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TITLE
Parasitic Radial Magnetic Forces
in the CRU-V Alternator

S/A NO 2209

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STATEMENT OF PROBLEM

Determine the resultant force on the CRU shaft due to mag-
netic pull between the rotor and stator under conditions
of eccentricity between rotor and stator centers.

ABSTRACT

The exact conditions of the problem are: (1) alternator
operating under no load, or the armature reaction field
has negligible effect; (2) the rotor is initially magne-
tized as a uniformly sinusoidal space distribution; and
(3) the eccentricity takes the form of the stator and
rotor centers not concentric, otherwise, perfect geometry.

The net force on the shaft is in the direction of the
point of minimum air gap. This force has a constant com-
ponent and a superimposed varying component of twice the
machine frequency. This varying component, however, is
negligibly small and can be ignored.

When appropriate allowances are made for leakage flux,
almost exact correlation is obtained between measured
static forces and the calculated net forces for most
values of eccentricity.

The estimated leakage reluctance for the path corresponding
to maximum flux density is only 3.7 times the uniform gap
reluctance at that point emphasizing the fact that rotor
leakage flux in the machine is quite significant.

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Introduction

The CRU-V shaft and bearings are subject to a number of forces. One of these is due to magnetic pull between the alternator stator and the permanent magnet rotor. If perfect geometric symmetry existed and the permanent magnet rotor was magnetized uniformly for all poles, the net force on the shaft would be zero. Nevertheless, even under these ideal conditions, vibration will be induced in the stator and rotor. In practice, the ideal conditions cannot be attained, and even if they could be, the high speed rotating shaft bends and so disturbs any geometric symmetry that exists when the shaft is at rest. In addition, the magnetization of the rotor is not a perfect sinusoid around the periphery, and the magnetic circuit is not perfectly linear so that when the alternator works under load, the resultant flux density distribution in the gap is a number of rotating harmonic fields varying in speed and direction. In general, this will induce a net force on the shaft in the form of an amplitude modulated rotating force vector.

Reference 5 is a discussion of the general problem of calculating the parasitic radial magnetic forces. A complete study of the problem necessitates the consideration of a number of cases consisting of various combinations of magnetic and geometric dissymmetries. In any case, the main problem is the description of the air gap flux density distribution as a function of space angle and time. This is made difficult by the presence of significant leakage fluxes.

Data taken at TRW (see References 3 and 7) give the static net force on the rotor as a function of eccentricity (distance between rotor and stator centers). However, no data is available for the dynamic case. The purpose of this paper is to predict, through analysis, the magnetic forces on the shaft under dynamic conditions.

Plan

The approach taken in the analysis is suggested by Covo (Reference 1), who calculated the radial force for the induction motor operating under no-load. Although the results of this paper are not directly applicable to permanent magnet machines, the general method and approach to a solution is useful.

The exact problem to be considered here corresponds to Case C of Reference 5. The rotor is perfectly magnetized and perfectly centered about the shaft. The stator is perfectly symmetrical about its center. However, the rotor and stator centers are not concentric (see Figure 1). There are no slots in the stator and leakage fluxes are ignored. In addition, the generator operates at no load (at least initially) so there is no armature reaction.

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There are three reasons for choosing this particular case. First, it is less complex than some other cases that could have been chosen and; therefore, there is a reasonable chance of solution. Second, and most important, it is expected that the direct or steady component of the force should correspond to the static measurements made by TRW and; therefore, the correlations of calculated and measured results can provide a basis for testing the assumptions made in the analysis. Thirdly, if the results are satisfactory, the analysis can later be extended to other cases if desired; cases which are more complex extensions of this one to be considered.

The general procedure of solution is the following steps:

1. Derive expressions for MMF drops across the gap and in the magnet in terms of gap flux density.
2. Solve for flux density.
3. Substitute expression for flux density into the force equation; all other parameters are known.

