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THE BUOYANT SURFACE JET DISCHARGING NORMAL TO AN AMBIENT FLOW OF VARIOUS DEPTHS

H. H. Carter
E. W. Schiemer
R. Regier

Reference 73-7 February 1973

Technical Report 81

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CHESAPEAKE BAY INSTITUTE

THE JOHNS HOPKINS UNIVERSITY

TECHNICAL REPORT 81

THE BUOYANT SURFACE JET DISCHARGING NORMAL
TO AN AMBIENT FLOW OF VARIOUS DEPTHS

by

H.H. Carter, E.W. Schiemer, and R. Regier

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D.W. Pritchard
Director

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List of Symbols (*continued*)

W	width of system
θ	excess temperature
α	angle between jet centerline and x-axis
ρ	fluid density
β	normal entrainment coefficient
γ	tangential entrainment coefficient
σ	cross-sectional area

Subscripts

o	orifice
a	ambient
p	peak value
1/2	where value is one-half of peak value
q	refers to jet velocity profile
θ	refers to excess temperature profile
e	entrainment
b	background

List of Symbols (*continued*)

W	width of system
θ	excess temperature
α	angle between jet centerline and x-axis
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Subscripts

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e	entrainment
b	background

I. Introduction

For about a decade, investigators at the Chesapeake Bay Institute have been engaged in a research program under AEC sponsorship; its major objective is to improve our capability to predict the consequences of discharging waste heat and/or radioisotopes into the coastal environment. Our approach to this problem has been both experimental and theoretical. On the theoretical side, we have developed simple analytical and numerical models for the prediction of the distribution of excess heat¹ and/or radioisotopes discharged with condenser cooling water. These models have been described in Okubo (1964); Okubo (1967a,b,c,d); Okubo (1968a,b,c); Sastry and Okubo (1968); Okubo and Karweit (1969); Okubo and Pritchard (1969); Pritchard (1969); Pritchard (1971); and Pritchard and Carter (1972). Experimentally, the far field physical processes of advection and turbulent diffusion which must be included in the analytical models have been and are being studied through field studies and the near field mechanical mixing in two low velocity circulating water flumes located at the Institute and at Hydronautics, Inc., Laurel, Maryland. These studies have been reported in Pritchard and Carter (1965); Carter (1968); and Carter (1969).

The purpose of this report is to make available preliminary results of flume studies carried out between June, 1971 and June, 1972. *This report should not be considered final*; our analysis of the tests covered herein show only too clearly that some important features requiring definition were not included in the original program. In addition, as we became more familiar with

¹Here excess heat means the difference between the heat that a given parcel of water would contain under conditions of a heated discharge and the heat that it would contain under "natural" conditions.

our experimental setup, it was deemed necessary to make certain improvements to the flume prior to resuming our program of measurement. The shortcomings of the testing program and the flume alluded to here are described in greater detail in the appropriate section of the report. The point to be made is that we consider the interpretations of the data contained herein to be preliminary and subject to continuous reevaluation as the program progresses.

The phenomenon being studied is shown conceptually in Figure 1.

Figure 1. Schematic drawing of a buoyant surface three-dimensional bent-over jet.

Figure 2. Schematic of CBI low velocity flume.

II. The Experimental Setup and Tests

Our experiments were performed in a specially constructed low velocity water flume at Hydronautics, Inc., Laurel, Maryland. The flume has a free surface test section approximately 10 feet wide, 1 foot deep and 40 feet long. There are baffles in the upstream sump to distribute the flow evenly throughout the width of the flume before it enters the test section and a honeycomb at the upstream end to straighten the flow. The pump is a Worthington Axial Flow Type KLD discharging into a 12" pipe. It is driven by a 7.5 HP Boston Ratiotrol (variable speed drive) motor. The combination of pump and motor is capable of producing water velocities of up to $0.56 \text{ feet sec}^{-1}$ in the test section. The foregoing is shown schematically in Figure 2. Although not shown in the figures and not used in the test program described herein, the flume has an anodized aluminum false bottom capable of being tilted so that the effect of bottom slope can be studied.

For the heated discharge, a model thermal plant was constructed which consisted of a pump, a flow control system, a rotometer, electric immersion heaters to simulate the condensers, and a temperature sensing and control system to maintain a constant temperature rise between intake and discharge. Front and back views of the model plant are shown in Figures 3 and 4. The plant is capable of producing flows of 5 gpm at a temperature rise of 20°F .

The discharge arrangement is shown in Figures 5 and 6. The 14 guage galvanized steel discharge canal is rectangular in cross section, $5 \frac{1}{2}$ inches wide and 5 inches deep with the discharge opening centered on one side of its

Figures 3 and 4. Front and rear views of model thermal plant.

Fig. 1 - View of the discharge tube and a hose, showing the in plate.

Figures 5 and 6. Photographs of jet discharge canal and associated piping (not in place).

94 inch length. The canal is lined with 1/8 inch closed cell neoprene foam lining to minimize heat losses. It was thought that considerable length would be required to smooth the 5 gpm flow and that the double ended arrangement would ensure symmetry of the emergent jet. At low flows, however, the heat loss through the bottom and sides of the canal was excessive and the canal was shortened to 32 inches. The opening is adjustable to 5 inches in width and is faired on the inside to straighten and smooth the flow prior to discharge. Heated water from the model power plant divides at a tee and is distributed to opposite ends of the canal where it is discharged omnidirectionally in the horizontal plane at low velocity via a flange and disc diffuser. Balanced flow was obtained by placing an identical orifice in each branch of a size dependent on flow rate. All piping was 5/8 inch garden hose.

There were three temperature measuring systems. The first system, shown in Figure 7, measured the temperature of the oncoming flow, i.e., the background temperature. In addition it detected any lateral variations in temperature of the oncoming flow. This system was mounted on the upstream end of the flume as shown in Figure 8. It consisted of a thermistor driven by a constant speed motor (72 rpm) through a gear belt reduction to produce a linear thermistor speed of 48 inches min^{-1} . It continuously traversed the oncoming flow at constant depth from side to side while measurements were being made.

