ARGONNE NATIONAL LABORATORY
9700 South Cass Avenue
Argonne, Illinois 60439

RESULTS OF MONTE CARLO CALCULATIONS
FOR ZPPR ASSEMBLY 2

by

F. L. Fillmore*

Applied Physics Division

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ABSTRACT

Results are presented from two Monte Carlo calculations made with the VIM code for ZPPR Assembly 2. Distinctive features of the VIM code and its cross-section library are described. The cross-section library is based on ENDF/B Version-II data. A description of the problem model used in the calculations is given as well as information concerning computer requirements for making these calculations. The Monte Carlo results are compared with experimental results where available and with results from a two-dimensional, diffusion-theory, multigroup calculation.

I. INTRODUCTION

Although most of the analysis for the fast-breeder-reactor critical experiments being performed at the Argonne National Laboratory has been, and probably will continue to be, made with diffusion- and transport-theory codes, the VIM-I Monte Carlo code,¹ being developed at AI (Atomics International), was obtained to aid in the analysis of these experiments. This report describes calculations for ZPPR Assembly 2 made with VIM using a cross-section library based on ENDF/B Version-II data. The VIM code development at AI is continuing, and the calculations described in this report were made with a version of VIM as it existed in May 1971.

In the analysis of experiments such as ZPPR using conventional multigroup methods, numerous approximations are necessary because of the heterogeneous nature of the assembly. Further approximations are made in constructing the multigroup cross-section library used in the calculations. Since the extent to which these approximations influence the calculated results is often not reliably known, it is desirable to have an alternate method available in which the approximations are greatly reduced. The Monte Carlo method has the capability of filling this need, but certain obstacles which are related to the capabilities of present-day computers can arise in applying the method to a large assembly such as ZPPR. With a naive approach, the storage requirement for the geometrical details and the cross-section library for such a problem can easily exceed 1 million words,

unless approximations similar to the ones used with multigroup methods are made. Moreover, since it is vital for the computing speed to be as fast as practical, all quantities used frequently in the calculation must be stored in the computer core. It is also necessary to avoid frequent searches through lengthy arrays and to avoid frequent repetition of time-consuming calculations of any quantity.

The VIM code was designed with the above points in mind. Advantage is taken of the repetitive nature of the drawer loadings to minimize the storage requirements for input data. The geometry subroutine is designed specifically for the plate-type geometry of the critical assemblies being analyzed so that the computing time in this subroutine can be reduced to a minimum. (Separate subroutines for spherical and cylindrical geometries are available for use in calculations involving these geometries.) Also, the data in the cross-section library are handled in such a way as to conserve storage space in the computer and to minimize computing time.

II. DESCRIPTION OF THE VIM-I CODE

A. General Features

A brief description of certain distinctive features of VIM is given next. Reference 1 contains a discussion of the physics of the collisions and the tracking of particles. The code is intended to be more of a research tool than a production code, and it is expected that the user will adapt certain features of the code, such as the dimensions and output edits, to accommodate the problem being studied. The following discussion applies primarily to the calculations made for ZPPR Assembly 2.

Since VIM was originally coded for use on the CDC-6600 computer, it was converted to run on the IBM-360/75 at the National Reactor Testing Station, Idaho. This conversion was complicated by the short word length of the IBM equipment. Reprogramming was required in several places to get around difficulties caused by the lack of adequate numerical precision. The dimensions of the code were increased to accommodate the ZPPR Assembly 2 problem which has a 51×49 -drawer matrix, two core regions, three blanket regions, and two reflector regions. Extensive use was made of DISK storage for the permanent storage of the VIM cross-section library and of certain output results which are subjected to detailed analysis after the Monte Carlo calculation is finished.

The basic unit of geometry in VIM is the drawer, and each drawer is divided into several zones which correspond to the matrix, plates, and cans of the actual assembly. At present there is provision for six different types of drawers and 150 different types of zones. Changing the dimensions is a minor task as long as computer storage capacity is not exceeded. A

complete three-dimensional description of the assembly including plate heterogeneity can be made with only minor geometrical approximations.

B. Cross-section Library

The VIM cross-section library is based on ENDF/B Version-II data and at present contains the following 13 isotopes: ^{238}U , ^{235}U , ^{239}Pu , ^{240}Pu , ^{241}Pu , Na, C, Fe, Cr, Ni, Mo, Al, and O. About 260 K bytes of storage are presently required for the entire library, and 590 K bytes are required for the code and library as structured for ZPPR. Each isotope has its own energy grid, which is tailored to meet its particular needs. This aids in minimizing the storage requirements for the library. The number of energy points for light isotopes ranges from about 450 to 800; about 2000 points are used for ^{238}U and ^{239}Pu . Cross-section data are entered for each energy point, and provision is made in the code for interpolating between points.

Resolved resonances are described by Doppler-broadened pointwise data. Unresolved-resonance cross sections are treated by a probability table method² in which the tables are based on many ladders of resonances constructed by sampling the statistical distributions for the unresolved-resonance parameters. At each selected energy in the unresolved-resonance range, a table is prepared that gives values for σ_{tot} , σ_{el} , σ_f , and the probability that the corresponding table entries for the cross-section values represents the cross sections in the neighborhood of the selected energy. The table entries are calculated from the ladders and represent the appropriate correlated averages for the cross-section types over many resonance ladders. In each table a unique value for each reaction type is associated with each probability entry in the table. Thus, the fluctuation of σ_{el} and σ_f for a given σ_{tot} is represented as an average over the ladders from which a table is constructed. The storage requirement for the tables is modest, being at present only 1230 words for the ^{239}Pu data which have tables at 30 different energies between 300 eV and 25 keV. The other four resonance isotopes require less space. When unresolved-resonance cross sections of an isotope are needed in VIM, they are obtained by generating a random number between zero and one that is used to pick from the appropriate tables the total, elastic scattering, and fission cross sections corresponding to this probability. Interpolation is not done at present.

The scattering angles for anisotropic elastic scattering are obtained by sampling probability distribution tables based on the Legendre expansion coefficients of the cross sections. A table of scattering angle versus the cumulative probability of scattering at this angle is constructed for selected energies. Each table contains 20 entries, and the fuel isotopes have 12-19 tables while the light isotopes have 30-67 tables, except for oxygen which has 184 tables. The use of these tables in place of the Legendre coefficients results in a considerable savings in computing time. The scattering angle

is obtained by generating a random number between zero and one that is used to pick the scattering angle from the appropriate table. Interpolation is not done at present for either angle or energy.

C. Edit

As stated earlier, it is intended that the VIM output edit be tailored to serve the requirements of the problem being solved. The basic mechanisms for monitoring quantities such as k , fission, absorption, leakage, neutron energy, flux, computing time, the number of histories, and collisions are supplied in the code. However, many of the details as to how the monitoring is to be accomplished and the printed output organized are left to the discretion of the user. This is especially true when a new problem that differs considerably from problems previously studied is being solved.

