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SOME COMMENTS ON
THE POWER-BALANCE PARAMETERS Q AND ϵ
AS MEASURES OF PERFORMANCE
FOR FUSION POWER REACTORS

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ABSTRACT

Power-producing fusion-reactor systems may require a complex of plasma-producing subsystems (containment, heating, and injection) and of energy-conversion subsystems (thermal and direct). To establish a basis for a comparison of these reactor power systems, generalized definitions based on power balance are proposed for the two important power-balance parameters: (1) Q (ratio of output to input power), and (2) ϵ (fractional circulating power). The values of Q and ϵ determined from the proposed generalized definitions are compared with values in the literature based on different definitions.

1. INTRODUCTION

A survey of analytical studies of power-producing fusion reactors¹ indicates that some reactor systems may require a complex of plasma-producing subsystems (containment, heating, and injection) and of energy-conversion subsystems (thermal and direct). The power-balance parameters used to characterize the behavior of some of these reactor systems are Q and ϵ . However, there are significant basic differences in the definitions as given by various authors. To establish a common basis for a comparison of these complex power systems, it has been found useful to define Q and ϵ based on a power balance of the total reactor system. The definitions are developed in Sec. 2; limit tests and other considerations that demonstrate the generalized nature of these definitions are applied in Sec. 3; the Q values computed from the various definitions are compared in Sec. 4; and concluding remarks are contained in Sec. 5.

2. DEFINITIONS

The relationship between the subsystems of a power reactor system is shown schematically in the basic power-flow diagram of Fig. 1. The power-balance equation for a reactor system may be written as the product form of the ratios of power output to power input for each of the subsystems

$$\frac{P_i}{P_c} \times \frac{P_o}{P_i} \times \frac{P_g}{P_o} \times \frac{P_c}{P_g} = 1, \quad (1)$$

where

$P_i \equiv$ power input to the power-source subsystem,

$P_o \equiv$ power output from the power-source subsystem,

$P_g \equiv$ gross electrical power output from the power-conversion subsystem,

and

$P_c \equiv$ circulating electrical power to the plasma-preparation subsystem.

Fig. 1

Basic Power-flow Diagram.
ANL Neg. No. 116-1080.

In Fig. 1, the quantity $P_c - P_i$ is the power loss of the plasma-preparation subsystem having an efficiency η_i . $P_o - P_g$ is the power loss of the conversion subsystem having an efficiency η_c , and $P_g - P_c$ is the net electrical power output P_e available for external distribution. With the power multiplication factor Q of the power source subsystem defined as

$$Q = \frac{P_o}{P_i} \quad (2)$$

and the fractional circulating power ϵ defined as

$$\epsilon = \frac{P_c}{P_g}, \quad (3)$$

the power-balance relation given by Eq. 1 may be written as

$$Q = \frac{1}{\epsilon \eta_i \eta_c}, \quad (4)$$

where $\eta_i = P_i/P_c$ and $\eta_c = P_g/P_o$. For an overall system efficiency η defined as the product of η_i and η_c , Eq. 4 may be further reduced to

$$Q = \frac{1}{\epsilon \eta}. \quad (5)$$

The Q factor as defined in Eq. 2 is the ratio of the total reactor power output P_o (across interface B) to the total power input P_i (across interface A), where

$$P_o = P_f + P_i, \quad (6)$$

and where P_f is the fusion power generated in the power-source subsystem. Note that the conservation of energy requires that the output power must include the input power. The parameter Q may also be written as

$$Q = 1 + \frac{P_f}{P_i}. \quad (7)$$

As defined here, Q corresponds to the quantity $R + 1$ used by Lawson² in developing the break-even criterion for a thermonuclear reactor. The parameter ϵ may also be written in the alternative useful forms

$$\epsilon = 1 - \frac{P_e}{P_g} \quad (8)$$

and

$$\epsilon = \frac{1}{1 + P_e/P_c}. \quad (9)$$

For the power-source subsystem alone, the lower-limiting condition for $P_o = P_i$ corresponds to $Q = 1$ in Eq. 2. If P_o exceeds P_i (i.e., the subsystem is a power producer), then $Q > 1$ and the upper-limiting value $Q = \infty$ occurs if $P_i = 0$ (with P_o finite). Therefore, the Q factor assumes a range of values