The second system, shown in the foreground of Figure 9, measured the lateral distribution of temperature across the jet flow at a preselected depth. In this case, the thermistor was driven along an aluminum I-beam by a constant speed motor (72 rpm) through a binary gear box which produced linear thermistor

speeds of 6.4 inches sec^{-1} to 0.0125 inches sec^{-1} . The thermistor travel at the ends of the beam was controlled by mechanical stops and switches and the direction of the travel changed by changing the direction of motor rotation. The beam, on which the thermistor traversed the flume, was mounted so that it could be rotated about a vertical axis which passed through the thermistor when it was at the end of the beam on the nearshore side, i.e., side of orifice or jet discharge. By rotating the beam, the thermistor could be driven at any angle with the x-axis from 0 to 90°. The thermistor beam support assembly was triangular and constructed from Nu-Rail fittings and aluminum pipe except for the hypotenuse which was another aluminum I-beam joined to the thermistor beam by a sliding block which was free to move along either beam. This entire assembly was mounted on a carriage so that it could be positioned longitudinally (x-direction) at any desired point in the test section. The result was a measuring system which could be positioned longitudinally and also rotated horizontally so that the direction of thermistor travel was approximately at right angles to the centerline of the jet flow.

All thermistors used were glass probe with measured time constants of 1/20 second in rapidly moving (3 feet per second) water. Temperatures were recorded on a 4 1/2" span Esterline Angus Speed Servo Recorder (background temperature) with a full scale response of 1/8 second and a 10" span Esterline Angus Speed Servo Recorder (jet flow measuring system) with a full scale response time of 2/10 seconds. The frequency response of these recorders is less than 10% attenuation at 4 Hz (4 1/2") and 3 Hz (10"). The third system, shown in Figure 10, was designed to measure the vertical temperature distribution at a fixed point (x,y). A constant speed motor turns a sprocket

Figure 7. Photograph of background temperature measuring system.

Figure 8. Photograph of upstream end of experimental setup showing recorders and background temperature measuring system.

Figure 9. The flume and jet flow temperature measuring system from the upstream end.

Figure 10. Photograph of vertical temperature distribution measuring system.

@ 10 rpm about a horizontal axis. The sprocket in turn drives an endless bicycle chain on which is attached a thermistor mount. This assembly was then mounted on three adjustable legs. As a result the thermistor is driven in a vertical plane at a rate of approximately $0.52 \text{ inches sec}^{-1}$ over a total vertical distance of 7.25 inches.

The test program that was carried out is shown in Table 1. Chart speeds of $2 \text{ inches min}^{-1}$ and $0.75 \text{ inches min}^{-1}$ were used for recording measurements in the jet and of the background respectively. The background thermistor was driven at a constant speed of $0.8 \text{ inches sec}^{-1}$; the beam thermistor was driven at various speeds from $0.4 \text{ inches sec}^{-1}$ to $1.6 \text{ inches sec}^{-1}$ depending on the location of the beam. Slower speeds were used, of course, as the beam was positioned closer to the discharge orifice.

TABLE 1. The Test Program.

Test	Q,	θ_o ,	D _w ,	h _o ,	b _o ,	A _r	α_o ,	q _o	u _a	R	R _e	F _r
Desig.	GPM	°F	Ft	Ft	Ft		°	Ft sec ⁻¹	Ft sec ⁻¹			
A	4	10	0.167	0.174	0.163	1.07	90	0.31	0.18	1.75	5,020	3.50
B	4	10	0.167	0.174	0.065	2.68	90	0.79	0.18	4.38	12,799	8.92
C	4	10	0.167	0.174	0.0325	5.35	90	1.58	0.18	8.78	25,517	17.78
D	2	10	0.167	0.174	0.0812	2.14	90	0.32	0.18	1.80	5,233	3.65
E	2	10	0.167	0.174	0.0325	5.35	90	0.79	0.18	4.37	12,718	8.86
F	2	10	0.167	0.174	0.0163	10.71	90	1.58	0.18	8.81	25,629	17.86
G	4	10	0.833	0.175	0.065	2.69	90	0.78	0.18	4.36	12,765	8.82
I	4	10	0.667	0.175	0.065	2.69	90	0.78	0.18	4.36	12,765	8.82
K	4	10	0.500	0.174	0.065	2.68	90	0.79	0.18	4.39	12,750	8.89
M	4	10	0.333	0.172	0.065	2.64	90	0.80	0.18	4.45	12,767	9.08
O	4	10	0.333	0.082	0.136	0.60	90	0.80	0.18	4.53	6,101	13.22
Q	4	10	0.333	0.250	0.045	5.56	90	0.79	0.18	4.47	18,436	7.46
S	4	10	0.667	0.169	0.065	2.60	90	0.81	0.18	4.52	12,765	9.29
T	4	10	0.667	0.167	0.065	2.57	90	0.82	0.18	4.57	12,766	9.46
V	4	10	0.833	0.167	0.065	2.57	90	0.82	0.18	4.57	12,766	9.46
W	4	10	0.833	0.167	0.065	2.57	90	0.82	0.18	4.57	12,766	9.46
Y	4	10	0.500	0.167	0.065	2.57	90	0.82	0.18	4.57	12,766	9.46
A-II	0.48	20	0.333	0.072	0.084	0.852	0	0.18	0.18	1.00	1,207	2.17
B-II	0.336	30	0.333	0.050	0.084	0.59	0	0.18	0.18	1.00	838	2.07

TABLE 1. (continued)

