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MAGNETIC MOMENTS AND VEHICLE TORQUES
IN THE SNAP 10A

AEC Research and Development Report

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IN THE SNAP 10A

By
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ATOMICS INTERNATIONAL
A DIVISION OF NORTH AMERICAN AVIATION, INC.

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CONTENTS

	Page
Abstract	5
I. Summary	7
II. Introduction	9
III. Basic Physical Relationships	10
IV. An Illustrative Example	13
V. Dipole Moments of the SNAP 10A	16
A. Dipole Moment of the Pump Magnet Assembly	16
B. Analytical Evaluation of the Pump Dipole Moment	16
C. Experimental Evaluation of the Pump Dipole Moment	17
D. Dipole Moment of the Thermoelectric Converter	20
E. Other Dipole Sources	22
F. Summary of Dipole Moments	24
VI. Magnetic Field of the Earth and Vehicle Torques	25
VII. Methods of Reducing Vehicle Torques	34
VIII. Conclusions	35
Nomenclature	36
References	37
Appendices	38
I. Theoretical Evaluation of the Pump Magnetic Moment	38
II. Experimental Data	40
III. Induced Dipole Moment of the "D" Tubes and the Resulting Torque	43
IV. The Magnetic Field of the Earth	44

TABLES

1. Pump Dipole Moments for Several Values of M_2	17
2. Tabular Summary of Magnetic Dipole Moments	24
3. Series Expansion of the Earth's Field	44

FIGURES

1. SNAP 10A System	8
2. Coordinates for Dipole Field	10
3. Dipole Model of the Earth's Field	13
4. Earth and Vehicle Coordinates	13
5. Sketch Showing Coordinate Deviations From the Desired Orientation Due to Magnetic Torques	15
6. Pump Magnetic Assembly	16
7. Experimental Equipment for the Determination of Pump Dipole Moments	18

FIGURES

	Page
8. Axes for Magnetic Measurements	18
9. Field Strength Data as a Function of Position, Experiment 1	18
10. Field Strength Data as a Function of Position, Experiment 2	20
11. Portion of Thermoelectric Converter Leg.	20
12. Complete Thermoelectric Converter	21
13. Sketch of Converter Current Flow.	22
14. Sketch of Converter Current Flow Showing Amperian Current Loops.	22
15. Cross-Section View of Thermoelectric Pump	23
16. Coordinate Systems	26
17. Yaw Axis Magnetic Field Component	27
18. Pitch Axis Magnetic Field Component	28
19. Roll Axis Magnetic Field Component	29
20. Yaw Torque Component	30
21. Pitch Torque Component	31
22. Roll Torque Component	32
23. Converter Circuit Design for Zero Dipole Moment.	35

ABSTRACT

Small magnetic dipole moments arise in the SNAP 10A nuclear electric power system due principally to the permanent magnet in the electromagnetic pump and current flowing in the thermoelectric converter. The dipole moments interact with the earth's magnetic field to produce torques on the orbiting vehicle. This report describes the nature of these torques, discusses the procedures which have been used to evaluate them, and discusses methods for reducing them.

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I. SUMMARY

The SNAP 10A 500-watt nuclear electric power system is designed to provide power to space vehicles while imposing no special attitude control requirements on the vehicle. The presence on the system of a permanent magnet and wires carrying relatively high currents does result in the generation of electromagnetic torques which can affect vehicle attitude. This report describes the nature of these torques, discusses the procedures which have been used to evaluate them, and discusses methods for reducing them if this is required.

Torques arise when magnetized bodies or current carrying conductors interact with an external magnetic field. The specific property of the body which determines the torque is its magnetic dipole moment. The torque is given as the vector cross product of the dipole moment into the magnetic flux density. Determination of magnetic torques requires a specification of the field of the earth and the dipole moments of the system.

The dipole moment of the magnet assembly in the NaK pump has been evaluated both analytically and experimentally, and has been found to be 95 ± 10 ampere meters² (AMP. M²). The dipole moment arising due to current flow in the thermoelectric converter has been evaluated analytically and found to be 15.3 ± 1 amp m². Several other possible sources of dipole moments have been examined and found to be substantially smaller than the above items. Since dipole moment is a vector, the direction must also be specified. The pump dipole moment is aligned with the +z axis of the vehicle. The converter dipole moment is aligned with the -x axis of the vehicle.

The magnetic field of the earth has been evaluated in the polar orbital path of the vehicle by a computer program based on a spherical har-

monic analysis. The program also provides a coordinate transformation into the vehicle coordinate system and computes the torque by the cross product operation. The field of the earth, as seen by the vehicle, is discussed in Section VI (Figures 17, 18, and 19) and the resulting torques are presented in Section VI (Figures 20, 21, and 22). The fields and torques are "nearly periodic" functions, having components at the earth rotational frequency, the orbital frequency, and multiples of the orbital frequency. The greatest torque occurs on the vehicle pitch (y) axis and is approximately sinusoidal at the orbital frequency with a peak amplitude of about 0.5 oz-in. A simplified dynamic analysis indicates that a vehicle pitch oscillation with a peak amplitude of about 9° will result. The normal vehicle attitude in SNAPSHOT is yaw (x) axis aligned with the local vertical and roll (z) axis along the forward velocity. Motions of this amount are not disruptive to the communications or other aspects of the SNAPSHOT program.

The simplest method for reducing the magnetic torques on the system is through the use of a counter torque magnet. This is a magnet whose dipole moment is equal to the vector sum of the system dipole moments. The counter torque magnet is aligned opposite to the net moment. The interaction between the earth's field and the counter torque magnet produces a torque which opposes or cancels that on the system itself. The location of the counter torque magnet is arbitrary, so that it can be selected for maximum convenience. It is estimated that a reduction of the magnetic torques by a factor of 10 is readily possible with this approach. Greater reduction of the torques would call for measurement accuracies which are difficult to attain. Other cancellation schemes are considered in the report.

It has been concluded that electromagnetic torques do not represent any limitation on the application of the SNAP 10A system or any potential limitation in advanced versions of the same concept.

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Figure 1. SNAP 10A System

II. INTRODUCTION

SNAP 10A is a 500-watt electrical power source for space vehicle applications. The system consists of a nuclear reactor heat source, a nuclear radiation shield, a liquid-metal heat-transfer medium, an electromagnetic pump, and a thermoelectric energy converter. Each of these components is mounted on a light weight structure designed to be mated to a launch vehicle. A conceptual drawing of the SNAP 10A is shown in Figure 1.

The initial flight and operation in space of the SNAP 10A system was successfully accomplished in the spring of 1965 in a demonstration program designated SNAPSHOT. The launch vehicle was an Atlas-Agena D, with the Agena being retained in orbit to provide the basic payload command, communications, and control functions. The SNAP 10A supplied power to the Agena to accomplish its basic functions and to enable the operation of several secondary payload experiments.

One of the basic features of the SNAP 10A is that it is a power supply which places no special requirements with respect to vehicle attitude on the vehicle it is supporting. It is desirable, also, that the system impose little or no torque loads on the parent vehicle. For the most part, the SNAP 10A design meets this condition as well. However, it has been found that the interaction of the field of the earth with the pump and converter does result in small torques on the system. Although these torques are small (< 1 oz-in.), these are of similar size to the loads which are commonly considered in attitude control systems design. This report describes the work which has been done to evaluate the magnetic torque loads and discusses some methods for reducing them if this should be necessary. On the SNAPSHOT program, it was found that the attitude control system could easily handle the loads and no special correction was used.

III. BASIC PHYSICAL RELATIONSHIPS

The basic physical relation describing the generation of torque due to the interaction of a magnetized body and an applied field is:⁽¹⁾

$$\vec{T} = \vec{m} \times \vec{B} \quad \dots(1)$$

In the MKS system, the factors of Equation 1 are expressed as follows:

$\vec{T}$ = torque in newton-meters

$\vec{m}$ = magnetic dipole moment in amp·m²

$\vec{B}$ = magnetic field strength in webers/m²

Any physical system giving rise to a magnetic field, either due to permanently magnetized materials or distributions of currents, maybe described by its magnetic dipole moment $\vec{m}$. This dipole moment has the property that it may be used to compute torques, through Equation 1, and it may be used to describe the field due to the system, at locations which are sufficiently far from the body. The field determination is based on the following relationship:

$$\vec{B} = \frac{\mu_0 |\vec{m}|}{4\pi r^3} (2 \cos \phi \vec{i}_r + \sin \phi \vec{i}_\phi) \quad \dots(2)$$

where

$\vec{B}$ = the field strength in webers/m²

μ_0 = the permittivity of free space = $4\pi \times 10^{-7}$ henries/meter

r = the distance from the dipole to the field point

ϕ = the co-latitude

$\vec{i}_r$ = a unit vector in the radial direction

$\vec{i}_\phi$ = a unit vector in the angular direction

These quantities are shown in Figure 2. Note that both $\vec{B}$ and $\vec{m}$ are vector quantities.

