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The Institute for Computer Research

The University of Chicago

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ICR Quarterly Report

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Table of Contents

Section I

- A. Maniac III Engineering Activity
- B. New Instruction--Link 7040 (C. K. Teng)
- C. The Man-Machine Project (C. C. J. Roothaan)
- D. The Man-Machine Project (C. C. J. Roothaan)

Section II

- A. Syntactic Analysis of Algol Programs Containing Errors
(C. L. Robinson)
- B. Matrix Operations (R. H. Miller)

Section III

- A. On The Construction Of Fixed Points And Solutions Of Nonlinear
Equations With Demicompact Mappings (W. V. Petryshyn)
- B. Proposal For a Large Michelson Stellar Interferometer (R. H. Miller)

MANIAC III ENGINEERING ACTIVITY

1. Connection to the IBM 7040 Computer.

The Maniac III end of this link is now complete except for some tidying-up, and has been tested as far as is possible from this end.

The other end of the link is not yet quite ready, some cables ordered for this being not yet available. The work involved was as follows:

- a) A Maniac III main frame instruction to initiate the transfer process has been installed (see Sect. IB, this report).
- b) Construction of the unit which serves as an interface between Maniac III and the 7040 has been completed, tested and connected. It contains various manual controls by which signals from the 7040 may be simulated, and these have been used for testing the system.
- c) The "queueing control" which apportions memory access times between Maniac III and the 7040 has been completed and installed. The queueing control has caused some difficulty because it introduces a variable delay in access to the Ampex memory by Maniac III.

In the case of Maniac III Disk File orders this trouble has been overcome (subject to final tests) by modifying these orders so that they send a warning signal to the queueing control a few microseconds before memory access is required.

In the case of the Univac 1004 printer no simple solution was possible--it would have involved a major redesign of the order logic in both Maniac III and the 1004, which was not justified in view of the approaching replacement of the 1004 with an IBM 360/30. We have temporarily accepted the programming restriction that, when the queueing control is connected, the 1004 print buffer must reside in the Maniac III 8K core. When there is no requirement for the 7040 link the queueing control may be disconnected by moving one plug, and there is then no restriction on the use of the 1004.

Since the queueing control gives Maniac III priority in memory access, it was possible to test the system under adverse conditions by temporarily wiring the queueing control to simulate continuous memory access requests from the 7040 while running Maniac III. Under these conditions the Ampex memory cycled every 2.5 microseconds.

2. Design of Interface Units for Connecting Future Peripherals.

- a) Internally these units will process a 48-bit word in three 16-bit sections, and they will therefore resemble computers with a 16-bit word-length and a three-word instruction.
- b) A digit-board has been designed which (on a single board) contains one bit of every internal register, one adder stage, one counter stage, a masking control and a central distributor with all necessary gates for communication between internal registers. Two or three of these boards are now being fabricated and will

be tested before ordering the sixteen such boards required for each interface unit.

- c) A control system has been devised based on a 4-phase clock.

A preliminary version of such a clock has been built and tested. The proposed control system seems well adapted to the design of important internal routines (e. g. for memory access) as independent units, which can be "called" by other routines as required.

- d) The control logic for a number of the internal routines has been designed and tested by simulation on Maniac III using an Algol program. Such routines are composed of a list of logical equations which may be used almost directly, either for circuit design or as input to the simulation routine. Two complete instructions have been designed and simulated in this way.

- e) A Maniac III machine-language program is being written to simulate the overall performance of an interface unit without taking account of details of the logical design. This will be used to check the design of the interface unit instruction set for consistency and usefulness, and also to simulate short programs. An Algol program would be excessively long and slow for this purpose.

- f) We now have enough experience with the logical design of this unit to begin the layout of the logic boards. At this time it

seems likely that logic boards will be large, that chassis wiring will be extensively standardized, and that the variable part of the logic wiring will mostly consist of jumpers or diodes inserted in various positions on the boards with a few special connections in the chassis back-wiring.

3. Queueing Control for Interface Units

The 7040 queueing control is not suitable for use with the interface units, as a more complex priority system will be required. Design of a new queueing control, which will eventually replace the 7040 queueing control, is in progress.

4. Power Supplies.

A power supply has been built to take part of the "-10V" load which at present uses three regulated supplies in parallel. The new supply is a 3-phase unregulated supply. It will be installed when some outstanding components have been delivered. This will release power supplies needed for completing the reorganization of the power supply system.

5. Reserved Addresses.

Plans have been made to change the code for reserved addresses to correspond with the bottom end of the Maniac III 8K memory. This will make core addresses in the 16K memory contiguous with those in the 8K. The change had been postponed until the work on the 7040 connection was completed. Appendix 2 lists the old and new reserved addresses.

6. Engineering Shutdowns.

There have been no major shutdowns this period. It has been possible to make the 7040 connections in stages, reducing the consecutive down-time to a day or two at each stage.

A troublesome fault in the Maniac III instruction register proved due to a change in a clamp voltage when new construction added an additional load. The design of the instruction register is unfortunately critical with respect to this voltage but change is difficult owing to space limitations.

A fault in the 1004 printer was corrected by Univac with some difficulty, causing about two days down-time.

A defective air-pressure switch on the Disk File unit that caused some down-time was eventually replaced by IBM.

MANIAC III USE

October 27, 1965-January 27, 1966

(Programmer and production use, debugging and production times are not separated in this table).

Engineering Computer time

Simulation of Interface Computer	82.05
Disk File Tests	48.44
Rewriting Engineering Tests (due in part to reserve address change)	48.44
Miscellaneous Engineering Tests (Including testing 7040 connection)	17.41
Systems work	88.79
Orthonormal properties of Latin Squares (Robinson)	74.4

I A-6

Astronomical calculations (Miller)	71.49
Numerical experiments (Ashenhurst)	13.4
Inter-nuclear cascades (Laitinen)	3.11
Miscellaneous	5.80

ENGINEERING PROGRESS REPORT

Appendix 2

The following list shows the changes which will be made in reserved address codes of Maniac III.

REGISTER/LOCATION	OLD ADDRESS	NEW ADDRESS
Z	3FF8	0000
U	3FF9	0001
R	3FFA	0002
S	3FFB	0003
G } G and H registers	3FFC	0004
H } will at a later date,	3FFD	0005
be made non-		
addressable		
V	3FFE	0006
W	3FFF	0007
T0	3FF0	0008
.	.	.
.	.	.
.	.	.
T7	3FF7	000F
Trap Locations {	3FED	0010
	3FEE	0011
	3FEF	0012
General Core		
Locations		0013--3FFF.

The trap locations will not be considered as reserved addresses for disk or tape orders. The disk order will continue to issue a stop command if a reserved address is encountered--in this case, a "wrap-around" spilling out the high end of memory.

NEW INSTRUCTION - LINK 7040

C. K. Teng

<u>Op Code</u>	<u>Mnemonic</u>	<u>Name</u>	<u>Format</u>
7a	L40	Link 7040	δ^L : Control Word address δ^R : Base address for data

Description: This instruction initiates information exchange between the IBM 7040 in the Computation Center and the new Maniac III memory (16K) via an interface subcomputer. Whenever the interface subcomputer is ready during the execution of this order, Maniac III prepares and sends the effective Control Word address and the effective base address for data (modification of both right and left address fields is allowed) to the interface subcomputer, which stores them into its internal address registers. At the same time the memory location following that containing the Control Word is cleared, since it is to receive a Report Word giving the condition and the status of the interface subcomputer and the 7040 after the information exchange. The nature of the information exchange operation is dictated by the Control Word, as interpreted by a program in the 7040.

δ^L gives the memory location which contains the Control Word to be sent to the 7040.

δ^L+1 gives the memory location which will receive the Report Word.

^R
6

gives the first address of the memory block in which the data from the 7040 will be stored, or in which data to be read to the 7040 is already stored (read/write choice is made on the basis of the Control Word).

Note: In the process of executing this instruction, a locked cyler step signifies that the interface subcomputer is not ready for the operation; the step is released as soon as interface subcomputer is not busy. Do not use the 8K memory.

THE MAN-MACHINE REPORT

C. C. J. Roothaan

Progress Reports Nos. 1-3

(for the period January 1, 1965 through September 30, 1965)

I. Introduction

Work on the Dual Machine System, the first effort defined for the Man-Machine Project, has centered around the general objective of optimizing, within the framework of a one-at-a-time job processor, the speed and facility of access to the computing power of the 7094. While this approach differs widely from multiprogramming, several facets of the multiprogram concept have influenced the design of the Dual Machine System.

The goal for the Dual Machine System, as outlined by Professor Roothaan and Mr. Worley, is minimum job turn-around time while maintaining throughput potential of the 7094 and compatibility with stand-alone 7094 systems.

In more concrete terms, the goal is a system in which job turn-around is so rapid as to make it unprofitable for a user to leave the building after submitting a short production run or debugging application. Achievement

* Work on this project, supported by a grant from the International Business Machines Corporation with C. C. J. Roothaan as principal investigator, is carried on under the joint auspices of the Institute and the Computation Center. The status of the project as of late 1965 is given in Reports Nos. 1-3, covering the period 1-1-65 to 9-30-65, and further progress is related in Report No. 4 (Sec. I-D), covering the period 10-1-65 to 12-31-65. These Reports are presented herewith; it is intended that subsequent progress reports on this project be included in the ICR Quarterly Reports on a regular basis.

of this goal would mean a job turn-around time on the order of twenty minutes for all jobs with a running time of under three or four minutes.

