MASTER

REACTION HISTORY NOTES

G. Roger Gathers

August 3, 1971

Prepared for U.S. Atomic Energy Commission under contract No. W-7405-Eng-48

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UCRL-51107

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REACTION HISTORY NOTES

Abstract

This report deals with the unclassified portions of the technology associated with high-speed diagnostic systems using traveling wave tube oscilloscopes. It includes a discussion of linear sys-

tems analysis for exponential signals, oscilloscope displays, system timing, derivation of commonly used results and an extensive data collection in the appendices.

Introduction

This document presents the unclassified portions of a classified handbook on the art and science of alpha diagnostics which has been in preparation for quite some time. This report deals with the technology of high speed diagnostics of general interest to anyone measuring very fast phenomena. The initial sections of this report deal with the peculiar difficulties encountered in recording fast exponential and Gaussian signals, a limited treatment of the mathematical basis of linear systems analysis and its appli-

cation to specific problems. This is followed by a discussion of the major oscilloscope displays including their uses and difficulties with specific examples. A section on the simultaneity dry run is then included with an example. Two appendices are included giving derivations of results of major interest and associated tables, graphs and nomographs.

Reference 1 is a document which has been written from a different point of view describing high-speed diagnostic techniques.

Recording Exponential and Gaussian Signals

Some sections of this material are marked with an asterisk and are intended for those familiar with modern operational mathematics using Fourier transforms and the Green's function formulation. These sections may be skipped if desired, without loss of continuity.

EXPONENTIAL SIGNAL ELECTRONIC PRINCIPLES

Systems designed to record exponential waveforms have a number of peculiar

features that set them apart from conventional equipment designed using Fourier analysis to represent signals by a superposition of sinusoids. A number of familiar concepts must either be modified or abandoned. Whereas sinusoidal circuit theory using complex analysis represents inductive and capacitive impedances as purely imaginary, all impedances appear to be purely real for an exponential signal. For sinusoids, $Z_R = R$, $Z_C = 1/j\omega C$ and $Z_L = j\omega L$, but for exponential signals we have

$Z_R = R$, $Z_C = 1/\alpha C$ and $Z_L = \alpha L$. Since the input current in the circuit containing L and C is continually rising, there is no opportunity for the energy stored in the electromagnetic fields to be returned to the circuit. These elements appear to be dissipative to the circuit. Since reactive elements cannot be arranged to cancel each other, there are no resonant circuits.

In general, the results for sinusoidal analysis can be adapted to the exponential case by replacing ω by α . The familiar concept of phase velocity associated with sinusoidal theory becomes meaningless, since there is no way to distinguish between amplitude changes and time delay with the purely exponential signal. Propagation velocity can only be defined as the velocity of propagation for a given signal magnitude. For a transmission line with losses, this constant-magnitude velocity is a function of α . Since time delay and amplitude changes cannot be distinguished, one cannot define amplifier gain in terms of the conventional input-output voltage ratio. The exponential signal also has the peculiar property of determining its own plate load impedance and gain in an amplifier. When a load resistance is applied in the plate circuit of an amplifier stage, it is in parallel with the shunt capacitance of the tube, and for large α , the load resistance is dominated by the capacitive impedance which is much less. The load impedance thus varies almost inversely with α for large α .

Conventional pulse amplifiers are not at all efficient for exponential signals. The design of pulse amplifiers is based on the requirement of maintaining uniform gain and phase shift over as wide a

frequency range as possible. Since all circuit elements are dissipative for exponential signals, the compensating elements in these amplifiers lead to inferior performance when compared with those using uncompensated circuits. Optimum amplification of exponential signals is obtained when the gain of the amplifier is maximized over as wide a frequency range as possible (without regard to uniformity of gain). An important requirement is linearity of the amplifier with respect to signal amplitude. If an amplifier begins to saturate, one cannot tell from the recorded signals whether the apparent change produced in $\alpha(t)$ is real or not. Efforts to use amplifiers to extend lower coverage limits at LLL have not been very successful and the difficulties are not well understood.

In some respects the behavior associated with exponential signals is simpler than for sinusoids. Whereas the input impedance of an improperly terminated transmission line excited by sinusoids has a cyclic variation with changes in length, the impedance for exponential excitation quickly and smoothly approaches the characteristic impedance of the line, regardless of the termination used.

Some of the engineering problems associated with transmission lines for exponential signals are more difficult however. Series blocking capacitors in lines must be carefully designed to have minimum stray inductance to prevent large signal attenuation by the voltage dividing action created for large α signals.

The principal effect of linear systems on exponential signals with constant α is to produce a reduction in the amplitude of

the signal without changing α . Linear circuits may thus be characterized by sensitivity reduction factors which are a function of α , and we will consider some examples. It is not our purpose to fully expound the character of exponential circuit theory here but rather to alert the reader to the significant modifications to familiar concepts involved. References 2 and 3 are given for the interested reader who wishes to pursue the matter further. References 4, 5, and 6 are also included for the design of cable attenuators and equalizers for the various systems. We will turn our attention to the concept of convolution used to characterize linear systems.

*LINEAR CONVOLUTION

We may define the basic concept of linear convolution to be

$$f_1 * f_2 = \int_{-\infty}^{+\infty} f_1(t - \tau) f_2(\tau) d\tau.$$

Convolution should be distinguished from correlation, defined as

$$\phi_{12}(t) = \int_{-\infty}^{+\infty} f_1(t + \tau) f_2(\tau) d\tau.$$

Now $\phi_{12}(t)$ is the cross correlation function for f_1 and f_2 . The significant difference is that correlation merely displaces the two functions in the integrand and sums the product without first reflecting one function in coordinates as convolution does.

It is of interest to examine the algebraic properties of convolution. We may

show that it is commutative. If in the definition we change variables $\tau' = t - \tau$, the result becomes

$$f_1 * f_2 = \int_{-\infty}^{+\infty} f_2(t - \tau') f_1(\tau') d\tau' = f_2 * f_1.$$

Since integration is linear, we have

$$\begin{aligned} f_1 * (f_2 + f_3) &= \int_{-\infty}^{+\infty} f_1(t - \tau) [f_2(\tau) + f_3(\tau)] d\tau \\ &= f_1 * f_2 + f_1 * f_3 \end{aligned}$$

and convolution is thus also distributive.

Now examine the associative properties.

Let

$$f_2 * f_3 = f_4 \text{ so that } f_1 * (f_2 * f_3) = f_1 * f_4$$

$$f_1 * f_4 = \int_{-\infty}^{+\infty} f_1(t - \tau_0) f_4(\tau_0) d\tau_0$$

$$= \int_{-\infty}^{+\infty} d\tau_0 f_1(t - \tau_0) \int_{-\infty}^{+\infty} d\tau f_2(\tau_0 - \tau) f_3(\tau)$$

$$= \int_{-\infty}^{+\infty} d\tau f_3(\tau) \int_{-\infty}^{+\infty} d\tau_0 f_1(t - \tau_0) f_2(\tau_0 - \tau)$$

Now consider

$$(f_1 * f_2) * f_3$$

and let

$$f_1 * f_2 = f_5$$

$$f_5 * f_3 = \int_{-\infty}^{+\infty} f_5(t - \tau_0) f_3(\tau_0) d\tau_0 = \int_{-\infty}^{+\infty} d\tau_0 \\ \times f_3(\tau_0) \int_{-\infty}^{+\infty} d\tau (t - \tau_0 - \tau) f_2(\tau).$$

Exchange variable τ , τ_0 in the integrals to give

$$f_5 * f_3 = \int_{-\infty}^{+\infty} d\tau f_3(\tau) \\ \times \int_{-\infty}^{+\infty} d\tau_0 f_1(t - \tau - \tau_0) f_2(\tau_0)$$

and change the variable in the inner integral

$$\tau_0 = \tau' - \tau, d\tau_0 = d\tau'.$$

Substitution gives

$$f_5 * f_3 = \int_{-\infty}^{+\infty} d\tau f_3(\tau) \int_{-\infty}^{+\infty} d\tau' f_1(t - \tau') \\ \times f_2(\tau' - \tau) = (f_1 * f_2) * f_3.$$

Comparison with the result for $f_1 * f_4$ shows that $f_1 * (f_2 * f_3) = (f_1 * f_2) * f_3$ and convolution is thus also associative.

The Dirac delta function will act as a unit operator for convolution:

$$f_1 * \delta = \int_{-\infty}^{+\infty} f_1(t - \tau) \delta(\tau) d\tau = f_1(t)$$

$$\delta * f_1 = \int_{-\infty}^{+\infty} \delta(t - \tau) f_1(\tau) d\tau = f_1(t)$$

$$\delta * \delta = \delta.$$

Now examine the existence of unique inverse operators. We want a unique function $f^{-1}(t)$ corresponding to every $f(t)$ such that $f * f^{-1} = f^{-1} * f = \delta(t)$. We will restrict ourselves to functions whose Fourier transform exists to satisfy uniqueness. We need f^{-1} such that

$$\int_{-\infty}^{+\infty} f(t - \tau) f^{-1}(\tau) d\tau = \delta(t).$$

If we transform the equation, we obtain

$$\frac{1}{\sqrt{2\pi}} \int_{-\infty}^{+\infty} dt e^{-i\omega t} \int_{-\infty}^{+\infty} d\tau f(t - \tau) f^{-1}(\tau) = \frac{1}{\sqrt{2\pi}} \\ = \frac{1}{\sqrt{2\pi}} \int_{-\infty}^{+\infty} d\tau f^{-1}(\tau) \int_{-\infty}^{+\infty} dt e^{-i\omega t} f(t - \tau).$$

If we change the variable in the inner integral

$$t' = t - \tau, dt' = dt,$$

we obtain

$$\frac{1}{\sqrt{2\pi}} \int_{-\infty}^{+\infty} dt e^{-i\omega t} f(t - \tau) \\ = e^{-i\omega \tau} \frac{1}{\sqrt{2\pi}} \int_{-\infty}^{+\infty} e^{-i\omega t'} f(t') dt' = e^{-i\omega \tau} F(\omega)$$

where $F(\omega)$ is the Fourier transform of $f(t)$. We thus have

$$\int_{-\infty}^{+\infty} d\tau f^{-1}(\tau) \frac{1}{\sqrt{2\pi}} \int_{-\infty}^{+\infty} dt e^{-i\omega t} f(t - \tau) \\ = \int_{-\infty}^{+\infty} d\tau f^{-1}(\tau) e^{-i\omega \tau} F(\omega) = \sqrt{2\pi} F(\omega) F^{-1}(\omega)$$

where $F^{-1}(\omega)$ is the Fourier transform of the inverse function. It follows that $F^{-1}(\omega) = 1/2\pi F(\omega)$ and a unique inverse will exist provided the Fourier transform of the function f does not vanish. For such functions the convolution process is associative, distributive, and commutative, has unique inverses, and has a unit operator. Therefore, we can build an algebra with convolution represented by multiplication. Algebraic theorems can be constructed as an analytical tool.

A difficulty encountered in engineering applications of convolution is associated with the necessary finite range of integration used. As a matter of convenience an alternate definition of convolution (Ref. 7) in use is

$$f_1 * f_2 = \lim_{T \rightarrow \infty} \frac{1}{T} \int_{-T/2}^{T/2} f_1(t - \tau) f_2(\tau) d\tau.$$

This has the advantage of a finite sampling interval but convolution with this definition will be commutative, associative and distributive only for periodic functions, and the question of the existence of unique inverses and unit operator becomes more difficult to analyze.

For our purposes we will be particularly interested in a class of functions which are identically zero for negative times. These clearly cannot be considered periodic. While unique inverses may exist for these functions, they may turn out to be representable only in terms of symbolic functions such as the Dirac delta function and its derivatives. As an example of this, we will consider a function commonly used to characterize the response of a fluor-photodiode combination where

$$g(t) = \begin{cases} \frac{1}{T_1 - T_2} \left(e^{-t/T_1} - e^{-t/T_2} \right) & t \geq 0 \\ 0 & t < 0 \end{cases}.$$

If we calculate the Fourier transform, we obtain

$$G(\omega) = \frac{1}{\sqrt{2\pi}} \int_{-\infty}^{+\infty} e^{-i\omega t} g(t) dt.$$

Evaluating the integral, we obtain

$$G(\omega) = \frac{1}{\sqrt{2\pi}(i\omega T_1 + 1)(i\omega T_2 + 1)}$$

and from our previous results, the transform of the universe is just

$$\begin{aligned} G^{-1}(\omega) &= \frac{1}{2\pi G(\omega)} = \frac{(i\omega T_1 + 1)(i\omega T_2 + 1)}{\sqrt{2\pi}} \\ &= \frac{[1 + i\omega(T_1 + T_2) - \omega^2 T_1 T_2]}{\sqrt{2\pi}}. \end{aligned}$$

Taking the inverse transform

$$g^{-1}(t) = \frac{1}{\sqrt{2\pi}} \int_{-\infty}^{+\infty} e^{i\omega t} G^{-1}(\omega) d\omega$$

we obtain

$$\begin{aligned} g^{-1}(t) &= \frac{1}{2\pi} \int_{-\infty}^{+\infty} e^{i\omega t} \\ &\quad \times [1 + i\omega(T_1 + T_2) - \omega^2 T_1 T_2] d\omega. \end{aligned}$$

Now

$$\frac{1}{2\pi} \int_{-\infty}^{+\infty} e^{i\omega t} d\omega = \delta(t) \text{ and}$$

$$\delta'(t) = \frac{d}{dt} \delta(t) = \frac{i}{2\pi} \int_{-\infty}^{+\infty} \omega e^{i\omega t} d\omega$$

$$\delta''(t) = \frac{-1}{2\pi} \int_{-\infty}^{+\infty} \omega^2 e^{i\omega t} d\omega.$$

Substitution gives

$$g^{-1}(t) = \delta(t) + (T_1 + T_2)\delta'(t) + T_1 T_2 \delta''(t).$$

We may verify that $g^*g^{-1} = \delta(t)$ to test the result where

$$g^*g^{-1} = \int_{-\infty}^{+\infty} g(t - \tau)g^{-1}(\tau)d\tau$$

$$g(t - \tau)$$

$$= \begin{cases} \frac{1}{T_1 - T_2} \left[e^{-(t-\tau)/T_1} - e^{-(t-\tau)/T_2} \right], & t \geq \tau \\ 0, & \tau > t \end{cases}.$$

Thus we have

$$g^*g^{-1} = \int_{-\infty}^t g(t - \tau)$$

$$\times [\delta(\tau) + (T_1 + T_2)\delta'(\tau) + T_1 T_2 \delta''(\tau)] d\tau$$

and now

$$\int_{-\infty}^t g(t - \tau)\delta(\tau)d\tau = g(t).$$

Using integration by parts and the property of the delta function, we obtain

$$\begin{aligned} \int_{-\infty}^t g(t - \tau)\delta'(\tau)d\tau &= g(0)\delta(t) \\ &- g(\infty)\delta(-\infty) + g'(t) \end{aligned}$$

but $g(0) = g(\infty) = 0$ so that the first two terms vanish. Similarly,

$$\begin{aligned} \int_{-\infty}^t g(t - \tau)\delta''(\tau)d\tau &= g(0)\delta'(t) - g(\infty)\delta'(-\infty) \\ &+ g'(0)\delta(t) - g'(\infty)\delta(-\infty) + g''(t) \end{aligned}$$

but $g'(\infty)$ also vanishes so that this reduces to $g'(0)\delta(t) + g''(t)$.

Combining results we obtain

$$\begin{aligned} g^*g^{-1} &= g(t) + (T_1 + T_2)g'(t) \\ &+ T_1 T_2 g''(t) + T_1 T_2 g'(0)\delta(t). \end{aligned}$$

It is a matter of algebra to show that $g(t) + (T_1 + T_2)g'(t) + T_1 T_2 g''(t) = 0$ and $T_1 T_2 g'(0) = 1$ so that $g^*g^{-1} = \delta(t)$ is indeed satisfied.

An interesting aspect of the result is that by allowing the inverse function to be represented by symbolic functions such as $\delta(t)$ we have included a case for which the Fourier transform of the response function vanishes at $\omega = \infty$. For functions whose Fourier transform vanishes only at discrete values of ω , it may be possible to define the inverse Fourier transform as a combination of delta functions and thus construct an inverse. The question of uniqueness would have to be reconsidered again, however.

An additional theorem may be deduced concerning the inverses for composite systems. We state the result: $(g_1 * g_2)^{-1} = g_1^{-1} * g_2^{-1}$ (i.e., convolution and inversion commute). This can be used to calculate the inverse for a composite system from the inverses of the components of the system. We first give a proof and then an example.

Let

$$g_3(t) = g_1 * g_2 = \int_{-\infty}^{+\infty} g_1(t - \tau) g_2(\tau) d\tau$$

and consider the Fourier transform

$$G_3(\omega) = \frac{1}{\sqrt{2\pi}} \int_{-\infty}^{+\infty} e^{-i\omega t} g_3 dt$$

$$G_3^{-1}(\omega) = \frac{1}{2\pi G_3(\omega)}$$

so that

$$\begin{aligned} g_3^{-1}(t) &= (g_1 * g_2)^{-1} \\ &= \frac{1}{\sqrt{2\pi}} \int_{-\infty}^{+\infty} e^{i\omega t} G_3^{-1}(\omega) d\omega \end{aligned}$$

or more explicitly,

$$\begin{aligned} (g_1 * g_2)^{-1} &= \frac{1}{\sqrt{2\pi}} \int_{-\infty}^{+\infty} e^{i\omega t} \\ &\times \left[\frac{1}{\sqrt{2\pi} \int_{-\infty}^{+\infty} e^{-i\omega t_0} (g_1 * g_2) dt_0} \right] d\omega. \end{aligned}$$

Now

$$\begin{aligned} \int_{-\infty}^{+\infty} e^{-i\omega t_0} (g_1 * g_2) dt_0 &= \int_{-\infty}^{+\infty} dt_0 e^{-i\omega t_0} \\ &\times \int_{-\infty}^{+\infty} g_1(t_0 - \tau) g_2(\tau) d\tau \\ &= \int_{-\infty}^{+\infty} d\tau g_2(\tau) \int_{-\infty}^{+\infty} dt_0 e^{-i\omega t_0} g_1(t_0 - \tau). \end{aligned}$$

Transforming variables in the inner integral and proceeding as before, the previous equation reduces to

$$\begin{aligned} \int_{-\infty}^{+\infty} e^{-i\omega t_0} (g_1 * g_2) dt_0 &= 2\pi G_1(\omega) G_2(\omega) \\ &= \frac{1}{2\pi G_1^{-1}(\omega) G_2^{-1}(\omega)}. \end{aligned}$$

Substituting, we obtain

$$(g_1 * g_2)^{-1} = \int_{-\infty}^{+\infty} e^{i\omega t} G_1^{-1}(\omega) G_2^{-1}(\omega) d\omega.$$

Taking inverse Fourier transforms, this becomes

$$\begin{aligned} (g_1 * g_2)^{-1} &= \frac{1}{2\pi} \int_{-\infty}^{+\infty} d\omega e^{i\omega t} \\ &\times \int_{-\infty}^{+\infty} dt_0 e^{-i\omega t_0} g_1^{-1}(t_0) \\ &\times \int_{-\infty}^{+\infty} dt_1 e^{-i\omega t_1} g_2^{-1}(t_1). \end{aligned}$$

Changing the order of integration, we obtain

$$\begin{aligned} (g_1 * g_2)^{-1} &= \int_{-\infty}^{+\infty} dt_0 g_1^{-1}(t_0) \\ &\times \int_{-\infty}^{+\infty} dt_1 g_2^{-1}(t_1) \frac{1}{2\pi} \int_{-\infty}^{+\infty} d\omega e^{i\omega(t-t_0-t_1)} \\ &= \int_{-\infty}^{+\infty} dt_0 g_1^{-1}(t_0) \int_{-\infty}^{+\infty} dt_1 g_2^{-1}(t_1) \\ &\times \delta(t - t_0 - t_1) = \int_{-\infty}^{+\infty} dt_1 g_2^{-1}(t_1) \\ &\times g_1^{-1}(t - t_1) = g_1^{-1} * g_2^{-1} \quad \text{Q.E.D.} \end{aligned}$$

As an example, we may consider the case

$$g_1(t) = \begin{cases} \frac{1}{T_1} e^{-t/T_1} & t \geq 0 \\ 0 & t < 0 \end{cases}$$

$$g_2(t) = \begin{cases} \frac{1}{T_2} e^{-t/T_2} & t \geq 0 \\ 0 & t < 0 \end{cases}$$

It is easy to show that

$$g_1 * g_2 = \frac{1}{T_1 - T_2} \left(e^{-t/T_1} - e^{-t/T_2} \right)$$

Proceeding as before, we obtain $g_1^{-1}(t) = T_1 \delta'(t) + \delta(t)$ and $g_2^{-1}(t) = T_2 \delta'(t) + \delta(t)$. Direct convolution of these inverses produces

$$g_1^{-1} * g_2^{-1} = \delta(t) + (T_1 + T_2) \delta'(t) + T_1 T_2 \delta''(t)$$

in agreement with the result we have already calculated for $(g_1 * g_2)^{-1}$.