Test	Q,	θ_o ,	D _w ,	h _o ,	b _o ,	A _r	α_o ,	q _o	u _a	R	R _e	F _r
Desig.	GPM	°F	Ft	Ft	Ft		°	Ft sec ⁻¹	Ft. sec ⁻¹			
C-II	0.336	30	0.333	0.050	0.084	0.59	0	0.18	0.18	1.00	838	2.07
D-II	0.336	30	0.333	0.048	0.084	0.59	0	0.18	0.18	1.00	804	2.12
E-II	0.336	30	0.333	0.048	0.084	0.59	0	0.18	0.18	1.00	804	2.12
F-II	1.06	10	0.167	0.167	0.083	2.00	0	0.18	0.18	1.00	2,799	2.07
G-II	4	10	0.168	0.179	0.065	2.75	90	0.77	0.18	4.26	12,786	8.38
H-II	4	10	0.167	0.185	0.043	4.35	90	1.14	0.18	6.32	19,551	12.43
I-II	2	10	0.168	0.175	0.146	1.20	90	0.17	0.18	0.97	2,770	1.91
J-II	4	10	0.168	0.178	0.146	1.22	90	0.34	0.18	1.91	5,635	3.80

$$R_e \equiv \frac{q_o h_o}{\nu} \quad \text{where } \nu \text{ is kinematic viscosity of water}$$

$$F_r \equiv \frac{q_o}{\sqrt{\frac{g \Delta \rho_o h_o}{\rho}}} \quad \text{where } g \text{ is the acceleration of gravity}$$

III. The Buoyant Surface Jet in a Cross Flow

Definition sketches are shown in Figure 11. For simplicity we shall resort to an integral analysis. A problem arises, however, when one attempts to define the width and depth of the jet. The offshore terminus is easily definable as the point where $q = u_a \cos \alpha$ or $\theta = \theta_b$. On the nearshore side, however, there is no ambient flow; in its place there is a large eddy with a counter current near the wall. This large eddy contains fluid with excess temperature above background that is being recirculated and possibly reentrained into the jet. This is shown in Figures 12 and 13. Our observations show that the profiles of q and θ are not symmetrical² about the centerline trajectory and are not similar as one proceeds downstream. However, it will be shown in a later section that if we consider only that part of the jet on the offshore side of the centerline, the profiles are similar and may be expressed in the form

$$q(\xi, \eta, z) = q_p(\xi) f_1\left(\frac{\eta}{\eta_{1/2}}\right) g_1\left(\frac{z}{z_{1/2}}\right), \text{ and}$$

$$\theta(\xi, \eta, z) = \theta_p(\xi) f_2\left(\frac{\eta}{\eta_{1/2}}\right) g_2\left(\frac{z}{z_{1/2}}\right)$$

for $\eta \geq 0$. Here f_1 , f_2 , g_1 and g_2 are the similarity functions; $\eta_{1/2}$ and $z_{1/2}$ are the positive values of η and z for which $\theta = \frac{1}{2} \theta_p$.

² The asymmetry is reduced as the depth of water, D_w , is increased relative to the orifice depth, h_o .

Figure 11. Definition sketches of the buoyant surface bent-over jet.

Figure 12. Idealized surface profile of excess temperature, θ/θ_p , and velocity, q/q_p , across (η -direction) the bent jet near the orifice (based on measurements). $R = 2$ (nominal) and $\alpha = 60^\circ$.

Figure 13. Horizontal distribution of excess temperature, θ/θ_0 , for $R = 4.93$. Test 11 in Carter (1969).

$b_q(\xi)$ and $h_q(\xi)$ may now be defined as

$$b_q(\xi) \equiv 2\eta_{1/2}(\xi) \int_0^{\infty} f_1 d\eta' \quad 1(a)$$

and

$$h_q(\xi) \equiv z_{1/2}(\xi) \int_0^{D_w} g_1 dz' \quad 1(b)$$

where

$$\eta' \equiv \eta/\eta_{1/2} \quad \text{and} \quad z' \equiv z/z_{1/2}$$

Similarly,

$$b_\theta(\xi) \equiv 2\eta_{1/2}(\xi) \int_0^{\infty} f_2 d\eta' \quad 1(c)$$

$$h_\theta(\xi) \equiv z_{1/2}(\xi) \int_0^{D_w} g_2 dz' \quad 1(d)$$

$$b_{q^2}(\xi) \equiv 2\eta_{1/2}(\xi) \int_0^{\infty} f_1^2 d\eta' \quad 1(e)$$

$$h_{q^2}(\xi) \equiv z_{1/2}(\xi) \int_0^{D_w} g_1^2 dz' \quad 1(f)$$

$$b_{\theta q}(\xi) \equiv 2\eta_{1/2}(\xi) \int_0^{\infty} f_1 f_2 d\eta' \quad 1(g)$$

and

$$h_{q\theta}(\xi) \equiv z_{1/2}(\xi) \int_0^{D_w} g_1 g_2 dz' \quad 1(h)$$

By defining $b(\xi)$ in terms of that part of the jet offshore from the centerline for use in the integral conservation relations we are essentially assuming that no significant *net* exchange of mass, momentum or heat takes place across a plane normal to the water surface and intersecting it along the jet centerline. The width, $b(\xi)$, and the depth, $h(\xi)$, in these relations are $b_0(\xi)$ and $h_0(\xi)$, respectively. Any inequalities introduced in this manner are considered to be included in either the entrainment coefficients or the drag coefficient; accordingly, we drop the subscript "0".

From here on we shall be concerned only with peak centerline values of q and θ ; accordingly, we drop the subscript "p".

