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ANALYSIS AND REDUCTION OF
PROTON-RECOIL DATA

by

E. F. Bennett, R. Gold,
and I. K. Olson

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Reactor Physics Division

January 1968

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TABLE OF CONTENTS

	<u>Page</u>
ABSTRACT.....	9
PREFACE.....	9
I. PROTON-RECOIL PROPORTIONAL-COUNTER RESPONSE FUNCTIONS	10
A. Introduction	10
B. Energy-independent Response-function Measurements	12
C. Energy-dependent Response Function	15
1. Experimental Measurements	15
2. Energy-density Transformations	20
D. Analytical Formation of the Response Function	24
1. Treatment and Parametrization of Experimental Data ..	24
2. Construction of Response Matrix	32
E. Summary	34
II. UNFOLDING FINITE-SIZE EFFECTS	36
A. Introduction	36
B. Iterative Unfolding of Proton-recoil Spectra	36
C. Integral Unfolding	41
D. Comparison of Unfolding Methods	42
III. REDUCTION OF PROTON-RECOIL PULSE-HEIGHT DISTRIBUTIONS TO NEUTRON-FLUX SPECTRA.....	44
A. Introduction	44
B. Energy per Ion Pair	45
C. Carbon and Nitrogen Recoils	45
IV. COMPUTER PROGRAMS	47
A. Introduction	47
B. The WERM Code	48
C. The ITERATE Code	57

TABLE OF CONTENTS

	<u>Page</u>
D. Discussion of PSNS Code and Test Problem	61
1. Description of PSNS Code	61
a. Computation of Proton Spectrum	62
b. Integral Response Unfolding	62
c. Energy Spacing and Neutron-flux Format	63
d. Slopes and Errors	63
e. Heavy-particle Recoils	64
f. Form of Punched Data	65
2. PSNS Test Problem	65
 APPENDIXES	
A. Error Estimates for Iterative Unfolding	68
B. WERM Code Listing	74
1. Printout of Program WERM	76
2. Printout of Subroutine MATFORM	82
3. Printout of Subroutine PRINTS(NUM)	84
4. Printout of Subroutine MATPRI	85
5. Printout of Subroutine VMM	86
6. Printout of Subroutine READY	87
7. Printout of Subroutine AIM	88
8. Printout of Subroutine FIRE	89
9. Printout of Subroutine DRESS	90
10. Printout of Subroutine STUFF	91
11. Printout of Subroutine MATMPY(M,N,H,G,S)	91
12. Printout of Subroutine FCN(M,G,F,B,M1)	92
C. ITERATE Code Listing	95
1. Printout of Program ITERATE	96
2. Printout of Subroutine MATSYM	98
3. Printout of Subroutine MAT1	98
4. Printout of Subroutine MPRINT(X,N)	99
5. Printout of Subroutine MAT2	100
D. PSNS Code and Flow Diagrams	102
1. Printout for Program PSNS	106
2. Printout for Function PLEG(N,ARG)	111
3. Printout of PSNS Output	111
ACKNOWLEDGMENTS	128
REFERENCES	129

LIST OF FIGURES

<u>No.</u>	<u>Title</u>	<u>Page</u>
1.	An Example of Spectral Distortion due to Finite-size Effects in Proton-recoil Proportional Counters	9
2.	Geometrical Variables Affecting Proportional-counter Response	13
3.	Pulse-height Spectrum of A^{37} from an Extremely Long Cylindrical Proportional Counter	14
4.	Pulse-height Spectrum A^{37} from a Counter with a Short Effective Length	14
5.	Relative Non-Gaussian Area versus Field-tube Diameter for Fixed Effective Length of 3.18 cm and Cathode Diameter of 1.59 cm	15
6.	Relative Non-Gaussian Area versus Effective Length for Fixed Field-tube Diameter of 0.056 cm and Cathode Diameter of 1.59 cm	15
7.	Geometric Characteristics of the 4π -hydrogen-recoil Proportional Counters	16
8.	The Thermal-neutron Exposure Facility in ATSR	17
9.	Response Distributions of Counter I at Total Pressure of 81.4 psi for Nitrogen Admixtures of 10 and 50%	18
10.	Response Distributions of Counter II at Total Pressure of 5.6 psi for Nitrogen Admixtures of 20 and 50%	19
11.	Ratio of Molecular Stopping Cross Sections in Nitrogen and Methane as a Function of Proton Energy	19
12.	Normalized Counter I Response Functions Corresponding to $E_0' = 2.26, 1.99, 1.63$, and 1.33 MeV Obtained from Measurements at 8, 10, 14, and 20 psi, Respectively	26
13.	Normalized Counter II Response Functions Corresponding to $E_0' = 2.87, 2.38$, and 2.11 MeV Obtained from Measurements at 3, 4, and 5 psi, Respectively	27
14.	The Multiplicative Transformation Factor $N \cdot \epsilon(E_0 - E) / N' \cdot \epsilon(E_0' - E')$	27
15.	Fourier-series Approximations of the Distorted Response Function $D(E', E_0')$ for Counter I at $E_0' = 0.626$ MeV	30
16.	Fourier-series Approximations of the Distorted Response Function $D(E', E_0')$ for Counter I at $E_0' = 3.00$ MeV	30

LIST OF FIGURES

<u>No.</u>	<u>Title</u>	<u>Page</u>
17.	Fourth-order Least-squares Polynomial Approximation of Relative Gaussian Area, $G_a(E_0')$, for Counter I.	31
18.	Fourth-order Least-squares Polynomial Approximation of Fourier Coefficient $a_3(E_0')$ for Counter I.	32
19.	Fourth-order Least-squares Polynomial Approximation of Fourier Coefficient $a_5(E_0')$ for Counter I.	32
20.	Interpolated (Renormalized) Response Function $R(E', E_i)$ Obtained from Parametrization Process for Counter I at Midbin Energy $E_i = 0.488$ MeV	33
21.	Interpolated (Renormalized) Response Function $R(E', E_i)$ Obtained from Parametrization Process for Counter I at Midbin Energy $E_i = 2.44$ MeV	33
22.	$1/E$ Test $\underline{X}$ Vector and $\underline{W}$ Vector for Counter I Delta-type Response Matrix of Order $n = 102$ with Mesh Size of $A = 19.53$ keV per Channel	37
23.	Norm of Residual Vector $N[R^{(m)}]$ as a Function of Iteration Number m for the $1/E$ Test Spectrum and Counter I Response Matrix	37
24.	Infinite-medium Proton-recoil Spectra Obtained from Same $\underline{W}$ Vector for Delta-type Response Matrix	38
25.	Infinite-medium Proton-recoil Spectra Obtained from Same $\underline{W}$ Vector for Triangular-type Response Matrix	39
26.	Comparison of Measured Proton-recoil Spectra from Counter Counter I_a (Aluminum Body, 50-psi Methane Filling) and Counter I_s (Stainless Steel Body, 80-psi Methane Filling)	40
27.	Comparison of Unfolded Infinite-medium Proton Spectra Obtained from Counters I_a and I_s	40
28.	Comparison of Neutron Spectra Using Iterative and Integral Unfolding	42
29.	Block Diagram of Logical Relationship among Computer Programs WERM, ITERATE, and PSNS	47
30.	Flow Chart A for WERM Code, Showing Treatment of Pressure-dependent Response Data.	49
31.	Flow Chart B for WERM Code, Showing Computations Leading to Parametrization of $G_a(E_0')$ and Fourier Coefficients $\{a_k(E_0')\}$	50

LIST OF FIGURES

<u>No.</u>	<u>Title</u>	<u>Page</u>
32.	Flow Chart C (MATFORM Subroutine) for WERM Code, Showing Formation of Matrix Elements	51
33.	Flow Chart for Program ITERATE	58
34.	Effect of Retention of Legendre Polynomial Terms on Test Spectrum	67
35.	General Flow Chart for PSNS Code	103
36.	PSNS Flow Chart A for Response-function Integration	104
37.	PSNS Flow Chart B for Slope-forming Routine	105

LIST OF TABLES

<u>No.</u>	<u>Title</u>	<u>Page</u>
I.	Sequence of Pressure Measurements for Counter I.	25
II.	Sequence of Pressure Measurements for Counter II.	26
III.	Output of Gaussian Fitting Routine (VMM Data).	52
IV.	Summary of Gaussian Data for Pressure Sequence	53
V.	Summary of Gaussian Parameters for Pressure Sequence	53
VI.	Least-squares Polynomial Coefficients for Fourier Coefficients and Gaussian Area	54
VII.	Printout of Upper Corner of Delta Response Matrix for Sample Problem	55
VIII.	Printout of Lower Corner of Delta Response Matrix for Sample Problem	56
IX.	Input Data for Iterative Unfolding of Sample Problem.	59
X.	Iterative Output for Iterations 2 and 10	60
XI.	Raw Data for First Set of PSNS Test Problem	65
B.I.	WERM Input	74
B.II.	Other Variables Used in WERM.	75
B.III.	WERM Sample Input.	92
C.I.	ITERATE Input.	95
C.II.	ITERATE Input.	101
D.I.	PSNS Input.	102
D.II.	Other Variables Used in PSNS.	103
D.III.	PSNS Input Data	113
D.IV.	PSNS Output Data	116

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ABSTRACT

A process for reducing 4π proton-recoil proportional-counter pulse-height distributions to fast-neutron energy spectra is described. Required corrections, including those associated with finite detector size, are analyzed in detail. This procedure provides corrected neutron spectra in the region from about 1 keV up to several MeV. The computer programs used in this treatment (WERM, ITERATE, and PSNS) are discussed, and a sample problem illustrates and explains the manner in which these codes function.

PREFACE

This report describes a treatment of experimental data obtained in fast-neutron spectral measurements by 4π -hydrogen-recoil proportional counting. It is instructive to indicate, at the outset, the extent of the correction that arises in a neutron spectrum due to the departure of the proportional-counter response from ideal.

Figure 1 is a fast-reactor spectrum measurement,¹ shown both with and without response-function corrections. (A complicated response function is associated with every detector of finite size.) The data have been smoothed to eliminate statistical fluctuations so that the general nature of the correction can be more readily observed. The uncorrected spectrum departs significantly from the true spectrum; the deviation is well in excess of other experimental errors. If spectrum measurements in fast reactors are to provide useful infor-

112-8539 Rev. 1

Fig. 1. An Example of Spectral Distortion due to Finite-size Effects in Proton-recoil Proportional Counters

mation concerning the quality of cross-section libraries or methods of calculation, a systematic error of the kind displayed in Fig. 1 is intolerable.

The complete correction, over the entire energy region, requires detailed experimental measurements that must be subsequently followed by an equivalent analytical effort. To describe the entire procedure, this report has been divided into four major parts, which deal with (1) the determination of proton-recoil proportional-counter response functions, (2) the unfolding of finite-size effects, (3) the reduction of proton-recoil data to neutron-flux spectra, and (4) the computer codes that perform these tasks, respectively. In Part I, the experimental methods used to obtain proportional-counter response functions are given and the (approximate) analytical representation of the response function is obtained. Part II uses the response function generated in Part I in a treatment of finite-size effects with both iterative and integral unfolding methods. Part III contains a number of detailed corrections, not involving finite-size effects, that arise in the process of reducing the corrected proton-recoil data (obtained from Part II) to neutron spectra. Part IV describes the computer codes WERM, ITERATE, and PSNS that have been developed to process raw proton data. A sample problem illustrates and explains the manner in which these codes function. The detailed FORTRAN listings of these codes are presented in Appendixes B-D.

I. PROTON-RECOIL PROPORTIONAL-COUNTER RESPONSE FUNCTIONS

A. Introduction

The determination of fast-neutron energy distributions by measurement of proton-recoil energy spectra is well established and exploited. This method is capable, in principle, of yielding accurate neutron-energy distributions, provided the corresponding proton spectra are determined precisely. In practice, limitations arise because of the finite sensitive volume of the detector. In fact, given a detector of finite sensitive volume, one cannot, in general, measure the desired proton spectrum precisely. The desired proton spectrum is by definition simply the infinite-medium proton spectrum, i.e., that spectrum attained in the limit of infinitely large sensitive volume. Accurate neutron spectra can be derived only from infinite-medium proton-recoil spectra.

The most apparent finite-size effect is related to the high-energy limit of the domain of applicability of the proton-recoil method with a given detector. This limit can be roughly defined as the point at which the range of a recoiling proton becomes comparable with the dimensions of the sensitive volume. In this energy region and beyond, proton energy losses outside this finite sensitive volume can lead to a significant distortion in the measured proton-recoil spectrum. The region in which the measured and infinite-medium proton-recoil spectra begin to differ significantly can be taken to define an inherent high-energy limitation of the proton-recoil method.

This example of finite-size effects is commonly called wall-and-end effect. Because of the important role this effect plays in defining the upper energy-response limit, it has received almost exclusive attention. Treatments of the wall-and-end-effect problem have centered upon the calculation of the probability distribution for the truncation of proton tracks in the walls and ends of the detector. These methods have used direct geometrical analysis²⁻⁶ (which usually leads to approximations), Monte Carlo techniques,^{7,8} or astute combinations of these computational approaches.⁹ More recently, an exact analytical treatment of this geometrical problem has been developed.¹⁰

These geometrically based formulations of the wall-and-end-effect problem comprise an approximate description of finite-size effects which shall be called the geometrical model. Although these approaches have demonstrated limited success, the large amount of emphasis and concentration on the geometrical model has tended to obscure the real nature of the problem. From a more general viewpoint, the presence of the wall-and-end effect implies the existence of an unfolding problem. Specifically, given the measured proton-recoil spectrum, what is really sought is the infinite-medium proton-recoil spectrum. Moreover, calculated probability distributions (within the framework of the geometrical model) can serve only to define (or approximate) the response function or response matrix of the detection system.¹¹

The treatment of the wall-and-end-effect problem in 4π hydrogen-recoil proportional counters used here has been formulated in this perspective. In this method, a wall-and-end-effect response matrix is determined from experimental measurements with the given recoil detector of interest. Several important advantages of this empirical formulation will become apparent in sequel. Perhaps the most crucial of these advantages lies in a more complete determination of the response matrix. Although it can be argued that the present experimental determinations may complement certain limited theoretical calculations (of the geometrical model), these measurements actually go much further. Indeed, these measurements contain a complete description of the spectral distortion, whereas the geometrical model cannot account for other sources of spectral distortion which can and often do arise. Often the basis for these additional sources of distortion may be obscure or unknown. The strength of the present empirical approach lies in the fact that these effects need not be calculated. Since all possible sources of distortion are present in the experimental measurements, these effects are automatically included in the determination of the response matrix.

"Tip effect" is perhaps the most important example of finite-size effects not included in the geometrical model. The existence of tip effect in all proportional counters can be traced, primarily, to the presence of inhomogeneities in the electric field strength at the extremities of the

counter.^{2,12} These inhomogeneities lead to nonuniform gas gain and therefore spectral distortion, which can become serious for smaller proportional counters. Consequently, for numerous applications that require proportional counters of small or miniature size, a comprehensive treatment must include a consideration of tip effect. For example, measurement of fast-reactor neutron spectra with in-core proportional-counter probes requires small sensitive volumes to keep the total event rate (signal plus background) within the processing rate capability of the electronic instrumentation.^{13,14} In such cases, tip effect can introduce a significant low-energy distortion.

The present method, which is based on experimentally determined response functions, divides quite naturally into two logical domains. For sufficiently low-energy events (i.e., ionization events of negligible range), the response function will be energy-independent. To obtain the energy-independent proportional-counter response function, we can introduce an appropriate soft ionizing radiative decay into the counter. The present measurements have used the A^{37} decay (2.82 keV) for this purpose. The origin of any spectral distortion that arises in counting this source must be due to tip effect.

In the second logical domain, the response function is energy-dependent. The energy-dependent behavior of the response function in this region can be determined from a sequence of measurements using the slow-neutron-induced $N^{14}(n,p)C^{14}$ reaction. From the viewpoint of this domain, the low-energy A^{37} -determined response function assumes an added significance. Namely, this measurement provides a low-energy boundary condition which must be satisfied by the energy-dependent response function.

Section B below considers the energy-independent response function obtained from the A^{37} experiments. Sections C and D discuss the experimental method used to obtain the energy-dependent response matrix. These sections describe the experimental measurements, the subsequent treatment of the data to determine response functions, the parametrization of the response function, and the construction of the response matrix.

B. Energy-independent Response-function Measurements

Proportional counters used for neutron spectroscopy have well-defined walls but poorly defined ends.^{2,12,15} At least three methods are in current use to provide an "effective end" (two ends per counter). The first method involves a field tube of small diameter relative to the cathode and aligned concentrically with the anode wire. The guard field tube is kept at a potential appropriate to the anode field and field lines terminate on the guard tube in the region near the ends, thereby preventing the collection of ionization that originates outside the guard tube. A simpler (and consequently more widely used) method is to increase the diameter of the anode wire in the end region. Gas multiplication depends very sensitively upon anode

diameter, and an increase by several times in the anode diameter will decrease multiplication by several orders of magnitude. This method was used exclusively for measurements described in this report and is especially well suited for counting tubes of small size. The third method, applicable to counters having spherical cathodes,¹⁵ uses careful placement of electrical insulation near ends. The tendency for the electric field to increase with decreasing cathode diameter is counteracted by the presence of the insulation.

In practice, and especially as the effective length of counters decreases, the response function will depart from ideal. It is not possible, even with the first method outlined above, to produce an ideal response, since lateral diffusion of ionization from proton recoils will occur and events near ends may be truncated regardless of the method of field shaping.

Figure 2 shows the parameters that can be varied in designing a cylindrical counter desensitized by an anode-diameter increase near the

112-8868

Fig. 2. Geometrical Variables Affecting Proportional-counter Response

ends. That a reduction in counter size (of d and/or l) to minimal values may be beneficial from an experimental viewpoint is clear from the following considerations. Sensitivity of counters to gamma background is nearly proportional to the total wall area, since most conversion electrons originate through interactions in the walls. The number of hydrogen

atoms in the effective counting region will determine the rate of proton recoils. Ordinarily resolution can be maintained in counters if gas pressure is increased in proportion to any reduction in diameter. Wall-and-end-effect distortion (which is governed by the ratio of gas pressure to counter diameter) will remain essentially unchanged if gas pressure is increased in proportion to any reduction in counter size. The upshot is that a reduction in counter size, accompanied by a corresponding increase in gas pressure, will improve the signal (proton-recoil)-to-background (gamma) rate while maintaining wall-and-end-effect problems (as well as detector resolution) fixed. The neutron sensitivity of the detector will also decrease, and this is desirable for experiments with a high neutron background (due to spontaneous fission) such as will be found in plutonium-fueled fast reactors. Where a high neutron sensitivity is desirable, counter size can be increased with an attendant improvement in internal electric field properties and a more nearly ideal response function.

The obvious way to test for response-function effects of the kind mentioned is to observe counter pulse-height spectra in response to monoionizing events; these events must be induced uniformly over the counter and have negligible track extension. The electron emitted by the decay of A^{37} (2.8 keV) by K capture is a convenient test source; it is inexpensive and can be added in small amounts to the normal filling gas.

Figure 3 is an A^{37} pulse-height spectrum obtained in an extremely long cylindrical proportional counter designed for cosmic-ray experiments.¹⁶

112-8919

Fig. 3. Pulse-height Spectrum of A^{37} from an Extremely Long Cylindrical Proportional Counter

influenced by gas type or multiplication. At low channels, electrons from the L-shell decay of A^{37} appear and cause the distributions to increase just above discriminator cutoff. In the analysis of distributions from A^{37} for response-function information, the L-shell component has been removed, and in effect distributions have been extrapolated linearly into the low-lying channels.

To examine more closely the influence of the counter parameters depicted in Fig. 2 upon the response function, we measured the A^{37} spectrum using a variety of counters. Various combinations of d , b , and ℓ (the anode a was fixed at 0.0254 mm) were tested, and in each case the pulse-height distribution resembled that of Fig. 4 but with relatively more or less of a contribution below the main peak. Each spectrum was fitted, over the peak, by a Gaussian distribution, and the area included in the non-Gaussian region was compared with the total area. This ratio (the relative non-Gaussian area, $1 - G_a$), which would be zero for an ideal counter (as shown in Fig. 3), varied with counter design. Figure 5 is a plot of this ratio versus the diameter, b , of the field desensitizing tube for a counter having an inside diameter, d , of

Contrast this result with Fig. 4, which displays the A^{37} measurement for a much smaller counter ($\ell = 5.0$ cm, $d = 1.59$ cm). Distortion effects are quite apparent in the smaller counter and indicate that the electric field near the ends does not change as abruptly as would be desired due to the excessive thickening of the anode near the ends even for $b = 0.0254$ cm. (Some lateral diffusion of electrons may also be contributing, although the two effects cannot be distinguished.) A similar distribution was observed for the smaller counter at widely differing gas pressures and with different gases, indicating that the distribution is of essentially

"mechanical" origin and is little

112-8918

Fig. 4. Pulse-height Spectrum of A^{37} from a Counter with a Short Effective Length

1.59 cm and an effective length, ℓ , of 3.18 cm. We see clearly the desirability of keeping b as small as possible (consistent with an adequate reduction in gas multiplication). Figure 6 presents the relative non-Gaussian area versus the effective length of a counter, ℓ , having a fixed (1.59-cm) inside diameter, d , and fixed (0.056-cm) field tube, b . As expected, the effects of distortion decrease markedly with increasing effective length.

112-8871

Fig. 5. Relative Non-Gaussian Area versus Field-tube Diameter for Fixed Effective Length of 3.18 cm and Cathode Diameter of 1.59 cm

112-9011

Fig. 6. Relative Non-Gaussian Area versus Effective Length for Fixed Field-tube Diameter of 0.056 cm and Cathode Diameter of 1.59 cm

In the above measurements, the separation s (see Fig. 2) was maintained at about a counter cathode diameter. The distributions were not sensitive to s with s chosen in excess of about one cathode radius.

C. Energy-dependent Response Function

1. Experimental Measurements

The basis of the present method lies in the determination of the response function of a given proportional counter to monoenergetic protons. To this end, we studied a sequence of measurements containing various total gas pressures in a single counter, all of which included an appropriate amount of nitrogen. The response distribution for each pressure was obtained from an exposure of the counter to a thermal-neutron flux. In such an environment, monoenergetic protons from the $N^{14}(n,p)C^{14}$ reaction arise throughout the entire volume of the counter. In this manner, a sequence of experimental response distributions is obtained, one for each total counter gas pressure used. The transformation of this sequence into a sequence of energy-dependent proton-response functions in a counter of fixed pressure is discussed in Section 2 below.

The present series of experiments were conducted using thermal neutrons in the graphite reflector of the Argonne Thermal Source Reactor (ATSR).¹⁷ For these measurements, two proportional counters were

constructed of aluminum bodies. Figure 7 shows the geometric characteristics of these counters. The smaller counter shown in Fig. 7a (Counter I) is an exact duplicate of a stainless steel proportional counter used for in-core fast-neutron spectra experiments in a plutonium-fueled critical assembly.¹ Aside from a shorter length, Counter II, as shown in Fig. 7b, is otherwise geometrically identical to a one-meter-long stainless steel proportional counter used in cosmic-ray neutron experiments.¹⁶ The length of Counter II is a compromise between the practical size limitations of the thermal-neutron exposure facilities and the desire to maintain a good approximation of the length-to-diameter ratio of the cosmic-ray neutron counter. Figure 8 shows the location and arrangement of Counters I and II in the ATSR graphite reflector.

The quality of thermal flux used in these experiments is important since precise pulse-height distributions are required for an adequate definition of the response matrix. In particular, the presence of an appreciable component of fast flux could result in an undesirable contribution of proton recoils in the measured pulse-height distributions. To guard against such a possibility, the counters were located in a position (as shown in Fig. 8) that possessed a cadmium ratio larger than 2000.* In addition, measurements were made with methane fillings only. The complete absence of nitrogen permits a direct examination of the background pulse-height distribution. Preliminary background distributions were obtained in this manner with stainless steel proportional counters. This investigation revealed significant background pulse-height distributions induced by thermal-neutron reactions in the stainless steel counter walls. To solve this problem for the proportional counters of interest, aluminum counters possessing identical geometric characteristics were constructed, as described above. In this manner, thermal-neutron-induced wall reactions and capture-gamma background were considerably reduced. Fast-reactor spectrum measurements¹ have normally been made using stainless steel counters, and any reactions involving counter walls have been observed to

Fig. 7. Geometric Characteristics of the 4π -hydrogen-recoil Proportional Counters.

* This cadmium ratio was measured with bare and cadmium-covered gold foils and is defined as the ratio of bare-to-cadmium-covered saturated activity.

(A) SIDE VIEW

(B) TOP VIEW

NOTES:

1. MOVABLE TABLE SUPPORTING GRAPHITE BLOCK NOT SHOWN
2. THERMAL COLUMN AND TOP DETAIL NOT SHOWN ON REACTOR
3. ALL DIMENSIONS IN CENTIMETERS

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Fig. 8. The Thermal-neutron Exposure Facility in ATSR

be at least three orders of magnitude below normal proton-recoil rates. For the response-function tests in highly thermal fluxes, however, an aluminum body was preferable.

In the energy-dependent response region, proton-recoil proportional counters are filled with methane. Consequently, it is desirable to include only a small relative percentage admixture of nitrogen in the pressure-dependent sequence of methane fillings. On the other hand, the intensity of the desired pulse-height distribution is controlled by the size of this nitrogen admixture. Under conditions of a few percent nitrogen admixture, background distributions could not be considered negligible at the low end of the required pressure sequence of measurements, in spite of the background reductions attained with the aluminum counters. Further investigation revealed, quite fortunately, that the experimental response was independent of the percentage nitrogen admixture within the experimental error of the individual response-function measurements. This behavior is demonstrated for Counters I and II, in Figs. 9 and 10, respectively. Figure 9 contains the response distributions obtained from Counter I at a total pressure of 81.4 psi for relative nitrogen admixtures of 10 and 50%. Figure 10 shows the response distributions of Counter II for a total pressure of 5.6 psi with relative nitrogen admixtures of 20 and 50%. These distributions are normalized to equal area. The only apparent differences that can be observed correspond to slight changes in the resolution attained. Such minor differences must be anticipated, since small changes in resolution are common and accompany successive counter fillings which maintain constant gas mixtures.

112-8936 Rev. 1

Fig. 9. Response Distributions of Counter I at Total Pressure of 81.4 psi for Nitrogen Admixtures of 10 and 50%

112-9021

Fig. 10. Response Distributions of Counter II at Total Pressure of 5.6 psi for Nitrogen Admixtures of 20 and 50%

These results, obtained with geometrically different counters and at different pressures, imply that the experimental measurements are generally independent of the percentage nitrogen admixture. This conclusion can be traced to the behavior of the molecular stopping cross section, $\epsilon(E)$, for protons in methane and nitrogen. Figure 11 shows that the stopping cross sections for these two gases are not significantly different below 700 keV. Here the ratio $\epsilon_{N_2}/\epsilon_{CH_4}$, obtained from the data of Whaling,¹⁸ is plotted as a function of proton energy. It can be seen that this ratio does not significantly deviate from unity. We can therefore use any convenient nitrogen admixture and still obtain a response function representative of methane. The present sequence of pressure measurements used nitrogen admixtures of approximately 50%. Under these conditions, background contributions were generally negligible.

112-8870 Rev. 1

Fig. 11. Ratio of Molecular Stopping Cross Sections in Nitrogen and Methane as a Function of Proton Energy

2. Energy-density Transformations

Consider a uniform-isotropic distribution of monoenergetic protons in a 4π -recoil proportional counter. It will become evident, in sequel, that the sensitive volume of the counter can be of arbitrary geometry. Let N be the molecular density of the counter gas, and E_0 the energy of the monoenergetic protons. Under such circumstances, we can introduce the track-length probability distribution of proton events $P(\ell, r)$, where ℓ is the track length and $r = r(E_0)$ is the range of the monoenergetic protons in the counter gas. Given monoenergetic protons of energy E_0 , $P(\ell, r)d\ell$ is the probability of proton-track lengths arising between ℓ and $\ell + d\ell$ in the sensitive volume of the detector.

The relation between the actual response function of the counter to monoenergetic protons of energy E_0 , $R(E, E_0)$, and this track-length probability distribution is (neglecting resolution broadening)

$$R(E, E_0) dE = R[E(\ell), E_0(r)] \frac{dE}{d\ell} d\ell; \quad (1a)$$

hence,

$$P(\ell, r) = R(E, E_0) \frac{dE}{d\ell}. \quad (1b)$$

Equations 1a and 1b imply the existence of a continuous function, $\ell(E, E_0)$, relating proton track length and proton energy loss, E , in the sensitive volume of the detector. This fundamental relation can be written in the form¹⁸

$$\ell(E, E_0) = \frac{1}{N} \int_{E_0 - E}^{E_0} \frac{d\xi}{\epsilon(\xi)}, \quad (2)$$

where $\epsilon(\xi)$ denotes, as before, the molecular stopping cross section. Equations 1 and 2 apply strictly only to events originating in the sensitive volume and escaping with track length ℓ . Some events originate in the region near the ends and may enter the effective volume and terminate there or continue on into a wall or into the opposite end. These events will all appear in the experimentally observed distributions. They will be few in number, relative to the totality of events, and it is not necessary to include this refinement in the treatment of relative stopping power given here.

According to the Bethe-Livingston-Bloch formula,¹⁹

$$\epsilon(\xi) = C_1 \frac{Z}{\xi} \left(\ln \frac{\xi}{Z} + C_2 \right), \quad (3)$$

where Z is the atomic number of the medium and C_1 and C_2 are constants. For protons,

$$C_1 = 2\pi e^4 \frac{m_p}{m_e} \quad (4a)$$

and

$$C_2 = \ln \left(\frac{4}{I_0} \cdot \frac{m_e}{m_p} \right), \quad (4b)$$

where e is the electron charge, m_e is the electron mass, m_p is the proton mass, and I_0 is an empirical constant.

In terms of Eq. 2, the proton range is given by

$$r(E_0) = \ell(E_0, E_0) = \frac{1}{N} \int_0^{E_0} \frac{d\xi}{\epsilon(\xi)}. \quad (5)$$

It follows that the track length ℓ can be expressed as the difference between two ranges. We have

$$\ell(E, E_0) = \frac{1}{N} \left[\int_0^{E_0} \frac{d\xi}{\epsilon(\xi)} - \int_0^{E_0-E} \frac{d\xi}{\epsilon(\xi)} \right]$$

or

$$\ell(E, E_0) = r(E_0) - r(E_0 - E). \quad (6)$$

Let us now consider simultaneous energy-density transformations $(E, N) \rightarrow (E', N')$, where E' and N' represent new energy and molecular-density variables, respectively. To obtain the response function from the pressure-dependent sequence of $N^{14}(n, p)C^{14}$ measurements, energy-density transformations will be used that leave the track length (and therefore the track-length probability distribution) invariant. In view of Eq. 6, the requirement that ℓ be invariant can be satisfied, provided the range, r , is invariant under this transformation. This condition can be written in the form

$$r'(E') = r(E) \quad (7a)$$

or, more specifically,

$$\frac{1}{N'} \int_0^{E'} \frac{d\xi}{\epsilon(\xi)} = \frac{1}{N} \int_0^E \frac{d\xi}{\epsilon(\xi)}. \quad (7b)$$

Conversely, if N , N' , and E are prescribed, then Eqs. 7a and 7b define a transformation $E \rightarrow E'$ which leaves ℓ invariant.

Using such a transformation, we find from Eq. 1b,

$$P(\ell, r) = R(E, E_0) \frac{dE}{d\ell} = R(E', E'_0) \frac{dE'}{d\ell'} \quad (8)$$

Hence the response function due to monoenergetic protons of energy E'_0 is given by*

$$R(E', E'_0) = \left(\frac{dE'}{d\ell'} \right)^{-1} \cdot \left(\frac{dE}{d\ell} \right) \cdot R(E, E_0), \quad (9)$$

where the Jacobians that arise in Eq. 9 can be determined from Eq. 2. We have**

$$\frac{dE}{d\ell} = N \cdot \epsilon(E_0 - E) \quad (10a)$$

and

$$\frac{dE'}{d\ell'} = N' \cdot \epsilon(E'_0 - E'). \quad (10b)$$

Using Eqs. 10a and 10b, Eq. 9 assumes the form

$$R(E', E'_0) = \frac{N \cdot \epsilon(E_0 - E)}{N' \cdot \epsilon(E'_0 - E')} \cdot R(E, E_0). \quad (11)$$

We can demonstrate that Eq. 11, together with the energy-density transformation equations (i.e., Eqs. 7a and 7b, provide a basis for the definition of the energy-dependent response function (or response matrix). To this end, let $R(E, E_0)$ be the observed pulse-height distribution due to monoenergetic protons of energy E_0 in a counter possessing gas density N . The transformed distribution provided by Eq. 11, $R(E', E'_0)$, corresponds to the response function due to monoenergetic protons of energy E'_0 in a counter possessing a molecular gas density N' . Consider, therefore, a sequence of measurements with different gas densities $\{N_i\}$, $i = 1, 2, \dots, n$, undertaken with a fixed value of E_0 . These measurements give rise to a sequence of transformations $(N_i, E_0) \rightarrow (N'_i, E'_0)$, $i = 1, 2, \dots, n$. For each transformation, the measured response distribution $R(E, E_0)$ is mapped into a response function $R(E', E'_0)$. In particular, if we choose

*The transformation properties of the response function $R(E, E_0)$ are therefore those of a relative scalar of weight one (or weight unity).

**These equations assume that the Jacobians do not vanish in the region of interest; this assumption is justified on physical grounds, cf. Eqs. 10a and 10b.

$N_i' \equiv N'$, $i = 1, 2, \dots, n$, then all transformed response functions $\{R_i(E', E_0')\}$ will correspond to a single counter gas density, namely N' . The set of response functions so obtained constitute a representation of the complete energy-dependent response, including wall-and-end effects, of the 4π hydrogen-recoil proportional counter (with molecular gas density N').

The empirical basis of the present method rests upon the relationship between the measured response distribution, $R(E, E_0)$, and the transformed response function, $R(E', E_0')$. According to Eq. 11, each transformed response function is simply obtained from a measured distribution by applying a multiplicative factor. This factor is the ratio of the two aforementioned Jacobians. If this multiplicative factor is a constant, i.e., independent of energy, then the measured and transformed response distributions coincide. In this event, the raw experimental data directly determine the complete response function. What actually occurs in practice is quite close to this case. Indeed, as will be demonstrated, the multiplicative factor is constant except in the region $E \rightarrow E_0$ (or correspondingly $E' \rightarrow E_0'$). Consequently, over almost the entire energy domain, we are actually using measured distributions directly. In this manner, all possible distortions that arise, such as the tip effect (see p. 11), are automatically included in the response function. Use of these empirically determined response distributions is also sufficient justification for not extending Eqs. 1 and 2 to include events entering the effective counting region from the ends. More specifically, it can be shown that the formulation provided here is applicable for all but a small portion of these end events. In fact, the present treatment is valid for all events entering the ends that lie outside the region $E \rightarrow 0$ (or correspondingly $E' \rightarrow 0$).

Before proceeding to the experimental measurements, we must stress that the underlying assumption of the present formulation, namely the existence of a continuous range-energy relation, rests upon a strong experimental and theoretical base. Range-energy tables for protons (in gases of interest) exist with at most 1 or 2% relative error.¹⁸ Conversely, any method that uses a given range-energy relation must possess the limitations inherent in this relation. Thus, the treatment of finite-size effects for electrons, rather than protons, is of considerably greater complexity. In contrast with the proton case, for electrons we can no longer assume that straggling is negligible. It is evident, however, that if such a lack of definition in the range-energy relation exists, it is an additional complexity not only for the present formulation but also for all other formulations.

It is now clear that the applicability of the present treatment extends to proportional counters of arbitrary geometry. However, the response matrices defined by the present thermal-neutron experiments are actually isotropic response matrices. From a rigorous viewpoint, these response matrices are directly applicable for unfolding finite-size

effects in counters of arbitrary geometry, only when the angular neutron flux is isotropic. However, if the counter possesses symmetry, the applicability of the isotropic response matrix may be extended to cases where the flux anisotropy coincides with the counter symmetry. In this event, the counter response is equivalent to what would arise in an isotropic flux. It follows that spherical counters may be used in any flux; cylindrical counters are allowed anisotropic flux in the radial direction, but not in the axial direction. Isotropic response matrices can also be used for nonisotropic angular flux, provided the angular dependence of the flux is known. In this event, a series of measurements can be carried out for selected angular orientations of the counter in the nonisotropic flux. A proper choice of angles and/or time-integrated neutron intensity at given orientations, for this sequence of measurements, can provide an integrated exposure equivalent to an incident isotropic neutron flux. The isotropic response matrix could then be used for unfolding the sum of this chosen sequence of measurements.

