37)$$

where for proton transition $\delta=1$, $\mu=\mu_p \text{ nm}$ and for neutron transitions $\delta=0$, $\mu=\mu_n \text{ nm}$. The statistical factor is defined as follows:

$$S(j_i, j_f, L) = \frac{Z_o^2(l'_i, j_i, l_f, j_f; \frac{1}{2}L)}{2j_i + 1}$$

$$= (2l'_i + 1)(2l_f + 1)(2j_i + 1)(2j_f + 1) \begin{pmatrix} l'_i & j_i & \frac{1}{2} \\ j_f & l_f & L \end{pmatrix} \begin{pmatrix} l'_i & l_f & L \\ 0 & 0 & 0 \end{pmatrix}$$

where $l'_i = l_i \pm 1$ according as $j_i = l_i \pm 1/2$. The values of F and G are given in the chart below.

	$j_i = l_i + \frac{1}{2}$		$j_i = l_i - \frac{1}{2}$	
	$j_f = l_f + \frac{1}{2}$	$j_f = l_f - \frac{1}{2}$	$j_f = l_f + \frac{1}{2}$	$j_f = l_f - \frac{1}{2}$
G	$j_i + j_f - L$	$-(L + j_f - j_i + 1)$	$-(L + j_i - j_f + 1)$	$-(j_i + j_f + L + 2)$
F	$\frac{1}{2}(j_i + j_f + L + 1)$	$\frac{1}{2}(L + j_i - j_f)$	$\frac{1}{2}(L - j_i + j_f)$	$\frac{1}{2}(j_i + j_f - L + 1)$

Again there are additional selection rules⁶⁸⁾ for magnetic transition: $\pi_{l_i} \pi_{l_f} = -(-1)^L$ and $|l_i - l_f| \leq (L - 1) \leq (l_i + l_f)$; the latter rule implies that in M1 transitions, the nucleon can not change its orbital angular momentum. Also for the special case when $L = |j_i - j_f|$, $F = L$ and $G = -1$.

If we evaluate the radial integral as for electric transitions, i.e., $I_{L-1} = \frac{3}{L+2}$ and consider the special case with $L = |j_i - j_f|$, the magnetic transition rate reduces to

$$T(\text{ML}) = \frac{0.193(L+1)}{L[(2L+1)!!]^2} \left(\frac{E_\gamma(\text{MeV})}{197.5} \right)^{2L+1} (\alpha \cdot 10^{-13} \text{ cm})^{2L-2} \left[\left(\frac{3}{L+2} \right)^2 \left(\mu L - \frac{L\delta}{L+1} \right)^2 \right] S(j_i, j_f, L) \cdot 10^{21} \text{ sec}^{-1}. \quad (4.38)$$

There are two commonly used units for transition strengths for magnetic transitions. Both are based on the assumption that the proton is responsible for the transition. For the Weisskopf unit⁶³⁾, the above formula is used with the term in the brackets replaced by $10\left(\frac{3}{L+3}\right)^2$ and S set equal to 1. For the Moszkowski unit⁶⁴⁾ the above formula is used as it is with $\delta=1$ and $\mu=\mu_p$.

A table of single proton transition rates and reduced transition probabilities is given in Appendix C. The formulae for multipolarities $L=1-5$ are given in an easy-to-use form. Both the Moszkowski and Weisskopf estimates are given.

4. Transitions Between Two Proton or Two Neutron States.

States in even-A nuclei are approximated by 2,4,6,... A particle states (depending on the sophistication of the model). Gamma-ray transition matrix elements must therefore be evaluated between states of an even number of nucleons. Here we give the factors by which the previous formulae (4.35) and (4.37) must be multiplied to obtain the transition rates between two particle states. Only the case where both particles are either protons or neutrons will be covered.

Since the multipole operators are one body operators, a non-vanishing transition matrix element is obtained if the initial and final states differ in the quantum number of at most one particle. Thus for a transition between two particle states there is a jumping and a non-jumping particle. Assuming j-j coupling to be valid, we denote the initial state by $|j_i j_n J_i\rangle$ and the final state by $|j_f j_n J_f\rangle$ where j_n refers to the non-jumping particle. The transition rates for electric and magnetic transition between two proton or two neutron states are obtained by multiplying the formulae (4.35) and (4.37) by⁶⁸⁾ the following factors:

$$nR = n(2j_i + 1)(2j_f + 1) \left\{ \begin{matrix} j_i & j_f & j_n \\ j_f & j_i & L \end{matrix} \right\}$$

where $n = 1$ when $j_i \neq j_n, j_f \neq j_n$
 $n = 2$ $j_i \neq j_f, j_i$ or j_f equal to j_n
 $n = 4$ $j_i = j_f = j_n$

Numerical values for this factor and the statistical factor are given for several cases of interest in table 4.5. Transitions in $^{140}_{58}\text{Ce}_{82}$ might be interpreted in terms of transitions between two proton states with the configurations given in the table.

Table 4.5

The statistical factors for transitions between states of two identical particles

Configurations		Transition multipolarity	n	R	S	nRS
$(g\frac{7}{2})^2_{J=2}$	$(g\frac{7}{2})^2_{J=0}$	E2	4	0.20	1.2	0.95
$(g\frac{7}{2})^2_{J=4}$	$(g\frac{7}{2})^2_{J=2}$	E2	4	0.20	1.2	0.95

6. The Effects of Center of Mass Motion and of Core Polarization

6a. Center of Mass Motion

The nucleon coordinates in the wave functions used to calculate single particle transition rates were relative to some arbitrary fixed point in space. Of course only the center of mass of the nucleus remains fixed, thus there is motion of this arbitrary point about the center of mass. A discussion of this effect has been given by several authors^{64,65,68,70}. It is found that the effect of center of mass motion (about the ficti-

tious fixed point) on single particle transition rates can be treated by simply introducing a correction factor to the single particle transition rates as given by Eqs. (4.35) and (4.37). The correction factor is generally ≈ 1 with the exception of electric dipole transitions⁶⁸⁾. In this case, the single neutron and single proton transition rates are approximately equal and reduced by a factor of 4 from the rates given by Eq. (4.35) for a proton transition. Thus for E1 transition the neutrons effectively acquire a charge $1/2 e$ by inducing motion of the charged core.

6b. Core polarization

The single particle transition rates are based on the strict single particle picture of the nucleus and as such take into account only the degrees of freedom of the extra core nucleons. It has been emphasized by B. Mottelson⁷¹⁾ that a more realistic model must take into account the polarization of the core due to the extra core nucleons. A detailed discussion of this problem may be found in ref.⁵²⁾. It is pointed out that the polarization effects manifest themselves most clearly for nuclei in which one particle or one hole polarizes a doubly magic core. It appears that in these cases the effect can be accounted for in a satisfactory way by the use of effective nucleonic charges and effective nucleonic g -factors characteristic of each multipole. For example, a comparison of experimental with theoretical transition rates on such nuclei, show that the E2 neutron rates require $e_{\text{effective}} = e$ whereas the proton rates require $e_{\text{effective}} = 2e$.