Work on the system has been channelled into three interrelated efforts: design of the 7040 Control Program; implementation of and experimentation with the 7040/7094 interface; and increased efficiency for the 7094 FORTRAN Monitor System (FMS).

II. 7040 Control Program

The major task necessary for the 7040/7094 Dual Machine System was the design of the 7040 Control Program. Under the supervision of Professor Roothaan and Mr. Kruskal, the development of the 7040 Control Program was divided into two main steps:

- 1) the construction of an operational model of the Dual Machine System which would serve as the Computation Center's operating system
- 2) the implementation of the final Dual Machine System in the light of experience gained by the design and observation of the operational model.

A. The Operational Model

Before any code for the Dual Machine System existed, two things were clear: First, a system could be designed immediately which would yield notably better service than the system which was in use at that time. (The system then in use was a stacked-input/stacked-output system in which the 7040 behaved somewhat like two 1401's used in normal off-line applications.) Secondly, the

opportunity of feedback from an actual operational model would prove invaluable in the design of the final Dual Machine System. For these reasons it was decided that a preliminary system be written and put into actual use.

Since this system was to be temporary, many corners were cut in its implementation. However, the preliminary system does have a complete dynamic priority scheme to determine the order in which jobs are to be sent to the 7094, along with a facile override for the operators of any aspect of 7040 job scheduling. In addition to the obvious advantages of priority scheduling, the priority scheme of the operational model relieves the operators of the numerous decisions which a computer is better equipped to make. At the same time, the priority mechanism substantially increases the operators' ability to control the system on the basis of those necessary but ill-defined value judgements best made by human beings.

The most glaring faults of the system are its low speed and its lack of an efficient priority scheme for scheduling job output. Both of these faults were tolerated in order to decrease programming time.

The system has been in operation for about four months.

B. The Final System

The final system is about two to three months from completion. In preliminary planning, it became apparent that the 7040 Control Program was essentially an aggregate of independent programs, and that a multiprogramming environment was best suited to such a structure. The multiprogramming supervisor for the Control Program is now complete. Select routines for

most devices have been completed. Those remaining to be done are in their final stage and will be finished shortly. The design of the dynamic priority scheme is virtually complete and programming for it will begin in a few days. The final Control Program should be in recognizable form (although not usable) in a month.

When complete the system will incorporate the following features:

- * A fast, clean easily expandable multiprogramming supervisor
- * A dynamic priority scheme for all system tasks
- * Operation of all unit-record input/output devices at maximum rated speeds and operation of magnetic tape at close to maximum speed
- * A disk organization based on the "task file" concept
- * Checkpoint and emergency recovery procedures to insure rapid and efficient system recovery after virtually any contingency
- * Remote job input/output via unit-record equipment attached to an on-line 1401
- * An assembly program--capable of assembling the system itself--operating under the multiprogramming supervisor
- * Utility programs operating under the multiprogramming supervisor
- * An expanded and consistent set of inquiry and response mechanisms for operator communication with the Control Program

Several of these features, and their analogues in the operational model, are discussed in further detail below.

C. Evolution of the Final 7040 Control Program

Supervisor

The preliminary 7040 control program operates as a job under the 7040 IBSYS basic monitor. While the use of IBSYS facilitated rapid construction of a working system, the problems inherent in operating a system control program as a "job" under a general stacked job processor soon became apparent. For example, it is obviously not desirable to "terminate" a system control program because of a random machine error. Nor is the time and space necessary to provide supervision for an extremely wide range of computer applications justified in the context of the highly specialized requirements of a "job" which is a system control program itself.

The final 7040 Control Program operates under a multi-programming supervisor developed by Mr. Kruskal. The supervisor was designed from the ground up to supply the multiprogram environment appropriate to the control program. The criteria for design were--above all--speed, and secondly ease of modification and expansion of programs operating under the supervisor.

The supervisor maintains tables describing independent programs, containing, for example, their priority and state of "readiness". An "unready" program is a program "waiting" for one of several "events".

The "ready" program with the highest priority is yielded control by the supervisor. This program retains control until the supervisor regains control either through the program's voluntary request to wait for a specified

event, or through a machine trap. On either of these contingencies, or, in general, whenever the supervisor gains control, event and program tables are updated and control is again passed to the highest priority "ready" program.

The supervisor also maintains real-time control of all programs through use of the 7040 storage clock. To provide insurance against programming errors or hardware malfunction no program is allowed under any circumstances to maintain control for longer than a specified real-time interval (a system assembly parameter).

In addition, the supervisor allows programs to associate a time limit with a request to wait for an event. A program in effect says to the supervisor: "I have reached a point where I am no longer able to continue useful work. Therefore I relinquish control to you and do not wish it returned until the occurrence of event E, or the passage of real-time interval T, whichever comes first".

Dynamic Priority Scheme

The priority mechanism in the operational model is embodied in the 7040 "Job Manager" subprogram developed by Mr. Williams. The Job Manager maintains a chained table of location, status, and priority of all input jobs in the system. On the basis of an up-to-the-second priority queue, the Job Manager provides access to the job with highest priority in the system at the moment the 7094 requests some word. In addition, the Job Manager provides facilities for holding back those jobs requiring some sort of setup until the operators have had time to make the necessary preparation. The Job Manager also processes operator requests for information about the status of jobs in the system, and requests to change the status of jobs.

The only major drawback of the priority scheme in the operational model

is that it applies only to job input. Printed output, in lieu of specific operator intervention of a rather gross nature (a sense switch option to "drop" printers from assignment of additional output), is routed to the printer which has the fewest lines to print at the time of output transmission from tape or direct-data. It is thus possible, indeed probable, that situations will arise in which all three printers are committed to printing very lengthy jobs, while a large backlog of short printing jobs accumulates.

The priority scheme of the final 7040 Control Program will incorporate a method for dynamic determination of priority of virtually every facet of systems activity: job input, job output, multiple copy printing and punching request, utility program output, special input and output setup requests.

Operating Speeds

The operating speeds of the various input/output devices under the operational model of the control program are, in general, unsatisfactory. Impaired operating speeds are the price paid for the general nature and wide flexibility (for our application, largely wasted) of the 7040 IBSIS Input/Output Control System.

Preliminary tests indicate that the select routines and multiprogramming supervisor of the final 7040 Control Program (both tailored specifically for our application) should yield operation of all input/output devices at virtually maximum rated speeds.

Disk Organization

In the preliminary control program, disk storage organization is based on the association of IBSYS symbolic units with various unit-record devices (printer files, reader files, etc.). A file table is maintained by the program which records the function (input, output), contents (empty, not-empty, full) and status (open, closed) of each of the symbolic disk units. Although workable, this system has

the fairly serious drawback of inefficient use of available disk storage. For example, the two symbolic units associated with a given printer have a combined capacity for storage of about 90,000 lines of output. If a job, or a series of jobs, assigned to a printer exceed this storage capacity, a "disk file overflow" occurs. All output transmission from tape or direct-data is inhibited until the printer can drain one of the symbolic units to make it available for additional output. This situation can arise even though the symbolic units associated with other printers are completely empty and idle.

In the case of the reader (job input) file, a similar overflow results in "garbage collection" in which unprocessed jobs are transcribed onto an alternate file to make room for additional input. Periodic "garbage collection" is inevitable; unfortunately, the operation stops all other processing for anywhere from a few seconds to several minutes.

The disk organization developed by Mr. Kosinski for the final 7040 Control Program is based on the concept of the "task file". A task file is a file of information associated with a specific task to be performed, such as a job to be sent to the 7094, or the output from such a job to be printed. The two distinct advantages of such a scheme are flexible and efficient utilization of available disk storage, and facile access to information about system tasks. A task file is, by definition, the exact size appropriate for the information contained. The length of a task is not restricted by arbitrary symbolic unit limits. Furthermore, the simple definition of such a file (location, length, status) imparts useful information about system "tasks", the Control Program's raison d'être, rather than information about arbitrary "symbolic units".

Even though magnetic disk is a "direct-access" storage device, it was felt that some thought should be given to the reduction of access-mechanism motion time. The storage allocation routines of the final 7040 Control Program's disk support are designed to allocate disk tracks sequentially whenever possible, alternating between the disk modules, and to cycle through the entire disk storage before returning to the original location. Thus, tracks allocated to a file at time (t) will not be needed again for reallocation until time $(t + T)$, where T is the period required to cycle through the entire storage capacity of the disk. This scheme at once decreases time spent on positioning the access-mechanism for the disk modules, renders less probably the need of frequent and time-consuming file purging, and affords the possibility, in the event of some failure, or re-executing a file's task for a time (T) after the file's tracks are allocated.

