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**LINEAR POLARIZATION OF
LOW-ENERGY BREMSSTRAHLUNG**

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CONTENTS

Abstract	v
I. Introduction	1
II. Theory	2
A. Classical Treatment of Bremsstrahlung Polarization	2
B. Quantum Mechanical Treatment of Bremsstrahlung Polarization	6
III. Previous Experimental Work	11
IV. Experiment	16
A. Apparatus	16
1. Electron Source	19
2. Target Chamber	21
3. Targets	26
4. Polarimeter	28
5. X-Ray Detector	35
6. Electronics System	35
B. Procedure	38
C. Experimental Data	39
D. Data Analysis	39
1. Energy Calibration of Multichannel Analyzer	39
2. Correction for Compton Scattering from the Ge(Li) Detector	41
3. Correction for Fluorescence Escape from the Ge(Li) Detector	41
4. Removal of Fluorescent Lines from the Spectrum	42
5. Correction for the Compton Polarimeter Energy Response	42
6. Normalization of Spectrum (Dead-Time Corrections)	43
7. Subtraction of Backgrounds	43
8. Calculation of the Difference of Spectra	43
9. Correction for Asymmetry Ratio of Polarimeter	44
10. Fitting a Polynomial to the Final Polarization Results	44

IV. Experiment (continued)	
E. Error Analysis	44
V. Results	49
A. k-Dependence of Polarization	62
B. Z-Dependence of Polarization	62
C. T_0 -Dependence of Polarization	68
D. θ -Dependence of Polarization	68
VI. Comparison with Theory and Previous Experiments	73
A. Experimental Results of Motz and Placious	73
B. Gluckstern and Hull's Calculation	76
C. Tseng and Prati's Calculation	78
D. Haug's Calculation	82
E. Sommerfeld's Theory	82
1. Kirkpatrick and Wiedmann's Calculation	82
2. Kulenkampff, Scheer and Zeitler's Relativistic Transformation	90
VII. Conclusions	90
Suggestions for Future Work	92
Appendices:	
A. Electron Energy Calibration	94
B. Ge(Li) Detector Efficiency Measurement	97
C. Target Thickness Measurement by X-ray Fluorescence Technique	102
D. Polarimeter Asymmetry Ratio, Efficiency and Energy Response	106
References	115

LINEAR POLARIZATION OF LOW-ENERGY BREMSSTRAHLUNG

ABSTRACT

The linear polarization of low-energy electron bremsstrahlung from thin targets ($\leq 50 \mu\text{g}/\text{cm}^2$) of Al, Cu, Ag and Au has been measured for incident electron energies of 50, 75 and 100 keV. The polarization was measured as a function of photon energy at four emission angles ($\theta = 22.5^\circ, 45^\circ, 90^\circ$ and 135°), using a Compton polarimeter with large asymmetry ratio (35 to 150) and a high-resolution Ge(Li) spectrometer (550 eV FWHM for 60-keV photons).

A brief discussion of the theoretical aspects of bremsstrahlung polarization, as well as a summary of calculational and experimental work to date, is presented. The Compton polarimeter is discussed in detail. The polarization data obtained here are compared with the predictions of various theories and, in particular, with the exact numerical bremsstrahlung calculation of Tseng and Pratt.

I. INTRODUCTION

An electron accelerated in the Coulomb field of an atomic nucleus radiates electromagnetic energy, called bremsstrahlung (braking radiation). Discovered by Roentgen in 1895, the phenomenon has been a subject of intense study throughout this century.*

Characteristics exhibited by bremsstrahlung include a continuous spectrum of photon energies from zero to the full kinetic energy of the electron, as well as linear and circular polarization. Linear polarization of low-energy electron bremsstrahlung is the subject of this investigation.

A photon beam is linearly polarized when there is a preferential orientation in space of the electric vector. Linear polarization of bremsstrahlung is a function of the kinetic energy of the incident electron T_0 , the energy of the emitted photon k , the angle of photon emission with respect to the direction of the incident electron θ , and the atomic number of the target Z .

Most bremsstrahlung calculations apply to single electron interactions, and must be compared with experimental results obtained using very thin targets. To calculate the characteristics of thick-target bremsstrahlung, complicated integration of the single interaction results over successive electron collisions is necessary, a procedure which has so far been unsatisfactory. The thin-target constraint and the lack of a high-efficiency, high-resolution x-ray spectrometer have been the principal reasons for the paucity of quantitative low-energy experimental results. However, the advent of the Ge(Li) detector provides an efficient, high-resolution x-ray spectrometer, and present-day thin-film technology allows preparation of targets thin enough to permit measurement using electrons of energy as low as 50 keV.

The purpose of this investigation was to apply these developments in measuring the linear polarization of thin-target, low-energy electron bremsstrahlung. The polarization was measured as a function of photon energy, electron energy, emission angle and atomic number.

*A review of bremsstrahlung calculations and measurements is presented by Massey et al. in Ref. 1.

The electron energy range was between 50 and 100 keV. All other parameters were varied widely ($Z = 13, 29, 47$ and 79 ; $\theta = 22.5^\circ, 45^\circ, 90^\circ$ and 135° ; and $k = 0$ to T_0) to provide a broad sampling of the dependence of polarization upon them. The experimental results are compared with several calculations.

II. THEORY

A. Classical Treatment of Bremsstrahlung Polarization

Classical analysis of the bremsstrahlung phenomenon is instructive since it is easily understood and illustrates many of the qualitative features of a more exact quantum mechanical treatment. Described classically, the accelerated electron is treated as a radiating electric dipole with components along each of the coordinate axes. Consider the geometry of Fig. 1. The radiation intensity observed at any point in space is the sum of intensities radiated from each of these three dipole components.

The radiation observed at a point in the x - z plane, at an angle θ relative to the incident electron direction, can be broken into two linearly polarized intensity components, one parallel to the x - z plane, $I_{\parallel}$, and one perpendicular to the plane, $I_{\perp}$. Since radiation is linearly polarized in the direction of the dipole from which it emanated, it is clear that $I_{\perp}$ will consist only of radiation from the y -component dipole while $I_{\parallel}$ will consist of the sum of contributions from both the x - and z -component dipoles. Considering the angular distribution of the intensity of dipole radiation [$I_j(\phi) = I_j \sin^2 \phi$, where ϕ is the angle between the j th dipole axis and the position vector at the point of observation] it can be seen that in the x - z plane

$$I_{\perp} = I_y \sin^2 90^\circ = I_y \quad (1)$$

and

$$I_{\parallel} = I_x \sin^2 \theta + I_z \sin^2 (90^\circ - \theta).$$

For an unpolarized electron beam there is axial symmetry so that $I_y = I_z$, and

$$I_{\parallel} = I_x \sin^2 \theta + I_y \cos^2 \theta. \quad (2)$$

Fig. 1. Polarized intensity components observed in the x-z plane due to a radiating charge located at the origin.

If $I_{\parallel} \neq I_{\perp}$, the photons are linearly polarized. A measure of the polarization $P(\theta)$ is defined* as

$$P(\theta) = \frac{I_{\perp} - I_{\parallel}}{I_{\perp} + I_{\parallel}}. \quad (3)$$

By this definition, the polarization P can range from +1 (complete polarization perpendicular to the electron-photon plane) to -1 (complete polarization parallel to this plane).

Substituting the expressions for $I_{\perp}$ and $I_{\parallel}$ into Eq. (3) gives

$$P(\theta) = \frac{\left(\frac{I_y}{I_x} - 1\right)}{\left(\frac{I_y}{I_x}\right)(2 \csc^2 \theta - 1) + 1}. \quad (4)$$

It can be seen that the polarization can be predicted by calculating the two dipole component intensities, I_x and I_y .

The knowledge that photons are polarized along the dipole axis may be used to describe some general features of bremsstrahlung linear polarization. The reaction plane is the plane containing the direction vectors of both the photon and the incident electron. Throughout this report the bremsstrahlung reaction plane will always be the horizontal plane. Consider the diagram of the initial, $\vec{p}_0$, and final, $\vec{p}$, electron momenta as shown in Fig. 2. Two cases are illustrated: (1) the emission of a high-energy photon, $k \approx T_0$, and (2) the emission of a low-energy photon, $k \approx 0$. The momentum difference $\vec{p} - \vec{p}_0$ is proportional to the acceleration vector, which in turn defines the electric dipole axis. For the high-energy photon case, $|\vec{p} - \vec{p}_0| \approx |\vec{p}_0|$, the acceleration has a large component parallel to $\vec{p}_0$, and the radiation, when observed in a direction perpendicular to $\vec{p}_0$, will be predominantly polarized parallel to $\vec{p}_0$, as shown. In the low-energy photon case, $|\vec{p} - \vec{p}_0| \ll |\vec{p}_0|$, the acceleration vector $\vec{p} - \vec{p}_0$ is predominantly perpendicular to $\vec{p}_0$, and the radiation will be polarized perpendicular to the reaction plane.

*Some authors, e.g., Ref. 2, define polarization as the negative of the above definition, i.e., $P = (I_{\parallel} - I_{\perp})/(I_{\parallel} + I_{\perp})$, but Eq. (3) is more common.

Fig. 2. Diagram of initial and final electron momenta in the bremsstrahlung interaction illustrating the major acceleration component (dipole axis).

This simple analysis reveals a general feature of low-energy bremsstrahlung polarization: perpendicular polarization at low photon energies and parallel polarization at high photon energies.

The details of this polarization reversal are dependent upon the extent to which the simple classical description of the process is valid. For example, as the incident electron energy is increased, the tendency is toward a larger transverse acceleration component and, hence, greater perpendicular polarization. Consequently, as electron energy is increased, the photon energy at which the cross-over from positive to negative polarization occurs also increases. At relativistic energies the classical model does not suffice, and additional polarization features are observed.

The screening of the nuclear charge by orbital electrons also complicates the simple picture discussed above. Qualitatively, screening effects tend to increase the parallel component of polarization by reducing the effective charge of the nucleus.

Other predicted features of the linear polarization of bremsstrahlung, such as its dependence upon Z , θ and T_0 , arise from the various assumptions of the calculational models used. The theoretical approaches applicable to this work will be discussed in the next section.

B. Quantum Mechanical Treatment of Bremsstrahlung Polarization

All derivations of the differential cross section $d\sigma$ for photon emission by the bremsstrahlung process consist essentially of the following. The transition probability per unit time,

$$w = \frac{2\pi}{\hbar} |H_{if}|^2 \rho_f,$$

is calculated where H_{if} is the matrix element for the transition of the system from its initial state i to its final state f , and ρ_f is the density of accessible final states. The cross section $d\sigma$ is obtained by normalizing to unit incident electron flux, i.e., by dividing by the velocity of the incident electron.

The important quantity to be evaluated is the matrix element H_{if} which is proportional to the following integral:

$$H_{if} \approx \int \psi_f^\dagger (\hat{e}_\gamma \cdot \vec{\alpha}) e^{-i\vec{k} \cdot \vec{r}} \psi_i d\tau$$

where $\hat{e}_\gamma$ is the unit polarization vector of the photon and $\vec{\alpha}$ is the Dirac matrix. The exponential results from expanding the magnetic vector potential $\vec{A}$, and ψ_i and ψ_f are the Dirac wave functions for the initial and final states of the electron, respectively. To derive an "exact" cross section, it is necessary to use "exact" wave functions to describe the electron in the screened Coulomb field of the nucleus. It is not possible to solve the Dirac wave equation in closed form for an electron in a Coulomb field, primarily because the wave function must be represented as an infinite series.^{3,4} However, various calculations which expand the wave function in a partial wave series and numerically solve the Dirac equation have been attempted.⁵ Also, various approximate wave functions and procedures have been used.^{6,7}

Bremsstrahlung cross-section calculations are relativistic or nonrelativistic, depending upon whether the Dirac $\{H = c\vec{\alpha} \cdot (\vec{p} - e\vec{A}) + \beta mc^2 + e\phi\}$ or Schrodinger $\{H = c[(\vec{p} - e\vec{A})^2 + m^2 c^2]^{1/2} + e\phi\}$ form of the Hamiltonian is used. Generally, one of three types of wave function is used, nonrelativistic Coulomb wave functions (Sommerfeld), relativistic Coulomb wave functions (Sommerfeld-Maue) or plane waves (Born approximation). Finally, various screening effects (atomic models) are considered.

Table 1 summarizes the bremsstrahlung linear polarization calculations and their salient features.

A brief discussion of the calculations listed in Table 1 follows:

Sommerfeld's Theory

Sommerfeld⁸ derived the matrix elements of the component dipoles in the bremsstrahlung process nonrelativistically, using the Schrodinger equation and assuming Coulomb wave functions. He assumed that the nuclear field is a pure Coulomb field, and that electron spin could be ignored. Because he neglected screening, his theory is not valid for low electron energies, ~ 1 keV, or for high atomic numbers.

He considered the wave systems of an electron approaching a nucleus and departing in a new direction with reduced speed, the lost energy being emitted as a photon. The radiation emitted by the accelerated electron is composed of the intensity components, I_x , I_y

Table 1. Bremsstrahlung polarization calculations.