In the ZPPR calculations, the neutron flux was monitored in a half-lethargy group structure over the inner and outer cores and over the inner and outer radial blankets using a track-length estimator. Provision was also made to monitor the flux over the central portions of the core regions. Examination of results from multigroup calculations shows that the neutron spectrum is essentially independent of position over the central eighth of the inner core, which is roughly the size of the monitoring region. The spectrum at the radial midpoint of the outer core is nearly the same as the integrated spectrum over the central part of the outer core over which the spectrum is monitored.

Neutron absorption, fission, and production rates were monitored over the core and blanket regions using an absorption estimator. In the interest of computing speed, only the total cross section is calculated while tracking a particle. Absorption and fission cross sections and ν are calculated only when an absorption occurs, thereby providing the means of estimating these reaction rates. In addition, the ^{238}U absorption rate and the fission rates of the uranium and plutonium isotopes were monitored over the central part of the inner core that was used to monitor the central flux spectrum. A collision estimator is used in order to improve the statistical precision of these results. Whenever a collision occurs in the central part of the inner core, the quantities $\sigma_x/\Sigma_{\text{tot}}$, where x denotes the reaction type, are calculated and tallied to estimate these reaction rates. The central reaction rates obtained in this way are correlated since the same set of collisions are used to estimate all the rates. The monitored flux and reaction rates are averages over the monitoring regions.

D. Restart and Initial Source

Restart capability for VIM, which enables a calculation to be extended by making a second computer run, was developed at ANL-Idaho. With restart capability, the user can examine the results of a run of moderate length to see if they are reasonable before committing himself to a

long run to obtain better convergence. The restart feature also allows a run that terminates abnormally due to computer malfunction or other abnormal conditions to be restarted at a point just before where the failure occurred.

Options were also provided to store the initial coordinates of all source particles on tape and to read the coordinates of the particles for the initial source from cards. The first time a given problem is run, the coordinates of the source particles for the first batch, the initial source, are generated in the code by means of a procedure that selects randomly from a uniform distribution of particles over the core of the assembly. For later runs, these coordinates can be selected from a converged distribution if the coordinates of all source particles have been saved from the first run.

III. ZPPR PROBLEM DESCRIPTION

A. Construction of the Model

Calculations were performed for both a heterogeneous and a homogeneous model of ZPPR Assembly 2. The heterogeneous model is based on the equal-core volume loading number 90 (Ref. 3) and contains over 41,000 zones, of which over 29,000 are in the inner and outer cores, six different drawer types, and 147 different zone types. Each half of the assembly contains 411 inner-core drawers, 209 outer-core drawers having two plutonium plates, 202 outer-core drawers having one plutonium plate, 375 inner-radial-blanket drawers, 450 outer-radial-blanket drawers, and 311 radial-reflector drawers. The core loading and the drawer front occupy the first 45.811 cm of the drawers. The axial blanket occupies the next 41.364 cm and includes the 0.720-cm spring gap consisting of the spring and the drawer end pieces between the front and back sections of the drawers. The axial reflector occupies the remainder of the drawers and was assumed to be 16.510 cm thick in the Monte Carlo model. The radial blanket occupies the first 87.175 cm of the drawers, and the radial reflector extends the full 103.685 cm. The axial reflector's thickness and composition is the average of the reflectors on each half of the assembly. The lattice pitch is 5.7836 cm high and 5.5244 cm wide.

The homogeneous model was formed by homogenizing the core, blanket, and reflector regions of the heterogeneous model. The procedure for deriving the heterogeneous model from the specifications for the as-built assembly is described next. Although every detail of the actual assembly cannot be represented precisely, most of the important details are represented in some way. The most important exception to this is that no void space was included in the model although about 9-12% of the volume of the lattice cell is void, depending on the region. Most of this void is in the space above each drawer. There are also seven channels for poison safety

rods which are withdrawn at criticality that are not included in the model. Since homogenizing the voids in the assembly decreases the leakage, the calculated k will be slightly higher than it would be if the voids were represented.

The key criterion in constructing the heterogeneous model was that it must reduce to essentially the same homogeneous atom densities as the ones obtained in homogenizing the actual assembly. These are given in Table X of Ref. 3. The procedure was to start with the nominal composition and dimensions of the various plates and then make minor adjustments to the atom densities until the correct homogenized atom densities were obtained. After the final adjustment, the homogenized atom densities of ^{238}U , ^{239}Pu , ^{240}Pu , ^{241}Pu , Mo, O, Na, and Fe agreed with the atom densities in Ref. 3 to at least 3 parts in 10,000. The agreement for the remaining isotopes was to at least 3 parts in 1000. The critical mass of $^{239}\text{Pu} + ^{241}\text{Pu}$ was 1,029.74 kg, whereas Table XVII of Ref. 3 gives 1030.52 kg, a difference of 8 parts in 10,000. The corrected value of k for the as-built assembly is 1.0007 using a conversion factor of $998 \text{ Ih} = 1\% \Delta k/k$.⁴

All the plates were assumed to be 5.08 cm high, to have the appropriate nominal thickness, and to extend the full length of the core or blanket regions, respectively, except that the drawer front was represented explicitly as a region 0.091 cm thick. The stainless steel cans for the sodium and plutonium were represented by plates 5.08 cm high and 0.0381 cm (0.015 in.) thick. All these plates in a given region were assumed to have the same composition. The stainless steel in the matrix and drawers was represented by plates running along the sides, tops, and bottoms of the drawer plates. The inner-core drawers contain a stainless steel plate 0.1588 cm (1/16 in.) thick in the bottom of the drawers. This material was included in the top and bottom plates, which represent the drawer plus matrix. The material in the spring gap was added to the plates that represent the cans in the blanket regions. Atom densities in the blanket regions were adjusted to include the spring-gap volume.

B. Cross-section Substitutions

Some of the less important isotopes shown in Table X of Ref. 3 are not contained in the VIM library. Rather than leave these out of the calculation, it was decided to make the following substitutions: The ^{242}Pu and ^{238}Pu were added to ^{240}Pu ; manganese and copper were added to iron; and silicon, phosphorous, and sulfur were added to aluminum. Americium-241 and hydrogen were omitted. The homogenized atom densities used in the VIM calculations are presented in Table I. The homogenized atom densities for an earlier version of ZPPR Assembly 2, which were used in a two-dimensional diffusion theory calculation,⁴ are also shown in Table I directly beneath the VIM atom densities.

TABLE I. Atom Densities for the VIM and 2D Diffusion-theory Homogeneous Problems
(The upper row for each isotope represents the VIM atom densities.)