D. Analytical Formation of the Response Function

1. Treatment and Parametrization of Experimental Data

We have pointed out that the present description can be implemented with the thermal-neutron-induced $N^{14}(n,p)C^{14}$ reaction. In this manner, however, the pure monoenergetic proton source required by this treatment is only approximately attained, since the C^{14} reaction product does carry off a small fraction of the available energy. To account for the contribution, the customary ionization-equivalent-energy calibration of the pulse-height scale is modified by subtracting the ionization equivalent energy of the C^{14} recoil (approximately 31 keV*) from the calibrated energy of each pulse-height channel. This small correction establishes a value of $E_0 = 584$ keV for the monoenergetic proton source used in the present series of measurements.

There exists an additional and important low-energy limitation inherent in all experimentally determined response distributions, namely that any measured distribution, such as $R(E, E_0)$, cannot be determined over the complete energy range $(0, \infty)$. In particular, a low-energy cutoff must be introduced to bias out both electronic noise and gamma-ray-induced events which arise in the lowest pulse-height channels. In the present measurements, $R(E, E_0)$ has been obtained below this cutoff by a smooth extrapolation of the measured pulse-height data in the channels above this cutoff.

* The ionization equivalent C^{14} recoil energy was determined from a comparison of the $N^{14}(n,p)C^{14}$ reaction Q-value and experimentally observed proportional-counter response to fast monoenergetic accelerator neutrons.

This treatment is not arbitrary and can be justified in terms of the foregoing analysis. Using Eqs. 8 and 10a, we can write

$$\lim_{E \rightarrow 0} \left[R(E, E_0) \cdot N \epsilon(E_0 - E) \right] = \lim_{\ell \rightarrow 0} P(\ell, r), \quad (12a)$$

or

$$\lim_{E \rightarrow 0} R(E, E_0) = \frac{\lim_{\ell \rightarrow 0} P(\ell, r)}{N \cdot \lim_{E \rightarrow 0} \epsilon(E_0 - E)} \quad (12b)$$

It follows from Eq. 12b that for small E , $R(E, E_0)$ will be a slowly varying function. It can be generally established that $P(\ell, r)$ is constant for $\ell \ll r$. Moreover, $\epsilon(\xi)$ is a smooth and slowly varying function for ξ in the neighborhood of E_0 . Consequently, smooth extrapolation of the experimental data into the region below the low-energy cutoff is justifiable.

Tables I and II summarize the sequence of pressure measurements carried out with Counters I and II, respectively. The values of E_0' listed in these tables correspond to methane operating pressures of 80 psi

TABLE I. Sequence of Pressure Measurements for Counter I

Filling No.	Resolution, ^a %	Nominal Pressure, psi	Transformed Energy, E_0' , MeV	Relative Gaussian Area, G_a
1	11.3	87	0.556	0.71
2	8.4	80	0.583	0.65
3	7.3	72	0.626	0.63
4	6.5	65	0.668	0.62
5	5.9	57	0.719	0.60
6	5.3	50	0.780	0.58
7	4.9	42	0.865	0.55
8	4.61	35	0.966	0.52
9	4.55	32	1.02	0.51
10	4.53	28	1.09	0.49
11	4.55	24	1.18	0.47
12	4.61	20	1.33	0.43
13	4.63	18	1.41	0.41
14	4.65	16	1.52	0.39
15	4.63	14	1.63	0.36
16	4.58	12	1.79	0.32
17	4.47	10	1.99	0.26
18	4.41	9	2.12	0.22
19	4.37	8	2.26	0.18
20	4.34	7	2.44	0.13
21	4.34	6	2.71	0.069
22	4.12	5	3.00	0.041

^aObserved relative full width half maximum (FWHM).

TABLE II. Sequence of Pressure Measurements for Counter II

Filling No.	Resolution, ^a %	Nominal Pressure, psi	Transformed Energy, E'_0 , MeV	Relative Gaussian Area, G_a
1	8.9	45	0.583	0.946
2	8.1	40	0.626	0.920
3	7.7	30	0.657	0.90
4	7.0	33	0.708	0.87
5	6.6	30	0.749	0.85
6	6.1	26	0.812	0.82
7	5.6	22	0.901	0.77
8	5.4	20	0.955	0.75
9	5.0	14	1.17	0.65
10	4.9	12	1.27	0.61
11	4.7	10	1.41	0.55
12	4.5	8.0	1.60	0.49
13	4.1	6.0	1.89	0.39
14	4.1	5.5	2.01	0.36
15	4.1	5.0	2.11	0.33
16	4.2	4.5	2.26	0.29
17	4.4	4.0	2.38	0.26
18	4.6	3.5	2.60	0.20
19	4.6	3.0	2.87	0.14
20	4.3	2.5	3.13	0.087

^aObserved relative full width half maximum (FWHM).

in Counter I and 45 psi in Counter II. These tables also contain the resolution (FWHM) and relative Gaussian area, G_a , obtained for each pressure measurement. Figures 12 and 13 show some examples of transformed response functions. Figure 12 presents Counter I results for the 8-, 10-, 14-, and

112-8915

Fig. 12. Normalized Counter I Response Functions Corresponding to $E'_0 = 2.26, 1.99, 1.63$, and 1.33 MeV Obtained from Measurements at 8, 10, 14, and 20 psi, Respectively

20-psi measurements corresponding to E'_0 values of 2.26, 1.99, 1.63, and 1.33 MeV, respectively. Figure 13 presents Counter II results for measurements at 3, 4, and 5 psi corresponding to E'_0 values of 2.87, 2.38, and 2.11 MeV, respectively. All transformed response functions have been unit-normalized; i.e., they satisfy the condition

$$\int_0^{\infty} R(E', E'_0) dE' = 1. \quad (13)$$

112-9018

Fig. 13. Normalized Counter II Response Functions Corresponding to $E_0^I = 2.87, 2.38$, and 2.11 MeV Obtained from Measurements at 3, 4, and 5 psi, Respectively

The actual behavior of the multiplicative transformation factor in Eq. 11,

$$\frac{N \cdot \epsilon(E_0 - E)}{N' \cdot \epsilon(E_0^I - E')},$$

has also been determined for the present experimental data. Figure 14 shows typical results for Counters I and II. As mentioned earlier, these data verify that the multiplicative transformation factor is essentially constant, except for $E \rightarrow E_0$ (or correspondingly $E' \rightarrow E_0^I$).

112-9071

(a)

(b)

Fig. 14. The Multiplicative Transformation Factor $N \cdot \epsilon(E_0 - E)/N' \cdot \epsilon(E_0^I - E)$: (a) Counter I at $E_0^I = 0.55, 1.02, 1.99$, and 3.00 MeV; and (b) Counter II at $E_0^I = 0.63, 0.96, 2.02$, and 3.13 MeV

The present measurements, or any other experimentally determined set of response distributions, can only serve to define the (energy-dependent) response function on a finite set of points in E'_0 -space. On the other hand, construction of the full response matrix requires the continuous behavior of the response function in E'_0 -space. To obtain such a continuous representation, we have introduced two analytical approximations for each measured response distribution. The need for two representations follows naturally from the existence of the distorted and undistorted portions of each distribution. The response function is partitioned accordingly to give

$$R(E', E'_0) = G(E', E'_0) + D(E', E'_0). \quad (14)$$

In this description, a Gaussian approximation, $G(E', E'_0)$, was introduced for the undistorted response. The function $D(E', E'_0)$, which denotes the distorted response, can now be regarded as the total response minus the undistorted Gaussian response. This is the same representation as was used in the analysis of the A^{37} spectra previously discussed.

In the neighborhood of the response peak, a Gaussian fit of the form

$$G(E', E'_0) = p(E'_0) \exp \left\{ -4 \ln 2 \left[\frac{E' - E'_0}{\Delta(E'_0)} \right]^2 \right\} \quad (15)$$

was determined for each measurement. In Eq. 15, $p(E'_0)$ and $\Delta(E'_0)$ are the peak value and full width at half maximum of the Gaussian fit. In practice, this fit is obtained by using only those data above the lower half-height energy value, i.e., in the region

$$E'_0 - \frac{\Delta(E'_0)}{2} \leq E'.$$

The relative Gaussian area, G_a , listed in Tables I and II was determined on the basis of this Gaussian representation. It is simply

$$G_a(E'_0) = \left(\frac{\pi}{4 \ln 2} \right)^{1/2} \cdot p(E'_0) \cdot \Delta(E'_0). \quad (16)$$

Given monoenergetic protons of energy E'_0 , $G_a(E'_0)$ represents (as implied above) the probability of undistorted events arising in the counter. The results summarized in Tables I and II demonstrate that this probability is a rapidly decreasing function of energy. In fact, the undistorted response vanishes rapidly above 3 MeV for both these detectors.

The resolution values quoted in Tables I and II have also been determined from this Gaussian representation. The results for both counters reveal a resolution of approximately 4.5% for $E'_0 \geq 1$ MeV. A gradual increase in resolution sets in below this energy. This behavior can be attributed to counter-gas impurities that contribute to broadening the resolution in the higher-pressure domain.

As pressure increases, proton tracks from the $N^{14}(n,p)C^{14}$ reaction are correspondingly reduced in length. In Counter I, for example, increasing total gas pressure in excess of about 90 psi does not produce significant additional changes in the shape of the pulse-height distribution. A substantial component of the response function which additional pressure increases will not reduce, still exists below the "Gaussian peak." The response distribution has reached an "asymptotic" shape; residual effects, especially "tip" effects, are entirely responsible for the non-Gaussian component of these asymptotic distributions. This may be independently verified by counting the soft (2.8-keV) electrons from A^{37} , as described in Part I.B. This radiation has no significant track extension, and the observed pulse-height distribution can only be distorted by nonideal electric field effects associated with the internal construction of the counter. When different intrinsic resolution and background effects are allowed for, the asymptotic (high-pressure limit) distribution from the $N^{14}(n,p)C^{14}$ reaction agrees very well with the distribution measured with the A^{37} source.

The distorted portion of the response, $D(E', E'_0)$, was approximated by expansion in a Fourier series. More specifically, the approximation

$$D(E', E'_0) \cong \sum_{k=1}^N a_{2k-1}(E'_0) \cos \frac{(2k-1)\pi E'}{2E'_0} \quad (17)$$

was introduced for $E' \leq E'_0$. This set of Fourier functions was chosen to ensure satisfaction of the boundary condition

$$D(E'_0, E'_0) = 0. \quad (18)$$

Experience with the present experimental data has demonstrated that adequate approximations can be obtained from this representation with six-term expansions ($N = 6$).

In addition to the representation of Eq. 17, the adequacy of two other representations (Legendre polynomials and least-squares polynomials) has been tested. The results obtained with the Legendre expansion were, in general, as adequate as those found with the Fourier series. However, the least-squares polynomial representation was not as satisfactory. Response matrices obtained from this representation did not produce a smooth interpolation of the proton response function for the subsequent iterative unfolding process.

It is clear why least-squares polynomial fitting is unsatisfactory; each polynomial argument is adjusted, in effect, relative to values assigned to the arguments of all the other polynomial terms. Slight deviations in a distribution may produce large changes in relative arguments. This does not occur in orthonormal expansions, since the amplitude of each member of the set is independently determined.

Two examples of the approximation afforded by Eq. 17 are presented in Figs. 15 and 16 for Counter I at $E_0' = 0.626$ and 3.00 MeV, respectively. These figures demonstrate the general trend that occurs; namely, the adequacy of the Fourier representation, Eq. 17, improves with increasing E_0' . This behavior is quite advantageous, since the distorted portion of the response function increases with increasing E_0' . Consequently, the adequacy of the Fourier representation improves as the (relative) distorted response area increases.

112-9015

Fig. 15. Fourier-series Approximations of the Distorted Response Function $D(E', E_0')$ for Counter I at $E_0' = 0.626$ MeV. The unnormalized Gaussian response at E_0' is displayed for orientation purposes only.

112-9016

Fig. 16. Fourier-series Approximations of the Distorted Response Function $D(E', E_0')$ for Counter I at $E_0' = 3.00$ MeV. The unnormalized Gaussian response at E_0' is displayed for orientation purposes only.

The basis for a continuous representation of the response function is available from Eqs. 15 and 17. To this end, the parameters $p(E_0')$ and $\Delta(E_0')$ of Eq. 15 and the Fourier coefficients $\{a_{2k-1}(E_0')\}$ are all regarded as functions of E_0' . Appropriate analytical approximations of these functions in E_0' -space will then provide the necessary continuous representation of the response function.

The process of determining any two-dimensional response function from experimental observations is subject to certain fundamental limitations. The very requirement of a smooth continuous representation of the response function (in E'_0 -space) limits the number of coefficients $\{a_k(E'_0)\}$ that can be used in any chosen representation of $D(E', E'_0)$. In principle, the set $\{a_k(E'_0)\}$ can be chosen large enough to obtain any desired degree of approximation to the measured data. (These data represent isolated measurements of the response functions at a limited number of energy points in E'_0 -space.) However, higher-order coefficients become sensitive not only to experimental error but also to computational procedure and even computer round-off error. It follows that the higher-order coefficients obtained from any interpolation method will rapidly lose physical significance with increasing order. If too many higher-order terms are retained, this sensitivity will be reflected in the interpolated response function leading to nonsmooth and generally inadequate results. The number of terms used in any given representation of $D(E', E'_0)$ must therefore be a judicious compromise between the complementary needs of employing a good approximation of the response function data and the requirement of a smooth, well-behaved, interpolated response function.

Instead of actually employing $p(E'_0)$ and $\Delta(E'_0)$ in the continuous representation of the Gaussian response, we used $G_a(E'_0)$ and $\Delta(E'_0)$. This choice is supported by the foregoing discussion, which points out the significance of $G_a(E'_0)$. Indeed, $G_a(E'_0)$ is the single most important physical observable associated with the Gaussian response. An adequate continuous approximation of $G_a(E'_0)$ was obtained with a fourth-order least-squares polynomial fit, and an example of this approximation is presented in Fig. 17.

112-8869

Fig. 17. Fourth-order Least-squares Polynomial Approximation of Relative Gaussian Area, $G_a(E'_0)$, for Counter I

In addition, a seventh-order least-squares polynomial fit was used to approximate the behavior of $\Delta(E'_0)$. The continuous representation for the parameter $p(E'_0)$ can then be obtained by using these two polynomial approximations in Eq. 16.

In a similar manner, a least-squares polynomial fit was obtained for each of the Fourier coefficients, $a_{2k-1}(E'_0)$. Satisfactory continuous representations were obtained with fourth-order polynomial approximations. Typical examples of these approximations are displayed in Figs. 18 and 19, depicting the behavior of the Fourier coefficients a_3 and a_5 , respectively, of Counter I.

A particularly advantageous aspect that accrued from the parametrization process was the elimination of erroneous or inconsistent experimental results. This was

accomplished by examination of any unusually high deviation between a given Fourier coefficient and the corresponding polynomial approximation. If such a deviation arose at a given energy E'_0 , the adequacy of all other least-squares approximations was examined at the same energy. In this manner, we uncovered systematic deviations which implied that the experimental data obtained at a given pressure were suspect and therefore the experiments were repeated.

112-9017

Fig. 18. Fourth-order Least-squares Polynomial Approximation of Fourier Coefficient $a_3(E'_0)$ for Counter I

112-9014

Fig. 19. Fourth-order Least-squares Polynomial Approximation of Fourier Coefficient $a_5(E'_0)$ for Counter I

Finally, the parametrized form of the energy-dependent response function must satisfy the low-energy boundary condition. This can be ensured by heavily weighting the parameter values associated with either the asymptotic (high-pressure) or the A^{37} distribution. In the least-squares polynomial fitting process of $\{a_k(E'_0)\}$ and $G_a(E')$, this heavy weighting will force the approximations to meet the boundary condition. This behavior is clearly demonstrated in Figs. 17-19.

2. Construction of Response Matrix

We can now outline the actual steps taken in the construction of a response matrix, using the energy-dependent response function that has evolved from the present numerical analysis of the experimental data.

Step 1: Given a response matrix of mesh size A , the midbin energies or midchannel energy points $\{E_i\}$ are generated. The response matrix mesh size A is just the calibrated energy width per pulse-height channel used in the recoil-proton spectral measurement of interest. Consequently,

$$E_i = A \cdot I_i, \quad i = 1, 2, \dots, n, \quad (19)$$

where I_i is the integral pulse-height channel number. The total number of channels so used, n , specifies the order of the response matrix.

Step 2: At each midbin energy, E_i , a response function, $R(E', E_i)$, is generated using the continuous parametrization described above. One cannot expect the set of interpolated response functions $\{R(E', E_i)\}$, as generated, to satisfy Eq. 13 exactly. In practice, the normalization condition is generally satisfied to within a few percent. Consequently, the interpolated response functions are, in turn, normalized so that Eq. 13 is satisfied exactly. In this renormalization process, the relative Gaussian area, $G_a(E_i)$, is held invariant, and the area of the distorted response function $D(E', E_i)$ is scaled to meet the unit-area requirement. In this manner, interpolated response functions retain as closely as possible the behavior of $G_a(E_0)$, an observable whose physical significance has been stressed. Figures 20 and 21 display interpolated (renormalized) response functions obtained for Counter I with a matrix mesh of $A = 19.53$ keV per channel at midbin energies $E_i = 0.488$ and 2.44 MeV, respectively.

112-8917 Rev. 1

Fig. 20. Interpolated (Renormalized) Response Function $R(E', E_i)$ Obtained from Parametrization Process for Counter I at Midbin Energy $E_i = 0.488$ MeV

112-8916 Rev. 1

Fig. 21. Interpolated (Renormalized) Response Function $R(E', E_i)$ Obtained from Parametrization Process for Counter I at Midbin Energy $E_i = 2.44$ MeV

Step 3: In the final step, the response matrix R is formed using the set of interpolated (renormalized) response functions. The response-matrix elements R_{ij} are computed from the customary definition,

$$R_{ij} = \int_{E_i - A/2}^{E_i + A/2} R(E', E_j) dE', \quad i, j = 1, 2, \dots, n. \quad (20)$$

We have developed a computer program for generating wall-effect response matrices (WERM). The WERM code accepts the pressure sequence of experimental measurements and forms the transformed normalized response functions. These transformed response functions are then parametrized to provide the interpolated response functions that are used to generate the desired response matrix.

Unfolding with the full-response matrix, as defined in Eq. 20, implies an attempt to remove not only systematic distortion but resolution broadening as well. Fortunately, a treatment of resolution effects is not actually required for proton-recoil data, since we must eventually use a numerical differentiation process to obtain the desired neutron spectrum. Consequently, additional options were included in the WERM code for the generation of two response matrices primarily representative of systematic distortion. These two response matrices (the triangular and delta type) are defined below.

Triangular Response Matrix

$$\left. \begin{aligned} R_{ij} &= 0 & i > j, \quad i = 1, 2, \dots, n; \\ R_{ij} &= \int_{E_i - A/2}^{E_i + A/2} R(E', E_j) dE' & i < j, \quad j = 1, 2, \dots, n; \\ R_{jj} &= \int_{E_j - \Delta(E_j)/2}^{\infty} R(E', E_j) dE' & j = 1, 2, \dots, n. \end{aligned} \right\} \quad (21)$$

Delta Response Matrix

$$\left. \begin{aligned} R_{ij} &= 0 & i > j, \quad i = 1, 2, \dots, n; \\ R_{ij} &= \int_{E_i - A/2}^{E_i + A/2} D(E', E_j) dE' & i < j, \quad j = 1, 2, \dots, n; \\ R_{jj} &= G_a(E_j) + \int_{E_j - A/2}^{E_j + A/2} D(E', E_j) dE' & j = 1, 2, \dots, n. \end{aligned} \right\} \quad (22)$$

E. Summary

In Sections A-D above, we have described a variety of experiments which reveal, in considerable detail, the actual counter response function and its variation with energy. These measurements do not, however, constitute a tractable representation of the response function from the viewpoint

of an unfolding problem. Hence, we have "parametrized" the response-function dependence upon energy by a least-squares polynomial fitting (in powers of energy) of the amplitudes of orthonormal functions derived from each experimental distribution.

The response function must also satisfy a "boundary condition" for sufficiently low proton energy. In this energy region, the response function ceases to depend upon energy and must agree with results obtained from A³⁷ electron counting (where ionization track extent is negligible).

Subsequently, when we speak of the "response function," it will be in the context of a mathematical function determined with arbitrary precision at all energies within the measurement range. We recognize that the parametrized response function will generally exhibit some deviation from the true response distribution; it will become apparent, however, that such deviations that do occur do not exert any significant influence in neutron spectroscopy.

II. UNFOLDING FINITE-SIZE EFFECTS

A. Introduction

Having determined a suitable response-function representation in Part I, we can now apply it to measurements of proton-recoil spectra. Two methods have been used for unfolding problems of the kind involved here. The first is by repeated iteration; through the response function, of an initial estimate for the unfolded spectrum; arresting conditions terminate the iteration when a satisfactory agreement with the measured distribution is reached. This method is capable of unfolding any appropriate response function and has been used elsewhere.¹¹

The second method (integral unfolding) consists of "high-to-low-energy" integration unfolding and is applicable where the response function retains a significant fraction in the "Gaussian" region when the high-energy portion of the spectrum (beyond the measurement limit) is known. This method has been used previously⁸ for unfolding measured proton-recoil spectra.

We will consider both unfolding techniques, and both, as used here, are subject to systematic error at the upper-energy measurement limit due to the neglect of neutrons that exceed this energy limit. Neutron spectra derived from data unfolded by both these techniques agree well, except near this measurement limit.

B. Iterative Unfolding of Proton-recoil Spectra

In terms of the response matrix, R , the matrix representation of the unfolding problem in the energy-dependent response region takes the form¹¹

$$\underline{W} = R\underline{X}, \quad (23)$$

where the $\underline{W}$ vector represents the measured proton-recoil data, and the appropriate solution, $\underline{X}$, is the (distortion-free) infinite-medium proton-recoil spectrum. Iterative unfolding was used for this system. The computer code, ITERATE, was used for these calculations.

We have already mentioned that use of the full response matrix, viz. Eq. 20, implies unfolding resolution effects that need not actually be treated (in view of the subsequent numerical differentiation process employed to extract neutron spectra). Consequently, this presentation will be confined chiefly to results obtained from delta-type response matrices. Iterative unfolding properties and results obtained with delta and triangular response matrices were virtually identical. A brief examination of iterative

112-8913 Rev. 1

Fig. 22. 1/E Test $\underline{X}$ Vector and $\underline{W}$ Vector for Counter I Delta-type Response Matrix of Order $n = 102$ with Mesh Size of $A = 19.53$ keV per Channel

The $\underline{W}$ vector generated from a chosen $\underline{X}$ test vector serves another extremely important function. It can be used to investigate convergence properties of the iterative unfolding method. Using this $\underline{W}$ vector, we can examine the sequence of iterates $\{\underline{X}^{(m)}\}$ and thereby directly inspect convergence to the chosen 1/E test spectrum. In addition, we can use the norm of the residual vector, $N[\underline{R}^{(m)}]$, as a measure of the rate of convergence. (The residual vector is defined as $\underline{R}^{(m)} = \underline{W} - \underline{W}^{(m)}$.)

Figure 23 displays the behavior of $N[\underline{R}^{(m)}]$ for the 1/E test spectrum and the $\underline{W}$ vector which have been depicted in Fig. 22. The rate of convergence is very rapid. Moreover, if we assume that the experimental error of the $\underline{W}$ vector is governed by Poisson statistics, the arresting condition is satisfied by the fifth iteration. All iterates, $\{\underline{X}^{(n)}\}$, beyond $n = 5$, are virtually indistinguishable from the 1/E test vector given in Fig. 22. Additional investigations

unfolding with the full response matrix demonstrated that convergence was slow and appropriate solutions could not be obtained as readily as with triangular- or delta-type response matrices.

To test the quality of a generated response matrix, we used a known smooth 1/E test spectrum as the $\underline{X}$ vector in Eq. 23. Multiplication of this test vector with the response matrix R yields an output vector $\underline{W}$, which can be examined for smoothness. Any deviation of the resulting $\underline{W}$ vector from a smooth monotone behavior implies a corresponding deficiency in the generated response matrix. Figure 22 presents the 1/E test $\underline{X}$ vector and the resulting $\underline{W}$ vector obtained from a delta response matrix of order $n = 102$ for Counter I ($A = 19.53$ keV per channel). The smooth continuous nature of this result indicates that a satisfactory R matrix has been constructed.

112-8911 Rev. 1

Fig. 23. Norm of Residual Vector $N[\underline{R}^{(m)}]$ as a Function of Iteration Number m for the 1/E Test Spectrum and Counter I Response Matrix. (See Fig. 22.) The bound depicted at 2.54×10^6 represents an arresting condition based on Poisson statistics.

were carried out to test the dependence of these results on the choice of the initial vector $\underline{X}^{(0)}$ (of the iteration process). It has been generally proven that this iteration method furnishes results that are independent of the choice of $\underline{X}^{(0)}$.¹¹ This behavior has been verified for this system by comparison of the cases $\underline{X}^{(0)} = \underline{W}$ and $\underline{X}^{(0)} = \text{constant}$. The convergence properties and solutions obtained with these two different initial vectors were identical. Appropriate solutions are, therefore, clearly determined for this system by iterative unfolding.

The same delta-type response matrix was employed to unfold actual proton-recoil data measured with Counter I. The $\underline{W}$ vector obtained with Counter I in the neutron spectrum of a fast critical assembly is presented in Fig. 24, along with the resulting unfolded, infinite-medium, proton-recoil spectrum. The measured $\underline{W}$ vector, which covers the energy region 0.488-2.50 MeV, becomes less reliable statistically at the higher-energy end of the spectrum. Note that corresponding statistical uncertainty is reflected in the unfolded solution in the very same higher-energy region. This behavior suggests that the relative error in the unfolded spectrum is approximately the same as the relative error in the measured $\underline{W}$ vector. Under the present conditions, this conjecture can, in fact, be more rigorously established. (See Appendix A in this regard.)

112-9020 Rev. 1

Fig. 24. Infinite-medium Proton-recoil Spectra Obtained from Same $\underline{W}$ Vector for Delta-type Response Matrix

Delta- and triangular-type response matrices were compared by generating a triangular response matrix (viz. Eq. 21) to treat the same Counter I proton-recoil data. This triangular response matrix was, therefore, of the same order $n = 102$ and possessed the same mesh size, $A = 19.53$ keV per channel. The infinite-medium proton-recoil spectrum obtained by unfolding with this triangular response matrix is presented in Fig. 25. Comparison with Fig. 24 reveals that the solutions obtained with the delta- and triangular-type response matrices are virtually identical. Only in the lowest and highest few channels is there any observable deviation between these solutions. The origin of these differences may be traced directly to the fact that the matrix must be of finite size, or correspondingly cover a finite energy region. Consequently, deviations at both the upper and lower extremities of these matrices will be emphasized due to the different definitions of the diagonal matrix elements (viz. Eqs. 21 and 22).

112-9019 Rev. 1

Fig. 25. Infinite-medium Proton-recoil Spectra Obtained from Same W Vector for Triangular-type Response Matrix

A more conclusive test of the ability of this procedure to deliver infinite-medium proton-spectra was devised. In this test, the same fast-neutron spectrum was measured with two different counters identical in all respects to the geometrical configuration of Counter I depicted in Fig. 7a. These counters differed only in that they were constructed of an

aluminum body (Counter I_a) and a stainless steel body (Counter I_s). In addition, Counters I_a and I_s were filled to methane pressures of 50 and 80 psi, respectively. These two detectors were then used to measure the same neutron spectrum in a fast-reactor critical assembly.

The proton-recoil data or $\underline{W}$ vectors obtained from these two detectors are compared in Fig. 26, where the measured spectra have been normalized to the same energy width per channel, hydrogen atom density, and experimental live-time. The data for Counter I_a cover the energy region 0.4-2.0 MeV, while those for Counter I_s cover the region 0.6-3.0 MeV. This choice was dictated by the fact that the Counter I (energy-dependent) response-function data would generate response matrices in the regions 0.4-2.0 MeV and 0.6-3.0 MeV for the pressure fillings used in Counters I_a and I_s , respectively. The difference in the observed proton-recoil spectra, as shown in Fig. 26, demonstrates the anticipated increase in distortion that arises in proton-recoil measurements when counter-gas pressure is reduced.

112-8920

Fig. 26. Comparison of Measured Proton-recoil Spectra from Counter I_a (Aluminum Body, 50-psi Methane Filling) and Counter I_s (Stainless Steel Body, 80-psi Methane Filling)

used to unfold the corresponding $\underline{W}$ vectors obtained with Counters I_a and I_s . The two infinite-medium proton spectra obtained from this process are compared in Fig. 27. The agreement is quite satisfactory over most of the overlapping energy region. Above approximately 1.6 MeV the two spectra begin to deviate, and this difference grows with increasing energy. The origin of this relatively minor deviation is, however, readily understood and again stems from the finite size of the response matrix. Truncation of the unfolding process for Counter I_a at 2.0 MeV implies that the neutron flux vanishes beyond 2.0 MeV and therefore leads to an overcorrection in the energy region immediately below 2.0 MeV. That is, the finite size of the matrix prevents proper

Delta-type response matrices were generated for Counters I_a and I_s and then

112-8914

Fig. 27. Comparison of Unfolded Infinite-medium Proton Spectra Obtained from Counters I_a and I_s

correction of the distortion from being continued indefinitely (in energy), and in the neighborhood of the high-energy truncation point, the excess proton data that arise from downscattering cannot be completely unfolded.

Thus, the pressure-dependent comparison serves as more than just a test of the validity of the entire unfolding procedure. It also provides a measure of the perturbation that arises at the higher end of the infinite-medium proton spectrum due to the finite size of the response matrix. For the present data, this perturbation is not negligible at energies extending below the high-energy truncation point. It follows that the infinite-medium proton spectra unfolded with finite-size response matrices cannot be used to provide neutron spectra that extend completely out to the high-energy truncation point. One must, instead, be satisfied with neutron spectra that terminate below this truncation point.

C. Integral Unfolding

The integral-unfolding procedure uses the fact that the non-Gaussian component of the finite-size detector response will lie entirely below the dominant peak in nearly every situation encountered. This fact can be used in a "high-low" integral correction procedure which corrects an observed distribution starting at the highest energies and working downward. Knowing the true proton spectrum, $X(E)$, at all energies higher than the energy at which the measured value, $W(E)$, is to be corrected, we can remove the distortion to $W(E)$ by using the response function in an integral over $X(E)$. This procedure has been used in previous work,⁸ and it has been incorporated into the PSNS code. The principal difficulty is that $X(E)$ must be given values in excess of the upper measurement limit where no information exists from the experiment. Fast-reactor spectra decrease rapidly above the maximum measurement energy (a few MeV), and if we take the neutron flux to be zero here, the distortion is small except near the limit; this point has been demonstrated in Section II.B.

Consistent with previous notation, let $R(E, E_0)$ denote the detector response to protons of energy E_0 . $R(E, E_0)$ is subject to the normalization condition given in Eq. 13 and is again partitioned into a dominant term $G_a \delta(E - E_0)$ and a non-Gaussian term consisting of a sum of orthogonal functions, where $1 - G_a$ is the fraction of the response in the non-Gaussian region. $W(E)$, the measured proton spectrum, can be expressed as

$$W(E) = \int_E^{\infty} X(E_0) R(E, E_0) dE_0. \quad (24)$$

Substituting the partitioned response function in Eq. 24 yields

$$W(E) = G_a X(E) + \int_E^\infty X(E_0) \left[\sum_n P_n(E/E_0) \right] dE_0, \quad (25)$$

where P_n are the orthogonal terms. Equation 25 can be rearranged to give

$$X(E) = \frac{W(E) - \int_E^\infty X(E_0) \left[\sum_n P_n(E/E_0) \right] dE_0}{G_a}, \quad (26)$$

The integral term in Eq. 26 involves only values of $X(E)$ above the energy E , and consequently the correction can be continued downward in energy.

D. Comparison of Unfolding Methods

To illustrate the effect of the two unfolding techniques used here upon a representative reactor spectrum measurement, Fig. 28 compares neutron spectra as derived from proton distributions corrected by both of the above-mentioned methods. Below 1 MeV, agreement is essentially perfect. Above 1 MeV, any discrepancy is less than 10%, except at the upper energy limit, where both results are subject to systematic errors.

112-9013

Fig. 28. Comparison of Neutron Spectra Using Iterative and Integral Unfolding

the rapidly falling nature of the neutron flux with increasing energy. The derived neutron flux is considerably more tolerant of response-function detail than can be expected in other spectral environments.

These conclusions are also illustrated by the results shown earlier in Fig. 1. The measured proton distribution is considerably altered as

The results in Fig. 28 are from a test problem that is explained in detail in Part IV of this report. The raw and unfolded proton spectra are considerably different in this energy region because of the strong energy dependence of the response function. (For example, see Fig. 24.) In spite of the large change in proton spectra at the higher energies caused by unfolding, neutron spectra (in fast reactors) tend to be only moderately influenced. This behavior was not anticipated at the outset of these investigations, and the reason apparently lies partly in

energy exceeds about 0.5 MeV by wall-and-end effects; yet the neutron spectra derived from the raw data and from the corrected data are similar.

Although we have considered the two unfolding techniques separately, both can be applied to the same spectrum measurement if so desired. For example, the response information may be expanded to include response distributions for energies above those used in Section II.B. Iterative unfolding would then be essential in the high-energy region; integral unfolding could, however, be applied at the lower energies.

III. REDUCTION OF PROTON-RECOIL PULSE-HEIGHT DISTRIBUTIONS TO NEUTRON-FLUX SPECTRA

A. Introduction

The proton-recoil method of neutron spectroscopy is distinguished by the fact that all proton energies below that of the incident neutron energy are equally probable in an n-p collision (at least for neutron energies below a few MeV). This "rectangular" response has to be "unfolded" from the proton distribution to arrive at the neutron spectrum. The unfolding operation is especially simple, however, consisting of differentiation only where no mitigating circumstances exist. The relationship that converts a proton-recoil distribution $X(E)$, to a distribution in neutron flux (per unit lethargy) $\phi(E)$, is

$$\phi(E) = -\frac{E^2}{NT\sigma_H(E)} \frac{dX(E)}{dE}, \quad (27)$$

where $\sigma_H(E)$ is the n-p scattering cross section, T is the time of accumulation of the H recoil distribution $X(E)$ (per unit energy), and N is the total number of hydrogen atoms in the effective counter.

Equation 27 is a well-established result, and the n-p cross section, $\sigma_H(E)$, can be represented accurately by²⁰

$$\sigma_H(E) = \frac{11010}{E + 4041} + \frac{2387}{E + 135.5} \quad (28)$$

with E in kilovolts and σ in barns. Any departure of n-p scattering from isotropic in the energy region (up to a few MeV) accessible to proton-recoil proportional counters is extremely small (not measurable as yet, in fact). The fitting procedures used place Eq. 28 within 2% of the observed n-p cross section, which has been studied extensively over the energy region of interest.

With a computer to handle the rather lengthy numerical effort, the rectangular response (and the attendant need to differentiate an experimental pulse-height histogram) is a trivial complication to spectroscopy. The existence of a smoothly varying, accurately known, cross-section energy dependence is the most attractive aspect of the proton-recoil method.

The response-function problem and means for eliminating its effect upon measured spectra have been discussed in Parts I and II. In Sections B and C below, we shall discuss corrections not involving the response function that are necessary in proton-recoil measurements; in particular, we shall discuss the energy loss per ion pair (W) variation with energy and the distortion of recoil-proton spectra arising from neutron scattering on heavy nuclei (carbon, nitrogen) which may, for various reasons, be present in the counting gas.

B. Energy per Ion Pair

If W , the energy expended by protons per ion pair created, were constant, proton energy and ionization would be proportional, the constant of proportionality being determined by a calibration. Gases of interest in proton-recoil work (hydrogen and methane) do not exhibit strict proportionality, however. There appears, approximately, to be a threshold energy that must be exceeded before W can be assumed constant.²¹ This threshold in hydrogen is about 0.3 keV, and to a good approximation it is the difference $E - 0.3$ keV that is proportional to pulse height. If E is in excess of 1 keV, this simple result should be adequate. In methane, the threshold is more nearly 4 keV, if interest is confined to energies above 100 keV or so. The assumption of a threshold cannot be justified except as an approximation to a much more complicated W variation for both gases. It would be desirable to have better models for the W energy dependence, and it is a straightforward procedure to incorporate any improvements into the numerical treatment.

Neutron spectrum measurements have been found to be least ambiguous if hydrogen is used as the counter gas below 100 keV and methane is used above this energy. The influence of W on results is minimized, the additional stopping power of methane can be used to extend measurements to several MeV, and the perturbation of the spectrum due to carbon recoils will be small.