$$1 \leq Q \leq \infty. \quad (10)$$

As shown in Fig. 1, P_g is the total power from the power-conversion subsystem (across interface C), and P_c is the power flow into the plasma-preparation subsystem. For a power-producing system, the range of P_c is $0 \leq P_c \leq P_g$; therefore from Eq. 3 the range of ϵ becomes

$$0 \leq \epsilon \leq 1. \quad (11)$$

The lower value corresponds to the condition that all the gross electrical power output is available for external distribution (i.e., the circulating power is zero). The upper value corresponds to a system in which the total gross electrical power output must be circulated to maintain the system (i.e., the net electrical power output is zero). If the system is not self-sustaining, the power deficiency must be made up by an external electrical power source in order to maintain operation. Under these circumstances, ϵ is greater than unity, corresponding to a negative P_e , which implies an input power from an external system. Present experimental plasma systems operate in this range.

The significance of Eq. 4 for Q is that it incorporates ϵ and the efficiencies of the various subsystems for the entire reactor system. The relation states that for a Q value determined by the characteristics of the power-source subsystem, the fractional circulating power can be reduced only by increasing the efficiencies of the plasma-preparation and/or power-conversion subsystems.

A more detailed power-flow diagram of a typical fusion-power-reactor system is shown in Fig. 2.* The complete reactor system is divided into four major subsystems: plasma preparation, power source, power conversion, and power distribution. The plasma-preparation subsystem is further subdivided into two minor subsystems, and the energy-conversion subsystem is divided into three minor subsystems.

Fig. 2. Typical Power-flow Diagram. ANL Neg. No. 116-743 Rev. 2.

*Not shown explicitly in the diagram but implicit in the efficiency parameters are the power requirements of components related to plant operation such as the vacuum system, coolant pumps, and magnet system.

The input power P_i into the power-source subsystem is the product of the circulating power P_c and the weighted efficiency η_i of the complex of the plasma-confinement and -heating subsystem and the injection-heating subsystem.

$$P_i = \eta_i P_c. \quad (12)$$

For the two plasma-preparation subsystems shown in Fig. 2,

$$\eta_i = f_H \eta_H + f_I \eta_I, \quad (13)$$

where f_H is the fraction of the circulating power necessary for the plasma-confinement and -heating subsystem having an efficiency η_H , and f_I is the fraction required to operate the injection-heating subsystem having an efficiency η_I , with $f_H + f_I = 1$.

The gross electrical power output is the sum of the power converted by the conversion subsystems and is the product of the output power P_o and the overall conversion efficiency η_c of the complex of conversion subsystems.

$$P_g = \eta_c P_o. \quad (14)$$

For the three power-conversion subsystems shown in Fig. 2,

$$\eta_c = f_T \eta_T + f_D \eta_D + f_M \eta_M, \quad (15)$$

where f_T is the fraction of the total power output P_o converted thermally with an efficiency η_T , f_D is the fraction converted by direct conversion with an efficiency of η_D , and f_M is the fraction converted by magnetic induction with an efficiency η_M , with $f_T + f_D + f_M = 1$.

The above treatment applied to the typical system of Fig. 2 is equally applicable to even more complex systems, with the overall system requirements still constrained by Eq. 4.

3. LIMIT TESTS

The power-balance relations derived from the above definitions are found to be consistent with limit tests for idealized systems. One such limit is obtained by considering an idealized reactor system in which all power-conversion and -injection subsystems are designed with 100% efficiencies; i.e., $\eta_i = \eta_c = 1$. In this limiting case, the Q factor from Eq. 4 becomes

$$\lim_{\eta_i, \eta_c \rightarrow 1} Q = \frac{1}{\epsilon}. \quad (16)$$

From Eq. 9, the Q factor in the above limit may also be written as

$$\lim_{\eta_i, \eta_c \rightarrow 1} Q = 1 + \frac{P_e}{P_c}, \quad (17)$$

which shows that for a given net electrical power output, high- Q -value reactor systems have relatively low circulating-power requirements.

For a break-even reactor system, that is, a zero net electrical power-producing system ($P_e = 0$), the critical Q factor Q_{crit} is obtained by taking the limit of Q as ϵ approaches unity,

$$\lim_{\epsilon \rightarrow 1} Q \equiv Q_{crit} = \frac{1}{\eta_i \eta_c} = \frac{1}{\eta}. \quad (18)$$

Equations 7 and 18 show the relationship between the system efficiency η and the power ratio P_f/P_i necessary to achieve a break-even system. As an example, for an overall efficiency of $\eta = 1/3$ as in the Lawson paper,² $Q_{crit} = 3$, so that Lawson's parameter $R = P_f/P_i = 2$; that is, the fusion power source must be twice the input power for a break-even condition.