a. The centerline trajectory. In a previous report (Carter, 1969), it was pointed out that because the buoyant surface bent-jet was neither free nor submerged, assumptions that the pressure field was hydrostatic and that the velocity and concentration profiles were similar, were inappropriate. The pressure forces were handled by regarding the jet stream as an obstruction in a uniform stream of velocity, u_a . That is, the pressure forces may be written in the form

$$\text{force} = \frac{\rho C_d \sigma u^2}{2}$$

where C_d is the coefficient of drag, σ is the projected cross-sectional area normal to the flow and u is a characteristic velocity of the flow defined in different ways by different investigators. In the previous report (Carter, 1969), the characteristic velocity u was considered to be identical to u_a . Motz and Benedict (1970) assumed u to be defined by $u = u_a \sqrt{\sin \alpha}$ where α is the angle between the jet and ambient current. Fan (1967) assumed u to

be defined by $u = u_a \sin \alpha$. The form of u is not trivial; in the latter two forms the drag force reduces to zero at some point and there will be no mechanism to return the trajectory to the nearshore, an observational fact that must be contended with if there is a farshore boundary effect. Most investigators agree that large differences in pressure exist in the early stages of development of the flow of the bent jet but that pressure differences are probably small in the major portion of the trajectory where the jet may be reasonably considered to be "free."³ As the farshore is approached, the actual trajectory, of course, departs from the trajectory of a free jet. Whatever their origin, pressure gradients normal to the initial direction of the jet will deflect the trajectory and parallel pressure gradients will accelerate or decelerate it. In addition, as the ambient flow is entrained it causes additional deflection, and/or deceleration or acceleration, depending on the initial orientation of the jet and ambient flow. The usual procedure is to assume forms for the forces due to entrainment and pressure that are physically reasonable and mathematically attractive. Each contains a free parameter such as an entrainment coefficient or a drag coefficient, the values of which are hopefully universal and obtained by fitting the trajectory or dilution model developed to data measured in a wind or water tunnel. The result of this approach has been a large number of models, differing just enough with respect to underlying assumptions and characteristics to be confusing if not misleading to the engineer who tries to apply them to a real situation. An excellent summary of recent formulations of analytical heated effluent models is contained in A.J. Policastro and J.V. Tokar (1972). Although physically these concepts of entrainment and pressure forces are valid, their ex-

³ The term "free" is understood to mean that the boundaries of the ambient fluid do not affect the jet zones being studied.

pression in terms that can be dealt with mathematically, while at the same time retaining sufficient physical reality so as to be extrapolatable to different conditions, is questionable. As an example, consider the conservation of normal momentum in integral form. It may be written as

$$\rho q^2 A \frac{d\alpha}{d\xi} = - u_a \sin \alpha \frac{d(\rho q A)}{d\xi} - \text{drag force per unit length.} \quad (2)$$

Without entrainment, tangential momentum is conserved and (2) becomes

$$\frac{d\alpha}{d\xi} = - \text{drag force per unit length} / (\rho q^2 A)_0. \quad (3)$$

For the slab $b(\xi)h(\xi)d\xi$ the drag per unit length may be written as

$$\frac{\rho C_d h u^2}{2} \quad \text{or we have}$$

$$\frac{d\alpha}{d\xi/b_0} = - \frac{C_d}{2} \left(\frac{u}{q_0} \right)^2 \quad (4)$$

If u is taken to be u_a , then

$$\frac{d\alpha}{d\xi/b_0} = - \frac{C_d}{2R^2} \quad (5)$$

which is an expression for the curvature of a circle.

On the other hand, if $u = u_a \sqrt{\sin \alpha}$, then

$$\frac{d\alpha}{d\xi/b_0} = - \frac{C_d}{2R^2} \sin \alpha \quad (6)$$

and the trajectory may be shown to be given by

$$\frac{y}{b_0} \frac{C_d}{2R^2} = \left\{ \frac{\pi}{4} - \frac{1}{2} \sin^{-1} \left(2 \exp \left(-2 \frac{C_d}{2R^2} \frac{x}{b_0} \right) - 1 \right) \right\} \quad (7)$$

Next if $u = u_a \sin \alpha$, then

$$\frac{d\alpha}{d\xi/b_o} = - \frac{C_d}{2R^2} \sin^2 \alpha \quad (8)$$

This equation may be integrated to obtain

$$\frac{y}{b_o} \frac{C_d}{2R^2} = \ln \left\{ 1 + \frac{x}{b_o} \frac{C_d}{2R^2} + \sqrt{\left(\frac{x}{b_o} \frac{C_d}{2R^2}\right)^2 + 2 \frac{x}{b_o} \frac{C_d}{2R^2}} \right\} \quad (9)$$

for the trajectory.

Finally, if $u = u_a (\sin \alpha)^{3/2}$ then

$$\frac{d\alpha}{d\xi/b_o} = - \frac{C_d}{2R^2} \sin^3 \alpha \quad (10)$$

from which one obtains

$$\frac{y}{b_o} \frac{C_d}{2R^2} = \sqrt{2 \frac{x}{b_o} \frac{C_d}{2R^2}} \quad (11)$$

for the trajectory. The effect of the form of u on the trajectory is shown in Figure 14 where equations (5), (7), (9), and (11) are plotted.

If we neglect drag, equation (2) reduces to

$$\rho q^2 A \frac{d\alpha}{d\xi} = - u_a \sin \alpha \frac{d(\rho q A)}{d\xi} \quad (12)$$

The conservation of tangential momentum is

$$\frac{d(\rho q^2 A)}{d\xi} = u_a \cos \alpha \frac{d(\rho q A)}{d\xi} \quad (13)$$

Figure 14. Sketch showing the effect of the form of u on the centerline trajectory.

Dividing (13) and (12) and integrating we obtain

$$\frac{v}{q_0} \frac{q}{q_0} \frac{A}{A_0} = 1 \quad (14)$$

That is, the flux of y-momentum is conserved in the absence of drag.

From (12) and (14) we have

$$\frac{d\alpha}{d\xi/b_0} = - \frac{\sin^2 \alpha}{R} \frac{d\left(\frac{q}{q_0} \frac{A}{A_0}\right)}{d\xi/b_0} \quad (15)$$

The conservation of mass is

$$\frac{dq}{d\xi} \frac{A}{A_0} = C u_e \quad (16)$$

where u_e is the entrainment velocity and C is a length over which entrainment is assumed to take place. A logical assumption for u_e according to Hoult and Weil (1970) is

$$u_e = \gamma (q - u_a \cos \alpha) + \beta u_a \sin \alpha \quad (17)$$

Here the entrainment is assumed to be due to two linearly additive processes which are proportional to the velocity shear both parallel and normal to the plume centerline. The constants of proportionality are γ and β respectively. Hoult and Weil (1970) estimate β to be about 0.6 and γ 0.11.