Summary and Conclusions

The net force on the shaft is calculated as a function of eccentricity between the rotor and stator centers. Initially, leakage fluxes are ignored, and the result consists of a force vector in the direction of the point of minimum air gap. The force is dependent on the instantaneous angle between the nearest magnetic pole center and the direction of the force vector; and it can be broken up into a constant component and a superimposed varying component. The frequency of the varying component is shown to be double the machine frequency. The varying component, however, is also shown to be negligibly small and can be ignored. Nevertheless, the constant component, which is expected to equal the measured static force turns out to be approximately six times smaller than the measured value for almost all values of eccentricity. It is concluded that this is due to the ignoring of leakage.

At this point, it is decided that since leakage is unknown and since the constant force vector should equal the measured value, the problem be essentially reversed. The leakage reluctance is determined by setting the constant component of the force equal to the measured value for a given eccentricity. Then, with this value of leakage reluctance, the total force (both the constant and varying components) is calculated for all eccentricities.

The result shows very close correlation between the constant components and corresponding measured values. In addition, the varying components are shown to be negligibly small.

The effective leakage reluctance at the point of maximum flux density (pole center) is calculated to be 3.7 times the gap reluctance for an uniform gap. This corroborates the fact that leakage fluxes in the CRU-V alternator are significant.

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The analysis is extended qualitatively to the case where the rotor center has an orbit about the stator center. For a given eccentricity, this results essentially in a rotating force vector of constant magnitude.

Because of the inherent self-regulation of the machine, full-load operation (maximum armature reaction) is not expected to produce a significant varying force component. At an eccentricity ratio of 0.5, the peak-to-peak variation is only 0.0031 percent of the constant component (21.5 pounds). This means that a 1000/1 increase in the variational component would be needed to produce a peak-to-peak variation of 0.67 pounds. Since the voltage regulation holds the normal flux change to +5 percent of no-load value, it is not expected that the varying component of the force under full-load conditions need be considered.

The analysis has shown that as leakage fluxes increase, the radial forces increase. This is due to the detrimental effect which leakage has on the inherent self-regulation of the permanent magnet.

Execution

In Figure 1, consider the closed path 1-2-3-4-1 about which the total magnetomotive force must equal zero.

$$F_{\theta} = F_{G\theta} + F_M + F_I + F_{12}$$

$$F_{\theta} = \text{total MMF}$$

$$F_{G\theta} = \text{gap MMF at angle } \theta$$

$$F_I = \text{MMF drop in stator iron}$$

$$F_M = \text{MMF drop in magnet iron}$$

$$F_{12} = \text{drop across path 1-2, the interface between adjacent poles. If the geometry was concentric, } F_{12} = 0, \text{ since the flux density midway between adjacent poles would be zero.}$$

Assumption 1 The MMF drop in the stator iron is small compared to the total since the flux density level is well below saturation. Henceforth, F_I is neglected.

Therefore, we have:

$$(1) \quad F_{\theta} = F_{G\theta} + F_M + F_{12}$$

$F_{G\theta}$ can be obtained by noting that in the differential area (dA):

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$$F_{G\theta} = R_{G\theta} d\phi \text{ where } R_{G(\theta)} = \text{gap reluctance at angle}$$

$$R_{G\theta} = \frac{g}{\mu_A (dA)} \text{ where } g = \text{gap length at angle}$$
$$\mu_A = \text{permeability of air} \approx \mu_0$$
$$\mu_0 = 3.19 \frac{\text{lines}}{\text{ampere-inch}}$$

$$d\phi_{\theta} = B_{\theta} dA \text{ where } B_{\theta} = \text{flux density at angle}$$