C. Carbon and Nitrogen Recoils

Some nitrogen is added to the counters during filling in order to calibrate the energy scale by exposure to a thermal-neutron flux, and the $N^{14}(n,p)C^{14}$ line at about 615 keV equivalent proton energy is observed. Some methane was also added to the hydrogen counter to quench ultraviolet emission and improve stability. As a consequence, carbon and nitrogen nuclei were present in the counters during measurements. The macroscopic scattering from these heavy nuclei is not insignificant relative to hydrogen and recoils involving these nuclei contribute to the observed spectrum. A correction for this effect, which is generally small, can be made and has been incorporated into the numerical analysis. We must replace the proton spectrum by the sum of $X(E)$ and $C(E)$, where $C(E)$ represents the spectrum of events originating from carbon recoils. (Similar arguments apply to nitrogen.) Consequently, the measured flux, $M(E)$, is actually given by

$$M(E) = -\frac{E^2}{N_H T \sigma_H(E)} \left[\frac{dX(E)}{dE} + \frac{dC(E)}{dE} \right], \quad (29)$$

where N_H is the hydrogen atom number, and $\sigma_H(E)$ is the corresponding cross section for scattering from hydrogen. The first term on the right

in Eq. 29 is the true spectrum, and to determine its value, we must make a correction involving the term $C(E)$. Scattering from carbon and nitrogen is known to be anisotropic for energies above 100 keV, and the variation of total cross section with energy is rather complicated. It will be assumed, however, that center-of-mass system scattering is isotropic for both elements and that total cross sections may be represented by the empirical results

$$\sigma_C(E) = \frac{5520}{E + 1150} \quad (30a)$$

and

$$\sigma_N(E) = \frac{1250}{E + 250}, \quad (30b)$$

with the energy E in keV and σ in barns.

The maximum energy that a carbon recoil can achieve is about 28% of the incident neutron energy. The relative (to protons) ionization created by a carbon fragment is about 0.75, however, and therefore the effective ionization of these heavy nuclei relative to protons is, about 20%. On the basis of these assumptions, the distribution of carbon-recoil events will become

$$C(E) = 5 \int_E^\infty \phi(E') \sigma_C(E') \frac{dE'}{(E')^2} \quad (31)$$

If Eq. 31 is used in Eq. 29, the result can be written as

$$\phi(E) = M(E) - \frac{N_C}{N_H} \frac{\sigma_C(5E)}{\sigma_H(E)} \phi(5E), \quad (32)$$

where N_C is the carbon atom number, and σ_C is the corresponding cross for scattering from carbon. The correction term involves the value of neutron flux at five times the energy at which the correction is made. This correction has been incorporated in the numerical treatment. It is clearly of limited accuracy and should not be used except when small in magnitude. Not only is the treatment of carbon scattering approximate, but it is certain that the relative value of W for heavy nuclei and protons will change appreciably, especially as energy decreases. The correction is, however, generally no more than a few percent, and to this extent Eq. 32 is probably adequate.

IV. COMPUTER PROGRAMS

A. Introduction

The three computer programs WERM, ITERATE, and PSNS are used in the evolution between a measured raw proton-recoil distribution and the final corrected neutron spectrum. Figure 29 shows the logical relationship among these programs. Using the experimental response data, the WERM code generates a parametrized response function for the PSNS code and a response matrix for ITERATE. The order and mesh size of this response matrix are dictated by the spectral measurement. ITERATE calls the response matrix, accepts the raw proton data, and provides the unfolded infinite-medium proton spectrum for the PSNS code. Figure 29 shows that an alternate unfolding path is available using the parametrized response function obtained from WERM in the integral unfolding routine of PSNS. Regardless of which path is followed, a routine in PSNS completes the proton-recoil response correction in the energy-independent region. The neutron spectrum and error are determined from the complete infinite-medium proton spectrum by using the PSNS code.

112-9119

Fig. 29. Block Diagram of Logical Relationship among Computer Programs WERM, ITERATE, and PSNS

Sections B, C, and D below contain a detailed exposition of the WERM, ITERATE, and PSNS programs, respectively. A sample problem has been chosen and is traced from beginning to end, i.e., from the raw proton data to the corrected neutron spectrum. A typical fast-reactor neutron-spectrum measurement with a detector similar to Counter I (viz. Fig. 7) has been chosen as this sample case. Required input data are discussed, and examples of computer output are given for this sample

problem. Different options that are available with these three codes are also described. Detailed input-data requirements and FORTRAN listings of WERM, ITERATE, and PSNS are contained in Appendixes B, C, and D, respectively.

B. The WERM Code

Flow Charts A, B, and C in Figs. 30-32 explain, in logical block-diagram form, the sequential functioning of the WERM (wall effect response matrix) program. The pressure-dependent response data are treated in Flow Chart A (Fig. 30). Here the pressure sequence of pulse-height distributions is treated one distribution at a time, in order of decreasing pressure. Each distribution is fitted to a Gaussian in the neighborhood of the response peak by calling the VMM (variable metric minimization) subroutine.²² Provision is made for bypassing the Gaussian fitting procedure if it has been previously performed with the same response data. In this event, the previously determined Gaussian parameters (X_1 , X_2 , and X_3) must be read into WERM as input data. Table III is a typical computer output of this Gaussian fitting procedure, which is available as an option in WERM.

Subtraction of this Gaussian leaves the distorted portion of the response data, which is treated by the energy-density transformations developed in Section I.C and depicted in Flow Chart A. The set of transformed energy values $\{E'_0\}$ (designated by EP(0) in the code output) as well as the relative Gaussian area, G_a , for the complete pressure sequence can be obtained in the form displayed in Table IV, and is printed out when the VMM Gaussian fitting procedure is bypassed. Table V summarizes other pertinent output data resulting from the Gaussian fitting routine. In the final step of Flow Chart A, the Fourier-series approximation of the transformed (difference) data is obtained.

Flow Chart B (Fig. 31) depicts the sequence of computations that lead to the parametrization of $G_a(E'_0)$ and the Fourier coefficients $\{a_k(E'_0)\}$. Table VI contains typical output of the least-squares polynomial (LSQPOL) coefficients obtained in fitting both $G_a(E'_0)$ and the set $\{a_k(E'_0)\}$. The mid-bin energies $\{EOF(I)\}$ for the response matrix are created using the matrix mesh A as well as the low and high pulse-height channel cutoffs (LOWCH and MAXCH) of the spectrum measurement. The interpolated Gaussian parameters $X_1[EOF(I)]$ and $X_2[EOF(I)]$, as well as the interpolated Fourier coefficients $\{a_k[EOF(I)]\}$, are formed and stored in computer memory. Then interpolated Fourier coefficients are scaled [by the scale factor SC(N)] to preserve the unit-area constraint for the interpolated response distribution. A printout option is available for all these interpolated parameters.

112-9124 Rev. 1

Fig. 30 Flow Chart A for WERM Code, Showing Treatment of Pressure-dependent Response Data

112-9125

Fig. 31. Flow Chart B for WERM Code, Showing Computations Leading to Parametrization of $G_a(E_0)$ and Fourier Coefficients $\{a_k(E_0)\}$

VARIABLE NOTATION:

AMEV = MATRIX MESH

EOF = MID BIN ENERGY

$$TCG = 0.8 * EOF$$
$$\text{ARG} = 2. * (\ln 2)^{\frac{1}{2}} / X_2$$
$$EOM1 = EOF(1) - AMEV/2$$
$$EOM2 = EOM1 + AMEV$$
$$RTEMP = [(\pi / \ln 2)^{\frac{1}{2}} * X_1 * X_2] / 4.$$

NDEL = DELTA MATRIX OPTION

NTRI = TRIANGULAR MATRIX OPTION

112-9126

Fig. 32. Flow Chart C (MATFORM Subroutine) for WERM Code, Showing Formation of Matrix Elements

TABLE III. Output of Gaussian Fitting Routine (VMM Data)

VARIABLE METRIC MINIMIZATION

N= 3 K= 5 E= 4.61220+001 P= 0.00000+000 DELTA= 1.00000+000

X= 4.61220+004 1.26600+001 1.05900+002

M

2.12724+009 0.00000+000 0.00000+000

0.00000+000 1.60276+002 0.00000+000

0.00000+000 0.00000+000 1.11302+000

IT 0 STEP 0 F= 1.32661+008

X= 4.61220+004 1.26600+001 1.05900+002

IT 1 STEP 0 F= 2.85055+007 GS= -1.27190+017

IT 2 STEP 0 F= 1.98695+007 GS= -3.77611+013

IT 3 STEP 0 F= 1.37997+007 GS= -1.56160+013

IT 4 STEP 0 F= 1.37769+007 GS= -5.08460+004

COLINEAR

IT 5 STEP 0 F= 1.37769+007 GS= -2.85931+000

X= 4.69911+004 1.20469+001 1.06191+002

RANDOM STEP 1 DELTA= 29.26187+023 GS= -12.26389+004

IT 0 STEP 1 F= 2.73496+007

X= 4.85494+004 1.17160+001 1.06219+002

FINAL VALUES

ERROR MATRIX

8.83644+002 -1.91988+005 1.58761+006

-1.91988+005 1.18013+008 -1.44311+009

1.58761+006 -1.44311+009 1.75182+009

DELTA= 2.74912+022 F= 1.37769+007 GS= -1.37562+002

X= 4.69911+004 1.20469+001 1.06191+002

G= -3.04164+001 -3.73257+002 -2.20706+003

Z	FM(Z)	FC(Z)	DF(Z)
1.0000000+002	2.2480000+004	2.2595886+004	-1.1588615+002
1.0100000+002	2.7714000+004	2.8084171+004	-3.7011182+002
1.0200000+002	3.3544000+004	3.3596953+004	-5.2952855+001
1.0300000+002	3.9426000+004	3.8685145+004	7.4085494+002
1.0400000+002	4.3757000+004	4.2874061+004	8.8293863+002
1.0500000+002	4.6122000+004	4.5735252+004	9.8674197+002
1.0600000+002	4.5933000+004	4.6958432+004	-1.0254317+003
1.0700000+002	4.5488000+004	4.6406856+004	-9.1885602+002
1.0800000+002	4.3163000+004	4.4142484+004	-9.7948372+002
1.0900000+002	4.0069000+004	4.0414521+004	-3.4552140+002
1.1000000+002	3.6375000+004	3.5614280+004	7.6011987+002
1.1100000+002	3.1682000+004	3.0207651+004	1.4743489+003
1.1200000+002	2.5909000+004	2.4661290+004	1.2477895+003
1.1300000+002	1.9901000+004	1.9378524+004	5.2247595+002
1.1400000+002	1.4219000+004	1.4656546+004	-4.3754649+002
1.1500000+002	9.1700000+003	1.0669613+004	-1.4996128+003
1.1600000+002	6.3560000+003	7.4760419+003	-1.1200419+003
1.1700000+002	4.3410000+003	5.0419768+003	-7.0097683+002
1.1800000+002	3.0680000+003	3.2729248+003	-2.0492476+002
1.1900000+002	2.2080000+003	2.0449244+003	1.6307561+002
1.2000000+002	1.5310000+003	1.2297718+003	3.0122816+002
1.2100000+002	1.0020000+003	7.1183262+002	2.9016738+002
1.2200000+002	6.7700000+002	5.9658592+002	2.8041408+002
1.2300000+002	4.6700000+002	3.1266829+002	2.5433171+002
1.2400000+002	0.0000000+000	1.0976762+002	-1.0976762+002
1.2500000+002	0.0000000+000	5.4532050+001	-5.4532050+001
1.2600000+002	0.0000000+000	2.6075668+001	-2.6075668+001
1.2700000+002	0.0000000+000	1.2001212+001	-1.2001212+001
1.2800000+002	0.0000000+000	5.3164393+000	-5.3164393+000

TOTAL COUNTS = 8.51296+005

RELATIVE GAUSS. AREA = 1.07854+001

EP(0) = 5.56447+001

TABLE IV. Summary of Gaussian Data for Pressure Sequence

Transformed Energy		Total Counts (Raw Data)	Relative Gauss. Area
N	EOP(N)		
1	0.55645	8.51296 + 005	0.707853
2	0.58336	8.21928 + 005	0.653221
3	0.62573	6.45134 + 005	0.658141
4	0.66758	7.44860 + 005	0.592866
5	0.71944	8.07871 + 005	0.592001
6	0.78043	1.15094 + 006	0.586428
7	0.86492	9.68506 + 005	0.541008
8	0.96616	6.48913 + 005	0.525489
9	1.01705	9.11205 + 005	0.506667
10	1.08827	7.78049 + 005	0.490199
11	1.18490	1.10599 + 006	0.457028
12	1.33164	7.24729 + 005	0.431909
13	1.40678	6.05536 + 005	0.422719
14	1.52059	6.51792 + 005	0.390157
15	1.63459	9.04207 + 005	0.361804
16	1.78693	8.34195 + 005	0.307011
17	1.99294	8.55170 + 005	0.259141
18	2.11579	7.87187 + 005	0.219275
19	2.25737	8.13380 + 005	0.182609
20	2.43928	7.46361 + 005	0.129182
21	2.70749	1.20477 + 006	0.070559
22	2.99581	9.96482 + 005	0.040649

TABLE V. Summary of Gaussian Parameters for Pressure Sequence

N	Fitted Gaussian Parameters			FITG(N)	Cosine Area
	X(3) = EOP, MeV	X(1)	X(2)		
1	0.55645	10.53630	0.063025	0.70686	1.46749 + 003
2	0.58336	11.45969	0.056523	0.68949	1.54074 + 003
3	0.62573	12.75326	0.048903	0.66388	1.17834 + 003
4	0.66758	13.69430	0.043942	0.64054	1.52295 + 003
5	0.71944	14.24327	0.040502	0.61408	1.62285 + 003
6	0.78043	14.03646	0.039226	0.58609	2.26009 + 003
7	0.86492	12.78666	0.040566	0.55215	2.07335 + 003
8	0.96616	10.91043	0.044562	0.51753	1.40471 + 003
9	1.01785	10.03106	0.047004	0.50189	2.07176 + 003
10	1.08827	8.98629	0.050424	0.48233	1.83011 + 003
11	1.18490	7.82119	0.055011	0.45799	2.69740 + 003
12	1.33164	6.47516	0.061264	0.42434	1.96915 + 003
13	1.40678	5.91034	0.064815	0.40777	1.68240 + 003
14	1.52059	5.15191	0.069761	0.38257	2.00034 + 003
15	1.63459	4.47528	0.074814	0.35640	2.92021 + 003
16	1.78693	3.67267	0.081579	0.31893	2.97256 + 003
17	1.99294	2.74492	0.089967	0.26287	3.38192 + 003
18	2.11579	2.26188	0.094232	0.22688	3.22596 + 003
19	2.25737	1.75419	0.098646	0.18420	3.64164 + 003
20	2.43928	1.17008	0.104740	0.13046	3.62261 + 003
21	2.70749	0.52098	0.117954	0.06541	6.45152 + 003
22	2.99581	0.31853	0.123515	0.04188	5.74316 + 003

TABLE VI. Least-squares Polynomial Coefficients for Fourier Coefficients and Gaussian Area

Normalized Gaussian Area Quartic Coefficients from LSQPOL					
B(1)	B(2)	B(3)	B(4)	B(5)	
1.34115 + 000	-1.75856 + 000	1.38794 + 000	-5.37377 - 001	7.40039 - 002	
Normalized Cosine Area Quartic Coefficients from LSQPOL					
A(N)	B(1)	B(2)	B(3)	B(4)	B(5)
A(1)	5.11530 - 001	-4.50416 - 002	1.87741 - 001	-1.09906 - 001	1.80101 - 002
A(2)	-6.66755 - 001	-3.66488 - 002	9.07005 - 001	-5.28011 - 001	8.68904 - 002
A(3)	1.00584 - 001	1.17607 + 000	-1.57990 + 000	6.85280 - 001	-9.74453 - 002
A(4)	4.54079 - 001	-2.60842 + 000	2.94575 + 000	-1.21460 + 000	1.69081 - 001
A(5)	-9.12900 - 001	3.50790 + 000	3.59598 + 000	1.41914 + 000	-1.92460 - 001
A(6)	1.20821 + 000	-4.08039 + 000	4.01857 + 000	-1.55552 + 000	2.08575 - 001

The process of generating a response matrix is described in Flow Chart C (Fig. 32), which considers the logical order of computations in subroutine MATFORM. The square matrix $R(I,J)$ is initialized as the null matrix and then is constructed column by column, working from Column 1 (LOWCH) up to Column n (MAXCH). In each column, the matrix elements are formed by working from Row 1 to Row n . Matrix elements for each column are computed by integration of the response function over fixed energy intervals. The parametrized response function associated with a given column is retrieved from computer storage, and the fixed energy interval of integration (viz. Eq. 20) is determined by the row location. The parametric representation of the response function changes with row location from the non-Gaussian or distorted region, to an overlap region, and then to the purely Gaussian region. The variable $TGG = 0.8 \text{ EOF}(J)$ has been introduced to define the start of the overlap region for Column J , i.e., that region in E_0 -space where both Fourier and Gaussian representations must be simultaneously employed. (This overlap region ends at the energy $\text{EOF}(J)$.)

The input parameters NDEL and NTRI allow different types of response matrices to be generated. For $NDEL = NTRI = 0$, the full-response matrix is constructed. The choice $NDEL \neq 0$, $NTRI = 0$ leads to a delta-response matrix; and $NTRI \neq 0$, $NDEL = 0$ yields a triangular-response matrix.

For most response matrices of interest, the order n is quite large. Hence provision has been made to store the matrix generated by subroutine MATFORM on magnetic tape. In this manner, the matrix is accessible and can be readily called by the program ITERATE. The entire matrix can be printed out. Tables VII and VIII present the upper and lower corners of the delta response matrix generated for the sample problem, where $A = 0.0179 \text{ MeV per channel}$ and $n = 98$ (i.e., $\text{LOWCH} = 31$, $\text{MAXCH} = 128$).

TABLE VIII. Printout of Lower Corner of Delta Response Matrix for Sample Problem

I/J	89	90	91	92	93	94	95	96
1	8.3875-003	9.4029-003	9.4172-003	9.4307-003	9.4436-003	9.4561-003	9.4685-003	9.4808-003
2	9.2571-003	9.2719-003	9.2854-003	9.2979-003	9.3096-003	9.3209-003	9.3318-003	9.3427-003
3	9.1271-003	9.1414-003	9.1542-003	9.1657-003	9.1763-003	9.1863-003	9.1958-003	9.2051-003
4	8.9975-003	9.0115-003	9.0236-003	9.0343-003	9.0438-003	9.0525-003	9.0605-003	9.0683-003
5	8.8681-003	8.8819-003	8.8936-003	8.9036-003	8.9121-003	8.9196-003	8.9263-003	8.9325-003
6	8.7387-003	8.7525-003	8.7640-003	8.7735-003	8.7813-003	8.7878-003	8.7932-003	8.7980-003
7	8.6091-003	8.6232-003	8.6347-003	8.6440-003	8.6513-003	8.6570-003	8.6614-003	8.6649-003
8	8.4792-003	8.4939-003	8.5058-003	8.5151-003	8.5222-003	8.5274-003	8.5310-003	8.5335-003
9	8.3489-003	8.3646-003	8.3771-003	8.3868-003	8.3940-003	8.3990-003	8.4022-003	8.4039-003
10	8.2182-003	8.2351-003	8.2486-003	8.2590-003	8.2667-003	8.2718-003	8.2749-003	8.2762-003
11	8.0870-003	8.1059-003	8.1203-003	8.1318-003	8.1403-003	8.1460-003	8.1493-003	8.1505-003
12	7.9554-003	7.9758-003	7.9923-003	8.0052-003	8.0149-003	8.0215-003	8.0254-003	8.0269-003
13	7.8235-003	7.8460-003	7.8645-003	7.8792-003	7.8904-003	7.8983-003	7.9033-003	7.9055-003
14	7.6914-003	7.7162-003	7.7370-003	7.7539-003	7.7670-003	7.7766-003	7.7830-003	7.7863-003
15	7.5593-003	7.5867-003	7.6100-003	7.6293-003	7.6447-003	7.6563-003	7.6645-003	7.6694-003
16	7.4275-003	7.4576-003	7.4836-003	7.5056-003	7.5235-003	7.5375-003	7.5479-003	7.5548-003
17	7.2962-003	7.3292-003	7.3580-003	7.3828-003	7.4035-003	7.4203-003	7.4331-003	7.4423-003
18	7.1659-003	7.2016-003	7.2334-003	7.2612-003	7.2849-003	7.3046-003	7.3203-003	7.3322-003
19	7.0368-003	7.0753-003	7.1101-003	7.1409-003	7.1678-003	7.1906-003	7.2093-003	7.2241-003
20	6.9094-003	6.9505-003	6.9882-003	7.0221-003	7.0522-003	7.0782-003	7.1008-003	7.1192-003
21	6.7842-003	6.8277-003	6.8681-003	6.9051-003	6.9383-003	6.9677-003	6.9930-003	7.0144-003
22	6.6616-003	6.7072-003	6.7501-003	6.7899-003	6.8263-003	6.8589-003	6.8877-003	6.9126-003
23	6.5420-003	6.5894-003	6.6346-003	6.6770-003	6.7162-003	6.7521-003	6.7843-003	6.8127-003
24	6.4259-003	6.4747-003	6.5217-003	6.5664-003	6.6084-003	6.6473-003	6.6828-003	6.7146-003
25	6.3137-003	6.3634-003	6.4118-003	6.4585-003	6.5029-003	6.5445-003	6.5831-003	6.6183-003
26	6.2059-003	6.2559-003	6.3053-003	6.3534-003	6.3998-003	6.4439-003	6.4853-003	6.5237-003
27	6.1028-003	6.1526-003	6.2024-003	6.2515-003	6.2994-003	6.3455-003	6.3895-003	6.4307-003
28	6.0047-003	6.0537-003	6.1033-003	6.1528-003	6.2017-003	6.2495-003	6.2955-003	6.3393-003
29	5.9120-003	5.9596-003	6.0083-003	6.0577-003	6.1070-003	6.1558-003	6.2034-003	6.2493-003
30	5.8247-003	5.8702-003	5.9176-003	5.9661-003	6.0153-003	6.0646-003	6.1132-003	6.1608-003
31	5.7431-003	5.7860-003	5.8312-003	5.8784-003	5.9268-003	5.9758-003	6.0250-003	6.0736-003
32	5.6672-003	5.7088-003	5.7494-003	5.7944-003	5.8414-003	5.8897-003	5.9387-003	5.9878-003
33	5.5969-003	5.6328-003	5.6721-003	5.7144-003	5.7593-003	5.8061-003	5.8543-003	5.9033-003
34	5.5322-003	5.5638-003	5.5993-003	5.6383-003	5.6805-003	5.7252-003	5.7719-003	5.8201-003
35	5.4728-003	5.4998-003	5.5310-003	5.5661-003	5.6049-003	5.6468-003	5.6914-003	5.7381-003
36	5.4184-003	5.4404-003	5.4669-003	5.4977-003	5.5326-003	5.5711-003	5.6130-003	5.6575-003
37	5.3688-003	5.3855-003	5.4070-003	5.4330-003	5.4634-003	5.4980-003	5.5364-003	5.5782-003
38	5.3233-003	5.3348-003	5.3509-003	5.3717-003	5.3973-003	5.4274-003	5.4618-003	5.5001-003
39	5.2816-003	5.2876-003	5.2983-003	5.3137-003	5.3341-003	5.3593-003	5.3892-003	5.4235-003
40	5.2429-003	5.2437-003	5.2489-003	5.2589-003	5.2737-003	5.2936-003	5.3185-003	5.3481-003
41	5.2066-003	5.2024-003	5.2022-003	5.2065-003	5.2158-003	5.2301-003	5.2496-003	5.2743-003
42	5.1720-003	5.1631-003	5.1578-003	5.1566-003	5.1601-003	5.1688-003	5.1826-003	5.2018-003
43	5.1384-003	5.1253-003	5.1151-003	5.1087-003	5.1066-003	5.1094-003	5.1174-003	5.1309-003
44	5.1049-003	5.0882-003	5.0737-003	5.0620-003	5.0548-003	5.0518-003	5.0539-003	5.0614-003
45	5.0708-003	5.0512-003	5.0329-003	5.0170-003	5.0044-003	4.9958-003	4.9921-003	4.9936-003
46	5.0352-003	5.0135-003	4.9922-003	4.9723-003	4.9550-003	4.9413-003	4.9318-003	4.9273-003
47	4.9974-003	4.9746-003	4.9509-003	4.9278-003	4.9055-003	4.8878-003	4.8729-003	4.8626-003
48	4.9566-003	4.9335-003	4.9086-003	4.8830-003	4.8582-003	4.8353-003	4.8154-003	4.7994-003
49	4.9120-003	4.8896-003	4.8645-003	4.8374-003	4.8100-003	4.7834-003	4.7591-003	4.7379-003
50	4.8630-003	4.8428-003	4.8181-003	4.7902-003	4.7613-003	4.7319-003	4.7037-003	4.6779-003
51	4.8090-003	4.7918-003	4.7689-003	4.7418-003	4.7118-003	4.6805-003	4.6493-003	4.6193-003
52	4.7493-003	4.7363-003	4.7164-003	4.6900-003	4.6612-003	4.6289-003	4.5954-003	4.5622-003
53	4.6835-003	4.6758-003	4.6600-003	4.6372-003	4.6090-003	4.5768-003	4.5421-003	4.5064-003
54	4.6112-003	4.6100-003	4.5994-003	4.5805-003	4.5549-003	4.5238-003	4.4890-003	4.4518-003
55	4.5321-003	4.5384-003	4.5342-003	4.5205-003	4.4985-003	4.4698-003	4.4359-003	4.3983-003
56	4.4460-003	4.4608-003	4.4640-003	4.4566-003	4.4396-003	4.4144-003	4.3825-003	4.3456-003
57	4.3528-003	4.3770-003	4.3887-003	4.3880-003	4.3777-003	4.3573-003	4.3287-003	4.2936-003
58	4.2525-003	4.2869-003	4.3080-003	4.3165-003	4.3128-003	4.2982-003	4.2742-003	4.2421-003
59	4.1492-003	4.1909-003	4.2219-003	4.2396-003	4.2444-003	4.2369-003	4.2186-003	4.1908-003
60	4.0311-003	4.0878-003	4.1302-003	4.1583-003	4.1724-003	4.1732-003	4.1617-003	4.1394-003
61	3.9105-003	3.9790-003	4.0330-003	4.0722-003	4.0965-003	4.1066-003	4.1033-003	4.0877-003
62	3.7838-003	3.8644-003	3.9304-003	3.9813-003	4.0168-003	4.0372-003	4.0430-003	4.0353-003
63	3.6514-003	3.7441-003	3.8226-003	3.8857-003	3.9330-003	3.9645-003	3.9806-003	3.9818-003
64	3.5139-003	3.6187-003	3.7096-003	3.7854-003	3.8451-003	3.8885-003	3.9157-003	3.9271-003
65	3.3718-003	3.4884-003	3.5919-003	3.6804-003	3.7531-003	3.8090-003	3.8482-003	3.8706-003
66	3.2298-003	3.3539-003	3.4696-003	3.5711-003	3.6568-003	3.7259-003	3.7776-003	3.8121-003
67	3.0765-003	3.2159-003	3.3432-003	3.4575-003	3.5565-003	3.6389-003	3.7039-003	3.7511-003
68	2.9247-003	3.0739-003	3.2131-003	3.3398-003	3.4520-003	3.5481-003	3.6267-003	3.6873-003
69	2.7710-003	2.9296-003	3.0796-003	3.2184-003	3.3436-003	3.4533-003	3.5458-003	3.6203-003
70	2.6162-003	2.7831-003	2.9432-003	3.0938-003	3.2313-003	3.3545-003	3.4611-003	3.5497-003
71	2.4609-003	2.6351-003	2.8043-003	2.9653-003	3.1153-003	3.2516-003	3.3722-003	3.4752-003
72	2.3057-003	2.4861-003	2.6634-003	2.8343-003	2.9957-003	3.1448-003	3.2791-003	3.3965-003
73	2.1513-003	2.3387-003	2.5209-003	2.7007-003	2.8727-003	3.0339-003	3.1815-003	3.3133-003
74	1.9982-003	2.1875-003	2.3773-003	2.5648-003	2.7464-003	2.9190-003	3.0795-003	3.2252-003
75	1.8468-003	2.0384-003	2.2329-003	2.4270-003	2.6172-003	2.8003-003	2.9728-003	3.1320-003
76	1.6977-003	1.8904-003	2.0881-003	2.2875-003	2.4852-003	2.6777-003	2.8619-003	3.0335-003
77	1.5512-003	1.7437-003	1.9433-003	2.1467-003	2.3506-003	2.5513-003	2.7455-003	2.9296-003
78	1.4074-003	1.5986-003	1.7988-003	2.0049-003	2.2136-003	2.4214-003	2.6247-003	2.8200-003
79	1.2666-003	1.4553-003	1.6548-003	1.8622-003	2.0754-003	2.2880-003	2.4993-003	2.7046-003
80	1.1298-003	1.3140-003	1.5116-003	1.7190-003	1.9334-003	2.1513-003	2.3693-003	2.5835-003
81	9.9414-004	1.1748-003	1.3693-003	1.5754-003	1.7905-003	2.0115-003	2.2347-003	2.4565-003
82	8.6238-004	1.0377-003	1.2280-003	1.4316-003	1.6461-003	1.8686-003	2.0958-003	2.3238-003
83	7.3340-004	9.0265-004	1.0878-003	1.2877-003	1.5003-003	1.7230-003	1.9526-003	2.1855-003
84	6.0696-004	7.6956-004	9.4878-004	1.1439-003	1.3534-003	1.5748-003	1.8054-003	2.0416-003
85	4.8274-004	6.3828-004	8.1085-004	1.0002-003	1.2054-003	1.4243-003	1.6544-003	1.8925-003
86	3.6037-004	5.0899-004	6.7394-004	8.5676-004	1.0565-003	1.2715-003	1.4998-003	1.7382-003
87	2.3943-004	3.8023-004	5.3795-004	7.1351-004	9.0682-004	1.1169-003	1.3419-003	1.5792-003
88	1.1947-004	2.5290-004	4.0274-004	5.7048-004	7.5657-004	9.6053-004	1.1810-003	1.4157-003
89	2.2260-001	1.2627-004	2.6815-004	4.2766-004	6.0583-004	8.0270-004	1.0174-003	1.2481-003
90	0.0000+000	2.1724-001	1.3397-004	2.8500-004	4.5470-004	6.4364-004	8.5146-004	1.0769-003
91	0.0000+000	0.0000+000	0.0000+000	1.4247-004	3.0329-004	4.8360-004	6.8358-004	9.0243-004
92	0.0000+000	0.0000+000	0.0000+000	2.0645-001	1.5169-004	3.2282-004	5.1410-004	7.2528-004
93	0.0000+000	0.0000+000	0.0000+000	0.0000+000				

C. The ITERATE Code

The program ITERATE had been previously developed for the general treatment of unfolding problems.¹¹ Some minor modifications have been introduced to make this program compatible with the WERM and PSNS codes, thereby facilitating operation of the overall system (see Fig. 29). The flow chart in Fig. 33 shows the logical sequence of computations in ITERATE that relate to this system.

The response matrix (that has been generated in WERM and stored on magnetic tape) is read from magnetic tape into ITERATE. An option for printout of this matrix from ITERATE is available and should be compared with the WERM printout (viz. Tables VII and VIII) to ensure proper transfer of the response matrix. The required input data consist of the matrix mesh A , $LOWCH$, $MAXCH$, and three vectors. The three input vectors are $\underline{W}$, the output of the detection system; $\underline{X1}$, the initial vector for the iteration process; and $\underline{ER}$, the error vector (used in forming the arresting condition). These input vectors and the associated set of midbin energies are listed in the ITERATE output as shown in Table IX. Here we find the $\underline{W}$ -vector (raw proton data) for the test sample problem (where $A = 0.0179$ MeV per channel, $LOWCH = 31$, and $MAXCH = 128$). The choice of a null-error vector creates a zero-arresting bound, and the iteration process continues for a specified total number of iterations. The chosen initial vector for the test problem is the constant vector $X1(I) = 1.0, I = 1, \dots, n$.

Printout and punchout of given members of the iteration sequence can be predetermined by fixing input-data parameters. Table X contains, for example, the results of the second iteration with the test problem. Here the $\underline{X}$ and $\underline{WPRIME}$ vectors are just $\underline{X}^{(2)}$ and $\underline{W}^{(2)}$, respectively. The $\underline{R}$ vector is the residual vector $\underline{R}^{(n)} = \underline{W} - \underline{W}^{(n)}$, where $n = 2$ for this case. The norm column is a running sum of the square of the residual-vector components. (Hence, the final value is just the square of the norm of the residual vector.)

Table X also contains the printout of the code for iteration number ten, the final iteration specified for the test problem. Since a null-error vector was chosen for the test problem, the arresting condition will not stop the iteration process. It will therefore continue until the maximum allowed number of iterations (specified by input data) has been reached. For this sample problem, the maximum allowed number of iterations was $n = 10$. This was determined by earlier investigations, which revealed rapid convergence and satisfactory solutions. This behavior can also be inferred from a comparison of the magnitudes of the components of the residual vector $\underline{R}$ for the second and tenth iterations given in Table X. Consequently, the vector $\underline{X}^{(10)}$ is the desired infinite-medium proton spectrum for the sample problem.