Far from the source $u = u_a$ and (17) becomes

$$u_e = u_a \sin \alpha \{ \gamma \tan \alpha + \beta \} \approx \beta u_a \sin \alpha \quad (18)$$

From (15), (16), and (18) we have that

$$\frac{d\alpha}{d\xi/b_o} = - \frac{\sin^3 \alpha}{R^2} \frac{C b_o}{A_o} \beta . \quad (19)$$

For a jet that is not depth limited, i.e., entrainment takes place through the bottom as well as the sides, C is an increasing function of downstream distance, ξ . As a result, the curvature at a given ξ will be larger and offshore penetration less than if C were constant. We wish to consider the limiting case with respect to maximum penetration in the offshore direction, which is the case when C is constant. For a bottom attached 2-dimensional bent surface jet with entrainment through both off- and onshore sides

$$\frac{C b_o}{A_o} = \frac{2 h_o b_o}{A_o} = 2$$

or (19.) becomes

$$\frac{d\alpha}{d\xi/b_o} = - \frac{2\beta}{R^2} \sin^3 \alpha . \quad (20)$$

This integrates to

$$\frac{y}{b_o} \frac{2\beta}{R^2} = \sqrt{2 \frac{x}{b_o} \frac{2\beta}{R^2} + C_1} \quad (21)$$

which is identical to equation (11) if $C_1 = 0$ and $4\beta = C_d$.

It has been shown that a hypothesis of drag without entrainment results in a model that conserves tangential momentum along a centerline trajectory that is parabolic in the limit. Whether the jet is 2- or 3-dimensional is unimportant; the absence of entrainment

is the essential factor. A hypothesis of entrainment without drag results in a model that conserves offshore or y-momentum along a centerline trajectory that is also parabolic in the limit. Intuitively we expect drag forces to be dominant close to the orifice or point of discharge with their effect becoming negligible when the relative velocity in the cross flow direction vanishes, i.e., $u = u_a$. The length of this zone or region may be found by considering equations (5), (7), (9), and (11). For small ξ it can be shown from these equations that

$$\frac{dy}{dx} = \tan \alpha \approx \frac{1}{\cos \alpha} = \frac{1}{\frac{y}{b_o} \frac{C_d}{2R^2}} \quad (22)$$

Now

$$\cos \alpha = \frac{u}{q} \approx \frac{u}{q_o} \approx \frac{u}{u_a} \frac{u_a}{q_o} \approx \frac{u}{u_a} \frac{1}{R} \quad (23)$$

or at the point where $\cos \alpha = 1/R$, relative longitudinal motion between the jet and the cross flow has essentially vanished. The distance between the orifice and this point is, from (22) and (23),

$$\frac{y}{Rb_o} \approx \frac{\xi}{Rb_o} \approx \frac{2}{C_d} \quad (24)$$

This is in general agreement with the length of the zone of maximum deflection as defined by Pratte and Baines (1967) and the momentum length scale l_m as defined by Hoult and Weil (1970). l_m is considered by Hoult and Weil to be the distance over which the jet trajectory is changed appreciably by the cross flow.

Beyond this initial zone, jet momentum is altered by entrainment alone with the centerline trajectory depending primarily on whether the entrainment is 1- or 2-dimensional. In the limit, however, the preferred trajectory seems to be parabolic (equation (21)). Both 2-dimensional entrainment and pressure gradients associated with the farshore boundary result in the trajectory departing from a parabolic path in the nearshore, i.e., y -direction. Although one can visualize generally how this will occur, there is no way *a priori* to uniquely formulate the forces involved. Accordingly, treating the general case by choosing a particular form for these forces and then measuring the free parameters γ , β , and C_d by experiment seems to involve more curve fitting than physics. Another complication is the great similarity in the two forces as formulated by various investigators. This is shown in Table 2 below.

Table 2

Limiting Values for the Entrainment and Drag
Forces per unit Length for the Surface Buoyant Bent Jet

	$\xi/b_o \rightarrow 0$	$\xi/b_o > R$
<u>Entrainment</u>	$- u_a^2 \rho h_o (R\gamma + \beta)2$	$- u_a^2 \rho C 2\beta \sin^3 \alpha$
<u>Drag</u>	$- u_a^2 \rho h_o \frac{C_d}{2}$	$- u_a^2 \rho h \frac{C_d}{2} (\sin \alpha)^{2n}$

In Table 2, the drag force per unit length is assumed to be of the form $\rho \frac{C_d}{2} h u^2$ where $u = u_a (\sin \alpha)^n$. The entrainment force per unit length is given by equations (12), (16) and (17).

Based on the foregoing analysis, it is concluded that the centerline trajectory of the two-dimensional buoyant surface jet in a cross flow will follow a parabolic trajectory given by

$$\frac{y}{b_o} = R \sqrt{2 \frac{x}{b_o}} \quad (25)$$

Equation (25) is derived from equation (21) using a value of 0.5 for β . Tests A through F, G-II, H-II, and J-II in Table 1 suggest that this is the proper value of $\beta \pm 20\%$. *Departures from this trajectory as the far-shore is approached are considered to be farshore boundary effects which are beyond the capability of present models to predict.* This boundary effect is predictable, however, if we plot

$$\frac{W}{b_o} / R \sqrt{2 \frac{x}{b_o}} \quad \text{versus} \quad \frac{y}{b_o} / R \sqrt{2 \frac{x}{b_o}}$$

where W is the distance between the near and farshores. This has been done for Tests A, D, and J-II in Figure 15 ($R = 2$ nominal); for Tests B, E, and G-II in Figure 16 ($R = 4$ nominal); and for Tests C, F, and H-II in Figure 17 ($R = 9$ nominal). It may be seen that there is essentially no dependence on R since the dashed curve, which has been fitted to the data by eye, is identical in all three figures.