LINEAR SYSTEM RESPONSE

Since we intend to use convolution to describe physical linear systems, we will be using functions representing the response of the system to a unit impulse

at zero time. These are called the response function of the system and are defined to be zero for negative times since causality forbids a response to precede the excitation in time. It is a common practice to express this property by truncating the lower limit of the convolution integral to zero. It may be of interest to observe that this removes the commutative property. Consider $f_1 * f_2$ where $f_2(t) = 0$ for $t < 0$. We may thus write

$$f_1 * f_2 = \int_0^{\infty} f_1(t - \tau) f_2(\tau) d\tau$$

The result is illustrated graphically in Fig. 1. Now change variables in the expression to give $\tau' = t - \tau$.

$$\begin{aligned} f_1 * f_2 &= \int_t^{-\infty} f_1(\tau') f_2(t - \tau') d\tau' \\ &= \int_{-\infty}^t f_2(t - \tau') f_1(\tau') d\tau' \end{aligned}$$

Again, we illustrate this graphically as shown in Fig. 2. If we now consider the commuted result we obtain

$$f_2 * f_1 = \int_0^{\infty} f_2(t - \tau) f_1(\tau) d\tau$$

Fig. 1. Illustration of convolution with restricted integral limits.

Fig. 2. Alternate representation of restricted convolution.

Fig. 3. Restricted convolution with commuted functions.

This is shown graphically in Fig. 3. It is evident that commutation fails. It can also be easily shown that the associative law

$$f_1 * (f_2 * f_3) = (f_1 * f_2) * f_3 \text{ fails.}$$

The usefulness of the response function of the system is that we can obtain the output of the system for arbitrary input by convolving the input function with the response function.

$$S_0(t) = \int_{-\infty}^{\infty} S_1(t - \tau)g(\tau)d\tau$$

$$S_0 = S_1 * g$$

where g = response function, S_1 = input function and S_0 = output function.

The physical reasonableness of the operation can be seen if we consider the input function to represent an infinite sequence of weighted unit impulses. Let

$g(t)$ represent the response of the system to a unit impulse at $t = 0$. Performing the convolution as described above we obtain

$$\int_{-\infty}^{+\infty} \delta(t - \tau)g(\tau)d\tau = g(t).$$

Now consider a signal consisting of a weighted sequence of unit impulses represented by a sum of weighted delta functions spaced in time

$$S_i(t) = \lim_{\substack{N \rightarrow \infty \\ \epsilon \rightarrow 0}} \sum_{j=0}^N a_j \delta(t - t_j), \quad \epsilon = t_{j+1} - t_j.$$

This is schematically indicated in Fig. 4. If we now convolve this with the response function, we obtain

$$\begin{aligned} S_i * g &= \lim_{\substack{N \rightarrow \infty \\ \epsilon \rightarrow 0}} \int_{-\infty}^{+\infty} \sum_{j=0}^N a_j \delta(t - \tau - t_j)g(\tau)d\tau \\ &= \lim_{\substack{N \rightarrow \infty \\ \epsilon \rightarrow 0}} \sum_{j=0}^N a_j \int_{-\infty}^{+\infty} \delta(t - \tau - t_j)g(\tau)d\tau \end{aligned}$$

Fig. 4. Representation of an input signal as a sequence of weighted unit impulses.

$$\begin{aligned} &= \lim_{\substack{N \rightarrow \infty \\ \epsilon \rightarrow 0}} \sum_{j=0}^N a_j g(t - t_j) = S_0(t). \end{aligned}$$

The output is thus seen to be the superposition of a weighted sequence of replicas of the response function with the spacing of the exciting impulses (see Fig. 5).

RESPONSE OF COMPOSITE SYSTEMS

Since most physical data transmission systems are composed of a sequence of rather distinct units, it can be useful to be able to derive a response function for the composite system, once the response functions of the individual units are determined. This would allow one to determine the response function of the components and be able to predict the response of any combination of them. In practice the integrals may not be expressible in a closed form and computer simulation would be required. In some cases one assumes an approximation to the unit response functions that allows the composite response function to be calculated

Fig. 5. Representation of the output signal as a sequence of weighted response functions.

in closed form. An example is the fluor-photodiode combination where two decaying exponentials are assumed to represent the response of the fluor and photodiode, respectively.

If we allow the first component of the system to be excited by the unit impulse, we can allow its response function to represent the input excitation of the second component. The output of the second component is represented by the convolution of its response function and the response function of the first component. Since the composite system was excited by a unit impulse, the result of the convolution is the response function of the combined system. This procedure can be extended indefinitely to an arbitrary number of components. We thus have $g(t) = g_1 * g_2 * g_3 * \dots * g_N$ for the response function of a system with N components.

As an example, we may consider the fluor-photodiode combination previously mentioned. The common approximation is to assume the fluor and photodiode each have an exponentially decaying response function for positive values of time

$$g_1(\tau) = \begin{cases} \frac{1}{T_1} e^{-\tau/T_1}, & \tau \geq 0 \\ 0, & \tau < 0 \end{cases};$$

Fig. 6. Convolution of two simple exponentials.

$$g_2(t) = \begin{cases} \frac{1}{T_2} e^{-t/T_2}, & t \geq 0 \\ 0, & t < 0 \end{cases}$$

$$g(t) = g_1 * g_2 = \int_{-\infty}^{+\infty} g_1(t - \tau) g_2(\tau) d\tau.$$

It is evident from Fig. 6 that we may truncate the limits

$$g(t) = \int_0^t \frac{e^{-(t-\tau)/T_1} e^{-\tau/T_2}}{T_1 T_2} d\tau.$$

Straightforward manipulation of the integral produces the result

$$g(t) = \frac{1}{T_1 - T_2} \left(e^{-t/T_1} - e^{-t/T_2} \right).$$

This function has a peak value $g(t_p)$ $= 1/T_2 (T_1/T_2)^{T_1/(T_2 - T_1)}$ occurring at time $t_p = T_1 T_2 / (T_1 - T_2) \log_e (T_1/T_2)$. For the case $T_1 \rightarrow T_2$, it follows that $T_p \rightarrow T_2$. A parametric study of the function has been made and a fit has been made to a number of standard detectors.

An additional case of interest is a composite coaxial transmission line made up of a number of distinct sections, each of

which has its own response function which we assume to be represented by that of an ideal lossless line. We will defer the detailed calculation to Appendix A and quote results here. The response function of the ideal lossless cable is

$$g(t) = \sqrt{\frac{\tau}{\pi}} \cdot \frac{e^{-\tau/t}}{t^{3/2}}$$

where τ is a constant characteristic of the cable.

In the equation

$$\tau = \left(\frac{L}{4}\right)^2 \frac{\epsilon_2 \mu_1}{\mu_2 \sigma_1} \left[\left(\frac{1}{a_1} + \frac{1}{a_2} \right) / \log_e \left(\frac{a_2}{a_1} \right) \right]^2$$

for a line of length L where a_1, a_2 are the radii of the inner and outer conductors of the line, σ_1 is the conductivity of the conductors, μ_1, μ_2 are the magnetic permeabilities of the conductors and dielectrics, respectively and ϵ_2 is the permittivity of the dielectric. It may be noted that τ is proportional to the square of the line length. Tables of cable characteristics appear in Appendix B giving τ as a function of length for a variety of standard cables. They are used in conjunction with the expression $\sqrt{\tau} = AL \times 10^{-7} \text{ sec}^{1/2}$ where L is the length of the cable in feet. The table gives values of A for the respective cables. It is shown in Appendix A that the response function of two ideal lossless cables with different τ in series has the same form as that for a single cable with an effective τ given by

$$\tau = \left(\sum_i \sqrt{\tau_i} \right)^2$$

where the τ_i are the values of τ for the component sections.

RESPONSE OF LINEAR SYSTEMS TO EXPONENTIAL AND GAUSSIAN INPUTS

A well known technique used in the solution of differential equations describing linear systems is to assume the solution consists of a combination of exponentials whose arguments are a linear function of the variable (i.e., $f(t) = Ae^{\alpha t}$ where α is a constant, and in general may be complex). It is not so often recognized that the method rests on the fact that such exponentials are eigenfunctions of the operators describing the linear system (i.e., the operators acting on the exponential produce the same exponential function multiplied by a characteristic constant). It is a consequence of this result that the action of a linear system on a purely exponential input produces an undistorted exponential output multiplied by a characteristic constant which in general depends upon the rate of rise of the exponential. For an exponential signal $e^{\alpha t}$ with constant α it is therefore meaningful to characterize the linear system by its eigenvalues, frequently called the sensitivity reduction factor in the present application.

For the portions of the reaction history α curve where α is a slowly varying function of time, the signals are nearly eigenfunctions of the system and therefore the distortion created by the linear transmission system is small. For the regions of the α curve where α becomes a rapid function of time, the signal consists of a broad superposition of eigenfunctions and thus the system cannot be even approximately characterized by a single eigenvalue.

It could be a useful procedure to perform a spectral analysis of the linear

system, expanding the signals in the exponential eigenfunctions. At present the corrections for the signal modifications by the linear system with nonexponential input are commonly made by computer deconvolution. Excellent reports on this technique have been written (Refs. 8 and 9). The technique has some inherent weaknesses. Since the physical measuring system acts as a low pass filter, the deconvolution process must compensate by acting as a high pass filter. Small random errors introduced into the data at each step of data processing introduce noise with appreciable high frequency components. The high frequency noise components are amplified by the deconvolution process along with the high frequency portions of the signal so that the result is quite noisy. In practice, a compromise is made to filter out some of this noise. An alternate method of correction is to use equalizers in the measuring system itself. Equalizers are high pass filters which are designed to compensate for the other low pass characteristics of the system over a wide band of frequencies at the price of considerable attenuation. In practice, the energy storage capabilities of the hardware available limits some practical equalizers to less than the ideal design.

We will now consider the eigenvalues or sensitivity reduction factors for some specific systems. As the first case, we will consider the fluor-photodiode combination for which we have assumed a phenomenological response function. The most straightforward approach is to apply the linear operator representing the system, convolution with its response function, to the exponential function which

is known to be an eigenfunction. The resulting multiplier of the exponential is the sensitivity reduction factor. We assume the appropriate response function is given by

$$g(t) = \begin{cases} \frac{1}{T_1 - T_2} \left(e^{-t/T_1} - e^{-t/T_2} \right), & t \geq 0 \\ 0, & t < 0 \end{cases}$$

Now,

$$L e^{\alpha t} = \int_{-\infty}^{+\infty} e^{\alpha(t-\tau)} g(\tau) d\tau = \lambda e^{\alpha t}$$

where λ is the sensitivity reduction factor and L is the linear integral operator representing the system under consideration. We thus have

$$\lambda = \int_{-\infty}^{+\infty} e^{-\alpha\tau} g(\tau) d\tau.$$

Substitution of the assumed $g(t)$ and straightforward integration produces the result

$$\lambda = \frac{1}{(1 + \alpha T_1)(1 + \alpha T_2)}.$$

This result is commonly used to characterize Compton diode, Cerenkov, and fluor detectors (discussed later); and curves with appropriate values of T_1, T_2 giving $\lambda(\alpha)$ are included in Appendix B.

We next consider the ideal lossless cable whose response function is

$$g(t) = \begin{cases} \sqrt{\frac{\tau_0}{\pi}} \frac{e^{-\tau_0/t}}{t^{3/2}}, & t \geq 0 \\ 0, & t < 0 \end{cases}$$

As before,

$$\lambda = \int_{-\infty}^{+\infty} e^{-\alpha\tau} g(\tau) d\tau.$$

We thus have

$$\lambda = \int_0^{\infty} e^{-\alpha\tau} \sqrt{\frac{\tau_0}{\pi}} \frac{e^{-\tau_0/\tau}}{\tau^{3/2}} d\tau$$

$$= \sqrt{\frac{\tau_0}{\pi}} \int_0^{\infty} \frac{e^{-\alpha\tau - \tau_0/\tau}}{\tau^{3/2}} d\tau$$

where τ_0 is the cable constant. If we make a change of variable $\tau = \tau_0/\mu^2$ we obtain

$$\lambda = -\frac{2}{\sqrt{\pi}} \int_0^{\infty} e^{-\mu^2 - (\alpha\tau_0)/\mu^2} d\mu$$

$$= \frac{2}{\sqrt{\pi}} \int_0^{\infty} e^{-\mu^2 - (\alpha\tau_0)/\mu^2} d\mu$$

but this is a standard integral found in tables, and the result reduces to

$$\lambda = e^{-2\sqrt{\alpha\tau_0}}$$

This is the sensitivity reduction factor for the ideal lossless cable with characteristic constant τ_0 . Curves of $\lambda(\alpha)$ for the ideal cable for a range of values of τ_0 appear in Appendix B. This sensitivity reduction factor for cables is sometimes called the cable transmission for constant α .

It is of interest to consider the response of cables and fluor-photo-multipliers to Gaussian inputs. If we fold a Gaussian with the response function of an ideal cable we obtain

$$S_1(t) = \kappa e^{-\beta(t-T)^2},$$

$$g(t) = \begin{cases} \sqrt{\frac{\lambda}{\pi}} \left(\frac{1}{t}\right)^{3/2} e^{-\lambda/t}, & t \geq 0 \\ 0, & t < 0 \end{cases}$$

$$S_0(t) = S_1 * g = \int_{-\infty}^{\infty} S_1(t - \tau) g(\tau) d\tau$$

$$= S_1(t) \sqrt{\frac{\lambda}{\pi}} \int_0^{\infty} \left(\frac{1}{\tau}\right)^{3/2} e^{[2\beta(t-T)\tau - \beta\tau^2 - \lambda/\tau]} d\tau.$$

Since the transmission

$$X(t) = \sqrt{\frac{\lambda}{\pi}} \int_0^{\infty} \left(\frac{1}{\tau}\right)^{3/2} e^{[2\beta(t-T)\tau - \beta\tau^2 - \lambda/\tau]} d\tau$$

is a function of time, it is evident that distortion of the Gaussian occurs. The integral is quite a difficult form to evaluate. The exact calculation has not been made.

If we consider the fluor-photomultiplier to be described by the two exponential response function, we can calculate a result for the detector transmission in closed form. Let

$$S_1(t) = \kappa e^{-\beta(t-T)^2}$$

$$S_0(t) = S_1 * g = \int_0^{\infty} \frac{\kappa}{T_1 - T_2} e^{-\beta(t-T-\tau)^2} \times \left(e^{-\tau/T_1} - e^{-\tau/T_2} \right) d\tau.$$

Complete the square in the exponents and let $t - T = t_0$ for convenience. Now,

$$e^{-\beta(t_0 - \tau)^2 - \tau/T_1} = e^{-f} e^{-(A+B\tau)^2}$$

where $B = \sqrt{\beta}$ and $A = 1/(2T_1\sqrt{\beta}) - t_0\sqrt{\beta}$,
 $f = (1/T_1)(t_0 - 1/4\beta T_1)$

$$\int_0^{\infty} e^{-\beta(t_0 - \tau)^2 - \tau/T_1} d\tau$$

$$= e^{-f} \int_0^{\infty} e^{-(A+B\tau)^2} d\tau.$$

$$\text{Let } \xi = A + B\tau$$

$$d\tau = \frac{d\xi}{B}$$

$$\tau \rightarrow 0 \Rightarrow \xi \rightarrow A, \tau \rightarrow \infty \Rightarrow \xi \rightarrow \infty$$

$$\therefore \int_0^{\infty} e^{-\beta(t_0 - \tau)^2 - \tau/T_1} d\tau = \frac{e^{-f}}{B} \int_A^{\infty} e^{-\xi^2} d\xi.$$

Now

$$\text{erf}(x) = \frac{2}{\sqrt{\pi}} \int_0^x e^{-x^2} dx$$

is the error function and $1 - \text{erf}(x) = \text{erfc}(x)$ is the complementary error function. Thus we may write

$$\int_A^{\infty} e^{-\xi^2} d\xi = \int_0^{\infty} e^{-\xi^2} d\xi - \int_0^A e^{-\xi^2} d\xi$$

$$= \frac{\sqrt{\pi}}{2} - \frac{\sqrt{\pi}}{2} \text{erf}(A) = \frac{\sqrt{\pi}}{2} \text{erfc}(A).$$

We thus have

$$\int_0^{\infty} e^{-\beta(t_0 - \tau)^2 - \tau/T_1} d\tau = \frac{e^{-f}}{B} \frac{\sqrt{\pi}}{2} \text{erfc}(A)$$

and similarly,

$$\int_0^{\infty} e^{-\beta(t_0 - \tau)^2 - \tau/T_2} d\tau = \frac{e^{-f'}}{B} \frac{\sqrt{\pi}}{2} \text{erfc}(A')$$

where

$$A' = \frac{1}{2T_2\sqrt{\beta}} - t_0\sqrt{\beta} \text{ and } f' = \frac{1}{T_2} \left(t_0 - \frac{1}{4\beta T_2} \right)$$

so that

$$S_0(t) = \frac{\kappa}{2(T_1 - T_2)} \sqrt{\frac{\pi}{\beta}}$$

$$\times [e^{-f} \text{erfc}(A) - e^{-f'} \text{erfc}(A')].$$

If we obtain an "instantaneous transmission" $X(t) = S_0(t)/S_i(t)$, we obtain

$$X(t) = \frac{\beta t_0^2}{2(T_1 - T_2)} \sqrt{\frac{\pi}{\beta}}$$

$$\times [e^{-f} \text{erfc}(A) - e^{-f'} \text{erfc}(A')]$$

but since

$$f + A^2 = \beta t_0^2 \text{ and } f' + A'^2 = \beta t_0^2$$

$$X(t) = \frac{1}{2(T_1 - T_2)} \sqrt{\frac{\pi}{\beta}}$$

$$\times [e^{A^2} \text{erfc}(A) - e^{A'^2} \text{erfc}(A')].$$

It is evident that distortion again occurs since X is a function of time through the factors A, A' .