Fig. 33. Flow Chart for Program ITERATE

TABLE IX. Input Data for Iterative
Unfolding of Sample Problem

ITERATIVE SOLUTION OF DETECTION SYSTEM

I	ENERGY, MEV	N VECTOR	ER VECTOR	X1 VECTOR
1	0.5549	8.5783+004	0.0000+000	1.0000+000
2	0.5728	7.8812+004	0.0000+000	1.0000+000
3	0.5907	7.2985+004	0.0000+000	1.0000+000
4	0.6086	6.7183+004	0.0000+000	1.0000+000
5	0.6265	6.1911+004	0.0000+000	1.0000+000
6	0.6444	5.7738+004	0.0000+000	1.0000+000
7	0.6623	5.3821+004	0.0000+000	1.0000+000
8	0.6802	5.0095+004	0.0000+000	1.0000+000
9	0.6981	4.6937+004	0.0000+000	1.0000+000
10	0.7160	4.3528+004	0.0000+000	1.0000+000
11	0.7339	4.0826+004	0.0000+000	1.0000+000
12	0.7518	3.8008+004	0.0000+000	1.0000+000
13	0.7697	3.6096+004	0.0000+000	1.0000+000
14	0.7876	3.3882+004	0.0000+000	1.0000+000
15	0.8055	3.1880+004	0.0000+000	1.0000+000
16	0.8234	2.9799+004	0.0000+000	1.0000+000
17	0.8413	2.8291+004	0.0000+000	1.0000+000
18	0.8592	2.6829+004	0.0000+000	1.0000+000
19	0.8771	2.5623+004	0.0000+000	1.0000+000
20	0.8950	2.4479+004	0.0000+000	1.0000+000
21	0.9129	2.3308+004	0.0000+000	1.0000+000
22	0.9308	2.1974+004	0.0000+000	1.0000+000
23	0.9487	2.1092+004	0.0000+000	1.0000+000
24	0.9666	2.0235+004	0.0000+000	1.0000+000
25	0.9845	1.9173+004	0.0000+000	1.0000+000
26	1.0024	1.8215+004	0.0000+000	1.0000+000
27	1.0203	1.7279+004	0.0000+000	1.0000+000
28	1.0382	1.6655+004	0.0000+000	1.0000+000
29	1.0561	1.6081+004	0.0000+000	1.0000+000
30	1.0740	1.5065+004	0.0000+000	1.0000+000
31	1.0919	1.4500+004	0.0000+000	1.0000+000
32	1.1098	1.3934+004	0.0000+000	1.0000+000
33	1.1277	1.3348+004	0.0000+000	1.0000+000
34	1.1456	1.2610+004	0.0000+000	1.0000+000
35	1.1635	1.2223+004	0.0000+000	1.0000+000
36	1.1814	1.1496+004	0.0000+000	1.0000+000
37	1.1993	1.1108+004	0.0000+000	1.0000+000
38	1.2172	1.0704+004	0.0000+000	1.0000+000
39	1.2351	1.0167+004	0.0000+000	1.0000+000
40	1.2530	9.8000+003	0.0000+000	1.0000+000
41	1.2709	9.5560+003	0.0000+000	1.0000+000
42	1.2888	9.0770+003	0.0000+000	1.0000+000
43	1.3067	8.6140+003	0.0000+000	1.0000+000
44	1.3246	8.3630+003	0.0000+000	1.0000+000
45	1.3425	7.9870+003	0.0000+000	1.0000+000
46	1.3604	7.6570+003	0.0000+000	1.0000+000
47	1.3783	7.2280+003	0.0000+000	1.0000+000
48	1.3962	6.9010+003	0.0000+000	1.0000+000
49	1.4141	6.6790+003	0.0000+000	1.0000+000
50	1.4320	6.3650+003	0.0000+000	1.0000+000
51	1.4499	6.1080+003	0.0000+000	1.0000+000
52	1.4678	5.8450+003	0.0000+000	1.0000+000
53	1.4857	5.6630+003	0.0000+000	1.0000+000
54	1.5036	5.4450+003	0.0000+000	1.0000+000
55	1.5215	5.2870+003	0.0000+000	1.0000+000
56	1.5394	5.0460+003	0.0000+000	1.0000+000
57	1.5573	4.8300+003	0.0000+000	1.0000+000
58	1.5752	4.5760+003	0.0000+000	1.0000+000
59	1.5931	4.3400+003	0.0000+000	1.0000+000
60	1.6110	4.2110+003	0.0000+000	1.0000+000
61	1.6289	4.1770+003	0.0000+000	1.0000+000
62	1.6468	4.0320+003	0.0000+000	1.0000+000
63	1.6647	3.9170+003	0.0000+000	1.0000+000
64	1.6826	3.7010+003	0.0000+000	1.0000+000
65	1.7005	3.5320+003	0.0000+000	1.0000+000
66	1.7184	3.3660+003	0.0000+000	1.0000+000
67	1.7363	3.1830+003	0.0000+000	1.0000+000
68	1.7542	3.1230+003	0.0000+000	1.0000+000
69	1.7721	2.9310+003	0.0000+000	1.0000+000
70	1.7900	2.8680+003	0.0000+000	1.0000+000
71	1.8079	2.7200+003	0.0000+000	1.0000+000
72	1.8258	2.7200+003	0.0000+000	1.0000+000
73	1.8437	2.4960+003	0.0000+000	1.0000+000
74	1.8616	2.4700+003	0.0000+000	1.0000+000
75	1.8795	2.2650+003	0.0000+000	1.0000+000
76	1.8974	2.2400+003	0.0000+000	1.0000+000
77	1.9153	2.1200+003	0.0000+000	1.0000+000
78	1.9332	2.0990+003	0.0000+000	1.0000+000
79	1.9511	1.9240+003	0.0000+000	1.0000+000
80	1.9690	1.8150+003	0.0000+000	1.0000+000
81	1.9869	1.7900+003	0.0000+000	1.0000+000
82	2.0048	1.6790+003	0.0000+000	1.0000+000
83	2.0227	1.6430+003	0.0000+000	1.0000+000
84	2.0406	1.5490+003	0.0000+000	1.0000+000
85	2.0585	1.4910+003	0.0000+000	1.0000+000
86	2.0764	1.4440+003	0.0000+000	1.0000+000
87	2.0943	1.4210+003	0.0000+000	1.0000+000
88	2.1122	1.2700+003	0.0000+000	1.0000+000
89	2.1301	1.2740+003	0.0000+000	1.0000+000
90	2.1480	1.2840+003	0.0000+000	1.0000+000
91	2.1659	1.1580+003	0.0000+000	1.0000+000
92	2.1838	1.1190+003	0.0000+000	1.0000+000
93	2.2017	1.0460+003	0.0000+000	1.0000+000
94	2.2196	9.5300+002	0.0000+000	1.0000+000
95	2.2375	1.0090+003	0.0000+000	1.0000+000
96	2.2554	9.6200+002	0.0000+000	1.0000+000
97	2.2733	9.1000+002	0.0000+000	1.0000+000
98	2.2912	8.5000+002	0.0000+000	1.0000+000

TABLE X. Iterative Output for Iterations 2 and 10

ITERATION NUMBER	X	WPRIME	R	ITERATION NUMBER	X	WPRIME	R	ITERATION NUMBER	X	WPRIME	R	ITERATION NUMBER	X	WPRIME	R	ITERATION NUMBER	X	WPRIME	R
1	5.4460+00	5.3108+00	5.2675+00	1	1.0677+00	9.2269+00	8.5783+00	2	1.0677+00	9.2269+00	8.5783+00	2	1.0677+00	9.2269+00	8.5783+00	2	1.0677+00	9.2269+00	8.5783+00
2	5.0807+00	4.9373+00	4.8437+00	3	1.9242+00	7.9500+00	7.2953+00	3	1.9242+00	7.9500+00	7.2953+00	3	1.9242+00	7.9500+00	7.2953+00	3	1.9242+00	7.9500+00	7.2953+00
3	4.7933+00	4.6173+00	4.5244+00	4	3.7094+00	6.7183+00	6.1913+00	4	3.7094+00	6.7183+00	6.1913+00	4	3.7094+00	6.7183+00	6.1913+00	4	3.7094+00	6.7183+00	6.1913+00
4	4.4793+00	4.3294+00	4.2444+00	5	3.1364+00	5.7500+00	5.3583+00	5	3.1364+00	5.7500+00	5.3583+00	5	3.1364+00	5.7500+00	5.3583+00	5	3.1364+00	5.7500+00	5.3583+00
5	4.1941+00	4.0794+00	3.9944+00	6	2.7192+00	4.9500+00	4.6183+00	6	2.7192+00	4.9500+00	4.6183+00	6	2.7192+00	4.9500+00	4.6183+00	6	2.7192+00	4.9500+00	4.6183+00
6	3.9787+00	3.8754+00	3.7904+00	7	2.4165+00	4.3500+00	4.0683+00	7	2.4165+00	4.3500+00	4.0683+00	7	2.4165+00	4.3500+00	4.0683+00	7	2.4165+00	4.3500+00	4.0683+00
7	3.7759+00	3.6824+00	3.5974+00	8	2.1605+00	3.8500+00	3.6183+00	8	2.1605+00	3.8500+00	3.6183+00	8	2.1605+00	3.8500+00	3.6183+00	8	2.1605+00	3.8500+00	3.6183+00
8	3.5731+00	3.4794+00	3.3944+00	9	1.9472+00	3.4000+00	3.2183+00	9	1.9472+00	3.4000+00	3.2183+00	9	1.9472+00	3.4000+00	3.2183+00	9	1.9472+00	3.4000+00	3.2183+00
9	3.3709+00	3.2754+00	3.1904+00	10	1.7641+00	3.0000+00	2.8183+00	10	1.7641+00	3.0000+00	2.8183+00	10	1.7641+00	3.0000+00	2.8183+00	10	1.7641+00	3.0000+00	2.8183+00
10	3.1680+00	3.0704+00	2.9854+00	11	1.6027+00	2.6500+00	2.4683+00	11	1.6027+00	2.6500+00	2.4683+00	11	1.6027+00	2.6500+00	2.4683+00	11	1.6027+00	2.6500+00	2.4683+00
11	2.9659+00	2.8654+00	2.7804+00	12	1.4572+00	2.3500+00	2.1683+00	12	1.4572+00	2.3500+00	2.1683+00	12	1.4572+00	2.3500+00	2.1683+00	12	1.4572+00	2.3500+00	2.1683+00
12	2.7638+00	2.6634+00	2.5784+00	13	1.3267+00	2.1000+00	1.9183+00	13	1.3267+00	2.1000+00	1.9183+00	13	1.3267+00	2.1000+00	1.9183+00	13	1.3267+00	2.1000+00	1.9183+00
13	2.5617+00	2.4614+00	2.3764+00	14	1.2082+00	1.8500+00	1.6483+00	14	1.2082+00	1.8500+00	1.6483+00	14	1.2082+00	1.8500+00	1.6483+00	14	1.2082+00	1.8500+00	1.6483+00
14	2.3596+00	2.2594+00	2.1744+00	15	1.0997+00	1.6500+00	1.4483+00	15	1.0997+00	1.6500+00	1.4483+00	15	1.0997+00	1.6500+00	1.4483+00	15	1.0997+00	1.6500+00	1.4483+00
15	2.1575+00	2.0574+00	1.9724+00	16	9.9827+00	1.4500+00	1.2583+00	16	9.9827+00	1.4500+00	1.2583+00	16	9.9827+00	1.4500+00	1.2583+00	16	9.9827+00	1.4500+00	1.2583+00
16	1.9554+00	1.8554+00	1.7704+00	17	8.9759+00	1.2500+00	1.0583+00	17	8.9759+00	1.2500+00	1.0583+00	17	8.9759+00	1.2500+00	1.0583+00	17	8.9759+00	1.2500+00	1.0583+00
17	1.7533+00	1.6534+00	1.5684+00	18	7.9672+00	1.0500+00	8.7883+00	18	7.9672+00	1.0500+00	8.7883+00	18	7.9672+00	1.0500+00	8.7883+00	18	7.9672+00	1.0500+00	8.7883+00
18	1.5512+00	1.4514+00	1.3664+00	19	6.9585+00	8.5000+00	7.3183+00	19	6.9585+00	8.5000+00	7.3183+00	19	6.9585+00	8.5000+00	7.3183+00	19	6.9585+00	8.5000+00	7.3183+00
19	1.3491+00	1.2494+00	1.1644+00	20	5.9498+00	6.5000+00	5.3083+00	20	5.9498+00	6.5000+00	5.3083+00	20	5.9498+00	6.5000+00	5.3083+00	20	5.9498+00	6.5000+00	5.3083+00
20	1.1470+00	1.0474+00	9.6244+00	21	4.9411+00	4.5000+00	3.8583+00	21	4.9411+00	4.5000+00	3.8583+00	21	4.9411+00	4.5000+00	3.8583+00	21	4.9411+00	4.5000+00	3.8583+00
21	9.2095+00	1.8413+00	1.6483+00	22	3.9323+00	3.5000+00	2.3183+00	22	3.9323+00	3.5000+00	2.3183+00	22	3.9323+00	3.5000+00	2.3183+00	22	3.9323+00	3.5000+00	2.3183+00
22	8.0143+00	1.6080+00	1.4154+00	23	2.9150+00	3.0000+00	1.7183+00	23	2.9150+00	3.0000+00	1.7183+00	23	2.9150+00	3.0000+00	1.7183+00	23	2.9150+00	3.0000+00	1.7183+00
23	7.0273+00	1.4077+00	1.2147+00	24	1.8967+00	2.5000+00	1.2183+00	24	1.8967+00	2.5000+00	1.2183+00	24	1.8967+00	2.5000+00	1.2183+00	24	1.8967+00	2.5000+00	1.2183+00
24	6.0393+00	1.2070+00	1.0140+00	25	9.8794+00	2.0000+00	1.0183+00	25	9.8794+00	2.0000+00	1.0183+00	25	9.8794+00	2.0000+00	1.0183+00	25	9.8794+00	2.0000+00	1.0183+00
25	5.0500+00	1.0060+00	8.8500+00	26	8.8500+00	1.5000+00	8.6500+00	26	8.8500+00	1.5000+00	8.6500+00	26	8.8500+00	1.5000+00	8.6500+00	26	8.8500+00	1.5000+00	8.6500+00
26	4.0600+00	8.8500+00	8.6500+00	27	7.8500+00	1.0000+00	7.6500+00	27	7.8500+00	1.0000+00	7.6500+00	27	7.8500+00	1.0000+00	7.6500+00	27	7.8500+00	1.0000+00	7.6500+00
27	3.0700+00	7.8500+00	7.6500+00	28	6.8500+00	500+00	6.6500+00	28	6.8500+00	500+00	6.6500+00	28	6.8500+00	500+00	6.6500+00	28	6.8500+00	500+00	6.6500+00
28	2.0800+00	6.8500+00	6.6500+00	29	5.8500+00	0+00	5.6500+00	29	5.8500+00	0+00	5.6500+00	29	5.8500+00	0+00	5.6500+00	29	5.8500+00	0+00	5.6500+00
29	1.0900+00	5.8500+00	5.6500+00	30	4.8500+00	0+00	4.6500+00	30	4.8500+00	0+00	4.6500+00	30	4.8500+00	0+00	4.6500+00	30	4.8500+00	0+00	4.6500+00
30	0.1000+00	4.8500+00	4.6500+00	31	3.8500+00	0+00	3.6500+00	31	3.8500+00	0+00	3.6500+00	31	3.8500+00	0+00	3.6500+00	31	3.8500+00	0+00	3.6500+00
31	0.0000+00	3.8500+00	3.6500+00	32	2.8500+00	0+00	2.6500+00	32	2.8500+00	0+00	2.6500+00	32	2.8500+00	0+00	2.6500+00	32	2.8500+00	0+00	2.6500+00
32	0.0000+00	2.8500+00	2.6500+00	33	1.8500+00	0+00	1.6500+00	33	1.8500+00	0+00	1.6500+00	33	1.8500+00	0+00	1.6500+00	33	1.8500+00	0+00	1.6500+00
33	0.0000+00	1.8500+00	1.6500+00	34	0.8500+00	0+00	0.6500+00	34	0.8500+00	0+00	0.6500+00	34	0.8500+00	0+00	0.6500+00	34	0.8500+00	0+00	0.6500+00
34	0.0000+00	0.8500+00	0.6500+00	35	0.0000+00	0+00	0.0000+00	35	0.0000+00	0+00	0.0000+00	35	0.0000+00	0+00	0.0000+00	35	0.0000+00	0+00	0.0000+00
35	0.0000+00	0.0000+00	0.0000+00	36	0.0000+00	0+00	0.0000+00	36	0.0000+00	0+00	0.0000+00	36	0.0000+00	0+00	0.0000+00	36	0.0000+00	0+00	0.0000+00
36	0.0000+00	0.0000+00	0.0000+00	37	0.0000+00	0+00	0.0000+00	37	0.0000+00	0+00	0.0000+00	37	0.0000+00	0+00	0.0000+00	37	0.0000+00	0+00	0.0000+00
37	0.0000+00	0.0000+00	0.0000+00	38	0.0000+00	0+00	0.0000+00	38	0.0000+00	0+00	0.0000+00	38	0.0000+00	0+00	0.0000+00	38	0.0000+00	0+00	0.0000+00
38	0.0000+00	0.0000+00	0.0000+00	39	0.0000+00	0+00	0.0000+00	39	0.0000+00	0+00	0.0000+00	39	0.0000+00	0+00	0.0000+00	39	0.0000+00	0+00	0.0000+00
39	0.0000+00	0.0000+00	0.0000+00	40	0.0000+00	0+00	0.0000+00	40	0.0000+00	0+00	0.0000+00	40	0.0000+00	0+00	0.0000+00	40	0.0000+00	0+00	0.0000+00
40	0.0000+00	0.0000+00	0.0000+00	41	0.0000+00	0+00	0.0000+00	41	0.0000+00	0+00	0.0000+00	41	0.0000+00	0+00	0.0000+00	41	0.0000+00	0+00	0.0000+00
41	0.0000+00	0.0000+00	0.0000+00	42	0.0000+00	0+00	0.0000+00	42	0.0000+00	0+00	0.0000+00	42	0.0000+00	0+00	0.0000+00	42	0.0000+00	0+00	0.0000+00
42	0.0000+00	0.0000+00	0.0000+00	43	0.0000+00	0+00	0.0000+00	43	0.0000+00	0+00	0.0000+00	43	0.0000+00	0+00	0.0000+00	43	0.0000+00	0+00	0.0000+00
43	0.0000+00	0.0000+00	0.0000+00	44	0.0000+00	0+00	0.0000+00	44	0.0000+00	0+00	0.0000+00	44	0.0000+00	0+00	0.0000+00	44	0.0000+00	0+00	0.0000+00
44	0.0000+00	0.0000+00	0.0000+00	45	0.0000+00	0+00	0.0000+00	45	0.0000+00	0+00	0.0000+00	45	0.0000+00	0+00	0.0000+00	45	0.0000+00	0+00	0.0000+00
45	0.0000+00	0.0000+00	0.0000+00	46	0.0000+00	0+00	0.0000+00	46	0.0000+00	0+00	0.0000+00	46	0.0000+00	0+00	0.0000+00	46	0.0000+00	0+00	0.0000+00
46	0.0000+00	0.0000+00	0.0000+00	47	0.0000+00	0+00	0.0000+00	47	0.0000+00	0+00	0.0000+00	47	0.0000+00	0+00	0.0000+00	47	0.0000+00	0+00	0.0000+00
47	0.0000+00	0.0000+00	0.0000+00	48	0.0000+00	0+00	0.0000+00	48	0.0000+00	0+00	0.0000+00	48	0.0000+00	0+00	0.0000+00	48	0.0000+00	0+00	0.0000+00
48	0.0000+00	0.0000+00	0.0000+00	49	0.0000+00	0+00	0.0000+00	49	0.0000+00	0+00	0.0000+00	49	0.0000+00	0+00	0.0000+00	49	0.0000+00	0+00	0.0000+00
49	0.0000+00	0.0000+00	0.0000+00	50	0.0000+00	0+00	0.0000+00	50	0.0000+00	0+00	0.0000+00	50	0.0000+00	0+00	0.0000+00	50	0.0000+00	0+00	0.0000+00
50	0.0000+00	0.0000+00	0.0000+00	51	0.0000+00	0+00	0.0000+00	51	0.0000+00	0+00	0.0000+00	51	0.0000+00	0+00	0.0000+00	51	0.0000+00	0+00	0.0000+00
51	0.0000+00	0.0000+00	0.0000+00	52	0.0000+00	0+00	0.0000+00	52											

The final iteration vector, $\underline{X}^{(n)}$, is automatically punched out by the ITERATE code for use in the PSNS code. This punchout is obtained whether or not the arresting condition has been satisfied. Consequently, the general applicability of this final vector for the PSNS code must be either predetermined or evaluated from the current ITERATE output. To this end, the code will print out any desired subsequence of results (Table X) so that convergence properties can be examined. For the sample problem presented here, every second iteration was listed in the ITERATE output.

D. Discussion of PSNS Code and Test Problem

1. Description of PSNS Code

Appendix D contains the PSNS (proton spectrum to neutron spectrum) code (CDC-3600 FORTRAN) together with input format and flow diagrams (Figs. 35-38). The code accepts a set of pulse-height distributions measured at various voltages with two counter types and provides a neutron spectrum as output. The need for both a hydrogen and methane-gas filling is justified on grounds of an extended energy response with minimum corrections (see Part III above). A calibration must be available to permit a scale (keV per channel) factor to be assigned to each voltage, and the number of hydrogen atoms present in the effective counting volume of each tube must be known. A multiplicity of voltage settings cannot be avoided without a large reduction in preamplifier noise (a possibility that does not appear to be imminent), if we wish to measure over the entire energy range. Changes in voltage bring successively overlapping regions of the spectrum within view of the electronics. The chief disadvantage of multiple data sets is an increased time requirement. If adequate neutron-flux levels exist (as will always be the case for reactor measurements) a full (8-hr) day may be necessary for a complete spectrum measurement with good statistics.

The code is straightforward, with the possible exception of the Legendre treatment of response functions; this point was discussed in Section I.D. The WERM code does not provide Legendre coefficients directly; it does provide punched data of the "transformed" response functions from the $N^{14}(n,p)C^{14}$ pressure-dependent sequence of measurements. The transformed data are equivalent to the counter response to mono-energetic protons, and simple codes (not presented here) perform the Legendre analysis and provide least-squares fitting of the Legendre coefficients to obtain a smooth, continuous dependence upon energy.

The Legendre analysis is applied to both the hydrogen and the methane counters, but energy dependence of the Legendre terms is only used for the methane counter. The break region, below which data from the hydrogen counter are used exclusively, is around 100 keV. For example, the hydrogen counter used in the test-problem measurement provided here contained 80-psi hydrogen and had a diameter of 1.6 cm.

It is not difficult to show that protons below 100 keV are subject to negligible wall-and-end effect. It is also important, however, to investigate the effect of the region of the neutron spectrum above the 100-keV limit. This region may contribute a significant number of events, through track truncation, to the spectrum below 100 keV. Low equivalent ionizing events can arise from the truncation of very energetic proton recoils, and if these events are significant, the response function for the hydrogen counter would have to be made energy-dependent in the same way as the code allows for the methane counter.

As a numerical test of an upper bound to this effect, an approximate energy-dependent response function for the hydrogen counter was constructed using data derived from the sequence of pressure-dependent measurements with the $N^{14}(n,p)C^{14}$ reaction discussed in Part I. The PSNS test problem (with modification) was run using both the energy-dependent and an energy-independent response for the hydrogen counter. At 100 keV the difference induced in neutron spectra was 3%, and this difference diminished rapidly as energy decreased. Because of the experimental method used,¹³ the true error below 100 keV is actually less than this calculated bound. In particular, the number of wall-and-end events recorded will be reduced by using the same pulse-shape discrimination intended primarily for the elimination of background gamma radiation.

These observations will certainly not apply other than to neutron spectra comparable to the dilute fast-reactor variety and to counters at least as high in pressure and of comparable size to those considered here. If harder spectra are studied, or if counter stopping power to protons is reduced, these effects would have to be re-examined.

The main features of the PSNS code are discussed in Sections a-f below.

a. Computation of Proton Spectrum. Raw proton spectra (number of counts per kilovolt per second per 10^{24} hydrogen atoms) are derived using the predetermined energy-scale calibration factors for each measurement and the experimental counting time. The hydrogen atom number will ordinarily not be the same for both counters.

b. Integral Response Unfolding. Raw proton spectra (RAPS) are first corrected for response-function effects by a numerical integration according to Eq. 26. Each set of data is corrected starting at the highest-energy set and working downward in energy. There may be an overlap of the energy region for different sets, and to simplify the integration, an energy (MNOLL) for each set is determined and the integration carried out only over the distribution in excess of this energy. The next higher-numbered set is then used to extend the integration to lower energies, terminating finally at the energy point to which the correction is being made. The response function appropriate to each energy in the integration is derived from input amplitudes of the first four Legendre polynomials,

$\{P_1 - P_4\}$. If the proton-spectrum point being corrected was taken with the methane counter, the Legendre amplitudes are energy-dependent (if energy exceeds ELIMIT). If the point under correction was taken with the hydrogen counter, the Legendre amplitudes are the fixed values CLHYD.

Generally, the response correction, as made here, fails near the upper energy extremity (see Section II.B). The fission-spectrum "tail" in fast reactors extends well beyond the region of measurement; failure of the correction near the limit of measurement will occur regardless of the manner in which a correction is made (iterative unfolding or the integral technique in PSNS).

Ordinarily, an iterative unfolding correction to at least the highest-energy data set is applied. The integral-response correction will then be skipped in PSNS by setting NUC equal to the number of sets that have been precorrected.

c. Energy Spacing and Neutron-flux Format. Most calculations of neutron spectra involve energy groups chosen to have a fixed fractional width. Flux per unit lethargy is then the same as group flux. For this reason, PSNS provides an output flux per unit lethargy at a sequence of edit energies having a fixed fractional spacing; the spacing is input as DINC in the PSNS program.

d. Slopes and Errors. The slope-taking half interval is determined by

$$\text{Half Interval} = RA + \frac{RB}{\sqrt{\text{energy}}},$$

where RA and RB must be provided as input data to the code. The resolution of a measurement can be related to the slope-taking interval as described in Ref. 14. As energy decreases, statistical effects broaden the intrinsic resolution of these detectors, and the slope-taking interval can be increased correspondingly by making RB nonzero. Each set of response-corrected data (PRAPS) is tested against the list of preformed edit energies, and if data in the set exist within plus and minus the slope-taking half-interval around an edit energy, the slope and slope error are determined.

The slope error is assigned on the basis of counting statistics in the data and is determined by using the uncorrected (RAPS) spectrum. If a response correction is made, the corrected proton spectrum (PRAPS) used in the determination of the corrected neutron spectrum will be different from the raw spectrum. The slope error is, consequently, scaled by the factor PRAPS/RAPS to allow for the correction.

When data are precorrected for response-function effects by ITERATE, the error determined in PSNS will not be the true error; this difference is discussed in Appendix A.

Each set of data is scanned from high to low energy, and measured flux values are assigned to those edit energies contained within the set. Two (or more) sets may be able to contribute to a given edit energy, in which case the weighted average is stored.

If the spacing of edit energies is less than the slope-taking interval, data in common will be used to determine the values of flux and error for adjacent energies. Correlations will occur, and it is not permitted to average fluxes and errors over energy intervals by the simple procedure used for combining counts in a counts-versus-channel number histogram, for example. The mean value of flux is correctly given by the mean value of the individual datum points, over the interval, but the error must be increased (by the square root of the ratio of the slope-taking interval to the spacing interval) over the usual error estimate for the mean. More explicitly, let UE denote the usual rms error for combining quantities ϕ_i ($i = 1, 2, \dots, N$). If the ϕ_i are uncorrelated, the mean is given by

$$\langle \phi \rangle = \frac{1}{N} \sum_{i=1}^N \phi_i,$$

and the error in the mean becomes

$$UE = \frac{1}{N} \sqrt{\sum_{i=1}^N (\text{err } \phi_i)^2},$$

where $\text{err } \phi_i$ is the error in each individual ϕ_i . If the ϕ_i are neutron fluxes determined by a slope-taking interval Δ with an energy spacing δ , the mean is the same as above. However, the error in the mean is greater than UE and is given by

$$\text{True Mean Error} = \sqrt{\Delta/\delta} \cdot UE.$$

If Δ is less than δ , the ϕ_i are not correlated and UE is the true error.

These arguments involving errors are important, for example, in a comparison of a measured spectrum with a group calculation. The group-energy width is ordinarily many times the energy spacing δ , and it is usually necessary to average a measurement in order to make a comparison with theory.

e. Heavy-particle Recoils. The correction for carbon and nitrogen recoils is made in PSNS as described in Part III. The fractional

correction for heavy-particle recoils at each edit energy, HPCOR, is listed in the output. This correction is uncertain for reasons given in Section III.C, and it is convenient to see its magnitude along with the flux and error.

f. Form of Punched Data. Values of the various computed quantities were initialized to zero in the code. If none of the data sets overlap a given edit energy, all quantities will be zero at that energy at the termination of the calculation. This will misrepresent the spectrum, and it is less ambiguous if the code removes these blank locations before printing or punching results.

The energy and the neutron flux and rms flux error are punched for subsequent use as input to special routines for plotting or for comparison with calculation.

2. PSNS Test Problem

To illustrate the various parameters used in the code, Table XI records a typical fast-reactor spectrum measurement and analysis. PSNS edits all the input material initially. The quantity C, which normally consists of the product of live-time and H-atom number, if absolute fluxes are desired, is given as a relative value for this test problem. The final computed fluxes are, consequently, not absolute.

TABLE XI. Raw Data for First Set of PSNS Test Problem

Initial Channel = 31
Final Channel = 128

85783	78812	72985	67183	61911	57738
53821	50095	46937	43526	40826	38008
36096	33882	31880	29799	28291	26829
25623	24479	23308	21974	21092	20235
19173	18215	17279	16655	16081	15065
14500	13934	13348	12610	12223	11496
11108	10704	10167	9800	9556	9077
8614	8363	7987	7657	7228	6901
6679	6365	6108	5845	5663	5445
5287	5046	4830	4576	4340	4211
4177	4032	3917	3701	3532	3366
3183	3123	2931	2868	2720	2720
2496	2470	2265	2240	2120	2059
1924	1815	1750	1679	1643	1549
1491	1444	1421	1270	1274	1284
1158	1119	1046	953	1009	962

The test spectrum consisted of a 64-channel pulse-height distribution for each set except the first. The first set covered proton energies above gamma background and did not use pulse-shape discrimination. All other sets used pulse-shape discrimination.

The range of usable data started well above channel one. When the lower-energy regions were measured, pulse-pileup distortion due to saturated ionizing events prevented a clear separation between proton and gamma events as analyzed by the discrimination circuit, unless events at small values of pulse height were excluded from the spectrum. In addition, pulse-amplitude overload caused some distortion in the last few channels (which were also excluded). These effects tend to worsen as the energy region analyzed diminishes. For example, the lowest-energy spectrum (4300 V on the hydrogen counter) extended up to only about 2.5 keV. Well over 90% of all the events registered were above this energy, and these events saturated the amplifiers, occasionally by as much as 10^3 . The amplifiers used were not especially well suited for counting under these circumstances, and considerably less distortion has been observed with a more recent design.

The input spectra consisted of measured pulse-height distributions, except for the first set, which was precorrected using ITERATE as described in Section II.B. The raw pulse-height distribution for the first set is included just before the results for the test problem (see Table XI). Statistical errors that apply to the ITERATE-corrected distribution are not those supplied by the PSNS code, and an appropriate factor (near unity), described in Appendix A, has to be applied. Neutron spectra derived from the ITERATE-corrected first set agree well with spectra derived using the raw pulse-height distribution for the first set in the PSNS code, except near the maximum energy (Fig. 23). The neglect of neutron flux in excess of the upper limit of measurement will introduce a serious systematic error in spectra computed near the limit, regardless of the method used for correcting spectra; this point was discussed in Part II.

The test-problem analysis leads to the same conclusions as those given in Part II. The inclusion of an energy-dependent response function has surprisingly little effect upon the derived neutron spectrum in the region (above 0.5 MeV) where a considerable distortion of the proton-recoil distribution occurs. This fact is undoubtedly due, in part, to the rapidly falling flux amplitude with increasing energy.

The effect of the non-Gaussian response upon the measurement can be observed by running the test problem (with the raw proton spectrum used in the first set) and then successively omitting orders in Legendre polynomials. The higher-order polynomial terms significantly change the shape of the non-Gaussian response distribution without altering the area under the distribution, which is given by the amplitude of the first polynomial term. The test problem was run with all terms set to zero (ideal counter response), retention of the leading Legendre term, retention of the

112-9012

Fig. 34. Effect of Retention of Legendre Polynomial Terms on Test Spectrum

first two terms, retention of the first three terms, and finally with all four terms. The root-mean-square deviation (each spectrum consisted of 147 energy points) of each spectrum from the spectrum retaining all four terms was calculated, and the result is shown in Fig. 34. Retention of the first two terms is sufficient, for this problem, to correct the distribution to one extremely close to the four-term result. Because of the integral nature of the response correction to proton distributions, the perturbation upon neutron spectra caused by the inclusion of the third and fourth terms is quite small.

The code edits the fractional correction that has been made to the neutron flux at each energy due to recoils from carbon and nitrogen [HPCOR(K)]. This correction is zero until an energy 20% of the maximum measured (see Section III.C) has been reached, since the code assumes that the neutron flux vanishes above the measurement region. Below about 500 keV, the correction becomes finite (approximately 2%). It increases to a maximum of about 6% near the break region between the methane and hydrogen counters and then drops to less than 2% as data used in determining the neutron spectrum are derived exclusively from the hydrogen counter. The correction increases with decreasing energy up to a maximum of 10% at a few keV. (This relatively large (percentage-wise) correction is due to the rapid reduction in neutron flux as energy decreases for the spectrum used as an example.) The significance of this correction made in the code is uncertain at low energies, because of lack of detailed knowledge of W for heavy nuclei. This correction can be reduced by at least half by removing nitrogen from the counter and replacing it by a considerably smaller amount of He^3 , which can also be used as the calibrating agent. For the test spectrum, a residual 5% maximum uncertainty in low-energy flux would still remain from carbon recoils. This error is comparable to statistics and quite possibly to other systematic errors existing in the spectrum at low energy.

APPENDIX A

Error Estimates for Iterative Unfolding

The elements of the output vector of a given detection system (the measured proton-recoil spectrum for the present system) will possess random statistical error. This fact implies the existence of random error in the iterative solution (i.e., the input vector). Expressions and estimates for this error are desirable. The present treatment will be applicable only within the context of two important assumptions. First, we must assume that the iterative unfolding method furnishes appropriate solutions for the given detection system of interest.¹¹ If appropriate solutions cannot be found, the method of iterative unfolding fails to determine the desired physical solutions, and for such cases error estimates cannot be regarded as meaningful. Second, it will be assumed that the response matrix is an exact representation of the systematic behavior of the detection system and therefore does not possess random statistical error. In many instances, this latter assumption is justified since the actual random error in the response matrix is negligible compared with that of the output vector.

Let the matrix representation of the detection system take the form¹¹

$$\underline{Y} = \underline{A}\underline{X}, \quad (\text{A.1})$$

where $\underline{A}$ is the response matrix (of order n) and $\underline{Y}$ is the output vector. In addition, let $\underline{E}$ represent the random error of the output vector $\underline{Y}$. Hence, we need generally consider systems of the form

$$\underline{Y} + \underline{E} = \underline{Y}' = \underline{A}\underline{X}. \quad (\text{A.2})$$

Under rather broad physical conditions,¹¹ application of iterative unfolding for Eqs. A.1 and A.2 will lead to the convergent sequences $\{\underline{Y}^{(m)}\}$ and $\{\underline{Y}^{(m)'}\}$, respectively. Namely,

$$\lim_{m \rightarrow \infty} \underline{Y}^{(m)} = \underline{Y}, \quad (\text{A.3a})$$

and

$$\lim_{m \rightarrow \infty} \underline{Y}^{(m)'} = \underline{Y}' = \underline{Y} + \underline{E}. \quad (\text{A.3b})$$

It follows from Eqs. A.3a and A.3b that, for sufficiently large m , $\underline{E}$ also represents the random error in $\underline{Y}^{(m)}$. In other words, any given iterate $\underline{Y}^{(m)}$, for sufficiently large m , possesses the same random statistical behavior attributable to the output vector $\underline{Y}$. As an example, consider the case in which the output vector $\underline{Y}$ is governed by Poisson statistics. This assumption is commonly used for many particle-detection systems

and counting experiments that arise in nuclear measurements. For this case, the variance $S_{y_i}^2 = y_i$, $i = 1, 2, \dots, n$. In this event, the variance $S_{y_i}^2(m)$ of $y_i^{(m)}$ is given by $S_{y_i}^2(m) = y_i^{(m)} = S_{y_i}^2 = y_i$, $i = 1, 2, \dots, n$, for sufficiently large m .

The variance $S_{x_i}^2(m)$ of any given element $x_i^{(m)}$, in the iterative solution $\underline{X}^{(m)}$, can be calculated directly from the matrix relation

$$\underline{Y}^{(m)} = A\underline{X}^{(m)}. \quad (\text{A.4})$$

We find

$$S_{x_i}^2(m) = \sum_{j=1}^n \left[(a^{-1})_{ij} \right]^2 S_{y_j}^2(m), \quad i = 1, 2, \dots, n, \quad (\text{A.5a})$$

and therefore, for sufficiently large m ,

$$S_{x_i}^2(m) = \sum_{j=1}^n \left[(a^{-1})_{ij} \right]^2 S_{y_j}^2, \quad i = 1, 2, \dots, n, \quad (\text{A.5b})$$

where $(a^{-1})_{ij}$ are the elements of the inverse matrix, A^{-1} . For Poisson statistics, Eqs. A.5a and A.5b become

$$S_{x_i}^2(m) = \sum_{j=1}^n \left[(a^{-1})_{ij} \right]^2 y_j^{(m)}, \quad i = 1, 2, \dots, n, \quad (\text{A.6a})$$

and

$$S_{x_i}^2(m) = \sum_{j=1}^n \left[(a^{-1})_{ij} \right]^2 y_j, \quad i = 1, 2, \dots, n. \quad (\text{A.6b})$$

These expressions are cumbersome and difficult to use since they involve elements of the inverse matrix, A^{-1} . Consequently, a less precise but more useful result, not involving A^{-1} , will be developed below. This analysis is based on the observation that most response matrices possess dominant diagonal elements and relatively small off-diagonal elements. Moreover, for diagonal response matrices, it follows from Eqs. A.4 and A.5a that the relative error in $\underline{X}^{(m)}$ is identical with that of $\underline{Y}^{(m)}$; hence it is identical with that of $\underline{Y}$. Therefore, we may qualitatively infer for dominantly diagonal response matrices that the relative error in $\underline{X}^{(m)}$ is approximately that of $\underline{Y}$.

To establish this conjecture rigorously, let the matrix A be partitioned in the form

$$A = A_d + A_0, \quad (\text{A.7})$$

where the matrices A_d and A_0 are defined as

$$(A_d)_{ii} = a_{ii}, \quad i = 1, 2, \dots, n, \quad (\text{A.8a})$$

$$\left. \begin{aligned} (A_0)_{ii} &= (a_0)_{ii} = 0, \quad i = 1, 2, \dots, n, \\ (A_0)_{ij} &= (a_0)_{ij} = a_{ij}, \quad i \neq j, \quad i, j = 1, 2, \dots, n. \end{aligned} \right\} \quad (\text{A.8b})$$

Consequently, A_d is simply a diagonal matrix whose elements coincide with the diagonal elements of A , and the matrix A_0 contains the remaining off-diagonal elements of A .

Taking differentials in Eq. A.4 and using Eq. A.7, we obtain

$$\underline{\delta Y} = (A_d + A_0) \underline{\delta X}. \quad (\text{A.9})$$

Since it has been assumed that m is sufficiently large to have established convergent results, the superscript m of the vectors $\underline{X}^{(m)}$ and $\underline{Y}^{(m)}$ has been omitted. In Eq. A.9, the differential vector $\underline{\delta Y}$ can be identified with the random error vector $\underline{E}$; i.e., $\delta y_i = e_i$, $i = 1, 2, \dots, n$. Let us form the ensemble average $\langle \delta y_i \cdot \delta y_{i+1} \rangle$, using the representation given in Eq. A.9. The quantities δy_i and δy_{i+1} are independent random variables. We therefore have

$$\begin{aligned} \langle \delta y_i \cdot \delta y_{i+1} \rangle &= \left\langle \left[a_{ii} \delta x_i + \sum_{j=1}^n (a_0)_{ij} \delta x_j \right] \left[a_{i+1, i+1} \delta x_{i+1} \right. \right. \\ &\quad \left. \left. + \sum_{k=1}^n (a_0)_{i+1, k} \delta x_k \right] \right\rangle = 0, \end{aligned} \quad (\text{A.10a})$$

which leads to

$$\langle \delta x_i \cdot \delta x_{i+1} \rangle = \frac{-1}{a_{ii} a_{i+1, i+1}} \sum_{j, k=1}^n (a_0)_{ij} (a_0)_{i+1, k} \langle \delta x_j \delta x_k \rangle \dots \quad (\text{A.10b})$$

Since the off-diagonal elements are no larger than n^{-1} , the right-hand side of Eq. A.10b will become negligible for sufficiently large n .