Checkpointing & Emergency Recovery

Use of checkpoint and emergency-recovery routines in the preliminary control program established their value as insurance against unpredictable system instabilities. These routines in the operational model, although somewhat inefficient due to the lack of centralized communication region incorporating all information necessary for recovery, virtually eliminate the one or two hour setback caused by hardware or systems failure under a heavy work load. On emergency recovery, the system, in a matter of seconds, resumes normal operation with all input and output intact to the nearest job boundary.

The final 7040 Control Program will incorporate essentially the same scheme for checkpointing and emergency recovery, albeit a good deal more

efficiently. All checkpointing, for example, will be completely overlapped with other processing.

Remote Processing

The operational model of the control program provides facilities for transmission of input and output tapes via cable connection to an "on-line" 1401 two blocks away at the Biological Sciences Computation Center of the University Hospitals. Considerable trouble was encountered in the development of the 7040 routines to handle the data transmission. After several weeks of exhaustive hardware and software checkout, it was discovered that most of the problems stemmed from an inconsistency in the 7040 IBSYS support for the on-line 1401.

The final Control Program will allow transmission directly to and from the unit-record equipment attached to the on-line 1401 in place of the tape-to-tape facilities now available. Job decks destined for processing by the 7094 will be read on the card reader at the Biological Sciences Computation Center in the same fashion as those decks read by the 7040's card readers.

Work is also in progress on a link between the 7040 Control Program and the MANIAC III computer at the University's Institute for Computer Research.

System Maintenance

The operational model of the control program is updated and assembled under 7040 IBSYS. Although the IBSYS facilities proved satisfactory in themselves for system maintenance, severe scheduling problems arose because of the need to have the 7040 available the greatest part of the day for normal system operation.

To eliminate this conflict, the final 7040 Control Program will incorporate a modified version of the IBM 8K 7040 Assembly Program operating under the

multi-programming supervisor. System assembly will be treated as simply another system task for the Control Program, and will in no way interfere with normal system operation. Miss Monheit is currently working on the System Assembler.

Utility Programs

Utility programs (tape dumps, card-to-tape, etc.) present much the same scheduling problem under the operational model as does system maintenance.

A set of utility programs operating under the final 7040 Control Program's multiprogramming supervisor will enable utility jobs to be treated in the same fashion as other system tasks. Utility job scheduling will be simplified, and for the most part, handled entirely by the dynamic priority mechanism.

III. 7040/7094 Interface

A. Role of Direct Data in the Operational Model

SIGN-ON, the 7094 FORTRAN Monitor System subprogram summoned for job-to-job transition, was modified by Mr. Kruskal to behave in the following fashion after completing a job while operating in the Direct Data Mode:

1. The monitor input tape is rewound. SIGN-ON interrogates the 7040 control program's sense-line interpreter to determine whether the 7040 is able to accept output.
2. If the 7040 control program indicates that it is so disposed, SIGN-ON rewinds the output tape for the job it has just completed, reads it, and transmits it to the 7040 over the direct data link. (The actual process involves many steps to synchronize the transmission and communicate modal information.) When the tape has been completely transmitted, it is rewound.
3. If the 7040 was not able to accept output, or if the output has already been sent, SIGN-ON will again interrogate the 7040 control program--this time requesting a job.
4. If the 7040 indicates that it does not have a job available, SIGN-ON waits briefly, then asks again.

5. If the 7040 has a job, it transmits it in blocks across the direct data link. (As with output transmission, several steps are required for each block transmitted.) SIGN-ON deblocks the records so received, reblocks them into the appropriate format, and transcribes them onto the monitor input tape.
6. When the job has been completely transmitted, SIGN-ON rewinds the input tape, informs the 7040 that it has successfully received the job, and commences to process the job in the same fashion as it would had it simply encountered the job serially on a stacked input tape.

This arrangement worked admirably during periods when the work load was moderate. However, there was a definite impairment of throughput potential inherent in the unoverlapped transmission of input and output between the two machines. Output transmission in particular (limited by the speed at which the 7040 could perform the complex deblocking and carriage-control interpretation) was very costly.

To offset the impaired throughput potential of the prototype system, software and operating procedures have been developed to enable the operators under peak work loads to bypass the normal job-at-a-time output transmission. (Provision was made for operators to request the 7040 control program to refuse to accept output from the 7094. The mechanics for such a refusal were already present in both machines to cover contingencies such as the 7040 encountering an output file overflow.)

Since the 7040 control program, as an integral part of its design, retained the ability to process output and input tapes in off-line fashion, processing of output from the 7094 can be overlapped with 7094 job-processing. The modified system, incorporating single-job input via direct data, and stacked-job output processing off-line, is able to keep pace with the heaviest work loads (as many as 500 jobs per day) while maintaining a turn-around time of about sixty minutes for all short jobs.

B. Alternative Forms of the 7040/7094 Interface Under Consideration

Professor Roothaan is currently investigating the feasibility of using program-switchable tape drives as an alternative form of the 7040/7094

interface. With this configuration, two input and output tapes would be used.

They would be switched under program control alternately between the 7040 and the 7094, yielding virtually complete overlap of all input and output transmission between the two machines. The trapping capabilities and sense-lines of the direct data link would be used for communication (as opposed to data transfer) between the machines.

Also under consideration is the possibility of the 7040 Control Program maintaining the 7094 FMS library on disk storage. Under this scheme, the 7094 FMS loader would normally receive library routines upon request across the direct data link from a 7040 librarian, rather than searching for them on the 7094 system tape. This application would drastically reduce the time now spent on the "library search" which is necessary for virtually every job specifying execution. In addition, because of the random-access nature of disk storage, restrictions imposed on the size of the 7094 monitor library by its present tape environment would be relaxed. The 7040/7094 library link would add a new dimension to the interface between the two machines: the 7040 would aid the 7094 in actual job-processing, in addition to its present services in job-to-job transition.

A proposal to break up the output transmission operation into two distinct phases is being considered. The first phase would be a nearly verbatim (record for record) transcription of output data onto disk storage. The second phase would

be a disk-to-disk operation involving deblocking and carriage control interpretation of the original records. This two-phase operation would allow output transmission to proceed at virtually tape speed since the most time consuming portions of the task would be delayed and overlapped with 7094 job processing.

IV. Modifications to the 7094 Fortran Monitor System

A. FASTRAN

Within a few days, Mr. Steiner will complete the final testing of the major modification of the 7094 FORTRAN Monitor System necessary for the replacement of the standard FMS FORTRAN II compiler with the Indiana University FASTRAN compiler. FASTRAN, while maintaining a very high degree of compatibility with the standard FORTRAN II compiler, was found in preliminary tests to compile some five to fifteen times faster than FORTRAN II. FASTRAN's high compilation speeds should appreciably decrease the machine-time expense of FORTRAN debugging. At the same time, since the majority of the users of the Dual Machine System are FORTRAN users, FASTRAN should substantially increase the throughput potential of the 7094.

The structure of the 7094 FMS system tape was modified so that the entire FASTRAN compiler and the FMS control card interpreter (SCAN) are co-resident in core storage. Under this arrangement, total time for a series of compilations is decreased significantly because the system tape need not be moved once the compilations begin.

In addition, Mr. Steiner has implemented a "LIST8" (equivalent assembler-level listing) feature which was not available with the original FASTRAN. Work is also in progress on a compilation option for a punched source deck of the assembler-level (FAP) equivalent of a FORTRAN source program. This feature should greatly

facilitate assembler-level modification of FORTRAN programs.

Several source statements have been introduced into FASTRAN which perform operations analogous to those of monitor control cards. The reduction of on-line printing occasioned by this feature will increase still further the overall speed of FORTRAN compilations.

Miss Monheit prepared the modified versions of the University of Chicago I/O routines to be used for FASTRAN compile-time input/output operations. One factor influencing the design of this I/O package was the possibility of its use by other processors which may be developed here in the future.

B. File

Miss Monheit has completed a new version of the University of Chicago FILE subroutine. FILE is a subroutine for concurrent buffered reading and writing of several tape files (roughly analogous to IBM's Input/Output Buffering System). The new version of File permits single as well as double buffering. In addition, provisions have been made for direct access to data in buffers and greater flexibility in the handling of unusual conditions.

THE MAN-MACHINE REPORT

C. C. J. Roothaan

Progress Report No. 4

(for the period 1 October 1965 through 31 December 1965)

1. 7040 Control Program

- a. Select routines and the disk support for the Control Program are being debugged.
- b. Work was begun on an extension of the Control Program's core storage allocator subprograms. The extension primarily involves the addition of a feature whereby double buffers are allocated whenever possible, with automatic reversion to single buffer allocation whenever demands for buffer space exceed available storage.
- c. Coding for the Control Program's system scheduler is in progress.

The scheduling algorithm is designed to yield "reasonable system overhead to as many jobs as possible, rather than merely giving optimal service to the shortest jobs in the system.

The scheduler will have provisions for priority scheduling of nearly every facet of system activity. Job printing, punching, and setup, as well as job execution, will be scheduled locally within "task" category, and globally for the job with which they are associated. Both specific task and total job scheduling, although largely independent of one another, will be coordinated toward "reasonable system overhead."