For a further discussion on core polarization for many particle configurations and other multipolarities ref.⁵²⁾ may be consulted. More data pertinent to this effect are much desired.

CHAPTER V

A COMPILATION AND DISCUSSION OF THE PROPERTIES OF NUCLEI WITH $N = 82$.

A. Evidence for Nuclear Structure at $N=82$.

1. Introduction

In this chapter we review evidence for special nuclear structure effects in the $N=82$ single closed shell nuclei. Before examining data on nuclear masses and excitation spectra, we mention by way of introduction one piece of evidence that was known before any other and which has played an important role in our ideas about nuclei. A histogram of the number of stable isotones as a function of N is shown in fig. 5.1. There are 7 stable nuclei⁷²⁾ found in nature with $N=82$, more than for any other N . Relative peaks occur at $N=20, 28$, and 50 . We may conclude that nuclear forces and nuclear structure are such that when there are exactly 82 neutrons (or some other magic number) an especially low-energy nucleon motion can

Fig. 5.1. Number of stable isotones as a function of N .

be achieved. In terms of the shell model, this extra stability is explained by the fact that the 82 neutrons fill a major shell which is separated by several MeV from the next highest shell.

2. Binding Energies

The one physical property of nuclei which probably has been more widely considered than any other is the nuclear mass. The property of mass is converted to the physical concept of binding energy $B.E.(Z,N)$ by the definition.

$$B(Z,N) = [ZM_p + NM_n - M(Z,N)] c^2$$

The free neutron and proton mass are denoted by M_n and M_p and the observed nuclear mass by $M(Z,N)$. It follows from this definition that the binding energy is the work required to completely separate a nucleus into Z protons and N neutrons. The work required to remove one neutron from a nucleus, $S_n(Z,N)$, is defined by the equation

$$S_n(Z,N) = B.E.(Z,N) - B.E.(Z,N-1)$$

Since the masses of many nuclei have been measured, and are tabulated (together with binding energies) in atomic mass tables⁵⁶⁾, it is possible to plot quantities such as $S_n(Z,N)$ as functions of Z and N .

One expects the extra stability of the $N=82$ nuclei to be reflected in $S_n(Z,N)$ vs N curves for a given isotope. Such curves fall into two groupings corresponding to nuclei with N even or odd. From the curves for nuclei around $N=82$ (fig. 5.2a), one sees that approximately 2 MeV more energy is required to separate a neutron from an even- N nucleus than from the adjacent odd- $(N-1)$ nucleus. This extra energy reflects the preference for pairwise coupling that nucleons in nuclei seem to have. A drop of ≈ 4 MeV in the neutron separation energies occurs between $N=82$ and $N=83$. Approximately 2 MeV of this drop can be attributed to the fact that the 82nd neutron has avail-

able a pairing energy which the 83rd neutron does not have. The other 2 MeV is attributed to the fact the 83rd neutron is more loosely bound to all the other nucleons.

To eliminate the effects of the pairing energy on the neutron separation energy, one should plot the differences $S_n(Z, N) - S_n(Z-2, N+2)$ as a function of N . (The A -dependence in these differences is removed since nuclei with the same A are compared). The two curves of fig. 5.2b show very strikingly how much more loosely the 83(84) neutron is bound compared to the 81(82) neutron. It is interesting to note that the stability associated with the $N=82$ nucleus does not arise from the fact that the 82nd neutron is much more difficult to separate than the 80th neutron, but from the fact that the 84th neutron is much easier to separate than the 82nd neutron. On this point nuclear shell structure appears to differ from atomic shell structure. The ionization potentials⁷⁹⁾ of the noble gases are considerably higher than those of the adjacent $(Z-1)$ atoms.

3. Systematics of the First 2+ Level

At the present there exists enough data on the first excited 2_1^+ state in even-even nuclei throughout the mass table for one to see systematic trends. In fig. 5.3 the energies of excitation for the 2_1^+ state are plotted against A . Data on nuclei with the same Z are connected by straight lines. Data points for single (double) closed shell nuclei are indicated with single (double) open circles. The figure is due to O. Nathan and S. G. Nilsson⁵²⁾.

A sharp increase in the excitation energies E_2 can be observed for nuclei having filled major neutron or proton shells. It is interesting to note that E_2 is 0.2 - 0.5 MeV larger for $N=50, 82$ nuclei than for $Z=50, 82$ nuclei. It is also interesting to note that $E_2 = 1.2$ MeV in ^{210}Po (doubly magic + 2 proton), whereas $E_2 = 0.8$ MeV in ^{210}Pb (doubly magic + 2 neutrons).

Fig. 5.2a. Neutron separation energies $S_n(Z, N)$ as a function of N . Lines connect data points belonging to the same isotopes.

Fig. 5.2b. Neutron separation energy differences $S_n(Z, N) - S_n(Z-2, N+2)$ as a function of N .

Fig. 5.3. The energy E_2 of the first excited $2+$ state in even-even nuclei. On the average E_2 decreases with increasing mass number as expected simply from the increase in size. This effect is brought out by the liquid drop model. The curve $0.2 \hbar\omega_2$ is shown ($\hbar\omega_2$ is the energy of the quadrupole, one phonon vibration). It can be seen that the energy E_2 rises sharply for nuclei in which major neutron or proton shells are filled. Data points corresponding to single(double) closed shell nuclei are indicated by single(double) open circles. The figure is due to O. Nathan and S. G. Nilsson⁵²).

Fig. 5.4. The reduced transition probability $B(E2)$ for the excitation of the first $2+$ state in even-even nuclei vs. mass number in units of the single proton estimate $B(E2; 0 \rightarrow 2+) = 0.3 A^{4/3} e^2 \text{fm}^4$. Single(double) open circles represent single(double) closed shell nuclei. Note that the enhancements in transition rates are relatively low for single-closed shell nuclei. The figure is due to O. Nathan and S. G. Nilsson⁵²).

The lifetimes of many of these first $2+$ states are also known and it is possible to look for systematic effects. The reduced transition probabilities in terms of the single proton estimate are shown in fig. 5.4. The effects of shell closure are again evident. We note that even though E2 transition rates for the $2_1+ \rightarrow 0+$ transitions in single-closed-shell nuclei are enhanced, the magnitude of this enhancement is relatively low.

There is some evidence⁵²⁾ that the excitation energy E_3 of the first 3^- level in scs is higher than in other even-even nuclei. Even though this effect is not as pronounced as for 2_1+ state, it may be regarded as further evidence for the existence of special shell structure in scs nuclei.

4. Ground State Electromagnetic Moments

Ground state spins and moments have played an important part in the development of nuclear shell theory, so it is natural to examine the data on nuclei in the $N=82$ region for shell structure effects. Unfortunately the data is so sparse, that only the Cs isotopes can be examined for systematic effects. The dipole moments⁷³⁾ μ of ^{55}Cs nuclei are plotted as a function of N in fig. 5.5a. The single proton Schmidt values are also shown. Nothing special can be noticed at $N=82$. The dipole moment data agree with generally observed trends, i.e., for $j = l - 1/2$, the experimental values are larger than Schmidt values, and for $J = l + 1/2$, the experimental values are less than the Schmidt values.