Isotope	Atoms/cm ³ x 10 ⁻²²							
	Inner Core	Outer Core	Inner Radial Blanket	Outer Radial Blanket	Inner Axial Blanket	Outer Axial Blanket	Radial Reflector	Axial Reflector
²³⁹ Pu	0.08433	0.12741	-	-	-	-	-	-
	0.08477	0.12757	-	-	-	-	-	-
²⁴⁰ Pu	0.01141	0.01724	-	-	-	-	-	-
	0.01123	0.01689	-	-	-	-	-	-
²⁴¹ Pu	0.00154	0.00232	-	-	-	-	-	-
	0.00174	0.00260	-	-	-	-	-	-
²³⁵ U	0.00123	0.00115	0.00243	0.00243	0.00153	0.00153	-	-
	0.00123	0.00116	0.00243	0.00243	0.00153	0.00153	-	-
²³⁸ U	0.55503	0.51915	1.09939	1.09939	0.69131	0.69341	-	-
	0.55723	0.52068	1.10242	1.10242	0.69369	0.69369	-	-
Mo	0.02306	0.03410	-	-	-	-	-	-
	0.02188	0.03285	0.00014	0.00018	0.00014	0.00014	-	0.00014
Na	0.87969	0.85643	0.63452	0.59147	0.85865	0.86550	-	0.44830
	0.88143	0.85986	0.63641	0.59302	0.86369	0.86369	-	0.89848
O	1.31123	1.17613	1.99661	1.99661	1.36527	1.37645	-	-
	1.31489	1.17808	2.00210	2.00220	1.37264	1.37264	-	-
Fe	1.28081	1.40714	0.72158	0.78465	1.11831	0.97836	7.21751	5.48290
	1.26947	1.39548	0.71109	0.77353	1.03197	1.03197	7.17735	3.34583
Cr	0.26997	0.25205	0.20106	0.21897	0.28626	0.24504	0.12051	0.34930
	0.27001	0.25239	0.20113	0.21910	0.26491	0.26491	0.12001	0.44841
Ni	0.12209	0.11602	0.09021	0.09921	0.12784	0.11032	0.05133	0.15590
	0.13158	0.12556	0.09984	0.10878	0.12837	0.12837	0.05904	0.19245
Mn	-	-	-	-	-	-	-	-
	0.02083	0.02008	0.01573	0.01743	0.02174	0.02174	0.05934	0.05167
C	0.00294	0.00232	0.10044	0.10048	0.00402	0.00340	0.05564	0.03870
	0.00061	-	0.09860	0.09864	0.00133	0.00133	0.05398	0.01914
Al	0.01407	0.01231	0.00969	0.01054	0.01524	0.01221	0.00907	0.02110
	-	-	-	-	-	-	-	-

C. Monte Carlo Features

The Monte Carlo calculations made for the two ZPPR models just described were of the analog type. That is, no variance-reduction techniques such as particle splitting and weight adjustment were used. All particles were started with unit weight, and their energy was selected from the fission spectrum of the fuel isotope from which they were produced. Each particle was tracked until it was absorbed, leaked out of the assembly, or its energy dropped below 10 eV.

The coordinates for the initial source were obtained from a preliminary 40,000-history calculation for ZPPR. Coordinates were selected randomly from the last 28,000 histories of this run and placed on punched cards which were read as input data. A constraint was imposed on the distribution of the initial-source particles so that the ratio of the number of particles in the inner core to the number in the outer core was 1.10 and the ratio of the number in the blankets to the number in the outer core was 0.07. These ratios are approximately equal to the corresponding ratios obtained with multigroup calculations. An initial source selected in this way should be quite well converged, eliminating the necessity of allowing time at the start of the computation for source settling.

The initial source contained 800 particles, as did each succeeding batch of particles. The source particles are tracked one by one until they are absorbed or leak out of the assembly at which time their space coordinates, the isotope identification number in which absorption occurred, and the ratio $v\sigma_f/\sigma_a$ for this isotope are stored in the computer. This information supplies the source particles for the next batch. This process is repeated batch by batch until the total number of histories desired have been processed.

The code is presently dimensioned for a maximum batch size of 1000. In earlier calculations made for ZPR-3 Assembly 48, at both ANL and AI, a batch size of 400 was used. Since the volume of ZPPR is several times larger than the volume of ZPR-3, it was decided to double the batch size for the ZPPR calculations. Whether this is the optimum batch size is not known, but it is suspected that bias may be introduced if the batch size is too small because there may be regions in the assembly which would not be adequately sampled. On the other hand, if the batch size is extremely large so that very few batches are involved in the calculation, a bias will be introduced unless the initial source has the correct spatial distribution. A further consideration in choosing the batch size is that the standard deviation of the final result is calculated by assuming that each batch gives an independent unbiased estimate of the quantity being calculated and making a statistical analysis of the individual batch results. It seems desirable to have at least 50-100 batches for this analysis to be meaningful. Since it was planned to do between 50,000 and 100,000 histories for each model, a batch size of 800 seemed to be a reasonable choice.

A total of 100,000 histories were done for each model, and as a matter of convenience the initial source was the same for both cases. This will introduce a negligible amount of statistical correlation between the two cases, because the random number cycles are different. The running time for 100,000 histories and the edits on the computer was 8 hr 3 min for the heterogeneous model and 5 hr 47 min for the homogeneous model. The difference in the running time is due to the very large difference in the number of boundary crossings made by particles in the two problems.

IV. RESULTS

This section presents the results from the two Monte Carlo calculations, and, as expected, there are certain noteworthy differences between them. In addition, results from a two-dimensional, multigroup, diffusion-theory calculation⁴ are presented for comparison with the homogeneous VIM results. Experimental results for central fission ratios⁵ and neutron-spectra measurements⁶ are also included.

A. Diffusion-theory Calculation

The diffusion-theory calculation was made for a homogeneous cylindrical model of an earlier loading, number 77, of ZPPR Assembly 2,⁷ instead of the loading 90 upon which the VIM calculation is based, so the two problems are not quite identical. The reason for using the loading 77 calculation for comparison with VIM results is that no calculation was made for loading 90 using unmodified ENDF/B Version-II data as was done for loading 77. The calculation was for the as-built assembly, which contained two spiked control rods in the inner core and had an experimental value of k slightly greater than unity. The multigroup cross-section library was based on ENDF/B Version-II data, and the resonance self-shielding calculation was made for a homogeneous mixture in each region of the assembly. This is the same type of resonance self-shielding that is obtained in the homogeneous VIM calculation. The atom densities used in the diffusion-theory calculation are presented in Table I beneath the ones used in the VIM calculation. Table II compares the region volumes, region heights, plutonium mass, and experimental k . The experimental k was obtained from the measured excess

TABLE II. Region Volumes, Region Heights,
Mass of $^{239}\text{Pu} + ^{241}\text{Pu}$, and Experimental k