2. 7040/7094 Interface

A feature to enable tapes to be switched under program control between the 7040 and 7094 has been ordered and should be delivered shortly. The program

switchable drives will permit the use of two job input and two job output tapes. The programmed "swapping" of input and output tapes between the two machines will afford complete overlap of input/output transmission with 7094 job processing. In addition to this increased 7094 throughput, the feature will insure that the 7040 has access to job output rapidly enough to yield efficient priority scheduling of job printing and punching. Coding for the support of this facility has been hampered by the need to circumvent its original purpose, namely, the switching of tapes between two channels on a 7094.

3. 7094 FORTRAN Monitor System

Work on the FASTRAN compiler was completed. The FORTRAN II compiler on the system tape was replaced by the FASTRAN compiler on December 1.

4. Systems Documentation

The systems documentation staff began work on a new, comprehensive, single-source system reference manual.

SYNTACTIC ANALYSIS OF ALGOL PROGRAMS CONTAINING ERRORS

C. L. Robinson

The compiler writer is faced with a difficult problem when he considers the compiler's reaction to syntactic errors in the input string. On the one hand he would like to produce as much information from a particular compilation attempt as he can, i. e., he would like to ignore the error, print a message and continue analysis, in the hopes of finding more errors; while on the other hand he has a certain obligation not to confuse the user with a misleading set of multiple error messages. His problem is further compounded by the realization that the error causing mistakes might, in itself, be syntactically correct: witness the results of the following "obvious" deletion of a semi-colon.

```
...END BEGIN REAL x, y; x:=5;
```

With these thoughts in mind and with a strong reluctance to give misleading information, the designers of the ICR Algol compiler initially had it terminate the user's compilation as soon as it found the first syntactic blunder. At this time it told the user the last correctly analyzed characters in the input string (in the above example REAL x, y;), but it made no attempt to announce the "cause" of the mistake or to continue analysis. This solution was grudgingly accepted

by professional users, who had rapid access to the machine; but it is obviously impractical when student users under a batch job system are considered.

In a seminar at the Argonne National Laboratory,¹ Mr. McKeeman of Stanford University mentioned that he had a reasonable amount of success in his precedence matrix compiler by, upon finding an error, sweeping the input pointer to the next semicolon or end while unstacking the operators back to the statement level.

It was decided to incorporate a similar method into our syntax directed technique for compilation.² As per McKeeman, our compiler forces the input pointer ahead to a point to the next END semicolon, or end-of-file. The push-down list recording unsatisfied routines (productions) is then unwound until the most recent call of unlabelled basic statement is found. This unlabelled basic statement routine is then forced to do a normal recursive exit and the syntactic analysis (but not the output generation) is allowed to proceed in normal fashion. At the end of compilation a final error message is issued and the job is terminated. If no covering unlabelled statement can be found, i. e., during the unwind operation the list of unsatisfied routines is found to be empty, the job is immediately thrown off the machine. Alternately basic statement could be entered at this point to force further compilation, but our reluctance to produce a chain of probably false error messages caused us to abandon that solution.

To further reduce unwarranted messages the error routine is forced to look also for the terminal character BEGIN. In this case

an exit of the statement routine is forced followed by a jump to its point of call so that it is re-entered immediately. This allows the declaration in nested blocks to occur as naturally as possible.

It is important to note that these changes cannot possibly cause the compiler to go into an infinite loop, although at first glance the existence of the empty or dummy statement in Algol provides a cause for worry. BEGIN can cause no trouble since the production unlabelled basic statement is immediately re-entered and BEGIN is a legal terminal character starting this production; therefore the input pointer is advanced. END, semicolon, and end-of-file always cause the compiler's error routine to exit (satisfy the production of) at least one routine, i. e., unlabelled basic statement. Therefore either these terminal characters will be satisfactorily recognized at some point, causing the input pointer to be advanced, or the termination condition of no covering call of unlabelled basic statement will be met.

Two pleasant surprises were encountered in implementing this change. In the first place the ease of making the change was extraordinary. Even granting that the changes required in modifying the compiler from the 1960 to the 1963 revised Algol took only one-half hour, it was amazing that this arbitrary remodeling of syntactic structure upon encountering an error should work; much less that it also took a half hour to install. The next surprise after discovering that it worked was that it worked so well. Although no extensive

experimentation with erroneous Algol programs has been done, and although input strings can be constructed which force several error messages to be produced before the compiler gets back on the track, the following test was performed. Using paper tape input the tape reader was stopped at random times and the tape was arbitrarily moved a small distance in either direction; this corresponds to either deleting or inserting characters in the middle of an Algol program (a likely keypunching error). In all cases the number of error messages produced exactly tallied with the number of errors in the program (i. e. , times that the tape was moved), and in 75% of the cases the compiler reached the end-of-file mark in the natural fashion (i. e. , the whole program was at least scanned).

This revision of the compiler induced one change in our card input language. Before, the input routine had two modes of operation while reading cards. Delimiters such as BEGIN, END etc. were set off by surrounding quote marks; the first quote caused the input routine to form block letters of all alphabet input received until the second quote was reached. The trouble arose when a quote mark was deleted or inserted by accident. In this case all the rest of the input stream was in the wrong case. This has now been eliminated by demanding that any non-alphabetic character end the case shift. Not only does this eliminate the problem of the unlimited scope of a case shift, but it also turns out to be no harder to keypunch.

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MATRIX OPERATIONS

R. H. Miller

This note was written to present my thoughts concerning what may be a reasonable way to study numerical analysis of functions of several variables. The general procedure envisioned is to form several functions from the several variables, in such a way that there is a non-singular mapping from the space of the arguments to the space of the functions. The possible computer representations of the argument are a set of points in the space of the arguments, and the range of values that might have one of these representations is the interior of a cell centered on the representative point. The process of calculating the function corresponds (if it could be done with mathematical precision) to a mapping of the points and cell boundaries into the function space. But the possible computer representations of the result is a different lattice of points and set of cells in the function space, and the problem of computer representation of the result is to find the point and cell size (for significance arithmetic) in the function space that best represents the transformed point and cell. In general, the transformed cell will have some strange shape and orientation. This viewpoint leads in a natural way to the concept of interval arithmetic but a large number of interval explorations might have to be carried out, and problems of monotonicity, etc. can be imagined. Over small regions like a cell size, the transformations are approximately linear in a sense of a differential geometry. It should be possible to diagonalize

the operation, and to study the resulting problem in the diagonal representation.

Unfortunately, although this picture is appealing, nothing can be derived from it that is not more easily derivable by other means. So far the notion appears sterile. However, it seemed of interest to work it out in detail for a case of low enough dimensionality to illustrate the ideas. A particularly simple example of functions of two variables is the case of conformal transformations of the complex plane. There the applications of the ideas seem straightforward.

A slightly more complicated situation arises with the matrix problem, which is here worked out in detail for the 2×2 matrix--specifically for the real symmetric 2×2 . Here again, all the results may be trivially established by other means. The only reason for describing it is that the results are rather pretty and suggestive of what a generalization to higher dimensionalities might look like. The object of this note is (hopefully) to simulate others to think along these lines.

No essential complication arises from the discussion of an Hermitian 2×2 matrix, but very little new appears, so the discussion will be mainly restricted to the real symmetric matrix, and the generalization will be indicated.

Typically in matrix problems, diagonalization and inversion are of primary interest. (The matrix of eigenvectors is usually wanted also). If a matrix can be diagonalized, its inversion is trivial. But diagonalization

is a special case of a similarity transformation, so the problem of interest is to find the subspace of all matrices that can be reached from the given matrix by a similarity transformation. For the real symmetric matrix, the class of similarity transformations may be restricted to the real orthogonal matrices.

Immediately the problem of a basis for representing the matrix arises. A basis is sought in terms of which any matrix may be represented by a set of scalar multiples: $M = \sum a_i A_i$. A

possible set would be $\begin{pmatrix} 1 & 0 \\ 0 & 0 \end{pmatrix}$; $\begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}$; $\begin{pmatrix} 0 & 0 \\ 0 & 1 \end{pmatrix}$, in terms of which the matrix $\begin{pmatrix} a & b \\ b & c \end{pmatrix}$ is $a \begin{pmatrix} 1 & 0 \\ 0 & 0 \end{pmatrix} + b \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix} + c \begin{pmatrix} 0 & 0 \\ 0 & 1 \end{pmatrix}$. The given matrix is then represented by a point in a 3-dimensional space with a , b , and c measured along its axes. When the similarity transformation by a real orthogonal matrix is carried out, the basis afforded by the matrices known to physicists as the Pauli matrices emerges as more natural. In this basis, select σ_i ; $i=0, 1, 2, 3$;

with σ_0 being the identity, σ_1 , diagonal, and the others given by

$\sigma_i \sigma_j = i \sigma_k$ ($i=\sqrt{-1}$; $i, j, k=1, 2, 3$). These matrices all obey $\sigma_i^2 = 1$, and form a hypercomplex number set or the quaternion algebra, whichever is preferred. The numbering differs from the

usual physicist's ordering. With the notation $M = \sum s_\alpha \sigma_\alpha$,

the numbers s_α can easily be obtained:

$$s_\alpha = \frac{1}{2} \text{Tr}(\sigma_\alpha M).$$

The desired representation is

$$\sigma_1 = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}; \sigma_2 = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}; \sigma_3 = \begin{pmatrix} 0 & -i \\ i & 0 \end{pmatrix};$$

and the original matrix is

$$\begin{pmatrix} a & b \\ b & c \end{pmatrix} = \begin{pmatrix} (S_0 + S_1) & S_2 \\ S_2 & (S_0 - S_1) \end{pmatrix};$$

(with an Hermitian matrix, the off diagonal elements are $S_2 \pm i S_3$; all S_α are real.) This representation of the matrix is obtainable from the a, b, c representation by a rotation by $\pi/4$ about the $b = S_2$ axis, illustrating that sensible representations are obtainable from one another by rotations of the representation space.