Therefore:

$$(2) F_{G\theta} = 0.313 g B_{\theta}$$

Next, F_{θ} will be determined. To do this, certain preliminaries are necessary. So, turn now to a consideration of the perfectly concentric configuration of Figure 2. From symmetrical considerations of the magnetic field, it is reasonable to assume the flux path at the point of maximum flux density to be as shown. If Ampere's Law is applied along this path, the result is:

$$\oint \vec{H} \cdot d\vec{\ell} = NI = 0$$

$$2 H_g g_N + 2 H_M r = 0$$

$$\text{where } H_g = \text{magnetic field strength in the gap}$$

$$H_M = \text{magnetic field strength at the terminals of the magnet}$$

$$r = \text{radius of the rotor}$$

$$g_N = \text{length of the even air gap}$$

If the vector $\vec{H}_M$ is defined opposite in direction to $\vec{H}_g$, then the following can be written:

$$H_M = \frac{H_g g_N}{r}$$

Since $B_{\max}$ is a known quantity (34300 lines/in²), using the relationship:

$$B_{\max} = \mu_0 H_g \text{ yields}$$

(3)

$$\frac{B_{\max}}{H_M} = \frac{\mu_0 r}{g_N} = \frac{3.19 \times 1.0625}{.015} = 225.96$$

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Equation (3) can be plotted on the B-H characteristic for Alnico V. In addition, the line of return can be drawn. The line of return is part of a minor hysteresis loop on which the stabilized magnet operates. To a good approximation, this is drawn parallel to the tangent of the major loop characteristic at $H = 0$. The line is drawn such that it intersects the air gap line at the operating point ($B = 34300$). (See Figure 3.)

As will be seen later, it is more convenient to work on the B-F plane. Since (F) is a linear function of (H), the transformation constant from H to F is needed. This is obtained from Equation (2) which defines the air gap line on the B-F plane. In the concentric case under consideration, $g = 0.015$ inches and; therefore, (F_A) the corresponding point to H_A in Figure 3 is determined. The transformation then is:

$$F = \frac{F_A}{H_A} H = 1.0625 H$$

The B-F plane is shown in Figure 4. The line of return is continued to its intersection with the (F) axis. It is dotted because, physically, it is unrealizable for the magnet to operate outside the major loop. Nevertheless, this is a useful representation, because the magnet can now be thought of as possessing an internal MMF generator (F_1) and an internal reluctance which results in (F_A) being present at the terminals of the magnet (and across the air gap). Since the original path of integration was the one that corresponded to maximum flux density, (F_1) can be thought of as $F_{\theta \max}$. At any other angle, the path on Figure 2 would correspond to a point of lower flux density. This would generate a line of return of the same slope, but intersecting the air gap line at the appropriately lower flux density and, hence, giving a smaller (in magnitude) F_1 .

In summary, the assumption is made that $F_{\theta \max}$ is the peak value of a sinusoidally varying (with angle) MMF generator, which, in turn, yields the sinusoidally distributed flux density. Referring to the nomenclature of Figure 1:

$$(4) \quad F_{\theta} = F_{\theta \max} \cos \frac{p}{2} (\alpha + \theta)$$

where p = number of poles

Assumption 2 The so-called internal MMF generator for the PM rotor varies sinusoidally with angle according to Equation (4), above, where $F_{\theta \max}$ is determined according to the preceding described method. Furthermore, this internal MMF distribution is assumed to remain the same for the eccentric rotor.

Figure 4 also shows that the internal MMF drop ($F_1 - F_A$) can be expressed as:

$$5) \quad F_{M_d} = S_R B_{\theta}$$

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where $\frac{1}{S_R}$ = slope of line of return

F_{M_0} = internal MMF drop in the magnet

Assumption 3 The internal MMF drop in the PM rotor remains equal to

$$[F_{M_0} = S_R B_0]$$

for the eccentric rotor as well as the concentric one.