	VIM	Diffusion Theory
Inner-core volume, liters	1203.16	1188.20
Outer-core volume, liters	1203.16	1188.14
Axial-blanket volume, liters	2172.74	2145.62
Inner-radial-blanket volume, liters	2088.98	2127.80
Outer-radial-blanket volume, liters	2506.78	2499.96
Axial-reflector volume, liters	1736.56	2132.90
Radial-reflector volume, liters	2060.58	2136.84
Core half-height, cm	45.811	45.810
Blanket half-height, cm	87.175	87.170
Reflector half-height, cm	103.685	107.490
Mass of $^{239}\text{Pu} + ^{241}\text{Pu}$, kg	1029.7	1022.1
k experimental	1.0007	1.0008

reactivity by applying corrections for source subcriticality, interface gap worth, temperature adjustment, and edge smoothing for the diffusion-theory model. The conversion factor⁸ $988 \text{ Ih} = 1\% \Delta k/k$ was used in making these corrections. The reason for the higher plutonium mass in the VIM calculation is that because of core-loading changes there are 411 drawers in each core in loading 90 as compared to 407 drawers in loading 77.

Some adjustments must be made to the diffusion-theory k before it can be compared with the homogeneous-VIM k . A transport-theory correction of 0.0024 must be added.⁴ The ^{241}Pu concentration was not adjusted to account for its decay from the time that the fuel was manufactured, as was done in the VIM calculation. Approximately 2.4 kg were lost due to decay, which corresponds to a reduction in k of about -0.0024.⁹ In the VIM calculations, the fission spectrum used is the one for the isotope in which fission occurs. In the diffusion-theory calculation, the ^{239}Pu fission spectrum was used for all fission. I estimate that this causes the diffusion-theory k to be high by about 0.0014. In comparing the plutonium mass with the presently accepted value for loading 77,³ we found that there is approximately 2.0 kg too much ^{239}Pu in the outer core of the diffusion-theory problem. This causes the calculated k to be high by about 0.00075.⁹ There are a few other minor differences between the VIM and the diffusion-theory problems, but it seems doubtful if any of them would cause a discrepancy in k of more than a few hundredths of a percent. When the above adjustments are applied to the calculated diffusion-theory $k = 0.9524$, the result is $k = 0.9503$ for comparison with the homogeneous-VIM k .

B. Comparison of k and Related Quantities

Table III presents various results from the Monte Carlo and the diffusion-theory calculations. The homogeneous-VIM k is lower than the adjusted-diffusion-theory k by nearly 2%, which is nearly five standard deviations in the VIM result. Some difference between these two results is expected, but the reason for such a large discrepancy is not clearly understood. To shed some light on this discrepancy, the neutron production by group in each region of the assembly was examined. The production in all regions is smaller in the VIM calculation than in the diffusion-theory calculation. The neutron production is monitored in VIM by means of an absorption estimator. Whenever an absorption occurs, the quantity $v\sigma_f/\sigma_a$ is calculated, multiplied by the weight of the particle being tracked, and added to the tally. The production is given by the final tally divided by the total number of histories, and k is the sum of the production in all regions. In Table IV, it is seen that about two-thirds of the discrepancy between the VIM and the diffusion-theory production comes from absorptions occurring below 2.0 keV. About 10% of the core production occurs below 2.0 keV, and about 25% occurs below 25 keV. Less than 4% of the production occurs in the blankets, and no significant discrepancy occurs there.

TABLE III. Results from the VDM and Diffusion-theory Calculations

Quantity	Heterogeneous VDM	Homogeneous VDM	Diffusion Theory
Number of histories	100,000	100,000	-
Computing time, min	483	347	-
k	0.9465	0.9322	0.9503 ^a
Standard deviation	±0.0038	±0.0038	-
Collisions per history	54.89	55.11	-
Leakage	0.0396	0.0384	0.0362
Total absorption	0.9623	0.9635	0.9638
Inner-core absorption	0.3739	0.3830	0.3785
Outer-core absorption	0.2920	0.2863	0.2899
Blanket absorption	0.2878	0.2868	0.2884
Reflector absorption	0.0086	0.0074	0.0070
Inner-core production	0.4671	0.4685	0.4719
Outer-core production	0.4441	0.4263	0.4403
Blanket production	0.0353	0.0373	0.0403
Inner-core flux	84.68	87.06	84.61
Outer-core flux	56.63	56.11	55.87
Inner-radial-blanket flux	25.32	25.35	25.86
Outer-radial-blanket flux	9.15	9.02	8.63
Axial-blanket flux	-	-	32.36
Average energy of source neutrons, MeV	2.062	2.064	-
Average energy of neutrons causing fission, keV	702	670	-
Median fission energy, keV	204	189	181

^aAdjusted value, as discussed in the text.

TABLE IV. Neutron Production by Neutrons Absorbed below the Indicated Energy

Group	Lower Energy, keV	Heterogeneous VDM			Homogeneous VDM			Diffusion Theory		
		Inner Core	Outer Core	Blankets	Inner Core	Outer Core	Blankets	Inner Core	Outer Core	Blankets
-	10,000.0	0.4671	0.4441	0.0353	0.4685	0.4263	0.0373	0.4718	0.4402	0.0403
1	6,065.3	0.4608	0.4381	0.0339	0.4626	0.4214	0.0352	0.4657	0.4349	0.0381
2	3,678.8	0.4438	0.4235	0.0299	0.4477	0.4086	0.0311	0.4509	0.4216	0.0336
3	2,237.3	0.4082	0.3899	0.0202	0.4132	0.3790	0.0211	0.4167	0.3913	0.0220
4	1,353.4	0.3661	0.3515	0.0102	0.3736	0.3439	0.0105	0.3758	0.3539	0.0112
5	820.9	0.3423	0.3227	0.0094	0.3509	0.3173	0.0095	0.3528	0.3288	0.0102
6	497.9	0.3046	0.2850	0.0091	0.3129	0.2817	0.0091	0.3178	0.2915	0.0099
7	302.0	0.2735	0.2542	0.0089	0.2812	0.2500	0.0087	0.2870	0.2592	0.0096
8	183.2	0.2369	0.2142	0.0085	0.2430	0.2113	0.0082	0.2488	0.2205	0.0092
9	111.1	0.1981	0.1739	0.0077	0.2033	0.1731	0.0077	0.2093	0.1816	0.0087
10	67.4	0.1644	0.1432	0.0068	0.1684	0.1414	0.0073	0.1757	0.1495	0.0081
11	40.9	0.1353	0.1163	0.0062	0.1386	0.1147	0.0067	0.1483	0.1237	0.0075
12	25.4	0.1147	0.0968	0.0057	0.1174	0.0954	0.0062	0.1279	0.1053	0.0069
13	15.0	0.0904	0.0750	0.0051	0.0923	0.0731	0.0054	0.1024	0.0810	0.0062
14	9.11	0.0733	0.0591	0.0043	0.0733	0.0570	0.0047	0.0824	0.0631	0.0056
15	5.53	0.0609	0.0481	0.0040	0.0618	0.0462	0.0043	0.0696	0.0521	0.0051
16	3.35	0.0535	0.0403	0.0037	0.0531	0.0389	0.0039	0.0607	0.0447	0.0047
17	2.04	0.0500	0.0373	0.0036	0.0497	0.0360	0.0038	0.0569	0.0415	0.0045
18	1.23	0.0361	0.0263	0.0030	0.0353	0.0245	0.0032	0.0418	0.0293	0.0038
19	0.749	0.0243	0.0163	0.0023	0.0216	0.0144	0.0022	0.0276	0.0187	0.0031
20	0.454	0.0147	0.0096	0.0014	0.0119	0.0080	0.0016	0.0163	0.0109	0.0024
21	0.275	0.0081	0.0056	0.0010	0.0063	0.0042	0.0012	0.0104	0.0069	0.0019
22	0.167	0.0033	0.0024	0.0004	0.0022	0.0015	0.0008	-	-	-
23	0.101	0.0017	0.0012	0.0003	0.0008	0.0007	0.0005	0.0022	0.0017	0.0010