Also, the special nature of $N=82$ nuclei is not apparent from a graph of the known electric quadrupole moments⁷³⁾ Q for Cs nuclei (fig. 5.5b). The values of the quadrupole moments are small (same magnitude as for the deuteron) which would seem to indicate that the nuclei are nearly spherical. It is interesting to note that Q changes signs between ^{133}Cs and ^{135}Cs . This is not expected in terms of the simple shell model since

Fig. 5.5a. Magnetic dipole moments of Cs nuclei.

Fig. 5.5b. Electric quadrupole moments of Cs nuclei.

in this model the $1g_{7/2}$ proton is responsible for Q in both nuclei. The small positive Q value for $^{137}_{58}\text{Cs}_{82}$ indicates that this nucleus has a slightly prolate shape.

B. Properties of Nuclei with $N=82$.

In the last section we saw that nuclei with 82 neutrons had special properties in relation to nuclei with different neutron numbers. Here we examine the properties of nuclei with $N=82$ as a function of proton number. Sixteen nuclei ($Z=50-65$) with $N=82$ and stable against nucleon emission have been found³³⁾. At present, data exist on the excitation spectra of 12 of these ($Z=54-65$).

1. Binding Energies

A graph of the binding energy/nucleon vs. Z is shown in fig. 5.6a. The average $B.E./A$ is ≈ 8.35 MeV. The decrease in $B.E./A$ with increasing Z is in agreement with the systematic trend⁷⁴⁾ for $B.E./A$ to decrease gradually with A for heavier nuclei. The zig-zag fluctuations of the curve reflect the pairing energy of even-even nuclei. The curves (fig. 5.6b) of the single proton separation energies $S_p(Z, N=82) = B.E.(Z, N=82) - B.E.(Z-1, N=82)$ show the pairing energy very clearly. The curves corresponding to Z -even and Z -odd are separated on the average by ≈ 2 MeV. It is interesting to note that the proton separation energy drops ≈ 4 MeV between $Z=55$ and $Z=63$. The increasing separation between the two curves with increasing Z indicates an increase in the pairing energy. This effect may be seen more clearly in a graph (fig. 5.7) of pairing energy vs. Z . We used as definition of the pairing energy (odd-even mass difference) the four-point formula⁷⁵⁾,

Fig. 5.6a. Binding energy per nucleon for isotones with $N = 82$.

Fig. 5.6b. Proton separation energies $S_p(Z,N)$ for isotones with $N = 82$.

Fig. 5.7. Proton pairing energies $P_p(Z, N)$ for the isotones with $N=82$. The theoretical values for the pairing energies, closely approximated by the lowest quasi-particle energies $E_{\alpha}(\text{min})$, are taken from the calculations of M. Rho.

$$P_p(Z,N) = \frac{1}{4} [B.E.(Z+1,N) - 3B.E.(Z,N) + 3B.E.(Z-1,N) - B.E.(Z-2,N)]$$

and the simpler three-point formula⁷⁵⁾,

$$P_p(Z,N) = \frac{1}{2} [S_p(Z+1,N) - S_p(Z,N)] = \frac{1}{2} [B.E.(Z+1,N) - B.E.(Z-1,N)] - B(Z,N)$$

The general increase in P_p with increasing Z observed for the $N=82$ nuclei is not observed for the $Z=28,50$ (Ni, Sn) s.c.s. nuclei¹⁰⁾. For the Sn nuclei the P_n vs Z curve oscillates⁵⁵⁾.

The quantity P_p is frequently used in theoretical calculations (see Chapter VI) to fix the strength of the two-nucleon residual interaction. It is approximately equal to the lowest single quasi-particle energy $E_\alpha(\min)$ of the adjacent even- $(Z-1)$ nucleus. Values of $E_\alpha(\min)$ calculated by Rho¹⁾ have also been plotted in fig. 5.7.

2. Excitation Spectra

The task of determining the energies, spins, parities, moments, and lifetimes of all states in the $N=82$ nuclei is far from complete. In fig. 5.8 we show the excitation spectra of these nuclei as they are known to date. A detailed reference chart for each nucleus is given in Appendix D. The data obtained through studies of radioactive decay, nuclear reactions, and atomic spectroscopy are included. The spectra are organized into two groups, characterized by even- Z and odd- Z . The ground state magnetic dipole moments, μ , and electric quadrupole moments, Q , of odd- A nuclei are given beneath the chemical symbol. Levels indicated with dotted lines and J^π values enclosed by parenthesis must be regarded as tentative.

A discussion of the salient features of the odd- Z and even- Z nuclei follows.

Fig. 5.8. Spectra of N=82 nuclei. Stable nuclei are indicated with heavy bars. Ground-state magnetic dipole moments and electric quadrupole moments are given for odd-Z nuclei underneath the chemical symbol. References are given in Appendix D. See ref. D.11) for more data on Ce-140.

Odd-Z, N = 82

The ground states of $^{135}_{59}\text{I}_{82}$, $^{137}_{55}\text{Cs}_{82}$, $^{139}_{57}\text{La}_{82}$ have $J\pi = 7/2+$ as expected from the shell model. The ground states of these nuclei should be rather pure single $1g_{7/2}$ quasi-particle states. The ground state of $^{141}_{59}\text{Pr}_{82}$ has (tentatively) $J\pi = 5/2+$, which agrees with the shell model ($2d_{5/2}$ qp). A tentative assignment⁷⁶⁾ (based on a log ft value and shape measurement) of $3/2+$ has been determined for the ground state of $^{145}_{63}\text{Eu}_{82}$. If this assignment is correct, theoretical calculations would be affected considerably since the $2d_{3/2}$ state is not expected to correspond to a ground state until $Z \geq 65$ (see e.g. ref. 1).

Not much can be said about the excited states since there is so little data. The observed $7/2+$ and $5/2+$ states should be nearly pure 1 qp states. The $1d_{3/2}$, $1h_{11/2}$, and $3s_{1/2}$ 1 qp states are expected to be present in the low-lying states of all the odd-Z nuclei. The identification of these states is altogether one of the most pressing needs in the further understanding of structure in the N=82 nuclei.

Even-Z, N = 82

The first excited 2_1+ state has been observed in five of the seven even-Z nuclei. The highest excitation energy, $E_2 = 1.67$ MeV, occurs in $^{144}_{62}\text{Sm}_{82}$. It would be of interest to know E_2 for $^{146}_{64}\text{Gd}_{82}$ and $^{136}_{54}\text{Xe}_{82}$ so as to give a better picture of E_2 vs. Z for the N=82 nuclei. In the $^{50}_{50}\text{Sn}$ nuclei E_2 reaches a maximum when there are 14 neutrons outside the N=50 major shell.

The lifetimes of the 2_1+ states in $^{138}_{56}\text{Ba}$, $^{140}_{58}\text{Ce}$, and $^{142}_{60}\text{Nd}$ are approximately equal ($\tau \approx 0.15\text{ps}$). These lifetimes correspond to an enhancement factor of ≈ 15 for the E_2 ground state transitions. In the Sn nuclei the enhancement factors are ~ 10 . These two factors are in agreement with the general trend⁵²⁾ for effective neutron charges to be less than effective pro-

ton charges and the trend for E2 enhancement factors to be smaller for closed-shell nuclei than for non-closed-shell nuclei.