For the combined limiting cases of 100% efficiencies and 100% fractional circulating power, the Q factor from Eq. 4 becomes

$$\lim_{\epsilon \rightarrow 1} \lim_{\eta_i, \eta_c \rightarrow 1} Q = 1. \quad (19)$$

This limiting Q value of unity represents a theoretical self-sustaining system in which the fusion power P_f approaches zero, and $P_i = P_o = P_g = P_c$.

4. COMPARISON WITH OTHER Q DEFINITIONS

Several other definitions of Q have been used in a variety of specific studies relating mostly to simple and minimum-B mirror machines. In a study of mirror systems incorporating direct-conversion subsystems, Post³ introduced the quantity Q_{EN} defined as

$$Q_{EN} = \frac{\text{Converted Nuclear Power Output}}{\text{Net Required Input Electrical Power}} = \frac{P_g - \eta_D P_i}{P_i/\eta_i - \eta_D P_i}, \quad (20)$$

where all terms are defined in Sec. 2. Note that the numerator of Eq. 20 represents the total converted electrical power P_g minus the direct-converted input power $\eta_D P_i$; the denominator represents the total input power into the plasma-preparation subsystem P_i/η_i minus the direct-converted input power. It is significant that the subtrahend in both the

numerator and denominator represents the main fraction of the circulating input power required to maintain the reactor system. In effect, Q_{EN} represents the Q for a subsystem of the total reactor system. As a result, Q_{EN} for the subsystem is greater than the Q given by Eq. 2 for the total system.

A consequence of the definition of Q_{EN} is that for the limiting case, in which the injection and conversion efficiencies approach unity, Q_{EN} becomes infinite independently of ϵ . This may be seen directly from Eq. 20, or by using the definitions of ϵ and P_i , given by Eqs. 3 and 12, respectively, in Eq. 20 to give

$$Q_{EN} = \frac{1 - \epsilon \eta_i \eta_D}{\epsilon(1 - \eta_i \eta_D)} \quad (21)$$

In this same limiting case, Q as written in Eq. 4 approaches the limit of $1/\epsilon$ as in Eq. 16, a result which is consistent with the basic power-flow diagram (Fig. 1) for a total reactor power system as discussed in Sec. 3. In the limit of zero net electrical power output, that is $\epsilon \rightarrow 1$, the critical Q_{EN} value becomes unity independently of subsystem efficiencies.

The definition of Q_{EN} does not provide a measure that is indicative of the total plasma-preparation power required to maintain the total reactor system operable. As a result, there is no corresponding measure of the capital investment in equipment required to handle the total circulating power, as will be discussed in Sec. 5.

The upper section of Table I lists the Q_{EN} values of Ref. 3 for the three fuel cycles (1) D-T, (2) D-D, and (3) D- ^3He , and for each the cases A and B corresponding to high and low collisional loss rates, respectively.

TABLE I. Values of Q and ϵ for Mirror Reactors

Post (Ref. 3)	D-T 300 keV		D-D 400 keV		D- ³ He 400 keV		
	A	B	A	B	A	B	
Q _{EN} (Ref. 3)	5.3	20.3	3.7	9.7	4.0	18.7	
Q (Eq. 2)	2.5	6.7	2.0	3.4	1.8	4.5	
ε	0.62	0.26	0.72	0.49	0.70	0.28	
Werner <i>et al.</i> (Ref. 4)					480 keV		
					Low Q	High Q	
Q _p (Ref. 4)						0.20	0.40
Q (Eq. 2)						1.22	1.44
ε						0.91	0.79
Cordey <i>et al.</i> (Ref. 5)		Min. B			Min. B		
		100 keV	300 keV	500 keV	500 keV		
Q _p (Ref. 5)	2.41	1.82	1.43	0.41			
Q (Eq. 2)	3.40	2.82	2.43	1.41			
ε	0.50	0.58	0.65	0.88			

Also listed are the values of Q determined from Eq. 2 using the data of Ref. 3. The corresponding values of ϵ computed from Eq. 3 have also been listed in Table I. The values of Q_{EN} are found to exceed Q by factors in the approximate range of 2-4.