Author	Characteristics	Screening	Regions of validity	Assumptions
Sommerfeld ⁸	Nonrelativistic	Neglects screening	1. Low Z 2. $0.06 < \beta_0 < 1$	1. Neglects electron spin 2. Neglects relativistic effects 3. Uses Coulomb wave functions
Kirkpatrick and Wiedmann ⁹	Graphical integration of Sommerfeld theory over all angles of recoiling electron	Same as above	Same as above	Same as above
Gluckstern and Hull ¹⁰	Relativistic	Performed with and without screening	1. $2\pi Z/137\beta_0 < 1$ 2. $2\pi Z/137\beta < 1$ 3. Screening not valid for high photon energy	1. Born Approximation (plane waves)
Fronsdale and Uberall ⁷	Relativistic	Includes screening		1. Born Approximation (plane waves) 2. Polarized incident electrons
	(Basically the same as Gluckstern and Hull except for more accurate screening)			
Olsen and Maximon ⁶	Extreme relativistic	Includes screening. Screening valid for all Z with accuracy: $\left(\frac{Z}{137}\right)^2 \frac{\ln(E/m_0c^2)}{E/m_0c^2}$	1. $T_0 > 20$ MeV 2. $T_0, T, k \gg m_0c^2$ 3. $P_0 \theta_0 \approx 1$	1. Relativistic Coulomb wave functions
Haug ¹¹	Relativistic	Neglects screening	1. Valid for all T_0 2. Low Z 3. Valid only for high energy limit of photon spectrum	1. Relativistic Coulomb wave functions (Sommerfeld-Maue)
Tseng and Pratt ⁵	Relativistic numerical calculation	Includes various models of screening	$5 \text{ keV} < T_0 < 1 \text{ MeV}$	1. Assumes spherical potential 2. Expands wave functions as partial wave series

and I_z , and these components are in turn proportional to the square of the corresponding dipole moments M_x , M_y and M_z . Sommerfeld calculated the dipole moments rigorously for a single electron incident upon a bare nucleus of charge Ze . To compare his results with experiment, it is necessary to integrate his dipole moments over all angles of the recoiling electron. This integration is formidable, and has not been done in closed form. Several approximations have been made, however. Elwert¹² and Weinstock¹³ independently integrated Sommerfeld's results by expanding the integrand in a series and integrating term by term. In at least one case¹² this involved applying further mathematical constraints on the regions of validity, and in both cases it required truncation of the series.

Kirkpatrick and Wiedmann⁹ felt the series expansion approach was invalid because no positive criterion had been found for evaluating the progress of convergence of the series, and the inaccuracy resulting from truncation could be high. Consequently, they evaluated Sommerfeld's matrix elements by performing the integration graphically. They then used these integrated matrix elements to obtain the corresponding intensity components, I_x and I_y ($= I_z$). They listed intensity components for specific cases of T_0 , k and Z , and in addition, obtained empirical formulas which approximated the values listed to within 5% in the worst case and within 2% on the average.

Since the Kirkpatrick and Wiedmann intensity components are calculated on the basis of the Sommerfeld theory, the results are valid only when the incident electron energy is low enough, i.e., $\beta \ll 1$, where β is the incident electron velocity relative to that of light. Kulenkampff et al.¹⁴ have supplemented Sommerfeld's theory by finding a transformation which accounts for relativistic corrections up to β values of 0.4 (~50 keV incident electron energy). The transformation is

$$P'(\theta) = P(90^\circ) \frac{(1 - \beta^2) \sin^2 \theta}{(1 - \beta \cos \theta)^2 - (\cos \theta - \beta)^2 P(90^\circ)} \quad (5)$$

where $P'(\theta)$ is the relativistically-corrected polarization and $P(90^\circ)$ is the 90° polarization value for a given photon energy calculated from Sommerfeld's theory.

To compare the results of the nonrelativistic and the "relativistically-corrected" Sommerfeld theory to measurements made in this investigation, a computer program was written to calculate nonrelativistic polarization using the Kirkpatrick and Wiedmann empirical formulas. "Relativistically-corrected" polarization was then calculated by applying Kulenkampff's transformation to the above. Comparisons are discussed in Section VI-E.

Calculation by Gluckstern and Hull

Gluckstern and Hull¹⁰ used the Born approximation to derive relativistic cross sections for bremsstrahlung linear polarization. Their results are valid only under the conditions that $2\pi Z/137\beta_0 \ll 1$ and $2\pi Z/137\beta \ll 1$ where β_0 and β are the velocities of the incident and recoiling electron, respectively, relative to the velocity of light. This limits applicability to cases of low atomic number and $\beta \approx 1$. The calculations have been made with and without screening corrections, but these corrections break down for the case of high-energy photon emission. Comparison is made with experimental results in Section VI-B.

Calculation by Haug

Haug¹¹ has relativistically calculated the linear polarization at the high-energy limit of the bremsstrahlung spectrum, using Sommerfeld-Maue wave functions¹⁵ for the incident and recoiling electron. These wave functions behave asymptotically like a plane wave plus outgoing and ingoing spherical waves. The calculation is valid for all electron energies, and for low-Z elements. However, he calculates polarization for atomic numbers as high as 79. His results are compared to experiment in Section VI-D. It is interesting to note that, contrary to the predictions of most theories,^{6,7} Haug finds that the linear polarization of bremsstrahlung depends upon the polarization state of the incident electron.

Calculation by Tseng and Pratt

Tseng and Pratt⁵ have calculated a relativistic bremsstrahlung cross section which is valid over an incident electron kinetic energy range of 5 keV to 1 MeV. The calculation assumes the atom to be a spherically-symmetric charge distribution of infinite mass, but alters the form of this charge distribution to include various models accounting for screening. The incoming and outgoing electron wave functions are expanded in a partial wave series and substituted into the Dirac equation which is then solved numerically. Matrix elements are then computed by numerical integration over these wave functions.

To date, Tseng and Pratt have published results pertaining only to differential bremsstrahlung cross sections integrated over all polarization states of the outgoing photons. However, their numerical technique readily predicts linear polarization. Their computer programs were obtained and modified for the Lawrence Livermore Laboratory's CDC 7600 computers. Results for specific cases of interest were calculated for comparison with measurement. Unfortunately, the computer time necessary to generate these results is very long, and therefore only a few representative cases have been calculated.

Other Calculations

Other calculations¹⁶ of bremsstrahlung polarization have been made but do not apply to the present work either because they merely confirm another, more complete, calculation or because they describe the bremsstrahlung process for much higher incident electron energies. They are referenced here only for completeness.

III. PREVIOUS EXPERIMENTAL WORK

Very few quantitative measurements of low-energy bremsstrahlung linear polarization have been accomplished to date. This is primarily because of the lack of an efficient high-resolution photon spectrometer. Also, thin targets were difficult to obtain until the last decade or so. The advent of the Ge(Li) detector in the late 1960's allows differential measurement of polarization as a function of photon energy from <5 keV to the bremsstrahlung endpoint energy.

The earlier measurements (mostly thick-target work) established the fact that bremsstrahlung exhibits predominantly parallel polarization. Some experiments showed qualitative agreement with some aspects of Sommerfeld's theory.¹⁷ The more recent experiment of Motz and Placious¹⁸ is the only work which has provided enough data to allow detailed comparisons with theory. However, the scope of their experiment was limited. To date no systematic investigation of bremsstrahlung polarization as a function of incident electron energy, target Z , photon energy and emission angle has been undertaken.

A summary of previous experimental work follows:

1905

Barkla,¹⁹ using the polarization sensitivity of the Compton scattering process, measured the polarization of x rays emitted at 90° to the electron beam from a thick-target x-ray tube. His measurement averaged polarization over the entire x-ray energy spectrum and showed a mean polarization of 5% parallel to the reaction plane. The thick targets and large dimensions of the scatterer and detector account for the low value of polarization observed.

1909

Bassler²⁰ confirmed that x rays were polarized with their electric vector in the plane of the incident electron beam. He also concluded that the mean polarization decreased as the electron energy was increased.

1910

Bassler²⁰ and Vegard²¹ showed that the average polarization was higher if the lower-energy photons were filtered out of the x-ray spectrum.

1923

Kirkpatrick²² used a balanced-filter technique to observe only those photons with energies near the high-energy limit. He concluded

that for 58-keV incident electron energy, the maximum polarization of the high-energy photons was only 10%.

1928

Ross²³ also used a balanced-filter technique to measure polarization at the high-energy limit of the spectrum and obtained results which disagreed with Kirkpatrick's. Ross found the polarization of these photons to be 100%.

1928

Wagner and Ott²⁴ attempted a measurement which was differential in photon energy by analyzing the continuous spectrum using a rock salt crystal. Their results disagreed with both Ross' and Kirkpatrick's in showing the high-energy polarization to be 47%. Targets of Fe, Cu, Ag and Pt were used, and no dependence upon atomic number was observed.

1929

Duane²⁵ performed the first thin-target experiment. He bombarded a Hg vapor target with 11.7-keV electrons and observed the x rays emitted at 90° to the beam. His measurements were not differential in photon energy, and he found a mean polarization of 47% over the entire spectrum.

1929

Kulenkampff²⁶ used targets that were thin relative to those of previous experimenters, but were still thick in an absolute sense. For electron energy of 38 keV, he found the mean polarization over the entire spectrum to be 45%, in agreement with Duane's results. Furthermore, using a balanced-filter technique, he observed 100% polarization at the high-energy limit.

1930

Dasannacharya,²⁷ using electrons of energy ~40 keV, measured the mean polarization of the entire x-ray spectrum from thick targets

as a function of target thickness. His targets ranged from $25\ \mu$ to 6 cm in thickness, all thick by absolute standards. The results indicated an exponential rise in polarization as target thickness decreased. A maximum polarization of 48% was observed. The shape of the polarization versus target thickness curve was independent of electron energy.

1934

Cheng¹⁷ used the balanced-filter technique to observe the x rays emitted from thick targets of Al, Cu, and W when bombarded by electrons of energies between 20 and 100 keV. He used filter combinations of Mo-Nb, Pd-Rh and W-Ta to measure polarization at 19.5, 23.8 and 68.5 keV, respectively. The major result of this experiment was the determination that, for a given x-ray energy, the polarization increased as the atomic number of the target decreased.

1936

Piston²⁸ used a balanced-filter technique to investigate the polarization of 68.5-keV photons from Al and Ag targets of 19 and $0.2\ \text{mg/cm}^2$ thickness, respectively. Incident electron energy was varied up to 102 keV. For $T_0 \approx 70\ \text{keV}$, he observed complete polarization near the high-energy limit of the spectrum.

1941

Boardman,²⁹ using targets of Ni, Ag and Pb with thicknesses of the order of $10\ \text{mg/cm}^2$, performed an experiment similar to Piston's and obtained the same results.

1954

Kulenkampff et al.³⁰ measured bremsstrahlung polarization by observing the directions and ranges of the recoiling photoelectrons when the bremsstrahlung was absorbed in a cloud chamber. Unfortunately, poor statistics resulted in very large experimental uncertainty.

1956

Motz³¹ used a NaI(Tl) spectrometer to measure polarization, differential in photon energy, for 1.0-MeV electrons incident on a 1 mg/cm² thick Al target and a 0.4 mg/cm² Au target. His results showed qualitative agreement with the predictions of the Born approximation calculation by Gluckstern and Hull.¹⁰

1958

Motz and Placious³² used the technique of Motz to measure the polarization at the high-energy limit as a function of bremsstrahlung emission angle. The incident electron energy was 500 keV and the targets were 4.3 mg/cm² of Be and 0.21 mg/cm² of Au. The results again showed qualitative agreement with the Gluckstern-Hull predictions.

1960

Motz and Placious¹⁸ summarized the theoretical calculations to that date. They also published experimental results of the polarization at the high-energy limit of the photon spectrum for $T_0 = 50$ and 100 keV and for targets of Be and Au. Their results were again compared with the Gluckstern-Hull calculations. The theory predicted too large a value of polarization, particularly at small forward angles.

1961

Kulenkampff and Zinn³³ measured the polarization of x rays emitted at 90° from low-Z targets bombarded with electrons of ~35 keV. The measurement was differential in photon energy, and the results showed qualitative agreement with predictions of Sommerfeld.⁸

1964

Huffman³⁴ measured the polarization of bremsstrahlung from a Hg vapor target as a function of incident electron energy, photon energy and angle of emission. The electron energies were 10, 15, 20, 25 and 30 keV. His results were compared with the predictions of

Sommerfeld, corrected for relativistic effects. The agreement between theory and experiment was quite poor.

1968

Scheer et al.³⁵ measured polarization as a function of photon energy and emission angle for 50-keV electrons on carbon. They also measured the polarization as a function of photon energy at an angle of 20° for 180-keV electrons incident on Al_2O_3 and Au. They show reasonable agreement with the predictions of Fronsda and Uberall.⁷

1970

Slivinsky³⁶ was the first experimenter to use high-resolution Ge(Li) and Si(Li) spectrometers to measure bremsstrahlung polarization. He measured the polarization of the x rays emitted by commercial x-ray machines (thick targets). He could not determine the polarization at the high-energy limit, but did show that polarization increases rapidly for high photon energies. He also observed a decrease in polarization with increasing electron energy.

IV. EXPERIMENT

The experiment performed here measured the bremsstrahlung polarization, as defined in Eq. (3), using a Compton polarimeter, as illustrated in Fig. 3. A detailed discussion of the technique follows.

A. Apparatus

The experimental apparatus consisted of: (1) a source of electrons, (2) a target chamber, (3) a thin target, (4) an x-ray polarimeter employing a Ge(Li) detector, and (5) electronics for monitoring the electron beam and acquiring polarization data. Figure 4 is a schematic drawing of the accelerator and target.

Fig. 3. Vector diagram illustrating the arrangement for measuring bremsstrahlung linear polarization using a Compton polarimeter.