Eccentric Rotor

Using the assumptions previously made, Equations (2, 4 and 5) are now substituted into Equation (1).

$$F_{\theta_{max}} \cos \frac{P}{2} (\alpha + \theta) = 0.313 g B_0 + S_R B_0 + F_{12}$$

In addition, the even air gap length (g_N) is very small compared to the mean diameter of the gap, so (g) can be written as:

$$g = g_N - \delta \cos \theta = g_N (1 - e \cos \theta)$$

where δ = eccentricity as shown in Figure 1

$$e = \frac{\delta}{g_N} = \text{eccentricity ratio}$$

Substitution into the preceding equation and solution for B_0 yields:

$$(6) \quad B_0 = \frac{F_{\theta_{max}} \cos \frac{P}{2} (\alpha + \theta) - F_{12}}{S_R + 0.313 g_N (1 - e \cos \theta)}$$

From examination and study of Reference 1, one can conclude that for eccentricity ratios of 0.5 or less, F_{12} is small enough to be neglected in a six-pole machine. In addition, it is considered unlikely that the eccentricity ratio could ever get worse than 0.5 in CRU-V.

Assumption 4 Eccentricity ratios of 0.5 or less will be considered and, therefore, F_{12} can be neglected.

From Figure 4, $F_{\theta_{max}} = 3900$ Amp-Turns
 $S_R = 0.1090$

Also, the number of poles is six and $g_N = 0.015$ inches. Using these results in Equation (6) gives:

$$(7) \quad B_0 = \frac{3900 \cos 3 (\alpha + \theta)}{0.1090 + .0047 (1 - e \cos \theta)}$$

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Equation (7) gives the expression for flux density as a function of angle from the X-axis (θ) with parameters eccentricity (e) and angle between the X-axis and the center of the magnetic pole (α). This equation is valid under these restated assumptions:

1. There is no-load on the generator, or the effect of the armature reaction field is zero.
2. There is no saturation of the stator iron and, hence, the MMF drop in this region is neglected.
3. Leakage fluxes are neglected (particularly those paths through the permeable shrink ring).
4. The normal sinusoidal distribution of MMF for the equivalent magnet source is maintained when the rotor becomes eccentric.
5. The internal MMF drop in the magnet remains the same linear function of flux density for both the concentric and eccentric case.
6. Only eccentricity ratios of 0.5 or less are considered, allowing the simplification which results in Equation (7).

At any point in the air gap, the radial force of attraction (from Reference 2) is:

$$(8) \quad P = 1.385 \times 10^{-8} B_o^2 \frac{\text{lbs}}{\text{in}^2}$$

P = pressure

B_o = flux density at the point

The force acting on a differential area in the gap is:

$$dF = P dA$$

$$dA = r l d\theta$$

where l = axial length of the machine

The net forces in the (x) and (Y) direction (see Figure 1) are then:

$$F_x = 1.385 \times 10^{-8} r l \int_0^{2\pi} B_o^2 \cos \theta d\theta$$

$$F_y = 1.385 \times 10^{-8} r l \int_0^{2\pi} B_o^2 \sin \theta d\theta$$

using $r = 1.0625$ inches

$l = 3.25$ inches

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and substituting Equation (7) into these expressions gives:

$$(9) \quad F_x = 0.7274 \int_0^{2\pi} \frac{\cos^2 3(\alpha + \theta)}{[0.109 + .0047 (1 - e \cos \theta)]^2} \cos \theta d\theta \quad \text{lbs}$$

$$(10) \quad F_y = 0.7274 \int_0^{2\pi} \frac{\cos^2 3(\alpha + \theta)}{[0.109 + .0047 (1 - e \cos \theta)]^2} \sin \theta d\theta \quad \text{lbs}$$

Inspection of Equation (10) shows that $F_y = 0$. This is due to the fact that an even function multiplied by an odd function is integrated over one period. This result is corroborated by observing the cancelling type symmetry in the (Y) direction in Figure 1. Therefore, the resultant force is in the X-direction, by Equation (9). Expand the denominator of Equation (9).

$$(11) \quad F_x = 0.7274 \int_0^{2\pi} \frac{\cos^2 3(\alpha + \theta)}{[0.118 + .0010 (1 - e \cos \theta) + 2.2 \times 10^{-5} (1 - e \cos \theta)^2]} \cos \theta d\theta$$