A more useful parameter, often called detector transmission for a Gaussian is defined to be the ratio of the output peak amplitude divided by the input peak amplitude. It is commonly used in making coverage estimates to ensure adequate oscilloscope deflection is available. This

parameter cannot be obtained as a particular value of the instantaneous transmission since the peaks occur at different times. We may find the time of the peak of the output signal, by solving for the time at which the first time derivative vanishes.

$$\frac{dS_o(t)}{dt} = 0 \text{ at } t = t_p.$$

The peak of the input signal occurs at time T . We thus have detector transmission $\chi \equiv S_o(t_p)/S_i(T)$. Using the definition

$$\operatorname{erfc}(A) = 1 - \frac{2}{\sqrt{\pi}} \int_0^A e^{-x^2} dx$$

the result

$$\frac{d}{dt} \operatorname{erfc}(A) = 2\sqrt{\frac{\beta}{\pi}} e^{-A^2}$$

and a similar result for A' , the equation for the time of the output signal peak becomes

$$\begin{aligned} 2e^{-f} \sqrt{\frac{\beta}{\pi}} e^{-A^2} + \frac{e^{-f'}}{T_2} \operatorname{erfc}(A') \\ = 2e^{-f'} \sqrt{\frac{\beta}{\pi}} e^{-A'^2} + \frac{e^{-f}}{T_1} \operatorname{erfc}(A) \end{aligned}$$

where

$$f = \frac{1}{T_1} \left(t_p - T - \frac{1}{4\beta T_1} \right),$$

$$A = \frac{1}{2T_1\sqrt{\beta}} - (t_p - T)\sqrt{\beta}$$

$$f' = \frac{1}{T_2} \left(t_p - T - \frac{1}{4\beta T_2} \right),$$

$$A' = \frac{1}{2T_2\sqrt{\beta}} - (t_p - T)\sqrt{\beta}.$$

It is evident that this is indeed a complicated equation to solve. The difficulty has been circumvented by using a computer to convolve a Gaussian and the two exponential response function for a range of parameters. The value of χ is calculated for each case. A set of curves for the fluor, Cerenkov, and Compton diode detectors giving χ as a function of FWHM (Full Width at Half Maximum) is given in the appendices.

The final recording apparatus in the system, the oscilloscope and its associated camera have a number of distinct contributions to recording errors which we will now discuss.

Oscilloscope Displays and Film Reading/Evaluation

This section describes the displays commonly used in recording high speed diagnostic data. Also included are the more familiar displays which are of interest to the Reaction History physicist. We will first consider the displays actually used to record the data, and then consider the special purpose displays used purely for oscilloscope cali-

bration and evaluation. Finally, we will examine the difficulties inherent in the use of both oscilloscopes and special displays. The discussion of the data displays is divided between those intended primarily for timing data and those intended to record the amplitude of the very fast signals associated with alpha data.

DATA DISPLAYS

Timing Displays

Linear Display

The linear display oscilloscopes are commonly used only for applications involving rather slow signals in the micro-second domain where the nonlinearities of the internal sweep generators do not present a serious problem. The most common application encountered by the Reaction History physicist is the neutron generator pulse. Figure 7 is an example showing the character of the neutron gen-

Fig. 7. Example of the display of a neutron generator pulse.

erator pulse. A marker pulse is located at the start of the trace for timing purposes. Figure 8 is an example of a cross timing oscilloscope with the marks located on the linear sweep. The sinusoid included at the bottom is provided for timing information and is written by a second beam in the oscilloscope. This timing system is commonly used by LASL rather than LLL.

Since the linear display is the most common display used with an oscilloscope, we will not consider it further, but rather assume the reader is quite familiar with its properties and limitations.

Raster Display

The raster display is only a minor modification of the linear display, used to extend the time interval covered by the oscilloscope without sacrificing time resolution. A full raster corresponds to a long linear sweep which has been divided into a number of discrete segments displayed as a series of vertically displaced lines on the oscilloscope face. A

Fig. 8. Example of a cross timing oscilloscope trace with linear trace and timing sinusoid.

finite retrace time is required for the oscilloscope to begin a new line of the raster, therefore, the time coverage of a single raster display is not continuous but composed of discrete nonconnected segments, with the gaps corresponding to the retrace periods. The gaps of time can be covered by the use of a second raster display, slaved to the first. The slave sweep is delayed 0.5 μ sec from the master raster display.

This is the procedure used with the L-10 oscilloscope used to record the X-unit signals and other data associated with HETT, zipper timing and pin signals. We will briefly review the character of the display used by the L-10 (Figs. 9 and

10 are examples). The sweep is from left to right and top to bottom when viewing the trace, with a sweep time of 2.5 μ sec/line. Internally generated marker pulses are placed on the trace at 0.5 μ sec intervals. The number of lines used in the raster is variable. Figure 11 is an example of a raster display presenting pin signals.

Dot Rossi Display

The Rossi displays represent the introduction of a new technique, used to partially overcome the difficulties associated with very fast linear displays. In the nanosecond time domain, the nonlinearities associated with conventional

Fig. 9. Example of a raster oscilloscope display. This is the master display, sweeping from left to right, and top to bottom with marker pulses located at 0.5 μ sec intervals.

Fig. 10. Raster slave display corresponding to the master display shows in Fig. 9. The sweep in this example was reversed, scanning from right to left, top to bottom.

sweep generators result in serious errors of timing if the horizontal position of the oscilloscope beam is assumed to be proportional to time. The Rossi display allows timing information to be added continuously to the display. A very stable sinusoidal signal of accurately measured high frequency is added to the approximately linear horizontal sweep voltage with a carefully controlled amplitude. The horizontally swept electron beam of the oscilloscope is by this means, made to partially retrace itself with each cycle of the sinusoidal signal, resulting in a figure which looks like a series of horizontally spaced dots created by the

trace intensification in the rewritten portions. One can thus associate each dot with a period of the sinusoidal component (which we will henceforth refer to as the Rossi signal). Figure 12 is an example of a Dot Rossi timing display. Marker pulses appear as the downward deflected signals near the beginning of the traces. The signal appears as the upward deflected portion on the upper trace. Figure 13 is an example of a cross timing oscilloscope presenting marker pulses with the Dot Rossi display. It should be immediately mentioned that the one to one correspondence of dots and Rossi periods holds only for the Dot display.

Fig. 11. Example of a raster oscilloscope display used to record PIN signals. The marker pulses are again at intervals of $0.5 \mu\text{sec}$. The rather slow signals correspond to the PINS. Sweep is from left to right and top to bottom.

We will consider the general character of the Rossi displays at this time, deferring detailed analysis to the appendices. We will describe some of the results in terms of dot width and spacing, parameters conveniently measured from the display. Figure 14 illustrates the definition of dot width and spacing. The dot width is the horizontal distance between the turnaround points enclosing the triply written portions of the trace, and the dot spacing is the horizontal distance between

successive forward or successive reverse turning points. The figure corresponds only to the Dot display, which contains only singly and triply written portions in the trace. We designate dot width and spacing by W_D and S_D , respectively. The horizontal deflection of the electron beam is given by

$$X = At + B \cos(\omega t + \phi)$$

where ω is the (angular) Rossi frequency and t is the time. It may be noted that

Fig. 12. Example of a Dot Rossi timing display. The marker pulses precede the signal at the left hand side of the trace.

Fig. 13. Cross timing oscilloscope trace using Dot Rossi display. The identity of the pulses is predetermined by polarity and sequence as well as width.

Fig. 14. Definition of dot width and spacing for the Dot Rossi display.

the initial beam position is determined by the phase angle ϕ . Analysis in the appendices shows that the dot spacing is determined by the slope of the ramp voltage, A , and the Rossi frequency. Since the Rossi frequency and dot spacing can be independently measured, we may determine the parameter A where

$$A = \frac{\omega}{2\pi} S_D.$$

The determination of the Rossi amplitude B is more difficult and involves the solution of a transcendental equation involving both dot width and spacing where

$$\frac{S_D}{2\pi B} = \cos \left[\pi \left(\frac{W_D}{S_D} \right) - \frac{2\pi}{S_D} \sqrt{B^2 - \left(\frac{S_D}{2\pi} \right)^2} \right].$$

The initial position of the electron beam is given by

$$X_0 = B \cos \phi.$$

The phase angle ϕ is thus determined by the point of the trace which is considered the initial point where

$$\phi = \cos^{-1} \left(\frac{X_0}{B} \right).$$

Solution of the equations for A , B and ϕ thus gives continuous timing information for the trace. A useful interpolation aid for hand reading of timing traces may be provided if we determine the fractions of a Rossi period which correspond to forward and reverse excursions of the electron beam.

If we let f represent the forward fraction of the Rossi period, analysis in the appendices shows that

$$\frac{W_D}{S_D} = \frac{1}{\pi} \tan [\pi(1 - f)] - (1 - f).$$

If we define $R = W_D/S_D$ and $\beta = \pi(1 - f)$, substitution produces the equation $\beta + \pi R = \tan \beta$ which is transcendental. Curves of f vs R have been calculated. For the Dot display, the forward fraction is approximately 0.6. It can be shown that

$$B = \frac{S_D}{2\pi \cos \beta}$$

so that solution of the equation $\beta + \pi R = \tan \beta$ also gives B . This equation is solved by a computer using an iterative technique in practice. It may be noted, however, that it is enough to solve the equation for a range of values of R and use the plotted curves of β vs R to obtain values of B once W_D and S_D are known.

The vertical deflection of the trace in a Rossi presentation corresponds to the input signal to be recorded by the oscilloscope. For the timing oscilloscope, this signal consists of a marker pulse near the beginning of the trace followed later by a sharply rising signal representing the time the applied signal rises above the threshold for the oscilloscope. Figure 15 illustrates schematically a typical trace for a Dot display. The marker pulse has (arbitrarily) been applied with a polarity opposite to that of the signal. The trace has been drawn with a slowly increasing vertical deflection to allow the path of the beam to be clearly indicated. In practice, the only deviations from the horizontal line occur at the marker pulse and signal breakout points. The trace appears to end a short distance from the point at which the signal breakout begins,

Fig. 15. Schematic of a dot display trace illustrating the method of reading. The portion of the trace between marker pulse and signal have been spread out vertically for the sake of clarity.

since the oscilloscope during dry runs is often not able to write the remainder of the rapidly rising signal.

We will illustrate with an example the procedure for timing the signal relative to the trailing half amplitude point of the marker pulse, indicated as point A. Points B, C, D, E, F, G, and H are forward turning points of the trace. The time from point B to point H represents six complete Rossi periods. The time from point A to the signal thus consists of six Rossi periods (or Dots) plus the increments of time between A and B, and between H and the appearance of the signal at Point I. The fraction

$$\left(\frac{L_F - X}{L_F} \right)$$

represents the fraction of a forward excursion corresponding to the time between points A and B. Since the forward excursion in a Dot presentation is approximately 0.6 Rossi period, the time from A to B must be approximately $(0.6) \times (L_F - X)/L_F$ Rossi period. Similarly, the time from point H to point I which

occurs during a reverse excursion is approximately $(0.4) \epsilon/L_R$ Rossi period. If we represent a Rossi period by D (Dot), the time from the trailing half amplitude of the marker pulse to the signal in dots is given by

$$t_I - t_A \approx \left(\frac{\epsilon}{L_R} \right) 0.4D + 6D + \left(\frac{L_F - X}{L_F} \right) 0.6D.$$

Table 1 lists some common Rossi frequencies and their corresponding periods. In the example indicated, if we assume $\epsilon/L_R = 0.7$, $(L_F - X)/L_F = 0.8$, and a Rossi frequency of 100 MHz, we obtain $t_I - t_A \approx 67.6$ nsec. When a timing oscilloscope uses the Dot presentation, it is indicated by the letter D beside the oscilloscope symbol on the one-line drawing.

Double Dot Rossi Display

If the amplitude of the Rossi signal is increased relative to the ramp deflection voltage, the amount of rewriting of the electron beam is increased. When a reverse turning point occurs behind the two previous forward turning points so that alternate portions of the horizontal trace are written either 5 times or 3 times, we have a display commonly called the Double Dot (DD) display. Figure 16 is an example of this display. A schematic example is shown in Fig. 17. Again, we have spread the pattern out vertically to indicate the nature of the

Table 1. Dot period for various Rossi frequencies.

f_R (MHZ)	D(nsec)
50	20
100	10
200	5

Fig. 16. Example of a double dot display trace.

beam motion. The pattern to the eye will appear as a series of dashes, corresponding to the regions where the beam writes 5 times, separated by regions where the beam only writes 3 times. A useful rule to remember is that the beam swings approximately three length units forward and then back two units in a Rossi period.

Fig. 17. Schematic of a double dot display trace. The trace has been spread out vertically for clarity.

L_F in the figure is ~ 3 dashes long. The forward fraction of the Rossi period is reduced somewhat from that of the Dot display to ~ 0.55 . Once again, we will time from the trailing half amplitude of the marker pulse to the signal at point I. This time is seen to be equal to the interval from A to C plus 5 Rossi periods from C to H, plus the interval from H to I. We have broken the interval from A to C into reverse and forward portions. The portion from A to B represents $(L_R - X)/L_R$ of a reverse fraction or $(0.45)(L_R - X)/L_R$ Rossi period. The portion from B to C represents one complete forward fraction, or 0.55 Rossi period. The interval from H to I is $(\epsilon/L_R)(0.45)$ Rossi period. If we assume

$$(L_R - X)/L_R = 0.3, \epsilon/L_R = 0.4,$$

and a Rossi frequency of 200 MHz, we thus have

$$t_I - t_A \approx [(0.3)(0.45) + 0.55 + 5 + (0.4)(0.45)] (5 \text{ nsec}) \approx 29.3 \text{ nsec}.$$

The Double Dot presentation is indicated on a one line drawing by DD beside the corresponding oscilloscope symbol.

The Double Dot presentation is more difficult to read and the equipment adjustment is more critical. A disadvantage of the Double Dot display is that fewer Rossi cycles can be presented in one full horizontal sweep of the oscilloscope. This requires lower Rossi frequency with an attendant decrease in timing precision. If the Rossi amplitude is set too high, triple dashes may result (7 writes in a dash with 5 writes between), and they are not

easily detected at a glance (Fig. 51 is an example of triple dashes). If the signal line has noise on it, the display may become very difficult to read (Fig. 45 is an example of a noisy display).

While the timing oscilloscopes using the Dot and Double Dot displays are primarily intended for timing purposes, it is occasionally possible to read some α data from them if the oscilloscope can write it and the Rossi frequency is appropriate.

Worm Display

This is rather a novelty. A sinusoid provides horizontal deflection while a linear ramp provides vertical deflection. Timing pulses appear as additional vertical deflection. Figure 18 is an example of a cross time worm display. Since this is not so commonly used we will not consider it further.

Alpha Data Displays

Plain Rossi Display

The plain Rossi is an extreme case in which the horizontal ramp voltage is completely eliminated. The Rossi signal provides horizontal deflection and the recorded signal provides vertical deflection of the electron beam. Figure 19 is an example. In general, this display must be self triggered to ensure that an adequate baseline will be produced by a number of Rossi cycles before the arrival of the signal without excessive overwriting. This produces a baseline that is occasionally difficult to locate accurately. This display has the advantage of simplicity but has at least one serious failing. The decreasing portion of the input signal

Fig. 18. Example of a crosstime worm display. The timing pulses are added to a linear ramp voltage giving vertical deflection while a sinusoid of precise frequency provides horizontal deflection.

may be written on top of the record for the increasing signal, making film reading difficult. This difficulty is normally overcome by gating the oscilloscopes off for the decreasing signal. If detector breakdown occurs, the noise may be written on top of the data, making the film very difficult to read.

Creep Rossi Display

The Creep Rossi was developed as a partial solution to the problem of overwriting with decreasing signal or noise. A small amount of ramp voltage is added to the Rossi signal so that the display

Fig. 19. Example of a plain Rossi display. Vertical deflection is provided by the data signal while the horizontal deflection is accomplished by a precision sinusoid. The trace becomes too faint to read in the upper portions because the electron beam is deflected too rapidly for sufficient excitation of the phosphors on the tube face.

moves slowly to the right. The data signal provides vertical deflection, as before. Figures 20 and 21 are examples. The advantage over the straight Rossi is threefold. First, as mentioned, the decreasing portions do not seriously overwrite the earlier part of the trace. Secondly, the timing of the display is less critical, since a greater number of Rossi cycles can appear before the data signal without poor baseline definition due to multiple overwriting.

Fig. 20. Example of a Creep Rossi display. Deflection is from left to right. The signal increases slowly and then accelerates, decaying again on the right hand side.

A final advantage is that a longer baseline is produced (which can be more easily located by optical filtering) allowing a more accurate rotational orientation of the figure about the axis of the oscilloscope tube. Most α data recorded by LLL uses this display. It is of interest at this point, to discuss a convenient way to measure an approximate α directly from the recorded trace in the field, without solving the Rossi equations for the

trace. Such a procedure is useful for making 6 hr report estimates of $\alpha_{\max}$.

If we assume the intensity of the γ -ray flux is proportional to the neutron population, $I_{\gamma}(t) = \beta N(t)$ and further assume

$$\frac{dN}{dt}(t) = \alpha(t) N(t)$$

where $N(t)$ is the neutron population as a function of time, we also have

Fig. 21. Example of a Creep Rossi display. The signal increases very rapidly in the early portions and then decelerates.

$$\frac{dI_{\gamma}(t)}{dt} = \beta \frac{dN(t)}{dt} = \alpha(t) \beta N(t) = \alpha(t) I_{\gamma}(t)$$

or

$$\frac{d \log_e I_{\gamma}(t)}{dt} = \alpha(t).$$

This can also be written as

$$\log_e \left(\frac{I_{\gamma}(t)}{I_{\gamma}(0)} \right) = \int_0^t \alpha(t) dt.$$

Now assume that the vertical deflection on the oscilloscope face is proportional to the voltage at the detector and to the intensity I_{γ} . If we consider the deflections D_1 and D_2 at times t_1 and t_2 we have

$$\log_e \left(\frac{D_2}{D_1} \right) = \int_{t_1}^{t_2} \alpha(t) dt.$$

As a matter of convenience, assume $t_2 - t_1 = T_R$, where T_R is one Rossi period. If α is assumed constant over one Rossi period, we have

$$\log_e \left(\frac{D_2}{D_1} \right) = T_R \alpha$$

or finally,

$$\alpha (\text{gen}/\mu\text{sec}) = f_R (\text{MHz}) \log_e \left(\frac{D_2}{D_1} \right).$$

Figure 22 illustrates the use of the expression. A plain Rossi display is shown as a matter of convenience. The deflections D_1 and D_2 have been chosen as two successive right turning points to give an interval of one Rossi period. It is significant to note that this procedure can also be used with the Creep Rossi display, so long as either two successive right or two successive left turning points are used. With the Creep display, the time between A and B in the figure is not half a Rossi period but rather a forward fraction. A nomogram solving the relevant equation has been constructed and appears in the appendices. The Creep display is indicated by CR beside the oscilloscope symbol on a one line drawing.

Ramp Rossi Display

The Ramp Rossi display is another modification of the plain Rossi, in which a slow ramp voltage is added to the vertical deflection signal. Figure 23 is an example. The purpose of the ramp is to prevent overwriting of the trace prior to the arrival of the signal. This allows an earlier detectable signal to be read from the film. The ramp deflection on the vertical sweep is subtracted from the total deflection during data reduction.

Fig. 22. Illustration of the method of determining approximate α directly from the trace using successive left or right turning points of the rising trace. Discussion is included in the text.

Since the upper voltage limit of the input signal that can be read on the oscilloscope is somewhat reduced by the bias voltage of the ramp however, the coverage range available on the oscilloscope is not much greater than that for the Creep display.