Fig. 4. Schematic drawing of electron accelerator and bremsstrahlung target chamber.

1. Electron Source

The following accelerator characteristics were necessary to accomplish this experiment: (1) well-defined beam energy, (2) sufficient beam current to establish reasonable bremsstrahlung intensities, and (3) sufficiently small beam diameter and divergence at the target. A source having some of these features was the LLL Statitron accelerator which provided a subnanosecond burst of electrons for detector impulse response studies. The accelerator was dismantled, moved to a new location, and rebuilt with many modifications to make it appropriate for this work. Presently, the accelerator is capable of delivering a steady-state current of $50 \mu\text{A}$ with energies ranging from a few keV to 125 keV. The beam diameter is not larger than 0.5 cm at the target. Electron energy is known to within $\pm 250 \text{ eV}$ (see Appendix A) and is stable to within 0.2%. Many characteristics of this accelerator have been described in detail by Lasher,³⁷ but a summary of the steady-state capability is in order here (refer to Fig. 4).

Electron Gun—A new electron gun was mounted on the accelerator for this work. This was necessary because the guns used in the pulsed mode are extremely expensive and were very sensitive to changing high-vacuum conditions encountered in the early stages of this experiment.

The gun used is a model EE-55 electron tube.* It consists of a planar, tungsten dispenser cathode and grid assembly, and is rated for a pulsed current delivery of several amperes. The gun was designed for short electron paths in a planar geometry, and had to be modified to deliver enough current to the target 2 meters away. The modification consisted of attaching a bell-shaped electrode to the gun to focus the electrons and allow passage to the accelerating column through an aperture in the gun anode. Without this addition to the gun, the beam current on target was insufficient to perform the experiment.

A voltage of up to 30 kV can be maintained across the electron gun. Beam current is controlled by varying the cathode heater power or the grid voltage.

*Manufactured by Machlett Laboratories, Inc.

Accelerator—Upon exit from the gun, the electrons are focused by two magnetic lenses. They are then accelerated in a 12-electrode linear accelerating column described by Lasner.³⁷ The voltage across the column is provided by a well-filtered, high-voltage supply with output capability of 0 to 150 kV at 5 mA. Voltage ripple is less than 0.01% (15 V at maximum output).

The kinetic energy of the electrons is the sum of the voltages on the electron gun and the accelerating column. The maximum variation in this energy is less than 0.2%. This stability is evidenced by the sharp endpoint energies obtained in bremsstrahlung spectra taken over very long time intervals, as seen in Fig. A-1 of Appendix A

A clean environment was needed to avoid poisoning the cathode by hydrocarbons from O-rings and organic pump fluids. Consequently, metal gaskets were used wherever possible, and the system was initially evacuated with cryopumps. A VacIon pump evacuated the system to base pressure of approximately 10^{-7} Torr. Operating pressure was typically 2×10^{-6} Torr or less.

Focusing and Momentum Analysis—In its previous location, the accelerator produced a beam of electrons that could only be used in the vertical direction. To provide a horizontal beam and allow adequate space for this experiment, and also to maintain energy stability, an analyzing magnet was added. The magnet was designed to bend a beam of 175-keV electrons through 90° with a 20-cm radius of curvature. This required a field strength of 76 gauss.

To avoid the large, cumbersome coil and pole pieces necessary if the magnet were mounted outside the beam pipe, the magnet was designed with the pole pieces themselves serving as vacuum barriers. Further, the design provided for a viewing port to allow visual observation along the horizontal beam axis (see Fig. 4). A simple degaussing circuit was built to eliminate hysteresis effects and obtain reproducible magnetic fields.

Positionable Al collimators with beveled edges were placed immediately before and after the analyzing magnet. They were mounted on a vacuum-tight metal bellows arrangement that allowed external adjustment without disturbing the vacuum. The surfaces of the

collimators were painted with a phosphor ($Zn_{12}SiO_4 Mn$) to permit observation of the beam position and spot size through the three accelerator viewing ports. These ports were covered with Pb-loaded glass. Additional fixed Al collimators were located nearer the target chamber.

Two additional focusing magnets were required to obtain the desired beam spot size on target (see Fig. 4). All beam pipes were wrapped with magnetic shielding to reduce the effect of the earth's magnetic field on the beam trajectory. Without this shielding it was impossible to deliver the beam to the target chamber.

2. Target Chamber

The target chamber was designed with the following features: (1) it was isolated from ground to serve as a Faraday cup for collecting the electron beam; (2) it allowed observation over a large range of bremsstrahlung emission angles (0° to 155°); (3) attenuation of the emerging x rays was minimal; and (4) it had a viewing port for observation of the electron beam spot on target. The chamber is seen in the photographs of Figs. 5, 6 and 7 and the schematic drawing of Fig. 8.

The chamber is a hollow right-circular cylinder of Al, 15 cm long and 10 cm in diameter. The targets are suspended on a steel rod which enters the chamber through a sliding Viton O-ring vacuum seal in the top plate. The rod can be raised or lowered to three positions which center the target, an empty target holder for background measurements or a quartz disc, on the beam axis. The quartz disc fluoresces under electron bombardment and allows observation of beam spot shape.

A single x-ray port, 2.54 cm high and subtending 265° of polar angle, allows continuous selection of the bremsstrahlung emission angle from $+110^\circ$ to -155° . The port is covered by a sheet of 0.025-cm-thick (47 mg/cm^2) Be which is transparent to x rays. The Be x-ray window and the O-ring which provides the vacuum seal around the port are held in place by an Al clamp which slips over the

Fig. 5. Experimental setup showing electron accelerator, target chamber, and Compton polarimeter.

Fig. 6. Compton polarimeter in position to observe bremsstrahlung emitted at 22.5° with the Ge(Li) detector positioned perpendicular to the bremsstrahlung reaction plane.

Fig. 7. Compton polarimeter in position to observe bremsstrahlung emitted at 135° , with the Ge(Li) detector positioned in the bremsstrahlung reaction plane. Polarimeter is partially disassembled to show scatterer.

Fig. 8. Schematic drawing of bremsstrahlung target chamber and Compton polarimeter.

chamber (Fig. 7). The electron beam penetrates the targets, but is totally absorbed in the Be window of the chamber.

The chamber has two lucite viewing ports sealed with O-rings. One is a 0.64-cm-diam opening located at the vertical center of the chamber wall at $+140^\circ$. The other is a 5-cm-diam opening in the bottom of the chamber and a mirror must be used to observe the target through this window (see Fig. 6).

The chamber is electrically isolated from the accelerator beam pipe (ground) by a Teflon spacer and Teflon bolt sleeves. It is isolated from the polarimeter table (ground) by a lucite cylinder which also supports the chamber from beneath.

Data runs were normalized to equal amounts of beam charge collected by the target chamber. Therefore, in addition to maintaining a high electrical impedance to ground, it was also necessary to insure that there was no low-energy secondary electron current collected by the chamber. This was accomplished by biasing the chamber at -90 V with respect to the accelerator beam pipe. The electrical isolation was adequate to maintain an impedance to ground of $>5 \times 10^8 \Omega$ and assure accurate current measurement.

The system was designed for the chamber to be pumped by the vacuum pumps on the accelerator. However, a valve was located on the drift tube to allow isolation of the chamber so that targets could be changed without bringing the accelerator to atmospheric pressure.

3. Targets

The major criterion for targets used in this study was that they be thin enough to minimize electron scattering effects. Four target materials were selected: Al ($Z = 13$), Cu ($Z = 29$), Ag ($Z = 47$) and Au ($Z = 79$), so that the dependence of polarization upon Z could be studied.

They were prepared by vacuum evaporation onto Parylene* substrates between 500 and 1000 Å thick. Uniformity across the foil was

* Parylene is the name given to a Poly-para-xylyene produced by Union Carbide. It is a CH_2 chain with a structure $[\text{CH}_2 - \text{C}_6\text{H}_4 - \text{CH}_2]_n$ where $n \approx 5 \times 10^3$.

insured by maintaining a large distance between the evaporation source and substrate during the evaporation.

Foil thickness was determined in two ways. First, during evaporation, the mass deposited on the substrate was monitored by simultaneous deposition onto a disc of known area mounted adjacent to the foil. The weight of this disc was measured during evaporation with a Cahn microbalance. Difficulties in applying this method during the early stages of the work resulted in large uncertainties ($\pm 30\%$) in target thickness. Later refinements in technique yielded foil thickness values accurate to within $\pm 10\%$.

A second method used to determine foil thickness involved measuring the K-shell x-ray fluorescence from a target when irradiated by a monoenergetic x-ray beam of known intensity. The K x rays from the target were detected with a Si(Li) spectrometer and recorded in a multichannel analyzer. Knowing the incident photon energy and flux, the K-shell photoelectric cross section, the K-shell fluorescence yield, and the detector geometry and efficiency, one can determine the number of target atoms present. The accuracy of this technique is also estimated to be within 10%. Appendix C discusses this fluorescence method in greater detail. Table 2 lists the target foils used and their respective thicknesses, as determined by both methods.

Table 2. Bremsstrahlung targets.

Target material	Thickness ($\mu\text{g}/\text{cm}^2$)	
	Microbalance	X-ray fluorescence
Al	50 ± 5	—
Cu	50 ± 5	52 ± 5
Ag	50 ± 15	38 ± 4
Au	50 ± 15	38 ± 4^a

^aUsed L-shell fluorescence.

The criteria used to insure that the targets were thin are based upon considering the energy loss and angular spread of the incident electrons and the effects of these phenomena upon the polarization measurement. The average energy lost by a 50-keV electron traversing a target of approximately $50 \mu\text{g}/\text{cm}^2$ is very small (~ 20 eV).³⁸ The

effect of the angular spread due to elastic scattering of the electrons in the target can be estimated by calculating the mean scattering angle and comparing it to the finite angular resolution of the polarimeter. The average number of elastic scattering events an electron of energy T_0 (keV) experiences in traversing a target of thickness ρx ($\mu\text{g}/\text{cm}^2$) and atomic number and mass Z and A , respectively, is given by³⁹

$$N = \frac{3Z^{4/3}}{AT_0} \rho x. \quad (6)$$

For the 12 combinations of energies and targets used, the typical value of N was between 2 and 2.5. The minimum number (best case) was 1.7 for 100-keV electrons on Al, and the maximum number (worst case) was 4 for 50-keV electrons on Au.

Plural scattering occurs when $1 < N < 20$. Keil *et al.*⁴⁰ have calculated angular distributions of plurally scattered electrons. Figure 9 shows a plot of the angular distribution of scattered-plus-uns scattered electrons calculated by Keil *et al.*⁴⁰ for the cases of $N = 2$ and 4. Nonelastic scattering is neglected since (1) the mean scattering angle is negligible compared to that of elastic scattering, and (2) the energy loss is negligible. From these plots it can be seen that, for the cases of $N = 2$ and 5, half of the scattered electrons are directed into cones of half-angle 1.5° and 3.8° , respectively. The angular resolution of the polarimeter was $\pm 5^\circ$ in most cases. Consequently, scattering in the foil was a smaller effect than the angular resolution of the polarimeter.

The Parylene target backings were $5\text{--}10 \mu\text{g}/\text{cm}^2$ thick. Considering the thickness of the backing relative to the target, the low atomic number and the fact that bremsstrahlung production varies as Z^2 , it is easily seen that the contribution to the measured bremsstrahlung resulting from the backing is negligible. This was verified by using only the Parylene backing in background measurements.

4. Polarimeter

The photon interaction phenomena that are sensitive to linear polarization, and which may be exploited in order to design a practical

Fig. 9. Angular distributions of scattered-plus-unscattered electrons traversing targets in which they experience a mean number of scattering events equal to 2 and 4.

polarimeter, are the photoelectric effect, photodisintegration of the deuteron, pair production, Compton scattering, and photofission. They are treated in reviews by Fagg and Hanna⁴¹ and McCallum and Vervier.⁴² Detailed consideration of these phenomena reveals that significant advantages can be obtained using the Compton scattering process at medium and low photon energies ($k \approx <1$ MeV).

Polarization Sensitivity of Compton Scattering—The sensitivity of the Compton scattering process to bremsstrahlung polarization is easily seen from the Klein-Nishina formula for the differential cross section,⁴³

$$\frac{d\sigma(\phi)}{d\Omega} = \frac{r_0^2}{2} \left(\frac{k}{k_0} \right)^2 \left(\frac{k}{k_0} + \frac{k_0}{k} - 2 \sin^2 \psi \cos^2 \phi \right), \quad (7)$$

where $r_0 = e^2/m_0c^2$ is the classical radius of the electron, ψ is the angle through which the incident photon is scattered, $d\Omega$ is the element of solid angle into which it is scattered, ϕ is the angle between the electric vector of the incident photon and the scattering plane (defined as the plane containing both the incident and scattered photon), and k_0 and k are the energies of the incident and scattered photon, respectively. In the above, the cross section has been integrated over all polarization states of the scattered photon. It is seen that the $\cos^2 \phi$ term is the polarization-sensitive part of the cross section, and results in a maximum for $\phi = \pi/2$ and a minimum for $\phi = 0$.

The initial and final photon energies are related by the expression

$$k = \frac{k_0}{1 + \frac{k_0}{m_0c^2} (1 - \cos \psi)}. \quad (8)$$

When $k_0 \ll m_0c^2$, $k \approx k_0$; and in Eq. (7), the term $k/k_0 + k_0/k$ is approximately 2. For $\psi = \pi/2$, the cross section then becomes

$$\frac{d\sigma(\phi)}{d\Omega} = r_0^2 \left(\frac{k}{k_0} \right)^2 (1 - \cos^2 \phi). \quad (9)$$

This implies an ideal response to polarization at low energies, i.e., a finite cross section for $\phi = \pi/2$ and zero cross section for $\phi = 0$. For values of ψ other than $\pi/2$, this ideal response is degraded.