This conclusion is based on physical conditions implied by the matrix representation of the detection system. Namely, the physical description implies that the correlation between any two elements x_i and x_ℓ decreases as $|i - \ell|$ increases. It follows that the correlation between δx_i and δx_ℓ is an even more rapidly decreasing function of $|i - \ell|$. We may therefore assume that, for sufficiently large n , the quantities $\{\delta x_i\}$ are independent random variables.

The validity of this assumption can also be demonstrated by using the relation

$$\delta x_i = \sum_{j=1}^n (a^{-1})_{ij} \delta y_j \quad (\text{A.11})$$

to obtain a direct comparison of $\langle (\delta x_i)^2 \rangle$ and $\langle \delta x_i \cdot \delta x_\ell \rangle$. We find

$$\langle (\delta x_i)^2 \rangle = \sum_{j=1}^n [(a^{-1})_{ij}]^2 \langle (\delta y_j)^2 \rangle, \quad (\text{A.12a})$$

and

$$\langle \delta x_i \cdot \delta x_\ell \rangle = \sum_{j=1}^n (a^{-1})_{ij} (a^{-1})_{\ell j} \langle (\delta y_j)^2 \rangle. \quad (\text{A.12b})$$

Introducing appropriate mean values, m_d^2 and m_0^2 , for the set of variances $\langle (\delta y_i)^2 \rangle$ that appear in these expressions, we have

$$\langle (\delta x_i)^2 \rangle = m_d^2 \sum_{j=1}^n [(a^{-1})_{ij}]^2, \quad (\text{A.13a})$$

and

$$\langle \delta x_i \cdot \delta x_\ell \rangle = m_0^2 \sum_{j=1}^n (a^{-1})_{ij} (a^{-1})_{\ell j}. \quad (\text{A.13b})$$

The resulting summations, which appear in Eqs. A.13a and A.13b, now have a simple interpretation. In particular, these sums represent elements of the matrix A^{-1} . $\tilde{A}^{-1} = (\tilde{A}A)^{-1}$. Consequently we can write

$$\langle (\delta x_i)^2 \rangle = m_d^2 [(\tilde{A}A)^{-1}]_{ii}, \quad (\text{A.14a})$$

and

$$\langle \delta x_i \cdot \delta x_\ell \rangle = m_0^2 [(\tilde{A}A)^{-1}]_{i\ell}. \quad (\text{A.14b})$$

It follows from Eqs. A.14a and A.14b that $\langle (\delta x_i)^2 \rangle \gg |\langle \delta x_i \cdot x_\ell \rangle|$ for dominantly diagonal response matrices. Moreover, Eq. A.14b establishes the nature of the rapid decrease in $\langle \delta x_i \cdot \delta x_\ell \rangle$ with increasing $|i - \ell|$. Thus, when A is dominantly diagonal, so is A^{-1} . Furthermore, for A dominantly diagonal, $\tilde{A}A$ will be even more dominantly diagonal. Consequently, as we move away from the dominant diagonal, the off-diagonal elements of $(\tilde{A}A)^{-1}$ can be expected to decrease rapidly.

Calculation of the variance of y_i can be significantly simplified by using the approximate independence of the $\{\delta x_i\}$. Manipulation of Eq. A.9 yields

$$\langle (\delta y_i)^2 \rangle = (a_{ii})^2 \langle (\delta x_i)^2 \rangle + \sum_{j=1}^n [(a_0)_{ij}]^2 \langle (\delta x_j)^2 \rangle. \quad (\text{A.15})$$

Using once more the fact that the off-diagonal matrix elements, $(a_0)_{ij}$, vanish at least as rapidly as n^{-1} , Eq. A.15 reduces to the approximate condition

$$\langle (\delta y_i)^2 \rangle \cong a_{ii}^2 \langle (\delta x_i)^2 \rangle \quad (\text{A.16})$$

for sufficiently large n . We can therefore write

$$\frac{\langle (\delta x_i)^2 \rangle^{1/2}}{x_i} \cong \frac{y_i}{a_{ii} x_i} \cdot \frac{\langle (\delta y_i)^2 \rangle^{1/2}}{y_i}. \quad (\text{A.17})$$

Hence, for dominantly diagonal response matrices of sufficiently large order, the relative error in an element $x_i \in \underline{X}$ is approximately equal to the relative error in the corresponding element $y_i \in \underline{Y}$. For Poisson statistics, Eq. A.17 reduces to

$$\langle (\delta x_i)^2 \rangle^{1/2} \cong \frac{(y_i)^{1/2}}{a_{ii}}. \quad (\text{A.18})$$

Some mention can be made of relaxing the condition that the response matrix be dominantly diagonal. To establish the (approximate) independence of the random variables $\{\delta x_i\}$, this condition may be too conservative for many applications. Since the inverse matrix elements, $(a^{-1})_{ij}$, can be positive or negative, we can generally anticipate a great deal of cancellation in the sum of Eq. A.13b for sufficiently large n . Consequently, the condition

$$\langle (\delta x_i)^2 \rangle \gg |\langle \delta x_i \delta x_\ell \rangle|$$

may actually hold in many applications under less restrictive assumptions. In this event, Eq. A.15 provides the basis for a matrix representation of an unfolding problem for the error estimates. Namely

$$\underline{S}_y = B \underline{S}_x, \quad (A.19)$$

where the elements of $\underline{S}_y$ and $\underline{S}_x$ are the variances

$$\left\{ \langle (\delta y_i)^2 \rangle \right\} \text{ and } \left\{ \langle (\delta x_i)^2 \rangle \right\},$$

respectively. The matrix elements b_{ij} of B are defined by

$$b_{ij} = (a_{ij})^2, \quad i, j = 1, 2, \dots, n. \quad (A.20)$$

This new unfolding problem is an analog of the original unfolding problem. Moreover, iterative unfolding is directly applicable since all pertinent physical conditions are obviously met.¹¹ The essential feature of this error-estimate unfolding problem is that the new response matrix elements are merely the square of the elements in the original response matrix (cf. Eq. A.20).^{*} For such cases, iterative unfolding can be used, in principle, to determine error estimates, and thereby we may again avoid the problems associated with both calculating and using inverse matrix elements.

^{*} Here, the applicability of Poisson statistics would imply an even closer analogy, since it follows that $\underline{S}_y = \underline{Y}$ for Poisson statistics.

APPENDIX B

WERM Code Listing

TABLE B.I. WERM Input

Card	Variable	Format	Description
1-4	E	12F6.4	Energy table.
5-8	RGE	12F6.4	Corresponding proton range table.
9	AMEV	E12.4	MeV per channel (matrix mesh).
	PN	E12.4	Molecular density for spectral measurement.
	TFACT	E12.4	Temperature corrected to 25.5°C.
	LOWCH	I6	Starting channel for matrix mesh.
	MAXCH	I6	Ending channel for matrix mesh.
	NTRI	I3	0 = no triangularization of matrix. + = triangularize.
	NDEL	I3	+ defines R(J,J) as normalized Gaussian area.
	NPNP	I3	- = magnetic-tape write of matrix. 0 = matrix print and test on NTEST to punch. (See NTEST.) + = Test on NTEST.
	NCOMB	I3	0 = combined fit of Gaussian and non-Gaussian. -,+ = Gaussian only. No more than 60 data channels.
	NPUNCH	I3	+, - = no punch of intermediate results. 0 = punch of intermediate results.
	NTRAN	I3	-,+ = do not print data transformation factors.
	NTEST	I3	+ = punch parameters for interpolated response function (NF modulo NTEST). -,0 = no punch.
	NVMM	I3	-,+ = X ₁ , X ₂ , X ₃ are read in and VMM is bypassed.
	NP	I3	+, - = no printout of coefficients and parameters.
	NPOT	I3	+, - = no punch of relative Gaussian area and cosine coefficient for LSQPOL on DDP-24 computer. 0 = punch.
10	Run Identification	20A4	
11	PNPRYM	E12.4	Molecular density for each pressure-response measurement.
	EOP(NC)	F6.0	Estimate of Peak channel.
	CRES	F6.0	Resolution of Gaussian.
	IV	I6	Number of channels down from peak to start of test for half-height.
12-24	Data for first run	I0F8.0	Highest pressure is input first.
25...	Repeat of Cards 10-24		No more than 30 pressure runs may be used.

TABLE B.II. Other Variables Used in WERM

Variable	Description
NC	Number of individual pressure-response measurements, ≤ 30 .
EOCO	(0.615 MeV/estimate of Gaussian peak channel.) Used to generate ionization equivalent energy scale.
ERG(I)	Energy (MeV) scale formed by $EOCO * I - 0.03075$, $I = 1, 128$. Subtraction of C^{14} recoil energy.
RELG(NC)	Relative Gaussian area formed from peak and width at half-height fitted by VMM subroutine.
DIF(I)	Differences formed by subtracting fitted and extrapolated Gaussian from data (I).
ERGP(I)	Transformed energy.
EOP(NC)	Center position of Gaussian peak in transformed energy for individual response measurement.
XI(2,NC)	Width at half-height of Gaussian in transformed energy for individual response measurement.
EP1(G1 or G2)	Polynomial representation of ϵ for data transformation with energy ≤ 0.4 MeV.
EP2(G1 or G2)	ϵ -function used in data transformation for energies > 0.4 MeV.
TRANS(I)	Ratio of transformed data to original data. (Transformation factor.)
AN(NC,I)	Fourier constants formed in non-Gaussian region for analytical function evaluation.
EOF(I)	Response function matrix energy scale formed from LOWCH to MAXCH times AMEV (midbin energies).
XIM(2,I)	Width of Gaussian at half-height for matrix formulation.
FITG(NC)	Least-squares polynomial-fitted RELG(I).
XI(1,NC)	Peak of Gaussian formed from FITG(I) and fitted XI(2,NC).
COSAREA(NC)	Integral of non-Gaussian area.
RN(NC)	Normalization factor formed from COSAREA and FITG for AN's.
FITGM(I)	Fitted relative Gaussian area for matrix formulation.
XIM(1,I)	Corresponding Gaussian peak for matrix usage.
ANM(I,J)	Fourier constants for matrix.
SC(I)	Scale factor formed from fitted normalized AN's to renormalize same.
R(I,J)	Response matrix elements.

1. Printout of Program WERM

```

PROGRAM WERM
C COSINE AND/OR GAUSSIAN FIT. IF BOTH, THEY OVERLAP. EFB/RG 8/67
C
  DIMENSION EOP(30),DATA(128),FM(60),EN(60),H(3,3),X(3),DIF(128),
  XDF(60),RN(30),W(30),RESID(30),AA(5,5),B(8),AN(30,10),XIM(2,128)
  X,ERR(60),G(3),S(3),XP(3),GP(3),T(3),GB(3),FITG(30),FITGM(128),
  XTRANS(128),XI(2,30),AG(5,5),AX(8,8),RELG(30),EOF(128),DAFT(128),
  XANM(128,10),YE(30),R(128,128),SC(128),COSAREA(128),RUN(30,20)
  DIMENSION E(47),RGE(47),RZ(47),ERG(128),RP(128),ERGP(128)
C
  COMMON H,X,G,S,XP,GP,T,GB,M,N,L,LS,M1,MS,NS,IT,GS,GSP,GTP,GSS,GTI,
  XGSB,F,FP,F0,EP,K,P,T0,RS,SL,Z,O,A,EL,DELTA,FB
C
  COMMON/XYZ/EN,FM,ERR,NG,DF
C
  COMMON/12345/EOF,R,ANM,NF,XIM,AN,XI,EOP,AMEV,HALFA,LC,NTRI,NP,NC,
  XFITG,COSAREA,SC,NDEL,SUM,RELG,ERG,ERGP,TRANS,NTRAN,B,JA,NVMM
C
  EP1(ZY) = 20.8198 + ZY *(893.3565 + ZY *(-1.41502E+4 + ZY *(1.0702
  X08E+5 + ZY *(-4.744126E+5 + ZY *(1.24538E+6 + ZY *(-1.784085E+6 +
  XZY * 1.070871E+6))))))
  EP2(ZX) = .2374144/ZX * (10. * LOGF(ZX) + 39.0654)
C
  1 FORMAT(20A4)
  2 FORMAT(6E12.5)
  3 FORMAT(10F8.0)
  4 FORMAT(1H110X20A4)
  5 FORMAT(10F8.6)
  7 FORMAT(E12.4,2F6.0,5I6)
  8 FORMAT(12F6.4)
  9 FORMAT(46H0 GENERATED ENERGY IS GREATER THAN 5 MEV,...= E10.4 )
  11 FORMAT(41H0 ADJUSTED PROTON RANGE IS GREATER THAN FB.5 /)
  12 FORMAT(3E12.4,2I6,10I3)
  17 FORMAT(19H MESH PT. = I4 )
C
C AMEV = MEV PER CHANNEL
C PN = MOLECULAR DENSITY FOR SPECTRAL MEASUREMENT
C TFACT = TEMPERATURE CORRECTION TO 25.5 DEGREES C.
C NTRI = 0 IS NO TRIANGULARIZATION, OTHERWISE TRIANGULARIZE
C NDEL = +,-,0 DEFINES N. G. A. AS R(J,J)
C NPMP = - INDICATES MAGNETIC TAPE WRITE AND PRINTOUT OF MATRIX
C NPMP = 0 MEANS MATRIX PRINTOUT AND TEST ON NTEST PARAMETER
C NPMP = + MEANS TEST ON NTEST WHICH IF +,- WILL PUNCH MESH PT. PARAM.
C NCOMB = 0 GIVES COMBINED FIT OF GAUSSIAN AND COSINE AREAS
C NCOMB = -,0 GIVES ONLY GAUSSIAN FIT, 60 CHANNELS MAY BE USED
C NPUNCH = +,-,0 MEANS NO PUNCHING OF INTERMEDIATE RESULTS FOR TESTING
C NPUNCH = 0 INDICATES PUNCH OF SAME
C NTRAN = -,0 MEANS DO NOT PRINT DATA TRANSFORMATION FACTORS
C NTEST = + MEANS PUNCH CARDS FOR EVERY NTEST(TH) ENERGY OF MESH
C NTEST = -,0 INDICATES NO PUNCHED CARDS OF MATRIX MESH
C NVMM = -,0 MEANS X1,X2,X3, ARE READ IN AS PARAMETERS AND VMM IS SKIPT
C NP = +,- MEANS NO PRINTOUT OF COEFF. AND PARAMETERS
C NOPT = -,0 MEANS NO PUNCH OF UNFITTED NORMALIZED AN'S AND RELATIVE
C GAUSS. AREAS FOR LSQPOL
C NOPT = 0 MEANS PUNCH
C
C PNPRYM = MOLECULAR DENSITY FOR EACH RESPONSE MEASUREMENT
C CRES = RESOLUTION OF GAUSSIAN
C IV = NO. OF CHANNELS DOWN FROM PEAK TO START TEST FOR HALF-HEIGHT
C

```

```

      NC = 1
      READ 8,E
      READ 8,RGE
      DO 10 I=1,47
10  RZ(I)= RGE(I)
      READ 12,AMEV,PN,TFACT,LOWCH,MAXCH,NTRI,NDEL,NPNP,NCOMB,NPUNCH,NTRA
      XN,NTEST,NVMM,NP,NOPT
      PRINT 39
39  FORMAT(1H1)
C
21  READ 1,(RUN(NC,I),I=1,20)
C
      IF(E0F,60)83,22
22  READ 7,PNPRYM,E0P(NC),CRES,IV
      READ 3,DATA
C
      E0C0 = .615/E0P(NC)
      RFACT = PN/PNPRYM
C
C  SHIFT DATA TO ELIMINATE NEGATIVE ENERGY FORMATION
C
      NX = 128
      J = 0
      DO 20 I=1,128
      ERG(I) = E0C0*I + 0.03075
      IF(ERG(I))23,23,24
23  ERG(NX) = 0.
      NX = NX + 1
      GO TO 20
24  J = J+1
      ERG(J) = ERG(I)
      DATA(J) = DATA(I)
20  CONTINUE
      NJ = J
      NSC = 128 - J
      J1 = J + 1
      DO 30 I=J1,128
30  DATA(I) = 0.
      SUM = 0.
      DO 40 I=1,NJ
40  SUM = SUM + DATA(I)
C
C  CORRECT PROTON RANGE FOR TEMP.
C
      DO 50 I=1,47
50  RGE(I)=RGE(I) * TFACT
C
C  FORM INTERPOLATED RANGE OF PROTONS (RP)
C
      DO 60 I=1,NJ
      DO 70 J=1,47
      IF(ERG(I) - E(J))25,26,70
25  RATIO = (ERG(I)-E(J-1))/(E(J)-E(J-1))
      RP(I) = RGE(J-1) + RATIO*(RGE(J)-RGE(J-1))
      GO TO 60
26  RP(I) = RGE(J)
      GO TO 60
70  CONTINUE
      IF(ERG(I) - E(47))60,60,27
27  PRINT 9, ERG(I)
60  CONTINUE

```

```

C
C   ADDITIONALLY ADJUST RANGE(RP) FOR PRESSURE
C
      DO 80 I=1,NJ
80  RP(I) = RP(I) * RFACT
C
C   TEST FOR START OF GAUSSIAN FIT AT ONE PT. UNDER HALF=HT.
C
      EOP(NC) = EOP(NC) - NSC
      LC = EOP(NC)
      IS = EOP(NC) - IV
      TEST = DATA(LC)/2.
      DO 90 I=IS,LC
      IF(DATA(I)-TEST)90,90,28
28  LH = I-1
      GO TO 29
90  CONTINUE
C
C   FORM ARRAYS FOR GAUSSIAN FIT
C
29  NG = 129-LH
      IF(NVMM)77,31,77
77  READ 2,X(1),X(2),X(3)
      GO TO 32
31  PRINT 4,(RUN(NC,I),I=1,20)
      DO 100 I=1,NG
      FM(I) = DATA(LH)
      EN(I) = LH
      ERR(I) = 1.
100  LH = LH + 1
      X(1) = DATA(LC)
      X(3) = EOP(NC)
      X(2) = X(3)*CRES
      EP = X(1)*1.0E-05
      DO 110 I=1,3
      DO 110 J=1,3
      IF(I=J)82,81,82
81  H(I,J) = (.01*X(I))**2
      GO TO 110
82  H(I,J) = 0.0
110  CONTINUE
      CALL VMM
C
C   FORM RELATIVE GAUSSIAN AREA BASED ON TOTAL COUNTS
C
32  FGA = X(1)*X(2)*1.06447
      RELG(NC) = FGA/SUM
      LC = X(3)
      IF(NC=MB)21,33,21
C
C   SUBTRACT FOR DIFFERENCES IN OVERLAP REGION WITH FITTED + EXTRAPOLATED
C   GAUSSIAN
33  I = LC + 1
34  I = I-1
      GAUS = X(1)*EXP(-4*.*69315*(X(3)-I)**2/(X(2)*X(2)))
      DIF(I) = DATA(I) - GAUS
      IF(I=1)35,35,34
35  CONTINUE
C
C   ENERGY TRANSFORMATION AFTER INTERPOLATION IN ENERGY AND RANGE TABLE

```

```

C FORM TRANSFORMED ENERGY(ERGP) AND DATA
C
  DO 120 I=1,NJ
    IF(RP(I) - RGE(47))37,120,36
36 PRINT 11,RGE(47)
    DATA(I) = 0.
    ERGP(I) = 0.
    GO TO 120
37 DO 400 J=2,47
    IF(RP(I)-RGE(J))38,38,400
38 RATIO = (RP(I)-RGE(J-1))/(RGE(J)-RGE(J-1))
    ERGP(I) = E(J-1) + RATIO*(E(J)-E(J-1))
    GO TO 120
400 CONTINUE
120 CONTINUE
    EOP(NC) = ERGP(LC) + (ERGP(LC + 1) - ERGP(LC))*(X(3)-LC)

C
C PRINT NC,EOP,SUM,RELG
C
  CALL PRINTS(1)
  XI(2,NC) = X(2)*EOP(NC)/X(3)

C
C DATA TRANSFORMATION ON DIFFERENCES
C
  C2 = EOP(NC)
  DO 130 I=1,LC
    G1 = .58425 * ERG(I)
    G2 = C2 - ERGP(I)
    IF(G1 = .4)42,42,43
42 IF(G2 = .4)44,44,45
43 IF(G2 = .4)46,46,47
44 DAFT(I) = DIF(I)/RFACT * EP1(G1)/EP1(G2)
    GO TO 48
45 DAFT(I) = DIF(I)/RFACT * EP1(G1)/EP2(G2)
    GO TO 48
46 DAFT(I) = DIF(I)/RFACT * EP2(G1)/EP1(G2)
    GO TO 48
47 DAFT(I) = DIF(I)/RFACT * EP2(G1)/EP2(G2)
48 IF(NTRAN)130,49,130
49 TRANS(I) = DAFT(I)/DIF(I)
130 CONTINUE

C
C PRINT ERG, ERGP, TRANS
C
  CALL PRINTS(2)

C
  IF(NPUNCH)55,53,55
53 PUNCH 2,X(1),XI(2,NC),EOP(NC),RELG(NC)
  PUNCH 3,(DAFT(I),I=1,LC)
  DO 140 I=1,LC
    IF(ERGP(I) = 1.0E-6)54,140,140
54 ERGP(I) = 1.0E-6
140 CONTINUE
  PUNCH 5,(ERGP(I),I=1,128)

C
C FORM COSINE APPROXIMATIONS. AN(NC,KI) OVER LC CHANNELS
C
55 NI = -1
  DO 150 KI = 1,6
    NI = NI + 2

```

```

      ARG = FLOATF(NI)*1.5708/EOP(NC)
      AUG = 2./(FLOATF(NI)*3.14159265)
      AN(NC,KI) = DAFT(I)*AUG * SIN( ARG*ERGP(I) ) * 2.0
      DO 380 I=2,LC
380  AN(NC,KI) = AN(NC,KI) + (DAFT(I)+DAFT(I-1))*AUG*(SIN( ARG*ERGP(I) )
      X=SIN( ARG*ERGP(I-1) )
      AN(NC,KI) = AN(NC,KI)+DAFT(LC)*AUG*2.0*(SIN( ARG*EOP(NC) )-SIN
      X( ARG*ERGP(LC) ) )
150  CONTINUE
      NC = NC + 1
      DO 160 I=1,47
160  RGE(I) = RZ(I)
      GO TO 21

C
      83 IF(NC@MB)57,56,57
      56 NF = MAXCH - LOWCH + 1
      J=1
      DO 170 I=LOWCH,MAXCH
      EOF(J) = AMEV*I
170  J = J + 1
      NC = NC + 1
      HALFA = AMEV*0.5

C
C  PRINT EOP,AN(I,J)
C
C      CALL PRINTS(3)
C
C  FIT N. G. A. AND X2,      USE THESE TO FORM REFITTED X1
C
      DO 180 J=1,NC
      W(J) = 1./XI(2,J)
180  YE(J)= XI(2,J)
      W(1) = 1./XI(2,1)*100.
      CALL LSQPOL(EOP,YE,W,RESID,NC,SUE,1,AX,B,8,NC,8)
      DO 190 I=1,NC
      W(I) = 1./RELG(I)
190  XI(2,I) = B(1)+EOP(I)*(B(2)+EOP(I)*(B(3)+EOP(I)*(B(4)+EOP(I)*
      X(B(5)+EOP(I)*(B(6)+EOP(I)*(B(7)+EOP(I)*B(8))))))
      W(1) = 1./RELG(1)*100.

C
      IF(NOPT)62,61,62
      61 DO 200 I=1,NC
200  PUNCH 2,EOP(I),W(I),RELG(I)
      62 DO 210 I=1,NF
      IF(EOF(I)-EOP(1))91,91,92
      91 XIM(2,I) = XI(2,1)
      GO TO 210
      92 XIM(2,I)= B(1)+EOF(I)*(B(2)+EOF(I)*(B(3)+EOF(I)*(B(4)+EOF(I)*
      X(B(5)+EOF(I)*(B(6)+EOF(I)*(B(7)+EOF(I)*B(8))))))
210  CONTINUE

C
      CALL LSQPOL(EOP,RELG,W,RESID,NC,SUE,1,AG,B,5,NC,5)
      DO 220 I=1,NC
      FITG(I) = B(1) +EOP(I)*(B(2)+EOP(I)*(B(3)+EOP(I)*(B(4)+EOP(I)
      X *B(5))))
      XI(1,I) = FITG(I)/(XI(2,I)*1.064467)

C
C  FORM NORMALIZED AREAS AND SCALE COSINE COEFFICIENTS,
C
      CONST = 2.*EOP(I)/3.14159265

```

```

COSAREA(I) = 0.
DO 230 J=1,6
230 COSAREA(I) = COSAREA(I) + AN(I,J)/(2*J-1)*(-1)**(J+1)*CONST
RN(I) = (1.-FITG(I))/COSAREA(I)
W(I) = 1.
DO 240 J=1,6
240 AN(I,J) = AN(I,J)*RN(I)
220 CONTINUE
IF(NOPT)64,63,64
63 DO 250 J=1,6
DO 250 I=1,NC
250 PUNCH 2,EOP(I),W(I),AN(I,J)
64 DO 260 I=1,NF
IF(EOP(I)-EOP(1))93,93,94
93 FITGM(I)=FITG(1)
GO TO 260
94 FITGM(I) = B (1)+EOP(I)*(B (2)+EOP(I)*(B (3)+EOP(I)*(B (4)+EOP(I)
X*B(5))))
260 XIM(1,I) = FITGM(I)/(XIM(2,I)*1.064467)
C
C PRINT EOP,XI(1,I),XI(2,I),FITG,COSAREA
C PRINT EOP,AN(I,J)
C PRINT NORM. G. A. LSQPOL QUARTIC COEFFICIENTS
C
CALL PRINTS(4)
C
DO LSQPOL ON SCALED COSINE COEFFICIENTS
C
DO 270 JA = 1,6
C
DO 280 I=1,NC
280 YE(I) = AN(I,JA)
W(1) = 100.
CALL LSQPOL(EOP,YE,W,RESID,NC,SUE,1,AA,B,5,NC,5)
DO 290 I=1,NC
290 AN(I,JA) = B(1) +EOP(I)*(B(2) +EOP(I)*(B(3) +EOP(I)*(B(4) + EOP(I)
X*B(5))))
DO 300 I=1,NF
IF(EOP(I)-EOP(1))67,67,68
67 ANM(I,JA) = AN(1,JA)
GO TO 300
68 ANM(I,JA) = B(1) + EOP(I)*(B(2) + EOP(I)*(B(3) + EOP(I)*(B(4) +
X*EOP(I)*B(5))))
300 CONTINUE
C
C PRINT LSQPOL COEFFICIENTS FOR AN'S
C
CALL PRINTS(5)
270 CONTINUE
C
C FORM COSINE AREA WITH FITTED COEFF., FORM SCALE FACTOR USING FITTED
C GAUSS. AREA AND ADJUSTED COSINE AREA.
C SCALE COSINE COEFFICIENTS, KEEPING UNIT AREA
C
DO 310 I=1,NC
COSAREA(I) = 0.
CONST = 2.*EOP(I)/3.14159265
DO 320 J=1,6
320 COSAREA(I) = COSAREA(I) +AN (I,J)/(2*J-1)*(-1)**(J+1)*CONST
SC(I) = (1.-FITG(I))/COSAREA(I)

```



```

      DO 330 J=1,6
330  AN(I,J) = AN(I,J)*SC(I)
310  CONTINUE
C
C    MATRIX PARAMETERS
C
      DO 340 I=1,NF
      COSAREA(I) = 0.
      CONST = 2.*EOF(I)/3.14159265
      DO 350 J=1,6
350  COSAREA(I) = COSAREA(I) + ANM(I,J)/((2*J-1)*(-1)**(J+1)*CONST
      SC(I) = (1.-FITGM(I))/COSAREA(I)
      DO 360 J=1,6
360  ANM(I,J) = ANM(I,J)*SC(I)
340  CONTINUE
C
C    PRINT MATRIX PARAMETERS
C
      CALL PRINTS(6)
C    FORM MATRIX ELEMENTS R(I,J) FROM EOF(J)
      CALL MATFORM
      IF(NPNP)73,74,75
73  READ(26,5) ZYX
      REWIND 26
      WRITE(26,5) ((R(I,J),J=1,NF),I=1,NF)
74  CALL MATPRI
75  IF(NTEST)57,57,76
76  DO 370 I=1,NF,NTEST
      PUNCH 17,I
      PUNCH 2,XIM(1,I),XIM(2,I),EOF(I)
370  PUNCH 2,(ANM(I,J),J=1,6)
57  CONTINUE
      REWIND 26
      END

```

2. Printout of Subroutine MATFORM

```

      SUBROUTINE MATFORM
      DIMENSION R(128,128),EOF(128),ANM(128,10),XIM(2,128),AN(30,10),
      XXI(2,30),EOP(30),FITG(30),COSAREA(128),SC(128)
      DIMENSION RELG(30),ERG(128),TRANS(128),B(8),ERGP(128)
C
      COMMON/12345/EOF,R,ANM,NF,XIM,AN,XI,EOP,AMEV,HALFA,LC,NTRI,NP,NC,
      XFITG,COSAREA,SC,NDEL,SUM,RELG,ERG,ERGP,TRANS,NTRAN,B,JA
C
      DO 3 I=1,NF
      DO 3 J=1,NF
3  R(I,J) = 0.0
C
      DO 123 J = 1,NF
      NMAT = 1
      I = 1
      TCG = 0.8*EOF(J)
      RTEMP = 0.25*SORTF(3.141593/0.693148)*XIM(1,J)*XIM(2,J)
      ARG = 2.*SORTF(0.693148)/XIM(2,J)
C

```

```

C  INITIALIZE BIN ENERGY LIMITS (EOM1 AND EOM2)
C
    EOM1 = EOF(1) - HALFA
    EOM2 = EOM1 + AMEV
C
C  TEST ON START WITH COSINE OR GAUSSIAN REGION
C
C  REGION 1
C
19 IF(EOM2 - EOF(J))1,1,2
1  C1 = 4.*EOF(J)/3,14159265
   C2 = 1.570796/EOF(J)
   R(I,J) = 0.0
   D018 L=1,6
   Z = 2 * L - 1
18 R(I,J) = R(I,J) + C1*ANM(J,L)/Z*COSF(C2*EOF(I)*Z)*SINF(C2*Z*HALFA)
   EOM1 = EOM2
   EOM2 = EOM1 + AMEV
   I = I + 1
   GO TO 19
C
C  REGION 2
C
C  TEST ON DELTA FUNCTION MATRIX
C
2 IF(NDEL)4,5,4
4  R(J,J) = 2.*RTEMP
   GO TO 123
5  ARGH1 = EOM1 - EOF(J)
   ARGH2 = EOM2 - EOF(J)
   M3 = 1
C
C  TEST ON SIGN OF ARGH1 AND ARGH2
C
   IF(ARGH2)7,6,6
6  IF(ARGH1)8,8,9
7  ARGH2 = -ARGH2
   ARGH1 = -ARGH1
   M3 = 2
   GO TO 9
C
8  ARGH1 = -ARGH1
   M3 = 3
C
9  ARG2 = ARG*ARGH2
   ARG1 = ARGH1*ARG
   TEMP1 = 1. - 1./((1.*ARG1*(.278393 + ARG1*(.230389 + ARG1*(.000972+
XARG1*0,078108))))**4
   TEMP2 = 1. - 1./((1.*ARG2*(.278393 + ARG2*(.230389 + ARG2*(.000972+
XARG2*0,078108))))**4
   GO TO (12,10,11),M3
10 R(I,J) = RTEMP*(TEMP1-TEMP2) + R(I,J)
   GO TO 110
C
C  DOWN THE SLOPE
C
11 TEMP1 = -TEMP1
C
   IF(NTRI)13,14,13

```

```

C
13 TEMP2 = 1.
GO TO 14
C
12 IF(NTRI)15,14,15
C
14 R(I,J) = RTEMP*(TEMP2-TEMP1) * R(I,J)
IF(R(I,J) - 1.0E+4)15,110,110
110 EOM1 = EOM2
EOM2 = EOM2 + AMEV
I = I + 1
C
GO TO (250,251), NMAT
250 IF(I-NF)2,2,15
251 IF(EOM2 - EOF(J))2,123,123
C
15 IF(EOF(J) - EOF(1))123,123,60
60 IF(TCG-EOF(1))61,61,62
62 DO 16 K=1,NF
IF(TCG-EOF(K))17,17,16
16 CONTINUE
C
FORM R(I,J) IN OVERLAP REGION
C
17 I=K-1
NMAT = 2
EOM2 = EOF(K) - HALFA
GO TO 110
61 I= 0
NMAT = 2
EOM2 = EOF(1) - HALFA
GO TO 110
C
123 CONTINUE
C
RETURN
END

```

3. Printout of Subroutine PRINTS(NUM)

```

SUBROUTINE PRINTS(NUM)
C
DIMENSION R(128,128),EOF(128),ANM(128,10),XIM(2,128),AN(30,10),
XXI(2,30),EOP(30),FITG(30),COSAREA(128),SC(128)
DIMENSION RELG(30),ERG(128),TRANS(128),B(8),ERGP(128)
C
COMMON/12345/EOF,R,ANM,NF,XIM,AN,XI,EOP,AMEV,HALFA,LC,NTRI,NP,NC,
XFITG,COSAREA,SC,NDEL,SUM,RELG,ERG,ERGP,TRANS,NTRAN,B,JA,NVMM
C
1 FORMAT(10H0      EOP( 12, 3H) = F10.5, 5X 14HTOTAL COUNTS = E13.5,3
XX2HRELATIVE GAUSS. AREA = F10.5 )
2 FORMAT(1H1 3X1H1 8X6HERG(1) 7X 7HERGP(1) 4X23H(1/RFACT)(EP(G)/EP(G
X)) /((1H 14,2F14.5,E25.5))
3 FORMAT(1H1 26X 19HCOSINE COEFFICIENTS /3X1HN 2X11HENERGY, MEV 7X
X4HA(1) 9X4HA(2) 9X4HA(3) 9X4HA(4) 9X4HA(5) 9X4HA(6) /
X(1H0 13,F13.5,6E13.5))
4 FORMAT(1H1 17X 26HFITTED GAUSSIAN PARAMETERS 26X 11HCOSINE AREA /
X6H0      N 2X 15HX(3) = EOP, MEV 10X 4HX(1) 10X 4HX(2) 8X 7HFITG(N)/
X(1H0 15,F17.5,F14.5,F14.6,F15.5,E15.5))

```

```

12 FORMAT(19H TRANSFORMED ENERGY 12X12HTOTAL COUNTS 15XHHRELATIVE /
X6X1HN 5X6HEOP(N) 15X10H(RAW DATA) 12X11HGAUSS. AREA //)
14 FORMAT(1H0I6,F11.5,E25.5,F23.6)
16 FORMAT(44H1 UNFITTED NORMALIZED COSINE COEFFICIENTS /
X1H02X1HN1X11HENERGY, MEV 9X4HA(1) 9X4HA(2) 9X4HA(3) 9X4HA(4) 9X
X4HA(5) 9X4HA(6) / (1H0 I3,F13.5,6E13.5))
13 FORMAT(1H1 10X 58H NORMALIZED GAUSSIAN AREA QUARTIC COEFFICIENTS
XFROM LSQPOL / 1H0 10X 4HB(1) 14X4HB(2) 14X 4HB(3) 14X 4HB(4) 14X
X4HB(5) //5E18.5,5(/),10X 58H NORMALIZED COSINE AREA QUARTIC COEFF
XICIENTS FROM LSQPOL /1H0 10X 4HA(N) 14X 4HB(1) 14X 4HB(2) 14X 4HB
X(3) 14X 4HB(4) 14X 4HB(5) / )
5 FORMAT(1H0 10X 2HA( I1, 1H) E20.5,4E18.5)
6 FORMAT(1H140X*MATRIX COEFFICIENTS, SCALING FACTORS, AND PARAMETERS
X* /1H02X1HN2X11HENERGY, MEV 9X4HA(1) 9X4HA(2) 9X4HA(3) 9X4HA(4) 9X
X4HA(5) 9X4HA(6) 6X5HSC(N) 9X4HX(1) 9X4HX(2) /
X (1H0I3,F13.5,6F13.8,3F13.6))

```

```

C
GO TO (10,20,30,40,50,60),NUM
10 IF(NP)72,39,72
39 IF(NVMM)41,42,41
41 IF(NC-1)43,43,44
43 PRINT 12
44 PRINT 14,NC,EOP(NC),SUM,RELG(NC)
RETURN
42 PRINT 1,NC,EOP(NC),SUM,RELG(NC)
RETURN
20 IF(NTRAN)72,51,72
51 PRINT 2, (I,ERG(I),ERGP(I),TRANS(I),I=1,LC)
RETURN
30 IF(NP)72,58,72
58 PRINT 3,(I,EOP(I),(AN(I,J),J=1,6),I=1,NC)
RETURN
40 IF(NP)72,65,72
65 PRINT 4,(I,EOP(I),XI(1,I),XI(2,I),FITG(I),COSAREA(I),I=1,NC)
PRINT 16,(I,EOP(I),(AN(I,J),J=1,6),I=1,NC)
PRINT 13,(B(I),I=1,5)
RETURN
50 IF(NP)72,88,72
88 PRINT 5,JA,(B(I),I=1,5)
RETURN
60 IF(NP)72,71,72
71 PRINT 6,(I,EOP(I),(ANM(I,J),J=1,6),SC(I),XIM(1,I),XIM(2,I),I=1,NF)
72 RETURN
END

```

4. Printout of Subroutine MATPRI

```

SUBROUTINE MATPRI
DIMENSION R(128,128),EOP(128),ANM(128,10),XIM(2,128),AN(30,10),
XXI(2,30),EOP(30),FITG(30),COSAREA(128),SC(128)
DIMENSION RELG(30),ERG(128),TRANS(128),B(8),ERGP(128)
COMMON/12345/EOP,R,ANM,NF,XIM,AN,XI,FOP,AMEV,HALFA,LC,NTRI,NP,NC,
XFITG,COSAREA,SC,NDEL,SUM,RELG,ERG,ERGP,TRANS,NTRAN,B,JA
PRINT 16
16 FORMAT(1H1 50X *RESPONSE MATRIX* ///1H0 *MID ENERGY (MEV)* //)
N1 = 1
N2 = 8
IF(N2-NF)2,2,1
1 N2 = NF

```

```

2 WRITE(61,3)(I,F0F(I),I=N1,N2)
3 FORMAT(1H02X3HI/J 2XI3,F9.4,7(I6,F9.4))
  DO 4 I = 1,NF
4 WRITE(61,5)I,(R(I,J),J=N1,N2)
5 FORMAT(I4,E16.4,7E15.4)
  IF(N2-NF)6,8,8
6 N1 = N1 + 8
  N2 = N2 + 8
  IF(N2 - NF)2,2,7
7 N2 = NF
  GO TO 2
8 RETURN
END

```

5. Printout of Subroutine VMM*

```

C      SUBROUTINE VMM
      VARIABLE METRIC MINIMIZATION    MONITOR ADAPTION
      DIMENSION H(3,3),X(3),G(3),S(3),XP(3),GP(3),T(3),GB(3)
      COMMON H,X,G,S,XP,GP,T,GB,M,N,L,LS,M1,MS,NS,IT,GS,GSP,GTP,GSS,GTI,
      XGSB,F,FP,F0,EP,K,P,T0,RS,SL,Z,Q,A,EL,DELTA,FB
      K = 5
      N = 3
      DELTA = 1.0
101 MS=0
      IF OVERFLOW FAULT 2161,2161
2161 IF DIVIDE CHECK 103,103
103 DO 1068 I = 1,N
1067 DO 1068 J=1,N
1068 H(J,I)=H(I,J)
109 WRITE OUTPUT TAPE 6,5
111 WRITEOUTPUTTAPE 6,6,N,K,EP,P,DELTA
112 WRITE OUTPUT TAPE 6,7,(X(I),I=1,N)
113 WRITE OUTPUT TAPE 6,8
114 DO 115 I=1,N
115 WRITE OUTPUT TAPE 6,9,(H(I,J),J=1,N)
116 M1=1
117 CALL FCN(N,G,F,X,M1)
118 L=1
1185 IT=0
119 WRITE OUTPUT TAPE 6,14,IT,MS,F
1195 WRITE OUTPUT TAPE 6,7,(X(I),I=1,N)
120 WRITE OUTPUT TAPE 6,4
121 CALL READY
122 L=L
123 GO TO (1393,159,137,126),L
124 L=2
125 GO TO 121
126 CALL AIM
127 L=L
128 GO TO(129,135,137),L
129 CALL FIRE
130 L=L
131 GO TO(135,132,126),L
132 L=1
133 CALL DRESS

```

*Special "equip" cards are needed to run this IBM 704 subroutine on CDC 3600.