The region absorptions in VIM are also monitored by an absorption estimator. Table III shows that the inner-core absorption in VIM is somewhat larger than in diffusion theory while the outer-core and blanket absorptions are a little lower. Both the neutron production and absorption distributions are consistent with the VIM inner-core flux being higher than the diffusion-theory inner-core flux. The fact that the neutron absorption by region agrees better than the neutron production by region suggests that there may be some trouble with the fission cross sections, especially at low energy.

The effect of heterogeneity on k , obtained by taking the difference between the two VIM results, is found to be $\Delta k = 0.0143 \pm 0.0054$, which is consistent with the value $\Delta k = 0.012$ obtained by multigroup methods.⁴ However, the accuracy of the VIM result leaves much to be desired and is a consequence of the two VIM calculations being statistically uncorrelated. A correlation coefficient of 0.080 was calculated for the individual batch results of the two VIM calculations, which corresponds to a probability of about 40% that the corresponding batches from the two calculations are completely uncorrelated. Although this alone does not prove that the two calculations are not correlated, it shows that any correlation that does exist between them is small. A correlated sampling method is clearly required if good accuracy is to be obtained for the heterogeneity effect.

Table V shows that there is no difficulty due to source settling in either VIM calculation since the value of k is nearly independent of the number of neutron histories neglected at the beginning of each run. This confirms that the initial source has very nearly the correct spatial distribution as expected in view of the manner in which it was obtained.

TABLE V. Value of k vs Number of Histories Neglected at the Beginning of the Calculation

Number of Histories Neglected	Heterogeneous k	Homogeneous k
0	0.9465	0.9322
800	0.9467	0.9323
1,600	0.9473	0.9327
2,400	0.9465	0.9325
3,200	0.9466	0.9323
4,000	0.9464	0.9318
8,000	0.9449	0.9314
12,000	0.9464	0.9325
16,000	0.9466	0.9322
20,000	0.9464	0.9329
24,000	0.9472	0.9350
28,000	0.9488	0.9344
32,000	0.9480	0.9338
36,000	0.9500	0.9328
40,000	0.9485	0.9315

Table VI presents the results of chi-square tests made on the individual batch results for k to determine if the parent distributions from which they were obtained are likely to be normal distributions. The tabulated quantity, P , is the probability that a value of chi-square larger than the calculated value for the sample could have been obtained merely as a result of statistical fluctuations. If the parent distribution from which a random sample is drawn is not normal, a small value P is likely to be obtained for the sample. However, even if the parent distribution is normal, a value of P that is close

to unity is unlikely because of the statistical fluctuations involved in drawing the sample. There is no fixed rule for determining when the value of P for the sample is so small that the parent distribution is probably not normal, but the criterion $P \leq 0.05$ is sometimes used for this purpose. Both VIM results easily pass this criterion for each number of degrees of freedom that were used in the chi-square test.

TABLE VI. Results of Chi-square Tests for VIM Calculations

Degrees of Freedom	$P = \int_{\chi^2}^{\infty} f(\chi^2) d\chi^2$	
	Heterogeneous	Homogeneous
2	0.19	0.15
4	0.49	0.19
6	0.11	0.46
10	0.29	0.20

C. Neutron Spectra

Tables VII and VIII present the integrated neutron-flux spectra over the core and radial-blanket regions from the Monte Carlo and diffusion-theory calculations. The statistical precision of the VIM results for the highest-energy group is only fair in the cores and is poor in the blankets because only a few of the neutron histories start at such a high energy. In the outer blanket, the precision is poor in the top three or four groups. At the low-energy end of the spectrum, the precision is good down to 275 eV, but is poor at energies below this and quite poor below 101 eV, because only a few of the neutron histories survive to such a low energy. For the present calculations, high precision is not required at the extreme ends of the neutron spectrum. If it becomes necessary to improve the precision here, this can be accomplished by particle-splitting or absorption-weighting techniques.

Above 500 keV, the heterogeneous-VIM flux is usually lower than the homogeneous-VIM flux, especially in the blanket regions. Below this energy, the VIM spectra are in good agreement down to 3.3 keV. Below 3.3 keV, the heterogeneous-VIM flux is higher than the homogeneous-VIM flux, except for an anomalous behavior around 500 eV in the blankets and 2 keV in the outer core. The discrepancy becomes more pronounced as energy decreases, but is not given very reliably below about 167 eV because of the poor statistical precision. The flux relationship at low energy may be accounted for in terms of resonance self-shielding. The effective cross section for the homogeneous model is larger than the one for the heterogeneous model, and this will reduce the homogeneous-VIM flux at low energy. I have not found an explanation for the behavior at high energy.