A measure of the sensitivity of a Compton scatterer to polarization is expressed by the asymmetry ratio R , defined as

$$R \equiv \frac{d\sigma(\phi = \pi/2)}{d\sigma(\phi = 0)}. \quad (10)$$

For the ideal, low-energy case, $R \rightarrow \infty$. However, as the incident photon energy k_0 increases, R decreases and its maximum occurs at $\psi < \pi/2$. Consequently, for a given k_0 there is an angle ψ_0 for which R is a maximum. For energies in the range up to 100 keV, $k/k_0 + k_0/k$ deviates from the value 2 by less than 1.5%. Therefore, for these energies R is a maximum near $\psi = \pi/2$.

Now assume a beam of x rays, linearly polarized in some arbitrary plane, to be incident upon a scatterer as indicated in Fig. 10. The total intensity of this beam may be divided into two components, $I_y (\propto E_y^2)$ and $I_z (\propto E_z^2)$, whose polarizations lie in orthogonal planes. If the detector is alternately positioned in the x-y and x-z planes, as shown, different counting rates will, in general, be observed. The difference of these rates, divided by their sum, is related to the x-ray polarization through the asymmetry ratio, R .

Let $\dot{N}_z$ and $\dot{N}_y$ be the counting rates observed with the detector in the x-z and x-y planes, respectively. Also let $d\sigma(0)$ and $d\sigma(\pi/2)$ be the Klein-Nishina differential cross section for $\phi = 0$ and $\phi = \pi/2$, respectively, where ϕ is the angle between the electric vector of the x-ray component and the scattering plane.

It is then seen that the counting rate observed by the detector in the x-z plane results from fractions of both components of the incident beam scattering into it, i.e.,

$$\dot{N}_z = I_z d\sigma(0) + I_y d\sigma(\pi/2). \quad (11)$$

We have assumed ideal geometry and 100% detector efficiency. Similarly,

$$\dot{N}_y = I_z d\sigma(\pi/2) + I_y d\sigma(0). \quad (12)$$

Fig. 10. Diagram for considering the measurement of photon polarization via Compton scattering.

From the above, it follows that

$$\frac{\dot{N}_z - \dot{N}_y}{\dot{N}_z + \dot{N}_y} = \frac{[d\sigma(\pi/2) - d\sigma(0)][I_y - I_z]}{[d\sigma(\pi/2) + d\sigma(0)][I_y + I_z]}. \quad (13)$$

Substitution of the asymmetry ratio, Eq. (10), and the polarization, $P = (I_y - I_z)/(I_y + I_z)$, into the above yields

$$\frac{\dot{N}_z - \dot{N}_y}{\dot{N}_z + \dot{N}_y} = \frac{R - 1}{R + 1} \cdot P \quad (14)$$

or

$$P = \frac{R + 1}{R - 1} \cdot \frac{\dot{N}_z - \dot{N}_y}{\dot{N}_z + \dot{N}_y}. \quad (15)$$

Thus, if the polarimeter asymmetry ratio is known, a measurement of the scattered photon intensity in two perpendicular planes determines the photon polarization.

In this experiment, the two orthogonal scattering planes are the horizontal and vertical planes. Also, the bremsstrahlung reaction plane (plane of the incident electron and the photon) is always the horizontal plane. Furthermore, there is a correspondence between the above quantities and the ones pertaining to the experiment, as follows:

$$\begin{aligned} \dot{N}_y &\rightarrow \dot{N}_\perp \\ \dot{N}_z &\rightarrow \dot{N}_\parallel \\ I_y &\rightarrow I_\perp \\ I_z &\rightarrow I_\parallel \end{aligned}$$

This discussion has considered only ideal geometry; the effects of finite detector and scatterer dimensions necessary to construct a practical polarimeter have not been included. For practical geometries, all equations and conclusions still apply except for Eq. (10). The polarimeter asymmetry ratio is now defined as

$$R(k_0) \equiv \frac{\int_{\Delta\phi \Delta\psi} \epsilon(k_0, \psi, \phi) \frac{d\sigma}{d\Omega}(k_0, \psi, \phi) d\Omega}{\int_{\Delta\phi \Delta\psi} \epsilon(k_0, \psi, \phi) \frac{d\sigma}{d\Omega}(k_0, \psi, \phi) d\Omega} \bigg|_{\phi=\pi/2}^{\phi=0} \quad (15)$$

where $\epsilon(k_0, \psi, \phi)$ is the efficiency for all processes occurring within the polarimeter other than the Compton scattering (e.g., photon absorption in the scatterer and detector efficiency). The integrals are taken over the finite angles $\Delta\phi$ and $\Delta\psi$ subtended by the scatterer and detector. The calculation of R for the three specific geometries used in this work is discussed in detail in Appendix D.

Features of This Polarimeter—The polarimeter designed for this work is shown in Figs. 6 and 7. A schematic drawing of it is shown in Fig. 8. It consists of a polyvinyltoluene (C_9H_{10}) scatterer and a Ge(Li) detector. The entire arrangement—scatterer, detector, etc., rotates about the polar and azimuthal axes. This was made possible by mounting all polarimeter components onto a single rigid Al framework which is attached to a vertically-standing machinist's index table. This system is in turn mounted on a large baseplate that can be rotated about a vertical axis passing through the x-ray target. This allows measurement of bremsstrahlung emission from $+110^\circ$ to -155° relative to the incident electron beam direction.

The scatterer is housed in an air-tight tubular chamber with 0.6 mg/cm^2 Mylar entrance and exit windows. Helium continually flows through this chamber to reduce attenuation of low-energy photons. The distances between the scatterer and target and the scatterer and detector can be changed, and the Compton scattering angle also can be varied. This permits the asymmetry ratio and polarimeter efficiency to be optimized for any photon energy and intensity.

The defining collimators for the polarimeter were 0.16-cm-thick Pb washers. Two diameters were used, 0.635 cm and 1.11 cm, depending upon the solid angle necessary to obtain adequate counting rates. Shielding (0.3 cm Pb sheet) was wrapped around the detector and scatterer housings to reduce background levels.

The scatterer for a low-energy Compton polarimeter should be of low atomic number to reduce photoelectric absorption of the incident and scattered photons. It should also be amorphous to eliminate the possibility of coherent interference phenomena perturbing the scattering. Organic hydrocarbons generally satisfy these criteria.

The scatterer for this polarimeter was machined from a commercially-available organic scintillator.* Two scatterers of different thicknesses, 0.6 cm and 1.0 cm, were used. For low-Z targets, at high energy, the 1-cm scatterer was needed to obtain a reasonable count rate.

Three combinations of collimation and scatterer thickness were used. The asymmetry ratio (also the efficiency and energy response) was calculated for all three cases using a computer program described in Appendix D. Asymmetry ratios varied from 35 to 200.

5. X-Ray Detector

The x-ray detector used in the polarimeter was a planar-type Ge(Li) detector having an active area of 0.95 cm X 0.95 cm and a depletion depth of 1.02 cm. When biased with 2500 V, it exhibited a resolution of ~550 eV for the 59.5-keV γ ray from ^{241}Am . The detector was mounted in vacuum behind a 12 mg/cm² Be window and was cooled by a cold finger extending from a dewar of liquid nitrogen. A 0.635-cm-diam collimator was positioned in front of the detector, just outside of the Be vacuum window.

6. Electronics System

Standard nuclear counting techniques and electronic circuits were used for recording data. A diagram of the overall electronics system is shown in Fig. 11.

Electron Energy Measurement—The electron energy was monitored by observing voltages applied both to the electron gun and to

* Composition of the scintillator was 97.5% polyvinyltoluene (C_9H_{10}) and 2.5% terphenyl ($\text{C}_{18}\text{H}_{14}$).

Preamplifier: LLL design, LE 15830-1

Amplifier: Canberra instruments, Model 1416

Oscilloscope: Tektronix, R64 7A, Type 102A amplifiers

Fig. 11. Diagram of electronics system.

the accelerating column. The electron gun voltage was measured with a 0-60 kV electrostatic voltmeter. The voltage applied to the accelerating column was determined by measuring the current through a resistor string in the power supply. These instruments were calibrated by determining the bremsstrahlung endpoint energy with the Ge(Li) spectrometer (see Appendix A).

Beam Current Integration—The electron beam current collected by the target chamber was integrated by a commercially-available current integrator (Elcor, Model A310B). The integrator was equipped with a built-in calibration unit, and its calibration was checked every few hours. The procedure consisted of driving a full-scale current through the indicator and integration circuits via a mercury cell for a fixed time. The total measured charge was compared to the product of the full-scale current and the time. Manual meter and circuit adjustments were made until the above comparison differed by less than 0.5%. Occasionally, the built-in calibration unit was shown to be operating properly by connecting an external dry cell and precision resistor to the input of the integrator.

Amplifiers—The preamplifier was an LLL design employing a two-stage, cooled FET. It was housed immediately adjacent to the detector with its FET inside the detector vacuum system.

The linear amplifier was made by Canberra Instruments (Model 1416). Pulse shaping of 2 μ sec was used to match the optimum input needs of the Nuclear Data multichannel analyzer.

Scalers—Two single-channel analyzers were used with scalars to monitor interesting x-ray energy intervals during successive runs. This provided an easy method for adjusting beam current to obtain proper counting rates, and also provided a running check of the overall progress of the experiment. For example, broken foils were immediately detected by a sharp drop in the scaler counting rate. In addition, real time measured by a digital clock triggered by the scalars was compared with the live time recorded in the multichannel analyzer to make dead-time corrections to the data.

Multichannel Analyzer—The multichannel analyzer was a Nuclear Data, Series 2200, 4096 channel analyzer system. Individual data runs were stored in separate quarters of the memory (1024 channels), allowing for informative comparisons by overlaying one upon another and displaying the result on a CRT. This was particularly useful in checking for gain shifts by overlaying the characteristic peaks observed in each of two runs, or for accelerator energy shifts by overlaying the high-energy endpoint of the bremsstrahlung spectrum.

B. Procedure

Each experimental run was conducted in the following manner: The accelerator was set at the proper energy and the beam spot size was observed, using the quartz disc in the target chamber. The bremsstrahlung target was then inserted into the beam, and the current was adjusted to provide an optimum detector counting rate. Beam current was never allowed to exceed 10 μA since a higher current would destroy the target.

With the detector in the horizontal (reaction) plane, data were recorded until a given amount of charge was collected by the Faraday cup (target chamber). The target was replaced by a background target, or empty target holder, and the process was repeated for a background measurement. With the accelerator off, the Ge(Li) detector and electronics system were calibrated using monoenergetic x rays from ^{241}Am and ^{57}Co sources. The detector was then rotated to the vertical plane and the signal, background, and calibration runs were repeated. A total of 192 accelerator runs were necessary, each lasting from 20 to 200 min.

The data were recorded in approximately 500 of the 1024 channels available in one-quarter of the memory of the multichannel analyzer. After four runs, they were transferred onto magnetic tape for reading into a CDC 6600 computer. In addition, the data were printed on paper tape, and the CRT display was photographed for cursory examination of the progress of the experiment.

C. Experimental Data

Photographs of raw data displayed on a CRT using a logarithmic ordinate scale are shown in Fig. 12 to illustrate typical signal-to-background levels encountered. The background count rate was higher for $T_0 = 100$ keV than for lower electron energies, but always less than 4% of signal levels. Apparent from these photographs is the sharpness of the high-energy end point of the spectrum, a feature which allows measurement of polarization even as the value of k/T_0 approaches unity, and the high resolution of the characteristic x-ray lines from the target, a feature which allows accurate removal of these lines from the spectrum.

D. Data Analysis

A program was written for use with a CDC 6600 digital computer to reduce the raw data. The operations performed by the program are listed in sequence below:

1. Energy calibration of multichannel analyzer.
 2. Correction of spectrum for Compton scattering from Ge(Li) detector.
 3. Correction of spectrum for fluorescent escape from Ge(Li) detector.
 4. Removal of target fluorescent lines from the spectrum.
 5. Correction for Compton polarimeter energy response.
 6. Normalization of spectrum (dead-time corrections).
 7. Subtraction of backgrounds.
 8. Calculation of the ratio of the difference and sum of perpendicular and parallel spectra, $(N_{\parallel} - N_{\perp})/(N_{\parallel} + N_{\perp})$.
 9. Calculation of final polarization by correcting the above for asymmetry ratio of the polarimeter.
 10. Fitting of polynomial to final polarization results.
- These operations will now be discussed in detail.

1. Energy Calibration of Multichannel Analyzer

The Ge(Li) detector and associated electronics system was calibrated using standard IAEA radiation sources of ^{241}Am and ^{57}Co .

Fig. 12. Photographs of raw data showing signal and background levels. Ordinate scale is logarithmic; each division is approximately a factor of ten.

These sources provided photons of energies 11.9, 13.9, 17.8, 20.8, 26.348, 59.543, 121.97 and 136.33 keV. The computer program selects the largest peaks in the calibration spectrum, fits these peaks with gaussian line shapes, and determines the centroid channel. It then assigns the appropriate energy to that channel and a straight line is fit to the resulting points using a least-squares technique. This line becomes the energy-versus-channel calibration for the run. A separate calibration was made for each run.