The third term in the denominator is a small fraction of 1 percent of the first term, so neglect this term. The result is:

$$(12) \quad F = F_x = 0.7274 \int_0^{2\pi} \frac{\cos^2 3(\alpha + \theta) \cos \theta}{1.188 \times 10^{-2} + 10^{-3} (1 - e \cos \theta)} d\theta$$

This integral will result in the net force as a function of:

1. Eccentricity ratio (e).
2. Angle between the center of the pole and the X-axis (α).

Solution of Equation (12) gives the results of Figure 5. These results are shown for the case ($\alpha = 0$). Notice from Equation (11) that because of the $\cos^2$ term, the solution will be in terms of (6α) whereas the flux density is proportional to (3α) (see Equation 7). Since (α) depends on rotor position, it can be seen that the force varies at double the machine frequency. As (α) is varied, the results do not change significantly from those of Figure 5. In fact, over the whole range of (α), the force only changes

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the fourth decimal place (e.g., for $e = 0.5$, F_x varies from 3.4485 to 3.4484 or 0.003 percent). The varying component is, therefore, negligible compared to the direct component; and the results of Figure 5 can be considered the direct component and independent of (α).

Figure 5, nevertheless, shows a major discrepancy between the calculated and measured values for the direct forces. The measured force is described in Reference 3 and merely copied here for comparison. The measured value for any eccentricity is approximately six times the calculated value. Therefore, a review of the assumptions is in order.

The most likely assumption to be unreasonable is that leakage fluxes can be ignored. Another possibility is that the magnetic path length in Figure 2 was chosen to be too large.

From Figure 2, the minimum possible path length is the straight line (chord) drawn between the centers of adjacent poles. This happens to be 0.53 inches per pole. Computation using 0.58 inches/pole (almost the minimum) yielded a result less than twice as great as the original case. In particular:

<u>Path Length</u>	<u>Eccentricity</u>	<u>Calculated Force</u>	<u>Measured Force</u>
1.0625 inches/pole	0.5	3.45 lbs	21.25 lbs
0.5800 inches/pole	0.5	6.10 lbs	21.25 lbs

Clearly, the chosen path length has little effect on the calculated result. Referring to Equation (6) and Figure 4, the path length determines the conversion factor from the B-H plane to the B-F plane. If the path is made smaller, the effect is to make F_{max} and S_R smaller; or the operating line of the magnet (Figure 4) intersects the (F) axis at a lower value while still intersecting the air gap line at the same point. It will be seen that introduction of leakage has the same effect on the B-F plane.

Leakage Reluctance

Consider Figure 6. Part (a) is the magnetic circuit of the case with no leakage flux and can be related to the flux path of Figure 2. Part (b) introduces a leakage reluctance (R_L) and Part (c) shows the effect on the source reluctance as seen from the terminals of the air gap reluctance. If leakage reluctance is introduced, the effect, as seen from the terminals of (R_g), can only be to lower the equivalent source reluctance ($R_{\text{equiv}} < R_M$). Working on a differential basis at the point of maximum flux density as at the beginning of the analysis, this means that the value of (S_R) must decrease with the introduction of leakage reluctance.

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$$SR_{(L)} = K_{Requiv} < K_{RM}$$

where $SR_{(L)}$ is used when leakage is considered. This is due to the fact that since everything is linear, SR varies directly as the source reluctance in Figure 6-c. Returning to Figure 4 and Equation (6), the effect of less leakage reluctance reflects the in lower (SR) and, hence, the line of operation intersects the (F) axis at a lower value while still intersecting the air gap line at the same point. As (SR) is made smaller, the net force gets larger.

The procedure now will be to find the proper value of (SR) and its corresponding $F_{\theta_{max}}$ (intersection of F axis), which will give a result for the calculation equal to the measured value. Using $\epsilon = 0.5$, (SR) will be chosen such that the net force comes out to be approximately 21.25 pounds. Then, using these conditions, the results of other eccentricity ratios will be checked against the measured values. Finally, the varying component of force (effect of varying ϵ) will be determined under the new conditions. Comparison of the final value of SR and the value used initially (when leakage was ignored) should give some idea of the relative magnitude of the leakage reluctance and gap reluctance.