The first excited 4_1^+ state has been identified only in ^{138}Ba and ^{140}Ce and in both cases the lifetimes has been measured. The $4_1^+ \rightarrow 2_1^+$, E2 transition is found to be retarded by ≈ 15 . Retardation factors of this order were obtained by Rho¹⁾ in a calculation which determined the 4_1^+ and 2_1^+ states to have the dominant 2 qp components $(g7/2)^2$, $J = 4$ and $(g7/2)^2$, $J=2$, respectively.

The 3_1^- state, described as an octupole vibration³⁸⁾, has been found in ^{138}Ba , ^{140}Ce , ^{142}Nd , and ^{144}Sm . The excitation energy of this state drops monotonically from $E_3=2.83$ MeV in ^{138}Ba to $E_3=1.75$ MeV in ^{144}Sm . In the even-N, Sn nuclei from ^{114}Sn to ^{122}Sn the energy E_3 fluctuates between 2.24 and 2.50 MeV. In both the N=82 nuclei and Sn isotopes, the E_3 transition rates for the $3_1^- \rightarrow 0_1^+$ transition have enhancement factors³⁸⁾ between 50 to 100 in the several cases where this quantity has been measured.

One excited 0_1^+ state has been found in ^{140}Ce , one in ^{142}Nd and two in ^{144}Sm . It is interesting to note that the excitations energies of the 0_1^+ and 2_1^+ states are so nearly equal ($E=1.67$ MeV) in ^{144}Sm that it is not known which corresponds to the first excited state. The character of the 0_1^+ states is not well understood. A calculation of the excitation energies of 0_1^+ states in N=82 nuclei was recently performed by Bès and Veje⁴³⁾ on the basis of the quasi-boson treatment⁴⁵⁾ of the pairing force. According to this calculation, there are four 0_1^+ states in each even-even nucleus and each is a combination of five 2 qp states.

States with $J^\pi \neq 0^+, 2^+, 3^-, 4^+$ have been identified only in ^{140}Ce . A discussion of the levels with $J^\pi = 3^+, 5^-, 6^+$ in ^{140}Ce is given in Chapter III.

C. Theoretical Descriptions of N=82 Nuclei

At present the most basic theoretical approach to the description of nuclear structure is that of the shell model with residual interactions. The single particle shell model states⁴⁸⁾ available to the spectroscopically active nucleons in nuclei with N=82 are shown below.

Fig. 5.9. Part of the spectrum of particle levels available to nucleons in heavier nuclei.

One sees that the magic number gaps in the particle spectrum are ~ 8 MeV whereas the spacing inside a band of levels is of the order of 10 times less. One expects⁴⁸⁾ that if the matrix elements of the residual two-nucleon forces are $\gtrsim$ mean spacing inside a band $<$ magic number gaps, there will be strong configuration mixing between levels of a band but little mixing

between bands. Thus for the $N=82$ s.c.s nuclei one expects the low-energy states to have proton configurations involving the $1g7/2$, $2d5/2$, $1h11/2$, $2d3/2$, $3s1/2$ states.

The determination of the n -proton eigenstates ($n=Z-50$) for some residual interaction is still a formidable task. The recognition of the fact that the most important effect of the residual interaction is to produce pairing correlations has led to the use of the quasi-particle approximation technique described in Chapter IV. Four different theoretical calculations^{1,8,10,43}) have been performed on the $N=82$ nuclei within this framework. Details of these computations are given by the authors. Some general remarks follow.

One difficulty which besets all the theoretical calculations on these nuclei is the fact that the single particle level spacings are not known well. This problem would be solved if an experimental investigation of the low-energy spectrum of $^{133}_{51}\text{Sb}_{82}$ could be made. Since the nearest stable nucleus is $^{136}_{54}\text{Xe}_{82}$, the study of ^{133}Sb is difficult. A calculational technique (inversion of the gap equations) which circumvents the need for the single particle level spacings has been developed by Gillet and Rho⁷⁷). As input, this computation requires only the single quasi-particle energy level spacings. These may be taken directly from the observed spectrum of the odd- Z nuclei. Calculations using this technique are in progress⁷⁸).

There is one general result which both Rho and Plastino et al. found. Once the strength of the two-body residual interaction has been determined by fitting to odd-even mass differences, the calculated excitation energies of the first 2_1^+ states do not agree well with the experimental values. To fit the 2_1^+ energy, Plastino et al. increase the force strength of the surface delta interaction by 38% which they justify in terms of the neglected core excitations. Rho, on the other hand, introduces a large spin-triplet component (from $t=0$ to $t=1$) into the two-body force for which he gives a plausibility argument in terms of the neglected tensor-force component

in the residual interaction. Both of these investigations suggest the need for further theoretical work.

D. Future Studies

Although the understanding of nuclear structure in the single-closed shell, $N=82$ nuclei has grown in the last several years, it is far from complete. In this section we make some specific suggestions for future investigations which will lead, hopefully, to a better understanding of these nuclei.

The need for data on the odd- Z nuclei is most pressing. The spins and parities of only 7 levels in these nuclei are known. There is no data at all on the excitation spectra of $^{135}_{55}\text{I}_{82}$, $^{143}_{61}\text{Pm}_{82}$, $^{145}_{63}\text{Eu}_{82}$ even though these nuclei are immediately adjacent to stable nuclei. The one-proton transfer reactions $(d, ^3\text{He})$ and $(^3\text{He}, d)$ are well suited for the investigation of the six odd- Z nuclei between $Z=53$ and $Z=63$. Such studies would give information on level energies and would help identify the proton configurations of the low-lying states. Radioactive decay scheme studies are needed and are feasible on the same six nuclei. The determination of gamma transition multipolarities, rates, and branching ratios would lead to spin and parity assignments and to a better understanding of the nature of the states. Logft values would also help identify⁵²⁾ the character of low-lying states. With these various types of investigations the identification of the one and three quasi-particle states may be possible.

Even though there are five stable even- Z nuclei with $N=82$, there is extensive data only on ^{140}Ce . A detailed investigation of the decay $^{138}\text{Cs} \rightarrow ^{138}\text{Ba}$ is much needed. Since many levels in ^{138}Ba are populated ($Q_\beta = 4.8$ MeV), it would be interesting and informative to compare this decay with the $^{140}\text{La} \rightarrow ^{140}\text{Ce}$ decay. The study of all the even- Z nuclei with inelastic scattering processes, such as the (p, p') and (α, α') reactions, is of interest. This should help identify the natural parity

parity states $0+$, $2+$, $3-$, $4+$, $5-$ in all the even- Z nuclei. The study of excited levels as populated through one-proton transfer reactions is also of interest. Such investigations should lead to a better knowledge of the proton configurations in the low-lying states.

Further theoretical studies should be carried out. The nature of the low-lying $0+$ states is not well understood. A completely satisfactory microscopic description of the low-lying 2_1+ and 4_1+ states and the negative parity $3-$, $5-$ states has not been given. Theoretical calculations of electromagnetic moments and transition rates are of interest.