With respect to the case where C is an increasing function of ξ , i.e., side and bottom entrainment, our data were found to fit the expression

$$\Delta h = 2.3 F_m^{1/3} u^{-2/3} x^{1/3} \quad (26)$$

Figure 15. Effect of the farshore boundary on the parabolic trajectory for $R = 2$ (nominal).

Figure 16. Effect of the farshore boundary on the parabolic trajectory for $R = 4$ (nominal).

Figure 17. Effect of the farshore boundary on the parabolic trajectory for $R = 9$ (nominal).

quite well. Equation (26) was derived by Briggs (1969) for a non-buoyant chimney plume in a crosswind. Here Δh is the plume rise above the chimney orifice (y in our terminology), F_m the momentum flux $((\rho q b h)_0 / \pi$ in our terminology), and u is the ambient wind speed (u_a in our terminology). Equation (26) when transformed into our terminology and form is

$$\frac{y}{b_0} = \frac{\pi}{2} R^{2/3} A_r^{1/3} \left(\frac{x}{b_0} \right)^{1/3} \quad (27)$$

where A_r is the aspect ratio defined as $A_r \equiv \frac{h_0}{b_0}$. If we normalize by $\sqrt{b_0 h_0}$ we obtain an expression which is independent of A_r or

$$\frac{y}{\sqrt{b_0 h_0}} = \frac{\pi}{2} R^{2/3} \left(\frac{x}{\sqrt{b_0 h_0}} \right)^{1/3} \quad (28)$$

Figure 18 is a plot of the centerline trajectory of Tests G, V, and W for a nominal R of 5, a nominal A_r of 2.6 and a nominal D_w , the water depth, to h_0 ratio of 5. Figure 19 is a plot of the centerline trajectory of Tests I, S, and T for a nominal R of 5, a nominal A_r of 2.6 and a nominal D_w to h_0 ratio of 4. Figure 20 is a plot of the centerline trajectory of Tests K and Y for a nominal R of 5, a nominal A_r of 2.6 and a nominal D_w to h_0 ratio of 3. Figure 21 is a plot of the centerline trajectory of Tests, M, O, and Q for a nominal R of 5, various A_r 's between 0.6 and 5.6 and a nominal D_w to h_0 ratio of 2. The solid curve on all figures is equation (28). It is clear from Figures 18 through 21 that for a R of 5, the various centerline trajectories show essentially no dependence on A_r or D_w/h_0 . This conclusion was reached not by any statis-

Figure 18. The centerline trajectory for the three-dimensional buoyant surface bent jet for $D_w/h_0 = 5$.

Figure 19. The centerline trajectory for the three-dimensional buoyant surface bent jet for $D_w/h_0 = 4$.

Figure 20. The centerline trajectory for the three-dimensional buoyant surface bent jet for $D_w/h_o = 3$.

Figure 21. The centerline trajectory for the three-dimensional buoyant surface bent jet for $D_w/h_0 = 2$ and various A_r 's.

tical test but rather by looking at the mean of the ratio $\frac{y}{\sqrt{b_o h_o}} / \frac{\pi}{2} R^{2/3} (\frac{x}{\sqrt{b_o h_o}})^{1/3}$ and visually at this ratio as a function of $\frac{x}{\sqrt{b_o h_o}}$ for the four sets of data plotted in Figures 18 through 21. No trend in this ratio was observed and the departures of the individual values and of the means from 1 were not considered to be significant, at least for purposes of prediction.

Another interesting feature shown in these figures is the apparent lack of farshore influences within the limits of the measurements in contrast to the two-dimensional situation depicted in Figures 15 through 17. Apparently, the three-dimensional jet is able to accept substantially all ambient fluid that is presented to its entrainment length C , thus greatly reducing drag since this force was not included in the derivation of equation (26) by Briggs.

b. The centerline excess temperature. In the absence of cooling, the conservation of excess temperature in integral form may be written as

$$\frac{d(\theta q A)}{d\xi} = 0 \quad (29)$$

This may be integrated to obtain

$$\frac{\theta}{\theta_o} \frac{q}{q_o} \frac{A}{A_o} = 1 \quad (30)$$

or beyond the zone of maximum deflection θ/θ_o and v/q_o have the same functional dependence on ξ . See equation (14).

For the two-dimensional buoyant surface bent over jet, it was shown previously that

$$y \propto x^{1/2}$$

Differentiating with respect to t , we obtain far from the source

$$\frac{dy}{dt} = v \propto x^{-1/2} \frac{dx}{dt} \propto u_a x^{-1/2} \propto \xi^{-1/2} \quad (31)$$

or

$$\frac{\theta}{\theta_o} \propto \left(\frac{\xi}{b_o} \right)^{-1/2} \quad (32)$$

Figure 22 is a plot of the centerline excess temperature θ/θ_o versus ξ/b_o for Tests A,B,C,D,E,F,G-II,H-II, and J-II. Tests D and J-II are for a nominal R of 2; Tests B, E and G-II for a nominal R of 4; Tests C and F for a nominal R of 9; Test H-II for a nominal R of 6. Although there is considerable scatter in the data, the data suggest that $\xi^{-1/2}$ is the proper power law for the decay of excess temperature along the centerline for the two-dimensional case. The solid line in the figure has been fitted by eye.

For the three-dimensional case, we found previously that

$$y \propto x^{1/3}$$

Differentiating with respect to t , we obtain where $u \approx u_a$

$$\frac{dy}{dt} = v \propto x^{-2/3} \frac{dx}{dt} \propto u_a x^{-2/3} \propto \xi^{-2/3} \quad (33)$$

or

$$\frac{\theta}{\theta_o} \propto \left\{ \frac{\xi}{\sqrt{b_o h_o}} \right\}^{-2/3} \quad (34)$$

Figure 22. The centerline excess temperature, θ/θ_0 , as a function of ξ/b_0 for the two-dimensional bent jet.