In the S_α -representation, a matrix that is orthogonally similar to the given matrix is described by:

$$\begin{aligned} S_0' &= S_0, \\ S_1' &= S_1 \cos 2\theta + S_2 \sin 2\theta, \\ S_2' &= -S_1 \sin 2\theta + S_2 \cos 2\theta, \end{aligned}$$

where θ is the angle of rotation in the orthogonal matrix. The orthogonal matrix may conveniently be described in terms of this basis, and the whole operation carried through algebraically:

$$R(\theta) = \sigma_0 \cos \theta + i \sigma_3 \sin \theta = \exp[i\theta \sigma_3].$$

All matrices that are orthogonally similar to the given matrix lie on a circle parallel to the $S_1 - S_2$ plane, and of radius $\sqrt{S_1^2 + S_2^2}$. The same result could be deduced by noting that the determinant of $\mathcal{M}$ is $S_0^2 - S_1^2 - S_2^2$. All matrices of zero determinant lie on the cone

$S_0^2 = S_1^2 + S_2^2$, and surfaces containing matrices of equal determinant are hyperboloids of revolution about the S_0 -axis. Diagonal representations are those that lie in the plane $S_2 = 0$. The circle of orthogonally similar matrices cuts this plane in two places; and the two cuts correspond to the 2! permutations of the eigenvalues.

Inverses lie on a plane containing the S_0 axis and the point representing the original matrix, as is evident since the same rotation diagonalizes both a matrix and its inverse, while rotations in this representation space go by twice the angle of the corresponding rotation in the space of the vectors. (This double-angle effect may be expected to hold in higher dimensionalities; to see this think of the matrix as a quadratic form.) The inversion process is therefore diagonal in this representation. If primed variables represent the inverse:

$$\begin{pmatrix} S_0' \\ S_1' \\ S_2' \end{pmatrix} = \frac{1}{\text{Determinant}} \begin{pmatrix} 1 & 0 & 0 \\ 0 & -1 & 0 \\ 0 & 0 & -1 \end{pmatrix} \begin{pmatrix} S_0 \\ S_1 \\ S_2 \end{pmatrix}.$$

If it is desired to study the numerical analysis of the inversion problem, the elements S_0 , S_1 , S_2 , or the determinant should separately be changed, not the elements a , b , or c . If the arithmetic is done correctly, only the corresponding S_0' should change. Any real process, like pivoting, naturally involves several steps and the same representation may not diagonalize each step. A numerical experiment should be directed toward identifying the "eigenvectors" of this kind of transformation.

The diagonalization process is itself diagonalized by a different representation, but similar arguments apply.

$$\begin{pmatrix} S_0' \\ S_1' - iS_2' \\ S_1' + iS_2' \end{pmatrix} = \begin{pmatrix} 1 & 0 & 0 \\ 0 & \frac{S_1 + iS_2}{\sqrt{S_1^2 + S_2^2}} & 0 \\ 0 & 0 & \frac{S_1 - iS_2}{\sqrt{S_1^2 + S_2^2}} \end{pmatrix} \begin{pmatrix} S_0 \\ S_1 - iS_2 \\ S_1 + iS_2 \end{pmatrix} = \begin{pmatrix} S_0 \\ \sqrt{S_1^2 + S_2^2} \\ \sqrt{S_1^2 + S_2^2} \end{pmatrix}$$

Since this is real, $S_2' = 0$ and a diagonal form has been found.

The generalization to Hermitian matrices is straightforward; a 4-dimensional representation results, with S_0 splitting off once more as an invariant subspace. In the 3-space orthogonal to S_0 , all matrices unitarily similar to the given matrix lie on the sphere $S_0^2 = S_1^2 + S_2^2 + S_3^2$. The value of the determinant is reminiscent of the metric of Lorentz transformations. The sphere cuts the S_1 axis ($S_2 = S_3 = 0$) at two places, corresponding to the permutation of eigenvalues.

The general 2×2 real matrix has no special features in this analysis -- it is merely a special case of the complex 2×2 matrix since it may have complex eigenvalues (if any). It can be represented by letting S_α be complex, in a 4-dimensional complex space.

The 2×2 problem has such beautiful features that it is not clear which of the many one should try to preserve on going to higher dimensionalities. While it is clear that any mathematical results should be independent of the specific representation chosen, it is also clear that an unfortunate choice of representation would obscure the problem unnecessarily. For example, it is not quite obvious beforehand that all matrices which have a given determinant should lie on a circle about the S_0 axis, because in the a, b, c representation a circle is not easily recognizable. Further, it is not quite obvious beforehand that the entire subspace with constant determinant would be spanned by orthogonally (or unitarily) similar matrices. In fact, with higher dimensionalities, there are extra invariants which are preserved under similarity

transformations. These will restrict the available subspace.

Diagonalization of the inversion process seems much more straightforward than for the class of similarity transformations. It can be worked out algebraically using Cramer's rule for almost any representation, and that representation can be (algebraically) diagonalized. It is still a formidable program for matrices much larger than 4×4 .

With 4×4 matrices, the representation known to physicists as the Dirac matrices (1, 2) may be useful representation. It too, has the properties of a hypercomplex number system.

It is not clear whether the higher dimension matrices will lead to representations possessing invariant subspaces. The identity is by itself a one-dimensional invariant subspace for all dimensionalities. (The remaining invariants are linear combinations of products of different eigenvalues, all with + signs, that are invariant under permutation of the eigenvalues. There are $\binom{n}{k}$ terms in each sum. They are the coefficients of $(-\lambda)^{n-k}$ in the characteristic equation for the eigenvalues. There are not likely to be invariant subspaces corresponding to combinations of matrix invariants other than the trace.) In any event, the space that can be reached by similarity transformations from a given matrix should cut the subspace of diagonal matrices (itself an n -dimensional subspace) in precisely $n!$ points, corresponding to permutations of the eigenvalues. The picture of this in high dimensionalities is not clear.

Another way to construct these operations would be to compose operations of higher dimensionality from operations on 2×2 submatrices. This is a frequently-used strategy for matrix operations.

I welcome discussion on this matter and hope that this note may stimulate others to think along similar lines. It just might be fruitful.

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ON THE CONSTRUCTION OF FIXED POINTS AND SOLUTIONS OF NONLINEAR EQUATIONS WITH DEMICOMPACT MAPPINGS*

W. V. Petryshyn

1. Introduction. Let C be a closed bounded convex subset of a Banach space X and let S be a completely continuous⁽⁺⁾ nonlinear mapping of C into C . Then, by the Schauder fixed point principle (i.e., Theorem 1 in [8]) S has at least one fixed point in C . Using this result and assuming also that X is a uniformly convex⁽⁺⁾ Banach space and that S is a contraction on C , Krasnoselsky [5] showed that for any $x_0 \in C$ the sequence $\{x_{n+1}\}$ determined by

$$(1) \quad x_{n+1} = \frac{1}{2} Sx_n + \frac{1}{2} x_n, \quad n = 0, 1, 2, \dots,$$

converges to a solution $x^* \in C$ of the equation $Sx = x$.

In [9] Schaefer extended Krasnoselsky's result in two directions. First, he showed that under the above conditions on X and S the sequence $\{x_{n+1}\}$ determined by the process

$$(2) \quad x_{n+1} = \beta Sx_n + (1 - \beta)x_n, \quad 0 < \beta < 1, \quad n = 0, 1, 2, \dots$$

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⁽⁺⁾For the precise definitions of the notions mentioned in the Introduction and the statements of the corresponding results see Section 2.

converges to a fixed point of S , and second, using Schauder's Theorem 2 [8], he proved that when X is a real Hilbert space H and when instead of complete continuity of S one assumes that S is weakly continuous⁽⁺⁾ on C , then $\{x_{n+1}\}$ determined by (2) converges weakly to a fixed point in C of S .