We have seen that the nuclear isotones with $N=82$ have properties favourable to experimental and theoretical investigations which could with further investigations lead to a better understanding of nuclear structure.

APPENDIX A

DATA ON DIFFRACTION PEAK PROFILES AND CRYSTAL REFLECTIVITY FOR A 2-METER CURVED-CRYSTAL SPECTROMETER WITH A $\text{Ge}(0\bar{2}2)$ CRYSTAL

Introduction

The available data on the diffraction properties of Ge crystals in use with curved-crystal spectrometers (ccs) is minimal. This fact can be attributed in part to the difficulty in elastically bending the Ge crystal lamina. It is desirable, however, to use Ge crystals for gamma-ray spectrometers due to the improved resolution widths and increased reflecting power^{12,13)} obtainable with this crystal as compared to crystals composed of lower Z atoms. Since the curved $\text{Ge}(0\bar{2}2)$ crystal has been used extensively in this laboratory for nuclear γ -ray energy and relative intensity measurements, data on the diffraction peak profiles and crystal reflectivity are of interest.

The UM ccs is described in detail in ref. 12). Data on diffraction peak profiles and crystal reflectivity of the $\text{Q}(310)$, $\text{Q}(112)$, $\text{Ge}(400)$, $\text{Ge}(0\bar{2}2)$ crystals are presented. The present study pertains to the $\text{Ge}(0\bar{2}2)$ crystal.

Line Shapes and Focal Aberrations

The 1st through 4th order diffraction peak profiles of the 87 keV gamma-ray in the decay of ^{160}Tb are shown in fig. A.1. The crystal area was approximately 4 cm^2 . The four profiles indicated with solid lines were obtained by aligning the source to minimize the fwhm of the 2nd order

peak. The other three profiles in this set were obtained without changing this source position. A graph of the fwhm vs order of these four peaks is shown in the inset (solid line). The profiles indicated with dashed lines will be discussed after examining the cause for line broadening.

The observed line broadening can be understood from the following consideration. Fig. A.2 shows two geometries of the spectrometer corresponding to the angle $\theta=0^\circ$ and some large Bragg angle θ . (In this spectrometer design the crystal is rotated through an angle θ , the detector and collimator through 2θ , while the source remains stationary.) It can be seen directly from fig. A.2 that the source displacement from the focal circle along the central ray (horizontal line through the source and crystal-pivot axis) is given by $\Delta R=2R(1-\cos\theta)$. For the present 2m ccs the displacement ΔR in cm is $\Delta R(\text{cm}) \approx 3.05 \times 10^{-2} (\theta^\circ)^2 = 1.8 \times 10^{-6} (\text{x s.d.})^2$. The values of ΔR for the peaks of fig. A.1 are shown with each profile and it is seen that the value $\Delta R = 2.05$ cm for the 4th order peak is large compared to the source dimension, 0.48 cm, parallel to the central ray. The relationship of the source to the focal circle for the four orders of reflection of the 87 keV γ -ray is shown in fig. A.3. The displacement of the source from the focal circle is drawn to the same scale as the source dimension parallel to the central ray. (The aperture angle 2α is not drawn to scale.) The divergent γ -rays emitted from the source into the angle 2α intercept the focal circle so as to give an effective source width W_n larger than the actual cross sectional width W_0 of the ribbon source. The width of the virtual image (see fig. A.2) of the γ -ray focused by the spectrometer has the same magnitude (assuming all other aberrations are negligible) as the effective source width since the magnification of the spectrometer is unity. The effective source widths on the focal circle are given approximately by the terms W_1, W_2, W_3, W_4 shown in fig. A.3. Each width W_n is a sum of two terms. The first term W_0 is

Fig. A.1. The 1st-4th order diffraction peak profiles of an 87 keV gamma-ray. The solid lines indicate the profiles obtained with the source held stationary. The dashed lines indicate the profiles obtained when the source is moved by an amount given by $\Delta R = 2R(1 - \cos\theta)$.

the effective geometrical cross section of the source ribbon. The second term is a function of the source displacement from the focal circle $\Delta R(\theta)$ and the total aperture angle 2α . The aperture angle is in itself a function of θ and decreases with increasing θ . However, the variations in 2α for the Bragg angles involved is negligibly small.

The variation of the effective source width as a function of order of reflection for the source-focal-circle arrangement shown in fig. A.3 is graphically displayed in the inset. The shape of this graph is different for each source-focal-circle arrangement. The dip in this graph is due to the fact that the focal circle crosses the source ribbon in 2nd order reflection for this arrangement.

Fig. A.2. Two geometries of the curved-crystal spectrometer which illustrate the line broadening shown in fig. A.1.

Fig. A.3. The relation of the source to the focal circle for the four orders of diffraction of the 87 keV γ -ray shown in fig. A.1.

The observed diffraction peak widths shown in fig. A.1 are easily understood in terms of the source-focal-circle arrangement shown in fig. A.3. Thus for the observed profile 87(+1) the source was positioned in front of the focal circle (such that the source to crystal distance was less than $2R$). For the profile 87(+2) the focal circle has moved across the source to give a slightly smaller effective width and a correspondingly smaller fwhm for the profile. The 3rd and 4th order profiles are much broadened since the source is now behind the focal circle (distances larger than $2R$ from the crystal). For small values of θ , the displacement ΔR increases with a θ^2 dependence.

The second set of profiles shown with dashed lines in fig. A.1 were obtained by displacing the source along the central ray by an amount $\Delta R_n - \Delta R_2$ for each order reflection. As can be seen, in each case the fwhm of the 2nd order profile was obtained (plotted with dashed line in inset, fig. A.1). The observed broadening of diffraction peak profiles is thus understood in terms of the variations in effective source width arising from the source-focal-circle displacement term $\Delta R = 2R(1 - \cos\theta)$.

It was possible to move the source along the central ray as required since the Pb source holder unit of the spectrometer¹²⁾ is mounted on an adjustable base. By means of two dovetail carriages, motion in the horizontal plane parallel and perpendicular to the central ray is possible.

The small displacements of the source required to bring the γ -rays into focus at large Bragg angles do not affect the energy measurements. In an energy measurement, a pair of diffraction peak profiles is obtained by reflecting from both sides of the crystal planes, and the separation distance of the two profiles is determined by overlapping the profiles. This separation distance $2x$ is related to the γ -ray energy through the calibration constant $E \cdot (x/n) = k$. If the source motion is entirely along a parallel to the central ray, the calibration constant remains unchanged.