Figure 23 is a plot of the centerline temperature, θ/θ_o , versus $\xi/\sqrt{b_o h_o}$ for Tests G, I, K, M, O, Q, S, T, V, and W. All are for a nominal R of 5 but for a range of D_w/h_o between 2 and 5 and a range of A_r between 0.60 and 5.56. The data in Figure 23 agree quite closely with equation (34) for $\xi/\sqrt{b_o h_o} < 40-100$. The solid line has been fitted by eye.

c. The lateral and vertical profiles of excess temperature, θ/θ_o .

Lateral profile data for the two-dimensional bent jet were obtained from Tests A, B, and C. All η statistics were developed from only that portion of the temperature record offshore from the centerline, i.e., + η direction, in accordance with our definitions of $b(\xi)$ given earlier (equations 1a, c, e, and g). Figures 24 through 26 are plots of these data. Although there is considerable scatter in the data, it is reasonably well represented by the relation

$$\frac{\theta}{\theta_p} = \frac{1}{2} \{1 + \cos(\frac{\pi}{2} \frac{\eta}{\eta_{1/2}})\} = f_2. \quad (35)$$

There is no apparent dependence on R or ξ . That is, if we consider only that portion of the jet offshore from the centerline, the lateral shape is apparently self preserving.

Let us now consider what the excess temperature, $\frac{\theta}{\theta_o}$, dependence on ξ and the centerline trajectory suggest the variation of A should be with increasing ξ for this case, i.e., the 2-dimensional bottom attached bent jet.

From equation (30) we have that

$$\frac{A}{A_o} = \frac{b}{b_o} = \frac{\theta_o}{\theta} \frac{q_o}{q} \quad (36)$$

Figure 23. The centerline excess temperature, θ/θ_0 , as a function of $\xi/\sqrt{b_0 h_0}$ for the three-dimensional bent jet.

Figure 24. Lateral (η) profiles of excess temperature, θ/θ_p , at various downstream distances, ξ/b_0 , for a R of 1.75 (two-dimensional bent jet).

Figure 25. Lateral (η) profiles of excess temperature, θ/θ_p , at various downstream distances, ξ/b_o , for a P of 4.38 (two-dimensional bent jet).

Figure 26. Lateral (η) profiles of excess temperature, θ/θ_p , at various downstream distances, ξ/b_0 , for a R of 8.78 (two-dimensional bent jet).

as h is constant. From equations (36), (1c), and (35) we have that

$$\frac{2\eta_{1/2}}{Rb_o} = \frac{\theta_o}{\theta} \frac{q_o}{q} \frac{1}{R} \approx \frac{\theta_o}{\theta} \cos \alpha \quad (37)$$

Early in the entrainment zone, but for $\frac{\xi}{Rb_o} > 0$, $\frac{\theta}{\theta_o}$ is constant (or nearly so) and $\frac{dx}{d\xi} \propto \xi$; ultimately $\frac{\theta}{\theta_o} \propto \xi^{-1/2}$ and $\frac{dx}{d\xi}$ equals 1 or we may tabulate as follows:

Table 3

The Variation of $b(\xi)$ with ξ (2-dimensional case)

	<u>Early</u>	<u>Late</u>
$\frac{\theta_o}{\theta}$	constant	$\xi^{1/2}$
$\frac{dx}{d\xi}$	ξ	1
$\frac{2\eta_{1/2}}{Rb_o}$	ξ	$\xi^{1/2}$

$\frac{2\eta_{1/2}}{Rb_o}$ has been plotted in Figure 27 as a function of $\frac{\xi}{Rb_o}$. The predicted functional dependence of $\frac{2\eta_{1/2}}{Rb_o}$ on ξ is in reasonable agreement with the data.

Similar data were obtained for the three-dimensional bent jet for both lateral and vertical profiles from Tests G, I, M, and Y. Figures 28 through 31 are plots of these data. Again there is considerable scatter but the data are reasonably well represented by equation (35) as before and the relation

$$\frac{\theta}{\theta_p} = \frac{1}{2} \left\{ 1 + \cos \left(\frac{\pi}{2} \frac{z}{z_{1/2}} \right) \right\} = g_2 \quad (38)$$

Figure 27. The width, $2\eta_{1/2}/Rb_0$, of the two-dimensional bent jet as a function of ξ/Rb_0 .

Figure 28. Lateral (η) profiles of excess temperature, θ/θ_p , at various downstream distances, $\xi/\sqrt{b_0 h_0}$, for p , $D_w/h_0 = 5$ (three-dimensional bent jet).

Figure 29. Lateral (η) profiles of excess temperature, θ/θ_p , at various downstream distances, $\xi/\sqrt{b_0 h_0}$, for $D_w/h_0 = 3$ (three-dimensional bent jet).

Figure 30. Lateral (η) profiles of excess temperature, θ/θ_p , at various downstream distances $\xi/\sqrt{b_0 h_0}$, for $D_w/h_0 = 2$ (three-dimensional bent jet).

Figure 31. Vertical (z) profiles of excess temperature, θ/θ_p , at various downstream distances, $\xi/\sqrt{b_0 h_0}$, for various ratios of D_w to h_0 (three-dimensional bent jet).

There is no apparent dependence on ξ and both the lateral shape offshore from the centerline and the vertical shape are self-preserving.

As before we shall consider what the excess temperature, θ/θ_o , dependence on ξ and the centerline trajectory suggest the variation of A should be with increasing ξ . That is,

$$\frac{A}{A_o} = \frac{b h}{b_o h_o} = \frac{\theta_o}{\theta} \frac{q_o}{q} \quad (39)$$

From equations (39), (1c), (1d), (35), and (38) we have that

$$\frac{2\eta_{1/2}}{\sqrt{b_o h_o}} \frac{z_{1/2}}{\sqrt{b_o h_o}} \approx \frac{\theta_o}{\theta} \frac{1}{R} \cos \alpha \approx \frac{\theta_o}{\theta} \frac{1}{R} \frac{dx}{d\xi} \quad (40)$$

In the vicinity of the orifice, $0 < \frac{\xi}{\sqrt{b_o h_o}} < R$, θ/θ_o is 1,

$\frac{dx}{d\xi} \propto \xi$, and $\frac{2\eta_{1/2}}{\sqrt{b_o h_o}} \propto \xi^{2/3}$ (see Fig. 32). As a result, $\frac{z_{1/2}}{\sqrt{b_o h_o}} \propto \xi^{1/3}$

from equation (40). We may divide the plume into two additional zones, one

where $R < \frac{\xi}{\sqrt{b_o h_o}} < 40-100$ and a second where $\frac{\xi}{\sqrt{b_o h_o}} > 40-100$. If so we may find the predicted behavior of $\frac{z_{1/2}}{\sqrt{b_o h_o}}$ from equation (40).