The purpose of this article is to extend these results in the following two directions. First, using the very recent⁽⁺⁾ result of Browder [2] concerning the existence of fixed points for noncompact contractions in a Hilbert space H , the proof of which was independently also given by De Prima [3], we prove that if T is a linear continuously invertible⁽⁺⁾ operator on H and S is a nonlinear mapping such that $\frac{1}{m}S$ maps a ball⁽⁺⁾ $B_r \subset H$ into itself, $\|Sx - Sy\| \leq \|Tx - Ty\|$ for all x and y in B_r , and the mapping $T^{-1}S$ is demicompact⁽⁺⁾ on B_r , then the sequence $\{x_{n+1}\}$ determined by the equation

$$(3) \quad Tx_{n+1} = \beta Sx_n + (1 - \beta)Tx_n, \quad 0 < \beta < 1, \quad n = 0, 1, 2, \dots,$$

converges to a solution $x^* \in B_r$ of the equation

$$(4) \quad Tx = Sx.$$

Second, if S is a demicompact contraction mapping of B_r into H such that

$$(5) \quad Sx - \lambda x \neq 0$$

for all x on the boundary ∂B_r of B_r and any $\lambda > 1$, then the sequence

$\{x_{n+1}\}$ determined by the modified process

$$(6) \quad x_{n+1} = \beta r_n Sx_n + (1 - \beta)x_n, \quad 0 < \beta < 1, \quad n = 0, 1, 2, \dots,$$

where the real numbers r_n , $n = 0, 1, 2, \dots$, are given by

$$(7) \quad r_n = \begin{cases} 1 & \text{if } \|Sx_n\| \leq r \\ \frac{r}{\|Sx_n\|} & \text{if } \|Sx_n\| \geq r; \end{cases}$$

converges to a fixed point $x^* \in B_r$ of S . Similar results are obtained

when instead of condition (5) we assume that S satisfies either the condition

$$(5_1) \quad \operatorname{Re}(Sx, x) \leq \|x\|^2, \quad x \in \partial B_r,$$

or the condition

$$(5_2) \quad \|Sx\| \leq \|x\|, \quad x \in \partial B_r.$$

It seems that the interesting and important features of the above iterative procedures lie in the fact that their convergence takes place under rather general conditions on the operators which do not insure the uniqueness of solutions. It is therefore reasonable to expect that

these procedures will prove to be useful in their application to problems in pure and applied mathematics which do not admit unique solutions.

The author is greatly indebted to Prof. F. E. Browder for his suggestions and interest in this paper, and for having made a preliminary draft of [2] available to him.

2. Construction of solutions and fixed points. Let P be a (not necessarily linear) operator mapping its domain $D_p (\subseteq H)$ into H . Following [1, 6, 7], we say that P is demicontinuous if $x_n \rightarrow x$ ($x_n, x \in D_p$) strongly in H implies $Px_n \rightarrow Px$ weakly in H ; P is weakly continuous if $x_n \rightarrow x$ weakly in H implies $Px_n \rightarrow Px$ weakly in H ; P is strongly continuous if $x_n \rightarrow x$ weakly in H implies $Px_n \rightarrow Px$ strongly in H ; P is compact if T maps every bounded set in D_p into a compact set in H ; P is completely continuous if T is continuous and compact; T is monotone if for all x and y in D_p

$$(8) \quad \operatorname{Re}(Px - Py, x - y) \geq 0,$$

strongly monotone if for all x and y in D_p and some $\eta > 0$

$$(9) \quad \operatorname{Re}(Px - Py, x - y) \geq \eta \|x - y\|^2,$$

and complex monotone if for all x and y in D_p and some $\tilde{\eta} > 0$

$$(10) \quad |(Px - Py, x - y)| \geq \tilde{\eta} \|x - y\|^2.$$

Let us observe in passing that an immediate consequence of

Browder's Lemma 1 in [1] is the following interesting fact.

Lemma 1. (*) Let P be a monotone demicontinuous mapping whose domain D_P is an open convex set in H , and let us assume that the null set N of P (i.e., $N = \{x \in D_P : Px = 0\}$) is not empty. Then N is a convex set.

Proof. Let x_1 and x_2 be any two points in N such that $x_1 \neq x_2$, and for any real t such that $0 < t < 1$ let $x = tx_2 + (1-t)x_1$. Then,

since $x_1, x_2 \in N$, D_P is convex, and P is monotone on D_P , $x \in D_P$ and for all y in D_P we have

$$\operatorname{Re}(Py, y - x_1) \geq 0 \quad \text{and} \quad \operatorname{Re}(Py, y - x_2) \geq 0.$$

Thus, multiplying the first inequality by $t > 0$ and the second by $(1-t) > 0$ and adding the resulting inequalities we get the desired inequality

$$\operatorname{Re}(Py, y - x) \geq 0,$$

which is valid for all y in D_P . Hence, by Lemma 1 in [1],

$Px = 0$, i.e., $x \in N$ and consequently, N is convex.

(*) Lemma 1 was originally formulated for the operator P of the form $P \equiv I - S$, where S is a contraction on D . But a subsequent discussion with Prof. F.E. Browder revealed its P validity in a much more general form stated here.

Let C be a closed bounded convex subset of H , and let ∂C be the boundary of C in H . A mapping P will be called a contraction on C if for all x and y in C

$$(11) \quad \|Px - Py\| \leq \|x - y\|.$$

In his very recent paper, Browder [2] (see also De Prima [3]), using the theory of monotone operators developed in [1, 6], proved the following useful theorem which we shall use in our discussion of the iterative methods (3) and (6)-(7).

Theorem B. If P is a contraction map of C into C , then P has at least one fixed point in C .

Theorem B was then used in [2] to present a systematic generalization, to a class of contraction operators in Hilbert space, of principal results in the fixed point and mapping theory for completely continuous operators. No indication was given, however, of how to construct a fixed point of a contraction mapping if we know its existence.

The purpose of this section is to discuss the method (3) and its modification (6)-(7) for the construction of a solution of equation (4) and a fixed point of S .

A bounded linear operator T on H will be continuously invertible if T has a bounded inverse T^{-1} defined on all of H . It follows that for a continuously invertible operator T there exists two constants $M > 0$ and $m > 0$ such that

$$(12) \quad m \|x\| \leq \|Tx\| \leq M \|x\| \quad x \in H.$$

Using Theorem B and Lemma 1 we prove the following lemma which we employ in the proof of Theorem 2 below.

Lemma 2. If T is a continuously invertible operator on H , S is a nonlinear operator such that $\frac{1}{m} [S(x)]$ maps the ball $B_r = \{x \in H: \|x\| \leq r, r > 0\}$ into itself, and

$$(13) \quad \|Sx - Sy\| \leq \|Tx - Ty\|, \quad x, y \in B_r,$$

then there exists at least one point x^* in B_r such that

$$(12) \quad Tx^* = Sx^*.$$

Furthermore, the set Z of all solutions of (12) is convex. However, if

$$(13i) \quad \|Sx - Sy\| < \|Tx - Ty\|$$

for any two points x and y in B_r such that $x \neq y$, then Z contains only one solution of (12).

Proof. If in H we introduce a new inner product and norm by

$$(14) \quad [x, y] = (Tx, Ty), \quad |x|^2 = [x, x], \quad x, y \in H,$$

and denote the resulting space by $\bar{H}$, then, since T is continuously invertible, $\bar{H}$ is a complete Hilbert space and the norm $\| \cdot \|$ in $\bar{H}$ is equivalent to the norm $\| \cdot \|$ in H . Furthermore, this and our conditions imply that $T^{-1}S$ is a contraction map of a closed bounded convex set B_r , considered as a subset of $\bar{H}$, into B_r . Indeed, for each y in B_r the corresponding solution $x = T^{-1}S$ belongs to B_r since $\|x\| = \|T^{-1}Sy\| \leq \frac{1}{m}\|Sy\| \leq r$ and, by (13) and (14),

$$(15) \quad \|T^{-1}Sx - T^{-1}Sy\| = \|Sx - Sy\| \leq \|Tx - Ty\| = \|x - y\|, \quad x, y \in B_r.$$

Thus, by Theorem B, $T^{-1}S$ has at least one fixed point $x^* \in B_r$ for which, of course, (12) is valid.

The validity of the second assertion of Lemma 2 follows from Lemma 1 since, in virtue of (15), the operator $P \equiv I - T^{-1}S$ is monotone on B_r , as a mapping in $\bar{H}$, and the zeros of P are the same as the solutions of (12). Let us remark that in our case here Lemma 1 remains valid even for a closed convex set, since for any x_1 and x_2 in Z the point $x = tx_2 + (1-t)x_1$ is always an interior point, or else we can extend $T^{-1}S$ to all of $\bar{H}$ and consider $P = I - T^{-1}S$ as a mapping of $\bar{H}$ into $\bar{H}$.

The proof of the last assertion is trivial, for if there were another solution z^* in Z such that $z^* \neq x^*$, then we would have the equality

$$\|Sx^* - Sz^*\| = \|Tx^* - Tz^*\| \text{ contradicting (13i).}$$

It is easy to see that in a Hilbert space the identity

$$\left\| \frac{x+y}{2} \right\|^2 + \left\| \frac{x-y}{2} \right\|^2 = 2 \left\{ \|x\|^2 + \|y\|^2 \right\}$$

implies the uniform convexity of H , i.e., to any $\varepsilon > 0$ there corresponds

a $\delta = \delta(\varepsilon, r) > 0$ such that $\left\| \frac{x+y}{2} \right\| < (1-\delta) \max \{ \|x\|, \|y\| \}$

whenever $x, y \in B_r$ and $\|x - y\| > \varepsilon$. In his investigation of method

(2), Schaefer [9] derived a much more refined inequality for a uniformly

convex Banach space; we use a special case of this here in the form of the following lemma.