The field near any body of finite dimensions containing current distributions and/or magnetic materials will generally be a more complex

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Figure 2. Coordinates for Dipole Field

function of position than that given in Equation 2. It is known, however, that any such system may be described in terms of its dipole moment, quadrupole moment, and multipoles of higher order, the field being obtainable by a series expansion in terms of these higher-order moments. These higher moments have the property that their field strengths drop off faster than the inverse cube relation with distance given by Equation 2 for the dipole. Consequently, at a point sufficiently far from the body, the field given by Equation 2 will be a close approximation to the true field. Conversely, if the observed field is following an inverse cube law with distance, it may be concluded that the observed field is in the "dipole region" of the body and Equation 2 may be used to evaluate the dipole moment and to extrapolate the field to greater distances.

In any case, only the dipole moment of a distribution of materials and currents causes a net torque when the system interacts with an externally applied field. The higher moments do not produce a torque.

The magnetic field due to a current carrying wire may be evaluated from the basic physical relationship,

$$dB = \frac{I \overline{ds} \times \overline{r}}{r^3} \quad \dots (3)$$

where

I = the current flowing in amperes

ds = an element of length along the wire

$\overline{r}$ = a vector from the wire element location to the field point.

This relationship may be used to derive the dipole moment of a closed current circuit,⁽¹⁾

$$\overline{m} = IA \overline{i}_n \quad \dots (4)$$

where I is the loop current, A is the area enclosed by the current loop, and $\overline{m}$ is directed along the positive normal ($\overline{i}_n$) to the current loop. Equations 3 and 4 will be used to evaluate the dipole moment of the converter wiring.

Permanently magnetized material may be described by its magnetization vector at each point in the body. The source of magnetization is believed to be atomic current loops in the material, each following Equation 4, which have been oriented along a desired direction by the application of an external field, and which retain the orientation even after removal of the field. The magnetization vector is given in terms of other field quantities by the relation

$$\overline{M} = \frac{\overline{B}}{\mu_o} + \overline{H} \quad \dots (5)$$

where

$\overline{M}$ is the magnetization in amp/m

$\overline{H}$ is the magnetic intensity in amp/m.

The actual values of each of these field quantities in ferromagnetic material is dependent on the prior treatment of the materials, and in

particular on the external fields to which it has been subjected. Property curves describing the values of B and H to be expected from a given treatment are available from the manufacturer of magnetic materials. However, a precise knowledge of M at every point in a magnetized body is rarely available.

In terms of the magnetization vector M , the dipole moment of magnetic material⁽²⁾ is given by

$$\overline{m} = \int_{Vol} \overline{M} dV \quad \dots (6)$$

where the integration is to be carried out over the volume of the material involved. Equation 6 is used in the theoretical evaluation of the dipole moment of the magnetic circuit of the electro-magnetic pump. The experimental evaluation is based on field strength measurements, and the use of Equation 2.

If a ferromagnetic material is placed in a magnetic field, the field will induce in the body an alignment of the crystals similar to that in a permanent magnet. The effect of this alignment is to produce flux density B in the body substantially greater than that existing in the surrounding free space. If the body is a sphere and the material substantially isotropic (e.g., soft iron), the field in the body will align itself with the exciting field. From basic field theory it is known that the magnetic intensity H is continuous across the boundary of the body so that

$$\overline{H}_i = \overline{H}_o = \frac{\overline{B}_o}{\mu_o} \quad \dots (7)$$

where

H_i = the field intensity in the body

H_o = the field intensity outside the body

B_o = the flux density outside the body

The flux density in the body is related to the magnetic intensity by

$$\bar{B}_i = \mu \bar{H}_i = \frac{\mu}{\mu_0} B_0 \quad \dots (8)$$

Therefore the magnetization in the body, $\bar{M}_i$ is

$$\bar{M}_i = \frac{B_i}{\mu_0} + H_i = \left(\frac{\mu}{\mu_0} + 1 \right) \frac{B_0}{\mu_0} \quad \dots (9)$$

Values of μ/μ_0 vary widely among different materials and with field strength. At low field strengths, the value for ferromagnetic materials may be several hundred.

Since in a sphere the field within the body is aligned with the external field, there is no net torque acting on the body to change its orientation. In nonspherical bodies, the process is more complicated. The field inside the body is influenced not only by the external field and the μ of the material, but also by the body geometry. Equation 9 is often modified by introducing a demagnetization factor N_D to describe the geometrical effect. Thus we have

$$\bar{M}_i = N_D \left(\frac{\mu}{\mu_0} + 1 \right) \frac{\bar{B}_0}{\mu_0} \quad \dots (10)$$

where N_D is dependent on the relative orientations of the body and the field. For long slender rods, N_D is substantially one for fields along the rod axis and very small for transverse fields. As a result of this, the magnetization of the rod is

due almost entirely to that component of the field along the rod axis, and is itself aligned with the rod axis.

Thus,

$$|\bar{M}_i| = N_D \left(\frac{\mu}{\mu_0} + 1 \right) \frac{B_0}{\mu_0} \cos \theta \quad \dots (11)$$

where θ is the angle between the field and the rod axis, and also between the resulting $\bar{M}_i$ and B_0 . Since $\bar{M}_i$ and $\bar{B}_0$ are not aligned, a torque will exist which will try to align the rod along the field lines.

Thus,

$$\begin{aligned} T &\cong |\bar{M} \times \bar{B}| = \left(\frac{\mu}{\mu_0} + 1 \right) V \frac{B_0^2}{\mu_0} \cos \theta \sin \theta \\ &= \frac{1}{2\mu_0} \left(\frac{\mu}{\mu_0} + 1 \right) V B_0^2 \sin 2\theta \quad \dots (12) \end{aligned}$$

where V is the volume of the rod.

Demagnetization factors for shapes other than rods are available in the literature.⁽³⁾ In general, blunt objects will have little torque exerted on them by this mechanism. Since the magnetization in the body is proportional to the applied field, the dipole moments obtained in this way are small compared to those obtainable with a comparable volume of permanent magnet material.

IV. AN ILLUSTRATIVE EXAMPLE

Before proceeding with a detailed evaluation of the magnetic torques in the SNAP 10A system, it is well to carry through the analysis for a very simplified but surprisingly accurate example. The objective in doing this is twofold: first, it will enable the full scope of the problem to be grasped without becoming overly embroiled in detail, and second, it will serve to determine the engineering significance of the problem.

The first step is to obtain an analytical model for the field of the earth. The simplest model describes the earth's field in terms of an equivalent dipole located at the center of the earth, having its axis aligned with the geographic polar axis, and having its "north pole" pointing to the south pole of the earth. The dipole moment of the earth is about 0.81×10^{23} amp m². (2) Figure 3 is a sketch of this model, showing the dipole as a bar magnet and giving the field components at a point in the orbital path of a satellite. The field components are given by the basic dipole Equation 2, where the radius is the distance from the earth's center to the satellite orbit.

The next step is to describe the vehicle in terms of its dipole moment. For the purposes

of this example, it will be assumed that all of the dipole moment is due to the electromagnetic pump and is equivalent to a dipole strength of 100 amp m^2 aligned with the +Z axis of the vehicle.

The torque on the vehicle is to be evaluated by using the basic equation of magnetic torque, Equation 1, where the field is that of the earth and the moment is that given above for the pump. However, before Equation 1 can be applied, it is necessary to express the earth's field in the vehicle coordinate system. If this can be done, the torque will also be expressed in terms of the vehicle coordinate systems, and thus can be used to evaluate the resulting vehicle motion.

Even if it is assumed that the vehicle attitude is not changing, the vehicle coordinate system is rotating and translating with respect to the earth's coordinates. Thus, when the earth's field is expressed in the vehicle coordinates, it will be time dependent, and can be expected to be periodic with period equal to the orbital period. For a circular polar orbit the coordinate transformation rules can be determined from the sketch of Figure 4.