If the source is displaced in a direction perpendicular to the central ray, the change in the calibration constant is given by $k - k' = k(1 - \cos\beta)$ where k' is the new calibration constant and β is the angular separation between the two central rays. This effect is illustrated in fig. A.4. The solid lines in this figure illustrate the manner in which the two-profile separation distance $2x$ is determined along the lead screw for one alignment of the source and crystal-pivot arm. The dashed lines illustrate the fact that the two-profile separation distance $2x$, as measured by the lead screw, is diminished by the factor $\cos\beta$ for a change in the alignment of the source and crystal-pivot arm. The relation of x to x' is given by $x' = x \cos\beta$, and the relation between the calibration constants is therefore $k' = (x'/x) \cdot k = k \cos\beta$. For a source displacement perpendicular to the central ray of magnitude 0.2 cm the fractional change in the calibration constant is $(k - k') / k = \frac{1}{2}\beta^2 = \frac{1}{2} \times 10^{-6}$. In most cases this change is negligibly small. In any case the value of β can always be made much smaller than 10^{-3} . This is achieved by moving the source along the perpendicular to the central ray, such that the peak position of some diffraction profile is at the position x (s.d.) calculated from the relation $E \cdot (x/n) = k$.

Crystal Reflectivity

There have been several investigations^{12,13)} directed towards determining the energy dependence in some order of reflection, n , of the crystal reflecting power $R(E, n)$ of various curved crystals. The determination of this function (or some closely related function) is necessary if the spectrometer is to be used in the measurement of relative γ -ray emission rates. There is another function, which we call the peak reflectivity $P(E, n, n' = 1)$, of interest for efficient use of a css. This function relates the maximum counting rate recorded by the spectrometer in the n th order

Fig. A.4. Diagram illustrating the change in the calibration constant caused by a change in the zero position along the lead screw.

diffraction peak of a gamma-ray with energy E , to the maximum counting rate in the 1st order diffraction peak of the same gamma-ray. Curves of this function for eleven gamma-rays in the energy range 0.087 - 1.597 MeV are shown in fig. A.5. The peak counting rates for these curves were determined to an overall accuracy of 5%. It was shown in the last section that there is a variation in the fwhm of diffraction peak profiles in different orders reflection, but that it is always possible to make the fwhm equal by moving the source along the central ray. The curves in fig. A.5 represent the relative peak counting rates for the case in which the fwhm for all order profiles of a gamma-ray have been made the same by moving the source.

Reflectivity curves are of interest when comparing the properties of crystals. However, one should exercise caution in using the presented curves for deducing detailed reflectivity properties of the $\text{Ge}(0\bar{2}2)$ planes. The relative peak counting rates that are recorded by the spectrometer depend not only on the crystal (and the particular planes) but also on the focusing properties of the collimator. The measured transmission¹²⁾ of the collimator is 50%. However, there may be small variations in this transmission as the collimator moves through the range $2\theta = 0^\circ$ to $2\theta = 20^\circ$. A possible check for variations in the collimator transmission can be obtained by comparing the peak counting rates between corresponding positive and negative orders of reflection. Such a comparison is shown in fig. A.6 for a 298 keV gamma-ray. The angular range through which the collimator moved is $2\theta = -7^\circ$ to $2\theta = +7^\circ$. Note that the positive order peaks have a peak counting rate about 5% less than the corresponding peak counting rates in the negative orders. However, the two reflectivity curves have nearly the same shape. Thus in this angular range the collimator appears to have a uniform transmission.

The reflectivity curves proved useful in the following practical situations:

Fig. A.5. Peak reflectivity curves for the Ge(022) planes.

Fig. A.6. The diffraction peak count rates for a 298 keV gamma ray in positive and negative orders reflection.

1. When one is studying short-lived isotopes, the use of these curves permits the most rapid determination of the optimum order for the measurements on a γ -ray.
2. When several γ -rays are focused at nearly the same Bragg angle, the use of these curves facilitates the determination of the best order reflection for the energy measurement of each γ -ray.
3. When one is measuring relative γ -ray emission rates, the use of these curves permits the peak counting rates in various orders reflection to be related to one particular order.

APPENDIX B

THE CALIBRATION OF A Ge(Li) GAMMA-RAY SPECTROMETER FOR RELATIVE INTENSITY MEASUREMENTS

Introduction

The usefulness of Ge(Li) γ -ray spectrometers for nuclear decay scheme studies is well established. Consequently, the calibration of a Ge(Li) spectrometer for accurate measurements of γ -ray energies and relative emission rates is of interest. A description of the calibration for relative detection efficiency is given here. The detection efficiency of γ -ray full energy peaks was determined using sources with well-measured relative γ -ray emission rates. Two Ge(Li) detectors with active volumes of $3 \text{ cm}^2 \times 0.5 \text{ cm}$ and $4 \text{ cm}^2 \times 0.5 \text{ cm}$ were calibrated in the energy range 80 - 3200 keV. An overall accuracy of 5% in the energy region from 500 - 3200 keV and an accuracy of 10% over the entire range was obtained. The relative areas of the single escape peak and double escape peak to the full energy peak as a function of energy are included.

Ge(Li) Spectrometer

The Ge(Li) spectrometers consisted of the following components: either an RCA detector, with depleted volume $3 \text{ cm}^2 \times 0.5 \text{ cm}$, mounted in a cryostat constructed at the UM machine shop or an ORTEC detector-cryostat with depleted volume $4 \text{ cm}^2 \times 0.5 \text{ cm}$; a Tennelec TC-130 (FET) pre-amp and TC-200 amplifier; and a Victoreen (SCIPP) 1600 channel pulse-height analyzer. The best resolution at low count rates was obtained with the 1st differentiator and integrator time constants set at $1.6 \mu \text{ sec}$ and the 2nd differen-

tiator time constant at 1 msec. The TC-130 and TC-200 units were modified to include pole-zero cancellation.

The source-detector arrangement used to obtain all the γ -ray spectra reported here is shown in fig. 2.1.

Determination of Peak Areas

A detailed view of two peaks in the ^{24}Na spectrum is shown in fig. B.1 for the purpose of illustrating the manner in which the peak areas were determined. A graph of this type showing the counting distribution in the ten or more channels adjacent to the base of the peak was plotted for each peak. The shape of the distribution in these ten channels provided the basis for drawing the straight line which separated from the total spectral distribution that part of the distribution which was to be considered the peak. The area A of the peak is defined by the sum $A = \sum (N_i - B_i)$, where N_i are the actual counts in a channel and B_i are the background counts in that channel as defined by the straight line. The area obtained in this way depends on the manner in which the line is drawn and it is therefore important to follow a consistent procedure. By examining the trend of the distribution in the ten or more channels near the base of the peak, it was generally possible to ascertain to within one or two channels where the curvature of the distribution changed markedly. This marked change in curvature established the two points through which the straight line was drawn. The ratio of peak areas (taken for a set of 4-6 standards' spectra) derived in this way showed an rms fluctuation of about 1%. Since the uncertainties due to counting statistics could be made negligibly small, these fluctuations are considered to reflect variations in judgment in selecting the two points determining the straight lines. These fluctuations are of about the same magnitude as the uncertainties in the relative γ -ray intensities used for the detection efficiency calibration. In view of this overall limit on the accuracy of the calibration, this

Fig. B.1. Enlarged view of two peaks in the ^{24}Na spectrum illustrating the manner in which peak areas were determined

method of determining peak areas was considered satisfactory. In complex spectra where the full-energy peaks are in general not as easily separated from the total distribution, care must be exercised to avoid making subjective errors. A possible check against this error is obtained by comparing the full energy peak profile after the background subtraction with a standard profile. Further, the extent of such a systematic error can be determined by drawing the separating straight line between different, but plausible, points at the base of the peak. From this range of peak areas one may deduce a reasonable limit on the uncertainty in that area.