6. Printout of Subroutine READY*

* Ibid. page 86.

```

212 M1=2
213 CALL FCN(N,GP,FP,XP,M1)
214 M=1
215 CALL MATMPY(M,N,S,GP,GSP)
216 IF(GSP)217,229,229
217 IF(FP-F)218,229,229
218 WRITE OUTPUT TAPE 6,1
231 FB=FP
232 DO 234 I=1,N
233 GB(I)=GP(I)
234 T(I)=XP(I)
220 IF(EL-2.0)221,223,223
221 L=3
222 RETURN
223 DELTA=2.0*DELTA
224 T0=1.0/SL
225 L=2
226 GO TO 222
227 L=1
228 GO TO 222
229 L=4
230 GO TO 222
1 FORMAT(10HOUNDERSHOT)
END

```

7. Printout of Subroutine AIM*

```

C SUBROUTINE AIM
  AIM (MONITOR ADAPTATION)
  DIMENSION H(3,3),X(3),G(3),S(3),XP(3),GP(3),T(3),GB(3)
  COMMON H,X,G,S,XP,GP,T,GB,M,N,L,LS,M1,MS,NS,IT,GS,GSP,GTP,GSS,GTT,
  XGSB,F,FP,F0,EP,K,P,T0,RS,SL,Z,Q,A,EL,DELTA,FB
300 Z=GS+GSP+3.0*(F=FP)/EL
301 T0=GS/Z
  TI=GSP/Z
302 Q=SQRTF(1.0=T0*TI)
  Q = ABSF(Q*Z)
  A=(GSP+Q*Z)/(GSP+GS+2.0*Q)
303 T0=(EL*(GSP+Z+2.0*Q)*A**2)/3.0
304 F0=FP-T0
305 CALL MATMPY(N,N,H,GP,T)
306 TP1=GSP/SL
307 DO 308 I=1,N
308 T(I)=-T(I)+TP1*S(I)
309 M=1
310 CALL MATMPY(M,N,T,GP,GTP)
311 IF(2.0*T0+GTP)317,312,312
312 TP1=1.0-A
313 DO 314 I=1,N
314 T(I)=A*X(I)+TP1*XP(I)
315 L=1
316 RETURN
317 IF(F+GTP/2.0)312,318,318
318 DO 319 I=1,N
319 T(I)=T(I)+XP(I)

```

*Ibid. page 86.

```

320 M1=2
321 CALL FCN(N,GB,FB,T,M1)
322 IF(FB-F0)323,312,312
323 WRITE OUTPUT TAPE 6,1
324 DO 325 I=1,N
325 S(I)=T(I)-XP(I)
326 M=1
327 CALL MATMPY(M,N,S,GB,GTT)
328 GTT=GTT-GTP
329 IF(GTT)335,330,330
330 GSS=GTT
331 SL=-GTP
332 EL=1.0
333 L=2
334 GO TO 316
335 L=3
336 GO TO 316
      1 FORMAT(9H0RICOCHET)
      END

```

8. Printout of Subroutine FIRE*

```

      SUBROUTINE FIRE
C      FIRE (MONITOR ADAPTATION)
      DIMENSION H(3,3),X(3),G(3),S(3),XP(3),GP(3),T(3),GB(3)
      COMMON H,X,G,S,XP,GP,T,GB,M,N,L,LS,M1,MS,NS,IT,GS,GSP,GTP,GSS,GTT,
      XGSB,F,FP,F0,EP,K,P,T0,RS,SL,Z,Q,A,EL,DELTA,FB
400 M1=2
401 CALL FCN(N,GB,FB,T,M1)
402 M=1
403 CALL MATMPY(M,N,S,GB,GSB)
404 TP1=MIN1F(F,FP)
405 IF(TP1-FB+EP)418,406,406
406 TP1=A/(1.0-A)
407 TP2=(1.0-A)/A
408 T0=GSB*(TP1+TP2)
409 IF(ABSF(T0)=Q)413,410,410
410 GSS=2.0*Q
411 L=1
412 RETURN
413 GSS=T0+2.0*Q
414 DO 415 I=1,N
415 G(I)=(GB(I)+G(I))*TP1+(GP(I)-GB(I))*TP2
416 L=2
417 GO TO 412
418 IF(F=FP)419,428,428
419 WRITE OUTPUT TAPE 6,1
420 EL=(1.0-A)*EL
421 FP=FB
422 GSP=GSB
423 DO 425 I=1,N
424 XP(I)=T(I)
425 GP(I)=GB(I)
426 L=3
427 GO TO 412

```

* Ibid. page 86.


```

428 WRITE OUTPUT TAPE 6,2
429 EL=EL*A
430 F=FB
431 GS=GSB
432 DO 434 I=1,N
433 X(I)=T(I)
434 G(I)=GB(I)
435 GO TO 426
1  FORMAT(10HOMOVE LEFT)
2  FORMAT(11HOMOVE RIGHT)
END

```

9. Printout of Subroutine DRESS*

```

C  SUBROUTINE DRESS
   DRESS (MONITOR ADAPTATION)
   DIMENSION H(3,3),X(3),G(3),S(3),XP(3),GP(3),T(3),GB(3)
   COMMON H,X,G,S,XP,GP,T,GB,M,N,L,LS,M1,MS,NS,IT,GS,GSP,GTP,GSS,GTT,
   XGSB,F,FP,F0,EP,K,P,T0,RS,SL,Z,Q,A,EL,DELTA,FB
   L=L
   GO TO(500,525,519,510),L
500 CALL MATMPY(N,N,H,G,X)
501 M=1
502 CALL MATMPY(M,N,X,G,T0)
503 TP1 = SL - GSS**2/T0 - EP
504 IF(TP1)524,505,505
505 DO 507 I=1,N
506 DO 507 J=1,N
507 H(I,J)=H(I,J)+X(I)*X(J)/T0
508 DELTA=DELTA*(EL*GSS/T0)
509 T0=EL/GSS
510 DO 512 I=1,N
511 DO 512 J=1,N
512 H(I,J)=H(I,J)+T0*S(I)*S(J)
519 F=FB
520 DO 522 I=1,N
521 G(I)=GB(I)
522 X(I)=T(I)
   IF OVERFLOW FAULT 531,2530
2530 IF DIVIDE CHECK 2528, 513
513 WRITE OUTPUT TAPE 6,1,IT,MS,F,GS
517 WRITE OUTPUT TAPE 6,4
518 IT=IT+1
5185 L=1
523 RETURN
524 WRITE OUTPUT TAPE 6,5
525 TP1=EL*SL/GSS
526 DELTA=DELTA*TP1
527 T0=(TP1-1.0)/SL
528 GO TO 510
2528 WRITE OUTPUT TAPE 6,206
2529 GO TO 532
531 WRITE OUTPUT TAPE 6,6
532 WRITE OUTPUT TAPE 6,7
533 DO 534 I=1,N
534 WRITE OUTPUT TAPE 6,8,(H(I,J),J=1,N)
535 WRITE OUTPUT TAPE 6,9,F,DELTA
536 WRITE OUTPUT TAPE 6,2,(X(I),I=1,N)

```

*Ibid. page 86.

```

537 WRITE OUTPUT TAPE 6,10,(G(I),I=1,N)
543 L=2
544 GO TO 523
1  FORMAT(4H0IT 14,7H STEP 14,4H F= E14,5,5H GS=E14,5)
2  FORMAT(3H0X= 8E14,5/(3H0 8E14,5))
4  FORMAT(2H0= -- -- -- -- --)
5  FORMAT(9H0C0LINEAR)
6  FORMAT(9H00VERFLOW)
206 FORMAT(13H0DIVIDE CHECK)
7  FORMAT(13H0ERROR MATRIX)
8  FORMAT(1H0 8E14,5/(1H08E14,5))
9  FORMAT(3H0F= E14,5,7H DELTA=E14,5)
10 FORMAT(3H0G= 8E14,5/(3H0 8E14,5))
END

```

10. Printout of Subroutine STUFF*

```

C      SUBROUTINE STUFF
      STUFF (MONITOR ADAPTATION)
      DIMENSION H(3,3),X(3),G(3),S(3),XP(3),GP(3),T(3),GB(3)
      COMMON H,X,G,S,XP,GP,T,GB,M,N,L,LS,M1,MS,NS,IT,GS,GSP,GTP,GSS,GTI,
      XGSB,F,FP,F0,EP,K,P,T0,RS,SL,Z,Q,A,EL,DELTA,FB
600 K=K+1
601 IF(K)617,602,602
602 MS=MS+1
      P = SQRTF(2, *F)
620 WRITE OUTPUT TAPE 6,1,MS,DELTA,GS
2603 T(1)=RANF (=1)-0,5
603 DO 604 I=2,N
604 T(I)=RANF (=1)-0,5
605 CALL MATMPY(N,N,H,T,S)
606 M=1
607 CALL MATMPY(M,N,S,T,TP1)
608 TP1=SQRTF(TP1)
609 EL=P/TP1
610 DO 611 I=1,N
611 X(I)=X(I)+EL*S(I)
612 M1=2
614 L=1
616 RETURN
617 L=2
618 MS=0
619 GO TO 616
1  FORMAT(13H0RANDOM STEP 14,8H DELTA=1E14,5,5H GS=E14,5)
END

```

11. Printout of Subroutine MATMPY(M,N,H,G,S)*

```

C      SUBROUTINE MATMPY(M,N,H,G,S)
      MATRIX MULTIPLICATION (MONITOR ADAPTATION)
      DIMENSION H(3,3),G(3),S(3)
702 DO 703 I=1,M
700 S(I)=0,0
      DO 703 J=1,N
703 S(I)=H(J,I)*G(J)+S(I)
704 RETURN
END

```

*Ibid. page 86.

12. Printout of Subroutine FCN(M,G,F,B,M1)*

```

SUBROUTINE FCN(M,G,F,B,M1)
  DIMENSION EN(60),B(3),ALF(60),C(60,3),G(3),FM(60),ERR(60),DF(60)
  X,S(3),XP(3),GP(3),T(3),GB(3),H(3,3),X(3)
  COMMON/XYZ/EN,FM,ERR,NG,DF
  IF(M1-2)8,10,260
  8 MM = M/3
  10 DO 110 I = 1,NG
    ALF(I) = 0.0
    DO 110 J = 1,MM
      T1 = (EN(I)-B(3*J))/B(3*J-1)**2
      T2 = EXPF(-2.77258872*T1*(EN(I)-B(3*J)))
      T3 = B(3*J-2)*T2
      ALF(I) = ALF(I) + T3
      C(I,3*J-2) = T2
      C(I,3*J) = T3*T1*5.54517744
  110 C(I,3*J+1) = C(I,3*J)*(EN(I)-B(3*J))/B(3*J-1)
      F = 0.0
      DO 140 I = 1,NG
        DF(I) = FM(I) - ALF(I)
  140 F = F + (DF(I)/ERR(I))**2
      DO 190 J = 1,M
        G(J) = 0.0
      DO 180 I = 1,NG
        G(J) = G(J) + DF(I)/ERR(I)**2*C(I,J)
  190 G(J) = -2.0*G(J)
      RETURN
  260 WRITEOUTPUTTAPE 6,4000
  4000 FORMAT(67H0          Z          FM(Z)          FC(Z)
    X      DF(Z)          )
      DO 280 I = 1,NG
  280 WRITEOUTPUTTAPE 6,5000,EN(I),FM(I),ALF(I),DF(I)
  5000 FORMAT(1H0  4E17,7)
      RETURN
      END

```

TABLE B.III. WERM Sample Input**

0.0	.005	.010	.015	.020	.025	.030	.035	.040	.045	.050	.055
.060	.065	.070	.075	.080	.085	.090	.095	.100	.150	.200	.250
.300	.350	.400	.450	.500	.550	.600	.650	.700	.750	.800	.850
.900	.950	1.0	1.5	2.0	2.5	3.0	3.5	4.0	4.5	5.0	
.0014	.0064	.0114	.0163	.0213	.0263	.0313	.0363	.0424	.0474	.0523	.0571
.0619	.0667	.0715	.0763	.0811	.0859	.0908	.0957	.101	.154	.218	.292
.376	.470	.574	.687	.809	.940	1.08	1.23	1.38	1.55	1.73	1.91
2.10	2.30	2.51	5.02	8.24	12.2	16.7	22.0	27.9	34.5	41.6	
.0179		.1356678*21		.188332		31	128	0	1	=1	0 1 1 0 0
				0	1						
9/67 16=1A 87 PSI											
0.148216*21		110.5	.12	7							
				87 PSI							
1000	1014	1028	1042	1056	1070	1084	1098	1112			
				1126							
1140	1154	1168	1182	1196	1210	1224	1238	1252			

*Ibid. page 86.

**Overflow (>72 columns) of input data printed on following line.

TABLE B.III. (Contd.)

1280	1294	1308	1266 1322	1336	1350	1364	1378	1392
1420	1434	1448	1406 1462	1476	1490	1504	1518	1532
1560	1574	1588	1546 1602	1616	1630	1644	1658	1672
1629	1631	1671	1686 1713	1702	1835	1808	1813	1963
1966	2042	2034	1921 2061	2128	2198	2255	2268	2303
2423	2569	2444	2392 2651	2754	2862	2767	3058	3376
3611	3648	3769	3365 4026	4080	4356	4591	4477	4885
5378	5743	6167	5013 6706	7096	7697	8260	9097	9782
11591	12968	15094	10642 18405	22480	27714	33544	39426	43757
45933	45488	43163	46122 40069	36375	31682	25909	19901	14219
6356	4341	3068	9170 2208	1531	1002	677	467	
9/67 1-1A 80 PSI .1356678*21 113.3 .09 7 80 PSI								
1013	1027	1040	1053 1133	1066	1080	1093	1106	1120
1146	1160	1172	1186 1266	1200	1213	1226	1239	1253
1279	1293	1306	1319 1399	1333	1346	1359	1372	1386
1412	1426	1439	1452 1532	1466	1479	1492	1505	1519
1545	1559	1572	1585 1567	1600	1593	1647	1629	1576
1524	1618	1639	1649 1791	1721	1688	1820	1697	1766
1910	1819	1950	1936 2195	1981	1946	2094	2178	2153
2319	2334	2449	2497 3049	2487	2673	2766	2733	2952
3192	3293	3582	3577 4421	3758	3908	4021	4067	4367
4675	5036	5218	5462 8932	5974	6477	6924	7613	8302
9554	10174	10757	11865 39002	13438	15692	19170	24031	31061
47653	53338	55554	54649 10417	49989	42720	33967	24316	16356
6975	4878	3295	2204	1377	818	547	418	
9/67 15-18 5 PSI* .8420290*19 114.2 .044 5 15 18								
23000	22725	22450	22175 20525	21900	21625	21350	21075	20800

*The above is only a sample of the total response function input data required.

20250	19975	19700	19425	19150	18875	18600	18325	17793
			17672					
17145	16885	16728	16333	15987	15831	15440	15052	14814
			14533					
14439	14236	13478	13734	13351	13035	12845	12591	12196
			12005					
11703	11531	11067	10940	10604	10156	9949	9661	9295
			9178					
9037	8759	8665	8387	8357	8100	7833	7588	7674
			7637					
7448	7205	7036	6776	6818	6581	6365	6252	6087
			5851					
5594	5450	5451	5264	5019	4816	4581	4380	4223
			3990					
3770	3591	3284	3137	3031	2785	2718	2637	2505
			2445					
2360	2220	2197	2230	2067	2025	1981	1973	1880
			1967					
1796	1777	1735	1695	1852	1793	1812	1939	2192
			2608					
3240	4477	6502	8458	8066	5401	2176	569	145
			62					
54	44	31	32	31	36	16	34	

APPENDIX C
ITERATE Code Listing

TABLE C.I. ITERATE Input

Card	Variable	Format	Description
1	NTYPE = 2	I6	Generate SME matrix via MAT2 from magnetic tape. ^a
2	N ≤ 120	I6	Matrix size, N x N.
	NPRINT	I6	1 = do not print SME matrix or symmetric SME matrix. 2 = do print SME matrix or symmetric SME matrix.
	NGEN	I6	1 = SME matrix input by rows. 2 = first row and first column input.
	MSYM	I6	1 = do not symmetrize SME matrix. 2 = symmetrize.
	ITMAX	I6	Output control parameter. Results printed IT modulo NOUT, where IT is the iteration number.
	NOUT	I6	
	NERR	I6	1 = input relative errors of W(I). Under this option, ER(I) is read in after X1(I). 2 = same relative errors for all W(I). 3 = error formed in program as $1/\sqrt{W(I)}$.
	NU	I6	Number of iteration sets to be punched.
	INCH	I6	Initial channel for energy formation.
	A	F12.0	MeV/channel (for energy printout).
3	[N1(I), I=1, NU] NU ≤ 9	I6	Which iteration numbers to punch for input to PSNS.
4,5	W(I), I=1, N	6F12.0	Components of output vector of detection system.
...	X1(I), I=1, N	6F12.0	Components of initial vector used to initiate iteration procedure.

^aEQUIP and MOUNT "Systems" cards (CDC-3600 Scope) are needed for magnetic-tape input of SME matrix via MAT2.

1. Printout of Program ITERATE

```

PROGRAM ITERATE
DIMENSION R(120),SUMN(120),XX(128)
COMMON /1/ SME(120,120) /2/ SS(120,120),WNU(120),W(120),ER(120),
XX(120),N,NPRINT,MSYM,ITMAX,NOUT,X1(120),N1(10),NU,INCH
BANK,(0),/1/, (ITERATE),/2/

C
C
1 READ 2, NTYPE
2 FORMAT(12I6)
IX = 1
IF(EOF,60)200,201
200 CALL EXIT
201 CONTINUE
GO TO (3,4)NTYPE
3 CALL MAT1
GO TO 5
4 CALL MAT2
5 CONTINUE
NNX = N + INCH = 1
NX = INCH = 1
DO 300 I = 1,NX
300 XX(I) = 0.
GO TO(7,6)MSYM
6 CALL MATSYM
GO TO 10

C
C      S MATRIX IS SYMMETRICAL
C
7 DO 8 I=1,N
8 WNU(I)=W(I)
DO 9 I=1,N
DO 9 J=1,N
9 SS(I,J)=SME(I,J)

C
C      FORM BOUND FOR ARRESTING CRITERION
C
10 BND=0.
DO 100 I=1,N
100 BND=BND+(ER(I)*WNU(I))**2
GO TO(13,11)NPRINT
11 PRINT 12,
12 FORMAT(1H17X17HORIGINAL S MATRIX//)
CALL MPRINT(SME,N)
13 GO TO(17,14)MSYM
14 GO TO(17,15)NPRINT
15 PRINT 16,
16 FORMAT(1H17X19HSYMMETRIC SS MATRIX//)
CALL MPRINT(SS,N)
17 CONTINUE

C
C      BEGIN ITERATION PROCEEDURE
C
PRINT 18,
18 FORMAT(1H1)
IT=1
NCONV=2

```

```

C
19 CONTINUE
C
C
22 SUM1=0.
   DO 24 I=1,N
     SUM=0.
     DO 23 J=1,N
23  SUM=SUM+SS(I,J)*X1(J)
     X(I)=X1(I)*WNU(I)/SUM
     W(I)=SUM
     R(I)=WNU(I)-W(I)
     SUM1=SUM1+R(I)**2
24  SUMN(I)=SUM1
     IF(SUMN(N)-BND)25,25,27
25  PRINT 26, IT,BND,SUMN(N)
26  FORMAT(1H0//9X50HARRESTING CRITERION SATISFIED ON ITERATION NUMBER
1  I6,//9X6HROUND=E11.4,3X5HNORM=E11.4,           //26X1HI9X1HX
211X6HWPRI12X1HR13X4HNORM//)
     NX = INCH
     DO 301 I=1,N
       XX(NX) = X1(I)
301  NX = NX + 1
       PUNCH 280,(XX(I),I=1,NNX)
       REWIND 26
       NCONV=1
       GO TO 30
27  IF(IX - NU)278,278,277
278  IF(IT - N1(IX))277,279,277
279  NX = INCH
     DO 302 I=1,N
       XX(NX) = X1(I)
302  NX = NX + 1
       PUNCH 280,(XX(I),I=1,NNX)
280  FORMAT(6F12.0)
     IX = IX + 1
277  IF(XMODF(IT,NOUT))34,28,34
C
28  PRINT 29, IT
29  FORMAT(1H0//9X17HITERATION NUMBER I6,//26X1HI9X1HX11X6HWPRI12X1H
1R13X4HNORM//)
C
30  DO 31 I=1,N
31  PRINT 32, I,X1(I),W(I),R(I),SUMN(I)
32  FORMAT(I27,4E15.4)
C
33  GO TO(1,34)NCONV
34  IT=IT+1
   IF(IT-ITMAX)35,35,37
35  DO 36 I=1,N
36  X1(I)=X(I)
   GO TO 19
37  PRINT 38, ITMAX,BND,SUMN(N)
38  FORMAT(1H0//9X40HARRESTING CRITERION NOT SATISFIED AFTER I6,2X11HI
1TATIONS.//9X6HROUND=E11.4,3X5HNORM=E11.4)
   REWIND 26
   GO TO 1
END

```


2. Printout of Subroutine MATSYM

```

SUBROUTINE MATSYM
  DIMENSION ST(120,120)
  COMMON /1/ SME(120,120) /2/ SS(120,120), WNU(120), W(120), ER(120),
  XX(120), N, NPRINT, MSYM, ITMAX, NOUT, X1(120)
  D01J=1,N
  D01I=1,N
1  ST(J,I)=SME(I,J)
C
C   FORM WNU=ST*W
C
  D02I=1,N
  SUM=0.
  D02J=1,N
  SUM=SUM+ST(I,J)*W(J)
2  WNU(I)=SUM
C
C   FORM SS=ST*SME
C
  D03I=1,N
  D03J=1,N
  SUM=0.
  D03L=1,N
  SUM=SUM+ST(I,L)*SME(L,J)
3  SS(I,J)=SUM
C
C
  PRINT4,
4  FORMAT(1H115X71HTHE S MATRIX HAS BEEN SYMMETRIZED AND A NEW W VECT
10R HAS BEEN GENERATED//26X1H15X9HW VECTOR)
  D05I=1,N
5  PRINT6,I,WNU(I)
6  FORMAT(127,E15.4)
C
  RETURN
  END

```

3. Printout of Subroutine MAT1

```

SUBROUTINE MAT1
  DIMENSION Y(120), S(120)
  COMMON /1/ SME(120,120) /2/ SS(120,120), WNU(120), W(120), ER(120),
  XX(120), N, NPRINT, MSYM, ITMAX, NOUT, X1(120)
  BANK, (MAT1), /2/
C
  READ1,N,M1,ISCALE,NPRINT,MSYM,MTYPE,ITMAX,NOUT,NERR
1  FORMAT(12I6)
  READ2,(Y(I),I=1,N)
  READ2,(S(I),I=1,N)
  READ2,(X1(I),I=1,N)
2  FORMAT(6F12.4)
  GO TO (20,30) NERR
20 READ 2, (ER(I), I=1,N)
  GO TO 50
30 READ 2, ERROR
  DO 40 I=1,N
40 ER(I)=ERROR
50 CONTINUE

```

```

C
C
      PRINT3,
3  FORMAT(1H115X38HITERATIVE SOLUTION OF DETECTION SYSTEM//26X1H17X5H
1S SET8X9HY VECT0R6X9HW VECT0R6X9HER VECT0R6X9HX1 VECTOR)
C
C      FORM W VECTORS
C
      D04I=1,N
4  W(I)=EXP(S(I))*S(I)**(1-M1)*Y(I)
      D05I=1,N
5  PRINT6,I,S(I),Y(I),W(I),ER(I),X1(I)
6  FORMAT(I27,5E15,4)
C
C
      READ2, FACTOR,X
      GOT0(11,7)ISCALE
7  D08I=1,N
8  S(I)=S(I)*FACTOR
      PRINT9,FACTOR
9  FORMAT(43H1THE S MATRIX ELEMENTS HAVE BEEN SCALED BY F11,6,./26X1H
117X5HS SET8X9HY VECT0R6X9HW VECT0R6X9HX1 VECTOR)
      D010I=1,N
      W(I)=EXP(S(I))*S(I)**(1-M1)*Y(I)
10 PRINT6,I,S(I),Y(I),W(I),X1(I)
11 CONTINUE
C
C      FORM S MATRIX
C
      D013I=1,N
      D013J=1,N
      A=J
      A=A*X
      B=A-1.
      SME(I,J)=S(I)**B
      GOT0(12,13)MTYPE
12 Q=GAMMA(A)
      SME(I,J)=SME(I,J)/Q
13 CONTINUE
      RETURN
      END

```

4. Printout of Subroutine MPRINT(X,N)

```

SUBROUTINE MPRINT(X,N)
DIMENSION X(120,120)
C
      K1 = 1
      K2 = 8
      IF(K2-N)2,2,1
1  K2 = N
2  WRITE(61,3)(I,I=K1,K2)
3  FORMAT(1H02X3HI/5XI3,7I14)
      D0 4 I=1,N
4  WRITE(61,5)I,(X(I,J),J=K1,K2)
5  FORMAT(I4,F15,4,7E14,4)
      IF(K2-N)6,8,8
6  K1=K1+8
      K2 = K2 + 8

```

```

      IF(K2-N)2,2,7
7 K2 = N
  GO TO 2
8 RETURN
  END

```

5. Printout of Subroutine MAT2

```

      SUBROUTINE MAT2
      COMMON /1/ SME(120,120) /2/ SS(120,120), WNU(120), W(120), ER(120),
      XX(120), N, NPRINT, MSYM, ITMAX, NOUT, X1(120), N1(10), NU, INCH
      BANK, (MAT2), /2/
      BANK, (0), /1/
      READ1, N, NPRINT, NGEN, MSYM, ITMAX, NOUT, NERR, NU, INCH, A
      READ 1, (N1(I), I=1, NU)
1  FORMAT(9I6, F12.0)
  GOT0(2,4)NGEN

C
C  READ ENTIRE S MATRIX BY ROWS
C
2 READ (26,3)((SME(I,J), J=1,N), I=1,N)
3 FORMAT(10F8.7)
  GOT06

C
C  CONSTRUCT S FROM FIRST ROW AND COLUMN
C
4 READ3, (SME(1,J), J=1,N)
  READ3, (SME(I,1), I=2,N)
  DO5 I=2,N
  DO5 J=2,N
5 SME(I,J)=SME(I-1,J-1)
10 FORMAT(6F12.0)
6 READ 10, (W(I), I=1,N)
  READ 10, (X1(I), I=1,N)
  GO TO (60,70,71), NERR
60 READ 10, (ER(I), I=1,N)
  GO TO 90
70 READ 10, ERROR
  DO 80 I=1,N
80 ER(I)=ERROR
  GO TO 90
71 DO 72 I = 1,N
72 ER(I) = 1./SQRT(W(I))
90 CONTINUE
  PRINT7,
7 FORMAT(1H125X38ITERATIVE SOLUTION OF DETECTION SYSTEM//26X1H14X
      X11HENERGY, MEV 5X9HW VECTOR 6X9HER VECTOR 6X9HX1 VECTOR )
  E = A*(INCH=1)
  DO8 I=1,N
  E = E + A
8 PRINT 9, I, E, W(I), ER(I), X1(I)
9 FORMAT(I27, F15.4, 3E15.4)
  RETURN
  END

```


APPENDIX D

PSNS Code and Flow Diagrams

TABLE D.I. PSNS Input

Card	Variable	Format	Description
1	PROB	18A4	Problem identification.
2	MN	I6	Total number of sets in the measurement.
	NMET	I6	Number of methane counter sets.
	NUC	I6	Number of sets precorrected for all response-function effects.
	RA	F6.0	Energy-independent slope-taking half-interval.
	RB	F6.0	Energy-dependent slope-taking half-interval factor.
	DINC	F6.0	Data computation increment used to form an energy sequence with fixed logarithmic spacing.
	ELIMIT	E12.5	Energy below which the response function for the methane counter is independent of energy.
	NP	I6	Option to punch final energies, fluxes, and errors. 0 is punch; 1 is no punch.
3	F1	E12.5	Energy cutoff for proton ionization in the methane counter.
	RC1	E12.5	Relative carbon-to-hydrogen atom ratio in the methane counter (0.25).
	RN1	E12.5	Relative nitrogen-to-hydrogen atom ratio for the methane counter.
	F2	E12.5	Energy cutoff for proton ionization in the hydrogen counter.
	RC2	E12.5	Carbon-to-hydrogen ratio, hydrogen counter.
	RN2	E12.5	Nitrogen-to-hydrogen ratio, hydrogen counter.
4	CPFT(L,1)	4E16.5	Third-order polynomial coefficients which fit the variation of Legendre amplitudes of the non-Gaussian response function with energy.
5	CPFT(L,2)	4E16.5	
6	CPFT(L,3)	4E16.5	
7	CPFT(L,4)	4E16.5	
8	CLHYD(L)	4E16.5	Amplitudes of the four Legendre polynomials that parametrize the non-Gaussian response function of the hydrogen counter.
9 ^a	EFB(I,J)	18A4	Set identifier, voltage, and/or counter type.
10 ^a	INCH(J)	I6	First data channel in the set to be used in the calculation. First data channel is always Channel 1.
	M(J)	I6	Total number of datum points in the set; also the last channel number.
	A(J)	E12.5	Energy (keV) per channel.
	C(J)	E12.5	Time (sec) multiplied by hydrogen atom number (units of 10^{24})
11 ^a ...	RAPS(I,J)	6F12.0	I equals 1 to M(J). Experimental pulse-height spectrum; transformed to the raw proton spectrum.

^aCards from 9 and on are used for additional sets up to MN. MN may be as high as 20 and the maximum M(J) is 128.

TABLE D.II. Other Variables Used in PSNS

Variable	Description
CHEN(I,J)	A(J) times the channel number plus the cutoff energy. CHEN is the energy (keV) of each datum point.
MNOLL(J)	Minimum nonoverlapping limits for each set; used in establishing limits for response-function integration.
PRAPS(I,J)	Proton spectrum corrected for response-function effects.
EDEN(I)	Energy points with fixed logarithmic spacing within energy limits of the data.
ERFL(I)	Absolute flux error (rms).
FLXL(1,I)	Corrected flux per lethargy using PRAPS(I,J).
FLXL(2,I)	Raw flux per lethargy using RAPS(I,J).
HPCOR(I)	Fractional correction to FLXL(1,I) from recoil events involving carbon and nitrogen.
CLEG(I)	Amplitudes of corresponding Legendre polynomials. These amplitudes are energy-dependent for the methane counter but not for the hydrogen counter.
PLEG(I)	Legendre polynomials defined in the interval 0 to 1. Only the first four are used.

