21, 215

RADIATIVE CORRECTIONS TO $P + P \rightarrow l^+ + l^- + \text{'ANYTHING'}$ AND
APPLICATION TO MUON - ELECTRON SYMMETRY

A. Soni

Columbia University, New York, N. Y. 10027

NOTICE

This report was prepared as an account of work sponsored by the United States Government. Neither the United States nor the United States Atomic Energy Commission, nor any of their employees, nor any of their contractors, subcontractors, or their employees, makes any warranty, express or implied, or assumes any legal liability or responsibility for the accuracy, completeness or usefulness of any information, apparatus, product or process disclosed, or represents that its use would not infringe privately owned rights.

This research was supported by the U. S. Atomic Energy Commission.

MASTER

DISTRIBUTION OF THIS DOCUMENT IS UNLIMITED

fy

DISCLAIMER

This report was prepared as an account of work sponsored by an agency of the United States Government. Neither the United States Government nor any agency Thereof, nor any of their employees, makes any warranty, express or implied, or assumes any legal liability or responsibility for the accuracy, completeness, or usefulness of any information, apparatus, product, or process disclosed, or represents that its use would not infringe privately owned rights. Reference herein to any specific commercial product, process, or service by trade name, trademark, manufacturer, or otherwise does not necessarily constitute or imply its endorsement, recommendation, or favoring by the United States Government or any agency thereof. The views and opinions of authors expressed herein do not necessarily state or reflect those of the United States Government or any agency thereof.

DISCLAIMER

Portions of this document may be illegible in electronic image products. Images are produced from the best available original document.

ABSTRACT

Radiative corrections, to order α^3 , are carried out to the lepton lines in the process $P + P \rightarrow l^+ + l^- + \text{'anything'}$. Expressions are derived which relate the radiative corrections, due to the emission of soft photons, and of hard photons from the lepton lines, to the observed cross section for production of electron or muon pairs. These expressions are used to predict the difference, based purely on electromagnetic interactions, between the cross sections for production of electron and muon pairs. As a specific example, computations are done using the BNL data for production of lepton pairs in hadron-hadron collisions.

RADIATIVE CORRECTIONS TO THE DECAY OF A VECTOR MESON^{1,2}

To perform the radiative corrections to the lepton lines, in the one photon exchange production of lepton pairs (Fig. 1) in the reaction,³

$$P + P \rightarrow l^+ + l^- + \text{'anything'}$$
(1)

we first carry out, for simplicity, the radiative corrections to the decay, into a lepton pair, of a massive vector meson of mass Q .

To the lowest order in α , the decay of the vector meson takes place, as shown in Fig. 2, with a transition probability per unit time, W_0 , given by

$$W_0 \rightarrow Q \propto \alpha/3$$
(2)

where α is the fine structure constant and terms of order m_l^2/Q^2 have been neglected. (m_l is the lepton mass)

The radiative corrections¹ to the process arise from diagrams (3) to (7). As usual, the contribution from real photon emission, Figs. 3 and 4, is divided into two regions, with photon energy $\omega < \omega_m$ (soft photon region), and $\omega > \omega_m$ (hard photon region). The contribution from the soft real photons is added to the virtual photon contribution arising from diagrams 5, 6 and 7 and the resultant correction to W_0 is given by (again letting $m_l^2/Q^2 \rightarrow 0$)

$$d_s(l) = -\frac{\alpha}{\pi} \left[4 \ln \frac{Q}{m_l} \ln \frac{Q}{2\omega_m} - 3 \ln \frac{Q}{m_l} - 2 \ln \frac{Q}{2\omega_m} \right. \\ \left. + 2 - \frac{\pi^2}{3} \right]$$
(3)

The contribution due to the real hard photons ($\omega > \omega_m$) is given by

$$\frac{dW_h}{dM^2} \rightarrow \frac{W_0}{Q^2} \frac{\alpha}{\pi} \frac{1+z^2}{1-z} \left[\ln \frac{M^2}{m_l^2} - 1 \right] \quad (4)$$

where M is the dilepton mass, and $z = M^2/Q^2$.

APPLICATIONS TO $P + P \rightarrow l^+ + l^- + \text{'anything'}$

We now apply the expressions given in the previous section to reaction (1). If $\frac{d\sigma_o}{dM^2}$ be the cross section for the process shown in Fig. 1, then the correction due to the soft photons changes it to $\frac{d\sigma_s(l)}{dM^2}$ given by

$$\frac{d\sigma_s(l)}{dM^2} = \frac{d\sigma_o}{dM^2} \left[1 + d_s(l) \right] \quad (5)$$

where $d_s(l)$ is given by (3) with $Q \approx M$ for $\omega_m \ll M$. If ω_m is the experimental energy resolution, then $\frac{d\sigma_s(l)}{dM^2}$ represents the cross section (to order α^3) for production of all lepton pairs which are not accompanied by any detectable photons emitted by the leptons.