This has been done and is tabulated below.

Table 4

The Variation of $z_{1/2}$ with ξ (3-dimensional case)

	$0 < \frac{\xi}{\sqrt{b_o h_o}} < R$	$R < \frac{\xi}{\sqrt{b_o h_o}} < 40-100$	$\frac{\xi}{\sqrt{b_o h_o}} > 40-100$
$\frac{\theta_o}{\theta}$	1	$\xi^{2/3}$	$\xi^{1/6}$
$\frac{dx}{d\xi}$	ξ	1	1

Table (continued)

	$0 < \frac{\xi}{\sqrt{b_o h_o}} < R$	$R < \frac{\xi}{\sqrt{b_o h_o}} < 40-100$	$\frac{\xi}{\sqrt{b_o h_o}} > 40-100$
$\frac{2\eta_{1/2}}{\sqrt{b_o h_o}}$	$\xi^{2/3}$	$\xi^{2/3}$	$\xi^{2/3}$
$\frac{z_{1/2}}{\sqrt{b_o h_o}}$	$\xi^{1/3}$	1	$\xi^{-1/2}$

$\frac{2\eta_{1/2}}{\sqrt{b_o h_o}}$ and $\frac{z_{1/2}}{\sqrt{b_o h_o}}$ have been plotted in Figures 32 and 33 as functions of

$\frac{\xi}{\sqrt{b_o h_o}}$. The vertical temperature structure was sampled with a chain-

driven thermistor which moved in a vertical plane between the water surface and the bottom whereas the lateral temperature distribution was sampled with a driven thermistor which moved in the horizontal plane just below the water surface. Because of difficulties associated with locating the plume centerline for the purpose of making the vertical measurements, i.e., the plume tends to meander much more in a horizontal plane than in a vertical plane, the determinations of $z_{1/2}$ are not considered to be of high quality. As a result, the vertical structure will be resampled during a later phase of the program with a more meaningful technique. The curves shown in Fig. 33 were, therefore, drawn arbitrarily to conform to Table 4 and the values of $z_{1/2}$ at the orifice. There are, of course, other possible interpretations of this poor quality data. The existence of the three regions together with the tendencies shown in each region are considered to be valid, however.

Figure 32. The width, $2\eta_{1/2}/\sqrt{b_0 h_0}$, of the three-dimensional bent jet as a function of downstream distance, $\xi/\sqrt{b_0 h_0}$, for various ratios of D_w to h_0 .

Figure 33. The depth, $z_{1/2}/\sqrt{b_o h_o}$, of the three-dimensional bent jet as a function of downstream distance, $\xi/\sqrt{b_o h_o}$, for various ratios of D_w to h_o .

Another interesting feature of Fig. 33 is the apparent dependence of $z_{1/2}$ on the depth of the system, D_w , at least for the depth range investigated. Apparently, the jet grows vertically until such time as it "feels" bottom. At this point increased bottom friction slows the jet until buoyant and inertial forces are of the same order, i.e., Froude Number of order 1. At this point vertical growth ceases. For the three tests, this appeared to take place at $z_{1/2} \approx 0.6 D_w$. The reduction in $z_{1/2}$ beyond this point is probably due to buoyant spreading.

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Appendix A. Ambient or far field turbulence in the flume.

Both theory and experiment for three-dimensional diffusion show that the centerline excess temperature decay in the absence of cooling from a finite source, $\theta_o Q$, is of the form

$$\theta \propto \frac{\theta_o Q}{x^n} \quad (1)$$

where the constant of proportionality depends upon the theoretical model chosen and n is of order 1. Here it is assumed that there is no excess velocity at the orifice or that

$$Q = q_o b_o h_o = u_a b_o h_o \quad (2)$$

In order to determine the level of ambient or far field turbulence in the flume, Tests A-II through F-II were carried out. In each of these tests, the discharge canal was placed facing in the x -direction on the centerline of the flume and Q , b_o and h_o adjusted so that $q_o = u_a$. The results of these tests are shown in Figure A-1. The solid line of slope -1, i.e., $n = 1$, has been fitted to the data by eye. This line, θ/θ_o versus x/b_o , may be scaled to the conditions applicable to Tests A through F and G-II through J-II as follows. (1) may be rearranged to give

$$\theta/\theta_o \propto \frac{Q}{b_o x/b_o} \quad (3)$$

The scaling factor for the data plotted in Figure A-1 is, for a given x/b_o

$$\frac{\theta}{\theta_o} \times \frac{Q}{.336} \times \frac{0.084}{b_o} = \text{scaling factor} \quad (4)$$

Figure A-1. The variation of the centerline-excess temperature, θ/θ_0 , as a function of downstream distance, x/b_0 , for the case where $q_0 = u_a$ (no excess momentum).

or (4) becomes upon substitution

$$\frac{\theta}{\theta_o} \times \left\{ \frac{h_o}{0.049} \right\} \times R = \text{scaling factor} \quad (5)$$

or we may tabulate as follows:

<u>TEST</u>	<u>R</u>	<u>SCALING FACTOR</u>
A	1.75	6.13
B	4.38	15.38
C	8.78	30.77
D	1.80	6.16
E	4.37	15.38
F	8.81	30.77
G-II	4.26	15.38
H-II	6.32	23.26
I-II	0.97	3.42
J-II	1.91	6.58

The ambient decay lines for Tests A, D, and J-II, B, E, and G-II, and C and F, respectively, are shown in Figure 22.