112-9120

Fig. 35. General Flow Chart for PSNS Code

112-9121 Rev. 1

Fig. 36. PSNS Flow Chart A for Response-function Integration

112-9122

Fig. 37. PSNS Flow Chart B for Slope-forming Routine

1. Printout for Program PSNS

PROGRAM PSNS
 REVISION OF AUGUST 1967
 PROTON SPECTRUM TO NEUTRON SPECTRUM

```

    DIMENSION RAPS(128,20),PRAPS(128,20),CHEN(128,20),EFR(20,18),PROB
    X(18),INCH(20),M(20),A(20),C(20),F(20),RC(20),RN(20),HPCOR(500),
    XFLXL(2,500),ERFL(500),EDEN(500),MNOLL(20),CLHYD(6),CPFT(6,6)
    COMMON FLXL,ERFL
    X,CLEG(6)
    4 FORMAT(2I6,2E12,5)
    300 FORMAT(4E16,5)
    733 FORMAT(6E12,4)
    2 FORMAT(6E12,5)
    55 FORMAT(///18A4)
    200 FORMAT(1H 16,E17,5,E18.5,E20,5)
    58 FORMAT(3I6,3F6.0,E12.5,16)
    5 FORMAT(18A4)
    100 FORMAT(6(//),3X31HPROGRAM OUTPUT IS LISTED BELOW. 4(//),3X8HCHAN.
    299 FORMAT(3(//),3X46HTHE SPECTRUM COMPUTATION INCREMENT WAS CHOSEN
    59 FORMAT(6F12,0)
    X/ 3X38HTHE SLOPE=TAKING HALF INTERVAL USED IS F6,3,2X 4HPLUS /
    X58HCHANNEL ENERGY RAW PROTON DIST, CORRECTED PROTON DIST, /)
    X 4(//),3X23HAW CUT-OFF ENERGY OF F6,3, 2X20HKEV FOR THE METHANE
    X/3X 6HTO BE F6,3,2X15HLETHARGY UNITS. ///
    X / 3X14HBEEN ASSUMED. /
    XF8.3,2X53HTIMES THE RECIPROCAL SQUARE ROOT OF ENERGY (IN KEV ),
    X / 3X 5HWAS F6,4,2X41HAND THE RATIO OF NITROGEN TO HYDROGEN WAS
    X 3(//),3X54HTHE RATIO OF CARBON TO HYDROGEN IN THE METHANE COUNTER
    X/3XF6.4,1X45H. THESE RATIOS FOR THE HYDROGEN COUNTER WERE F6,4,
    400 FORMAT(3(//),2X1HN2X13HRAW FLUX/LTY,2X11HEDIT ENERGY 3X 9HFLUX/LTY.
    X / 3X 5HAND F6,4,2X13HRESPECTIVELY, )
    500 FORMAT(18H INITIAL CHANNEL =14,10X15HFINAL CHANNEL =14,/(10F7.0))
    X3X10HFLUX ERROR 2X12HHEAVY RECOIL/(14,5E13,4))
    501 FORMAT(1(//),1H 11X28H(ENERGY(KEV) PER CHANNEL) = 18XE12,3,/1H 11X
    X46HC(TIME(SEC) X H-ATOM NO, (10 TO THE 24)) = E12,3,/ )
    21 FORMAT(1H1 10X28HANL=RP393X (PSNS) AUG, 1967 )
    X / 3X13HCOUNTER AND F6,3, 2X33HKEV FOR THE HYDROGEN COUNTER HAS
    X / 3X56HPROPORTIONAL COUNTERS, VARIOUS CORRECTIONS ARE APPLIED
    22 FORMAT(3(//),3X 48HPROGRAM TO PROCESS PROTON RECOIL DATA FROM
    X / 3X46HINCIDENT NEUTRON FLUX SPECTRUM IS DERIVED. )
    X / 3X56HTO REMOVE NON-IDEAL COUNTER RESPONSE EFFECTS AND THE
    23 FORMAT(3(//),3X20HTHIS PROBLEM IS FOR )
    24 FORMAT(3(//),3X11HATOTAL OF 14,2X29HSETS OF DATA (INCLUDING BOTH
    X/ 3X58HVOLTAGE AND COUNTER CHANGES) WERE ACCUMULATED FOR THIS
    X/ 3X22HEXPERIMENT. THE FIRST 14,2X28HSETS REFER TO THE PREDOMINA-
    X/ 3X52HNTLY METHANE COUNTER COVERING THE HIGHER ENERGIES, )
    25 FORMAT(3(//),3X 14,2X41HSETS (IN ORDER OF DECREASING ENERGY) HAVE
    X/ 3X55HBEEN PRE-CORRECTED FOR ALL NON-IDEAL RESPONSE EFFECTS, )
    26 FORMAT(3(//),3X47HTHE ENERGY LIMIT FOR THE METHANE COUNTER BELOW
    X/ 3X57HWHICH THE RESPONSE FUNCTION IS NOT DEPENDENT UPON ENERGY
    X/3X 4HIS E12,5,2X42HKEV. THE PREDOMINANTLY HYDROGEN COUNTER IS
    X/ 3X58HNOT USED AT ENERGIES SUFFICIENTLY HIGH TO PERMIT THE
    X/ 3X37HRESPONSE TO BECOME ENERGY DEPENDENT, )
    27 FORMAT(3(//),3X47HTHE FOUR POLYNOMIAL TERMS USED TO EXPRESS THE
    X/ 3X57HENERGY DEPENDENCE OF THE AMPLITUDE OF THE FOUR LEGENDRE
    X/ 3X57HFUNCTIONS WHICH PARAMETRIZE THE ENERGY DEPENDENT RESPONSE
    X/ 3X37HFUNCTION FOR THE METHANE COUNTER ARE / )
  
```

```

16 FORMAT(3(/),3X47HTHE FOUR LEGENDRE FUNCTION AMPLITUDES USED TO
X/ 3X57HPARAMETRIZE THE RESPONSE FUNCTION FOR THE PREDOMINANTLY
X/ 3X27HHYDROGEN FILLED COUNTER ARE / )
17 FORMAT(6(/),3X33HA SUMMARY OF INPUT DATA FOLLOWS. / )
35 READ 5,(PR08(I),I=1,18)
   IF(E0F,60)36,37
37 READ 58,MN,NMET,NUC,RA,RB,DINC,ELIMIT,NP
   READ 2,F1,RC1,RN1,F2,RC2,RN2
   READ 300,((CPFT(L,K),L=1,4),K=1,4)
   READ 300,(CLHYD(L),L=1,4)
   PRINT 21
   PRINT 22
   PRINT 23
   PRINT 5,(PR08(I),I=1,18)
   PRINT 24,MN,NMET
   PRINT 25,NUC
   PRINT 299,DINC,RA,RB,F1,F2,RC1,RN1,RC2,RN2
   PRINT 26,ELIMIT
   PRINT 27
   PRINT 300,((CPFT(L,K),L=1,4),K=1,4)
   PRINT 16
   PRINT 300,(CLHYD(L),L=1,4)
   PRINT 17
   DO 10 J=1,MN
     READ 5,(EFB(J,K),K=1,18)
     PRINT 55,(EFB(J,K),K=1,18)
     READ 4,INCH(J),M(J),A(J),C(J)
     PRINT 501,A(J),C(J)
     MAXCHAN = M(J)
     INCHAN = INCH(J)
     READ 59,(RAPS(I,J),I=1,MAXCHAN)
     PRINT 500,INCH(J),M(J),(RAPS(I,J),I=INCHAN,MAXCHAN)
C
C   FORM CHANNEL ENERGY AND PROTON ENERGY DISTRIBUTION
C
     AA = A(J)
     IF(J=NMET)409,409,411
409 FF=F1
     GO TO 412
411 FF=F2
412 DO 10 I=1,MAXCHAN
     CHEN(I,J) = AA*I + FF
     10 RAPS(I,J) = 1.0/(C(J)*AA)*RAPS(I,J)
C
C   DEFINE MINIMUM NON-OVERLAPPING LIMITS (MNOLL(J)) FOR EACH SET
C
     J = 1
63 K = INCH(J)
     IF(J+1-MN)40,40,62
40 IF(CHEN(K,J)=CHEN(M(J+1),J+1))67,68,68
67 K = K + 1
     IF(K=M(J))40,68,68
68 MNOLL(J) = K
     J = J + 1
     GO TO 63
62 MNOLL(MN) = INCH(MN)
C
C   RESPONSE FUNCTION INTEGRATION FOR BOTH COUNTERS ALLOWING ENERGY
C   DEPENDENCE ON THE METHANE COUNTER AT HIGHER ENERGIES
C

```

```

      DO 170 I=1,MN
      KA = INCH(I)
      K0 = M(I)
      DO 170 K=KA ,K0
170  PRAPS(K,I) = RAPS(K,I)
      I = 1+NUC
      79 N = M(I)
      90 J = 1
      X = 0.0
      86 MNLL = MN0LL(J)
      L = M(J) - 1
      88 ARG = CHEN(N,I)/CHEN(L,J)
      IF(I=NMET)191,191,192
192  DO 193 KL=1,4
193  CLEG(KL)=CLHYD(KL)
      GO TO 194
191  IF(CHEN(L,J)=ELIMIT)97,98,98
      97 U = ELIMIT
      GO TO 95
      98 U = CHEN(L,J)
      95 DO 96 KL=1,4
      96 CLEG(KL)=CPFT(1,KL)+U*(CPFT(2,KL)+U*(CPFT(3,KL)+U*(CPFT(4,KL))))
194  S=(CHEN(L+1,J)-CHEN(L,J))*0.5*(PRAPS(L,J)+PRAPS(L+1,J))/CHEN(L,J)
      DO 99 KL=1,4
      99 X = X + S*CLEG(KL)*PLEG(KL,ARG)
      L = L - 1
      IF(I=1)80,93,80
      93 IF(N - M(1))80,84,80
      84 X = 0.0
      80 IF(I - J)92,91,92
      92 IF(MNLL=L)88,88,81
      91 IF(N=L)88,88,89
      81 ARG = CHEN(N,I)/CHEN(M(J+1),J+1)
      IF(I=NMET)181,181,182
182  DO 183 KL=1,4
183  CLEG(KL)=CLHYD(KL)
      GO TO 184
181  IF(CHEN(MNLL,J) = ELIMIT)76,83,83
      76 U = ELIMIT
      GO TO 78
      83 U = CHEN(MNLL,J)
      78 DO 71 KL=1,4
      71 CLEG(KL)=CPFT(1,KL)+U*(CPFT(2,KL)+U*(CPFT(3,KL)+U*(CPFT(4,KL))))
184  S=0.5*(PRAPS(M(J+1),J+1)+PRAPS(MNLL,J))*(CHEN(MNLL,J)-CHEN(M(J+1),
      X J+1))/CHEN(M(J+1),J+1)
      DO 72 KL=1,4
      72 X = X + S*CLEG(KL)*PLEG(KL,ARG)
      J = J + 1
      GO TO 86
      89 PRAPS(N,I) = (RAPS(N,I) - X)/(1.-CLEG(1))
      N = N - 1
      IF(INCH(I)=N)90,90,77
      77 I = I + 1
      IF(I=MN)79,79,14

```

C
C
C

FORM EDIT ENERGIES HAVING FIXED LOGARITHMIC SPACING

```

14 EDEN(1) = CHEN(M(1),1)/(1.+(RA+RB/SQRTF(CHEN(M(1),1)))*1.01)
   TEST = CHEN(INCH(MN),MN)
   DO 50 I=2,500

```

```

EDEN(I) = EDEN(I=1)/(1.* DINC)
ED = EDEN(I)
IF(ED-ED*(RA + RB/SQRTF(ED))*1.01 - TEST)15,15,50
50 CONTINUE
15 NED = I=1
DO 201 I = 1,NED
  WPCOR(I)=0.0
  ERFL(I) = 0.0
  FLXL(2,I) = 0.0
201 FLXL(1,I) = 0.0

C
C  DEFINE RANGE OF DATA FOR SLOPE-TAKING
C
  J=1
699 K = 1
601 IF(CHEN(M(J),J) = EDEN(K))602,602,603
602 K = K + 1
  GO TO 601
604 K = K + 1
603 EDIT = EDEN(K)
  TEST1 = EDIT * EDIT*(RA + RB/SQRTF(EDIT))
  TEST2 = EDIT * EDIT*(RA + RB/SQRTF(EDIT))
  IF(CHEN(M(J),J) = TEST1)604,604,605
605 IF(CHEN(INCH(J),J) = TEST2)606,28,28
606 ITEST = M(J)
623 IF(CHEN(ITEST,J) = TEST1)670,669,669
669 ITEST = ITEST - 1
  GO TO 623
670 N2 = ITEST + 1
  ITEST = INCH(J)
614 IF(CHEN(ITEST,J) = TEST2)615,615,616
615 ITEST = ITEST + 1
  GO TO 614
616 N1 = ITEST - 1

C
C  DETERMINE SLOPES AND ERRORS
C
  L = N1
  T1 = 0.0
  T2 = 0.0
  T3 = 0.0
  T4 = 0.0
  TP1 = 0.0
  TP3 = 0.0
  TP5 = 0.0
687 T1 = T1 + CHEN(L,J)*PRAPS(L,J)
  T2 = T2 + CHEN(L,J)
  T3 = T3 + PRAPS(L,J)
  T4 = T4 + CHEN(L,J)**2
  TP1 = TP1 + CHEN(L,J)*RAPS(L,J)
  TP3 = TP3 + RAPS(L,J)
  TP5 = TP5 + RAPS(L,J)*CHEN(L,J)**2
  L = L + 1
  IF(N2 = L)688,687,687
688 T2SQ = T2**2
  V = N2 - N1 + 1
  SLOP1 = (T1*V - T2*T3)/(T4*V - T2SQ)
  SLOP2 = (TP1*V - T2*TP3)/(T4*V - T2SQ)
  ERSI = SQRTF((TP5*V*V+TP3*T2SQ-2,*TP1*T2*V)/(C(J)*A(J)))/(T4*V-T2SQ)

```

```

      X *PRAPS(N1,J)/RAPS(N1,J)
      SIGE = 11010./(EDIT*4041.) + 2387./(EDIT+135,5)
      TEMP = EDIT**2/SIGE
C
C  STORE FLUXES AND ERRORS WITH REGARD TO OVERLAP
C
      IF(ERFL(K))38,39,38
38  DEN = 1./((ERFL(K)**2) + 1./((TEMP*ERSL)**2))
      FLXL(1,K) = (FLXL(1,K)/(ERFL(K)**2) - TEMP*SLOP1/((TEMP*ERSL)**2))
      X/DEN
      FLXL(2,K) = (FLXL(2,K)/(ERFL(K)**2) - TEMP*SLOP2/((TEMP*ERSL)**2))
      X/DEN
      ERFL(K) = 1./SQRT(DEN)
      GO TO 41
39  FLXL(1,K) = -TEMP*SLOP1
      FLXL(2,K) = *TEMP * SLOP2
      ERFL(K) = TEMP*ERSL
C
C  CORRECT FOR CARBON AND NITROGEN RECOILS
C
41  ET = 5.*EDIT
      IF(ET - EDEN(1))101,102,102
101  DO 70 I = 1,NED
      IF(ET - EDEN(I))70,70,405
405  IF(ERFL(I))105,105,401
401  IF(ERFL(I-1))105,105,103
103  FEE5 = FLXL(1,I-1)+(ET-EDEN(I-1))/(EDEN(I)-EDEN(I-1))*(FLXL(1,I)
      X -FLXL(1,I-1))
      GO TO 105
70  CONTINUE
102  FEE5 = 0.0
105  IF(J=NMET)402,402,403
402  RC=RC1
      RN=RN1
      GO TO 404
403  RC=RC2
      RN=RN2
404  H = (RC*(5520./(ET+1150.)) + RN*(1750./(ET+250.)))/SIGE
      FLXL(1,K) = FLXL(1,K)-H*FEE5
      IF(HPCOR(K))406,406,407
406  HPCOR(K)=H*FEE5/FLXL(1,K)
      GO TO 604
407  HPCOR(K) = .5*(HPCOR(K)+H*FEE5/FLXL(1,K))
      GO TO 604
28  J = J + 1
      IF(J - MN)699,699,29
C
C  DELETE EMPTY DATA LOCATIONS
C
29  L = 1
506  IF(ERFL(L))503,504,503
503  L = L + 1
      IF(L - NED)506,506,505
504  NED = NED + 1
      DO 502 K = L,NED
      ERFL(K) = ERFL(K + 1)
      FLXL(1,K) = FLXL(1,K+1)
      FLXL(2,K) = FLXL(2,K+1)
      HPCOR(K)=HPCOR(K+1)

```

```

502 EDEN(K) = EDEN(K+1)
    GO TO 29
C
C   PRINT AND PUNCH RESULTS
C
505 PRINT 100
    J=1
    33 I = M(J)
    31 PRINT 200, I, CHEN(I, J), RAPS(I, J), PRAPS(I, J)
    I=I+1
    IF(I=INCH(J)) 32, 31, 31
    32 J = J+1
    IF(MN-J) 34, 33, 33
    34 PRINT 400, (I, FLXL(2, I), EDEN(I), FLXL(1, I), ERFL(I), HPGOR(I), I=1, NED)
    IF(NP) 73, 73, 35
    73 I = 1
    74 PUNCH 733, EDEN(I), FLXL(1, I), ERFL(I), EDEN(I+1), FLXL(1, I+1), ERFL(I+1)
    I = I + 2
    IF(I=NED) 74, 75, 35
    75 PUNCH 733, EDEN(I), FLXL(1, I), ERFL(I)
    GO TO 35
    36 CONTINUE
    END

```

2. Printout for Function PLEG(N, ARG)

```

    FUNCTION PLEG(N, ARG)
    GO TO (1, 2, 3, 4), N
    1 PLEG = 1,
    RETURN
    2 PLEG = (2, *ARG = 1, ) * 1.73205
    RETURN
    3 PLEG = (6, *ARG * (ARG - 1, ) + 1, ) * 2.23607
    RETURN
    4 PLEG = (-1, *ARG * (12, *ARG * (30, -ARG * 20, ))) * 2.64575
    RETURN
    END

```

3. Printout of PSNS Output

ANL-RP393X (PSNS) AUG, 1967

PROGRAM TO PROCESS PROTON RECOIL DATA FROM
PROPORTIONAL COUNTERS. VARIOUS CORRECTIONS ARE APPLIED
TO REMOVE NON-IDEAL COUNTER RESPONSE EFFECTS AND THE
INCIDENT NEUTRON FLUX SPECTRUM IS DERIVED.

THIS PROBLEM IS FOR
PSNS TEST PROBLEM, TYPICAL FAST REACTOR SPECTRUM

A TOTAL OF 11 SETS OF DATA (INCLUDING BOTH VOLTAGE AND COUNTER CHANGES) WERE ACCUMULATED FOR THIS EXPERIMENT. THE FIRST 3 SETS REFER TO THE PREDOMINANTLY METHANE COUNTER COVERING THE HIGHER ENERGIES.

1 SETS (IN ORDER OF DECREASING ENERGY) HAVE BEEN PRE-CORRECTED FOR ALL NON-IDEAL RESPONSE EFFECTS.

THE SPECTRUM COMPUTATION INCREMENT WAS CHOSEN TO BE 0.050 LETHARGY UNITS.

THE SLOPE-TAKING HALF INTERVAL USED IS 0.050 PLUS 0.150 TIMES THE RECIPROCAL SQUARE ROOT OF ENERGY (IN KEV).

A W CUT-OFF ENERGY OF 4.000 KEV FOR THE METHANE COUNTER AND 0.290 KEV FOR THE HYDROGEN COUNTER HAS BEEN ASSUMED.

THE RATIO OF CARBON TO HYDROGEN IN THE METHANE COUNTER WAS 0.2500 AND THE RATIO OF NITROGEN TO HYDROGEN WAS 0.0100 . THESE RATIOS FOR THE HYDROGEN COUNTER WERE 0.0250 AND 0.0250 RESPECTIVELY.

THE ENERGY LIMIT FOR THE METHANE COUNTER BELOW WHICH THE RESPONSE FUNCTION IS NOT DEPENDENT UPON ENERGY IS $5.52800E-02$ KEV, THE PREDOMINANTLY HYDROGEN COUNTER IS NOT USED AT ENERGIES SUFFICIENTLY HIGH TO PERMIT THE RESPONSE TO BECOME ENERGY DEPENDENT.

THE FOUR POLYNOMIAL TERMS USED TO EXPRESS THE ENERGY DEPENDENCE OF THE AMPLITUDE OF THE FOUR LEGENDRE FUNCTIONS WHICH PARAMETRIZE THE ENERGY DEPENDENT RESPONSE FUNCTION FOR THE METHANE COUNTER ARE

-0.25384E-01	0.58488E-03	-0.18894E-06	0.29466E-10
0.29185E 00	-0.14058E-03	-0.11469E-06	0.21748E-10
-0.20740E 00	0.59950E-03	-0.38492E-06	0.73273E-10
-0.23164E 00	0.42971E-03	-0.25025E-06	0.45359E-10

THE FOUR LEGENDRE FUNCTION AMPLITUDES USED TO PARAMETRIZE THE RESPONSE FUNCTION FOR THE PREDOMINANTLY HYDROGEN FILLED COUNTER ARE

0.21800E 00	0.87000E-01	-0.12900E-01	-0.69100E-01
-------------	-------------	--------------	--------------

TABLE D.III. PSNS Input Data

CH4 COUNTER AT 3200V, TEST PROBLEM

A(ENERGY(KEV) PER CHANNEL) = 1.7900E 01
 C(TIME(SEC) X H=ATOM NO, (10 TO THE 24)) = 5.3900E 02
 INITIAL CHANNEL = 31 FINAL CHANNEL = 128

92269. 85163. 79500. 73561. 68113. 64069. 60191. 56389. 53282. 49572.
 46839. 43747. 42031. 39676. 37561. 35192. 33735. 32279. 31210. 30187.
 29044. 27523. 26795. 26060. 24870. 23825. 22770. 22294. 21882. 20522.
 20049. 19541. 18958. 18017. 17798. 16815. 16544. 16216. 15568. 15277.
 15248. 14659. 14082. 13977. 13559. 13229. 12638. 12273. 12145. 11774.
 11525. 11243. 11150. 10948. 10887. 10590. 10342. 9972. 9631. 9566.
 9754. 9621. 9558. 9191. 8942. 8685. 8362. 8398. 8018. 8025.
 7755. 7950. 7409. 7508. 6998. 7089. 6846. 6803. 6486. 6252.
 6173. 6066. 6091. 5883. 5812. 5783. 5853. 5364. 5546. 5763.
 5346. 5326. 5132. 4822. 5274. 5190. 5067. 4885.

CH4 COUNTER AT 3500V, TEST PROBLEM

A(ENERGY(KEV) PER CHANNEL) = 1.0270E 01
 C(TIME(SEC) X H=ATOM NO, (10 TO THE 24)) = 1.2350E 02
 INITIAL CHANNEL = 19 FINAL CHANNEL = 64

80906. 74217. 68142. 62743. 58312. 53624. 50561. 47054. 43334. 40572.
 38012. 35385. 33007. 31325. 29028. 27852. 26135. 24328. 22766. 21969.
 20814. 19581. 18635. 17688. 16684. 16022. 15274. 14506. 13740. 13190.
 12466. 12068. 11193. 10972. 10609. 10016. 9525. 9186. 8786. 8470.
 8044. 7704. 7410. 7192. 6772. 6485.

CH4 COUNTER AT 3750V, TEST PROBLEM

A(ENERGY(KEV) PER CHANNEL) = 5.3500E 00
 C(TIME(SEC) X H=ATOM NO, (10 TO THE 24)) = 1.3300E 02
 INITIAL CHANNEL = 19 FINAL CHANNEL = 64

118315. 110072. 101906. 94413. 87902. 81725. 76573. 72758. 68772. 64953.
 61714. 58923. 55367. 52384. 49885. 47327. 45636. 43408. 41651. 39930.
 38487. 36862. 35214. 33892. 32520. 31105. 29951. 29038. 27811. 26999.
 25555. 24802. 23846. 22970. 22149. 21769. 20873. 20126. 19439. 18888.
 18092. 17547. 16911. 16639. 15948. 15280.

H2 COUNTER AT 3250V, TEST PROBLEM

A(ENERGY(KEV) PER CHANNEL) = 2.1700E 00
 C(TIME(SEC) X H=ATOM NO, (10 TO THE 24)) = 8.7400E 01
 INITIAL CHANNEL = 19 FINAL CHANNEL = 60

94804. 90134. 85301. 81014. 76850. 73415. 70615. 68100. 64561. 62370.
 59590. 56992. 55211. 52509. 50589. 48866. 46669. 44850. 43409. 41784.
 40334. 38914. 37687. 36856. 34984. 34054. 33057. 32154. 31595. 30578.
 29428. 28676. 27741. 26371. 26022. 25196. 24324. 23886. 23130. 22080.
 22019. 21296.

TABLE D.III. (Contd.)

H2 COUNTER AT 3400V. TEST PROBLEM

A(ENERGY(KEV) PER CHANNEL) = 1.2900E 00
 C(TIME(SEC) X H=ATOM NO, (10 TO THE 24)) = 8.6800E 01
 INITIAL CHANNEL = 19 FINAL CHANNEL = 60

88423, 84817, 79706, 75763, 73483, 71852, 69845, 68220, 65525, 63663,
 61611, 59939, 58159, 56028, 54473, 53373, 51031, 50120, 47898, 46861,
 45041, 44294, 43051, 42454, 41000, 40475, 39477, 38365, 37236, 36070,
 34956, 34609, 33384, 32731, 31953, 30418, 29704, 29368, 28230, 27632,
 26896, 26438,

H2 COUNTER AT 3550V. TEST PROBLEM

A(ENERGY(KEV) PER CHANNEL) = 7.4800E-01
 C(TIME(SEC) X H=ATOM NO, (10 TO THE 24)) = 9.6200E 01
 INITIAL CHANNEL = 19 FINAL CHANNEL = 60

88190, 85373, 81798, 79652, 77223, 74833, 72873, 70799, 68622, 66736,
 64474, 62818, 61157, 59283, 57493, 55614, 53511, 51916, 50270, 49118,
 48479, 47436, 46592, 46288, 45882, 44822, 44656, 43172, 42436, 42013,
 40810, 40358, 39164, 38237, 37655, 36983, 35633, 34952, 34416, 33591,
 32810, 32337,

H2 COUNTER AT 3700V. TEST PROBLEM

A(ENERGY(KEV) PER CHANNEL) = 4.3600E=01
 C(TIME(SEC) X H=ATOM NO, (10 TO THE 24)) = 8.5500E 01
 INITIAL CHANNEL = 25 FINAL CHANNEL = 60

57074, 54944, 53481, 52084, 50663, 49361, 48384, 47248, 46491, 45385,
 43706, 43329, 42449, 41648, 41007, 40391, 39544, 39186, 38183, 38120,
 36786, 36595, 35496, 34852, 34389, 34039, 33216, 32662, 31997, 31335,
 30735, 29837, 29760, 28862, 27989, 27451,

H2 COUNTER AT 3850V. TEST PROBLEM

A(ENERGY(KEV) PER CHANNEL) = 2.4000E=01
 C(TIME(SEC) X H=ATOM NO, (10 TO THE 24)) = 1.1240E 02
 INITIAL CHANNEL = 25 FINAL CHANNEL = 60

58108, 57186, 56082, 54853, 53600, 52749, 51596, 51319, 49950, 49826,
 49038, 48183, 47444, 47144, 46343, 45404, 44886, 44331, 43245, 42985,
 42138, 41327, 40738, 40291, 39213, 38735, 38166, 37665, 37413, 36133,
 35631, 34982, 34419, 33929, 33697, 32893,

TABLE D.III. (Contd.)

H2 COUNTER AT 4000V. TEST PROBLEM

A(ENERGY(KEV) PER CHANNEL) = 1.3080E-01
 C(TIME(SEC) X H-ATOM NO, (10 TO THE 24)) = 1.1110E 02
 INITIAL CHANNEL = 25 FINAL CHANNEL = 60
 41082, 40631, 40145, 39609, 38862, 38441, 38217, 37831, 36851, 36923,
 36300, 35480, 35395, 35243, 34775, 34241, 33855, 33588, 33497, 33026,
 32378, 32054, 32339, 31468, 30762, 30952, 30488, 30321, 29906, 29335,
 29001, 28652, 28368, 27911, 27499, 27158.

H2 COUNTER AT 4150V. TEST PROBLEM

A(ENERGY(KEV) PER CHANNEL) = 6.9300E-02
 C(TIME(SEC) X H-ATOM NO, (10 TO THE 24)) = 1.0630E 02
 INITIAL CHANNEL = 25 FINAL CHANNEL = 60
 25821, 25824, 25244, 25266, 25017, 24657, 24659, 24382, 23789, 23865,
 23701, 23239, 22964, 23073, 22476, 22444, 22174, 22076, 22003, 22164,
 21674, 21576, 21437, 21145, 21483, 20890, 20874, 20659, 20414, 20233,
 20275, 19986, 19785, 19602, 19291, 19157.

H2 COUNTER AT 4300V. TEST PROBLEM

A(ENERGY(KEV) PER CHANNEL) = 3.6900E-02
 C(TIME(SEC) X H-ATOM NO, (10 TO THE 24)) = 1.0550E 02
 INITIAL CHANNEL = 25 FINAL CHANNEL = 60
 18169, 17904, 17357, 17122, 17128, 17022, 16505, 16457, 16129, 16275,
 16205, 16139, 15847, 16003, 15593, 15452, 15267, 15166, 14943, 14601,
 14943, 14826, 14614, 14695, 14427, 14532, 14028, 14060, 14075, 14060,
 14068, 13725, 13775, 13774, 13272, 13716.

TABLE D.IV. PSNS Output Data

CHAN.	CHANNEL ENERGY	RAW PROTON DIST.	CORRECTED PROTON DIST.
128	2.29520E 03	5.06317E-01	5.06317E-01
127	2.27730E 03	5.25181E-01	5.25181E-01
126	2.25940E 03	5.37930E-01	5.37930E-01
125	2.24150E 03	5.46636E-01	5.46636E-01
124	2.22360E 03	4.99788E-01	4.99788E-01
123	2.20570E 03	5.31918E-01	5.31918E-01
122	2.18780E 03	5.52026E-01	5.52026E-01
121	2.16990E 03	5.54099E-01	5.54099E-01
120	2.15200E 03	5.97320E-01	5.97320E-01
119	2.13410E 03	5.74828E-01	5.74828E-01
118	2.11620E 03	5.55965E-01	5.55965E-01
117	2.09830E 03	6.06648E-01	6.06648E-01
116	2.08040E 03	5.99393E-01	5.99393E-01
115	2.06250E 03	6.02399E-01	6.02399E-01
114	2.04460E 03	6.09758E-01	6.09758E-01
113	2.02670E 03	6.31316E-01	6.31316E-01
112	2.00880E 03	6.28725E-01	6.28725E-01
111	1.99090E 03	6.39815E-01	6.39815E-01
110	1.97300E 03	6.48003E-01	6.48003E-01
109	1.95510E 03	6.72257E-01	6.72257E-01
108	1.93720E 03	7.05113E-01	7.05113E-01
107	1.91930E 03	7.09570E-01	7.09570E-01
106	1.90140E 03	7.34756E-01	7.34756E-01
105	1.88350E 03	7.25324E-01	7.25324E-01
104	1.86560E 03	7.78185E-01	7.78185E-01
103	1.84770E 03	7.67923E-01	7.67923E-01
102	1.82980E 03	8.23997E-01	8.23997E-01
101	1.81190E 03	8.03785E-01	8.03785E-01
100	1.79400E 03	8.31770E-01	8.31770E-01
99	1.77610E 03	8.31045E-01	8.31045E-01
98	1.75820E 03	8.70431E-01	8.70431E-01
97	1.74030E 03	8.66699E-01	8.66699E-01
96	1.72240E 03	9.00177E-01	9.00177E-01
95	1.70450E 03	9.26815E-01	9.26815E-01
94	1.68660E 03	9.52623E-01	9.52623E-01
93	1.66870E 03	9.90662E-01	9.90662E-01
92	1.65080E 03	9.97191E-01	9.97191E-01
91	1.63290E 03	1.01098E 00	1.01098E 00
90	1.61500E 03	9.91491E-01	9.91491E-01
89	1.59710E 03	9.98228E-01	9.98228E-01
88	1.57920E 03	1.03357E 00	1.03357E 00
87	1.56130E 03	1.07192E 00	1.07192E 00
86	1.54340E 03	1.09763E 00	1.09763E 00
85	1.52550E 03	1.12841E 00	1.12841E 00
84	1.50760E 03	1.13473E 00	1.13473E 00
83	1.48970E 03	1.15567E 00	1.15567E 00
82	1.47180E 03	1.16531E 00	1.16531E 00
81	1.45390E 03	1.19454E 00	1.19454E 00
80	1.43600E 03	1.22034E 00	1.22034E 00
79	1.41810E 03	1.25880E 00	1.25880E 00
78	1.40020E 03	1.27206E 00	1.27206E 00
77	1.38230E 03	1.30990E 00	1.30990E 00
76	1.36440E 03	1.37115E 00	1.37115E 00

TABLE D.IV. (Contd.)

75	1.34650E 03	1.40535E 00	1.40535E 00
74	1.32860E 03	1.44868E 00	1.44868E 00
73	1.31070E 03	1.45956E 00	1.45956E 00
72	1.29280E 03	1.51937E 00	1.51937E 00
71	1.27490E 03	1.58042E 00	1.58042E 00
70	1.25700E 03	1.58342E 00	1.58342E 00
69	1.23910E 03	1.61358E 00	1.61358E 00
68	1.22120E 03	1.68075E 00	1.68075E 00
67	1.20330E 03	1.71474E 00	1.71474E 00
66	1.18540E 03	1.74283E 00	1.74283E 00
65	1.16750E 03	1.84472E 00	1.84472E 00
64	1.14960E 03	1.86741E 00	1.86741E 00
63	1.13170E 03	1.96495E 00	1.96495E 00
62	1.11380E 03	2.02537E 00	2.02537E 00
61	1.09590E 03	2.07803E 00	2.07803E 00
60	1.07800E 03	2.12705E 00	2.12705E 00
59	1.06010E 03	2.26801E 00	2.26801E 00
58	1.04220E 03	2.31071E 00	2.31071E 00
57	1.02430E 03	2.36005E 00	2.36005E 00
56	1.00640E 03	2.46940E 00	2.46940E 00
55	9.88500E 02	2.57771E 00	2.57771E 00
54	9.70600E 02	2.70105E 00	2.70105E 00
53	9.52700E 02	2.77723E 00	2.77723E 00
52	9.34800E 02	2.85269E 00	2.85269E 00
51	9.16900E 02	3.01033E 00	3.01033E 00
50	8.99000E 02	3.12880E 00	3.12880E 00
49	8.81100E 02	3.23483E 00	3.23483E 00
48	8.63200E 02	3.34563E 00	3.34563E 00
47	8.45300E 02	3.49654E 00	3.49654E 00
46	8.27400E 02	3.64756E 00	3.64756E 00
45	8.09500E 02	3.89310E 00	3.89310E 00
44	7.91600E 02	4.11231E 00	4.11231E 00
43	7.73700E 02	4.35640E 00	4.35640E 00
42	7.55800E 02	4.53426E 00	4.53426E 00
41	7.37900E 02	4.85474E 00	4.85474E 00
40	7.20000E 02	5.13801E 00	5.13801E 00
39	7.02100E 02	5.52254E 00	5.52254E 00
38	6.84200E 02	5.84457E 00	5.84457E 00
37	6.66300E 02	6.23864E 00	6.23864E 00
36	6.48400E 02	6.64058E 00	6.64058E 00
35	6.30500E 02	7.05973E 00	7.05973E 00
34	6.12600E 02	7.62440E 00	7.62440E 00
33	5.94700E 02	8.23997E 00	8.23997E 00
32	5.76800E 02	8.82692E 00	8.82692E 00
31	5.58900E 02	9.56344E 00	9.56344E 00
64	6.61280E 02	5.11296E 00	5.21594E 00
63	6.51010E 02	5.33924E 00	5.54290E 00
62	6.40740E 02	5.67038E 00	5.91988E 00
61	6.30470E 02	5.84226E 00	6.06942E 00
60	6.20200E 02	6.07406E 00	6.29742E 00
59	6.09930E 02	6.34212E 00	6.56952E 00
58	5.99660E 02	6.67800E 00	6.92709E 00
57	5.89390E 02	6.92714E 00	7.15982E 00
56	5.79120E 02	7.24251E 00	7.47568E 00

TABLE D.IV. (Contd.)

55	5.68850E 02	7.50979E 00	7.72021E 00
54	5.58580E 02	7.89691E 00	8.11752E 00
53	5.48310E 02	8.36445E 00	8.63121E 00
52	5.38040E 02	8.65065E 00	8.92242E 00
51	5.27770E 02	8.82489E 00	9.06002E 00
50	5.17500E 02	9.51476E 00	9.87369E 00
49	5.07230E 02	9.82856E 00	1.01793E 01
48	4.96960E 02	1.03994E 01	1.08174E 01
47	4.86690E 02	1.08330E 01	1.12642E 01
46	4.76420E 02	1.14370E 01	1.19271E 01
45	4.66150E 02	1.20425E 01	1.25809E 01
44	4.55880E 02	1.26322E 01	1.32023E 01
43	4.45610E 02	1.31542E 01	1.37229E 01
42	4.35340E 02	1.39457E 01	1.45887E 01
41	4.25070E 02	1.46924E 01	1.53799E 01
40	4.14800E 02	1.54382E 01	1.61553E 01
39	4.04530E 02	1.64104E 01	1.72139E 01
38	3.94260E 02	1.73210E 01	1.81716E 01
37	3.83990E 02	1.79494E 01	1.87387E 01
36	3.73720E 02	1.91809E 01	2.00869E 01
35	3.63450E 02	2.06056E 01	2.16633E 01
34	3.53180E 02	2.19593E 01	2.31147E 01
33	3.42910E 02	2.28865E 01	2.39737E 01
32	3.32640E 02	2.46975E 01	2.59745E 01
31	3.22370E 02	2.60237E 01	2.72943E 01
30	3.12100E 02	2.78986E 01	2.93036E 01
29	3.01830E 02	2.99698E 01	3.15240E 01
28	2.91560E 02	3.19882E 01	3.36209E 01
27	2.81290E 02	3.41658E 01	3.58729E 01
26	2.71020E 02	3.70988E 01	3.90571E 01
25	2.60750E 02	3.98638E 01	4.19356E 01
24	2.50480E 02	4.22787E 01	4.42714E 01
23	2.40210E 02	4.59749E 01	4.82156E 01
22	2.29940E 02	4.94684E 01	5.17757E 01
21	2.19670E 02	5.37251E 01	5.62206E 01
20	2.09400E 02	5.85149E 01	6.12160E 01
19	1.99130E 02	6.37887E 01	6.66701E 01
64	3.46400E 02	2.14743E 01	2.20407E 01
63	3.41050E 02	2.24130E 01	2.32890E 01
62	3.35700E 02	2.33842E 01	2.43776E 01
61	3.30350E 02	2.37664E 01	2.46778E 01
60	3.25000E 02	2.46603E 01	2.56494E 01
59	3.19650E 02	2.54262E 01	2.64417E 01
58	3.14300E 02	2.65449E 01	2.76908E 01
57	3.08950E 02	2.73192E 01	2.84715E 01
56	3.03600E 02	2.82847E 01	2.94949E 01
55	2.98250E 02	2.93346E 01	3.06174E 01
54	2.92900E 02	3.05938E 01	3.20025E 01
53	2.87550E 02	3.11278E 01	3.24138E 01
52	2.82200E 02	3.22816E 01	3.36362E 01
51	2.76850E 02	3.35128E 01	3.49448E 01
50	2.71500E 02	3.48563E 01	3.63845E 01
49	2.66150E 02	3.59146E 01	3.74287E 01
48	2.60800E 02	3.79439E 01	3.97385E 01

TABLE D.IV. (Contd.)