The hard photon emission corrects $\frac{d\sigma_o}{dM^2}$ to $\frac{d\sigma_h(l)}{dM^2}$ given by, from (4),

$$\frac{d\sigma_h(l)}{dM^2} = \frac{\alpha}{\pi} \int_{(M+\omega_m)^2}^{Q_{\max}^2} \frac{d\sigma_o}{dQ^2} \frac{dQ^2}{Q^2} \frac{1+z^2}{1-z} \left[\ln \frac{M^2}{m_l^2} - 1 \right] \quad (6)$$

The total cross section, to order α^3 , is then given by

$$\frac{d\sigma_{\text{obs}}(l)}{dM^2} = \frac{d\sigma_s(l)}{dM^2} + \frac{d\sigma_h(l)}{dM^2}$$

and it is free of ω_m since the dependence on ω_m can be shown to cancel between the two contributions. We can invert this relation and obtain an expression for $\frac{d\sigma_o}{dM^2}$ in terms of $\frac{d\sigma_{\text{obs}}(l)}{dM^2}$, that is

$$\frac{d\sigma_o}{dM^2} = \left[1 - d_s^i(l) \right] \frac{d\sigma_{obs}(l)}{dM^2} - \frac{\alpha}{\pi} \left[\ln \frac{M^2}{m_l^2} - 1 \right] \times$$

$$\int_{M^2}^{Q_{max}^2} \frac{dQ^2}{Q^2 - M^2} \left[(1 + z^2) \frac{d\sigma_{obs}(l)}{dM^2} - \frac{2 d\sigma_{obs}(l)}{dM^2} \right] \quad (7)$$

where

$$d_s^i(l) = -\frac{\alpha}{\pi} \left\{ -3 \ln \frac{M}{m_l} + 2 - \frac{\pi^2}{3} \right\} + \frac{2\alpha}{\pi} \left(\ln \frac{M^2}{m_l^2} - 1 \right) \times$$

$$\ln \frac{Q_{max}^2 - M^2}{M^2} \quad (8)$$

The total radiative corrections $d_t(l)$, defined as,

$$d_t(l) = \frac{\frac{d\sigma_{obs}(l)}{dM^2} - \frac{d\sigma_o}{dM^2}}{\frac{d\sigma_o}{dM^2}} \quad (9)$$

is now simply obtained from (7) in terms of $\frac{d\sigma_{obs}(l)}{dM^2}$. Finally, we can use (7) to get an expression for $\frac{d\sigma_{obs}(l)}{dM^2}(l')$ and hence for $d_t(l')$ in terms of $\frac{d\sigma_{obs}(l)}{dM^2}$.

In Fig. 9 we show the total radiative corrections for electron and muon pairs. To do so the BNL data³ for muon pair production (reaction (1)) was used.

TEST OF MUON-ELECTRON SYMMETRY

The relations derived above yield simple expressions for testing muon-electron symmetry in hadronic collisions. For this purpose, we introduce asymmetry parameters ϵ_s and ϵ , as a measure of the difference, based purely on electromagnetic interactions, in the electron and muon pair production cross sections.

We define ϵ_s as

$$\epsilon_s = \frac{\frac{d\sigma_s(e)}{dM} - \frac{d\sigma_s(\mu)}{dM}}{\frac{d\sigma_s(e)}{dM} + \frac{d\sigma_s(\mu)}{dM}} \quad (10)$$

Using (5) and (3) we get

$$\epsilon_s \approx \frac{3\alpha}{2\pi} \ln \frac{m}{m_e} \left[1 - \frac{4}{3} \ln \frac{M}{2\omega_m} \right] \quad (11)$$

which is about -10% for $\frac{\omega_m}{M} = \frac{1}{2}\%$ and goes down to -4% for $\frac{\omega_m}{M} = 5\%$.

Similarly, we introduce another asymmetry parameter, ϵ , defined as

$$\epsilon = \frac{\frac{d\sigma_{\text{obs}}(e)}{dM} - \frac{d\sigma_{\text{obs}}(\mu)}{dM}}{\frac{d\sigma_{\text{obs}}(e)}{dM} + \frac{d\sigma_{\text{obs}}(\mu)}{dM}} \quad (12)$$

On substitution into (12) of the $o(\alpha^3)$ expressions for cross sections derived in the previous section, we get the lowest order in α contribution to ϵ . This⁴ contribution we term as ϵ_1 , and compute it using, again, the BNL data on reaction (I). Fig. 10 shows a graph of ϵ_1 . It is seen to vary considerably over the dilepton mass spectrum.

FURTHER REMARKS

1. Since the C asymmetric process for production of lepton pairs, via an interference between one photon exchange (C - odd lepton pair) and those two photon exchange diagrams which produce a C - even lepton pair, does not contribute to $\frac{d\sigma_{\text{obs}}}{dM}$, ϵ_1 is the complete contribution, to order α^3 in the cross section, to ϵ .