TABLE VII. Calculated Flux Spectra for Cores

Group	Lower Energy, keV	Inner-core Flux				Outer-core Flux			
		Homogeneous VIM		Heterogeneous VIM ÷ Homogeneous VIM	Diffusion Theory ÷ Homogeneous VIM	Homogeneous VIM		Heterogeneous VIM ÷ Homogeneous VIM	Diffusion Theory ÷ Homogeneous VIM
		Flux ^a	Percent Standard Deviation			Flux ^a	Percent Standard Deviation		
1	6065.3	0.00274	3.87	1.035	1.075	0.00367	3.37	0.895	0.941
2	3678.8	0.81174	1.59	0.976	1.011	0.01390	1.51	0.998	1.005
3	2231.3	0.02891	1.15	0.963	1.018	0.03297	1.18	0.982	1.016
4	1353.4	0.84173	1.06	0.975	1.018	0.04812	0.96	0.978	1.002
5	820.9	0.85178	0.98	0.980	0.982	0.05875	1.00	0.980	0.977
6	497.9	0.09641	0.85	0.997	0.987	0.10411	0.92	0.989	0.982
7	302.0	0.88968	0.84	1.006	1.028	0.09468	0.94	1.002	1.030
8	183.2	0.11503	0.87	1.000	1.009	0.12739	0.86	1.002	1.015
9	111.1	0.12256	0.83	0.987	0.986	0.11858	0.83	1.010	1.016
10	67.4	0.10222	0.82	0.996	1.005	0.09691	0.92	1.000	1.026
11	40.9	0.08371	0.81	1.007	0.964	0.07800	0.95	1.020	0.984
12	24.8	0.05930	0.84	0.998	0.953	0.05532	0.92	0.993	0.936
13	15.0	0.05563	0.91	0.998	0.954	0.06415	1.01	0.969	0.939
14	9.11	0.04457	1.01	0.988	1.061	0.04143	1.21	0.999	1.016
15	5.53	0.02627	1.15	1.006	1.005	0.02361	1.43	1.002	0.976
16	3.35	0.01524	1.31	1.022	1.075	0.01301	1.65	1.087	1.010
17	2.04	0.00887	1.59	1.111	1.003	0.00522	1.91	0.998	0.963
18	1.23	0.01406	1.43	1.033	1.040	0.01371	1.83	1.057	1.006
19	0.749	0.01076	1.79	1.078	1.053	0.00840	2.33	1.099	1.038
20	0.454	0.00570	2.65	1.146	1.208	0.00420	3.51	1.192	1.208
21	0.275	0.00262	3.82	1.331	1.847	0.00195	5.04	1.271	1.076
22	0.167	0.00100	4.76	1.534	1.75	0.00067	7.53	1.634	1.66
23	0.101	0.00034	9.26	1.784		0.00028	12.13	1.589	
24	0.061	0.00009	18.95	2.839	3.3	0.00008	18.40	2.626	2.8
25	0.037	0.00002	34.40	3.678		0.00002	42.75	4.228	

^aNormalized to unity over the region for all groups.

TABLE VIII. Calculated Flux Spectra for Radial Blankets

Group	Lower Energy, keV	Inner-radial-blanket Flux				Outer-radial-blanket Flux			
		Homogeneous VIM		Heterogeneous VIM ÷ Homogeneous VIM	Diffusion Theory ÷ Homogeneous VIM	Homogeneous VIM		Heterogeneous VIM ÷ Homogeneous VIM	Diffusion Theory ÷ Homogeneous VIM
		Flux ^a	Percent Standard Deviation			Flux ^a	Percent Standard Deviation		
1	6065.3	0.00155	6.78	0.752	0.949	0.00078	16.37	0.507	0.805
2	3678.8	0.00572	3.92	0.897	0.991	0.00289	8.87	0.772	0.788
3	2231.3	0.01456	2.81	0.893	1.017	0.00680	5.94	0.884	0.951
4	1353.4	0.02108	2.30	0.947	1.008	0.00986	5.55	0.957	0.937
5	820.9	0.02705	2.01	1.005	1.020	0.01305	4.81	0.992	0.970
6	497.9	0.06743	1.79	0.986	1.046	0.04182	3.21	0.931	1.040
7	302.0	0.06693	1.33	1.000	1.018	0.04405	2.71	0.995	1.035
8	183.2	0.09965	1.32	1.004	1.020	0.07549	2.43	0.991	1.020
9	111.1	0.11280	1.37	0.997	1.005	0.09532	2.33	0.995	1.015
10	67.4	0.10765	1.27	1.012	1.012	0.10197	2.14	1.014	1.024
11	40.9	0.09987	1.13	1.009	0.968	0.10464	1.96	0.974	0.970
12	24.8	0.08161	1.17	0.994	0.903	0.09346	1.94	0.971	0.908
13	15.0	0.08277	1.37	1.010	0.963	0.10103	2.08	1.021	0.976
14	9.11	0.06329	1.49	0.974	0.981	0.07734	2.21	1.008	1.017
15	5.53	0.04100	1.44	0.996	0.954	0.05489	2.53	0.997	0.957
16	3.35	0.02473	1.95	1.016	0.962	0.03619	2.45	1.039	1.058
17	2.04	0.08043	2.23	1.030	1.068	0.01615	3.17	1.092	0.847
18	1.23	0.02492	2.26	1.015	1.000	0.03654	2.99	1.053	0.957
19	0.749	0.01812	2.78	1.053	1.019	0.02798	3.33	1.043	1.015
20	0.454	0.01251	3.01	0.960	1.086	0.02162	3.76	0.946	1.041
21	0.275	0.00780	4.45	1.005	1.068	0.01489	4.73	1.097	0.988
22	0.167	0.00431	5.36	1.178	1.437	0.01032	5.39	1.051	1.158
23	0.101	0.00236	6.42	1.235		0.00628	6.12	1.147	
24	0.061	0.00104	8.39	1.411	2.59	0.00336	8.93	0.994	1.83
25	0.037	0.00060	12.77	1.324		0.00214	12.54	1.180	

^aNormalized to unity over the region for all groups.

The diffusion-theory flux in group 1 is higher than the homogeneous-VIM flux in the inner core, but is lower than the VIM flux in the outer core and the blankets. In the outer blanket, this situation continues through the top four or five groups. As energy decreases, the agreement is generally good, except around 25 and 2 keV, down to 750 eV in the cores and the inner blanket and to 275 eV in the outer blanket. The discrepancy near 25 keV can be at least partially attributed to an inadequate treatment of the 34.1-keV ^{56}Fe p-wave resonance by the ETOE code, which converts the ENDF/B data to the format required by MC². This resulted in much too large a contribution to the cross section by this resonance.¹⁰ Although the width of this resonance is only 36 eV, the MC² ultrafine-group flux output shows it to give about the same flux depression as the ^{56}Fe 28-keV s-wave resonance, which has a 1.5 keV width. The incorrect iron cross section in group 12 causes the diffusion-theory flux in this group to be too low. The disagreement around 2 keV appears odd and may be related somehow to the 2.85-keV sodium resonance, which is represented by pointwise data in the VIM library. Below 1.2 keV, the diffusion-theory flux is usually higher than the homogeneous-VIM flux. Note that the two lowest-energy diffusion-theory groups are one lethargy unit wide.