In Equations (6 and 7), a new value for SR is used. However, since with $\epsilon = 0$, $B_0 = 34300$, a new value for $F_{\theta_{max}}$ is determined. The resulting equation is then substituted into Equation (8), and, as before, the result is an equation of the same form as (12).

In order that the direct component of force be about equal to the measured static value:

$$SR_{(L)} = 0.015$$

$$F_{\theta_{max}(L)} = 675.71 \text{ amp-turns}$$

This gives for Equation (6):

$$(13) \quad B_0 = \frac{675.71 \cos 3(\alpha + \theta)}{0.015 + 0.0047(1 - \epsilon \cos \theta)}$$

Substitution into Equation (8) and then solution for the force component in the (X) direction gives:

$$(14) \quad F = F_x = 0.0218 \int_0^{2\pi} \frac{\cos^2 3(\alpha + \theta) \cos \theta}{\left[2.25 \times 10^{-4} + 1.41 \times 10^{-4}(1 - \epsilon \cos \theta) + 2.2 \times 10^{-5}(1 - \epsilon \cos \theta)^2 \right]} d\theta$$

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Solution of Equation (14) yields the results of Figure 7 for the case ($\alpha = 0$). Once again the alternating or varying component of the force is insignificant (force is essentially independent of α), so the force can be considered a constant vector in the X-direction. The measured values for the static case again are shown in Figure 7 for comparison. Table I, below, shows the effect of (α) on the net force.

Table I

	$F_{X_{max}} (\alpha = 0)$	$F_{X_{min}} (\alpha = 30^\circ)$	% Variation
$l = 0.5$	21.498625	21.497783	0.0031 %
$l = 1.0$	45.967301	45.937248	0.0608 %

In the analysis, the equations were based on eccentricity ratios of 0.5 or less. Correlation at ($e > 0.5$), nevertheless, turns out to be reasonable.

Comparison of Leakage and Gap
Reluctances for Uniform Gap

For the case in which leakage was ignored, $S_R = 0.109$. In matching the calculated and measured values, $S_{R(L)} = 0.015$. From the preceding discussion and Figure 6:

S_R corresponds to R_M

$S_{R(L)}$ corresponds to R_{equiv}

Therefore, from Figure 6:

$$S_{R(L)} = \frac{S_L S_R}{S_L + S_R} \quad \text{where } S_L \text{ corresponds to } R_L$$

$$S_L = \frac{S_{R(L)} S_R}{S_R - S_{R(L)}} = \frac{0.015 \times 0.109}{0.109 - 0.015} = 0.0174$$

From Figure 4:

$$S_{G_N} = 0.313 g = 0.313 \times 0.015 = 0.0047$$

where S_G corresponds to R_{G_N}

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$$\frac{S_L}{S_{GN}} = \frac{.0174}{.0047} = \frac{R_L}{R_{GN}}$$
$$\frac{R_L}{R_{GN}} = 3.7$$

Therefore, for the uniform air gap, the leakage reluctance at the point of maximum flux density is only 3.7 times the gap reluctance.

Extension to the Case of an Orbiting Shaft

Suppose now that all conditions remain the same except that the center of the shaft now has a circular orbit. This corresponds to Case D in Reference 5. In Figure 1, this case would exist if the X-axis rotated about the stator center. Careful inspection will show that since the net force vector is in the X-direction, this case gives a rotating force vector at the angular speed of the X-axis about the stator center. Since the varying component of the force is negligible, the result is a constant rotating force vector. However, if the varying component were significant, the effect of the orbiting shaft would be to change the frequency of variation. The frequency of variation is dependent on the rate of change of (α), which is referred to the X-axis. When the X-axis is stationary, it has been found that the frequency of force variation is twice the machine frequency. It follows then that if the X-axis rotates, the frequency of force variation will be twice the difference between the angular velocities of the rotor and the X-axis. There should be no effect on the magnitude of the force providing the eccentricity ratio remains constant.