2. If the cross section for production of lepton pairs was assumed to scale,^{5,6}

that is, $\frac{d\sigma_o}{dT} = \frac{F(T)}{s}$, then from the expressions obtained in the previous sections

$$\begin{aligned}\epsilon_1 &\approx \frac{d\sigma(e) - d\sigma(\mu)}{2} \\ &= \ln \frac{m_\mu}{m_e} f(T)\end{aligned}\quad (13)$$

where $T = M/\sqrt{s}$, so that ϵ_1 also scales and the plot of ϵ_1 in Fig. 10 then holds for all values of $s \gg m_{\text{nucleon}}^2$.

3. α^4 CONTRIBUTION TO ϵ .

A notable contribution to ϵ comes from the two⁷photon exchange diagram of Fig. 8. The cross section, as $s \rightarrow \infty$, of this $\alpha(\alpha^4)$ process has a logarithmic dependence on the lepton mass and for $\frac{M^2}{s} \ll 1$ it contributes up to a few percent to ϵ . This contribution is termed⁸ $\epsilon^{2\gamma}$ and is shown in Fig. 10 for $s = 60 \text{ GeV}^2$ and is seen to be very small compared to ϵ_1 .

Unless s is very very large, the two photon exchange contribution to the asymmetry is never too large. For example, for $s = 3200 \text{ GeV}^2$, $\epsilon^{2\gamma}$ is $\sim +3\%$ for $T \approx 0.04$ ($M = 2 \text{ GeV}$) and drops to $+1\%$ for $T \approx .18$ ($M = 10 \text{ GeV}$). It is interesting to note that $\epsilon^{2\gamma}$ and ϵ_1 are of opposite signs, that is, the order α^3 radiative cross section for production of muon pairs in reaction (1) is greater than^{that} for production of electrons, but in order α^4 , the two photon process contribution (of Fig. 8) is just the other way round. $\epsilon^{2\gamma}$ also drops sharply with increase in M or T unlike ϵ_1 .

ACKNOWLEDGEMENT

I am sincerely grateful to Professor Norman Christ for his sponsorship of this problem.

REFERENCES AND FOOTNOTES

1. F. Ehlitzky and H. Mitter, *Nuovo Cimento*, Vol. LV A, N. 1; 181 (1968) and references therein.
2. This is to acknowledge the help of Professor L. M. Lederman in carrying out a similar calculation, namely, "Radiative corrections to the decay of an indefinite metric photon" (unpublished).
3. J. Christenson et al., *Phys. Rev. Lett.* 25, 1523 (1970)
4. That is, $\epsilon = \epsilon_1 + \dots$ contributions from higher order radiative corrections to the lepton pair production cross section.
5. J. D. Bjorken and E. A. Paschos, *Phys. Rev.* 185, 1975 (1969)
6. S. D. Drell and T. M. Yan, *Phys. Rev. Lett.* 25, 316 (1970)
7. K. Fujikawa, *Nuovo Cimento*, 12 A, 83 (1972), *ibid.*, 12 A, 117 (1972)
V. M. Budnev et al., *Phys. Letters* 39 B, 526 (1972)
A. Soni, Columbia University Preprint CO-2271-3
8. $\epsilon^{2\gamma}$ is computed in the last paper of reference 7.

FIGURE CAPTIONS

Figure 1. The reaction $P + P \rightarrow l^+ + l^- + \text{'anything'}$ in lowest order in α .

Figure 2. Decay of a massive vector meson into a lepton pair in lowest order in α .

Figures 3 to 7. Radiative corrections, to order α^3 , to the lepton lines in Figs. 1 and 2.

Figure 8. Two photon exchange production of α/C -even lepton pair in the reaction $P + P \rightarrow l^+ + l^- + \text{'anything'}$. The thick lines in the figure represent the unobserved final hadronic states.

Figure 9. Total radiative corrections, defined by equation (9), extracted from the BNL data of reference 3, are shown for production of electron and muon pairs.

Figure 10. ϵ_1 , the contribution from $\mathcal{O}(\alpha^3)$ cross section to the asymmetry ϵ (defined by equation (12)) is shown for the BNL experiment of reference 3 taking $s = 60 \text{ GeV}^2$. However, if the cross section is assumed to scale, then this curve should hold for all values of s (see equation (13)). For comparison, we also show $\epsilon^{2\gamma}$ for $s = 60 \text{ GeV}^2$ computed in the last paper of reference 7. This is the contribution to the μ -e asymmetry arising from Fig. 8 and calculated in the leading log s approximation as $s \rightarrow \infty$ holding M^2/s fixed.

Figs. 1 to 8

Fig. 9

Fig. 10