2. Correction for Compton Scattering from the Ge(Li) Detector

When monoenergetic photons are incident upon a Ge(Li) detector, a certain fraction will be absorbed via the photoelectric effect while another, smaller, fraction will be scattered out of the detector, depositing a portion of the photon energy in the crystal. The result is that counts appear in low-energy channels rather than in the higher, full-energy channel associated with the photopeak.

This effect has been measured by Slivinsky⁴⁴ and calculated by Smith⁴⁵ for a detector geometry similar to the one used here. Their results were applied to correct the present data to energies as low as 2 keV. The magnitude of the correction is a few percent or less in all cases, and affects only the low-energy channels where $k < T_0/4$.

3. Correction for Fluorescence Escape from the Ge(Li) Detector

Most of the Ge K x rays produced in the detector via photoelectric interactions of the incident x rays will be reabsorbed. Some will escape, however, and carry away approximately 10 keV of energy ($K_\alpha = 9.976$ keV and $K_\beta = 10.984$ keV). The result of this phenomenon is the occurrence of two peaks in the pulse-height spectrum approximately 10 keV below the photopeak. Actually, these peaks are unresolved in the spectrum and appear as a single "escape peak." The magnitude of this "escape peak" relative to the photopeak is a function of incident photon energy, and was experimentally measured in the determination of detector efficiency described in Appendix B. Figure B-3 shows a plot of the escape-peak-to-photopeak ratio as a function of energy.

The program makes a channel-by-channel correction for the escape peak, starting at the high-energy end of the spectrum and working downward, using the measured escape-peak-to-photopeak ratio as a function of energy. The correction is negligible for energies above 50 keV, is small between 15 and 50 keV because the spectrum shape is decreasing with increasing energy, but can reach values of from 10 to 50% at energies below 10 keV.

4. Removal of Fluorescent Lines from the Spectrum

The characteristic x rays of the bremsstrahlung target are unpolarized and it is necessary to remove these lines from the spectrum. We attempted to interpolate between the bremsstrahlung counts on each side of the lines, leaving a continuous spectrum. The uncertainty in this interpolation, however, was significant. Therefore, all data within the energy intervals of the characteristic lines were discarded and the final polarization in this region was obtained from interpolation of the polarization in other regions of the spectrum. To do this, the program sets the counts in all channels within the specified interval to zero. This is done after the corrections for Compton scattering and Ge K x-ray escape are made.

5. Correction for the Compton Polarimeter Energy Response

The energy response of the Compton polarimeter was calculated, and is discussed in detail in Appendix D. The polarimeter has two effects upon the bremsstrahlung spectrum. The first is to shift the photon energy due to the Compton scattering according to Eq. (8). The second is to broaden the photon energy due to the finite dimensions of the scatterer and detector.

It is seen in the Appendix that the width of the energy response function is small enough to allow correction of the scattered energy spectrum, to obtain the unscattered spectrum, simply by applying Eq. (8). This is accomplished by multiplying k by the ratio k_0/k . Of course, the ratio k_0/k is not a linear function of energy, and therefore the multiplication is not done channel-by-channel. The width of each energy interval must be adjusted by the derivative of k

with respect to k_0 . The program makes this nonlinear multiplication correction to the scattered spectrum to obtain the true bremsstrahlung spectrum incident upon the polarimeter.

6. Normalization of Spectrum (Dead-Time Corrections)

All spectra are normalized to the same total incident beam charge and corrected for analyzer dead-time by multiplying the counts in each channel by a constant. This normalization constant is

$$K = \frac{Q_0}{Q} \times \frac{T}{\tau} \quad (17)$$

where

Q_0 = some arbitrary reference charge to which each run was normalized,

Q = total integrated beam current incident on target,

T = actual elapsed time of run,

τ = total analyzer "live time" during run.

The dead-time contribution to the normalization was always $<0.2\%$.

7. Subtraction of Backgrounds

Background data undergo the same corrections as signal data, and are then subtracted from the signal plus background data channel by channel.

8. Calculation of the Difference of Spectra

The corrected spectra are then combined, channel by channel, to form the following ratio:

$$\frac{N_{\parallel} - N_{\perp}}{N_{\parallel} + N_{\perp}},$$

where $N_{\parallel}$ are the corrected counts corresponding to the polarimeter detector lying in the reaction plane, and $N_{\perp}$ are the corrected counts corresponding to the polarimeter detector perpendicular to the reaction plane.

9. Correction for Asymmetry Ratio of the Polarimeter

The asymmetry ratio R of the polarimeter is a function of photon energy. This asymmetry ratio affects the apparent photon polarization, and as was discussed in Section IV-A-4, is used as a correction to the recorded data in the following manner:

$$P(k) = \frac{N_{\parallel} - N_{\perp}}{N_{\parallel} + N_{\perp}} \cdot \frac{R(k) + 1}{R(k) - 1} \quad (18)$$

where P is the polarization, $N_{\perp}$ and $N_{\parallel}$ are the corrected counts, and $R(k)$ is the asymmetry ratio of the polarimeter, all for energy k . This correction is applied channel by channel.

Three different polarimeter geometries were employed during the course of these measurements, and the asymmetry ratio for each is plotted as a function of photon energy in Appendix D. It is seen that R is large enough for all three cases, so that the correction $(R + 1)/(R - 1)$ ranges between 1.06 and 1.01. For ease in applying this correction in the computer program, each of the three asymmetry ratios were approximated by the straight lines indicated in Fig. D-4. The largest error in the correction ratio $(R + 1)/(R - 1)$, resulting from this approximation is less than 0.3%, and the typical error is <0.1%.

10. Fitting a Polynomial to the Final Polarization Results

The data analysis program plots polarization P as a function of photon energy k for a given target material, electron energy, and angle of emission. These data are fit to polynomials of degree 1 through 4 using a standard least-squares technique. The value of χ -square divided by the number of degrees of freedom is then computed as a measure of the goodness of each fit.

E. Error Analysis

The polarization P is defined as a ratio of intensity components of the bremsstrahlung, i.e.,

$$P = \frac{I_{\perp} - I_{\parallel}}{I_{\perp} + I_{\parallel}} \quad (19)$$

However, as derived from experimental data, the polarization is

$$P = \frac{N_{\parallel} - N_{\perp}}{N_{\parallel} + N_{\perp}} \cdot \frac{R + 1}{R - 1} \cdot f \quad (20)$$

or

$$P = N \cdot K \cdot f \quad (21)$$

where $N = (N_{\parallel} - N_{\perp}) / (N_{\parallel} + N_{\perp})$ and $N_{\perp}$ and $N_{\parallel}$ are counts in a particular energy interval; $K = (R + 1) / (R - 1)$ is the polarimeter asymmetry correction factor; R is the polarimeter asymmetry ratio; and f is the ratio of the true polarization to the value of the polynomial fit (f equals unity for a perfect fit to the data).

The total uncertainty, ΔP , in the polarization is given by

$$\frac{\Delta P}{P} = \sqrt{\left(\frac{\Delta N}{N}\right)^2 + \left(\frac{\Delta K}{K}\right)^2 + \left(\frac{\Delta f}{f}\right)^2} \quad (22)$$

where ΔN , ΔK and Δf are the individual uncertainties in the factors of Eq. (21). Each of these uncertainties will now be discussed separately.

ΔK

The polarimeter asymmetry correction $K = (R + 1) / (R - 1)$ is a function only of the asymmetry ratio R . Consequently, the uncertainty ΔK is due solely to the uncertainty ΔR , and the dependence is given by

$$\Delta K = \frac{\partial K}{\partial R} \cdot \Delta R = \frac{2\Delta R}{(R - 1)^2} \quad (23)$$

and

$$\left(\frac{\Delta K}{K}\right)^2 = \left(\frac{2\Delta R}{R^2 - 1}\right)^2 \quad (24)$$

The uncertainty in R is estimated at <25%. The ratio R was calculated as described in Appendix D, and no experimental verification of this was possible. However, experimental verification of another feature of the polarimeter predicted by the same calculation (shape of the energy response function to monoenergetic, unpolarized photons) gave confidence that a 25% uncertainty in asymmetry ratio is conservative.

Since the correction factor K is small (<6% in the worst case), a large uncertainty in R still yields a small uncertainty in final polarization. For example, the 25% uncertainty in R results in an error in the polarization of <2% for $R = 35$ (the worst case).

Δf

The uncertainty Δf is a measure of the quality of the polynomial fit to the final polarization data. The magnitude of this uncertainty is obtained as a numerical by-product of the least-squares calculation. Implicit in Δf are the uncertainties due to counting statistics for both the signal and background spectra, as well as uncertainties due to the fit itself. Values for $\Delta f/f$ are typically about 1%, but in cases of extremely poor statistics, such as obtained at the endpoints of the spectrum at backward angles, or in cases of near-zero polarization ($N_{\perp} \approx N_{\parallel}$) the uncertainty is as large as 15%.

ΔN

The uncertainty ΔN results from the uncertainties in $N_{\perp}$ and $N_{\parallel}$, the measured and corrected counts at each energy. Since $N_{\perp}$ and $N_{\parallel}$ are independent quantities,

$$\Delta N^2 = \left(\frac{\partial N}{\partial N_{\perp}} \Delta N_{\perp} \right)^2 + \left(\frac{\partial N}{\partial N_{\parallel}} \Delta N_{\parallel} \right)^2 \quad (25)$$

or

$$\Delta N = \frac{2N_{\perp}N_{\parallel}}{(N_{\perp} + N_{\parallel})^2} \sqrt{\left(\frac{\Delta N_{\perp}}{N_{\perp}} \right)^2 + \left(\frac{\Delta N_{\parallel}}{N_{\parallel}} \right)^2} \quad (26)$$

where $N_{\perp}$, $N_{\parallel}$, $\Delta N_{\perp}$, $\Delta N_{\parallel}$ and, consequently, ΔN , are functions of the photon energy. Furthermore,

$$\left(\frac{\Delta N}{N} \right)^2 = \frac{4N_{\parallel}^2N_{\perp}^2}{(N_{\perp}^2 - N_{\parallel}^2)^2} \left[\left(\frac{\Delta N_{\perp}}{N_{\perp}} \right)^2 + \left(\frac{\Delta N_{\parallel}}{N_{\parallel}} \right)^2 \right]. \quad (27)$$

The uncertainties which contribute to N do so through the terms $\Delta N_{\perp}$ and $\Delta N_{\parallel}$, i.e., $\Delta N_{\perp}^2 = \sum_i (\Delta N_{\perp})_i^2$ and $\Delta N_{\parallel}^2 = \sum_i (\Delta N_{\parallel})_i^2$. A discussion of each of these contributions $(\Delta N_{\perp})_i$ and $(\Delta N_{\parallel})_i$, follows.

Electron Energy

The accelerator voltage was determined to 1.1% and was stable to <0.2%. Since polarization varies slowly with electron energy, resulting uncertainty is negligible.

Target Thickness

The uncertainty in target thickness is estimated at <10%. However, the discussion in Section IV-3 establishes that the targets used here were sufficiently thin to have no effect upon polarization. This conclusion was based upon comparing the small mean angle of electron scattering in the target to the larger angular spread of the polarimeter.

Kulenkampff and Zinn³³ measured polarization as a function of target thickness and extrapolated to zero thickness to obtain true polarization. Their results, if applicable to this work, indicate that a correction of 5% should be made to the present data for the worst case ($T_0 = 50$ keV, $Z = 79$). However, since their measurements were made only for Au at 90° and also were strongly a function of polarimeter geometry, it is not clear to what extent their corrections apply here. Consequently, no correction to the data was made, but a systematic uncertainty of between zero and 2% has been estimated based on their findings.

Bremsstrahlung Emission Angle

The uncertainty in determining the bremsstrahlung emission angle θ is negligible. The spread in θ , $\Delta\theta$, was accounted for in the calculation of the polarimeter asymmetry ratio, but still has the effect of averaging the bremsstrahlung polarization over its extent. The $\Delta\theta$ subtended by the polarimeter was 7° or 10° , depending upon the particular collimator used. Since polarization varies slowly and smoothly over this small angular spread, the error is <1.5%. This was

estimated using the shape of the angular dependence of polarization as calculated by Tseng and Pratt.

Polarimeter Azimuthal Angle

The uncertainty in the azimuthal angle, ϕ , is negligible and its spread, $\Delta\phi$, is accounted for in the calculation of polarimeter asymmetry ratio.

Beam Current Integration

The accuracy of the current integrator is $\pm 2\%$. However, only its precision affects the uncertainty of the polarization since the measured value of integrated current is used to normalize one run against another; the absolute value is of no consequence. The precision of the integrator was continually checked by integration of a known current for a fixed time. The variations were $< 0.5\%$ of the mean value.

Dead-Time Corrections

Dead-time corrections are always $< 0.2\%$. The uncertainty in this correction is negligible.

Ge(Li) Detector Efficiency

The absolute efficiency of the Ge(Li) detector was measured (Appendix B) and is believed accurate to within $\pm 3\%$. However, the ratio $(N_{\parallel} - N_{\perp}) / (N_{\parallel} + N_{\perp})$ is independent of detector efficiency, and so this uncertainty has no effect upon polarization.

Fluorescence Escape Correction

The uncertainty in the fluorescence escape peak correction is $< 10\%$ at high energies (50 keV), where the correction is small, and considerably less at low energies (15 keV), where the correction becomes significant. At 5 keV, where the correction to the counts can be as large as 50% the resulting uncertainty in polarization is still $< 5\%$.