Lemma S. If x and y are any points of B_r and $\beta > 0$ is such that $1 - \beta > 0$, then to any $\varepsilon > 0$ there corresponds a $\delta = \delta(\varepsilon, \beta, r) > 0$ with the property that for any $x, y \in B_r$ with $\|x - y\| > \varepsilon$ we have

$$(16) \quad \|\beta x + (1 - \beta)y\| < (1 - \delta) \max \{ \|x\|, \|y\| \}.$$

It turns out that to prove the convergence of the process (3)

and its modification (6)-(7) we need to impose an additional mild condition on the operator S .

A nonlinear operator P will be called demicompact if it has the property that whenever $\{u_n\}$ is a bounded sequence and $\{g_n\} \equiv \{u_n - Pu_n\}$ is a strongly convergent sequence, then there exists a subsequence $\{u_{n_i}\}$ which is strongly convergent.

Theorem 1. The class of demicompact operators P contains, among others, the following classes of operators.

(a) The class of compact operators (and, in particular, of completely continuous and strongly continuous operators).

(b) The class of all operators P such that $-P$ is quasi-compact.

(c) The class of operators P which satisfy either the condition

$$(17) \quad \operatorname{Re}(Px - Py, x - y) \leq a \|x - y\|^2, \quad (1 - 2a) > 0$$

or the condition

$$(18) \quad \operatorname{Re}(Px - Py, x - y) \leq a \|Px - Py\|, \quad (1 - 2a) \geq 0.$$

(d) The class of operators P for which $(I - P)^{-1}$ exists and is continuous on its range $R(I - P)$.

Proof. (a) The proof of assertion (a) is trivial. Indeed, if $\{u_n\}$ is a bounded and $\{g_n\} \equiv \{u_n - Pu_n\}$ is a strongly convergent sequence, then the compactness of P implies that $\{Pu_n\}$ contains a strongly convergent subsequence $\{Pu_{n_i}\}$. But then the sequence $\{u_{n_i}\} = \{g_{n_i} + Pu_{n_i}\}$ is strongly convergent.

(b) Assertion (b) is obvious since it is one of the four conditions defining a quasi-compact operator. For the discussion of the latter class of operators see the paper by Kaniel [4].

(c) Let $\{u_n\}$ be a bounded sequence so that $\{g_n\} = \{u_n - Pu_n\}$ is a Cauchy sequence. Then, by (17) or (18),

$$\begin{aligned} \|g_m - g_n\|^2 &= \|u_m - u_n - (Pu_m - Pu_n)\|^2 = \|u_m - u_n\|^2 - 2\operatorname{Re}(u_m - u_n, Pu_m - Pu_n) \\ &\quad + \|Pu_m - Pu_n\|^2 \geq (1-2a) \|u_m - u_n\|^2 + \|Pu_m - Pu_n\|^2 \end{aligned}$$

or

$$\|g_m - g_n\|^2 \geq (1-2a) \|Pu_m - Pu_n\|^2 + \|u_m - u_n\|^2.$$

Since $\{g_n\}$ is a Cauchy sequence, the last inequalities imply that $\{u_n\}$ is itself a Cauchy sequence.

(d) Let $\{u_n\}$ be a bounded sequence so that $g_n = u_n - Pu_n = (I-P)u_n$ is strongly convergent. But then, since $(I-P)^{-1}$ is continuous, $(I-P)^{-1}g_n = u_n$ converges strongly in H .

Remark 1. The class of operators P for which the conditions of Theorem 1(d) are satisfied includes, as a subclass, the demicontinuous operators P for which either the inequality (17) is valid with $a < 1$ or the inequality

$$(19) \quad |(Px - Py, x - y)| \leq b \|x - y\|^2, \quad 0 < b < 1,$$

since the operator $(I-P)$ is demicontinuous and strongly monotone in the

first case and complex monotone in the second. In both cases the operator $(I-P)^{-1}$ exists and is continuous, and, in fact, $R(I-P) = H$.

Using arguments similar to those in [5, 9] we now prove the theorem which establishes the convergence of the process (3).

Theorem 2. Let T be a continuously invertible operator on H , and let S be a nonlinear operator such that $\frac{1}{m} [S(x)]$ maps the ball B_r into itself and for all x and y in B_r

$$(20) \quad \|Sx - Sy\| \leq \|Tx - Ty\|.$$

If $T^{-1}S$ is demicompact on B_r , then for any $x_0 \in B_r$ the sequence

$\{x_{n+1}\}$ determined by the process (3) converges to a solution

$x^* \in Z \subset B_r$ of the equation (4).

Proof. Let us first observe that, in view of the conditions of Theorem 2, Lemma 2 implies that the set Z of solutions of (4) is not empty and the sequence $\{x_{n+1}\}$ determined by (3) belongs to B_r ; moreover, for each n

$$(21) \quad \|Tx_n - Sx_n\| \leq \|Tx_{n-1} - Sx_{n-1}\|, \quad n = 1, 2, 3, \dots$$

The last assertion follows from (3), the equality $\|Tx_{n+1} - Tx_n\| = \beta \|Tx_n - Sx_n\|$, and the fact that, by (3) and (20),

$$\|Tx_{n+1} - Tx_n\| = \|\beta (Tx_n - Sx_n) + (1 - \beta) (Sx_n - Sx_{n-1})\| \leq \|Tx_n - Tx_{n-1}\|.$$

Furthermore,

$$(22) \quad \lim_n \|Tx_n - Sx_n\| = 0.$$

Indeed, assuming that to the contrary $\lim_n \|Tx_n - Sx_n\| > \varepsilon_0 > 0$ for some $\varepsilon_0 > 0$, and letting $v_n = Tx_n - Tz$ and $w_n = Sx_n - Tz$, where z is an arbitrary element of Z , we see that in virtue of (21) and (3) we obtain the relations $\|w_n - v_n\| > \varepsilon_0$ for $n \geq 0$ and

$$\begin{aligned} \|Tx_{n+1} - Tz\| &= \|\beta(Sx_n - Sz) + (1-\beta)(Tx_n - Tz)\| \\ &= \|\beta w_n + (1-\beta)v_n\|. \end{aligned}$$

Hence, by Lemma S, there exists a $\delta_0 = \delta(\varepsilon_0, \beta, Mr) > 0$ so that

$$(23) \quad \|Tx_{n+1} - Tz\| < (1 - \delta_0) \|Tx_n - Tz\|$$

and, therefore, by (20) and (23)

$$(24) \quad \|Tx_n - Sx_n\| \leq \|Tx_n - Tz\| + \|Sz - Sx_n\| < 2(1 - \delta_0) \|Tx_{n-1} - Tz\|.$$

The inequalities (23) and (24) imply the relation (22). Hence the contradiction of our assumption is obtained.

Let us now observe that, by (22) and (12), $g_n \equiv x_n - T^{-1}Sx_n$ converges strongly to 0. Since $\{x_n\}$, being in B_r , is bounded and $T^{-1}S$ is demicompact

on B_r , there exists a strongly convergent subsequence $\{x_{n_i}\}$ such that $x_{n_i} \rightarrow x^* \in B_r$ and $Tx_{n_i} \rightarrow Tx^*$ strongly in H . Hence, by (20) and (22), $\|Sx_{n_i} - Sx^*\| \leq \|Tx_{n_i} - Tx^*\| \rightarrow 0$ as $n_i \rightarrow \infty$, and $Tx^* = Sx^*$, i. e., $x^* \in Z$. Since for every n

$$\begin{aligned} \|Tx_{n+1} - Tx^*\| &= \|\beta(Sx_n - Sx^*) + (1-\beta)(Tx_n - Tx^*)\| \leq \beta\|Sx_n - Sx^*\| \\ &\quad + (1-\beta)\|Tx_n - Tx^*\| \leq \|Tx_n - Tx^*\| \end{aligned}$$

we conclude that the entire sequence $\{x_{n+1}\}$ converges to x^* , and thus complete the proof of Theorem 2.

Corollary 1. If S is a demicompact contraction from B_r to B_r then the sequence $\{x_{n+1}\}$ determined by (2) converges to a fixed point $x^* \in B_r$ of S .

Proof. If S maps B_r into B_r , then Corollary 1 is a special case of Theorem 2 with $T = I$.

Remark 2. The assertion of Corollary 1 remains valid for general closed bounded convex subsets C of H , since the fact that B_r is a ball was only used to prove that $T^{-1}S$ maps B_r into itself. Consequently, Theorem 1(a) implies that the results of Krasnoselsky and Schaefer are special cases of Corollary 1.

Theorem 3. Let ∂B_r be the boundary of B_r and let S be a demicompact contraction map of B_r into H such that

$$(5) \quad Sx - \lambda x \neq 0 \text{ for all } x \in \partial B_r \text{ and any } \lambda > 1.$$

Then the sequence $\{x_{n+1}\}$ determined by the process (6)-(7) converges to a fixed point $x^* \in B_r$ of S .