Horizontal deflection of the Ramp Rossi display is provided by the Rossi signal alone. The oscilloscope is triggered about 10 Rossi cycles before the earliest detectable signal is expected. At some time prior to the arrival of the signal, a very short marker pulse can be

Fig. 23. Example of a Ramp Rossi display. The trace is rather poorly focused. A linear ramp is added to the data signal to provide vertical deflection. Horizontal deflection is provided by the Rossi sinusoid.

applied to the vertical input. When used as a timing oscilloscope, the mark is very short, and the oscilloscope is not easily cross timed with the Dot displays without the aid of a computer. The point of signal breakout is difficult to locate accurately by eye, as it is rising from a portion of the trace which is itself rising. The Ramp Rossi display should always be self triggered.

An additional serious disadvantage of the Ramp Rossi display is the error introduced in the subtraction of the ramp voltage. It is difficult to make an accu-

rate fit to the ramp signal for this purpose, and the ramp may not be consistent on successive sweeps.

The display can also rewrite on itself in the same manner as the plain Rossi display. The timing discrepancies are difficult to deal with and the display requires a lot of work by the bunker crew to adjust it properly. This display is not recommended.

We have previously mentioned the factors to be considered in the choice of f_R/α for a display. The Rossi frequency should be chosen so that signals with the expected α will produce an easily read figure on the oscilloscope face. The common choice is made such that $2 \leq f_R/\alpha \leq 3$. Higher values in this interval are generally preferable since greater time resolution is possible and the Rossi frequency is further removed from the Fourier components of the data. This becomes particularly important when Rossi feed-through into the vertical deflection system occurs.

Historically, the Rossi display has been read with hand controlled equipment. The data is converted to digital form by using a microscope with cross hairs to follow the recorded trace on the film. The lateral position of the crosshairs is mechanically determined with digital registers. When the crosshairs are centered on the trace, a button is pressed by the operator to record the digital position. The sequence of such positions forms the digital record of the display. This procedure is still in use where automated methods cannot be used because of poor trace definition or excessive overwriting problems.

Most data recorded in Rossi displays is read by a programmable film reader

(abbreviated PFR), consisting of a small computer with CRT display and associated optical equipment for scanning the film. The PFR used by EG&G in Las Vegas uses a photomultiplier to view a CRT face with the data film as an intermediate filter. A precise raster is swept on the CRT face at a slow rate. When the spot of light is in a position corresponding to a point of the recorded data trace, the transmitted light received by the photomultiplier is changed. The relative position of the indicated point is determined by the timing of the raster. The system includes an additional transmission filter using optical polarization to superimpose a reference grid on the data trace. The signals for the grid are recorded by a separate photomultiplier with light extracted by a beam splitter in the optical path. The computer software allows any unit of the grid to be displayed on the operator's CRT at the console. Interaction between computer and operator is by a light pen and associated selector switches. The digital resolution provided by the PFR is much greater than that for the hand reading technique and the results are less influenced by operator judgment, since the digital location of the data trace is determined using statistical weighting methods to smooth the curve. As a further advantage, the PFR is a considerably faster system to use. However, one experiences difficulties when the film shows poor focus, contrast, or has excessive overwriting. In such instances, it is common practice to use the handreading system.

The digital representation of the Rossi display is used as the input data for another code which solves the Rossi equa-

tions and uses the resulting amplitude vs time data to calculate $\alpha(t)$. The output of this code provides a graphical display of the $\alpha(t)$ curve. Since the absolute calibration of the Rossi display depends on the deflection voltages of the recording CRT at each point of the data trace, it is necessary to record the characteristics of each CRT for use with the corresponding data traces. This is presently accomplished using calibration rasters made in the field for each tube. In the following sections we will discuss calibration displays and then consider typical problems encountered in the field.

CALIBRATION DISPLAYS

DC Calibration Rasters

The dc calibrations are made by placing a series of dc voltage steps on the vertical sweep while making a horizontal sweep at each step. The result is a series of lines across the face of the oscilloscope that reveal local and overall distortions introduced by the tube. Tubes with gross distortions are eliminated. The calibration traces of those that are retained for data are used to correct the recorded data for oscilloscope distortions. Figure 24 is an example of a good calibration raster. While the top and bottom lines show distortion, the central regions (where the data will be displayed) show lines that are straight and parallel.

A measure of confusion of terminology has prevailed concerning the names of certain lines in these displays. The reader may observe that there is a large gap dividing the lines into two groups. The bottom line of the upper group has traditionally been called the "fireline" at

Fig. 24. Example of a satisfactory dc calibration raster. While the top and bottom lines show distortion, the central region where the data will be displayed show lines that are straight and parallel.

LLL. The line below it, across the gap is called the baseline. Practice at EG&G has been to call these the baseline and

prebaseline, respectively. To compound the confusion, an EG&G manual (Ref. 10) calls these the positioning and prebase

lines. We will use the LLL terminology here. A small gap in the baseline, called a fiducial mark is formed by placing a piece of tape on the oscilloscope face to aid in superposition of the dc calibration film and shot film. This will be seen in subsequent figures.

The calibration voltages used in the dc calibrations are listed on the one-line drawing and are grouped by oscilloscope type, with the 507 and 3445 oscilloscopes together. The 507 and 3445 oscilloscopes use a nominal increment of 20 volts between lines (other than the baseline and fireline for which it is 40 volts). An increment of 5 volts is used for the 517 oscilloscope and 1 volt is used for the 3344 oscilloscope.

Pulse Calibrations

Pulse calibrations are used to check attenuator values and find system discontinuities. Figure 25 is an example of an attenuation check. Successive sweeps were made with attenuators removed between sweeps. The amplitude ratios of the pulses serves to check the attenuator calibrations.

Figure 26 shows a system problem. Either the pulser gave an improper output or there is a gross reflection in the system. We will consider a few other sample problems revealed by pulse calibrations. Figure 27 illustrates a portion of a one-line drawing showing two groups of oscilloscopes connected to a detector line through a tee. Figure 28 shows the pulse seen on oscilloscope 51 with no

Fig. 25. Example of a pulse calibration trace used to check attenuator values using multiple sweeps with successive attenuators removed.

Fig. 26. Example of an attenuator check by pulse calibration showing a system problem. Either the pulser malfunctioned or a gross reflection exists in the system.

Fig. 27. One line drawing used to illustrate system problems in the text. The traces shown in Figs. 28 and 29 were taken from oscilloscope 51.

appreciable delay between oscilloscopes 51 and 52. The small bump immediately after the main pulse is a reflected signal from oscilloscope 52.

The bumps near the end of the trace are signals which are reflected from the oscilloscopes to the tee and back again. Figure 29 shows the same system with a 30 nsec cable delay inserted between the two oscilloscopes. The reflected signal from oscilloscope 52 now arrives at oscilloscope 51 well after the signal. In each case, a 50 nsec cable delay was placed between the tee and oscilloscopes 51. It is evident that sufficient delay

Fig. 28. Pulse calibration showing interscope reflection between oscilloscopes 51 and 52. The small bump immediately after the main pulse is a reflected signal from oscilloscope 52. See Fig. 27 for the wiring diagram.

Fig. 29. Pulse calibration with the system shown in Fig. 27. The trace is the same as that shown in Fig. 28 except that a 30 nsec delay has been inserted between oscilloscopes 51 and 52, moving the reflected signal away from the data on the trace.

cable must be located between the tee and the first oscilloscopes so that reflections do not return at times of interest after the signal rise.

Reflections may also occur at attenuators in the system. The following experiment illustrates this problem. Figure 30 illustrates a situation with an

Fig. 30. One line drawing used to illustrate problems discussed in the text. Figures 31 and 32 correspond to oscilloscope 51 in this diagram.

Fig. 31. Trace seen on oscilloscope 51 of Fig. 30, with the attenuating pad located too close to the tee. The bump on the trailing edge of the pulse is caused by reflection between these components.

attenuating pad located too close to a tee so that reflections between the two components arrive at the oscilloscope shortly after the rise of the signal pulse. Figure 31 shows the result seen on oscilloscope 51. On the trailing edge of the signal pulse at about 12% of the maximum amplitude, a reflection signal appears. The delay between the leading edge of the

signal pulse and the reflection signal is just twice the transit time between the tee and the attenuator. When the attenuator is located between the pulser and the integrator, well removed from the tee, the result is shown in Fig. 32. The reflection signal has been removed. It is evident that yet another reflection remains.

Fig. 32. Trace seen on oscilloscope 51 of Fig. 30, with greater delay between the attenuator and tee. The bump on the trailing edge is much reduced. Since a bump still appears to be present, there is evidently either an additional problem or a distorted pulse output.

Fig. 33. Pulse calibration in a dual helix oscilloscope with one helix unused, but terminated in its characteristic impedance at only one end. The bump on the trailing edge represents energy coupled from the active helix into the unused helix, reflected from the unterminated end and then coupled back into the active helix.

Another problem can occur if an unused helix in an oscilloscope is not properly terminated. The two helices of the oscilloscope cross couple with each other to a certain degree. Figure 33 shows the result when one end of the unused helix is terminated in its characteristic impedance while the other end is left open. A bump appears at about 10% of the maximum amplitude on the trailing edge of the signal pulse. This represents energy coupled to the unused helix, reflected from the open end of the helix and then coupled back to the original helix. The reflected pulse on the unused helix does not reappear again because it is dissipated at the terminated end. Figure 34 shows the result when both ends of the unused helix are unterminated. The coupled energy reflects back and forth between the unterminated ends of the unused helix, coupling energy back into the signal circuit with each pass.

We will now turn to problems relating to limitations of the electron optics of the tubes and associated sweep circuits.

DISPLAY PROBLEMS

Tube Characteristic Problems

A number of common difficulties that are characteristic of the tubes are non-orthogonality of the electron optics, poor writing capability, astigmatism, poor focus and cross talk between the vertical and horizontal deflection systems. An additional serious problem called blooming occasionally occurs and is not well understood. In this problem, the overall background intensity of light from the tube phosphors suddenly increases. If this happens during the recording of the data, the trace may have too little contrast to be read properly. It is common practice to use relatively high intensity settings on the oscilloscope to avoid this

Fig. 34. Pulse calibration in a dual helix oscilloscope with both ends of the unused helix unterminated. Energy coupled from the active helix reflects repeatedly from the unterminated ends of the unused helix, coupling energy back into the active helix on each transit.

Fig. 35. DC calibration raster showing barrel distortion, a severely distorted top line, and astigmatism where the trace becomes broad.

problem. We will consider a few examples of these problems. Figures 35, 36, and 37 show a number of these problems. Figure 35 shows barrel distortion and a severely distorted top line. Astigmatism

is also very evident, particularly in the lower left corner where the trace width becomes very broad. Figure 36 also shows barrel distortion and astigmatism. Note the fiducial mark on the baseline.

Fig. 36. DC calibration raster showing barrel distortion and astigmatism, particularly in right hand corners. Note the fiducial mark on the baseline (fifth from the bottom).

Figure 37 is a rather extreme example. The baseline and all lines below it show distortion, the top lines are bowed and astigmatism is very pronounced in the upper right corner.

Figure 38 shows a 3445 oscilloscope which is a poor writer. (The 3445 is normally a good writer.) The trace has adequate intensity in the early horizontal portions but quickly becomes too faint to read as the signal rises.

Fig. 37. DC calibration raster showing extreme distortion of lines and pronounced astigmatism in the upper right corner.

Fig. 38. Creep Rossi display on a 3445 oscilloscope which is a poor writer. The trace rapidly becomes too faint to read while the initial portions have good intensity.

Fig. 39. Severe Rossi distortion in a creep display. The trace appears badly pinched on the left hand side. The trace is also poorly focused.

Fig. 40. Rossi distortion in a creep display. The pinching is to the right in the lower portions. This is also a poorly focused trace.

Electronic Problems

Rossi Distortion

This has been treated as an electronic problem but it need not always be the case. Rossi waveform distortion will produce the difficulty, but it can also be caused by interaction between the deflection systems of the CRT. The problem commonly called "pinching" is thought to be an example of this. Figure 39 shows a rather severe example in a Creep display. The left hand turning points appear pinched while the right hand turning points are rounded. This is a rather poorly focused example. Figure 40 is another example in which it is not so evident. Figure 41 shows pinching in a plain Rossi

Fig. 41. Straight (plain) Rossi display showing Rossi distortion. The lower right turning points are pinched.

Fig. 42. Rossi distortion in a straight (plain) Rossi display. This example shows pinching on the left near the bottom and on the right side near the top. The problem is therefore caused by electron optics difficulties within the tube, since pinching due to waveform distortion would be consistently on either the left or the right side.

display. Figure 42 is a more extreme case. The pinching is very pronounced in the upper part of the trace, and it is the right-hand turning points which are pinched. Near the bottom, the left hand sides are pinched. This is necessarily a tube problem since waveform distortion would show a consistent result.

Rossi Feedthrough

Figures 43 and 44 show very extreme

examples of feedthrough. In each case, the feedthrough signal is a high-frequency Rossi signal radiating into the lines from another oscilloscope. This is the high frequency "rick-rack" like signal superimposed on the main signal.

System Noise

Figure 45 is an example of a very noisy trace. The timing information is

Fig. 43. An extreme example of Rossi feedthrough. A high-frequency oscilloscope elsewhere in the system is radiating its Rossi signal onto the data signal lines. It appears here, superimposed on the data in a lower-frequency display giving a "rik-rak" like effect to the trace. The problem is rarely this obvious.

very difficult to read. There is also evidence of a marker pulse reflection in the lower trace.

Improper Adjustments and Miscellaneous System Difficulties

Figure 46 is an example of a poorly focused oscilloscope. The trace is quite wide. Figure 47 shows a Dot display in which the dots are too small. They are little more than a periodic hesitation in the trace. Figure 48 shows the Double Dot display with the same problem. In both of these cases the Rossi amplitude needs to be increased. More is not always better however, and Figs. 49 and 50 show the Dot and Double Dot displays at the other extreme. The Rossi ampli-

tude is so large that the pattern has an almost unreadable smeared appearance.

Figure 51 illustrates an important pitfall. The Rossi amplitude has been increased to the point that the trace is now a triple dot rather than a double dot. If assumed to be a Double Dot display, this would contribute a major error in system timing.

The reader can appreciate from the figure how difficult it is to spot this problem. It is revealed by examination of the timing marks near the beginning of the trace or the figure representing the signal pulse. A vertical line passed through portions of the pulse crosses the trace seven times.

Fig. 44. Another example of extreme Rossi feedthrough.

Fig. 45. An example of an extremely noisy trace. Try reading the timing information on the upper trace.

Fig. 46. A Creep Rossi display very poorly focused.

Fig. 47. A Dot Rossi display with inadequate Rossi amplitude. The reverse portions of the dots are scarcely more than hesitations.

Fig. 48. A Double Dot Rossi display with inadequate Rossi amplitude.

Fig. 49. A Dot display with excessive Rossi amplitude. The trace appears smeared since the portions of the trace which are singly written are almost vanishingly small.

Fig. 60. A Double Dot display with excessive Rossi amplitude.

Fig. 51. A Triple Dot display caused by excessive Rossi amplitude. Note that a vertical line drawn through portions of the trace is crossed seven times.

Fig. 52. A Double Dot display in which a reflection of the marker pulse appears.

Figure 52 shows a DD display in which a reflection of the marker pulse appears immediately after the marker pulse. Figure 53 shows an erratic intensifier pulse length (note the variable length of the trace lines).

This is by no means an exhaustive account of the electronic difficulties encountered in the field but is intended to give the reader a reasonable sample of the difficulties encountered with the CRT systems.

Fig. 53. A DC calibration raster showing erratic intensifier pulse length. Note the erratic length of the lines.

Simultaneity Dry Runs

In the coverage of phenomena with dynamic range larger than that of any single appropriate detector it is necessary to use a group of detectors to cover various portions of the phenomena and then construct the total behavior from the measurement of the parts. In the reconstruction process, the relative timing of the parts is needed. This is commonly accomplished by means of timing marks placed on the recording oscilloscopes assigned to the different detectors through carefully measured delays. The simultaneity dry run serves to check that all the time delays in the system have been correctly accounted for. We will illustrate the dry run with a specific example. Since the beginner can seldom be expected to have sufficient familiarity with a one-line drawing, we have chosen to use an example with considerably more than the minimum detail required, in order to give a more realistic illustration of the practical situation.

ONE LINE DRAWING

Figure 54 (parts a and b) represents a somewhat simplified extract from a typical one line. It is by no means complete, omitting the types of cables used, trigger generator voltage levels, etc. as well as other detectors and oscilloscopes of the system. We will discuss the major features shown before proceeding with our example.

Part a of the figure shows three detectors of the Compton Diode type, each connected to a group of oscilloscopes. One oscilloscope for each detector is

used for timing purposes. These are oscilloscopes 19, 29, and 39. Each is a type 3445 oscilloscope, using the Double Dot (DD) display. The sweep time, Rossi frequency and relevant trigger/marker generators are indicated on each of these (e.g., for oscilloscope 39, triggered by TG-3, the sweep time is $0.4 \mu\text{sec}$, the Rossi frequency is 100 MHz and the timing mark is provided from marker generator 3). The oscilloscopes for each detector are shown divided into two strings by a tee following a delay line. Appropriate attenuators are included in each string, with their attenuation shown as a voltage ratio. These are indicated by rectangles in the lines. Each string is terminated in the characteristic impedance of its cables. In our example, it is shown as a rectangle at the end of the line, designating 100 ohms. (Fifty ohm terminations are designated by a half round symbol. Examples may be seen in part b of the figure where the upper lines to oscilloscopes XT-1 and XT-2 are terminated with 50 ohms impedance.)

The numbers in parenthesis shown on the lines are nominal cable lengths in nanoseconds. Oscilloscopes 14 and 16 show a letter L with a left to right arrow above them. This indicates that the lower trace of these oscilloscopes is used, swept from left to right. The remainder of the oscilloscopes use the upper trace with left to right sweep. In each detector line, the time delay from the tee to the timing oscilloscope for simultaneity is indicated separately (e.g., for detector 2 it is 21.31 nsec). The reader may notice that the connecting

Fig. 64. Excerpt of a cno line drawing to illustrate the simultaneity dry run. a) Detector and oscilloscope lines.

Fig. 54. Excerpt of a one line drawing to illustrate the simultaneity dry run. b) Marker pulse lines.

cables do not make up the total delay for the signal. The internal delay of the oscilloscope must also be accounted for. The delay specified from the tee to the timing oscilloscope is the total delay from the tee to the center of the helix of the timing oscilloscope.

In our example, each oscilloscope chain is self triggering (i.e., all oscilloscopes assigned to a detector are triggered using signals generated by that detector). The trigger generators are shown with double tees in the lines from the detectors. Each unit is a dual system except for the power supplies to provide redundancy. The first half of the dual system to generate a trigger pulse activates the oscilloscope sweep and corresponding marker generator. Sometimes oscilloscopes are triggered using signals from a detector other than the one they are connected to. This is called a program trigger which has a time delay based on expected behavior of the signals from the detectors.

A number of vertical lines have been drawn through the cables between the detectors and trigger generators. These represent points of the cable system where connectors are used. The patch panel is located in the bunker and allows dry runs entirely internal to the bunker to be made substituting an electronic simulator in place of the remainder of the cables and detectors. The ground plane indicates the point where the cable system is grounded. The collar splice rack is located at the top of the hole at ground zero. This is the point of connection with the downhole cables, which are connected to the detectors by a short cable called the jumper. Detector lines 2

and 3 have equalized cables, with the equalizers located in the surface cable run. They are shown as attenuators. The time delays between the patch panel and the oscilloscope chain tees are shown above each line. The time delay from the patch panel to the detectors are shown boxed, to indicate that these times are the delays to be used only during the simultaneity runs. For these runs a simulator is substituted for the detectors with their jumper cables using extra delays to mockup the expected time sequence of signals from the detectors. The result is such that simultaneous signals originating at the detectors shown, delayed by the amounts specified in the boxes would arrive at the patch panel with approximately the same sequence and relative timing as that expected when the actual experiment is performed. With this artifice, the "excitation" of each of the three detectors shown may be treated as a single physical event in time. This is an important concept which we will use in our example.