Correction for Escape of Compton-Scattered Photons

The uncertainty in this correction can be as large as 50% at low energies. Since the magnitude of the correction is <2%, the consequent uncertainty in the polarization ranges from negligible to <2% at a few keV.

Polarimeter Energy Response

The effect of the energy response of the polarimeter is to shift and broaden the apparent energy of the incident photons. The final data are corrected for the shift. The broadening was shown to be negligible by examining the line shapes of fluorescent x rays observed by the polarimeter. Therefore, no correction was applied to the data for this effect.

The magnitudes of $\Delta K/k$, $\Delta f/f$ and $\Delta N/N$ were calculated for various cases and substituted into Eq. (22) to obtain the final uncertainty in the polarization. This uncertainty, $\Delta P/P$, is indicated as error bars on some of the data points. Error bars have only been calculated for representative cases.

V. RESULTS

The raw spectral data were processed according to the procedure discussed in Section IV-D to determine linear polarization as a function of photon energy, with Z , T_0 , and θ as parameters.

Experimental results for the 48 cases measured are presented in Fig. 13a-1. Polarization is plotted as a function of both photon energy (k/T_0 , photon energy relative to the incident electron energy), and emission angle θ . Positive values indicate polarization perpendicular to the bremsstrahlung reaction plane, and negative values polarization parallel to the reaction plane. Shaded regions at the high- or low-energy ends of some plots indicate extrapolations of the data. Dashed segments of the grid lines indicate regions of the spectra where characteristic x-ray lines of the target appeared and polarization results were interpolated. Also, for cases in which reasonable agreement with theory was obtained, calculated points are indicated on the plots.

Fig. 13a. Linear polarization plotted as a function of both photon energy k/T_0 , and emission angle θ . Photon energy is expressed relative to the incident electron energy, T_0 . Positive (negative) values indicate perpendicular (parallel) polarization. Shaded regions indicate extrapolations of the polarization data, and dashed-line segments indicate interpolation of same.

Fig. 13b.

Fig. 13c.

Fig. 13d.

Fig. 13e.

Fig. 13f.

Fig. 13g.

Fig. 13h.

Fig. 13i.

Fig. 13j.

● Tseng and Pratt

Fig. 13k.

Fig. 131.

Polynomials of degree 1 through 4 were fit to the data using the least-square technique. A measure of the goodness of the fit was evaluated by determining the value of χ^2 square divided by the number of degrees of freedom. With very few exceptions, the fit was well determined. Over 50% of the cases had χ^2/f values between 0.8 and 1.2, and 70% had values between 0.7 and 1.3. The number of degrees of freedom was typically 400. The data in Fig. 13 are plots of the best polynomial fits obtained in each case. Table 3 lists the degrees and coefficients of these polynomials.

A. k-Dependence of Polarization

The qualitative dependence of linear polarization upon photon energy was predicted in Section II by considering a classical description of bremsstrahlung. High-energy photons tend to be polarized parallel to the reaction plane, and low-energy photons perpendicular to this plane. This dependence was observed in all 48 cases presented in Fig. 13.

B. Z-Dependence of Polarization

In the bremsstrahlung process, the Coulomb field of the nucleus may alter the direction of the incident electron before the photon is emitted. Referred to as the Coulomb effect,^{*} this results in a decrease in the observed polarization. The magnitude of this effect increases with Z and predicts an inverse Z-dependence of polarization.

The experimental data substantiate this Z-dependence for both parallel and perpendicular polarization. This is illustrated in Figs. 14a through 14d which show polarization as a function of both k/T_0 and Z. These curves include four combinations of θ (45° and 90°) and T_0 (50 keV and 100 keV). They clearly illustrate the trend for all energies and angles. The low-energy end of the spectrum is seen to exhibit its most dramatic dependence upon Z at angles for which the polarization is a maximum, i.e., $\theta \approx 45^\circ$ for $T_0 = 100$ keV and

^{*}In general, the Coulomb effect refers to the difference between the exact and the Born Approximation bremsstrahlung cross section (see Refs. 4 and 18).

Fig. 14a. Linear polarization plotted as a function of both relative photon energy k/T_0 , and atomic number Z .

Fig. 14b.

Fig. 14c.

Fig. 14d.

Table 3. Coefficients of polynomial fits to experimental data $P(k/T_0)$
 $= a + b(k/T_0) + c(k/T_0)^2 + d(k/T_0)^3$, where k/T_0 = photon
 energy relative to the incident electron energy.

T_0 (keV)	Z	θ (deg)	n^a	a	b	c	d
50	13	22.5	3	0.0657	-0.525	0.595	-0.866
50	13	45	2	0.0611	-1.02	0.0655	—
50	13	90	1	0.0376	-0.995	—	—
50	13	135	2	-0.138	0.238	0.8175	—
50	29	22.5	2	-0.0178	0.0316	-0.635	—
50	29	45	2	0.00919	-0.805	0.012	—
50	29	90	1	-0.0127	-0.795	—	—
50	29	135	2	-0.0948	-0.0116	-0.465	—
50	47	22.5	2	-0.00337	-0.202	-0.273	—
50	47	45	2	0.0192	-0.945	0.239	—
50	47	90	1	-0.0947	-0.630	—	—
50	47	135	2	-0.121	-0.111	-0.233	—
50	79	22.5	1	0.0122	-0.415	—	—
50	79	45	2	-0.102	-0.765	0.2473	—
50	79	90	1	-0.272	-0.384	—	—
50	79	135	2	-0.0977	-0.358	-0.0273	—
75	13	22.5	2	0.0604	-0.421	-0.404	—
75	13	45	2	0.167	-1.31	0.253	—
75	13	90	1	0.141	-1.05	—	—
75	13	135	2	-0.0648	-0.0029	-0.534	—
75	29	22.5	2	-0.0141	-0.1275	-0.544	—
75	29	45	2	0.153	-1.22	0.254	—
75	29	90	1	0.0654	-0.870	—	—
75	29	135	2	-0.0799	0.124	-0.574	—
75	47	22.5	2	-0.00343	0.0140	-0.434	—
75	47	45	2	0.102	-1.01	0.199	—
75	47	90	1	-0.0123	-0.780	—	—
75	47	135	1	-0.0219	-0.338	—	—
75	79	22.5	1	-0.0102	-0.416	—	—
75	79	45	2	-0.00692	-1.07	0.419	—
75	79	90	1	-0.209	-0.488	—	—
75	79	135	1	-0.122	-0.221	—	—
100	13	22.5	2	0.115	-0.689	-0.170	—
100	13	45	2	0.266	-1.70	0.547	—
100	13	90	1	0.136	-1.06	—	—
100	13	135	2	-0.0746	0.178	-0.732	—
100	29	22.5	3	0.151	-0.711	0.413	-0.595
100	29	45	2	0.214	-1.44	0.408	—
100	29	90	1	0.135	-0.967	—	—
100	29	135	2	-0.0743	0.0682	-0.439	—
100	47	22.5	1	0.100	-0.693	—	—
100	47	45	2	0.187	-1.53	0.628	—
100	47	90	1	0.0983	-0.924	—	—
100	47	135	2	-0.0489	-0.230	-0.139	—
100	79	22.5	1	0.0345	-0.577	—	—
100	79	45	2	0.0736	-1.32	0.576	—
100	79	90	2	-0.0274	-1.03	0.398	—
100	79	135	1	-0.0869	-0.330	—	—

^a n = the degree of the polynomial fit.

$\theta \approx 90^\circ$ for $T_0 = 50$ keV. The Z -dependence of the high-energy limit is less dependent upon angle.

C. T_0 -Dependence of Polarization

The dependence of linear polarization upon the energy of the incident electron is illustrated in Figs. 15a through 15d. Polarization is plotted as a function of both k/T_0 and T_0 . Results have been shown only for $Z = 13$, but they are similar for all Z .

It is apparent that at low photon energies the perpendicular component of the polarization increases with T_0 , and most dramatically at angles of peak polarization, i.e., $\theta = 45^\circ$ and 90° . This T_0 -dependence is anticipated because the Coulomb effect, mentioned earlier, decreases with increasing electron energy. Also, as the electron becomes more relativistic the transverse component of acceleration is enhanced, thus increasing the perpendicular component of the bremsstrahlung. For high-energy photons, no dependence of polarization upon T_0 for $T_0 = 50$ to 100 keV was observed.

D. θ -Dependence of Polarization

The angular dependence of the polarization is seen in Fig. 13. These show polarization as a function of both k/T_0 and θ . For both low-energy and high-energy photons, the polarization is highest between 45° and 90° and falls off at 22.5° and 135° . Peak polarization is predicted at 90° by the classical description presented earlier, but, due to relativistic effects, occurs at a smaller angle as T_0 increases. This trend is seen in the experimental results upon comparison of the low-energy perpendicular polarizations of Al at θ equal to 45° and 90° for T_0 of 50 and 100 keV.

Fig. 15a. Linear polarization plotted as a function of both relative photon energy k/T_0 , and incident electron energy T_0 .

Fig. 15b.

Fig. 15c.

Fig. 15d.

VI. COMPARISON WITH THEORY AND PREVIOUS EXPERIMENT

A. Experimental Results of Motz and Placius

Very little quantitative experimental data are available on the linear polarization of bremsstrahlung in general and, with the exception of the measurements by Motz and Placius (MP),¹⁸ virtually none to which valid comparison with this work can be made. Furthermore, comparison with the MP results may be made in only isolated cases. Motz and Placius discuss polarization by considering the high-energy and low-energy regions of the photon spectrum separately. For the purpose of comparison, the same distinction shall be made here.

Figures 16 and 17 show the MP results for the polarization at the high-energy limit of the spectrum ($k/T_0 = 0.9$) as a function of θ for $Z = 4$ and 79 and $T_0 = 50$ and 100 keV. Also shown are the predictions of the Gluckstern and Hull¹⁰ calculation. Corresponding results of this work are plotted for comparison for $Z = 13, 29, 47$ and 79, $\theta = 22.5^\circ, 45^\circ, 90^\circ$ and 135° . The agreement between experiments is extremely good for $Z = 79$, except at $\theta = 90^\circ$ where the present results more closely match those predicted by calculation. Higher polarizations were obtained for $Z = 13, 29$ and 47 than the MP results show for $Z = 4$. This is not expected in view of the inverse Z dependence of polarization, and may result from the better energy resolution of the present work. The present results exhibit excellent qualitative agreement with the shape of the polarization-versus-emission angle curve predicted by theory.

It is useful to consider the peak polarization for a given k and T_0 . This peak polarization is obtained from data on the angular dependence of polarization. Figures 18 and 19 show MP results and Gluckstern-Hull predictions for the peak polarization of the low-energy region of the photon spectrum ($k/T_0 = 0.1$) as a function of T_0 , for $Z = 13$ and 79. Note that this polarization is predominantly perpendicular to the reaction plane (positive in value). For the case of $Z = 79$, Motz and Placius have constructed a semiempirical curve extending downward in energy to 200 keV. The present results for $T_0 = 50$ and 100 keV are in excellent agreement with this curve and allow extrapolation to $T_0 < 50$ keV.

Fig. 16. Linear polarization of the high-energy limit of the spectrum, $k/T_0 = 0.9$, as a function of angle for $T_0 = 100 \text{ keV}$.

Fig. 17. Linear polarization of the high-energy limit of the spectrum, $k/T_0 = 0.9$, as a function of angle for $T_0 = 50$ keV.

Fig. 18. Peak linear polarization of the low-energy end of the spectrum, $k/T_0 = 0.1$, as a function of incident electron energy for $Z = 13$.

Fig. 19. Peak linear polarization of the low-energy end of the spectrum, $k/T_0 = 0.1$, as a function of incident electron energy for $Z = 79$.

Finally, Motz and Placious have constructed a family of semi-empirical curves which describe the behavior of the peak linear polarization as a function of T_0 for $k/T_0 = 0.1, 0.4, 0.6$ and 0.9 , and for $Z = 13$ and 79 . Again these curves are terminated on the low-energy end at $T_0 = 200$ keV. Figures 20 and 21 show that the results of the present measurements are in excellent agreement, and again allow extrapolation to $T_0 < 50$ keV.

Since the measurements here were made only at angles of 22.5° , 45° and 90° , peak polarization values had to be estimated by interpolation between data points. The shape of the angular distribution of polarization as predicted by Tseng and Pratt was used for this interpolation.

B. Gluckstern and Hull's Calculation

Polarization values predicted by the Gluckstern-Hull¹⁰ (GH) calculation were taken from the paper by Motz and Placious.¹⁸ Comparisons of polarization versus angle for $k/T_0 = 0.9$ and $T_0 = 50$ and 100 keV are shown in Figs. 16 and 17. Qualitative agreement in shape is obtained, but predictions are generally too large. The GH results are for an unscreened calculation. Experimentally measured peak polarization as a function of incident electron energy differs markedly from calculation, particularly at high Z (see Fig. 18 and 19).

Figure 22 shows peak polarization as a function of photon energy for $Z = 13$, $T_0 = 50, 75$ and 100 keV. Qualitative agreement with GH (unscreened) is obtained over a large portion of the spectrum in the 100 -keV case, but not otherwise.

C. Tseng and Pratt's Calculation

Polarization values predicted by the Tseng and Pratt⁵ (TP) calculation were computed. Their computer program was obtained and converted for use on the CDC 7600 computers at LLL. Since each combination of k/T_0 , Z and T_0 required three successive computer programs and significant computer time, it was not possible to

Fig. 20. Peak linear polarization as a function of incident electron energy for $Z = 13$ and $k/T_0 = 0.1, 0.4, 0.6$, and 0.9 .