47	2.55450E 02	3.90851E 01	4.08458E 01
46	2.50100E 02	4.08095E 01	4.27049E 01
45	2.44750E 02	4.20926E 01	4.39548E 01
44	2.39400E 02	4.37144E 01	4.56311E 01
43	2.34050E 02	4.57031E 01	4.77647E 01
42	2.28700E 02	4.76312E 01	4.97853E 01
41	2.23350E 02	4.94892E 01	5.16802E 01
40	2.18000E 02	5.18052E 01	5.41466E 01
39	2.12650E 02	5.40890E 01	5.65288E 01
38	2.07300E 02	5.61169E 01	5.85319E 01
37	2.01950E 02	5.85356E 01	6.10122E 01
36	1.96600E 02	6.10049E 01	6.35133E 01
35	1.91250E 02	6.41361E 01	6.68382E 01
34	1.85900E 02	6.65126E 01	6.91062E 01
33	1.80550E 02	7.01075E 01	7.29305E 01
32	1.75200E 02	7.36196E 01	7.65692E 01
31	1.69850E 02	7.78118E 01	8.10258E 01
30	1.64500E 02	8.28094E 01	8.64456E 01
29	1.59150E 02	8.67318E 01	9.03307E 01
28	1.53800E 02	9.12838E 01	9.49480E 01
27	1.48450E 02	9.66510E 01	1.00523E 02
26	1.43100E 02	1.02253E 02	1.06265E 02
25	1.37750E 02	1.07614E 02	1.11541E 02
24	1.32400E 02	1.14855E 02	1.19138E 02
23	1.27050E 02	1.23536E 02	1.28415E 02
22	1.21700E 02	1.32686E 02	1.38038E 02
21	1.16350E 02	1.43217E 02	1.49172E 02
20	1.11000E 02	1.54693E 02	1.61183E 02
19	1.05650E 02	1.66278E 02	1.72919E 02
60	1.30490E 02	1.12286E 02	1.19356E 02
59	1.28320E 02	1.16096E 02	1.23813E 02
58	1.26150E 02	1.16420E 02	1.23486E 02
57	1.23980E 02	1.21956E 02	1.29807E 02
56	1.21810E 02	1.25943E 02	1.34114E 02
55	1.19640E 02	1.28252E 02	1.36248E 02
54	1.17470E 02	1.32850E 02	1.41279E 02
53	1.15300E 02	1.37205E 02	1.45962E 02
52	1.13130E 02	1.39045E 02	1.47396E 02
51	1.10960E 02	1.46269E 02	1.55676E 02
50	1.08790E 02	1.51198E 02	1.60969E 02
49	1.06620E 02	1.55164E 02	1.64981E 02
48	1.04450E 02	1.61227E 02	1.71624E 02
47	1.02280E 02	1.66589E 02	1.77310E 02
46	1.00110E 02	1.69537E 02	1.79851E 02
45	9.79400E 01	1.74298E 02	1.84658E 02
44	9.57700E 01	1.79555E 02	1.90037E 02
43	9.36000E 01	1.84458E 02	1.94899E 02
42	9.14300E 01	1.94329E 02	2.06035E 02
41	8.92600E 01	1.98710E 02	2.10062E 02
40	8.70900E 01	2.05180E 02	2.16678E 02
39	8.49200E 01	2.12667E 02	2.24503E 02
38	8.27500E 01	2.20312E 02	2.32427E 02
37	8.05800E 01	2.28880E 02	2.41418E 02
36	7.84100E 01	2.36478E 02	2.49049E 02

TABLE D.IV. (Contd.)

35	7.62400E 01	2.46069E 02	2.59098E 02
34	7.40700E 01	2.57653E 02	2.71545E 02
33	7.19000E 01	2.66738E 02	2.80630E 02
32	6.97300E 01	2.76862E 02	2.90874E 02
31	6.75600E 01	2.91108E 02	3.06195E 02
30	6.53900E 01	3.00499E 02	3.15090E 02
29	6.32200E 01	3.14197E 02	3.29274E 02
28	6.10500E 01	3.28855E 02	3.44428E 02
27	5.88800E 01	3.40408E 02	3.55337E 02
26	5.67100E 01	3.59067E 02	3.75038E 02
25	5.45400E 01	3.72328E 02	3.87495E 02
24	5.23700E 01	3.87092E 02	4.01535E 02
23	5.02000E 01	4.05203E 02	4.19403E 02
22	4.80300E 01	4.27158E 02	4.41911E 02
21	4.58600E 01	4.49762E 02	4.64653E 02
20	4.36900E 01	4.75245E 02	4.90488E 02
19	4.15200E 01	4.99868E 02	5.14549E 02
60	7.76900E 01	2.36113E 02	2.47168E 02
59	7.64000E 01	2.40203E 02	2.51668E 02
58	7.51100E 01	2.46776E 02	2.58672E 02
57	7.38200E 01	2.52117E 02	2.64054E 02
56	7.25300E 01	2.62280E 02	2.75549E 02
55	7.12400E 01	2.65281E 02	2.77825E 02
54	6.99500E 01	2.71657E 02	2.84369E 02
53	6.86600E 01	2.85366E 02	3.00216E 02
52	6.73700E 01	2.92314E 02	3.07329E 02
51	6.60800E 01	2.98146E 02	3.12941E 02
50	6.47900E 01	3.09086E 02	3.25003E 02
49	6.35000E 01	3.12185E 02	3.26950E 02
48	6.22100E 01	3.22134E 02	3.37579E 02
47	6.09200E 01	3.32547E 02	3.48698E 02
46	5.96300E 01	3.42630E 02	3.59283E 02
45	5.83400E 01	3.52561E 02	3.69557E 02
44	5.70500E 01	3.61474E 02	3.78409E 02
43	5.57600E 01	3.66163E 02	3.81749E 02
42	5.44700E 01	3.79148E 02	3.95583E 02
41	5.31800E 01	3.84480E 02	3.99500E 02
40	5.18900E 01	3.95581E 02	4.10676E 02
39	5.06000E 01	4.02252E 02	4.16055E 02
38	4.93100E 01	4.18506E 02	4.33548E 02
37	4.80200E 01	4.27768E 02	4.41929E 02
36	4.67300E 01	4.47612E 02	4.63662E 02
35	4.54400E 01	4.55748E 02	4.70213E 02
34	4.41500E 01	4.76664E 02	4.92901E 02
33	4.28600E 01	4.86488E 02	5.01158E 02
32	4.15700E 01	5.00375E 02	5.14390E 02
31	4.02800E 01	5.19407E 02	5.33942E 02
30	3.89900E 01	5.35304E 02	5.49192E 02
29	3.77000E 01	5.50236E 02	5.62914E 02
28	3.64100E 01	5.68562E 02	5.80666E 02
27	3.51200E 01	5.85191E 02	5.95910E 02
26	3.38300E 01	6.09260E 02	6.20294E 02
25	3.25400E 01	6.23772E 02	6.32049E 02
24	3.12500E 01	6.41696E 02	6.47782E 02

TABLE D.IV. (Contd.)

23	2.99600E 01	6.56262E 02	6.58829E 02
22	2.86700E 01	6.76625E 02	6.76894E 02
21	2.73800E 01	7.11839E 02	7.13438E 02
20	2.60900E 01	7.57484E 02	7.62560E 02
19	2.48000E 01	7.89689E 02	7.93540E 02
60	4.51700E 01	4.49390E 02	4.59972E 02
59	4.44220E 01	4.55963E 02	4.67164E 02
58	4.36740E 01	4.66817E 02	4.78586E 02
57	4.29260E 01	4.78282E 02	4.90719E 02
56	4.21780E 01	4.85730E 02	4.97646E 02
55	4.14300E 01	4.95194E 02	5.07082E 02
54	4.06820E 01	5.13955E 02	5.20314E 02
53	3.99340E 01	5.23294E 02	5.37385E 02
52	3.91860E 01	5.31382E 02	5.44764E 02
51	3.84380E 01	5.44265E 02	5.58176E 02
50	3.76900E 01	5.60858E 02	5.76208E 02
49	3.69420E 01	5.67140E 02	5.80926E 02
48	3.61940E 01	5.83858E 02	5.98871E 02
47	3.54460E 01	5.89736E 02	6.02819E 02
46	3.46980E 01	5.99964E 02	6.12216E 02
45	3.39500E 01	6.20588E 02	6.34757E 02
44	3.32020E 01	6.22895E 02	6.33721E 02
43	3.24540E 01	6.37625E 02	6.48449E 02
42	3.17060E 01	6.43268E 02	6.51412E 02
41	3.09580E 01	6.47492E 02	6.52456E 02
40	3.02100E 01	6.59222E 02	6.62987E 02
39	2.94620E 01	6.73716E 02	6.76912E 02
38	2.87140E 01	6.82596E 02	6.83510E 02
37	2.79660E 01	6.98606E 02	6.99079E 02
36	2.72180E 01	7.21480E 02	7.23223E 02
35	2.64700E 01	7.43646E 02	7.46206E 02
34	2.57220E 01	7.72872E 02	7.77912E 02
33	2.49740E 01	7.98984E 02	8.05268E 02
32	2.42260E 01	8.23860E 02	8.30648E 02
31	2.34780E 01	8.49903E 02	8.57092E 02
30	2.27300E 01	8.72986E 02	8.79293E 02
29	2.19820E 01	8.96000E 02	9.00937E 02
28	2.12340E 01	9.27435E 02	9.32834E 02
27	2.04860E 01	9.53645E 02	9.57469E 02
26	1.97380E 01	9.83899E 02	9.86672E 02
25	1.89900E 01	1.01272E 03	1.01339E 03
24	1.82420E 01	1.03996E 03	1.03740E 03
23	1.74940E 01	1.07317E 03	1.06834E 03
22	1.67460E 01	1.10693E 03	1.09919E 03
21	1.59980E 01	1.13675E 03	1.12417E 03
20	1.52500E 01	1.18643E 03	1.17361E 03
19	1.45020E 01	1.22558E 03	1.20844E 03
60	2.64500E 01	7.36386E 02	7.34824E 02
59	2.60140E 01	7.50818E 02	7.51801E 02
58	2.55780E 01	7.74237E 02	7.78239E 02
57	2.51420E 01	7.98326E 02	8.05392E 02
56	2.47060E 01	8.00392E 02	8.04260E 02
55	2.42700E 01	8.24481E 02	8.31187E 02
54	2.38340E 01	8.40576E 02	8.47732E 02

TABLE D.IV. (Contd.)

53	2.33980E 01	8.58335E 02	8.66252E 02
52	2.29620E 01	8.76174E 02	8.84710E 02
51	2.25260E 01	8.91035E 02	8.99189E 02
50	2.20900E 01	9.13112E 02	9.22713E 02
49	2.16540E 01	9.22501E 02	9.29822E 02
48	2.12180E 01	9.34921E 02	9.40646E 02
47	2.07820E 01	9.52197E 02	9.57499E 02
46	2.03460E 01	9.81678E 02	9.89735E 02
45	1.99100E 01	9.86802E 02	9.90591E 02
44	1.94740E 01	1.02259E 03	1.03043E 03
43	1.90380E 01	1.02428E 03	1.02641E 03
42	1.86020E 01	1.05118E 03	1.05442E 03
41	1.81660E 01	1.06079E 03	1.06005E 03
40	1.77300E 01	1.08351E 03	1.08222E 03
39	1.72940E 01	1.10003E 03	1.09620E 03
38	1.68580E 01	1.11723E 03	1.11077E 03
37	1.64220E 01	1.13871E 03	1.13056E 03
36	1.59860E 01	1.16232E 03	1.15276E 03
35	1.55500E 01	1.17243E 03	1.15741E 03
34	1.51140E 01	1.21747E 03	1.20639E 03
33	1.46780E 01	1.24714E 03	1.23524E 03
32	1.42420E 01	1.26745E 03	1.25168E 03
31	1.38060E 01	1.29792E 03	1.28068E 03
30	1.33700E 01	1.32413E 03	1.30374E 03
29	1.29340E 01	1.35906E 03	1.33742E 03
28	1.24980E 01	1.39718E 03	1.37456E 03
27	1.20620E 01	1.43465E 03	1.41019E 03
26	1.16260E 01	1.47390E 03	1.44733E 03
25	1.11900E 01	1.53104E 03	1.50646E 03
60	1.46900E 01	1.21934E 03	1.19682E 03
59	1.44500E 01	1.24915E 03	1.23273E 03
58	1.42100E 01	1.25775E 03	1.23820E 03
57	1.39700E 01	1.27591E 03	1.25580E 03
56	1.37300E 01	1.29678E 03	1.27674E 03
55	1.34900E 01	1.32084E 03	1.30161E 03
54	1.32500E 01	1.33945E 03	1.31935E 03
53	1.30100E 01	1.38690E 03	1.37375E 03
52	1.27700E 01	1.39624E 03	1.37918E 03
51	1.25300E 01	1.41481E 03	1.39624E 03
50	1.22900E 01	1.43591E 03	1.41631E 03
49	1.20500E 01	1.45363E 03	1.43186E 03
48	1.18100E 01	1.49359E 03	1.47560E 03
47	1.15700E 01	1.51016E 03	1.48913E 03
46	1.13300E 01	1.53199E 03	1.50914E 03
45	1.10900E 01	1.56206E 03	1.53938E 03
44	1.08500E 01	1.59345E 03	1.57099E 03
43	1.06100E 01	1.60309E 03	1.57447E 03
42	1.03700E 01	1.64335E 03	1.61679E 03
41	1.01300E 01	1.66392E 03	1.63356E 03
40	9.89000E 00	1.68313E 03	1.64824E 03
39	9.65000E 00	1.71793E 03	1.68252E 03
38	9.41000E 00	1.74763E 03	1.70984E 03
37	9.17000E 00	1.75875E 03	1.71302E 03
36	8.93000E 00	1.78614E 03	1.73668E 03

TABLE D.IV. (Contd.)

35	8.69000E 00	1.81784E 03	1.76542E 03
34	8.45000E 00	1.84705E 03	1.79054E 03
33	8.21000E 00	1.85165E 03	1.78380E 03
32	7.97000E 00	1.90240E 03	1.83569E 03
31	7.73000E 00	1.91266E 03	1.83532E 03
30	7.49000E 00	1.95540E 03	1.87604E 03
29	7.25000E 00	1.98695E 03	1.90188E 03
28	7.01000E 00	2.03340E 03	1.94617E 03
27	6.77000E 00	2.07896E 03	1.98857E 03
26	6.53000E 00	2.11988E 03	2.02424E 03
25	6.29000E 00	2.15406E 03	2.05046E 03
60	8.13000E 00	1.86886E 03	1.79692E 03
59	8.00720E 00	1.89232E 03	1.82413E 03
58	7.87640E 00	1.92067E 03	1.85271E 03
57	7.74560E 00	1.95212E 03	1.88508E 03
56	7.61480E 00	1.97167E 03	1.90205E 03
55	7.48400E 00	1.99568E 03	1.92459E 03
54	7.35320E 00	2.01867E 03	1.94562E 03
53	7.22240E 00	2.05796E 03	1.98729E 03
52	7.09160E 00	2.08652E 03	2.01497E 03
51	6.96080E 00	2.09801E 03	2.02061E 03
50	6.83000E 00	2.12994E 03	2.05216E 03
49	6.69920E 00	2.11686E 03	2.02597E 03
48	6.56840E 00	2.16545E 03	2.07846E 03
47	6.43760E 00	2.22538E 03	2.14507E 03
46	6.30680E 00	2.20577E 03	2.10966E 03
45	6.17600E 00	2.22807E 03	2.12768E 03
44	6.04520E 00	2.27266E 03	2.17392E 03
43	5.91440E 00	2.30507E 03	2.20421E 03
42	5.78360E 00	2.31133E 03	2.20074E 03
41	5.65280E 00	2.32971E 03	2.21252E 03
40	5.52200E 00	2.35627E 03	2.23448E 03
39	5.39120E 00	2.39302E 03	2.26911E 03
38	5.26040E 00	2.42522E 03	2.29754E 03
37	5.12960E 00	2.43568E 03	2.29779E 03
36	4.99880E 00	2.44153E 03	2.29188E 03
35	4.86800E 00	2.49796E 03	2.35031E 03
34	4.73720E 00	2.54083E 03	2.39085E 03
33	4.60640E 00	2.53587E 03	2.36980E 03
32	4.47560E 00	2.60331E 03	2.44090E 03
31	4.34480E 00	2.62987E 03	2.45907E 03
30	4.21400E 00	2.64529E 03	2.46247E 03
29	4.08320E 00	2.67426E 03	2.48273E 03
28	3.95240E 00	2.72566E 03	2.53106E 03
27	3.82160E 00	2.76255E 03	2.56007E 03
26	3.69080E 00	2.79599E 03	2.58394E 03
25	3.56000E 00	2.82703E 03	2.60398E 03
60	4.44800E 00	2.60053E 03	2.42750E 03
59	4.37870E 00	2.61871E 03	2.44818E 03
58	4.30940E 00	2.66093E 03	2.49309E 03
57	4.24010E 00	2.68577E 03	2.51559E 03
56	4.17080E 00	2.71306E 03	2.54105E 03
55	4.10150E 00	2.75229E 03	2.58156E 03
54	4.03220E 00	2.74659E 03	2.56442E 03

TABLE D.IV. (Contd.)

53	3.96290E 00	2.77116E 03	2.58588E 03
52	3.89360E 00	2.80442E 03	2.61823E 03
51	3.82430E 00	2.83360E 03	2.64512E 03
50	3.75500E 00	2.83578E 03	2.63726E 03
49	3.68570E 00	2.91627E 03	2.72929E 03
48	3.61640E 00	2.87039E 03	2.65940E 03
47	3.54710E 00	2.91003E 03	2.69880E 03
46	3.47780E 00	2.92890E 03	2.71136E 03
45	3.40850E 00	2.94220E 03	2.71659E 03
44	3.33920E 00	3.00872E 03	2.78955E 03
43	3.26990E 00	2.98686E 03	2.74917E 03
42	3.20060E 00	2.99677E 03	2.74929E 03
41	3.13130E 00	3.01008E 03	2.75356E 03
40	3.06200E 00	3.04673E 03	2.78744E 03
39	2.99270E 00	3.05107E 03	2.77972E 03
38	2.92340E 00	3.13211E 03	2.86974E 03
37	2.85410E 00	3.11732E 03	2.83674E 03
36	2.78480E 00	3.15465E 03	2.87014E 03
35	2.71550E 00	3.21736E 03	2.93550E 03
34	2.64620E 00	3.23963E 03	2.94854E 03
33	2.57690E 00	3.22931E 03	2.91953E 03
32	2.50760E 00	3.30981E 03	3.00620E 03
31	2.43830E 00	3.34741E 03	3.03723E 03
30	2.36900E 00	3.34714E 03	3.01924E 03
29	2.29970E 00	3.39601E 03	3.06355E 03
28	2.23040E 00	3.42981E 03	3.08785E 03
27	2.16110E 00	3.42682E 03	3.06449E 03
26	2.09180E 00	3.50556E 03	3.14497E 03
25	2.02250E 00	3.50515E 03	3.12335E 03
60	2.50400E 00	3.52329E 03	3.27067E 03
59	2.46710E 00	3.40924E 03	3.12247E 03
58	2.43020E 00	3.53819E 03	3.27695E 03
57	2.39330E 00	3.53845E 03	3.26640E 03
56	2.35640E 00	3.52560E 03	3.23893E 03
55	2.31950E 00	3.61371E 03	3.34027E 03
54	2.28260E 00	3.61166E 03	3.32590E 03
53	2.24570E 00	3.61551E 03	3.31889E 03
52	2.20880E 00	3.61166E 03	3.30183E 03
51	2.17190E 00	3.60344E 03	3.27905E 03
50	2.13500E 00	3.73290E 03	3.43199E 03
49	2.09810E 00	3.70593E 03	3.38439E 03
48	2.06120E 00	3.77477E 03	3.45905E 03
47	2.02430E 00	3.75396E 03	3.41870E 03
46	1.98740E 00	3.80842E 03	3.47434E 03
45	1.95050E 00	3.83848E 03	3.49835E 03
44	1.91360E 00	3.75063E 03	3.37143E 03
43	1.87670E 00	3.83848E 03	3.46912E 03
42	1.83980E 00	3.89576E 03	3.52719E 03
41	1.80290E 00	3.92170E 03	3.54474E 03
40	1.76600E 00	3.96923E 03	3.58945E 03
39	1.72910E 00	4.00545E 03	3.61921E 03
38	1.69220E 00	4.11076E 03	3.73669E 03
37	1.65530E 00	4.07069E 03	3.66761E 03
36	1.61840E 00	4.14570E 03	3.74528E 03

TABLE D.IV. (Contd.)

35	1.58150E 00	4.16265E 03	3.74807E 03
34	1.54460E 00	4.18063E 03	3.75167E 03
33	1.50770E 00	4.14313E 03	3.68398E 03
32	1.47080E 00	4.22738E 03	3.77165E 03
31	1.43390E 00	4.23971E 03	3.76669E 03
30	1.39700E 00	4.37252E 03	3.91505E 03
29	1.36010E 00	4.39975E 03	3.92732E 03
28	1.32320E 00	4.39821E 03	3.90213E 03
27	1.28630E 00	4.45857E 03	3.95542E 03
26	1.24940E 00	4.59908E 03	4.11000E 03
25	1.21250E 00	4.66715E 03	4.17036E 03

N	RAW FLUX/LTY.	EDIT ENERGY	FLUX/LTY.	FLUX ERROR	HEAVY RECOIL
1	6.8667E 02	2.1783E 03	6.8667E 02	4.3104E 01	0.0000E-01
2	6.8660E 02	2.0746E 03	6.8660E 02	4.3919E 01	0.0000E-01
3	9.7301E 02	1.9758E 03	9.7301E 02	4.0819E 01	0.0000E-01
4	1.1102E 03	1.8817E 03	1.1102E 03	4.2206E 01	0.0000E-01
5	1.0148E 03	1.7921E 03	1.0148E 03	3.9571E 01	0.0000E-01
6	1.0145E 03	1.7068E 03	1.0145E 03	4.1155E 01	0.0000E-01
7	7.9464E 02	1.6255E 03	7.9464E 02	3.8311E 01	0.0000E-01
8	8.4231E 02	1.5481E 03	8.4231E 02	4.0068E 01	0.0000E-01
9	7.8965E 02	1.4744E 03	7.8965E 02	3.6928E 01	0.0000E-01
10	1.0196E 03	1.4042E 03	1.0196E 03	3.4181E 01	0.0000E-01
11	1.0624E 03	1.3373E 03	1.0624E 03	3.6392E 01	0.0000E-01
12	9.2458E 02	1.2736E 03	9.2458E 02	3.9477E 01	0.0000E-01
13	9.9794E 02	1.2130E 03	9.9794E 02	3.1317E 01	0.0000E-01
14	1.1146E 03	1.1552E 03	1.1146E 03	3.4031E 01	0.0000E-01
15	1.1339E 03	1.1002E 03	1.1339E 03	3.1611E 01	0.0000E-01
16	1.1457E 03	1.0478E 03	1.1457E 03	3.4830E 01	0.0000E-01
17	1.1661E 03	9.9791E 02	1.1661E 03	3.2529E 01	0.0000E-01
18	1.2432E 03	9.5039E 02	1.2432E 03	3.0391E 01	0.0000E-01
19	1.1662E 03	9.0513E 02	1.1662E 03	2.7920E 01	0.0000E-01
20	1.2617E 03	8.6203E 02	1.2617E 03	3.1871E 01	0.0000E-01
21	1.4957E 03	8.2098E 02	1.4957E 03	2.9544E 01	0.0000E-01
22	1.6385E 03	7.8189E 02	1.6385E 03	3.5030E 01	0.0000E-01
23	1.7155E 03	7.4465E 02	1.7155E 03	3.2938E 01	0.0000E-01
24	1.8532E 03	7.0920E 02	1.8532E 03	3.0816E 01	0.0000E-01
25	1.8427E 03	6.7542E 02	1.8427E 03	2.9006E 01	0.0000E-01
26	2.0189E 03	6.4326E 02	2.0189E 03	2.7501E 01	0.0000E-01
27	2.0217E 03	6.1263E 02	2.0221E 03	2.9652E 01	0.0000E-01
28	1.8414E 03	5.8346E 02	1.8520E 03	6.9597E 01	0.0000E-01
29	1.8298E 03	5.5567E 02	1.8982E 03	6.5943E 01	0.0000E-01
30	1.8121E 03	5.2921E 02	1.9388E 03	6.1158E 01	0.0000E-01
31	1.9768E 03	5.0401E 02	2.1407E 03	5.8255E 01	0.0000E-01
32	1.9049E 03	4.8001E 02	2.0312E 03	5.3973E 01	0.0000E-01
33	1.9873E 03	4.5715E 02	2.1114E 03	6.2273E 01	0.0000E-01
34	2.0103E 03	4.3538E 02	2.0867E 03	5.8191E 01	2.0912E-02
35	2.0064E 03	4.1465E 02	2.0670E 03	5.3748E 01	2.1370E-02
36	2.0785E 03	3.9491E 02	2.1303E 03	5.0774E 01	2.9461E-02
37	2.1346E 03	3.7610E 02	2.2002E 03	5.9436E 01	3.2641E-02
38	2.2480E 03	3.5819E 02	2.3318E 03	5.5393E 01	2.8338E-02

TABLE D.IV. (Contd.)

39	2.1726E 03	3.4114E 02	2.2042E 03	5.2225E 01	3.0034E-02
40	2.1428E 03	3.2489E 02	2.1870E 03	3.8402E 01	2.3790E-02
41	2.2251E 03	3.0942E 02	2.2527E 03	3.5996E 01	2.4267E-02
42	2.2085E 03	2.9469E 02	2.2357E 03	4.2587E 01	2.2518E-02
43	2.2224E 03	2.8065E 02	2.2310E 03	3.9465E 01	2.9571E-02
44	2.1635E 03	2.6729E 02	2.1587E 03	3.6018E 01	3.2679E-02
45	2.1904E 03	2.5456E 02	2.1606E 03	3.6584E 01	2.8144E-02
46	2.1707E 03	2.4244E 02	2.1401E 03	3.1449E 01	3.0528E-02
47	2.2636E 03	2.3089E 02	2.2356E 03	3.1256E 01	3.2939E-02
48	2.2066E 03	2.1990E 02	2.1578E 03	2.8848E 01	3.4523E-02
49	1.9652E 03	2.0943E 02	1.9362E 03	5.9763E 01	3.9927E-02
50	1.9368E 03	1.9945E 02	1.8940E 03	5.5260E 01	4.1624E-02
51	2.0380E 03	1.8996E 02	2.0161E 03	5.1553E 01	4.1657E-02
52	2.1491E 03	1.8091E 02	2.1727E 03	3.7682E 01	3.6353E-02
53	2.1924E 03	1.7230E 02	2.2250E 03	4.3704E 01	3.8478E-02
54	2.1032E 03	1.6409E 02	2.0880E 03	5.3217E 01	4.8559E-02
55	2.0313E 03	1.5628E 02	1.9779E 03	3.7314E 01	5.6123E-02
56	1.9804E 03	1.4884E 02	1.9020E 03	4.5470E 01	6.1120E-02
57	1.9409E 03	1.4175E 02	1.8584E 03	4.1659E 01	6.7471E-02
58	2.1339E 03	1.3500E 02	2.0969E 03	2.9601E 01	5.9506E-02
59	2.3552E 03	1.2857E 02	2.3542E 03	3.6529E 01	5.7956E-02
60	2.2732E 03	1.2245E 02	2.2688E 03	2.9917E 01	3.4055E-02
61	2.2281E 03	1.1662E 02	2.2173E 03	2.7582E 01	3.2605E-02
62	2.2005E 03	1.1106E 02	2.3360E 03	5.5756E 01	8.9143E-03
63	2.0310E 03	1.0578E 02	2.1174E 03	6.1154E 01	1.0111E-02
64	1.7310E 03	1.0074E 02	1.7401E 03	5.6168E 01	1.3630E-02
65	1.8223E 03	9.5941E 01	1.8550E 03	6.4001E 01	1.2212E-02
66	1.8818E 03	9.1373E 01	1.9348E 03	5.8723E 01	1.2228E-02
67	1.8529E 03	8.7022E 01	1.8999E 03	5.4213E 01	1.2374E-02
68	1.8355E 03	8.2878E 01	1.8776E 03	5.0080E 01	1.2476E-02
69	1.8868E 03	7.8931E 01	1.9382E 03	4.6442E 01	1.2523E-02
70	1.8182E 03	7.5172E 01	1.8576E 03	5.3880E 01	1.3571E-02
71	1.8682E 03	7.1593E 01	1.9113E 03	3.0811E 01	1.4369E-02
72	1.7996E 03	6.8184E 01	1.8274E 03	3.4831E 01	1.4451E-02
73	1.7632E 03	6.4937E 01	1.7692E 03	3.2414E 01	1.4714E-02
74	1.6764E 03	6.1845E 01	1.6702E 03	2.9774E 01	1.6068E-02
75	1.6185E 03	5.8900E 01	1.5965E 03	3.4623E 01	1.6734E-02
76	1.4451E 03	5.6095E 01	1.3735E 03	2.6869E 01	1.8913E-02
77	1.3943E 03	5.3424E 01	1.3222E 03	2.4644E 01	1.8869E-02
78	1.4633E 03	5.0880E 01	1.4159E 03	2.6954E 01	1.7861E-02
79	1.5179E 03	4.8457E 01	1.4938E 03	2.4784E 01	1.7079E-02
80	1.4692E 03	4.6149E 01	1.4347E 03	2.2725E 01	1.8666E-02
81	1.3745E 03	4.3952E 01	1.3392E 03	3.7395E 01	1.9501E-02
82	1.4049E 03	4.1859E 01	1.3698E 03	2.6889E 01	1.8053E-02
83	1.3903E 03	3.9866E 01	1.3571E 03	2.8246E 01	1.8367E-02
84	1.2426E 03	3.7967E 01	1.1817E 03	2.5717E 01	2.2172E-02
85	1.1562E 03	3.6159E 01	1.0672E 03	2.5529E 01	2.5724E-02
86	1.0190E 03	3.4437E 01	9.0543E 02	2.3242E 01	3.0289E-02
87	8.5779E 02	3.2798E 01	7.1344E 02	2.2583E 01	3.5345E-02
88	7.1822E 02	3.1236E 01	5.4954E 02	2.4455E 01	4.3299E-02
89	8.2682E 02	2.9748E 01	7.0534E 02	2.2459E 01	3.3079E-02
90	1.0552E 03	2.8332E 01	1.0100E 03	2.0767E 01	2.3436E-02
91	1.1851E 03	2.6983E 01	1.1776E 03	1.9119E 01	2.3192E-02
92	1.2850E 03	2.5698E 01	1.3160E 03	3.1491E 01	2.2819E-02

TABLE D.IV. (Contd.)

93	1.2314E	03	2.4474E	01	1.2249E	03	2.2474E	01	2.4654E-02
94	1.0823E	03	2.3309E	01	1.0430E	03	2.0597E	01	2.7945E-02
95	1.0031E	03	2.2199E	01	9.3413E	02	2.0136E	01	3.2485E-02
96	9.3368E	02	2.1142E	01	8.5610E	02	1.8446E	01	3.2080E-02
97	8.8100E	02	2.0135E	01	8.0179E	02	1.9514E	01	2.8414E-02
98	8.1228E	02	1.9176E	01	7.2058E	02	1.7845E	01	3.4016E-02
99	7.4849E	02	1.8263E	01	6.3604E	02	1.4893E	01	3.9315E-02
100	7.3382E	02	1.7393E	01	6.2359E	02	1.6086E	01	3.8918E-02
101	7.3079E	02	1.6565E	01	6.2845E	02	1.5644E	01	3.8684E-02
102	7.5968E	02	1.5776E	01	6.8811E	02	2.6487E	01	3.7345E-02
103	7.6002E	02	1.5025E	01	6.9318E	02	2.4325E	01	3.5684E-02
104	7.1860E	02	1.4309E	01	6.5291E	02	2.2201E	01	3.9068E-02
105	6.9646E	02	1.3628E	01	6.2644E	02	2.0561E	01	3.9063E-02
106	7.9205E	02	1.2979E	01	7.3917E	02	1.5952E	01	3.4363E-02
107	7.5607E	02	1.2361E	01	7.0447E	02	1.4652E	01	3.2864E-02
108	7.0998E	02	1.1772E	01	6.5603E	02	2.1254E	01	3.2834E-02
109	7.0332E	02	1.1212E	01	6.5340E	02	2.2595E	01	2.8479E-02
110	6.2620E	02	1.0678E	01	5.6148E	02	2.0722E	01	3.2036E-02
111	5.7718E	02	1.0169E	01	5.0334E	02	1.8914E	01	3.8363E-02
112	5.1466E	02	9.6852E	00	4.2823E	02	1.7321E	01	4.7634E-02
113	4.5237E	02	9.2240E	00	3.5530E	02	1.5764E	01	5.5253E-02
114	4.2211E	02	8.7848E	00	3.2415E	02	1.4411E	01	5.6735E-02
115	3.9234E	02	8.3665E	00	2.9525E	02	1.5469E	01	6.3828E-02
116	4.0404E	02	7.9681E	00	3.1598E	02	1.4191E	01	5.9120E-02
117	4.1639E	02	7.5886E	00	3.4248E	02	1.5537E	01	4.7618E-02
118	4.5501E	02	7.2273E	00	3.8586E	02	9.8011E	00	3.9597E-02
119	4.4871E	02	6.8831E	00	3.8849E	02	1.4196E	01	3.2245E-02
120	4.0553E	02	6.5553E	00	3.4192E	02	1.4519E	01	2.8916E-02
121	3.6539E	02	6.2432E	00	2.9857E	02	1.4952E	01	2.5729E-02
122	3.4983E	02	5.9459E	00	2.8384E	02	1.3673E	01	3.4866E-02
123	3.0504E	02	5.6627E	00	2.2715E	02	1.4157E	01	6.2270E-02
124	2.8608E	02	5.3931E	00	2.0713E	02	1.3031E	01	7.9700E-02
125	2.8526E	02	5.1363E	00	2.1013E	02	1.1881E	01	8.7712E-02
126	2.7200E	02	4.8917E	00	1.9973E	02	1.0807E	01	8.5932E-02
127	2.5254E	02	4.6588E	00	1.8418E	02	1.1361E	01	7.9505E-02
128	2.5383E	02	4.4369E	00	1.9131E	02	1.0394E	01	6.8668E-02
129	2.2602E	02	4.2256E	00	1.6427E	02	9.4615E	00	7.3405E-02
130	2.4878E	02	3.8328E	00	1.9623E	02	1.0483E	01	5.1815E-02
131	1.9954E	02	3.6503E	00	1.3843E	02	9.4725E	00	6.4992E-02
132	1.7087E	02	3.4764E	00	1.0694E	02	9.5018E	00	8.2600E-02
133	1.5994E	02	3.3109E	00	9.7692E	01	9.6460E	00	9.1331E-02
134	1.6811E	02	3.1532E	00	1.1267E	02	8.8350E	00	8.6692E-02
135	1.6051E	02	3.0031E	00	1.0759E	02	7.9993E	00	9.1467E-02
136	1.7982E	02	2.8601E	00	1.3590E	02	8.2195E	00	6.8287E-02
137	1.7196E	02	2.7239E	00	1.2941E	02	7.4643E	00	6.9066E-02
138	1.5372E	02	2.5942E	00	1.0718E	02	6.7644E	00	9.8179E-02
139	1.4426E	02	2.4706E	00	9.9326E	01	6.9928E	00	1.0103E-01
140	1.4760E	02	2.1342E	00	1.0118E	02	7.1663E	00	7.9184E-02
141	1.5115E	02	2.0326E	00	1.0882E	02	6.6248E	00	6.6008E-02
142	1.5439E	02	1.9358E	00	1.1692E	02	6.5309E	00	5.2287E-02
143	1.4678E	02	1.8436E	00	1.1155E	02	5.9265E	00	4.5547E-02
144	1.3055E	02	1.7558E	00	9.2509E	01	5.3691E	00	5.0138E-02
145	1.3637E	02	1.6722E	00	1.0238E	02	5.3835E	00	4.1375E-02
146	1.2249E	02	1.5926E	00	8.7935E	01	5.3494E	00	5.1593E-02
147	1.1915E	02	1.5168E	00	8.5666E	01	4.9450E	00	5.7463E-02

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