Armature Reaction

The preceding analysis was based on the generator operating no-load. When the generator operates under load, the armature reaction field must be considered as a component of the gap flux density distribution.

Qualitatively speaking, when the alternator works under full load conditions (maximum armature reaction), the voltage regulation is within +5 percent of the no-load value. This voltage, in turn, is dependent on the flux. Therefore, the flux/pole cannot change more than 5 percent normally. From Table I, the varying component is only 0.0031 percent for $\epsilon = 0.5$. If the varying component grew to 1000 times larger, the peak-to-peak variation for this case would only be 0.67 pounds. It is not expected that this can occur with this kind of voltage regulation.

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List of Symbols

B_{θ}	Air gap flux density at angle (θ).
B_{max}	Maximum air gap flux density for uniform machine (34.3 kilolines/inches ²).
e	Eccentricity ratio.
F_{θ}	Total equivalent magneto-motive force at angle (θ) (source MMF).
$F_{g\theta}$	MMF across the gap at angle (θ).
F_I	MMF drop in the stator iron.
F_{12}	MMF drop across path 1, 2 in Figure 1.
F_M	MMF drop within the permanent magnet.
F_{omax}	Maximum equivalent source MMF (leakage ignored).
F_o	Equivalent source MMF (leakage considered).
F	Radial magnetic force.
$F_{x,y}$	Force in the X or Y direction.
g	Gap length
g_N	Length of uniform gap (0.015 inches).
H_g	Magnet field intensity in the gap.
H_M	Magnetic field intensity at the terminals of the magnet.
I	Current (general).
l	Length of rotor (3.25 inches).
N	Number of turns (general).
P	Magnetic pressure or stress.
P	Number of poles.
r	Radius of the rotor (1.0625 inches).

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- R_G Reluctance of the gap at angle ( ).
- R_M Internal reluctance of the magnet.
- R_L Leakage reluctance.
- R_{equiv} Equivalent source reluctance (R_M and R_L in parallel).
- S_R Ratio of the total MMF drop, with the exception of the gap MMF, to the gap flux density (see Figure 4).
- $S_{R(L)}$ The value of S_R when modified by the inclusion of leakage.
- S_G Ratio of gap MMF to gap flux density.
- α Angle between X-axis and the center of nearest pole.
- θ Angle on XY plane referenced to X-axis.
- δ Eccentricity.
- ϕ Flux.
- μ_o Permeability of a vacuum (3.19 lines/amp-inch).

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Fig. 1

The Eccentric Rotor Machine

Coordinate System (XY) fastened to stator

α = angle between X axis and center of magnetic pole at any time

θ = general angle referred to X axis

δ = eccentricity

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Fig. 2
Concentric Machine
Showing Flux Path at point of Maximum Flux Density

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Fig 4

B-F Characteristic for Line Integral of Fig 2

$F_g = \text{Air Gap MMF} = 161.28 \text{ Amp Turns}$

$F_i = \text{Equivalent MMF Source}$

$F_i - F_g = F_m = \text{MMF Drop internally in the Magnet}$

$F_i = F_{0 \text{ MAX}} = 3900 \text{ Amp-Turns}$

$S_R = 0.1090$

$$\frac{B}{F} = \frac{1}{.3139}$$

$$\text{Slope} = \frac{1}{S_R}$$

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Fig. 5

Force
(F_x)
pounds

Net Radial Force (F_x)
vs
eccentricity ratio ($\frac{\delta}{g_N}$)
for $\alpha = 0$

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eccentricity ratio

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Fig. 6
Magnetic Equivalent Circuit

- a) Leakage Reluctance Ignored
- b) Effect of Leakage Reluctance
- c) Thevenin Equivalent Circuit

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$R_{equiv.} < R_M$

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Fig 7

Net Radial Force (F_x)

vs

Eccentricity Ratio ($\frac{e}{g_H}$)for $\alpha = 0$ (Leakage Flux Accounted For)

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