Fig. 21. Peak linear polarization as a function of incident electron energy for $Z = 79$ and $k/T_0 = 0.1, 0.4, 0.6$, and 0.9 .

Fig. 22. Peak linear polarization as a function of photon energy, k/T_0 , for $Z = 13$, and $T_0 = 50, 75$, and 100 keV. Comparison with Gluckstern and Hull calculation.

consider every combination measured. Consequently, the following representative cases were selected:

T_0 (keV)	Z	k/T_0
50	13	0.4, 0.6, 0.8
50	79	0.4, 0.6, 0.9
75	13	0.6, 0.8, 0.9
75	79	0.4, 0.6, 0.8, 0.9
100	13	0.6, 0.8, 0.9
100	79	0.4, 0.6, 0.8, 0.9

Figures 23a through 23f show both TP and present experimental results for polarization versus angle. Also, TP results are indicated on Figs. 13a, b, c, j, k, l.

For $T_0 = 100$ keV and $Z = 13$, agreement is excellent; and for $T_0 = 100$ keV and $Z = 79$, it is reasonable. However, for lower electron energies, $T_0 = 75$ and 50 keV, the calculated values qualitatively agree in shape but are consistently higher than experiment.

D. Haug's Calculation

Haug¹¹ has calculated the polarization at the high-energy limit of the spectrum for $T_0 = 100$ keV and $Z = 4$ and 79. Figure 24 compares his results with experiment, and shows reasonable agreement at all angles for $Z = 79$. At lower Z , the experimental polarization is less for $\theta = 22.5^\circ$ and 135° than interpolation between Haug's results for $Z = 4$ and 79 would imply. Haug's values are for an unscreened nucleus.

E. Sommerfeld's Theory

1. Kirkpatrick and Wiedmann's Calculation

A program was written to compute the polarization as calculated by Kirkpatrick and Wiedmann⁹ (KW). Agreement with experiment was generally poor, as expected, since the Sommerfeld calculation does not include screening or relativistic effects. However, reasonable agreement was obtained in some cases, but only at forward angles (22.5° or 45°). Also, agreement generally tended to improve for low

Fig. 23a. Linear polarization as a function of angle with parametric dependence upon photon energy, k/T_0 . Solid lines are calculated results of Tseng and Pratt.

Fig. 23b.

Fig. 23c.

Fig. 23d.

Fig. 23e.

Fig. 23f.

Fig. 24. Linear polarization of the high-energy limit of the spectrum, $k/T_0 = 1.0$, as a function of angle for $T_0 = 100$ keV. Solid lines are the results of Haug's calculation.

T_0 and low Z , again as expected. KW points are indicated only for cases in which reasonable agreement was obtained (Figs. 13a, 13b, 13d through 13h).

2. Kulenkampff, Scheer and Zeitler's Relativistic Transformation

The computer program which calculated KW bremsstrahlung polarization also applied the Kulenkampff et al.¹⁴ relativistic transformation to the results. This transformation is valid only for electron energies < 40 keV, and should not apply to the energies employed here. However, for forward angles and $Z = 79$, agreement was obtained for $T_0 = 50, 75$ and 100 keV. These cases are indicated in Fig. 13j.

VII. CONCLUSIONS

The purpose of this work was to conduct a systematic experimental investigation of the linear polarization of low-energy electron bremsstrahlung. Linear polarization was measured as a function of photon energy over a wide range of emission angles and target Z . The results allow interpolation of polarization for all values of θ , Z , and T_0 ($50 \text{ keV} < T_0 < 100 \text{ keV}$). Quantitative comparison of present data with previous experimental results, where possible, has produced good agreement.

The observed polarization dependence upon the variables k , T_0 , θ and Z is in qualitative agreement with the predictions of theory. Quantitative comparison with the results of individual theories illustrates that some calculations are more successful at predicting polarization than others. A summary of these comparisons follows.

1. The quality of agreement varies from fair to excellent. No pattern is obvious, but trends in the comparisons may be seen in Fig. 25. Only cases of good agreement have been indicated on this figure. In all other cases, agreement was poor.

Fig. 25. Chart showing the agreement obtained between experimental results and theoretical predictions. Only instances of good agreement are indicated.

2. The numerical calculation of Tseng and Pratt is more successful than other calculations at predicting polarization, particularly for low Z . It should be emphasized that TP calculations were carried out only for $Z = 13$ and 79 (shaded regions on Fig. 25).
3. The Born approximation calculations (such as Gluckstern and Hull or Fronsdaal and Uberall) generally predict polarization values larger than observed. However, the GH calculation of peak polarization (including screening) shows fair agreement with experiment for low Z .
4. Sommerfeld predictions, whether relativistically transformed or not, cannot generally be applied to this electron energy region. The few instances of agreement obtained may be fortuitous.

Suggestions for Future Work

During the course of this work it has become apparent that several related experiments would be useful for testing the theory more completely. These are:

1. Measurement of polarization emphasizing the low k/T_0 region of the spectrum.

Observed bremsstrahlung intensities fall off at the low-energy end of the spectrum. Also, polarimeter efficiency is decreased severely for low photon energy. Consequently, experimental results are less certain and more difficult to obtain. Polarization in this region, however, is most sensitive to the screening of the nucleus by orbital electrons, and its determination should be helpful in guiding the calculational treatment of screening.

2. Measurement of polarization for $T_0 < 50$ keV.

Little quantitative bremsstrahlung polarization data for very low electron energy ($T_0 < 50$ keV) exists. However, both the Sommerfeld theory and Tseng and Pratt calculations apply in this region. The difficulty in performing an experiment at these energies is primarily one of obtaining thin targets. An obvious approach would be to attempt the measurement using gas targets and a high-resolution Si(Li) detector.

3. Measurement of polarization using polarized electrons.

The calculation by Haug¹¹ indicates a dependence of linear polarization upon the spin orientation of the incident electron. Born approximation calculations^{6,7} do not.

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APPENDIX A—ELECTRON ENERGY CALIBRATION

The total voltage applied to the accelerator is the sum of the voltages applied to the electron gun and the accelerating column. An electrostatic voltmeter* is used to measure the gun voltage, and a precision resistance string, built into the 150-kV power supply, is used to measure the voltage across the accelerating column.

The accuracy and precision of the voltage readings were established by an energy calibration procedure based upon measurement of the endpoint energy of thin-target bremsstrahlung spectra generated by the electron beam. The spectra were observed with a high-resolution Ge(Li) detector, and the endpoints were well-defined (see Fig. A-1).

The bremsstrahlung was generated in Au and Mo targets of $\sim 20 \mu\text{g}/\text{cm}^2$ thickness and observed at 45° to the incident electron direction. The procedure was repeated for different voltage settings over a period of many days. The detector and multichannel pulse-height analyzer were calibrated with radioactive sources of ^{241}Am and ^{57}Co . A plot of bremsstrahlung endpoint energy versus accelerator voltage settings is shown in Fig. A-2.

Errors in the measurement result from: (1) uncertainty in reading the voltage meters, $\pm 1.0\%$; (2) uncertainty in determination of the bremsstrahlung endpoint, $\pm 0.15\%$; and (3) uncertainty in analyzer calibration, $\pm 0.01\%$. The resultant electron energy is determined to within $\pm 1.1\%$.

*The voltmeter is a model ESH, made by Sensitive Research Instrument Corporation, and has a range of 0-60 kV.

$T_0 = 75 \text{ keV}$ $Z = 42$

Target thickness: $20 \mu\text{g}/\text{cm}^2$

Fig. A-1. Thin-target bremsstrahlung spectra observed by a Ge(Li) spectrometer. Ordinate scale is logarithmic, each division is approximately a factor of ten.

Fig. A-2. Bremsstrahlung endpoint energy as a function of accelerator voltage settings. Voltage setting refers to the sum of the voltages indicated by the electrostatic voltmeter and the meter on the control panel of the Sorensen voltage supply.

APPENDIX B—Ge(Li) DETECTOR EFFICIENCY MEASUREMENT

The detector used in the polarimeter was a planar-type Ge(Li) detector having an active area of $0.95 \text{ cm} \times 0.95 \text{ cm}$ and a depletion depth of 1.02 cm . A thin ($\leq 40 \mu\text{g}/\text{cm}^2$) layer of Au was deposited on the surface of the detector, and the entire crystal was mounted in vacuum behind a $12 \text{ mg}/\text{cm}^2$ Be window. With 2500 V bias, the detector had a resolution of $\sim 550 \text{ eV}$ for the 59.5-keV x ray of ^{241}Am .

The technique employed to measure its efficiency was the same as that reported in detail by Slivinsky and Ebert,⁴⁶ and will be discussed only briefly here (see Fig. B-1). The bremsstrahlung x-ray output from a 150-kV , 12-mA , Picker x-ray machine excites a fluorescer whose characteristic radiation is then K-edge-filtered and used as a source of monoenergetic photons. The filter is chosen so that its K absorption edge lies between the energies of the K_α and K_β x rays of the fluorescer. In this manner, the fluorescer K_β x-ray is preferentially attenuated and, for this measurement, contributed $<1\%$ of the total K radiation. Figure B-2 shows an example of Dy fluorescent emission ($K_\alpha = 45.7 \text{ keV}$, $K_\beta = 52.4 \text{ keV}$) before and after passing through a Sm ($K_{\text{abs}} = 46.7 \text{ keV}$) filter. Table B-1 is a listing of the fluorescer-filter combinations used and the corresponding photon energies obtained.

Table B-1. Fluorescer-filter combinations and photon energies obtained.

Fluorescer	Filter	Prefilter ^a	Photon energy
Fe	—	—	6.4
Cu	Ni	—	8.0
Mo	Nb	—	17.4
Pr	La	Ta	35.9
Dy	Sm	Ta	45.7
Pb	Pt and Au	Ta	74.2
U	Th	Er and Pb	97.1

^aPrefilters were placed between the x-ray tube and fluorescer in some cases to attenuate the low-energy bremsstrahlung which scatters from the fluorescer and appears as "fluorescent" radiation to the poor-resolution NaI(Tl) spectrometer.

Fig. B-1. Experimental arrangement for measuring Ge(Li) detector efficiency.

Fig. B-2. Photograph of Dy fluorescent x-ray emission before and after K-edge filtration with Sm. Left half of photograph shows unfiltered spectrum with large K $_{\beta}$ line intensity. Right half of photograph shows same spectrum after filtration. The K $_{\beta}$ line is completely removed.

The absolute intensities of the x rays were measured using a totally absorbing NaI(Tl) spectrometer (4 mm thick $\times$ 2.5 cm diam for $h\nu \leq 45.7$ keV, and 2.5 cm thick $\times$ 2.5 cm diam otherwise). The Ge(Li) detector was placed in the same location and geometry configuration as the NaI(Tl) spectrometer and its efficiency compared to that of the 100% efficient NaI. Efficiencies above 100 keV were measured using the 122- and 136-keV x rays from a calibrated IAEA ^{57}Co source.

Figure B-3 shows a plot of the detector efficiency as a function of photon energy. The lower portion of the upper curve is the photopeak contribution to the efficiency while the upper portion is the sum of the counts in the photopeak plus the Ge escape peak. Also plotted is the escape-peak-to-photopeak ratio, EP/PP , as a function of photon energy. It can be seen that the detector is essentially black (99% efficient) from 20 to 70 keV, and the contribution of the escape peak becomes negligible above 50 keV. These results are in excellent agreement with those measured by Slivinsky and Ebert.⁴⁶

Since the effect of detector efficiency cancels out in a polarization measurement, the efficiency information, per se, is not used. However, the escape-peak-to-photopeak ratio as a function of energy is applied as a correction to the raw data.

Fig. B-3. Ge(Li) detector efficiency and escape-peak-to-photopeak ratio, EP/PP, as functions of photon energy.

APPENDIX C—TARGET THICKNESS MEASUREMENT BY X-RAY FLUORESCENCE TECHNIQUE

A technique was developed for determining the mass thickness (mass per unit area) of a thin target by measuring its K x-ray yield when excited by monoenergetic x rays. The method is accurate to better than $\pm 10\%$ and depends upon knowledge of the K-shell photoelectric cross section and fluorescence yield for the target material.

The source of monoenergetic photons is the same as that described in Appendix B.* The target whose thickness is to be measured is excited by these photons, and the resulting fluorescent emission is observed by a Si(Li) spectrometer. Considering the geometry shown in Fig. C-1, it is seen that the count rate of the Si(Li) spectrometer will be

$$\dot{N}_{\text{Si}} = \int_0^{\rho T / \cos 45^\circ} \phi A e^{-\frac{\mu_0}{\rho} \rho x} \left(\frac{\mu_k}{\rho} \right) e^{-\frac{\mu}{\rho} \rho x} \omega \frac{\Omega_{\text{Si}}}{4\pi} \epsilon_{\text{Si}} \rho dx \quad (\text{C-1})$$

where

$\phi \left(\frac{\text{photons}}{\text{cm}^2\text{-sec}} \right)$ = the photon flux at the defining collimator,

$A \text{ (cm}^2\text{)}$ = area of defining collimator,

$\frac{\mu_0}{\rho} \left(\frac{\text{cm}^2}{\text{gm}} \right)$ = total mass attenuation coefficient of the target material for the incident photons of energy $h\nu_0$,

$\frac{\mu_k}{\rho} \left(\frac{\text{cm}^2}{\text{gm}} \right)$ = mass absorption coefficient for photoionization of the K-shell of the target for photons of energy $h\nu_0$,

$\frac{\mu}{\rho} \left(\frac{\text{cm}^2}{\text{gm}} \right)$ = total mass attenuation coefficient of the target material for photons of energy equal to the target K x-ray energy, $h\nu$,

* A detailed description of this technique for routinely producing monoenergetic photons with energy $< 100 \text{ keV}$ is presented in Ref. 47.