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Figure 3. Dipole Model of the Earth's Field

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Figure 4. Earth and Vehicle Coordinates

From the figures, the following conclusions can be drawn. First, the radial component of the earth's field is always aligned with the vehicle yaw (x) axis; second, there is no component of the earth's field along the vehicle pitch (y) axis; and third, the " φ " component of the earth's field is either parallel or anti-parallel to the vehicle roll (z) axis. Since the magnetic dipole moment of the vehicle is aligned with the roll (z) vehicle axis, the cross product equation for torque reduces to,

$$\overline{T} = M_z B_r \hat{i}_y \quad \dots (13)$$

that is, the torque vector has only a pitch axis component and is proportional to the product of the pump dipole moment and the radial field component.

The vehicle torque can now be expressed as a function of time by expressing the radial component of the earth's field as a function of the orbital angle φ and replacing φ by $\omega_o t$, the orbital angular velocity times time.

Thus,

$$B_r = \frac{-2\mu_o |m_e|}{4\pi r^3} \cos \varphi \quad \dots (14)$$

and

$$T_y = \frac{-2\mu_o |m_e| |m_z|}{4\pi r^3} \cos \omega_o t \quad \dots (15)$$

where T_y is the torque exerted on the pitch axis of the vehicle. As expected, the torque is periodic with an angular frequency equal to the orbital angular frequency, and is also proportional to the product of the earth's dipole moment and the vehicle dipole moment. Using the values for these quantities as given above, and assuming an orbit attitude of 700 N. Mi, the torque is evaluated numerically to be

$$T_y = -0.522 \cos 0.0567 t \quad \dots (16)$$

where the torque is given in oz-in., and time is in minutes.

In evaluating the above torque, it has implicitly been assumed that the vehicle attitude corresponds to the desired orbital attitude. In fact, this torque and other disturbances will cause vehicle motions which in turn result in changes in the actual torque. If the motions are small, which is certainly desirable, the actual torque will not deviate too much from that computed by neglecting this effect. In order to illustrate the significance of these torques, a computation of vehicle motions on the above basis using a simple model of the vehicle can be made. It is assumed that the pitch axis motion is uncoupled to the roll-yaw motion, and that the system dynamics are adequately described by a dumbbell gravity gradient stabilized model. With these assumptions, the equation describing pitch axis motion⁽⁴⁾ is,

$$\frac{1}{3\omega_o^2} \frac{d^2\theta_p}{dt^2} + \theta_p = \frac{T_p}{3\omega_o^2 I_p} \quad \dots (17)$$

where θ_p is the angular deviation from the vertical, I_p is the vehicle moment of inertia, and T_p is the torque exerted on the pitch axis of the vehicle.

The differential equation may be solved for the sinusoidal steady state motion as,

$$\theta_p = |\theta_p| \cos \omega_o t \quad \dots (18)$$

where $|\theta_p|$ is the peak deviation from the equilibrium (desired) position and is given as

$$|\theta_p| = \frac{T_y}{2I_p \omega_o^2} \quad \dots (19)$$

Using the numerical values of Equation 16, together with a value of 10,000 slug ft² for I_p , the peak angular motion is found to be 8.75°.

It may be difficult at first to grasp the nature of the motion computed above. The vehicle oscillates about its equilibrium position, completing one oscillation each orbit. A sketch of the vehicle attitude at various positions in orbit is given in Figure 5. As can be seen, the maximum deviations occur over the poles, and the deviation from the desired orientation goes to zero over the equator.

The motion described above is not negligible, but may not cause any important problems. This question can only be answered by a systems analysis which includes the effect on the vehicle mission of motion of this sort. Regarding the accuracy of the above estimate, it should be noted that the Agena vehicle used in SNAPSHOT has gyroscopic damping systems which may damp the computed motion somewhat. However, the physical reason for such a small torque resulting in significant vehicle motion is that

the torque is periodic and has a period near a natural vehicle resonance. The damping system is hard pressed to counteract this type of input.

The above remarks raise an important question: "What can be done to effectively reduce the vehicle motions if they turn out to be too large?" Fortunately, a simple solution exists. If another permanent magnet is installed on the vehicle such that its dipole moment is equal to that of the pump, but oriented to be antiparallel to the pump moment, the torque on this magnet will just cancel that on the pump and no net torque will be exerted on the vehicle. The advantage of this approach lies in the fact that the "counter torque magnet" need not be located near the offending magnet. Only its magnitude and alignment must be carefully controlled. The subject of torque cancellation and the problems associated with it will be discussed further in Section VII.

Figure 5. Sketch Showing Coordinate Deviations From the Desired Orientation Due to Magnetic Torques

V. DIPOLE MOMENTS OF THE SNAP 10A

A study of the SNAP 10A system has revealed several potential sources of dipole moments in the system. These may be listed in order of importance as follows:

- 1) Pump magnet assembly
- 2) Current flowing in converter wiring
- 3) Drum drive motor magnets
- 4) Current flowing in pump thermoelectric circuits
- 5) Permeable piping materials
- 6) Control and instrument wiring.

Quantitative evaluation of each of these sources was undertaken. The methods used and the results obtained are described in the succeeding paragraphs.

A. DIPOLE MOMENT OF THE PUMP MAGNET ASSEMBLY

Since a preliminary evaluation indicated that the pump magnet assembly is the principal source of vehicle dipole moments, considerable attention has been given to the evaluation of the moment. Two methods have been used. The first is basically analytical, although measurements are used to "fit" the model. The second is experimental.

B. ANALYTICAL EVALUATION OF THE PUMP DIPOLE MOMENT

The analytical approach is based on a direct appeal to Equation 6, i.e.,

$$\bar{m} = \int_{\text{vol}} \bar{M} dV$$

A sketch of the pump magnet assembly is shown in Figure 6. In order to apply Equation 6, it is necessary to know the state of magnetization at all points in the magnet assembly. The only measurement which sheds light on this is a

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Figure 6. Pump Magnetic Assembly

measurement of the flux in the gap of the magnetic circuit. It may be assumed that the field in the iron at the edge of the gap is about the same as the gap field. The magnet design equations^(5,6) provide an estimate of the flux at the neutral axis of each magnet (the point opposite the gap), namely

$$|M_1| = F |M_3| \quad \dots (20)$$

where M_1 is the neutral axis magnetization and M_3 is the magnetization in the iron near the gap. For the assembly used in the SNAP 10A pump, the factor F is about 3.6. With the gap flux used in the SNAP 10A pump, Equation 20 predicts a flux at the neutral axis which is about 90% of the saturation flux in Alnico 5.

It is evident from Equation 20 that the field strength in the magnet drops off rapidly as we proceed from the neutral axis to the gap, due to leakage of flux from the magnet limbs. The analytical model used assumes that the flux magnitude drops off as the square of angle from the neutral axis to the top of the pole piece, and then linearly with length in the iron. It is further assumed

that at each point in the material the magnetization vector is perpendicular to the magnet cross-section. Since the relative amounts of leakage from the magnet limbs themselves and from the pole pieces is unknown, an additional parameter is included to permit parametric evaluation. The parameter is M_2 , the magnetization at the uppermost (and lowermost) portion of the magnet limbs.

The total dipole moment is given by

$$m_t = 2m_m - m_I \quad \dots (21)$$

where m_m is the dipole moment of each magnet limb and m_I is that of the pole piece. For the magnet limbs we have (see Appendix 1 for details)

$$m_m = M(2A_m R) \quad \dots (22)$$

where

$$M = M_2 \left(1 - \frac{8}{\pi^2} \right) + \frac{8}{\pi^2} \frac{M_1}{M_2} \quad \dots (23)$$

A_m is the magnet cross-sectional area and R is the magnet radius. For the iron we have

$$m_I = M_I(2\ell_p A_I) \quad \dots (24)$$

where

$$M_I = \frac{M_2 + M_3}{2} \quad \dots (25)$$

ℓ_p is the length of one pole piece and A_I is the cross-sectional area of the iron at the gap.

The following parameter values apply,

$$M_3 = 2.03 \times 10^5 \text{ amp/m (based on gap flux)}$$

$$M_1 = 3.6 M_3 \quad \text{(from design equation)}$$

$$A = 1.38 \text{ in.}^2$$

$$R = 1.82 \text{ in.}$$

$$\ell_p = 1.57 \text{ in.}$$

$$A_I = 1.69 \text{ in.}^2$$

The value of M_2 may range from M_1 to M_3 . Numerical evaluation of the dipole moment for a range of values yields the results of Table 1

TABLE 1
PUMP DIPOLE MOMENTS
FOR SEVERAL VALUES OF
 M_2

M_1/M_2	m_m	m_I	m_T
1	62.3	43.1	81.5
1.5	58.0	30.0	86.0
2.0	56.6	24.7	88.5
3.0	54.4	19.5	89.3
3.6	53.9	17.7	90.1

The above results indicate a general insensitivity of the net dipole moment to the value of M_2 . Based on this analysis, a value of $m_T = 87.1$ amp m^2 is recommended (this is the arithmetic mean). The dipole moment is aligned with the pole piece and "points" toward the north magnet pole.