The Relative Detection Efficiency Curve

The number of counts A in a full-energy peak is related

to the emission rate of a γ -ray with energy E , intensity I from a source at a distance p from the detector, by the expression

$$A(E, p) = \eta'(E, p) \cdot \alpha(E, p) \cdot \Omega(p) \cdot I \cdot t ,$$

where I is the isotropic, γ -ray emission rate/ster, $\Omega(p)$ the solid angle subtended by the detector, t the counting time, and $\alpha(E, p)$ the attenuation factor representing the effects of absorbing layers between the source and detector. The function $\eta'(E, p)$ represents the fraction of γ -rays in the solid angle that are detected in the full energy peak. It is written as a function of p since edge effects are included in this term. In calibrating a particular source-detector arrangement it is desirable and convenient to combine the detection efficiency and attenuation terms into one function $\eta(E, p) = \eta'(E, p) \cdot \alpha(E, p)$. The ratio of two values of this function for γ -ray energies E_1, E_2 can be written

$$R(E_1, E_2, p) = \frac{\eta(E_1, p)}{\eta(E_2, p)} = \frac{A(E_1, p)}{A(E_2, p)} \cdot \frac{I_2}{I_1} .$$

The function $R(E_1, E_2, p)$ represents the relative full-energy peak detection efficiency for a particular source-detector arrangement. The dependence of $R(E_1, E_2, p)$ on p can be investigated experimentally by determining the ratio $A(E_1, p)/A(E_2, p)$ as a function of source to detector distance. It was found that the relative areas for peaks in the standards' spectra obtained with the source-detector arrangement shown in fig. 2.1 were independent of p in the range $6 \text{ cm} \leq p \leq 20 \text{ cm}$, to within the accuracy that the area ratios were determined. The dependence for other values of p was not determined since the available source strengths were not convenient. Henceforth, the function $R(E_1, E_2, p)$ will be written $R(E_1, E_2)$ with the above range of p implied. The calibration of a source-detector arrangement is effected when the function $R(E_i, E_j)$ is determined for all values

E_1, E_j within a certain range.

The calibration curves for the RCA and ORTEC detectors are shown in fig. B.2. The calibration range is approximately 80 - 3200 keV. The relative efficiency values of these two curves at the ^{22}Na ($E = 511$ keV) point are in the same ratio as the full energy peak areas in the spectra obtained with the two detectors at equal source to detector distances and equal counting times. It is interesting that the value of this ratio, 2.4, seems to be inconsistent with the ratio of volumes, 1.3, based on the figures supplied by the manufacturers.

The two curves were obtained by using a pair-point method to be described. This method relies on the values of relative γ -ray emission rates of pairs of γ -rays in a group of standards' sources. Since there exist several standard sources with two γ -rays having a relative emission rate known to about 1% or better, the pair-point method leads to an accuracy in the calibration which is somewhat better than is generally obtainable using sources with calibrated source strengths. In table B.1 are listed the data pertinent to the efficiency calibration for the ORTEC detector. In column one and two the sources and their γ -ray energies are listed. In column three the relative γ -ray emission rates with their uncertainties are given. The related references for these values are given in column four. In column five are listed the relative full-energy-peak areas, $A(E_1)/A(E_2)$, for each pair of γ -rays. (For the case of ^{22}Na , a 1% correction was applied to account for absorption in the 0.13 cm thick Al surrounding the source, which assured that all the positrons would annihilate at the source.) The uncertainties in the area ratios are rms deviations in the 3 to 6 runs taken with each source. Each source was run at various source-to-detector distances in the range 6-20 cm. In column six the relative detection efficiencies $\eta(E_1)/\eta(E_2)$ of each pair of γ -rays is listed. This number is obtained by simply dividing the relative peak

Fig. B.2. The relative detection efficiency curves for two Ge(Li) detectors. The dimensions of the depleted volumes were provided by the manufacturer.

areas (col 5) by the relative emission rates (col 3).. The uncertainty in the relative detection efficiency of each pair is due to two independent contributions, the uncertainty in the relative γ -ray emission rate and the uncertainty in the relative areas.

Using the relative detection efficiency numbers of column six in table B.1 (ORTEC detector) the efficiency curve was drawn on semi-log paper using the following graphical method. The ²²Na points were first placed on the graph. The ⁶⁰Co points were placed next. These two points are used to determine the curvature of the curve at the ²²Na ($E_{\gamma} = 1.275$ MeV) point. On semi-log paper the efficiency curve appears to follow a straight line in this energy interval. Next the curve was extended to slightly higher energies so as to determine the position of the ²⁴Na ($E_{\gamma} = 1.369$ MeV) point, which in turn also determined the position of the

^{24}Na (2.75 MeV) point. The ^{88}Y ($E_\gamma = 0.90$ MeV) and ^{88}Y ($E_\gamma = 1.84$ MeV) points were now placed so that a smooth curve could be drawn through all the data points. The ^{46}Sc ($E_\gamma = 0.89$ MeV) and ^{46}Sc ($E_\gamma = 1.12$ MeV) points were used as a check, and were in good agreement with the curve. The low energy portion of the curve was obtained using the ^{180}Hf data. The ^{180}Hf ($E_\gamma = 448$ keV) point was placed first by extending the finished portion of the curve to 448 keV. Once the ^{180}Hf ($E_\gamma = 448$ keV) point is placed, the other ^{180}Hf points are fixed, thus extending the curve down to 90 keV. The curve was drawn to 80 keV using the ^{133}Ba points.

The uncertainty in the efficiency curve constructed in this graphical manner depends on the uncertainty in the detection efficiency of each pair of calibration points, and on the number of points that are available to position the curve, and the extent to which these points overlap. Thus in the region between 0.50 to 2.8 MeV in which there are ten data points, the maximum overall uncertainty is estimated to be about 5%. The uncertainty in the relative detection efficiency for smaller energy intervals is of course smaller, and for the case in which the relative detection efficiency of two γ -rays with energies close to that of a pair of points used in the calibration is desired, the uncertainty is about 1% (in the interval 0.5 to 2.8 MeV). In the interval 80 to 500 keV the uncertainty in the curve is larger, due to the uncertainty in the relative intensities of the ^{180}Hf γ -rays (3-7%). Thus the overall uncertainty in this portion of the efficiency curve is about 10% while for smaller intervals it is less.

It should be pointed out that the relative efficiency curve, obtained in this way represents the overall relative efficiency of the source-detector system, as shown in fig. 2.1, and includes the effects of the various absorbing layers.

This completely empirical method of obtaining the relative detection efficiency curve for a particular detector has the following advantages. It circumvents the need for

solid angle calculations as well as calculations for attenuation requiring a knowledge of various photon interaction cross-sections. As more calibration points become available, these data can be easily included in the calibration to further reduce the uncertainty in the curve. This method is simple and yet permits relative γ -ray intensity measurements with good accuracy.