The approximation mentioned above is always present in a triggered system, since the time interval required for the signal to reach a triggering level depends directly on its rate of rise. Since the rise time of the simultaneity pulser will in general be different from that of the experimental signal, the triggering intervals will not be identical. This is also a significant problem in the measurement of the cable lengths. The attenuators are removed from the system during these measurements to prevent trigger interval errors. The attenuation would modify the rise times of signals. Since the cables themselves have different losses,

the problem is not entirely removed. The cable lengths are measured by sending pulses down the length of each cable and measuring the two way transit time. The result indicated on the one line is determined by the average of ~10,000 pulses. This is accomplished using a gated electronic clock with counting registers. The reader interested in further details may consult a report by John Glasgow.¹¹

Now consider part b of Fig. 54. This deals entirely with the timing system. Three marker generators of the system with their associated fan-outs are shown (marker generator 1A and its fan-out were omitted as unnecessary for our example). We will consider marker generator 1 (MG-1) as representative.

Two delay line coils with a nominal length of 30 nsec are shown. The marker is also a dual system with one delay line for each half of it. In operation, a voltage step is sent down the delay line and reflected from the other end with opposite polarity. The reflected signal returns to the exciting end of the line and terminates the step, producing a pulse with a nominal length of 60 nsec. The tees with the 50 ohm terminations serve to eliminate a long trailing edge on the pulse by loading the marker generator. The pulse is fed through an attenuator to the fan-out to provide isolation.

Marker pulses are sent from the fan-out to the various oscilloscopes with delays as indicated. The observant reader may have noticed that oscilloscopes 19, 29, and 39 have 50 ohm terminations on the marker lines while the signal lines in part a of the figure show 100 ohm terminations. Deflection of the oscilloscope beam by the signal and

marker pulses are accomplished by different structures in the tube and hence the lines involved can have different impedance. Signal deflection is accomplished by the helix, while the marker pulse is applied to electrostatic deflection plates.

The resulting isolation of the signal and marker systems prevents propagation of marker pulses to other oscilloscopes on the signal lines. The physical separation of these deflection structures introduces an additional time delay corresponding to the transit time of the electron beam between them. This delay is customarily accounted for by including it in the delay between the marker fan-out and the oscilloscope (e.g., the delay of 14.13 nsec indicated between marker generator 1 fan-out and oscilloscope 19 includes the delay between marker plates and helix in oscilloscope 19). The 3445 oscilloscope uses the KR-1M tube for which this delay is 2.4 ± 0.1 nsec. The 3344 oscilloscope may use either the KR-22 tube or the KR-3M tube. The delay for the latter is 1.0 ± 0.1 nsec.

The marker pulses which appear on the cross time oscilloscopes XT-1 and XT-2 are applied from the marker fan-outs through diode fan-ins (DFI). These serve to prevent pulses reflected from the cross time oscilloscopes from returning to the fan-outs, since the reflected pulses have opposite polarity. Further isolation between fan-outs and fan-ins is provided by the attenuators in these lines. They serve to damp reflections from the fan-ins themselves. The two cross time oscilloscopes are shown with both 50 ohm and 100 ohm terminated lines. The L-12 oscilloscope is provided with inputs which

allow either termination. The identification of the pulses on these oscilloscopes is shown on the one line beside them.

EXAMPLE OF A TYPICAL SIMULTANEITY DRY RUN

Using the simultaneity delays in boxes on the one-line, as mentioned above, we can pretend that the "excitation" of the detectors shown occurs as a simultaneous event. The interval of time between this event and the occurrence of another distinct event, such as the departure of the trailing half amplitude point of the pulse from marker generator 3 fan-out will have a unique value. The algebraic sum of the time delays in the system, starting from any detector and ending at marker generator 3 fan-out should thus give this value. This calculation can be performed for each detector. If the result for any single detector is markedly different from the results for the other detectors, it is most probable that one of the delays used in calculating the result for that detector is incorrect. Two basic rules for the algebraic sign of a particular delay in going from a detector along the cables to a standard reference point, such as the fan-out of marker generator 3 may be adopted:

- a) If the direction of travel on a line is in the direction of signal (or marker pulse) propagation, treat the delay as positive.
- b) If going from an earlier to a later event on an oscilloscope trace, treat the delay as positive.

Now consider our example with the above sign convention. The delays from detector AX-1 to the patch panel, from

the patch panel to the oscilloscope chain tee, and from the tee to oscilloscope 19 are all in the direction of signal propagation and thus are considered as positive, giving a total delay of $2091.15 \text{ nsec} + 573.33 \text{ nsec} + 7.74 \text{ nsec} = 2672.22 \text{ nsec}$. The signal appears on oscilloscope 19 after the mark from marker generator 1, so that we are traveling from a later to an earlier event on this oscilloscope. This delay is therefore considered to be negative. Figure 55 shows the trace from oscilloscope 19. The display is Double Dot with a Rossi frequency of 50 MHz. The interval from signal break-out to the trailing half amplitude point of the pulse is -336.0 nsec . The delay from oscilloscope 19 to MG-1F.0. is -14.13 nsec since we are traveling in the opposite direction to the marker pulse propagation direction on this line. In going from MG-1F.0. to DFI-2, we are going in the propagation direction, hence this delay is $+209.59 \text{ nsec}$. Similarly, the delay from DFI-2 to cross time oscilloscope XT-1 is $+17.93 \text{ nsec}$. The delay from the trailing half amplitude point of marker pulse 1 to the trailing half amplitude point of marker pulse 3 on XT-1 is forward in time, hence it is positive. The trace is shown in Fig. 56. The presentation is a 100 MHz Dot Rossi, hence the delay may be found to be $+268.1 \text{ nsec}$. The delays from XT-1 to DFI-3 and from DFI-3 to MG-3F.0. are both negative since we are traveling against the propagation direction. These delays are -17.05 nsec and -124.24 nsec , respectively. The total delay from detector AX-1 output to departure of the trailing half amplitude point of the pulse from MG-3F.0. is thus $2672.22 \text{ nsec} - 336.0 \text{ nsec} - 14.13 \text{ nsec} + 209.59 \text{ nsec}$

Fig. 55. Timing trace observed on oscilloscope 19. The display is a 50 MHz Double Dot.

+ 17.93 nsec + 268.1 nsec - 17.05 nsec
 - 124.24 nsec = +2676.42 nsec.

A similar calculation from detector AX-2 to the same point gives 2409.22 nsec
 + 457.87 nsec + 21.31 nsec - 157.4 nsec
 - 88.39 nsec + 84.36 nsec + 17.93 nsec
 + 71.5 nsec - 17.05 nsec - 124.24 nsec
 = 2675.11 nsec.

The reader may follow this sequence of delays from detector AX-2 to oscilloscope 29, from signal breakout to the trailing half amplitude point of mark 2 on oscilloscope 29, and then to MG-2F.0.,

then to DFI-2, from DFI-2 to XT-1, from the marker pulse 2 trailing half amplitude point to the marker pulse 3 trailing half amplitude point on XT-1, then from XT-1 to DFI-3 and finally from DFI-3 to MG-3F.0.

The calculation for detector AX-3 gives 2426.09 nsec + 371.03 nsec + 21.10 nsec - 120.3 nsec - 24.11 nsec = 2673.81 nsec. The traces for oscilloscopes 29 and 39 are shown in Figs. 57 and 58. The largest discrepancy between the three results is 2.61 nsec. This is a

Fig. 56. 100 MHz Dot Rossi display from cross time oscilloscope XT-1.

rather typical result. The agreement is seldom better than 1 nsec since this involves considerable precision in reading the oscilloscope traces to fractions of a Rossi period, and cumulative triggering interval errors caused by variations in signal rise times may be this great. In following this example, the reader may have noticed a certain awkwardness in searching through all the data on the one-line for the desired numbers. To alleviate this problem somewhat, a number of useful aids (timing sheets) have been developed. They are supplied conveniently as note pads. Figure 59 illustrates the common marker timing sheet used with detectors AX-1 and AX-2 following our example. The appropriate numbers are extracted from the one-line drawing and placed on the sheet. The appropriate time intervals read from the oscilloscope faces are entered on the respective symbols. With the data thus collected, the sequence of delays with proper algebraic sign are quite evident.

An alternate technique (cross timing) for simultaneity runs closes loops taking the total algebraic delay from one detector to the next.

Since the detectors are assumed to be excited simultaneously, the algebraic sum should vanish. Figure 60 illustrates one type of cross timing sheet showing the loop closures for detector AX-1 to detector AX-2 and for detector AX-2 to detector AX-3.

The method of common marker timing previously discussed has some distinct advantages over cross timing. Cross timing introduces much duplication of effort since the delays on a detector line may be calculated several times. Notice

Fig. 57. Timing trace observed on oscilloscope 29 as a 100 MHz Double Dot display.

also that the size of the largest discrepancy is more readily apparent with common marker timing. Our cross timing example shows discrepancies of 1.3 nsec. However, a cross timing calculation between detectors AX-1 and AX-3 would show 2.61 nsec.

A further advantage of common marker timing is that it is more readily evident which line contains the error when a dis-

crepancy is significant. An additional cross timing sheet is useful when the two detectors have timing oscilloscopes using the same marker generator, so that it is not necessary to go through a cross-time oscilloscope. Figure 61 shows an example. As an exercise, the reader may perform the above calculations using cross-time oscilloscope XT-2 which is provided to give redundancy and thus

Fig. 58. Timing trace observed on oscilloscope 39 as a 100 MHz Double Dot display.

greater reliability to the system. Figure 62 shows the trace from XT-2. The

reader can judge his skill in film reading by the size of the discrepancies.

Fig. 59. An example of a common marker timing sheet.

Notes:

1. Unless otherwise noted, times are in nanoseconds.

$$\Delta t = 1.31$$

$$\Delta t = 1.3$$

Fig. 60. An example of a cross timing sheet.

Fig. 61. An additional type of cross timing sheet.

Fig. 62. Trace from cross time oscilloscope XT-2. The display is a 50 MHz Dot display.

Acknowledgments

I wish to acknowledge the contributions of many people to this document. Valuable suggestions concerning the text were made by J. N. Shearer, J. C. Watson, T. Hamm, Jr., L. L. Moon, L. P. Altbaum (who also contributed the oscilloscope data table), and D. M. Buchla (who also contributed the α -nomogram, and the mean free path tables). The cable τ

nomogram and cable transmission curves were contributed by A. L. Jorgensen, the detector characteristic tables by J. H. Mallett, and the neutron and γ -ray transmission nomogram is due to C. A. Anderson. The lead and paraffin filter curves were originated by G. C. Roth. The author accepts any aberrations as his own.

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Appendix A Derivations

RESPONSE FUNCTION OF AN IDEAL LOSSLESS COAXIAL CABLE

With a frequency dependent inductance $L \neq 0$ and a frequency dependent C , the telegrapher's equation may be written

$$\frac{\partial^2 I}{\partial z^2} - LC \frac{\partial^2 I}{\partial t^2} - (LG + RC) \frac{\partial I}{\partial t} - RGI = 0$$

where I is the current, R and C are the resistance and capacitance per unit length of line, G is the leakage conductance between conductors of the line, and z is a coordinate along the line. Assume that for arbitrary current input to the line we can express the current in the line as

$$I(z, t) = \frac{1}{\sqrt{2\pi}} \int_{-\infty}^{\infty} \tilde{I}(\omega) e^{-i[\omega t - k(\omega)z]} d\omega$$

where $k(\omega)$ expresses the frequency dispersion characteristics of the line

$$\frac{\partial I}{\partial t} = \frac{1}{\sqrt{2\pi}} \int_{-\infty}^{\infty} \tilde{I}(\omega) (-i\omega) e^{-i[\omega t - k(\omega)z]} d\omega$$

$$\frac{\partial^2 I}{\partial t^2} = \frac{1}{\sqrt{2\pi}} \int_{-\infty}^{\infty} \tilde{I}(\omega) (-\omega^2) e^{-i[\omega t - k(\omega)z]} d\omega$$

$$\frac{\partial^2 I}{\partial z^2} = \frac{1}{\sqrt{2\pi}} \int_{-\infty}^{\infty} \tilde{I}(\omega) (-k^2) e^{-i[\omega t - k(\omega)z]} d\omega.$$

Substitution in the telegrapher's equation thus gives

$$\frac{1}{\sqrt{2\pi}} \int_{-\infty}^{\infty} \tilde{I}(\omega) e^{-i[\omega t - k(\omega)z]} [-k^2 + LC\omega^2 - (LG + RC)(-i\omega) - RG] d\omega = 0$$

but since this is to be true for arbitrary current, the quantity in brackets must vanish, so that

$$k^2(\omega) = LC\omega^2 + i\omega(LG + RC) - RG = -(R - i\omega L)(G - i\omega C).$$

If we neglect G, we can write

$$k(\omega) \cong \sqrt{i\omega C (R - i\omega L)}.$$

Now for the coaxial line, we have (Ref. 12)

$$R = \frac{1}{\sqrt{4\pi}} \left(\frac{1}{a_1} + \frac{1}{a_2} \right) \sqrt{\frac{i\mu_1}{\sigma_1}} \quad (\text{ohms/meter})$$

$$L = \frac{\mu_2}{2\pi} \left[\log_e \left(\frac{a_2}{a_1} \right) + \frac{1}{\sqrt{4\pi}} \frac{1}{\mu_2} \sqrt{\frac{\mu_1}{i\sigma_1}} \left(\frac{1}{a_1} + \frac{1}{a_2} \right) \right] \quad (\text{Henry/meter})$$

and

$$C = \frac{2\pi\epsilon_2}{\log_e \left(\frac{a_2}{a_1} \right)}$$

where a_2 , a_1 are the radii of the outer and inner conductors of the line, f is the operating frequency, σ_1 is the conductivity of the conductors, μ_1 and μ_2 are the magnetic permeabilities of the conductors and dielectric, respectively, and ϵ_2 is the permittivity of the dielectric.

It may be seen that L may be broken into a constant term and a frequency dependent term

$$L = L_1 + L_2(\omega).$$

Separate the frequency dependence of L so that

$$k(\omega) = \sqrt{i\omega C (R - i\omega L_1 - i\omega L_2)} = \sqrt{i\omega RC + \omega^2 L_1 C + \omega^2 L_2 C} = \omega \sqrt{L_1 C} \sqrt{1 + \frac{i\omega RC + \omega^2 L_2 C}{\omega^2 L_1 C}}.$$

If we approximate the second root by the first two terms of a Taylor expansion, we obtain

$$k(\omega) = \frac{\omega}{v} + \frac{1}{2} \sqrt{\frac{C}{L_1}} (iR + \omega L_2)$$

where we have used the relation

$$\sqrt{L_1 C} = \left[\frac{\mu_2}{2\pi} \log_e \left(\frac{a_2}{a_1} \right) \frac{2\pi \epsilon_2}{\log_e \left(\frac{a_2}{a_1} \right)} \right]^{1/2} = \sqrt{\mu_2 \epsilon_2}$$

and $\sqrt{\mu_2 \epsilon_2} = 1/v$ gives propagation velocity on the line. Now

$$R = \frac{1}{\sqrt{4\pi}} \left(\frac{\mu_1}{\sigma_1} \right)^{1/2} \left(\frac{1}{a_1} + \frac{1}{a_2} \right) \left(\frac{\omega}{2\pi} \right)^{1/2}$$

$$L_2 = \frac{1}{2\pi} \frac{1}{\sqrt{4\pi}} \left(\frac{\mu_1}{\sigma_1} \right)^{1/2} \left(\frac{1}{a_1} + \frac{1}{a_2} \right) \left(\frac{\omega}{2\pi} \right)^{1/2}$$

thus

$$(iR + \omega L_2) = \frac{1}{\sqrt{4\pi}} \left(\frac{\omega}{2\pi} \right)^{1/2} \left(\frac{\mu_1}{\sigma_1} \right)^{1/2} \left(\frac{1}{a_1} + \frac{1}{a_2} \right) (1 + i)$$

and

$$\sqrt{\frac{C}{L_1}} = 2\pi \left(\frac{\epsilon_2}{\mu_2} \right)^{1/2} \frac{1}{\log_e \left(\frac{a_2}{a_1} \right)}$$

so that

$$\frac{1}{2} \sqrt{\frac{C}{L_1}} (iR + \omega L_2) = \frac{1}{2} \left(\frac{\epsilon_2 \mu_1}{\mu_2 \sigma_1} \right)^{1/2} \left(\frac{1}{a_1} + \frac{1}{a_2} \right) \frac{\sqrt{\omega}}{\log_e \left(\frac{a_2}{a_1} \right)} \frac{(1+i)}{\sqrt{2}},$$

Define

$$K = \frac{1}{2} \left(\frac{\epsilon_2 \mu_1}{\mu_2 \sigma_1} \right)^{1/2} \left(\frac{1}{a_1} + \frac{1}{a_2} \right) \frac{1}{\log_e \left(\frac{a_2}{a_1} \right)},$$

then

$$\frac{1}{2} \sqrt{\frac{C}{L_1}} (iR + \omega L_2) = K \sqrt{i\omega}$$

and

$$k(\omega) = \frac{\omega}{v} + K \sqrt{i\omega}.$$

The expansion of the root above assumes

$$\frac{i\omega RC + \omega^2 L_2 C}{\omega^2 L_1 C} \ll 1$$

or

$$iR + \omega L_2 \ll \omega L_1.$$

Now

$$(\omega L_2 + iR) = \frac{(1+i)}{\sqrt{4\pi}} \left(\frac{f\mu_1}{\sigma_1} \right)^{1/2} \left(\frac{1}{a_1} + \frac{1}{a_2} \right)$$

and

$$\omega L_1 = \mu_2 f \log_e \left(\frac{a_2}{a_1} \right).$$

The inequality thus reduces to

$$\sqrt{f} \gg \frac{(1+i)}{\mu_2 \sqrt{4\pi}} \left(\frac{\mu_1}{\sigma_1} \right)^{1/2} \left(\frac{1}{a_1} + \frac{1}{a_2} \right) \frac{1}{\log_e \left(\frac{a_2}{a_1} \right)}.$$

Both the real and imaginary parts of the complex number must be small. If we assume copper conductors, $\sigma_1 \sim 5.8 \times 10^7$ mho/meter. Let $a_2/a_1 = 2.36$ to correspond to an impedance of ~ 51.5 ohms, and assume $\mu_1 = \mu_2 = 4\pi \times 10^{-7}$ Henry/meter. With these assumptions,

$$f \gg \frac{1}{4\pi\sigma_1\mu_1} \left[\left(\frac{1}{a_1} + \frac{1}{a_2} \right) \frac{1}{\log_e \left(\frac{a_2}{a_1} \right)} \right]^2$$

and for $a_2 = 1$ in., we have $f \gg 25.9$ Hz. The approximation is thus an excellent one in practice. We thus have $k(\omega) = \omega/v + K\sqrt{i\omega}$ with the frequency dependence explicitly expressed.