C. EXPERIMENTAL EVALUATION OF THE PUMP DIPOLE MOMENT

Because of the large size of the dipole moment that was obtained by the above analysis, and because of the assumptions necessary to obtain results analytically, it was necessary to verify experimentally the dipole moment value. The experimental method used was to map the flux density field around the pump. As indicated in Section IV, at distances sufficiently far from any magnetized body, this field will show an inverse cube law dependence on distance from the body. If data can be obtained in this region of the field, Equation 2 can be used to evaluate the dipole moment of the body producing the field.

Some care must be taken in setting up an experimental evaluation based on the above

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Figure 7. Experimental Equipment for the Determination of Pump Dipole Moments

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Figure 8. Axes for Magnetic Measurements

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Figure 9. Field Strength Data as a Function of Position, Experiment 1

approach. The field strengths that must be measured are quite low. For example, if the actual dipole moment is 100 amp m^2 , the field strength at a distance of one-half meter from the pump would be only 1.6 gauss. Since the earth's field is about 0.4 gauss, a potential error of 25% exists in the method. The instruments used in the evaluation used Hall effect probes to measure the field. The probe is sensitive to the net flux passing through it. The earth's field was eliminated by making pairs of measurements at a given distance from the center of the body, one on the north pole side and the other on the south pole side. In one case the earth's field will add to the pump field, and in the other case it will subtract. Averaging the two readings eliminates the effect of the earth's field.

In addition to the above problems, it is necessary to perform the experiment in an environment that is relatively free of stray fields or ferrous metal bodies. The experiment was conducted on a wooden table in the center of a "screen" room about 15 ft square. The room is designed to shield out electromagnetic fields. It was found that very little attenuation of the earth's field was produced by the mu-metal skin on the walls. However, the room was relatively free of disturbing influences other than the earth's field.

The experimental procedure was "checked out" by evaluating the dipole moment of a small bar magnet. The magnet was not a "standard," but it was possible to estimate its dipole moment analytically with somewhat more a priori confidence than could be placed on the pump magnet analysis.

Two experiments were made on actual system components. The first was made on the magnet assembly only. A Radio Frequency Laboratories Model 1890 Gaussmeter was used. The lowest range scale on this meter is from 0 to 1 gauss

The second experiment was made on a complete pump, and an RFL Model 2000 gaussmeter was used with a lowest range of from 0 to 1/10 of a gauss. A photograph of this experimental setup is shown in Figure 7.

Data were taken along three axes through the pump as shown in Figure 8. On each axis, the probe was placed perpendicular to the axis. Field strength measurements were made at several different distances, measured from the center of the pump to the probe. With this "normal" orientation, the probe measures the radial field component. Data from the two experiments, as taken along the A-A' axis are shown in Figures 9 and 10. Here a log-log plot has been used and a straight line with a slope of -3 put through the data. As can be seen from both figures, all the data points fall close to the line, indicating that the pump field is substantially that of a dipole over the full range of the experiment. Any point on the line can then be used to estimate the dipole moment. By applying suitable conversion factors to Equation 2, a data reduction formula can be obtained in the form

$$m = \frac{0.0082 B_r r^3}{\cos \theta} \quad \dots (26)$$

where m is the dipole moment in $\text{amp} \cdot \text{m}^2$, B_r is the measured field strength in gauss, r is the distance from probe to pump center in inches, and θ is the angle of the measurement axis from the pump north-south line. Based on the above expression, the data of Figure 7 indicates a dipole moment of 87 and that of Figure 8 a value of 103 amp m^2 . The angular data yield similar results, with small differences being attributed to the difficulty of measuring the angle with precision. A tabulation of data and computations is presented in Appendix II. Based on the experiment, the recommended value of pump dipole moment is 95 amp m^2 , which is

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Figure 10. Field Strength Data as a Function of Position, Experiment 2

in fair agreement with the theoretical evaluation. Based on the scatter in the data, differences between the two experiments, and differences between the analysis and the experimental results, it is expected that the above recommended value is within $\pm 10\%$ of the value for the flight system.

D. DIPOLE MOMENT OF THE THERMOELECTRIC CONVERTER

The SNAP 10A uses a direct radiating thermoelectric energy conversion system to produce electric power from thermoenergy. The heat from the nuclear core is transferred to the hot junction of the TE elements by the circulating liquid-metal heat-transfer medium. Figure 11 shows the basic thermoelectric modules, and Figure 12 shows the full converter assembly on a flight system. The individual thermocouples are essentially series connected (the actual connection uses both series and parallel paths to provide redundancy) and in normal operation, current flows over the surface of the conical structure much as is shown in Figure 13.

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Figure 11. Portion of Thermoelectric Converter Leg

The vehicle torque, due to interaction of the earth field with the current flow, may be obtained readily by perceiving the actual current flow as current loops. Such a system of current loops is shown in Figure 14, and is known as Amperian construction. Note that each of the loops is closed, and is considered to carry a current whose value is one-half of the actual converter current. Where two loops have a common branch, the branch current is found by adding the loop currents vectorially. Thus, adjacent loops on the conical surface always add, but at the top and bottom, the loops on the conical surface alternately add to or subtract from the top and bottom loops. The result is that the configuration of current loops produces a net current distribution which is identical to that flowing in the converter, and therefore must have the same dipole moment as the actual converter. The converter dipole moment is thus readily found by adding vectorially the dipole moments of the individual current loops.

Consider first the current loops drawn on the conical skin. These can be paired by taking loops on opposite sides of the cone. As long as the number of legs is even, as it must be to have

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Figure 12. Complete Thermoelectric Converter

both electrical terminals at the same end of the converter, the paired loops have dipole moments which are equal in magnitude and opposite in direction. Hence, their dipole moments cancel, and it follows that the current paths on the converter skin do not contribute a net dipole moment to the system.

Next, consider the top and bottom current loops. As shown, each of these flows counter-

clockwise. The dipole moments are given by product of the loop area, and one-half of the converter current, and they add to give a net dipole moment along the $-X$ axis of the system. The upper loop area is 0.265 m^2 and the lower loop area is 1.215 m^2 . If the converter current is taken as 18 amp, the total dipole moment is 13.3 amp m^2 . A small increase in this value, necessitated by the presence of permeable material in the system, will be discussed later.

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Figure 13. Sketch of Converter Current Flow

E. OTHER DIPOLE SOURCES

Two of the six sources of dipole moments on the SNAP 10A system have been discussed in the preceding sections. The remaining four, namely pump current loops, drum drive motors, control and instrumentation wiring, and permeable piping material, have been found to be of much less importance than those discussed above. Each of these will be discussed below.

The electromagnetic pump used in the SNAP 10A system has an integral current supply based on thermoelectric energy conversion. Heat is taken directly from the pumped fluid and is rejected to space. The thermoelectric elements are designed to produce high current at very low voltage. A current of about 400 amp flows through the NaK perpendicular to the magnetic field, the interaction between the two, producing the pumping pressure. The dipole moment due to the current flowing through the NaK and the thermoelectric elements may be evaluated based on a current loop model. Even though the area involved is very small, the

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Figure 14. Sketch of Converter Current Flow Showing Amperian Current Loops

high current potentially suggests a substantial dipole moment. Actually, two current circuits are used to produce the total current flow. These circuits (Figure 15) are on opposite sides of the pump throat, and have the NaK as a common link. Viewed from above, the loop current flow is clockwise in one and counterclockwise in the other, the net result being a dipole moment of zero, assuming balanced circuits. On this basis the pump current loop contribution is neglected.

The startup of the SNAP 10A system is accomplished by stepping in two control drums which form a portion of the reflector around the nuclear core. Especially designed dc motor actuators are used for this stepping function. Each of the motors contains a small permanent magnet and an iron field path containing appropriately arranged gaps. As a result of the iron return paths surrounding the magnet, and the relatively small size of the magnet, the leakage field around the motor is quite small. Since it

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Figure 15. Cross Section View of Thermoelectric Pump

is this leakage field which must be characterized by a dipole moment to estimate the torque, the dipole moment is also small. Based on field strength measurements similar to those described for the pump, it is estimated that the dipole moment contribution of the motors is about 1 amp m^2 each, oriented along the +X axis of the system. Thus, this effect can be subtracted from the dipole of the current loops, but changes the resulting value by only 15%.