One may compare this curve with the theoretical curves obtained by the Monte Carlo method. If the efficiency curve for the ORTEC detector (0.5 cm depletion depth, 2.3 cm diameter) is compared with the theoretical intrinsic efficiency curves of ref. B.6) for the 0.5, 0.8 cm depletion depth detectors (1.8 cm diameter), it is observed that the curvature in the region 400 - 1500 keV are very nearly the same. In the region below 400 keV the theoretical curves rise more sharply than the experimental curve. Absorbing layers account for a part of this deviation. At 200 keV, 6-7% of the γ -rays are absorbed.

When analyzing complex spectra it is sometimes useful to know the ratios of the single and double escape peak areas to the full energy peak areas. Two such curves are shown in fig. B.3 for the ORTEC detector. These data were not compared with the results of ref. B.6) since these curves are expected to be strongly dependent on detector geometry. One does not expect such a strong dependence for relative photopeak efficiency curves.

Fig. B.3. Curves showing the single- and double-escape peak to full-energy peak area ratios for the ORTEC detector.

Table B.1

Data on the detection efficiency calibration for the Ortec detector

Source	E MeV	Rel. emission rates ^{a)} I_1 / I_2	Ref.	Full-energy peak area ratios ^{b)} $A(E_1) / A(E_2)$	Full-energy peak efficiency ratios $\eta(E_1) / \eta(E_2)$
(1)	(2)	(3)	(4)	(5)	(6)
²² Na	0.511 1.274	1.81 (1 ± 0.01)	1	8.60 (1 ± 0.01)	4.75 (1 ± 0.014)
²⁴ Na	1.368 2.753	1.00 (1 ± 0.01)	2	3.39 (1 ± 0.003)	3.39 (1 ± 0.01)
⁶⁰ Co	1.173 1.333	1.00 (1 ± 0.01)	1	1.205 (1 ± 0.01)	1.205 (1 ± 0.014)
⁸⁸ Y	0.898 1.836	0.940 (1 ± 0.01)	3	3.04 (1 ± 0.01)	3.23 (1 ± 0.014)
⁴⁶ Sc	0.889 1.120	1.00 (1 ± 0.01)	1	1.46 (1 ± 0.01)	1.36 (1 ± 0.014)
^{180(m)} Hf	0.093 0.215	0.200 (1 ± 0.035)	4	0.925 (1 ± 0.011)	4.63 (1 ± 0.04)
	0.215 0.332	0.88 (1 ± 0.05)	4	2.31 (1 ± 0.013)	2.63 (1 ± 0.05)
	0.332 0.448	1.15 (1 ± 0.07)	4	2.10 (1 ± 0.011)	1.83 (1 ± 0.07)
¹³³ Ba	0.081 0.356	0.52 (1 ± 0.04)	5	9.07 (1 ± 0.003)	17.4 (1 ± 0.04)

a) A 1% uncertainty in the relative emission rates has been adopted except for transitions in ^{180(m)}Hf and ¹³³Ba.

b) The area ratio for ²²Na has been corrected for attenuation in the 0.13 cm thick Al surrounding the source.

References for Appendix B

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APPENDIX C

Gamma-ray transition rates for single proton transitions; Weisskopf and Moszkowski estimates.

	$F(LE_\gamma)$	$B(\sigma L)$	$T_{sp}(\text{proton})$ in sec^{-1}
E1	$7.00 \times 10^{59} E_\gamma^3$	$0.0645 \times A^{2/3} S e^2 \text{fm}^2$	$1.04 \times 10^{14} A^{2/3} E_\gamma^3 S$
M1		$1.67 (1.79) \times \mu_0^2 S$	$2.98 (3.20) \times 10^{13} E_\gamma^3 S$
E2	$5.30 \times 10^{79} E_\gamma^5$	$0.0595 \times A^{4/3} S e^2 \text{fm}^4$	$7.26 \times 10^7 A^{4/3} E_\gamma^5 S$
M2		$6.22 (1.65) \times A^{2/3} S \mu_0^2 \text{fm}^2$	$8.40 (2.23) \times 10^7 A^{2/3} E_\gamma^5 S$
E3	$2.46 \times 10^{99} E_\gamma^7$	$0.0594 \times A^2 S e^2 \text{fm}^6$	$3.37 \times 10^1 A^2 E_\gamma^7 S$
M3		$13.81 (1.65) \times A^{4/3} S \mu_0^2 \text{fm}^4$	$8.67 (1.04) \times 10^1 A^{4/3} E_\gamma^7 S$
E4	$7.32 \times 10^{118} E_\gamma^9$	$0.0630 \times A^{8/3} S e^2 \text{fm}^8$	$1.06 \times 10^{-5} A^{8/3} E_\gamma^9 S$
M4		$25.5 (1.75) \times A^2 S \mu_0^2 \text{fm}^6$	$4.76 (0.326) \times 10^{-5} A^2 E_\gamma^9 S$
E5	$1.49 \times 10^{138} E_\gamma^{11}$	$0.0693 \times A^{10/3} S e^2 \text{fm}^{10}$	$2.38 \times 10^{-12} A^{10/3} E_\gamma^{11} S$
M5		$43.4 (1.93) \times A^{8/3} S \mu_0^2 \text{fm}^8$	$1.65 (0.0732) \times 10^{-11} A^{8/3} E_\gamma^{11} S$

$$T_{sp} = F(L, E_\gamma) B(\sigma L) = \frac{8\pi(L+1)}{L[(2L+1)!!]^2} \frac{\hbar}{e} B(\sigma L); e = 4.80 \times 10^{-10} \text{ esu}; \mu_0 = 0.505 \times 10^{-23}$$

erg/gauss = lnm; lfm = 10^{-13} cm; $a = 1.2 \times A^{1/3}$ fm; E_γ in MeV. Weisskopf units for electric transitions are the same as Moszkowski units with $S=1$. For magnetic transitions, numerical factors in parenthesis correspond to Weisskopf unit; also $S=1$ for Weisskopf unit.

Physical properties	Nucleus $Z = 50 - 65$ $N = 82$															
	Sn 50	Sb 51	Te 52	I 53	Xe 54	Cs 55	Ba 56	La 57	Ce 58	Pr 59	Nd 60	Pm 61	Sm 62	Eu 63	Gd 64	Tb 65
Ground state																
M				1	1	1	1	1	1	1	1	1	1	1	1	1
$T_{1/2}(\beta)$	2	2	2	2	2	2	2	2	2	2	2	2	2	2	2	2
Ω, Q, μ, J				2,3		2,3		2,3		2,3		2,3		4		
Excited States																
Nuclear Reactions							5,6,7	8,9	5,6	7,8,9	5,6,7		5,6			
Radioactivity					2	10	2	2,11 20	2,11,15	2,11, 12	2,11					
lifetimes of excited st.						2	2,13	2	2,11,13, 14	2	2,13					
Theoretical calculations			16,17 18,19	17,19	16,17 18,19	17,19	16,17 18,19	17,18 19	16,17 18,19	17,18 19	16,17 18,19	17,18 19	16,17 19	17,19	16,17 19	

Table D.1. References for data on $N=82$ nuclei. References for work before January 1966 may be found in ref. 33).

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