In an earlier survey study of simple mirror machines, Post⁶ introduced the quantity Q defined as the ratio of total nuclear power released to the injection-particle energy escaping,*

$$Q_P = \frac{\text{Reactor Nuclear Power Output}}{\text{Total Reactor Power Input}} = \frac{P_f}{P_i} \quad (22)$$

where P_f and P_i are defined in Sec. 2. This definition involves only part of the power flow across interface B of Fig. 2, and consequently Q from Eq. 7 differs from Q_P by an additive constant of unity. In terms of ϵ and subsystem efficiencies, Eq. 22 becomes

$$Q_P = \frac{1 - \epsilon\eta_i\eta_c}{\epsilon\eta_i\eta_c} \quad (23)$$

The above definition of Q was used in a study of minimum-B mirror machines by Fowler and Rankin⁷ and in a review paper by Fowler.⁸ Most recently, this definition was used as the figure of merit in the mirror reactor power system studies by Werner *et al.*,⁴ by Cordey *et al.*,⁵ and in a slightly modified form by Sweetman.⁹

The difference between the Q_P of Eq. 22 and the Q of Eq. 2 is that the injection power, P_i , is not included in the accounting for the total power output; Q_P corresponds to Lawson's factor R . A consequence of the above definition is that in the limiting case of 100% injection and conversion efficiencies, Q_P approaches the value $(1 - \epsilon)/\epsilon$ which is equivalent to the ratio of the net electrical power output to the circulating power. Although this may appear not to be inconsistent with the corresponding limiting value $Q = 1/\epsilon$ in Eq. 16, it does not represent the total power flow across interface B between the power source and the power-conversion subsystems of Fig. 1 or 2.

In the limiting case of zero net electrical power output, $\epsilon \rightarrow 1$, the critical Q_P approaches $(1 - \eta)/\eta$, rather than $1/\eta$ as in Eq. 18. For the combined limiting case of $\epsilon \rightarrow 1$ and $\eta \rightarrow 1$, a self-circulating power balance (that is, reactor output power equal to reactor input power) is more appropriately described by a Q value of unity, whereas the Q_P approaches zero.

Table I lists the Q_P values reported in the studies of Werner *et al.*,⁴ and Cordey *et al.*,⁵ using the definition given by Eq. 22. Values of Q

*The notation Q_P is introduced to distinguish this definition from the Q defined by Eq. 2.

determined from Eq. 2 are included for comparison. Also listed are values of ϵ calculated from Eq. 3 using data in the references cited.

5. REMARKS

The trend noted in Table I (from Ref. 5) for the D-T fuel cycle is that Q and ϵ reflect the increase in circulating-power requirements as the initial plasma-temperature conditions are increased. The Ref. 3 studies also indicate that a reduction in the end losses of a mirror machine (case B versus case A) results in higher- Q -value reactor systems with corresponding reduction in circulating-power requirements. The operating and capital costs of the reactor systems should be expected to relate to the power parameters. To a first approximation, total power costs can be assumed to scale directly with the gross electrical power output P_g . Consequently, the normalized capital costs are proportional to the normalized power P_g/P_e and in turn are related to ϵ through Eq. 8 as

$$\text{Normalized Capital Costs} \propto \frac{P_g}{P_e} = \frac{1}{1 - \epsilon}.$$

Therefore, high capital costs are to be expected for those systems having high fractional circulating-power requirements.

The definitions of the power-balance parameters Q and ϵ presented in this report are applicable to complex systems involving any combination of energy conversion, plasma formation, and injection devices, and can be used as measures of performance common to all fusion-reactor systems, including closed magnetic systems or open-ended magnetic systems. Some typical values of Q and ϵ listed in the survey¹ for closed systems are as follows: stellarators, $Q \approx 13$, $\epsilon = 0.22$; tokamaks, $7 \leq Q \leq 100$, $0.03 \leq \epsilon \leq 0.3$. Typical values for open-ended systems other than the mirror machine are as follows: θ -pinch, $Q \approx 4.4$, $\epsilon = 0.36$; astron $Q \approx 8.5$, $\epsilon = 0.2$.

In summary, the power-balance parameters defined in this study serve two purposes: (1) to establish a common basis for comparison of the fundamentally different approaches to a fusion-power-reactor system (including fusion by laser ignition); and (2) to serve eventually as feedback information to help orient and guide research and development efforts toward optimum total systems within each basic concept and for different fuel cycles.

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