It should be noted that under conditions of actual stepping, the electromagnets in the motor will be energized and will change the dipole moment. However, the SNAP 10A control system is active only during the initial portion of the flight, when a reaction jet control system is available to ensure proper vehicle attitude. In addition, the activation is only momentary, so the effect on vehicle attitude will be small.

Ferromagnetic materials have been generally avoided in the SNAP 10A system. However, one significant source of this type of material is in the converter NaK tubes (known as D tubes). These tubes are made of 405 stainless steel, and constitute a total weight of about 28 lb and volume of about 100 in^3 . These tubes produce two effects of interest here. First, the tubes lie within the current loops of the converter and are approximately parallel to the resulting dipole moment. The tubes form, in effect, a permeable

core for the electromagnet represented by the current loops. Second, the tubes, being long and thin, behave as rods of permeable material in the earth's field, as discussed in Section III. Thus, there arises both a constant dipole moment, due to the converter field, and a changing dipole moment due to the earth's field.

To quantitatively evaluate the first effect, it is assumed that the converter is a solenoid and has a core consisting primarily of vacuum but also of a small amount of material of permeability μ_i much larger than that of free space. The dipole moment of such a configuration is then given by

$$m = m_o \left(1 + \frac{\mu_i A_i}{\mu_o A_o} \right) \quad \dots (27)$$

where A_i is the area of high permeability, A_o is the "free space" area (in this case substantially equal to the total area) and m_o is the dipole moment found if the permeable materials is neglected. No data has been found on the low field strength permeability of 405 SS, but based on the high-field-strength data, and low-field-strength data on other materials, μ_i/μ_o is estimated to be about 200. Since A_i/A_o is 0.00113, the above formula predicts an increase in dipole moment due to this effect of 22.6%. The actual effect will be even smaller since a solenoid model was used which implied that the field due to the current was parallel everywhere to the tubes. In practice, the field will not line up with the tubes and the resulting increase in dipole moment will be less than that suggested above. An actual increase of 15% is used in subsequent calculations.

As was indicated in Section III, the induced dipole moment resulting from the action of the earth's field on permeable materials is small. The method of including the effect is to assume that the radial component of the earth's field induces a dipole moment in the tubes of magnitude

$$m = \frac{B_r}{\mu_o} \left(1 + \frac{\mu_i}{\mu_o} \right) V_T \quad \dots (28)$$

where V_T is the tube total volume. This amounts to neglecting the 9° inclination of the tubes and assuming the demagnetizing factor N_D is unity. Thus, we have a time varying dipole moment which may be treated otherwise in the same manner as any dipole moment. The peak value of the dipole moment is about 0.8 amp. m^2 . The peak torque resulting from this is evaluated in Appendix III to be $0.86 \times 10^{-4} \text{ oz-in.}$ and on this basis the effect is neglected in the remainder of this work.

The present SNAP 10A design contains a substantial amount of diagnostic instrumentation which will enable a careful evaluation of the system performance in orbit. This wiring, in addition to the startup and control system, constitutes a potential source of magnetic dipole moments. However, the expected current flow under normal operation is not large and the

component placement results in closely bundled wires. It has therefore been assumed that the individual dipole moment contributions will be small and will tend to cancel due to the "random" orientation resulting from normal component placement considerations.

F. SUMMARY OF DIPOLE MOMENTS

A tabular summary of the results of this inventory of dipole moments is given in Table 2.

TABLE 2
TABULAR SUMMARY OF MAGNETIC
DIPOLE MOMENTS

Source	M_x	M_y	M_z
Pump Magnet	0	0	95
Converter Current	-13.3	0	0
"D" Tubes	-2.0	0	0
Drum Drive Motors	+2.0	0	0
Pump Current	0	0	0
Instrument Wiring	0	0	0
Totals	-13.3	0	95

VI. MAGNETIC FIELD OF THE EARTH AND VEHICLE TORQUES

Given a description of the system in terms of its dipole moment, the problem of determining the vehicle torques may be divided into three steps. First, the field of the earth must be specified in terms of the earth's coordinate system, at the orbital altitude; second, a coordinate transformation must be made to express the field in vehicle coordinate system, which is in constant motion with respect to the earth; and third, the vector cross product of the system dipole moment with the earth's field must be carried out at each point in the orbit. Each of these problems is discussed below.

In the illustrative example of Section III, the field of the earth was described by a simple dipole, located at the center of the earth and aligned with the true north-south geographic axis. In fact, this is really a zeroth order approximation to the true field of the earth. A first-order model of the earth's field would assume that the dipole was still centered in the earth, but would recognize that the magnetic poles are misaligned from the geographic poles by about 11° of latitude and 70° of longitude. With this model, analytical calculations of the vehicle torques are still possible but the resulting expressions are cumbersome and recourse must be had to approximate forms or numerical evaluation. In view of this, it seems worth while to skip this step and, except for checking purposes, to proceed directly to a detailed model, requiring digital computation, but permitting high accuracy in the representation. Such a model is provided by spherical harmonic analysis, a method originally proposed by Gauss in the 18th century.^(7, 8)

Qualitatively, spherical harmonic analysis describes the field in terms of a series expansion in altitude, latitude, and longitude. The coefficients of the series are evaluated by fitting

the series to experimental observations of the earth's field. The expansion used here was taken from Reference 9 and is given as:

$$W = a \sum_{n=1}^{\infty} \sum_{m=0}^{m=n} \left(\frac{a}{r}\right)^{n+1} \left(g_n^m \cos m\lambda + h_n^m \sin m\lambda \right) P_n^m(\cos \theta) \quad \dots (29)$$

where

W = the geomagnetic potential function

a = the radius of the earth

r = the distance from the center of the earth to the orbital altitude

λ = east longitude

θ = co-latitude

g_n^m, h_n^m = the gaussian coefficients

$P_n^m(\cos \theta)$ = an associated Legendre function of degree n and order m , of the partly normalized type introduced by A. Schmidt.

The field quantities are obtained from the potential by taking the gradient of V . Thus we have

$$B_r = \frac{\partial W}{\partial r} \quad \dots (30)$$

$$B_\theta = \frac{1}{r} \frac{\partial W}{\partial \theta} \quad \dots (31)$$

$$B_\phi = -\left(\frac{1}{r \sin \theta}\right) \frac{\partial W}{\partial \phi} \quad \dots (32)$$

where B_r is the component along the radius toward the center of the earth, B_θ is the component in an easterly direction.

In the present work, the first four terms of the outer sum were retained. This results in terms depending on orbital radius to the -3 through

-6 power being retained. The gradient operation and evaluation of the series coefficients were performed manually and the resulting expressions were programmed for machine evaluation. (The field expansions are given in Appendix IV.)

A polar orbit is used in the .SNAPSHOT program. The orbit must be long lived to permit time for the decay of the radioactive fission products. The nominal orbit is circular with a 700 N. Mi altitude. The SNAP 10A-Agena combination is a long slender vehicle. The long axis (x axis) is aligned with the local vertical and the direction of travel is defined by the +z vehicle axis. Attitude stabilization is provided by gravity gradient and damping gyros. Therefore, the coordinate transformation from the earth system to the vehicle system is relatively simple and is illustrated in Figure 16. As noted in this figure, and in the illustrative example of Section III, the symmetry of the functions involved permit a simplification which enables the series to be written directly as a function of time, with latitude increasing with an angular frequency equal to the orbital frequency, and longitude increasing at the earth's rotational frequency.

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Note: It is possible to simplify coordinate conversion by letting the range of θ be $0 \leq \theta \leq 360^\circ$, identifying unit vectors according to the rule $i_x = -i_r$, $i_\theta = -i_z$, $i_\phi = -i_y$, and ignoring the fact that ϕ is normally increased by 180° going over the South Pole.

Figure 16. Coordinate Systems

Once the coordinate transformation rules have been established, it is possible to express the field as a function of time in the vehicle coordinate system. Figures 17, 18, and 19 contain plots of the field obtained from a computer display device. The period shown covers about one day. The function is not quite periodic (it is "almost periodic" in the sense described in Reference 10), since the orbital frequency is not an integral multiple of the earth's rotational frequency. The predominant frequency in the x axis field (Figure 17) is the orbital frequency. The same is true for the roll axis (z) field (Figure 16C). However, the pitch axis field shows a primary component at twice the orbital frequency, and also a strong component (or "average" level) at the earth's rotational frequency (Figure 18). The dipole model in Section III does not even hint at this effect, but it is predicted by the first order dipole model discussed in the beginning of this section.