Fig. C-1. Experimental arrangement for measuring target thickness using x-ray fluorescence technique.

ω = K-shell fluorescence yield of the target material,

Ω_{Si} (ster) = solid angle subtended by the Si(Li) spectrometer at the target,

ϵ_{Si} = efficiency of the Si(Li) detector for incident photons of energy $h\nu$ (includes transmission of any material between the target and the detector),

$\rho \, dx \left(\frac{\text{gm}}{\text{cm}^2} \right)$ = element of target thickness,

$\rho T \left(\frac{\text{gm}}{\text{cm}^2} \right)$ = total target thickness.

The flux, ϕ , incident on the defining collimator is determined by measuring the flux at large distance away from the source with a NaI(Tl) spectrometer. Relying on the isotropic nature of the source, the flux at the defining collimator is given by:

$$\phi = \frac{R^2}{r^2} \frac{1}{\epsilon_{\text{NaI}}} \frac{\dot{N}(\text{NaI})}{A(\text{NaI})} \quad (\text{C-2})$$

where

$\dot{N}(\text{NaI}) \left(\frac{\text{counts}}{\text{sec}} \right)$ = count-rate measured by the NaI(Tl) spectrometer,

$A(\text{NaI})(\text{cm}^2)$ = area of the NaI(Tl) spectrometer,

ϵ_{NaI} = efficiency of the NaI(Tl) spectrometer for incident photons of energy $h\nu_0$ (includes transmission of any material in the beam between the target position and the spectrometer),

R, r (cm) = distances indicated in Fig. C-1.

Substituting this expression for ϕ into Eq. (C-1) gives

$$\dot{N}_{\text{Si}} = \frac{R^2}{r^2} \frac{\dot{N}(\text{NaI})}{A(\text{NaI})} \frac{A\omega \epsilon_{\text{Si}}}{\epsilon_{\text{NaI}}} \frac{\Omega_{\text{Si}} \mu_k}{4\pi \rho} \int_0^{\rho T / \cos 45^\circ} e^{-\left(\frac{\mu_0 + \mu}{\rho}\right) \rho x} \rho \, dx. \quad (\text{C-3})$$

Integrating and solving for the target mass thickness, ρT , gives

$$\rho T = \frac{\cos 45^\circ}{\left(\frac{\mu_0}{\rho} + \frac{\mu}{\rho}\right)} \left[\ln F - \ln \left(F - \frac{\dot{N}(\text{Si})}{\dot{N}(\text{NaI})} \right) \right] \quad (\text{C-4})$$

where

$$F = \frac{A}{A(\text{NaI})} \frac{R^2}{r^2} \frac{\epsilon_{\text{Si}}}{\epsilon_{\text{NaI}}} \frac{\Omega_{\text{Si}}}{4\pi} \frac{\left(\frac{\mu_k}{\rho}\right) \omega}{\left(\frac{\mu_0}{\rho} + \frac{\mu}{\rho}\right)} \quad (\text{C-5})$$

Obviously, for thin targets, Eq. (C-1) can be approximated by

$$\dot{N}_{\text{Si}} \approx \phi A \frac{\mu_k}{\rho} \frac{\rho T}{\cos 45^\circ} \omega \frac{\Omega_{\text{Si}}}{4\pi} \epsilon_{\text{Si}} \quad (\text{C-6})$$

and Eq. (C-4) by

$$\rho T \approx \frac{N_{\text{Si}} \cos 45^\circ 4\pi}{\phi A \frac{\mu_k}{\rho} \omega \Omega_{\text{Si}} \epsilon_{\text{Si}}} \quad (\text{C-7})$$

where ϕ is still given by Eq. (C-2)

Uncertainties associated with this technique result from uncertainties in the attenuation coefficients and fluorescence yields used,* as well as from spectral impurities in the photon beam. An error of 5% is assigned to each of these three sources with a resulting total uncertainty of approximately $\pm 9\%$.

* Attenuation coefficients used in this experiment were taken from McMaster et al.,⁴⁸ and fluorescence yields were taken from Fink et al.⁴⁹

APPENDIX D—POLARIMETER ASYMMETRY RATIO, EFFICIENCY AND ENERGY RESPONSE

A computer program has been written to calculate the efficiency, asymmetry ratio, and energy response function of a Compton polarimeter such as the one described here. The program has been designed to apply generally to the geometry shown in Fig. D-1. It assumes that completely polarized photons enter the polarimeter and are scattered from a given volume element of the scatterer into a given volume element of the detector. It then integrates these scattering events over all volume elements of the scatterer and detector.

Summary of Computer Program

Step 1

The incident photon flux is exponentially attenuated in the scatterer to a particular volume element. The total-minus-coherent attenuation coefficient of the scatterer material is used for this calculation. Coherent scattering is not considered since this scattering is mainly in the forward direction, and the photons are assumed to still enter the volume element.

Step 2

The probability of interaction in the scatterer volume element is computed and then multiplied by the ratio of differential-to-total cross sections to find the photon fraction scattered incoherently and the fraction scattered coherently. The incoherent and the coherent differential cross sections are modified by the appropriate form-factors and scattering functions, respectively.

Step 3

The scattered photon flux is exponentially attenuated in the scatterer as it exits. The total attenuation coefficient of the scattering material is used.

All parameters indicated on the sketch are variable.
Also variable are the following:

1. Scatterer and detector materials
2. No. of volume elements of scatterer and detector (grid sizes)

Fig. D-1. Polarimeter geometry applicable to computer program for calculating asymmetry ratio, efficiency and response function.

Step 4

The scattered photon is then exponentially attenuated in the Ge(Li) detector to the given volume element. The total-minus-coherent attenuation coefficient for Ge is used.

Step 5

The probability of photon interaction in the volume element of the detector is computed.

Step 6

The program sums over all combinations of scatterer and detector volume elements which are physically possible for a given source and collimator geometry.

Furthermore, the scattered photons are sorted according to energy, and a response function of the polarimeter for a given incident photon energy is obtained. The integral of this response function is the efficiency of the polarimeter and the ratio of efficiencies for $\phi = \pi/2$ and $\phi = 0$ is the asymmetry ratio.

Mathematical Foundation of the Program

The asymmetry ratio of an ideal Compton polarimeter was defined in Section IV-A-4 as

$$R = d\sigma(\phi = \pi/2)/d\sigma(\phi = 0). \quad (D-1)$$

For a polarimeter with finite dimensions the asymmetry ratio is defined analogously, i.e.,

$$R = \frac{\text{polarimeter efficiency for } \phi = \pi/2}{\text{polarimeter efficiency for } \phi = 0} = \frac{\epsilon_{\pi/2}}{\epsilon_0}. \quad (D-2)$$

The efficiency of a polarimeter of azimuthal orientation ϕ for observing a completely polarized photon source of energy k_0 and intensity of 1 photon/ster is given by the equation:

$$\begin{aligned} \epsilon_{\phi} = & \int_{V_{\text{scat}}} \int_{V_{\text{det}}} \Delta\Omega_s e^{-\mu'x} (1 - e^{-\mu\Delta x}) \frac{d\sigma_i}{\mu} S e^{-\underline{\mu}y} e^{-\underline{\mu}''z} \\ & \times \left(1 - e^{-\underline{\mu}'''\Delta z}\right) \Delta\Omega_D + \int_{V_{\text{scat}}} \int_{V_{\text{det}}} \Delta\Omega_s e^{-\mu'x} \\ & \times (1 - e^{-\mu\Delta x}) \frac{d\sigma_c}{\mu} F^2 e^{-\underline{\mu}y} e^{-\underline{\mu}''z} (1 - e^{-\underline{\mu}'''\Delta z}) \Delta\Omega_D \quad (\text{D-3}) \end{aligned}$$

where the first double-integral term is the efficiency considering incoherent scattering only, and the second double-integral term is the efficiency considering coherent scattering only. Obviously, the coherently and incoherently scattered photons will have different energies, and the proper attenuation coefficients must be used. The symbols in the above equation refer to quantities as follows:

$\Delta\Omega_s$ = solid angle subtended by the volume element of the scatterer;

$\Delta\Omega_D$ = solid angle subtended by the volume element of the detector;

μ' = total-minus-coherent attenuation coefficient of the scatterer material for energy k_0 ;

$\underline{\mu}, \underline{\mu}''$ = total attenuation coefficient of the scatterer material for energy k_0, k , respectively;

$\underline{\mu}'', \underline{\mu}'''$ = total-minus-coherent attenuation coefficient of Ge for energy k_0, k , respectively;

$\underline{\mu}''', \underline{\mu}''''$ = total attenuation coefficient of Ge for energy k_0, k , respectively,

$d\sigma_i$ = differential (Klein-Nishima formula) incoherent scattering cross section (expressed in units of a linear attenuation coefficient, i.e., cm^{-1}); this factor contains the ϕ dependence explicitly;

$d\sigma_c$ = differential coherent scattering cross section (expressed in units of a linear attenuation coefficient, i.e., cm^{-1}); this factor contains the ϕ dependence explicitly;

S = scattering function for modifying Klein-Nishima cross section to include binding effects of the electron to the atom (values taken from Keating and Vineyard⁵⁰);

F^2 = the square of the form-factor for modifying the differential coherent scattering cross section to account for coherent interference from the orbital electrons of the atom (values taken from Shibata⁵¹)

$x, y, z, \Delta x, \Delta z$ = geometrical parameters from Fig. D-1;

$V_{\text{scat}}, V_{\text{det}}$ = volume of the scatterer and detector, respectively.

The program computes $\epsilon_{\pi/2}$ and ϵ_0 by numerically integrating Eq. (D-3) over all volume elements of the scatterer and area elements of the detector using a CDC 6600 computer. Although the program is designed to integrate over the volume of the Ge(Li) detector, in practice the integration was performed only over the detector area. A polarization measurement is independent of detector efficiency, and therefore the efficiency was not computed. Grid sizes were chosen to divide the scatterer into 10^5 volume elements ($<1 \text{ mm}^3$) and the detector into 225 area elements ($\sim 0.44 \text{ mm}^2$). The grid size was checked by continually rerunning the program using successively smaller volume and area elements until the effect upon results was negligible.

The program does not account for multiple photon scattering in the scatterer. It is difficult to calculate the error resulting from this but, as stated previously, experimental verification of the calculated polarimeter response to monoenergetic photons allows confidence that errors are small.

Results

Three different polarimeter geometries were used in the present experiment. They differed in scatterer solid angle and thickness, and were characterized by the following parameters (refer to Fig. D-1).

- Case 1: $r_1 = 9.52$ cm $R = 13.18$ cm $D = 1.27$ cm $\alpha = 45^\circ$
 $r_2 = 5.32$ cm $T = 1.0$ cm $d = 0.635$ cm $t = 1$ cm $\Delta \ll t$
- Case 2: $r_1 = 9.52$ cm $R = 13.18$ cm $D = 0.635$ cm $\alpha = 45^\circ$
 $r_2 = 5.32$ cm $T = 1.0$ cm $d = 0.635$ cm $t = 1$ cm $\Delta \ll t$
- Case 3: $r_1 = 9.52$ cm $R = 3.18$ cm $D = 0.635$ cm $\alpha = 45^\circ$
 $r_2 = 5.32$ cm $T = 0.6$ cm $d = 0.635$ cm $t = 1$ cm $\Delta \ll t$

Response functions, efficiencies, and asymmetry ratios as a function of energy were calculated for each of the three cases. Figure D-2 shows a typical calculated response function. The higher-energy peak is the contribution due to coherent scattering, while the lower, main peak is due to incoherent scattering. Figure D-3 shows a plot of efficiency, ϵ , as a function of photon energy, k_0 , for both the perpendicular ($\phi = \pi/2$) and parallel ($\phi = 0$) configurations of each case. Figure D-4 shows a plot of the asymmetry ratio $R = \langle \epsilon_{\pi/2} / \epsilon_0 \rangle$ versus energy for each case.

The dips in the asymmetry ratios at low photon energies are unexplained, but are of little consequence because of the very small effect the asymmetry ratio correction has upon data below 10 keV. The "worst case" uncertainty resulting from this dip is $<0.6\%$. For ease in applying the $(R + 1)/(R - 1)$ correction to the data using the computer program described in Section IV-D, the calculated polarimeter asymmetry ratios were approximated by the straight lines shown on the graph. This approximation resulted in an error of $<0.2\%$ in all cases.

Fig. D-2. Polarimeter response to monoenergetic incident photons of energy 22.1 keV.

Fig. D-3. Polarimeter efficiency as a function of photon energy for the three geometries used.

Fig. D-4. Polarimeter asymmetry ratio as a function of photon energy for the three geometries used.

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Page 28

2nd paragraph, line 8 should read: "... N = 2 and 4, half ..."

Page 44

Eq. (19), denominator should be: " $I_{\perp} + I_{||}$."

Page 91

Fig. 25, please add following legend to figure:

T = Tseng and Pratt calculation (results available only for cases indicated by shaded regions).

S = Sommerfeld theory (Kirkpatrick and Wiedmann calculation).

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