We then have

$$I(z, t) = \frac{1}{\sqrt{2\pi}} \int_{-\infty}^{\infty} \bar{I}(\omega) e^{-i\left(\omega t - \frac{\omega z}{v}\right)} e^{iKz\sqrt{i\omega}} d\omega.$$

If we excite the free end of the line with an impulse at zero time, we have $I(0, t) = \delta(t)$ and $I(z, t) = g(z, t)$ thus

$$\delta(t) = \frac{1}{\sqrt{2\pi}} \int_{-\infty}^{\infty} \bar{I}(\omega) e^{-i\omega t} d\omega$$

and therefore, applying the inverse transform, we obtain $\bar{I}(\omega) = 1/\sqrt{2\pi}$ and thus

$$g(z, t) = \frac{1}{2\pi} \int_{-\infty}^{\infty} e^{-i\omega\left(t - \frac{z}{v}\right)} e^{iKz\sqrt{i\omega}} d\omega.$$

Now let $t' = t - z/v$, $p = -i\omega$, $d\omega = i dp$ and $i\sqrt{i\omega} = -\sqrt{p}$ so that $iKz\sqrt{i\omega} = -Kz\sqrt{p}$

$$g(z, t) = \frac{i}{2\pi} \int_{-i\infty}^{-i0} e^{pt'} e^{-Kz\sqrt{p}} dp = \frac{1}{2\pi i} \int_{-i\infty}^{+i\infty} e^{pt'} e^{-Kz\sqrt{p}} dp.$$

This, however, is just the inverse Laplace transform of $e^{-Kz\sqrt{p}}$

$$L^{-1}(e^{-k\sqrt{p}}) = \frac{k}{2(\pi t^3)^{1/2}} e^{-k^2/4t}, \quad k > 0$$

$k = Kz$ and thus we have

$$g(z, t) = \frac{Kz}{2\sqrt{\pi}} \frac{e^{-K^2 z^2/4\left(t - \frac{z}{v}\right)}}{\left(t - \frac{z}{v}\right)^{3/2}}$$

which may be seen to be a propagating solution.

If we define $\tau = K^2 z^2/4$ and define the time origin so that $t = 0$ when the signal first arrives at the observation point, we obtain

$$g(t) = \sqrt{\frac{\tau}{\pi}} \frac{e^{-t/\tau}}{t^{3/2}}$$

with

$$\tau = \left(\frac{\ell}{4}\right)^2 \left(\frac{\epsilon_2 \mu_1}{\mu_2 \sigma_1}\right) \left[\left(\frac{1}{a_1} + \frac{1}{a_2}\right) \frac{1}{\log_e \left(\frac{a_2}{a_1}\right)}\right]^2$$

for a line of length ℓ . As an additional exercise of interest we will show that the case of Heavisides's "distortionless line" cannot be achieved with a coaxial line.

If we consider the case $RC = LG$ and assume R , L , G are independent of frequency, we can readily calculate the response function by taking the Laplace transform of the telegrapher's equation with the current and voltage initially zero everywhere. Excitation with the unit impulse $\delta(t)$ produces the signal

$$g(z, t) = e^{-\sqrt{RG} z} \delta(t - \sqrt{LC} z).$$

The impulse thus propagates along the line with velocity $v = 1/\sqrt{LC}$ and is exponentially damped by the dissipative losses. When one includes the frequency dependence of R , L however, $RC = LG$ does not produce a distortionless line. If we consider

$$I(z, t) = \frac{1}{\sqrt{2\pi}} \int_{-\infty}^{\infty} \bar{I}(\omega) e^{-i[\omega t - k(\omega)z]} d\omega$$

as before, and let $G = RC/L$, we obtain $k^2(\omega) = \omega^2 LC + 2i\omega RC - R^2 C/L$. If we consider $L = L_1 + L_2$, we can write

$$k(\omega) = \omega \sqrt{L_1 C} \left[1 + \frac{\omega^2 L_2 C + 2i\omega RC - R^2 C/(L_1 + L_2)}{\omega^2 L_1 C} \right]^{1/2}.$$

If we define $v = 1/\sqrt{L_1 C} = 1/\sqrt{\mu_2 \epsilon_2}$ and expand the root above as before, we obtain

$$k(\omega) \approx \frac{\omega}{v} + \frac{1}{2} \sqrt{\frac{C}{L_1}} (\omega L_2 + 2iR - R^2/\omega L),$$

Substituting the previously defined values for K, L_1 , L_2 and v this reduces to

$$k(\omega) \cong \frac{\omega}{v} + K \frac{(1+2i)}{\sqrt{2}} \sqrt{\omega} + \frac{K^2 \mu_2}{\pi \left(\frac{1}{v^2} + \frac{\sqrt{2}}{\pi^2} \frac{K}{v\sqrt{\omega}} \right)}.$$

With $I(0,t) = \delta(t)$ as before, we obtain $\bar{I}(\omega) = 1/\sqrt{2\pi}$ and thus

$$g(z,t) = \frac{1}{2\pi} \int_{-\infty}^{\infty} e^{-i\omega \left(t - \frac{z}{v}\right)} e^{i \frac{Kz}{\sqrt{2}} (1+2i)\sqrt{\omega} + \frac{iK^2 \mu_2 z}{\pi \left(\frac{1}{v^2} + \frac{\sqrt{2}}{\pi^2} \frac{K}{v\sqrt{\omega}} \right)}} d\omega.$$

As before, let $p = -i\omega$, $t' = t - z/v$. The result now reduces to

$$g(z,t) = \frac{1}{2\pi i} \int_{-i\infty}^{+i\infty} e^{pt'} \left[e^{i \frac{Kz}{\sqrt{2}} (1+2i)\sqrt{ip} + \frac{iK^2 \mu_2 z}{\pi \left(\frac{1}{v^2} + \frac{\sqrt{2}}{\pi^2} \frac{K}{v\sqrt{ip}} \right)}} \right] dp$$

and this is just the inverse Laplace transform of the quantity in brackets. Now for a distortionless line we would expect a solution of the form $g(z,t) = e^{-bz} \delta(t - az)$ with a, b real. If we let $t' = t - z/v$ as above, this can be written

$$e^{-bz} \delta\left(t' + \frac{z}{v} - az\right) = e^{-bz} \delta(t' - a'z)$$

with

$$a' = a - \frac{1}{v}.$$

Therefore, let us simply write $g(z,t) = e^{-bz} \delta(t' - az)$ and absorb the velocity term in a. Now

$$L[e^{-bz} \delta(t' - az)] = e^{-bz} L[\delta(t' - az)] e^{-bz} e^{-azp} = e^{-(ap+b)z}$$

but we have found

$$L[g(z,t)] = e^{\frac{iKz}{\sqrt{2}}(1+2i)\sqrt{ip} + \frac{iK^2\mu_2^2 z}{\pi\left(\frac{1}{v^2} + \frac{\sqrt{2}K}{\pi^2 v\sqrt{ip}}\right)}}.$$

It is evident that distortion must occur.

RESPONSE FUNCTION FOR TWO IDEAL LOSSLESS COAXIAL CABLES IN SERIES

Frequent occasions arise to treat a compound coaxial cable as two ideal cables with different τ in series. We will show that the result is again a lossless cable with an effective τ . Let us describe the two cables with the response functions

$$g_1(t) = \sqrt{\frac{\tau_1}{\pi}} \frac{e^{-\tau_1/t}}{t^{3/2}}, \quad g_2(t) = \sqrt{\frac{\tau_2}{\pi}} \frac{e^{-\tau_2/t}}{t^{3/2}}.$$

Following the reasoning previously used in the section "Recording Exponential and Gaussian Signals," page 1, we assume a δ function input to the first part of the system so that

$$S_0(t) = \int_0^t \delta(t - \tau) g_1(\tau) d\tau = g_1(t).$$

This in turn excites the next part of the system so that

$$S'_0(t) = \int_0^t S_0(t - \tau) g_2(\tau) d\tau = \int_0^t g_1(t - \tau) g_2(\tau) d\tau.$$

But since the system was excited by a δ function, we may write $S'_0(t) = g_{12}(t)$, the response function of the compound system.

Now by considering the Laplace transform of a function

$$L[f(t)] = \int_0^\infty e^{-st} f(t) dt,$$

we can derive a useful relation concerning the Laplace transform of a convolution integral

$$L \left[\int_0^t f(t - \tau) g(\tau) d\tau \right] = \int_0^\infty e^{-st} \int_0^t f(t - \tau) g(\tau) d\tau dt.$$

Consider the unit step function

$$u(t - \tau) = \begin{pmatrix} 1 & t \geq \tau \\ 0 & t < \tau \end{pmatrix}$$

$$f(t - \tau) u(t - \tau) g(\tau) = \begin{pmatrix} f(t - \tau) g(\tau), & t \geq \tau \\ 0 & , \quad t < \tau \end{pmatrix}.$$

We can thus include the step function $u(t - \tau)$ in the above convolution integral and extend the limit t to ∞ . This is demonstrated below.

Consider the integral

$$\int_0^t S_i(t - \tau) g(\tau) d\tau$$

and the curves describing the character of the integrand in Fig. A-1. If we assume that the input signal function S_i is zero for all negative times (it was caused by an excitation at time zero), then $S_i(t - \tau)$ is zero for $\tau > t$ as shown in the figure.

We thus can extend the limit of the integral to ∞ , therefore

$$\begin{aligned} L \left[\int_0^t f(t - \tau) g(\tau) d\tau \right] \\ = \int_0^\infty e^{-st} \int_0^\infty f(t - \tau) u(t - \tau) g(\tau) d\tau dt. \end{aligned}$$

Reversing the order of integration gives

$$\begin{aligned} L(f * g) \\ = \int_0^\infty d\tau g(\tau) \int_0^\infty dt e^{-st} f(t - \tau) u(t - \tau) \end{aligned}$$

Fig. A-1. Graphical illustration of the convolution integral. Integration is over the shaded region.

but since $u(t - \tau) = 0$ for $t < \tau$ we may raise the lower limit of the inner integral to τ and drop $u(t - \tau)$ for the resulting integral

$$L(f * g) = \int_0^{\infty} d\tau g(\tau) \int_{\tau}^{\infty} dt e^{-st} f(t - \tau).$$

Now change variables $t - \tau = y$, $dy = dt$ so that

$$L(f * g) = \int_0^{\infty} d\tau g(\tau) \int_0^{\infty} dy e^{-s(y+\tau)} f(y) = \int_0^{\infty} d\tau g(\tau) e^{-s\tau} \int_0^{\infty} e^{-sy} f(y) dy$$

thus $L(g * f) = L(g)L(f)$. Now $g_{12}(t) = g_2 * g_1$ so that $L(g_{12}) = L(g_1) L(g_2)$

$$L(g_1) = \sqrt{\frac{\tau_1}{\pi}} \int_0^{\infty} e^{-st - \tau_1/t} \frac{dt}{t^{3/2}}$$

but from transform tables* we have

$$L\left(\frac{k}{2\sqrt{\pi}} \frac{e^{-k^2/4t}}{t^{3/2}}\right) = e^{-k\sqrt{s}} \quad (k > 0).$$

Let

$$\tau_1 = \left(\frac{k}{2}\right)^2 \text{ or } k = 2\sqrt{\tau_1},$$

then

$$L(g_1) = L\left(\sqrt{\frac{\tau_1}{\pi}} \frac{e^{-\tau_1/t}}{t^{3/2}}\right) = e^{-2\sqrt{\tau_1 s}}$$

for $2\sqrt{\tau_1} > 0$. Similarly

$$L(g_2) = e^{-2\sqrt{\tau_2 s}}, \quad 2\sqrt{\tau_2} > 0.$$

Thus we have

*C. R. C. Standard Mathematical Tables, (Chemical Rubber Publishing Co., Cleveland, Ohio) 12th ed., p. 332.

$$L(g_{12}) = e^{-2\sqrt{s}(\sqrt{\tau_1} + \sqrt{\tau_2})}, \quad \sqrt{\tau_1}, \sqrt{\tau_2} > 0.$$

Let $\sqrt{\tau_{12}} = \sqrt{\tau_1} + \sqrt{\tau_2}$, then

$$L(g_{12}) = e^{-2\sqrt{s}\tau_{12}}, \quad \sqrt{\tau_{12}} > 0.$$

Taking the inverse transform, we have the result

$$g_{12}(t) = \sqrt{\frac{\tau}{\pi}} \frac{e^{-\tau/t}}{t^{3/2}}$$

where

$$\tau = (\sqrt{\tau_1} + \sqrt{\tau_2})^2$$

and the compound cable again looks like an ideal cable with an effective τ .

Consider a compound cable composed of n sections of length ℓ and nearly equal $\tau_i = \tau_\ell$, so that $\tau \approx n^2 \tau_\ell$. If we consider the case where $\ell \rightarrow 0$ and $n \rightarrow \infty$, the cable approaches a single cable of response time τ , but $\tau < \infty$ as $n \rightarrow \infty$ so that the τ for a very short cable must be approximately zero, a result in agreement with physical intuition. To see this, notice that the maximum of $g(t)$ is

$$\left(\frac{3}{2}\right)^{3/2} \frac{e^{-3/2}}{\sqrt{\pi\tau}} \quad \text{and occurs at time } t = \frac{2}{3} \tau.$$

As $\tau \rightarrow 0$, $t \rightarrow 0$ and the pulse becomes infinitely large, exactly as expected for δ function excitation at time zero.

It may also be noted that the area under the function $g(t)$ (using the gamma function integral) is unity, precisely the result expected for a lossless cable since that is the normalization of the δ function.

THE ROSSI EQUATIONS

The Rossi presentations discussed in the "Simultaneity Dry Run" section (p. 52) are used to add continuous timing information to signals recorded on fast oscilloscopes. We will derive the relations which allow extraction of the timing information from the figure traced on the oscilloscope face. We will refer to characteristic lengths in the

figure as dot width and spacing to correspond to the usage with oscilloscopes which record only timing information, but the results are of general applicability.

Let X be the horizontal position of the electron beam of the oscilloscope on the tube face. The horizontal deflection signals consist of a superposition of a linear ramp and a sinusoid. The horizontal beam position is thus described by

$$X = X_0 + At + B \cos(\omega t + \phi).$$

For convenience, we will abbreviate $\omega t + \phi = \theta$. Now X_0 , A , B , ω , ϕ are characteristic constants of the trace, which we wish to determine from the figure. We define dot width (W_D) and dot spacing (S_D) as in Fig. A-2. The points a, b, c represent turnaround points of the trace where the beam is momentarily at rest. For these points, we write

$$\frac{dX}{dt} = A - B\omega \sin(\omega t + \phi) = 0$$

so that

$$\left(\frac{A}{B\omega}\right) = \sin \theta_a = \sin \theta_b = \sin \theta_c.$$

Fig. A-2. Definitions of dot width (W_D) and dot spacing (S_D) for the dot display.

Fig. A-3. Graphical solution of Rossi equations.

We represent the solution graphically in a plot of θ vs $(A/B\omega)$ shown in Fig. A-3.

We have no ramp for $(A/B\omega) = 0$. This is the case of the plain Rossi presentation. No solution exists for $|A/B\omega| > 1$ since the value of X never becomes stationary. All useful solutions occur with $0 \leq |A/B\omega| \leq 1$. For this range, θ_a can always be taken to lie in the first quadrant of a polar plot. We illustrate the relationships between θ_a , θ_b , and θ_c in Fig. A-4. It is evident from both Figs. A-3 and A-4 that $(\theta_c - \theta_b) > (\theta_b - \theta_a)$ and $\theta_b = \pi - \theta_a$, $\theta_c = 2\pi + \theta_a$. This is rewritten as

$$\theta_b + \theta_a = \pi, \quad \theta_c - \theta_a = 2\pi$$

or

$$\omega(t_b + t_a) + 2\phi = \pi \quad (1)$$

$$\omega(t_c - t_a) = 2\pi. \quad (2)$$

Now we may express dot width and spacing with the expressions

$$S_D = X_c - X_a = A(t_c - t_a) + B \cos \theta_c - B \cos \theta_a$$

$$W_D = X_a - X_b = A(t_a - t_b) + B \cos \theta_a - B \cos \theta_b.$$

Now from Fig. A-4 it is evident that

Fig. A-4. Phase diagram used in solution of Rossi equations.

$$\cos \theta_a = \cos \theta_c \text{ and } \cos \theta_b = -\cos \theta_c$$

we may thus write

$$S_D = A(t_c - t_a) \quad (3)$$

$$W_D = A(t_a - t_b) + 2B \cos (\omega t_a + \phi). \quad (4)$$

Combining Eqs. (2) and (3), we have

$$A = \left(\frac{\omega}{2\pi} \right) S_D.$$

The rate of rise of the ramp is thus determined by the Rossi frequency and dot spacing alone. Now

$$\cos \theta_a = \sqrt{1 - (A/B\omega)^2}$$

or

$$B \cos \theta_a = \sqrt{B^2 - (A/\omega)^2}$$

so that

$$W_D = A(t_a - t_b) + 2\sqrt{B^2 - (A/w)^2}.$$

Substituting for A, we have

$$W_D = S_D \frac{w}{2\pi} (t_a - t_b) + 2\sqrt{B^2 - (S_D/2\pi)^2}.$$

Separating the time terms,

$$\frac{2\pi W_D}{S_D} - \frac{4\pi}{S_D} \sqrt{B^2 - \left(\frac{S_D}{2\pi}\right)^2} = w(t_a - t_b) \quad (5)$$

but from Eq. (1), $w(t_a + t_b) = \pi - 2\phi$. Adding equations gives

$$2\omega t_a = 2\pi \left(\frac{W_D}{S_D}\right) - \frac{4\pi}{S_D} \sqrt{B^2 - \left(\frac{S_D}{2\pi}\right)^2} + \pi - 2\phi$$

Now from Eq. (2), we have $2\omega t_a = 2\omega t_c - 4\pi$ so that

$$2(\omega t_c + \phi) = 5\pi + 2\pi \left(\frac{W_D}{S_D}\right) - \frac{4\pi}{S_D} \sqrt{B^2 - \left(\frac{S_D}{2\pi}\right)^2} = 2\theta_c.$$

From this result, we obtain

$$\sin \theta_c = \sin \left[\frac{5\pi}{2} + \pi \left(\frac{W_D}{S_D}\right) - \frac{2\pi}{S_D} \sqrt{B^2 - \left(\frac{S_D}{2\pi}\right)^2} \right] = \cos \left[\pi \left(\frac{W_D}{S_D}\right) - \frac{2\pi}{S_D} \sqrt{B^2 - \left(\frac{S_D}{2\pi}\right)^2} \right].$$

But

$$B = \frac{A}{w \sin \theta_c} = \frac{S_D}{2\pi \sin \theta_c}$$

or

$$\sin \theta_c = \frac{S_D}{2\pi B}.$$

Combining equations, we have

$$\frac{S_D}{2\pi B} = \cos \left[\pi \left(\frac{W_D}{S_D} \right) - \frac{2\pi}{S_D} \sqrt{B^2 - \left(\frac{S_D}{2\pi} \right)^2} \right]. \quad (6)$$

If we choose X_0 as the starting point of the trace, at which $t = 0$ then $\cos \phi = 0$ and we may choose $\phi = \pi/2$.

We thus see that the solution for the parameter B involves a transcendental equation. In practice this is commonly solved by computer methods, although a graphical solution could be devised.