The series expansion discussed above provides the capability of computing the field acting at the vehicle location at each point of its orbit. This information may be combined with the dipole moments of the vehicle to find the torques acting at each instant. The required equation was given in Section III and may be expanded as follows:

$$\vec{T} = \vec{m} \times \vec{B} = \begin{vmatrix} \vec{i}_x & \vec{i}_y & \vec{i}_z \\ m_x & m_y & m_z \\ B_x & B_y & B_z \end{vmatrix} \quad \dots (33)$$

where the torque components obtained by determinant evaluation are:

$$T_x = m_y B_z - m_z B_y \quad \dots (34)$$

$$T_y = m_z B_x - m_x B_z \quad \dots (35)$$

$$T_z = m_x B_y - m_y B_x \quad \dots (36)$$

Figure 17. Yaw Axis Magnetic Field Component

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Figure 18. Pitch Axis Magnetic Field Component

Figure 19. Roll Axis Magnetic Field Component

SNAP IOA MAGETIC TORQUES 600N.M.

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Figure 20. Yaw Torque Component

Figure 21. Pitch Torque Component

Figure 22. Roll Torque Component

Actually, the torque computations require knowledge of the vehicle attitude with respect to the desired attitude. In the present work it is assumed that the vehicle remains substantially in the desired attitude. This, in effect, amounts to providing an estimate of the torque loads that must be countered by the vehicle attitude control system to prevent undesired vehicle motion.

Equations 34, 35, and 36 above have been implemented in the same computer program which evaluates the magnetic field. The torque loads on the vehicle, due to interaction of the vehicle moments with the earth's field are shown in Figures 20, 21, and 22. A period of one day is shown. The dipole moments used in the run shown were $m_z = 91.8 \text{ amp m}^2$ and $m_x = -15.9 \text{ amp m}^2$, which are substantially the same as the values obtained in Section VI. A rapid scan of the figures will reveal that the greatest torques occur on the pitch (y) axis (Figure 21) and that these torques have a strong periodic component at the orbital frequency, although the distortion produced by the presence of other harmonics is evident. The peak torque is about 0.5 oz-in., which is substantially the same as was predicted by the

illustrative example of Section IV. The second greatest torque component is that on the yaw (x) axis, Figure 20, where strong components occur at the earth's rotational frequency and at twice the orbital frequency. The peak torque is shown about 0.12 oz-in., or a factor of about 4 lower than that on the pitch axis. The average "amplitude" of the component at twice the orbital frequency is about 0.05 oz-in. The roll (z) axis torque component, Figure 22, is quite small, and again shows primary components at the earth's rotational frequency and twice the orbital frequency.

The curves shown in Figures 20, 21, and 22, are felt to be the end item of this work. Properly, the study should go on to demonstrate that the flight vehicle can satisfactorily maintain the desired orbital attitude with the torque loads shown in these figures. For the SNAPSHOT program, this is the role of the vehicle supplier, who has complete information on the attitude control system. Since the SNAP 10A is designed to impose a minimum of interface requirements on the carrier vehicle and payload, methods of reducing or eliminating these torque loads are discussed in Section VII.

VII. METHODS OF REDUCING VEHICLE TORQUES

As was indicated in Section IV, even the small torques produced by interaction of the earth's field with vehicle dipole moments of the order of those on the SNAP 10A system can produce motions on the order of 10° from desired attitude. If this amount of motion is disruptive to communications or other vehicle experiments, the motion can be reduced by using an attitude control system. An active attitude control system, such as a reaction jet system, could virtually eliminate motion from these torques. For long-term flights, however, the resulting energy expenditures may be too high. Passive attitude stabilization systems, such as control moment gyros, on the other hand require little energy input, but also provide relatively light damping, and the vehicle motions for periodic torques such as magnetic torques may be close to those for undamped systems.

As an alternative to feedback control, one can attempt to cancel the undesired torques. In the case of magnetic torques, this implies cancelling of the dipole moments. The simplest procedure is to locate on the vehicle a permanent magnet having a dipole moment equal in size to the net vehicle moment, and oriented so as to cancel this moment. For the present system, such a magnet would weigh about 3 lb and can be located anywhere on the vehicle.

There are several problems associated with this approach. First, it is very difficult to experimentally verify that true cancellation has been obtained, since the system field will behave as a dipole only at substantial distances from the system, and at these distances the fields will be very small. It is, therefore, necessary to rely on calculations to size the magnet and determine the required orientation. Second, during the mission lifetime the original dipole moments may change due to component degrada-

tion. It is virtually impossible to ensure continuous cancellation under these conditions.

An improvement to the cancellation can be obtained by including on the vehicle, an electromagnet capable of generating moments along each of three perpendicular axes. By suitably setting the current in each axis, it is possible to obtain a range of magnitude and angle of the resulting dipole moment at only a modest energy expenditure. Use of such a device as a trimming adjustment to a permanent magnet cancellation scheme has been reported in the literature.⁽¹¹⁾ In this case, the adjustment was done by a ground command system after vehicle attitude data had been examined.

For advanced, higher-power versions of the SNAP 10A system, such as might be used on large unmanned, or manned experimental laboratories, the magnetic torques are potentially much greater. The relative importance of the dipole moment due to the converter increases rapidly with power. For example, if vehicle diameter and output voltage are roughly the same as in the present system, the converter dipole moment increases in proportion to the system power. Pumps on higher-power systems would also be larger. Fortunately, it is possible to apply the dipole moment cancellation technique at the design phase in these cases and keep the residual moment very low. In the case of the pump, advanced systems will probably use multiple pumps. In this case, if an even number of pumps is used, it will always be possible to orient them so that the dipole moments are in opposition and thus obtain "natural" cancellation.

Two moment cancellation schemes for the thermoelectric converter have been examined. The simplest method is called the counter torque ring. In this case, a current loop is put in series with the converter itself, so that all of the

converter current flows through the loop. The loop can be mounted on the same structure as the TE elements or can be placed in another region, possibly a lower-temperature region. The plane of the loop must be parallel to the upper and lower planes of the converter and the loop area must be equal to one-half the sum of the upper and lower areas. Actually, a multiple turn counter torque loop could be used to reduce the loop area. The relationship between loop area and number of turns is

$$A_{ctl} = \frac{1}{2N}(A_U + A_L) \quad \dots (37)$$

where A_{ctl} is the counter torque loop area, N is the number of turns, and A_U and A_L are the upper and lower loop areas.

The second method involves breaking the converter into two separate circuits which are paralleled to obtain the final output current. The two circuits are interleaved to produce the torque cancellation. Figure 23 shows a view of the converter with this arrangement. The method may present some fabrication problems. Another factor of importance is that this approach involves

a loss of redundancy in the circuit cross connections, and a corresponding loss of reliability. However, in higher-power systems, the degree of circuit paralleling will be high, and the reliability reduction involved in breaking the system into two main circuits will not be great

Figure 23. Converter Circuit Design for Zero Dipole Moment

VIII. CONCLUSIONS

The following conclusions have been drawn from this work

- 1) By applying basic principles of field theory, it has been possible to evaluate the dipole moments of the SNAP 10A system, and determine the torques which will result in a nominal orbit. The accuracy with which these are known is about $\pm 10\%$.

- 2) The vehicle motions resulting from these torques will not adversely affect the SNAPSHOT program mission.

- 3) Simple methods exist for reducing these torques by a factor of 10 or more for future SNAP 10A missions and for advanced systems based on the same concept.

NOMENCLATURE

Symbols

A = area
B = magnetic flux density, webers per square meter
ds = element of arc length
F = magnet design factor, dimensionless
H = magnetic intensity, amperes per meter
I = electric current, amperes
i = unit vector, subscript denotes axis
K = constant of proportionality
l = length
M = magnetization, amperes per meter
m = magnetic dipole moment, ampere meter²
N = number of turns
N_D = demagnetization factor, dimensionless
R = radius
r = radius, also used as a subscript
T = torque
t = time
V = volume
W = geomagnetic potential function
x = coordinate axis, used as subscript
y = coordinate axis, used as subscript
z = coordinate axis, used as subscript

Greek Symbols

θ = theta, angle or angular coordinate axis
 μ = mu, magnetic permeability, henries per meter
 π = pi, 3.1415926
 ϕ = phi, angle or angular coordinate axis
 ω = omega, angular velocity, radians per unit time

Note: Certain symbols denote vector quantities. These are shown with a bar (-) above the symbol, e.g. $\vec{B}$.