Proof. Let R be the retraction map of H on B_r given by

(25)

$$Ru = \begin{cases} u & \text{if } \|u\| \leq r \\ \frac{ru}{\|u\|} & \text{if } \|u\| \geq r. \end{cases}$$

It is easy to see that R is a contraction of H into B_r and that $S_1 \equiv RS$ is a contraction map of B_r into B_r . Hence, by Theorem B, S_1 has at least fixed point $\tilde{x}$ in B_r so that $S_1 \tilde{x} = \tilde{x}$. But then, in view of our condition (5) and the definition of S_1 , it is not hard to see that $\tilde{x}$ is also a fixed point of S (see Theorem 3 in [2]).

Now in view of (25) and the definition of S_1 the process (6)-(7) can be written in the form

$$(25) \quad x_{n+1} = \beta S_1 x_n + (1 - \beta) x_n, \quad n = 0, 1, 2, \dots$$

Consequently, in view of Corollary 1, the validity of Theorem 3 will be established if we prove that the operator S_1 is demicompact on B_r .

This will be shown in the following lemma.

Lemma 3. If S is a demicompact contraction map of B_r into H , then $S_1 = RS$ is also demicompact on B_r .

Proof. Let $\{u_n\}$ be an arbitrary sequence in B_r so that

$$(26) \quad g_n' \equiv u_n - S_1 u_n \rightarrow g'$$

strongly in H . In virtue of the demicompactness of S , to show that under these conditions there exists a strongly convergent subsequence

$\{u_{n_i}\}$ it is sufficient to show that $g_{n_k}' \equiv u_{n_k} - S u_{n_k}$ converges strongly for some subsequence $\{u_{n_k}\}$.

Let $\psi(t)$ be a real-valued continuous function defined by

$$(27) \quad \psi(t) = \begin{cases} 1, & 0 \leq t \leq r \\ \frac{r}{t}, & t \geq r. \end{cases}$$

Since $\{S u_n\}$ is a bounded set there exists a weakly convergent subsequence $\{S u_{n_i}\}$ and a real number $\alpha \geq 0$ such that

$$(28) \quad \|S u_{n_i}\| \rightarrow \alpha, \quad n_i \rightarrow \infty.$$

We have to distinguish three possible cases.

Case 1. If $\alpha < r$ and $\epsilon = \frac{r - \alpha}{2}$, then, in view of (28), there corresponds an integer $N = N(\epsilon) > 0$ such that for $n_i \geq N$

$$\alpha - \epsilon < \|S u_{n_i}\| < \alpha + \epsilon = \frac{\alpha + r}{2} < r.$$

Thus in this case infinitely many elements $S u_{n_i}$ (for which $n_i \geq N$)

belong to the interior of B_r . Consequently, (26) and the definition of S_1 imply that

$$g'_{n_i} \equiv u_{n_i} - S_1 u_{n_i} = u_{n_i} - Su_{n_i} \equiv g_{n_i}$$

is strongly convergent. By demicontinuity of S , there exists a strongly convergent subsequence $\{u_{n_k}\}$.

Case 2. If $\alpha = r$, then $\psi(\|Su_{n_i}\|) \rightarrow 1$ and there exists a sequence of numbers $\{\epsilon_{n_i}\}$ with $\epsilon_{n_i} \rightarrow 0$, as $n_i \rightarrow \infty$, such that $\psi(\|Su_{n_i}\|) = 1 + \epsilon_{n_i}$ and

$$g'_{n_i} \equiv u_{n_i} - S_1 u_{n_i} = u_{n_i} - \psi(\|Su_{n_i}\|) Su_{n_i} = u_{n_i} - Su_{n_i} - \epsilon_{n_i} (Su_{n_i}).$$

Thus, $g_{n_i} \equiv u_{n_i} - Su_{n_i} = g'_{n_i} + \epsilon_{n_i} Su_{n_i}$ converges strongly, since g'_{n_i} is strongly convergent and $\epsilon_{n_i} \rightarrow 0$ and Su_{n_i} is bounded (and hence

$\epsilon_{n_i}(Su_{n_i}) \rightarrow 0$). Hence there exists a strongly convergent subsequence

$$\{u_{n_i}\}.$$

Case 3. If $\alpha > r$, then $\psi(\|Su_{n_i}\|) \rightarrow \frac{r}{\alpha} \equiv \tilde{\alpha} < 1$ and there exists a sequence $\{\epsilon_{n_i}\}$ with $\epsilon_{n_i} \rightarrow 0$ as $n_i \rightarrow \infty$, such that $\psi(\|Su_{n_i}\|) = \tilde{\alpha} + \epsilon_{n_i}$ and

$$g'_{n_i} \equiv u_{n_i} - S_1 u_{n_i} = u_{n_i} - \tilde{\alpha} Su_{n_i} - \epsilon_{n_i} (Su_{n_i}).$$

Thus, as in case 2, $\tilde{g}_{n_i} \equiv u_{n_i} - \tilde{\alpha} Su_{n_i} = g'_{n_i} + \epsilon_{n_i} (Su_{n_i})$ is strongly

convergent. Furthermore, using the fact that S is a contraction and that $0 < \tilde{\alpha} < 1$, we get in this case the relation

$$\|\tilde{g}_{n_i} - \tilde{g}_{n_k}\| \geq \|u_{n_i} - u_{n_k}\| - \tilde{\alpha} \|Su_{n_i} - Su_{n_k}\| \geq (1 - \tilde{\alpha}) \|u_{n_i} - u_{n_k}\|,$$

i. e. $\{u_{n_i}\}$ is a Cauchy sequence.

Thus the proof of Theorem 3 now follows from Corollary 1 and

Lemma 3.

To show that the conclusion of Theorem 3 remains valid when the condition (5) is replaced by either the condition (5₁) or the condition (5₂), it is sufficient to show that both of these conditions imply our general condition (5).

Lemma 4. The conditions (5₁) and (5₂) imply the condition (5).

Proof. Since clearly (5₂) implies (5₁) it is therefore sufficient to show that (5₁) implies (5). Suppose, to the contrary, that (5₁) holds and for some $\lambda_0 > 1$ and $x_0 \in \partial B_r$ we have $Sx_0 = \lambda_0 x_0$. Then $\text{Re}(Sx_0, x_0) = \lambda_0 \|x_0\|^2$. Hence, for (5₁) to hold, we must have $\lambda_0 \leq 1$. Therefore, the assumption that for some $\lambda_0 > 1$ and $x_0 \in \partial B_r$ we have $Sx_0 = \lambda_0 x_0$ leads to contradiction of (5₁). Thus we obtain the following theorem as a particular case of Theorem 3.

Theorem 4. If S is a demicompact contraction map of B_r into H such that either the condition (5₁) or the condition (5₂) is satisfied, then the sequence $\{x_{n+1}\}$ determined by the process (6)-(7) converges to a fixed point $x^* \in B_r$ of S .

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PROPOSAL FOR A LARGE MICHELSON STELLAR INTERFEROMETER

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A report entitled "Measurement of Stellar Diameters" has been prepared and circulated. Since its main emphasis is astronomical, it does not seem appropriate to include much detail concerning the proposed interferometer here. A few copies of the report are available and a copy can be sent upon request.

Briefly, the report contains a review of the history and operation of the Michelson Stellar Interferometer, with mention of the Intensity Interferometer of Hanbury Brown and Twiss. In a comparison of the two instruments, it is pointed out that, while both provide information on the diameters of stars or starlike objects, the Intensity Interferometer is much simpler from an engineering point of view and is much less sensitive to atmospheric turbulence ("scintillation" or "seeing"), but its brightness limitation at about magnitude 3.5 severely limits the number and kinds of objects that can be studied. On the other hand, recent technological developments seem to make a large Michelson Stellar Interferometer appear possible, and its less severe brightness limitation makes it attractive in spite of the engineering difficulties. The Michelson Stellar Interferometer may be able to work with objects as faint as the quasistellar sources (13th, 16th magnitude and fainter; 10 magnitudes is a factor of 10^{-4} in brightness).

It appears that previous attempts to build and operate Michelson Stellar Interferometers have been hampered by engineering difficulties, and by a lack of enthusiasm resulting from the limited number of objects accessible for study by the interferometers that could be built (20 and 50 feet). The report contains proposals for avoiding these problems and suggests a feasibility study leading to the construction of a one-kilometer interferometer that should be able to resolve quasistellar sources if it can be made to work on such faint objects.

Any Michelson Stellar Interferometer requires length measurements to within a few wavelengths of visible light. The requirement is not relaxed with a very large instrument. The length measurements are to be made by laser fringe counters, and the operation of the entire instrument would require 20-30 such counters. Compensation for the earth's rotation requires dynamic control allowing fringe counting rates of about 30,000 per second. This implies a very elaborate (computer) control and servo system which is not mentioned in the report. Its implementation seems manageable, although difficult.

Recognition and measurement of faint fringe patterns that change and move erratically due to "seeing" presents an interesting detection problem that is not treated in detail in the report.

The report is intended as the scientific justification for undertaking such a project. If it is well received, a feasibility study taking

1-2 years will be proposed, leading to a proposal for the final project.

The project would push existing technology far beyond its present

limits in several areas, and could have many useful byproducts.