The forward and reverse fractions of each Rossi cycle can be determined from the $(A/B\omega)$ vs $\sin^{-1}(A/B\omega)$ plot once A and R are determined. We can also write a transcendental equation for these. Define F and R as the forward and reverse fractions, respectively and note that $R = 1 - F$. Now

$$R = \frac{t_b - t_a}{t_c - t_a} = \frac{\omega}{2\pi} (t_b - t_a),$$

but from Eq. (5)

$$\omega(t_b - t_a) = \frac{4\pi}{S_D} \sqrt{B^2 - \left(\frac{S_D}{2\pi} \right)^2} - 2\pi \left(\frac{W_D}{S_D} \right)$$

so that

$$R = (1 - F) = \frac{2}{S_D} \sqrt{B^2 - \left(\frac{S_D}{2\pi} \right)^2} - \left(\frac{W_D}{S_D} \right). \quad (7)$$

$$\pi(1 - F) = \frac{2\pi}{S_D} \sqrt{B^2 - \left(\frac{S_D}{2\pi} \right)^2} - 2\pi \left(\frac{W_D}{S_D} \right).$$

Since the cosine is an even function, we may substitute this result in Eq. (6) to give

$$\frac{S_D}{2\pi B} = \cos [\pi(1 - F)] \quad (8)$$

but from Eq. (7) we can also write

$$\left[(1 - F) + \left(\frac{W_D}{S_D} \right) \right]^2 \left(\frac{S_D}{2} \right)^2 = B^2 - \left(\frac{S_D}{2\pi} \right)^2$$

or

$$B = \left(\frac{S_D}{2\pi} \right) \sqrt{1 + \pi^2 [(1 - F) + W_D/S_D]^2}.$$

Combining with Eq. (8) gives

$$\frac{1}{\cos [\pi(1 - F)]} = \sqrt{1 + \pi^2 [(1 - F) + (W_D/S_D)]^2}.$$

Let $\beta \equiv \pi(1 - F)$, $\alpha = W_D/S_D$, then

$$\frac{1}{\cos \beta} = \sqrt{1 + (\beta + \pi\alpha)^2}$$

which reduces to

$$\beta + \pi\alpha = \pm \tan \beta.$$

Introduce the parameter $y = \tan \beta$, $y = \pm(\beta + \pi\alpha)$ and consider a graphical representation of the simultaneous solution of these equations shown in Fig. A-5. Since

$\beta = \pi(1 - F)$ and $1/2 \leq F \leq 1$, only $0 \leq \beta \leq \pi/2$ need be considered. Only the first quadrant solution y_+ is physically interesting. It can be seen from the figure that the parametric equations are well conditioned for an iterative solution by computer over much of the interesting interval. Taking the +root we may thus write the transcendental equation as

$$\frac{W_D}{S_D} = +\frac{1}{\pi} \tan [\pi(1 - F)] - (1 - F).$$

We may illustrate the solutions for the Dot, Double Dot Rossi, Plain Rossi, and

Creep Rossi presentations in a plot of $(A/B\omega)$ vs $\sin^{-1} (A/B\omega)$ shown in Fig. A-6. We may see from Eq. (8) that

$$(A/B\omega) = \cos \beta.$$

Fig. A-5. Schematic illustration of solution of equations for forward fraction.

Fig. A-6. Schematic illustration of typical solutions of Rossi equations.

The determination of β thus defines a vertical line at constant $(A/B\omega)$. The vertical lines labeled CR, DD, and D are the approximate regions of the solution for the Creep, Double Dot, and Dot Rossi presentations. The vertical axis $(A/B\omega) = \epsilon$ represents the solution for the Plain Rossi presentation. At $(A/B\omega) = 1$, the electron beam just comes to rest periodically without rewriting. The region $(A/B\omega) > 1$ corresponds to the situation where the beam never stops but is only decelerated and accelerated periodically.

Solutions with $(A/B\omega) < 0$ are physically the same as the positive solutions with the only distinction being the introduction of a phase shift of π in the Rossi signal. Since the vertical coordinates in the figure are proportional to time, we can see that the forward fraction may be determined directly from the figure by the line lengths $\theta_c - \theta_b$ and $\theta_c - \theta_a$. Some representative solutions appear in Table A-1. For timing scopes it is easy to make a sliderule which solves the transcendental equation $\pi\alpha + \beta = \tan \beta$, by plotting $y = \tan \beta$ for the first quadrant of β with a suitable scale compression on the y axis. The slide of the rule will be a straight line crossing the $\tan \beta$ plot at an appropriate angle (45° if no scale compression is used).

Table A-1. Forward fractions for various Rossi displays.

Presentation	α	β	F	(A/B ω)
R	∞	0.5π	0.50	0
CR	6.60	0.484π	0.516	0.05
DD	1.56	0.45π	0.55	0.156
D	0.58	0.40π	0.60	0.309

The vertical scale can be labeled with values of $\pi\alpha$, ($\alpha = W_D/S_D$) and the horizontal scale with values of β/π ($\beta/\pi = 1 - F$). To use the rule, one determines W_D and S_D from the recorded oscilloscope trace, then enters the corresponding value of $\pi\alpha$ on the rule by setting the slide to it. The intersection of the slide line and the $\tan \beta$ trace then gives the corresponding value of β/π on the other scale. This gives the forward fraction F. The Rossi equation for the beam motion is then readily solved by means of

$$A = f_R (\text{HZ}) S_D$$

$$B = \frac{A}{2\pi f_R (\text{HZ}) \cos \beta}.$$

It is not practical to use this method for the Creep presentations, however, since β will correspond to an excessively large value of the tangent and the errors in the $\tan \beta$ plot will be significant.

The character of the $\tan \beta$ plot near $\beta \sim \pi/2$ illustrates an experimentally known fact. A small change in the amplitude of the Rossi signal will make a significant change in the appearance of the traced figure in a Creep presentation.

Appendix B

Additional Detector, Oscilloscope, Cable, Shield, and Filter Data

DETECTORS

Figure B-1 shows the time delay for the Gaussian input to various detectors. The sensitivity reduction factor vs alpha for various detectors is shown in Fig. B-2. Transmission of various detectors for Gaussian signals is shown in Fig. B-3. Detector characteristics are listed in Tables B-1 and B-2.

Detector Nomenclature Conversion Table

Late in 1970, the nomenclature of detectors was revised. Table B-3 is a list of the changes. The drawing numbers are EG&G numbers giving fabrication details.

Fig. B-1. Time delay for Gaussian input to various detectors.

Fig. B-2. Signal reduction factor (transmission) at constant α for various detectors.

Fig. B-3. Transmission of various detectors for Gaussian signals.

Table B-1. Detector characteristics.

Detector	Head	γ -sens	14 MeV	γ -sens	Black	Linear	Trans	Signal	Connector		Dry	Colli-	Comments
		$\left(\frac{A/\gamma \text{ MeV}}{\text{cm}^2 \text{ sec}}\right)$	n-sens $\left(\frac{A/n \text{ MeV}}{\text{cm}^2 \text{ sec}}\right)$	14 MeV n-sens	paper $\left(\frac{A/\gamma \text{ MeV}}{\text{cm}^2 \text{ sec}}\right)$				Signal	HV			
L-24	DT-1(PM)	1×10^{-13}	4.4×10^{-14}	2.3		1-1.5	18 ± 1	-	GHV(F) ^a	GHV(F)	TLP ^b	4	L-24 LOS materials 0.25 in., Al, 6 in. MEL 150C fluor or lucite, 6.25 in. Al. Lucite reduces γ -sens by 100, n-sens by 300.
	(3-7117's)	2×10^{-11}	4.4×10^{-12}										
	DT-1A(PM)	2×10^{-13}	4.4×10^{-14}	2.3		1.5-3	22 ± 1	-	GHV(F)	GHV(F)	TLP	4	
	(3-96603's)	2×10^{-11}	4.4×10^{-12}										
	DT-3A(PD)	1×10^{-19}	4.4×10^{-20}	2.3	1×10^{-20} same for lucite	20-30	1	+	GHV(F)	GHV(F)	TLP	4	FW 114A gives 4 times sensitivity
	(2 in., FW114)	1×10^{-17}	4.4×10^{-18}										
	DT-4A(PD)	1×10^{-18}	4.4×10^{-19}	2.3	3×10^{-19} same for lucite	40-70	1	+	GHV(F)	GHV(F)	TLP	4	DT-14 head uses HV cable as capacitor
	(5 in., FW-127)	4×10^{-16}	4.4×10^{-17}										
L-24S	DT-11(PD)	1×10^{-19}				20-30	1	+	GHV(F)	GHV(F)	TLP	2	L-24S LOS materials, 0.25 Al, 3.5 in. MEL 150C or lucite 0.25 Al. Lucite reduces γ -sens by 100. DT-13 head uses cable as capacitor.
	(2 in., FW114)	1×10^{-17}											
NPM-20	1-7117 per channel	5×10^{-14}	Variable			0.2	~ 18	-	GHV(F)	GHV(F)	TLP	none	Sensitivity varied by applied bias.
RG-213		$\sim 10^{-21}$ $\left(\frac{A/\text{meter}}{\gamma \text{ MeV}}\right)$ $\left(\frac{A/\text{meter}}{\gamma \text{ MeV}}\right)$ $\text{cm}^2 \text{ sec}$											
7/8 FF		$\sim 10^{-20}$ $\left(\frac{A/\text{meter}}{\gamma \text{ MeV}}\right)$ $\left(\frac{A/\text{meter}}{\gamma \text{ MeV}}\right)$ $\text{cm}^2 \text{ sec}$											

^aGremar high voltage (female).^bTest light pulser.

Table B-2. Detector characteristics.

Detector	γ -sens (A/ γ MeV) (cm ² sec)	14 MeV n-sens (A/n MeV) (cm ² sec)	γ MeV sens 14 n-sens	Linear current (A)	Trans time (nsec)	Signal polar- ity γ n	Connector			Dry run system	Colli- mation (in.)	Comments
							Signal	Dry run	Vac. ion			
L-22	5×10^{-20} (1 arm)	8×10^{-22} (1 arm)	63	1-200	1	+	ST(F) ^a	ST(F) or GHV(F) ^c	GHV(M) ^b	Second arm	4	L-22 LOS materials, 1/16 in. Al, 1/4 in. Fe, 1/16 in. Al. Comes with arm(s) or 6 ft 8 in. (10 nsec) RG-214 pigtail(s), second arm gives 1/2 sensitivity.
L-25A	6×10^{-20}	2×10^{-22}	300	1-200	1	+	GHV(M)	GHV(M)	GHV(M)	Second arm	4	L-25, 125 series LOS materials, 1/16 in. Ag, 1/16 in. Ta, 1/8 in. Al, 1/16 in. Al. Other L-22 comments apply.
L-125	6×10^{-20}	1.2×10^{-21}	50	1-200	1	+	GHV(M)	None	None	None	4	ITT ^d sealed L-25A. Comes with 6 ft 6 in. (10 nsec) RG-214 pig- tail.
L-125A	3×10^{-20}	1×10^{-22}	300	1-200	1	+	GHV(M)	GHV(M)	None	Second arm	4	ITT sealed dual arm L-25A. Comes with 6 ft 6 in. (10 nsec) RG-214 pigtails.
L-25S	1.5×10^{-20}										2	
ON-10-L	5.7×10^{-20}			~100	1	-	GHV(F)		None		4	LOS materials, 1/8 in. Al, 2 in. Ni, 1/2 in. Al, 2-1/2 in. polybutene oil.
ON-11-L	1.7×10^{-20}			~100	1	-	GHV(F)		None		4	LOS materials, 1/8 in. Al, 1/4 in. Ni, 1/8 in. Al, 1.119 in. poly- butene 32X.
VCD 131	3×10^{-21}				1	+	GHV(M)	None	None	None	7/8	Same LOS materials as L-25, L-125
VCD 130	1.5×10^{-21}				1	+	GHV(M)	GHV(M)	None	Second arm	7/8	Same as VCD 131 with second arm.

^aStoddart Tamar (female).^bGremer high voltage (male).^cGremer high voltage (female).^dInternational telephone and telegraph.

Table B-3. Detector nomenclature conversion table.

Old title	New title	Drawing No.
L-24 (CD-12)	MHD-24	NF-2075
L-24S (CD-4)	MHD-22	ND-2826
L-26	PDD-26	ND-4467
N-PD-56	PDD-56	NF-1989
N-PM-17	PMD-17	NF-2132
N-PM-18	PMD-18	NF-2138
N-PM-20A	PMD-20	NF-2137
N-PM-42	PMD-42	NF-2087
DT-1 (DT-33)	PMH-1	NF-2072
DT-1A (DT-34)	PMH-3	NF-2073
DT-3D	PDH-3	NF-2070
DT-30	PDH-4	NF-2071
DT-11	PDH-11	NF-2136
DT-13	PDH-13	NF-2135
DT-14	PDH-14	NF-1279
DT-4	PDH-4	NF-2071
Solid State	SSD-1	NC-2293
L-125	VCD-125	NF-50293
L-125A	VCD-126	NF-50292
X-RD-7	XRD-7	ND-4659
X-RD-9	XRD-9	ND-4539
N-LP-9	DLP-9	ND-2650
N-LP-11	DLP-11	ND-3481
N-LP-12	DFO-12	ND-2680
N-LP-13	DFO-13	ND-2724
N-LP-14	DFO-14	ND-2725
N-LP-19	DFO-19	ND-3164
N-LP-40	DFO-40	ND-4519
N-LP-41	DFO-41	ND-4531
N-LP-18	DFO-18	ND-3020
N-LP-28	DLP-28	ND-2689
N-LP-29	DLP-29	ND-4512
N-CB-16	DCC-16	ND-4547
N-CB-17	DCC-17	ND-4703

DIAGNOSTIC INSTRUMENTATION OSCILLOSCOPE SPECIFICATIONS

Table B-4 lists the diagnostic instrumentation oscilloscope specifications.

Table B-4. Diagnostic instrumentation oscilloscope specifications.

Type oscilloscope	RM507	RM517	EG&G 3344	EG&G 3445	546		L12
					(Ampli- fied, type 1A2)	(Direct access)	
CRT type	507	517	<u>KR-3M</u> KR-22	KR-1M			
Bandpass (MHz) (3 db down)	~400	~300	~2.5 K	~1.0 K	50	~250	~275
Frequency for 10% reduction in sensitivity (MHz)	~350	~270	~600	~500	~30	—	~140
α (gen/ μ sec) for 10% reduction in sensitivity	~250	~200	~500	~500	—	—	~100
Maximum vertical de- flection (cm)	6	4	1.2	5.4	6	6	4 cm/ beam with 2 cm overlap
Useful vertical deflection (cm)	5.6	3.2	1.0	4.2	5.5	5.5	4 cm/ beam with 2 cm overlap
Vertical sensi- tivity (V/cm)	50	17	10	85	50 m-20	$\frac{15}{45}$	17
Trace width (in.)	0.008- 0.012	0.008- 0.012	0.002- 0.003	0.004- 0.006	0.008	0.008	0.008- 0.012
Vertical field (TW)	120-190	120-190	130-200	330-490	~300	~300	120-190
Sensitivity (V/TW)	1.0- 1.5	0.35- 0.52	0.6- 0.08	0.86- 1.3	1.0 m- 0.4	$\frac{0.3}{0.9}$	0.35- 0.52
<u>Minimum readable voltage</u>							
10 TW creep	15 V	5 V	0.6 V	13 V	10 mV- 4 V	$\frac{3 \text{ V}}{9 \text{ V}}$	5 V
3 TW dot	4.5 V	1.5 V	0.18 V	4 V	3 mV- 1.2 V	$\frac{0.9 \text{ V}}{0.27 \text{ V}}$	1.5 V
0.5 TW lin	0.7 V	0.25 V	0.03 V	0.7 V	0.5 mV- 0.2 V	$\frac{0.15 \text{ V}}{0.45 \text{ V}}$	0.25 V
CRT writing speed (maxi- mum) TW/sec	5×10^{10}	5×10^{10}	$\frac{3 \times 10^{11}}{1 \times 10^{12}}$	2×10^{11}	1×10^{11}	1×10^{11}	5×10^{10}
Horizontal scan (cm)	10	8	1.5	8	8	8	7
Horizontal sweep sensitivity (V/cm)	70	70	55	115	40	40	70

CABLES

Cable Transmission Curves for Constant Alpha

Cable transmission curves for constant alpha are shown in Figs. B-4, B-5, and B-6.

Cable τ Nomogram and Associated Data for Various Cables

Table B-5 lists characteristics of commonly used cables. An approximate relationship for the τ of a cable exists in the form $\sqrt{\tau} = A \times L \times 10^{-7} \text{ (sec)}^{1/2}$ where L is the length of the cable in feet, and A is approximately constant over a limited frequency range. The response function of two ideal lossless cables with different τ connected in series has the same form as that for a single cable with an effective τ given by

$$\tau = \left(\sum_{i=1}^2 \sqrt{\tau_i} \right)^2$$

where the τ_i are the values of τ for the component sections. The table lists A values for three frequencies. The nomogram (Fig. B-7) is used by connecting the proper A value and cable length with a straight edge. The $\sqrt{\tau}$ or τ is read where the straight edge crosses the center scale.

The nomogram may be constructed following the instructions given below:

Initial Scale Layout

Draw three equally spaced, equally long parallel lines to serve as the scales of the nomogram as in Fig. B-8. All scales will be logarithmic. The A and length scales will each contain two decades. The A scale should run from 0.03 to 3 as shown. In the figure, we will let the length of one decade on the A scale serve as the standard of length denoted by L. The length scale should also contain two decades and for convenience run from 80 to 8000.

Calibration of the τ and $\sqrt{\tau}$ Scales

Draw construction lines (1) and (2). The portion of the τ scale between points (a) and (b) cut off by the construction lines divides off two decades on the τ scale and should physically be of length L/2. Bisecting this segment (a) - (b) at (c) produces the length of one decade on the τ scale, which should physically be L/4 long. The $\sqrt{\tau}$ scale may then be completed, filling in decades of physical length L/2. Notice that the τ and $\sqrt{\tau}$ scales also serve as a handy device for estimating square roots of numbers.

Fig. B-4. Coaxial cable transmission for constant α signals.

Fig. B-5. Coaxial cable transmission for constant α signals.

Fig. B-6. Coaxial cable transmission for constant α signals.

Table B-5. Coaxial cable characteristics.

Type	Size	Impedance	Conductor	"A" 10 MHz	"A" 100 MHz	"A" 1 GHz
RF-1	—	50	Cu-Cu	0.565	0.617	0.822
RG-213	—	50	Cu-Cu	0.565	0.617	0.822
Foamflex	1/2	100	Cu-Al	0.282	0.304	0.365
	7/8	100	Cu-Al	0.164	0.182	0.236
	1/2	50	Cu-Al	0.257	0.279	0.340
	7/8	50	Cu-Al	0.154	0.169	0.229
Styroflex	7/8	100	Cu-Al	0.144	0.153	0.175
	7/8	50	Cu-Al	0.144	0.143	0.164
	1-5/8	50	Cu-Al	0.0740	0.0747	0.0914
	3-1/8	50	Cu-Al	0.0380	0.0390	0.0534
Foam Heliax	1/2	50	Cu-Al	0.277	0.305	0.3698
	7/8	50	Cu-Al	0.154	0.163	0.216
	1-5/8	50	Cu-Al	0.0976	0.111	0.159
	1/2	50	Cu-Cu	0.2465	0.266	0.349
	7/8	50	Cu-Cu	0.139	0.149	0.205
	1-5/8	50	Cu-Cu	0.0842	0.0975	0.154
Air Heliax	7/8	50	Cu-Al	0.134	0.1365	0.147
	1-5/8	50	Cu-Al	0.0740	0.0747	0.0811
	7/8	50	Cu-Cu	0.119	0.130	0.1387
	1-5/8	50	Cu-Cu	0.0657	0.0682	0.0719
NHJ-5	7/8	50	Cu-Cu	0.118	0.128	0.149

Fig. B-7. Coaxial cable γ nomogram.

FIG. B-6. Construction method for cable γ nomogram.
See text for discussion.

Fig. B-9. Average α nomogram. Scales are connected with a straight edge using quantities indicated at the bottom of the figure.