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APPENDIX 1

THEORETICAL EVALUATION OF THE PUMP MAGNETIC MOMENT

In order to apply Equation 6 to the evaluation of the dipole moment of the pump magnet it is necessary to specify the state of magnetization M at all points within the volume of magnets and the pole piece. The only experimentally available data on this is a field strength measurement in the gap. This value is nominally 2560 gauss, which corresponds to an M value in the iron at the edge of the gap of

$$M_g \cong \frac{B_g}{\mu_0} = 2.03 \times 10^5 \text{ amp/m}$$

since H in the iron (or in the magnet) is very small. In the determination of the required magnet size, the pump designers made an estimate, based on the magnet geometry, of the ratio of the gap flux to the "neutral axis" flux. The "neutral axis" flux is the flux in the magnet at the point opposite the gap. From this factor we can obtain the flux at the magnet neutral axis. Thus,

$$\frac{M_{NA}}{M_g} = F = 3.6 \quad \dots A1$$

It is known (Reference 10) that due to leakage the flux drops from its maximum value at the neutral axis to its minimum value at the gap. The above experimental and design information fixes these two values. In order to evaluate the integral given by Equation 6 a functional dependence of M on position between these values has been assumed. This information is defined in terms of Figure 6. The M values in the magnet limbs are taken as

$$M = M_\psi = M_1 - a\psi^2 \quad \dots A2$$

where

$$a = \left(\frac{2}{\pi}\right)^2 (M_1 - M_2) \quad \dots A3$$

That is, we have assumed M to be uniform across the magnet cross section, and lined up along the curvature of the magnet (ψ direction). The variation of magnitude has been taken as a quadratic function of angle. This indicates that leakage is greatest near the neutral axis, which is in agreement with available magnet design information.⁽¹⁰⁾

For convenience, a linear relationship was taken to describe variation of M in the iron pole piece. That is,

$$M = M_2 - bl \quad \dots A4$$

where l is distance from the top of the pole piece, and

$$b = \frac{M_2 - M_3}{l_p} \quad \dots A5$$

where l_p is the length of the pole piece. In terms of the previously defined quantities, $M_1 = M_{NA}$, and $M_3 = M_g$. The value of M_2 is not known, except that it must be intermediate between M_1 and M_3 . It will be retained as a parameter in the calculation until it can be shown that the magnetic moment is insensitive to its exact value.

From the symmetry of the magnetic circuit it is evident that the net moment M is a vector in the $+z$ direction. The other components will cancel as the integration is carried out. In evaluating Equation 6, we then consider only components along the z axis. For one magnet limb, we obtain

$$m = \int \vec{M} dV = 2 \int_0^{\pi/2} M_\psi \cos \psi AR d\psi \quad \dots A6$$

where A is the cross sectional area of the magnet, and we have integrated over the upper

quadrant, and doubled the result. If Equation 2 is substituted and the integration carried out we find

$$m_m = \bar{M}(2AR) \quad \dots A7$$

where

$$\bar{M} = M_2 \left[\left(1 - \frac{8}{\pi^2} \right) + \frac{8}{\pi^2} \frac{M_1}{M_2} \right] \quad \dots A8$$

Equation A7 has the following physical interpretation. The curved magnet limb has a dipole moment equivalent to that of a bar magnet having the same cross sectional area and a length equal to the magnet diameter, and having a uniform magnetization given by $\bar{M}$. $\bar{M}$ is seen to be an

average value for the magnet which places heavier weight on the neutral axis value than on the end value.

Since a linear variation of M with position in the pole piece has been assumed, the dipole moment of the two pole pieces can be written immediately as

$$\bar{m}_I = 2\ell_p \bar{M}_I A_I \quad \dots A9$$

with

$$|\bar{M}_I| = \frac{M_2 + M_3}{2} \quad \dots A10$$

and where A_I is the gap cross-sectional area.

The total magnetic dipole moment of the magnet is given by:

$$m_T = 2m_m - m_I \quad \dots A11$$

APPENDIX II **EXPERIMENTAL DATA**

RAW EXPERIMENTAL DATA

Observed Distance d_o	B_1	B_2	B_3	B_4	B_5	B_6
1	61	60	82	86	172	164
2	33	32	53	45	95	94
3	19	19.5	31	31	61	54
4	11.7	12.2	19.2	20	38	32.5
5	8.35	8.8	13.2	13.7	24.5	22.5
6	6.1	5.8	9.3	9.0	17.5	15.8
7	4.7	4.05	6.5	6.5	12.4	11.5
8	3.55	3.3	5.4	4.85	9.3	8.7
9	2.8	2.55	4.2	3.8	7.1	6.5
10	2.3	2.0	3.4	3.05	5.6	5.1
11	1.88	1.62	2.65	2.4	4.5	4.1
12	1.64	1.29	2.1	2.0	3.6	3.3
13	1.43	1.08	1.85	1.55	3.0	2.7
14	1.24	0.87	1.6	1.3	2.5	2.25
15	1.08	0.73	1.38	1.05	2.05	1.89
16	0.96	0.59	1.2	0.88	1.82	1.6
17	0.87	0.47	1.05	0.79	1.56	1.34
18	0.77	0.42	0.92	0.65	1.36	1.13
19	0.70	0.33	0.85	0.53	1.19	1.0
20	0.645	0.28	0.80	0.44	1.05	0.86
10	2.25	2.2	3.0	3.0	5.5	5.15

NOTES:

Axis Location

Meter type Radio Frequency Laboratories Gaussmeter
Model 1890. Meter reading without magnet present
0.27 Gauss.

B_1 is in Gauss
 D_o is in inches

DATA REDUCTION

d_o	dc	B_1	B_2	B_u	$(d_c)^3$	m
1	4.44	61.	60.	60.5	87.6	76.
2	5.44	33.	32.	32.5	162.	75.2
3	6.44	19.	19.5	19.3	268.	74.
4	7.44	11.7	12.2	12.0	413.	70.9
5	8.44	8.35	8.8	8.58	603.	74.1
6	9.44	6.1	5.8	5.95	841.	71.6
7	10.4	4.7	4.05	4.38	1125.	70.7
8	11.4	3.55	3.3	3.43	1485.	72.7
9	12.4	2.8	2.55	2.68	1910.	73.4
10	13.4	2.3	2.0	2.15	2400.	73.7
11	14.4	1.88	1.62	1.75	2986.	74.9
12	15.4	1.69	1.29	1.47	3652.	76.7
13	16.4	1.43	1.08	1.26	4411.	79.4
14	17.4	1.24	0.87	1.06	5268	79.8
15	18.4	1.08	0.73	0.905	6230.	80.7
16	19.4	0.96	0.59	0.78	7306.	81.6
17	20.4	0.87	0.47	0.67	8490.	81.4
18	21.4	0.77	0.42	0.595	9800.	83.5
19	22.4	0.70	0.33	0.515	11239.	82.4
20	23.4	0.645	0.28	0.463	12813.	84.9

FORMULAS

$$dc = d_o + 3.44$$

$$B_1 = (B_1 + B_2)/2$$

$$\overline{m} = 0.0143 \overline{B} d_c^3$$

$$\text{Mean value } \overline{m} = 76.9$$

EXPERIMENTAL DATA
(Experiment 2)

Distance* (in.)	B ₁ § †	B ₂	B ₃	B ₄	B ₅	B ₆
10					12.2	12.6
11					9.4	9.8
12					7.25	7.5
13	3.42	3.64	3.14	3.3	5.6	5.9
14	2.68	2.80	2.5	2.66	4.45	4.68
15	2.13	2.33	2.0	2.17	3.6	3.63
16	1.79	2.01	1.68	1.83	2.9	3.13
17	1.47	1.69	1.39	1.58	2.36	2.58
18	1.21	1.4	1.15	1.32	2.05	2.17
19	1.01	1.21	0.965	1.13	1.73	1.92
20	0.86	1.04	0.806	1.0	1.46	1.65
00					0.058	

NOTES:

*Distance from field point to center of pump throat.

†Field strength in Gauss, measured with Radio Frequency Laboratories Model 2000 Gaussmeter.