The integrated VIM neutron-flux spectra over the central portions of the inner and outer cores are presented in Tables IX and X. Comparison is made with the diffusion-theory spectra and experimental spectrum

TABLE IX. Calculated and Experimental Flux Spectra at Center of Inner Core

Group	Lower Energy, keV	Homogeneous VIM		Heterogeneous VIM ÷ Homogeneous VIM	Diffusion Theory ÷ Homogeneous VIM	Heterogeneous VIM ÷ Experiment	Diffusion Theory ÷ Experiment
		Flux ^a	Percent Standard Deviation				
1	6065.3	0.00323	8.54	1.028	0.973	-	-
2	3678.8	0.01226	4.13	1.066	1.036	-	-
3	2231.3	0.03004	3.47	0.975	1.001	-	-
4	1353.4	0.04457	2.76	0.959	1.020	1.072	1.141
5	820.9	0.05540	2.79	0.974	0.981	0.895	0.901
6	497.9	0.10200	2.36	1.007	0.998	1.109	1.099
7	302.0	0.06454	2.29	1.021	1.043	1.021	1.042
8	183.2	0.12117	2.10	0.990	1.026	1.020	1.057
9	111.1	0.13023	1.87	0.989	0.995	1.011	1.017
10	67.4	0.11070	1.99	1.004	0.997	0.915	0.908
11	40.9	0.09116	2.06	0.993	0.952	0.925	0.887
12	24.8	0.06354	2.01	1.023	0.959	0.976	0.915
13	15.0	0.07009	2.32	1.005	0.961	1.093	1.045
14	9.11	0.04738	2.43	1.033	1.055	1.043	1.065
15	5.53	0.02898	2.77	0.982	0.982	0.961	0.981
16	3.35	0.01666	3.40	1.012	1.062	0.984	1.033
17	2.04	0.00632	3.78	1.099	1.005	0.779	0.716
18	1.23	0.01725	4.20	1.023	1.044	1.150	1.173
19	0.749	0.01068	5.06	1.164	1.140	-	-
20	0.454	0.00596	6.97	1.228	1.233	-	-
21	0.275	0.00261	9.99	1.411	1.208	-	-
22	0.167	0.00106	16.02	1.212	-	-	-
23	0.101	0.00042	22.91	1.142	1.61	-	-
24	0.061	0.00021	41.75	1.050	-	-	-
25	0.037	0.00001	76.02	0.839	1.28	-	-

^aNormalized to unity for groups 4-18.

TABLE X Calculated and Experimental Flux Spectra at Center of Outer Core

Group	Lower Energy, keV	Homogeneous VIM		Heterogeneous VIM	Diffusion Theory	Heterogeneous VIM	Diffusion Theory
		Flux ^a	Percent Standard Deviation	Homogeneous VIM	Homogeneous VIM	Experiment	Experiment
1	4065.3	0.00434	5.02	0.055	0.099	-	-
2	3678.8	0.00508	2.95	0.079	0.095	-	-
3	2231.3	0.03602	2.22	1.094	1.051	-	-
4	1353.4	0.05425	1.42	0.982	1.024	1.118	1.145
5	828.9	0.06756	1.42	0.974	0.957	0.941	0.925
6	497.9	0.11573	1.39	0.988	0.980	1.117	1.108
7	302.0	0.30981	1.40	1.085	1.038	1.003	1.055
8	183.2	0.12732	1.37	1.040	1.020	1.034	1.044
9	111.1	0.12838	1.35	1.001	1.009	0.907	1.005
10	67.4	0.10307	1.46	0.980	1.023	0.856	0.904
11	40.9	0.07935	1.61	1.060	1.013	0.929	0.888
12	24.8	0.05534	1.57	1.032	0.967	0.968	0.900
13	15.0	0.06592	1.79	0.967	0.946	1.114	1.090
14	9.11	0.04288	1.97	0.971	1.005	1.048	1.075
15	5.53	0.02347	2.55	1.023	0.988	0.905	0.934
16	3.35	0.01405	2.89	1.047	0.991	0.923	0.999
17	2.04	0.00508	4.07	1.042	0.979	0.881	0.828
18	1.23	0.01388	3.07	1.044	0.914	1.112	1.065
19	0.749	0.00653	4.49	1.048	0.980	-	-
20	0.454	0.00429	4.16	1.087	1.069	-	-
21	0.275	0.00190	8.81	1.128	0.934	-	-
22	0.167	0.00061	14.40	1.418	-	-	-
23	0.101	0.00024	27.12	1.248	1.34	-	-
24	0.061	0.00005	52.49	2.547	-	-	-
25	0.037	0.00004	108.00	8.883	2.7	-	-

^aNormalized to unity for groups 4-18.

measurements⁶ obtained near the center of the VIM monitoring regions. The comparison of the calculated spectra shows the same general features as the corresponding comparison over the entire core regions as discussed above. The VIM standard deviations for the central regions are larger than the ones for the full core regions because there is more scatter in the individual batch results for the smaller monitor regions. The comparison with experiment shows a little better agreement for the heterogeneous-VIM spectra than for the diffusion-theory spectra, but the disagreement is significant in several places. A correlation is observed between the calculated-to-experiment ratios for the inner- and outer-core regions. This suggests that at least part of the discrepancy between calculation and experiment is caused by shortcomings in the cross-section data.

It is of interest to know the fraction of the neutron collisions occurring in each energy group. A separate 200-history run was made using special monitoring to obtain this information. The group fractions are presented in Table XI. It was found that 66.4% of the collisions occur in the cores, 30.8% in the blankets, and 2.8% in the reflectors. Moreover, 31% of all the collisions occur at energies below 25 keV.

TABLE XI Fraction of Neutron Collisions Occurring in Each Group from a 10 955 Collision Sample

Group	Lower Energy, keV			Group	Lower Energy, keV		
		Cores	Blankets			Cores	Blankets
1	4065.3	0.001	-	13	15.0	0.095	0.049
2	3678.8	0.004	-	14	9.11	0.061	0.049
3	2231.3	0.016	0.002	15	5.53	0.026	0.021
4	1353.4	0.018	0.003	16	3.35	0.022	0.016
5	828.9	0.023	0.004	17	2.04	0.014	0.010
6	497.9	0.049	0.034	18	1.23	0.014	0.013
7	302.0	0.055	0.014	19	0.749	0.010	0.006
8	183.2	0.065	0.024	20	0.454	0.006	0.005
9	111.1	0.066	0.032	21	0.275	0.003	0.004
10	67.4	0.068	0.031	22	0.167	0.001	0.003
11	40.9	0.051	0.034	23	0.101	0.001	0.002
12	24.8	0.053	0.032				

D. Central-reaction-rate Ratios

Central reaction rates are monitored in VIM over the central 13% of the inner core by means of a collision estimator. As noted earlier, the neutron spectrum is essentially independent of position over this region. The reaction-rate ratios relative to ^{239}Pu are presented in Table XII. Standard deviations were not calculated, but an estimate of the precision which was made by examining the fluctuation of each ratio over the last 50,000 histories indicates a precision of about 1/4% for $^{235}\text{U}_f/^{239}\text{Pu}_f$, 1/2% for $^{238}\text{U}_a/^{239}\text{Pu}_f$, and 1% for the others.