ALPHA NOMOGRAM

The alpha nomogram (Fig. B-9) can be used to estimate average alpha from a Rossi display using the amplitude ratio of successive right or left turning points as shown on the diagram.

The corresponding Rossi frequency on the left hand scale is connected with the proper ratio on the right hand scale with a straight edge and the corresponding alpha value is read where the straight edge crosses the center scale. The left hand numerals of the frequency scale are associated with the left hand numerals of the alpha scale.

The nomogram may be constructed following the instructions below.

Construction of the Frequency Scale

Lay off three parallel lines equally separated from each other. Mark the top of the frequency scale at point (a) at a convenient point and label it 20 on the left, as in Fig. B-10. Mark a similar point (b) at the bottom of the line and label it 200. Use the length L between the 20 and 200 marks (one decade) as the standard of length for the rest of the nomogram. Add the points (c) and (d) and label them 50 and 100 respectively following the specified fraction of the standard length. These fractions are easily computed as follows. Since L represents one decade, its length corresponds to $\log 10 = 1$. The length from 20 to 50 is then given by $\log (50) - \log (20) = 0.39794$.

Calibration of the Alpha Scale

From the point (d) on the frequency scale, draw construction line (1) at right angles to the scale (this is convenient rather than necessary). Label the point (e) with e (the base of natural logarithms) on the ratio scale. Add construction lines (2), (3), and (4). Since the nomogram is to solve the equation

$$\alpha = f \ln (D_2/D_1),$$

when the ratio $D_2/D_1 = e$, we have $\alpha = f$. Points (f), (g), (h), and (i) thus have the same labels as points (a), (c), (d), and (b), respectively.

This calibrates the alpha scale. It remains to fill it in. Since the scale lines are equally spaced, one decade on the alpha scale (which is also logarithmic) has a length equal to half the length of a decade on the frequency scale.

Since point (h) is at the end of a decade on the alpha scale, it is convenient to measure calibration distances from this point ($\alpha = 100$). The length to the location on the scale for $\alpha = X$ from $\alpha = 100$ is one α decade length multiplied by

$$\log_{10} X - \log_{10} 100 = \log_{10} X - 2.$$

Fig. B-10. Construction method for α nomogram. See text for discussion.

The length is thus $L(\frac{1}{2} \log_{10} X - 1)$. (Negative lengths are measured upward from $\alpha = 100$ since the scale is increasing downward.) This can also be written

$$L \left(\frac{\ln X}{\ln 100} - 1 \right)$$

in terms of natural logarithms.

Calibration of the Ratio Scale

Draw the construction line (5) through points (b) and (g). Line (5) intersects the ratio scale at point (j) where

$$D_2/D_1 = e^{1/4}.$$

This calibrates the ratio scale. Note that the distance between points (b) and (c) should be equal to the distance between points (c) and (j) since the scales are equally spaced. (If this is not obvious, consider the angles θ and ϕ . In each case, the line between (g) and (i) represents half the projected lengths on the other scales.)

Since $\alpha = f \ln (D_2/D_1)$, we have $\ln \alpha = \ln f + \ln \ln (D_2/D_1)$ so that the ratio scale will be double logarithmic. It is convenient to measure distances on this scale from the value e since $\ln \ln e = 0$.

Let the length on the scale to the value $D_2/D_1 = y$ be denoted by ξ . We have $\xi = N \ln \ln y$ where N is an adjustable normalizing constant. Since at (j) we have $y = e^{1/4}$ and $\xi = L(\log 200 - \log 50) = L \log 4$ from the frequency scale, we have $L \log 4 = N \ln \ln (e^{1/4}) = -N \ln 4$ so that

$$N = -\frac{L \log 4}{\ln 4} = -\frac{L}{\ln 10}$$

and we have

$$\xi = -\frac{L}{\ln 10} \ln \ln y.$$

Distances measured upward from (e) are negative since the scale increases downward.

SHIELD AND FILTER DATA

Tables of Mean Free Paths and Transmission Factors for Neutrons and γ Rays in Various Materials

The tables are in two parts. Table B-6 gives mean free path in cm. The first column gives the density assumed for each material in gm/cm^3 . The next three columns correspond to γ -rays of energy specified above each column. The final column corresponds to 14.1 MeV neutrons. Table B-7 allows estimates of transmission to be

Table B-6. Table of mean free paths (cm).

Material	$\rho(\text{gm/cm}^3)$	1.7 MeV γ	3.0 MeV γ	4.4 MeV γ	14.1 MeVn
Ag	10.5	2.14	2.54	2.73	3.96
Al	2.70	7.90	10.36	12.47	9.48
Au	19.3	1.08	1.22	1.20	3.14
Be	1.84	12.60	17.3	21.5	5.46
C	1.58	13.12	17.93	21.9	9.61
Graphite	2.25	9.22		15.39	6.76
CH	1.06	18.18	25.15	30.70	10.08
CH ₂	0.92	19.57	26.9	33.68	9.32
Cu	8.94	2.48	3.10	3.47	4.67
Fe	7.86	2.77	3.52	4.00	4.68
Li	0.534	45.0	61.7	77.2	14.8
LiH	0.75	27.5	37.4	46.9	8.23
LX-04	1.89	10.71	14.65	17.82	8.92
Ni	8.90	2.41	3.02	3.37	4.10
Pb	11.34	1.80	2.05	2.06	5.61
Pb-CH ₂	4.0	4.83	6.28	6.34	7.06
U-238	18.9	1.00	1.17	1.20	3.61
W	19.3	1.08	1.26	1.26	2.99

Table B-7. Table of transmission factors.

X	e^{-X}	X	e^{-X}	X	e^{-X}	X	e^{-X}
0.05	0.9512	0.60	0.5488	1.30	0.2725	2.40	0.0807
0.10	0.9048	0.65	0.5220	1.40	0.2466	2.50	0.0821
0.15	0.8607	0.70	0.4966	1.50	0.2231	2.60	0.0743
0.20	0.8187	0.75	0.4724	1.60	0.2019	2.70	0.0672
0.25	0.7788	0.80	0.4493	1.70	0.1827	2.80	0.0608
0.30	0.7408	0.85	0.4274	1.80	0.1653	2.90	0.0550
0.35	0.7047	0.90	0.4066	1.90	0.1486	3.00	0.0498
0.40	0.6703	0.95	0.3867	2.00	0.1353	3.50	0.0302
0.45	0.6376	1.00	0.3679	2.10	0.1225	4.00	0.0183
0.50	0.6065	1.10	0.3329	2.20	0.1108	4.50	0.0111
0.55	0.5769	1.20	0.3012	2.30	0.1003	5.00	0.0067

made assuming exponential attenuation with depth. The transmission is of the form $T = e^{-t/\lambda}$ where t is the material thickness in cm and λ is the appropriate mean free path, also in cm.

As an example, consider the attenuation of 3.0 MeV γ -rays by 5.19 cm of Be with density 1.84 gm/cm³. The appropriate mean free path is 17.3 cm, and the transmission is 0.7408. The attenuation of compound shields may be estimated as the product of such transmissions computed for each material.

Neutron and γ -Ray Transmission Nomogram

It is sometimes necessary to approximately equalize the γ -ray and neutron transmissions of a shield or filter. This nomogram (Fig. B-11) is a useful aid in giving approximate thicknesses of Pb and CH₂ necessary for particular transmissions of 1.7 MeV or 4.4 MeV γ -rays and 14.1 MeV neutrons. It can also be used to determine the transmission of 1.7 MeV or 4.4 MeV γ -rays and 14.1 MeV neutrons through given thicknesses of Pb and CH₂. The nomogram is used by connecting the desired γ -ray and neutron transmissions with a straight edge and reading the thicknesses of Pb and CH₂ necessary to give these transmissions on the scales labeled with the appropriate γ -ray energy. We will illustrate with three examples.

- a) We require a transmission of 10^{-5} for both 14.1 MeV neutrons and 1.7 MeV γ -rays. The required thicknesses are 12.8 cm Pb and 86 cm CH₂.
- b) We require a transmission of 10^{-6} for 4.4 MeV γ -rays and 10^{-5} for 14.1 MeV neutrons. The required thicknesses are 24.4 cm Pb and 66.8 cm CH₂.
- c) We have 20 cm Pb and 80 cm CH₂. The transmissions for 1.7 MeV γ -rays, 4.4 MeV γ -rays, and 14.1 MeV neutrons are $\sim 2.51 \times 10^{-7}$, 5.65×10^{-6} , and 5.29×10^{-6} , respectively.

If the nomogram is not constructed by the proper method, small drafting errors will be magnified. The following instructions illustrate a way to construct it so that the drafting errors correspond to the worst case. The absolute precision is then improved for the rest of the nomogram.

Construction of the Initial Scales

Draw the two transmission scales of equal length and parallel so that their ends form the corners of a rectangle as in Fig. B-12. Divide each of these lines logarithmically with seven decades each, as shown. Note that the left scale increases upward while the right hand scale increases downward. This forms the basic framework for the nomogram.

Location of Zero Thickness Line

Connect the upper left and lower right corners of the transmission scales with construction line (1). Since these corners each correspond to complete transmission, the zeros of all material thickness scales will be on this line.

Fig. B-11. Neutron and γ -ray transmission nomogram. Transmission scales are connected with a straight edge and the appropriate material thicknesses are read from the inner scales.

Fig. B-12. Construction method for neutron and γ -ray transmission nomogram. See text for discussion.

Location of the Zeros of the Thickness Scales

Pb Thickness Scales

Consider a filter entirely of CH_2 , 150 cm thick. Using mean free path data, one may calculate the transmission for 14.1 MeV neutrons to be 1.023×10^{-7} . This is located at point (a), 0.9996 L from the bottom of the right hand scale, where L is the length of seven decades. The transmission for 4.4 MeV γ -rays is 1.164×10^{-2} and is

located 0.2763 L from the top of the left hand scale and is shown as point (b). Construction line (2) intersects line (1) at point (c). This is the zero point of the Pb thickness scale for 4.4 MeV γ -rays.

The transmission of the filter for 1.7 MeV γ -rays is 4.691×10^{-4} and is located at point (d), 0.4755 L from the top of the left hand scale. The intersection of construction lines (3) and (1) at point (e) is the zero point of the Pb thickness scale for 1.7 MeV γ -rays.

CH₂ Thickness Scales

Consider a filter entirely of Pb, 30 cm thick. The transmission for 14.1 MeV neutrons is 4.760×10^{-3} , located 0.3318 L from the bottom of the right hand scale at point (f). The transmission for 4.4 MeV γ -rays is 4.735×10^{-7} located 0.9035 L from the top of the left scale, at point (g). The intersection of construction lines (4) and (1) at (h) is the zero point of the CH₂ thickness scale for 4.4 MeV γ -rays.

Now consider a Pb filter, 25 cm thick. The transmission for 14.1 MeV neutrons is 1.16×10^{-2} located at point (i), 0.2765 L from the bottom of the right hand scale. The transmission for 1.7 MeV γ -rays is 9.29×10^{-7} located at point (j), 0.8817 L from the top of the left scale. The intersection of construction lines (5) and (1) at (k) is the zero point of the CH₂ thickness scale for 1.7 MeV γ -rays.

Calibration of the Thickness Scales

Pb Scales

Draw scale lines (6) and (7) parallel to the transmission scales. The intersections with lines (4) and (5), respectively, form the 30 cm point of the 4.4 MeV γ scale and the 25 cm point of the 1.7 MeV γ scale.

CH₂ Scales

Draw scale lines (8) and (9) parallel to the transmission scales. The intersections with lines (2) and (3), respectively, form the 150 cm point of the 4.4 MeV γ -ray scales. Since the thickness scales are linear, they may be filled in with a linear variable scale.

Lead and Paraffin Filter Transmission Curves for Neutrons and γ Rays

The curves given in Fig. B-13 assume purely exponential attenuation with distance in the medium. With this assumption, they are essentially as good as the mean free paths listed. They are useful for making shielding estimates and are not intended to be used for final data reduction.

SOLID ANGLE ATTENUATION CURVES

The solid angle in steradians subtended by unit area of detector face at distance R is given by

$$\Omega = \frac{1}{4\pi R^2}.$$

If Ω_0 corresponds to unit radius $R_0 = 1$ in the chosen units of length, then $\Omega_0 = 1/4\pi$ and we may write

$$\ln \left(\frac{\Omega}{\Omega_0} \right) = -2 \ln \left(\frac{R}{R_0} \right)$$

or

$$\ln (4\pi\Omega) = -2 \ln R,$$

Fig. B-13. Lead and paraffin filter transmission curves for neutrons and γ -rays.

Fig. B-14. Solid angle attenuation curves.

$$\ln \Omega = -2 \ln R - \ln 4\pi.$$

The plot of Ω vs R is thus linear in a log-log plot, and the line plotted from the above equation is the same, regardless of units. The left hand scale of Fig. E-14 gives Ω in steradians if the detector face has unit area with the unit of length used for R . Eight decades of Ω are given, corresponding to four decades of R . The right hand scale gives values of $1/R^2$ in the square of the length unit used for R . This is used with the curve for $1/R^2$.

ATOMIC NUMBERS, WEIGHTS AND DENSITIES OF ELEMENTS AND MATERIALS

The data for the elements was taken from the C. R. C. Handbook of Chemistry and Physics (40th ed.) and is included here as Table E-8. Where the atomic weight is in parenthesis, it represents the most stable isotope. Densities connected by a hyphen represent a range corresponding to various allotropic forms of the material. Those separated by commas correspond to well-known forms such as in the case of phosphorus. The densities given correspond to white, red, and black phosphorus, respectively. The additional materials at the end were taken from a collection by N. Bailey and are useful for making approximations.

Table B-8. Atomic numbers, weights and densities of elements and materials (at. wt. values in parenthesis correspond to the most stable isotope).

Name	Symbol	At. No.	International at. wt. (1959)	Density
Actinium	Ac	89	(227)	10.07 (calc)
Aluminum	Al	13	26.98	2.699
Americium	Am	95	(243)	11.7
Antimony	Sb	51	121.76	6.691
Argon (gas-STP)	Ar	18	39.944	1.7837 g/l
Arsenic	As	33	74.92	5.73
Astatine	At	85	(210)	—
Barium	Ba	56	137.36	3.50
Berkelium	Bk	97	(247)	—
Beryllium	Be	4	9.013	1.85
Bismuth	Bi	83	208.99	9.747
Boron	B	5	10.82	2.37
Bromine	Br	35	79.916	3.12
Cadmium	Cd	48	112.41	8.65
Calcium	Ca	20	40.08	1.55
Californium	Cf	98	(251)	—
Carbon (amorphous)	C	6	12.011	1.8-2.1
Cerium	Ce	58	140.13	6.67-8.23
Cesium	Cs	55	132.91	1.873
Chlorine (liq.)	Cl	17	35.457	1.56
Chromium	Cr	24	52.01	7.18-7.20
Cobalt	Co	27	58.94	8.9
Copper	Cu	29	63.54	8.96
Curium	Cm	96	(247)	~7
Dysprosium	Dy	66	162.51	8.536
Einsteinium	Es	99	(254)	—
Erbium	Er	68	167.27	9.051
Europium	Eu	63	152.0	5.259
Fermium	Fm	100	(257)	—
Fluorine (liq.)	F	9	19.00	1.108
Francium	Fr	87	(223)	—
Gadolinium	Gd	64	157.26	7.895, 7.8
Gallium	Ga	31	69.72	5.907
Germanium	Ge	32	72.60	5.323
Gold	Au	79	197.0	19.32
Hafnium	Hf	72	178.50	13.29
Helium (gas-STP)	He	2	4.003	0.177 g/l

Table B-8 (contd)

Name	Symbol	At. No.	International at. wt. (1959)	Density
Holmium	Ho	67	164.94	8.803
Hydrogen	H	1	1.0080	0.08988 g/l
Indium	In	49	114.82	7.31
Iodine (solid)	I	53	126.91	4.93
Iridium	Ir	77	192.2	22.42
Iron	Fe	26	55.85	7.874
Krypton	Kr	36	83.80	3.733 g/l
Lanthanum	La	57	138.92	5.98-6.186
Lead	Pb	82	207.21	11.35
Lithium	Li	3	6.940	0.534
Lutecium	Lu	71	174.99	9.872
Magnesium	Mg	12	24.32	1.738
Manganese	Mn	25	54.94	7.21-7.44
Mendelevium	Md	101	(256)	—
Mercury	Hg	80	200.61	13.546
Molybdenum	Mo	42	95.95	10.22
Neodymium	Nd	60	144.27	6.80, 7.004
Neon (gas-STP)	Ne	10	20.183	0.89990 g/l
Neptunium	Np	93	(237)	18.0-20.45
Nickel	Ni	28	58.71	8.902
Niobium	Nb	41	92.91	8.57
Nitrogen (liq.)	N	7	14.008	0.808
Nobelium	No	102	(254)	—
Osmium	Os	76	190.2	22.57
Oxygen (liq.)	O	8	16.000	1.14
Palladium	Pd	46	106.4	12.02
Phosphorus	P	15	30.975	1.82, 2.20, 2.25-2.69
Platinum	Pt	78	195.09	21.45
Plutonium (α)	Pu	94	(244)	19.84
Polonium (α)	Po	84	(209)	9.32
Potassium	K	19	39.100	0.862
Praseodymium	Pr	59	140.92	(α)6.782, (β)6.64
Promethium	Pm	61	(145)	—
Protactinium	Pa	91	(231)	15.37 (calc)
Radium	Ra	88	(226)	5?
Radon (gas-STP)	Rn	86	(222)	9.73 g/l
Rhenium	Re	75	186.22	21.02
Rhodium	Rh	45	102.91	12.41

Table B-8 (contd)

Name	Symbol	At. No.	International at. wt. (1959)	Density
Rubidium (solid)	Rb	37	85.48	1.532
Ruthenium	Ru	44	101.1	12.41
Samarium	Sm	62	150.35	(α)7.533, (β)7.40
Scandium	Sc	21	44.96	2.992
Selenium (gray)	Se	34	78.96	4.79
Silicon	Si	14	28.09	2.33
Silver	Ag	47	107.873	10.50
Sodium	Na	11	22.991	0.971
Strontium	Sr	38	87.63	2.54
Sulfur	S	16	32.066	2.07, 1.957
Tantalum	Ta	73	180.95	16.6
Technetium	Tc	43	(97)	11.50 (calc)
Tellurium	Te	52	127.61	6.24
Terbium	Tb	65	158.93	8.272
Thallium	Tl	81	204.39	11.85
Thorium	Th	90	(232)	~11.66
Thulium	Tm	69	168.94	9.332
Tin	Sn	50	118.70	5.75, 7.31
Titanium	Ti	22	47.90	4.54
Tungsten	W	74	183.86	19.3
Uranium	U	92	238.07	~18.95
Vanadium	V	23	50.95	6.11
Xenon (gas-STP)	Xe	54	131.30	5.887 g/l
Ytterbium	Yb	70	173.04	(α)6.977, (β)6.54
Yttrium	Y	39	88.91	4.45
Zinc	Zn	30	65.38	7.133
Zirconium	Zr	40	91.22	6.53 (calc)
Air (STP)	—	7.2	28.97	1.293 g/l
CH	—	7	13.018	1.06
CH ₂	—	8	14.026	0.92
LiH	—	4	7.95	0.75
Tuff	—	—	—	1.763
Concrete	—	—	16	2.3
PEH-9404	—	~56	100	1.84
PEH-94-0	—	~51	100	1.869
Epon	—	—	18	2.3
Coral Sand	—	—	—	1.34

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Printed in U.S.A. Available from the National Technical Information Center, National Bureau of Standards, U. S. Department of Commerce, Springfield, Virginia 22151. Price: \$12.00. Stock No. 22151.

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