§Axes system as shown below:

APPENDIX III INDUCED DIPOLE MOMENT OF THE "D" TUBES AND THE RESULTING TORQUE

The induced dipole moment is proportional to the radial component of the earth's field as,

$$\bar{m} = K_1 B_r \bar{i}_x = m_x \bar{i}_x$$

where

$$K_1 = \frac{V_T}{\mu_o} \left(1 + \frac{\mu_1}{\mu_o} \right) = 2460 \frac{\text{amp-m}^4}{\text{weber}}$$

for the values given in the text. Using the dipole model for the earth's field, the field in the orbital path may be expressed as,

$$\bar{B} = K_2 (2 \cos \varphi \bar{i}_r + \sin \varphi \bar{i}_\varphi)$$

where

$$K_2 = 1.57 \times 10^{-5} \text{ webers/m}^2$$

or in the vehicle system

$$\bar{B} = K_2 (2 \cos \varphi \bar{i}_x + \sin \varphi \bar{i}_z)$$

The torque is given as,

$$\bar{T} = \bar{m} \times \bar{B}$$

$$= \begin{vmatrix} \bar{i}_x & \bar{i}_y & \bar{i}_z \\ 2K_1 K_2 \cos \varphi & 0 & 0 \\ 2K_2 \cos \varphi & 0 & K_2 \sin \varphi \end{vmatrix}$$

$$= 2K_1 K_2^2 \sin \varphi \cos \varphi \bar{i}_y$$

i.e., the torque is on the pitch axis of the vehicle. this may be written as,

$$T_y = K_1 K_2^2 \sin 2\varphi$$

so that the torque varies at a frequency given by twice the orbital frequency and has a peak amplitude of

$$\begin{aligned} |T_y| &= K_1 K_2^2 = 0.606 \times 10^{-6} \text{ newton-meters} \\ &= 0.858 \times 10^{-4} \text{ oz-in.} \end{aligned}$$

APPENDIX IV THE MAGNETIC FIELD OF THE EARTH

Equations used for describing the Earth's field are given in Table 3. The coordinate system used is presented in Figure 16.

TABLE 3
SERIES EXPANSION OF THE EARTH'S FIELD
(Sheet 1 of 2)

$$\vec{B} = B_x \vec{i}_x + B_y \vec{i}_y + B_z \vec{i}_z$$

$$B_x = 2 \left(\frac{3959}{r} \right)^3 [-0.303 \cos \theta - 0.0238 \cos \varphi \sin \theta + 0.0535 \sin \varphi \sin \theta]$$

$$+ 3 \left(\frac{3958}{r} \right)^4 [-0.003225 - 0.009675 \cos 2\theta + 0.028319 \cos \varphi \sin 2\theta - 0.016628 \sin \varphi \sin 2\theta$$

$$+ 0.005023 \cos 2\varphi - 0.005023 \cos 2\varphi \cos 2\theta + 0.001949 \sin 2\varphi - 0.001949 \sin 2\varphi \cos 2\theta]$$

$$+ 4 \left(\frac{3959}{r} \right)^5 [0.005175 \cos \theta + 0.008625 \cos 3\theta - 0.002863 \cos \varphi \sin \theta - 0.014315 \cos \varphi \sin 3\theta$$

$$- 0.000184 \sin \varphi \sin \theta - 0.000942 \cos \varphi \sin 3\theta + 0.008859 \cos 2\varphi \cos \varphi$$

$$- 0.008859 \cos 2\varphi \cos 3\theta - 0.002566 \sin 2\varphi \cos \theta + 0.002566 \sin 2\varphi \cos 3\theta$$

$$+ 0.008063 \cos 3\varphi \sin \theta - 0.002688 \cos 3\varphi \sin 3\theta - 0.002253 \sin 3\varphi \sin \theta$$

$$+ 0.000751 \sin 3\varphi \sin 3\theta]$$

$$+ 5 \left(\frac{3959}{r} \right)^6 [0.001659 + 0.003688 \cos 2\theta + 0.006453 \cos 4\theta + 0.000712 \cos \varphi \sin 2\theta$$

$$+ 0.002490 \cos \varphi \sin 4\theta + 0.000316 \sin \varphi \sin 2\theta + 0.001107 \sin \varphi \sin 4\theta$$

$$+ 0.001803 \cos 2\varphi + 0.002404 \cos 2\varphi \cos 2\theta - 0.004207 \cos 2\varphi \cos 4\theta$$

$$- 0.000335 \sin 2\varphi - 0.000447 \sin 2\varphi \cos 2\theta + 0.000783 \sin 2\varphi \cos 4\theta]$$

$$B_y = \left(\frac{3959}{r} \right)^3 [-0.0238 \sin \varphi - 0.0535 \cos \varphi]$$

$$+ \left(\frac{3959}{r} \right)^4 [0.0981 \sin \varphi \cos \theta + 0.0576 \cos \varphi \cos \theta + 0.020092 \sin 2\varphi \sin \theta$$

$$- 0.007794 \cos 2\varphi \sin \theta]$$

$$+ \left(\frac{3959}{r} \right)^5 [-0.017177 \sin \varphi - 0.028629 \sin \varphi \cos 2\theta + 0.001102 \cos \varphi + 0.001837 \cos \varphi \cos 2\theta$$

$$+ 0.035438 \sin 2\varphi \sin 2\theta + 0.010264 \cos 2\varphi \sin 2\theta + 0.016126 \sin 3\varphi$$

$$- 0.016126 \sin 3\varphi \cos 2\theta + 0.004506 \cos 3\varphi - 0.004506 \cos 3\varphi \cos 2\theta]$$

$$+ \left(\frac{3959}{r} \right)^6 [0.006404 \sin \varphi \cos \theta + 0.004981 \sin \varphi \cos 3\theta - 0.002846 \cos \varphi \cos \theta$$

$$- 0.002214 \cos \varphi \cos 3\theta + 0.007212 \sin 2\varphi \sin \theta - 0.016827 \sin 2\varphi \sin 3\theta$$

$$- 0.001342 \cos 2\varphi \sin \theta - 0.003131 \cos 2\varphi \sin 3\theta]$$

$$B_y = \begin{cases} -B_\varphi & \text{If satellite is in southbound portion of orbit} \\ +B_\varphi & \text{If satellite is in northbound portion of orbit} \end{cases}$$

TABLE 3
 SERIES EXPANSION OF THE EARTH'S FIELD
 (Sheet 2 of 2)

$$\begin{aligned}
 B_{\theta} = & \left(\frac{3959}{r}\right)^3 \left[+ 0.303 \sin \theta - 0.0238 \cos \varphi \cos \theta - 0.0535 \sin \varphi \cos \theta \right] \\
 & + \left(\frac{3959}{r}\right)^4 \left[0.01935 \sin 2\theta + 0.056638 \cos \varphi \cos 2\theta + 0.033255 \sin \varphi \cos 2\theta \right. \\
 & \quad \left. + 0.010046 \cos 2\varphi \sin 2\theta + 0.002598 \sin 2\varphi 2\theta \right] \\
 & + \left(\frac{3959}{r}\right)^5 \left[- 0.005175 \sin \theta - 0.025875 \sin 3\theta - 0.002863 \cos \varphi \cos \theta - 0.042945 \cos \varphi \cos 3\theta \right. \\
 & \quad - 0.000184 \sin \varphi \cos \theta - 0.002756 \sin \varphi \cos 3\theta - 0.008859 \cos 2\varphi \sin \theta \\
 & \quad + 0.026578 \cos 2\varphi \cos 3\theta + 0.002566 \sin 2\varphi \sin \theta - 0.007698 \sin 2\varphi \sin 3\theta \\
 & \quad + 0.008063 \cos 3\varphi \cos \theta - 0.008063 \cos 3\varphi \cos 3\theta - 0.002253 \sin 3\varphi \cos \theta \\
 & \quad \left. + 0.002253 \sin 3\varphi \cos 3\theta \right] \\
 & + \left(\frac{3959}{r}\right)^6 \left[- 0.007375 \sin 2\theta - 0.025813 \sin 4\theta + 0.001423 \cos \varphi \cos 2\theta + 0.009961 \cos \varphi \cos 4\theta \right. \\
 & \quad + 0.000632 \sin \varphi \cos 2\theta + 0.004427 \sin \varphi \cos 4\theta - 0.004808 \cos 2\varphi \sin 2\theta \\
 & \quad \left. + 0.016827 \cos 2\varphi \sin 4\theta + 0.000894 \sin 2\varphi \sin 2\theta - 0.003131 \sin 2\varphi \sin 4\theta \right] \\
 & \hspace{15em} (0 \leq \theta \leq 180^\circ)
 \end{aligned}$$

$$B_Z = \begin{cases} -B_{\theta} & \text{If satellite is in southbound portion of orbit} \\ +B_{\theta} & \text{If satellite is in northbound portion of orbit} \end{cases}$$