TABLE XII. Central-reaction-rate Ratios

Quantity	Heterogeneous VIM	Homogeneous VIM	Diffusion Theory	Experiment ^a
$^{235}\text{U}_f/^{239}\text{Pu}_f$	1.0868	1.0779	1.1447	1.067 ± 0.014
$^{238}\text{U}_f/^{239}\text{Pu}_f$	0.02123	0.02257	0.02591	0.0215 ± 0.0005
$^{240}\text{Pu}_f/^{239}\text{Pu}_f$	0.1890	0.2126	0.1954	0.1818 ± 0.0047
$^{241}\text{Pu}_f/^{239}\text{Pu}_f$	1.5047	1.4502	1.4503	-
$^{238}\text{U}_a/^{239}\text{Pu}_f$	0.1946	0.1893	0.1892	-

^aSee Ref. 5.

The heterogeneous-VIM results for ^{235}U and ^{240}Pu fission are a little higher than experiment, but the discrepancy is only a little more than the estimated standard deviation in the differences. The agreement for ^{238}U fission is very good. The homogeneous-VIM result for ^{235}U fission is in good agreement with experiment, but the results for ^{238}U and ^{240}U fission are high. Since these isotopes have fission thresholds, their fission ratios relative to ^{239}Pu fission can be very sensitive to the neutron spectrum, whereas the ^{235}U fission ratio relative to ^{239}Pu fission would be much less sensitive to the spectrum. The homogeneous-VIM results are consistent with the VIM flux ratios presented in Table IX. Below 3.3 keV, the homogeneous flux is much lower than the heterogeneous flux; at high energy, the fluxes are nearly the same. Table IV shows that more than 10% of the fissions in the inner core occur at energies below 3.3 keV. The smaller ^{239}Pu fission at low energy in the homogeneous-VIM calculation causes a substantial increase in the calculated fission ratios of the threshold materials.

The diffusion-theory fission ratios for ^{235}U and ^{238}U relative to ^{239}Pu are quite high, and the cause of this is not understood.

V. DISCUSSION

The values of k obtained with the VIM code for ZPPR Assembly 2 support the conclusion already established by multigroup calculations that

the ENDF/B Version-II data produces quite low values of k for typical fast-breeder-reactor assemblies. On the other hand, the central fission ratios obtained from the VIM calculation for the heterogeneous ZPPR model are in fairly good agreement with experimental results. This suggests that the ENDF/B Version-II fission-cross-section ratios may be fairly accurate, although the actual values of the cross sections may be low over at least part of the energy range. Comparison of calculated and experimental neutron spectra in Tables IX and X shows several areas of significant disagreement. Furthermore, it can be seen that there is a correlation between the calculated-to-experiment ratios for the inner and outer cores. In view of this correlation and the evident shortcomings in the Version-II cross-section data, it seems likely that at least a part of the discrepancy between calculated and experimental neutron spectra is caused by the cross-section data.

The value of $k = 0.9322 \pm 0.0038$ for the homogeneous-VIM calculation reported in Table III is in excellent agreement with the value $k = 0.9337 \pm 0.0034$ obtained recently at Atomic International¹¹ with VIM for the ZPPR homogeneous benchmark problem, which is being studied in connection with the ENDF/B data testing program. The AI code contains monitoring for multigroup cross sections which is not yet included in the ANL code. Comparison of the multigroup cross sections generated in the AI VIM calculation with the ones generated by MC² for the diffusion-theory calculation shows significant differences for the absorption and fission cross sections in the resonance-energy region. For energies where the ENDF/B cross-section data are smooth, the agreement of the multigroup cross sections is usually good and is within a few parts in 1000 or better most of the time. It is believed that much of the discrepancy between the VIM and the diffusion-theory results for k can be accounted for by differences in the cross-section libraries used in the two calculations. There should be some difference between the two results, but the difference of nearly 2% seems surprisingly large. The reason for apparent differences in the cross-section libraries is not understood.

The results presented in this report show the present capability for making meaningful Monte Carlo calculations with the VIM code for typical fast breeder reactors. Quantities that can be calculated satisfactorily with the present code include k , multigroup flux spectra, central-reaction-rate ratios, and neutron absorption, fission, and production rates for core and blanket regions. Monitoring regions for flux and reaction rates must be fairly large to obtain satisfactory precision. It also appears that it would be possible to obtain the breeding ratio and calculate the worth of a bank of control rods with satisfactory precision providing a fixed source calculation is made which will result in there being a correlation between the calculations with rods inserted and with rods removed. Such calculations have not yet been performed.

The running time on the IBM-360/75 computer for the ZPPR VIM problem was several hours, and 80% of the available core storage was used.

This shows that it would not be very practical to attempt a VIM calculation for a problem of this size with a smaller or slower computer. However, computers are now coming into use that are more than five times as fast and have more than twice the core capacity of the IBM-360/75. It is clear that the capability is therefore available to do even larger problems than the one described in this report in a reasonable amount of computing time.

VI. FUTURE PLANS

Coding to enable monitoring multigroup cross sections and P_0 transfer matrices in VIM has been obtained from AI and is being added to the code at ANL-Idaho. A procedure for obtaining transport cross sections will be developed so that the multigroup cross sections generated by VIM can be used to make diffusion- and transport-theory calculations.

A fixed-source capability for VIM was developed at ANL-Idaho which enables two similar VIM calculations to use the same source particles. This capability will be extended so that the same random-number cycle is initiated at the beginning of corresponding histories in the two calculations. This provides a degree of naive correlation between the results of two such calculations, because many of the neutron histories in them are identical. Only those histories that encounter a physical difference existing between the two problems, such as the presence or absence of a control rod, contribute to the eigenvalue difference for the two calculations, thereby reducing its statistical uncertainty. The statistical analysis of such results requires further study.

To calculate small sample worths, reactivity coefficients, and heterogeneity effects satisfactorily, it will be necessary to employ adjoint Monte Carlo methods and complete correlation techniques which use the same random walk (particle tracks and collisions) in two Monte Carlo calculations. Particle splitting and absorption weighting techniques may also be required for certain applications. These methods are being developed for VIM at AI and will be implemented in the ANL code as they become available. The capability of dealing with a rodded-zone geometry is also needed and can probably be supplied by AI.

A VIM cross-section library based on ENDF/B Version-III data will be placed in service at ANL-Idaho when it is received from AI. This library will be used in making VIM calculations for ZPPR Assembly 3 to calculate the worth of control-rod banks and to obtain VIM-generated multigroup cross sections for use in supplementary multigroup calculations for this assembly.

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