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Large-Spin Rotational States of Deformed Nuclei*

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In a recent letter¹⁾, Stephens, Lark and Diamond have strikingly demonstrated the increase of moment of inertia, $\mathcal{I}$, as one progresses to higher angular momentum states for a given nucleus. They have investigated the ground state band of several rare earth even-even nuclei, in some cases observing all levels up to $I = 18$. The effect of increasing I can result in a two- to three-fold increase in $\mathcal{I}$ over this range. Since the effective moment of inertia determined from the $I = 2 - I = 0$ level spacing is normally about one-third to one-half of the rigid body value, one sees that by the time the $I = 18$ level is reached, the rigid body $\mathcal{I}$ is almost attained.

There are many alternative explanations of this phenomenon. Mottelson and Valatin²⁾ have outlined one such possibility within the framework of nuclear pairing theory. One effect of the rotational motion is to reduce the effective pairing force strength, G , from the no rotation value. Since it has been shown that the pairing force can account for the reduction of $\mathcal{I}$ from the naively expected rigid value³⁾, one would expect that an increase in the angular velocity ω (corresponding to larger I values) would result in less pairing (smaller G_{eff}) and consequently lead to an increase in $\mathcal{I}$ toward the rigid body value. In addition, a critical value of G generally exists such that pairing solutions do not occur for $G_{\text{eff}} < G_{\text{critical}}$. This indicates that the energy gap should be completely destroyed for values of I corresponding to $\omega > \omega_{\text{critical}}$.

The connection between $\mathcal{I}$ and the energy gap is illustrated in the case of the $K^\pi = 0^+$ band which occurs at low excitations in the deformed regions. This band may be based on a particular two-quasiparticle state or on a more collective linear superposition of such states^{4), 5)}. In either case, the energy gap should be smaller (reduced by blocking) than it is in the low-lying levels of the ground state band. Such an effect is observed; typically

one finds $\mathcal{I}(K^\pi = 0^+) \sim 1.7 \mathcal{I}_{\text{gnd}}^{(5)}$.

Another explanation of the dependence of $\mathcal{I}$ on the degree of rotation comes from the theory of Davydov and Chaban⁶⁾. They consider the coupling between the β vibrations and the rotational motion which leads to a centrifugal stretching of the nucleus and a corresponding increase in $\mathcal{I}$ as the rotational frequency is increased. The authors of Ref. (1) use this theory to obtain a good two parameter fit to their data.

The main purpose of this paper is to point out still another and perhaps simpler explanation for this effect. The moment of inertia is usually calculated from the cranking model⁷⁾. One begins with a rotating, deformed, self-consistent potential well. The Hamiltonian is then transformed to a reference frame rotating with the nucleus. In this system, a new term appears, $H' = -\omega J_x$. If this term is treated as a perturbation, then upon transforming back to the lab co-ordinates one obtains to lowest order

$$\Delta E = \omega^2 \sum_m \frac{|\langle m | J_x | 0 \rangle|^2}{E_m - E_0} \quad (1)$$

Since ΔE must correspond to the rotational energy ($\mathcal{I}_0 \omega^2 / 2$), one can solve for $\mathcal{I}_0$,

$$\mathcal{I}_0 = 2 \sum_m \frac{|\langle m | J_x | 0 \rangle|^2}{E_m - E_0} \quad (2)$$

This is the usual cranking model result. It is proposed here, however, that the next higher order term in this perturbation expansion is important. Retaining terms of order ω^4 , one has

$$E = \frac{1}{2} (\mathcal{J}_0 + 3 C \omega^2) \omega^2 \quad (3)$$

where C can be written as

$$C = 2 \sum_{mnp} \frac{\langle 0 | J_x | n \rangle \langle n | J_x | m \rangle \langle m | J_x | p \rangle \langle p | J_x | 0 \rangle}{(E_n - E_0)(E_m - E_0)(E_p - E_0)} \quad (4)$$

$$= 2 \mathcal{J}_0 \sum_m \frac{|\langle 0 | J_x | m \rangle|^2}{(E_m - E_0)^2} .$$

This calculation also leads to the result

$$\langle J_x \rangle = \omega (\mathcal{J}_0 + 2 C \omega^2) . \quad (5)$$

Upon comparing Eqs. (3) and (5), one sees that different effective values of $\mathcal{J}$ enter in calculations of the rotational energy and the angular momentum. Thus it seems as though a unique moment of inertia does not exist in the usual sense when one considers terms of higher order in ω .

The present model is based on the two equations

$$E = \frac{1}{2} \omega^2 (\mathcal{J}_0 + 3 C \omega^2) \quad (3)$$

and

$$[I(I+1)]^{\frac{1}{2}} = \omega (\mathcal{J}_0 + 2 C \omega^2) . \quad (5')$$

The parameter ω may be eliminated from these, leading to a cubic equation in E involving only two parameters, $\mathcal{J}_0$ and C ,

$$\frac{128 C^2}{\mathcal{J}_0^3} \frac{E^3}{I(I+1)} - \frac{32 C}{\mathcal{J}_0} \frac{E^2}{I(I+1)} + 2 \mathcal{J}_0 \frac{E}{I(I+1)} + \frac{72 C}{\mathcal{J}_0^2} E^{-1} - \frac{27 C}{\mathcal{J}_0^3} I(I+1) = 0 . \quad (6)$$

The third and fifth terms (those not containing C) lead directly to the simple rotational spectrum, $E = I(I + 1)/2\mathcal{I}_0$.

A better idea of the effect of C on the spectrum may be obtained by solving Eq. (6) by iteration,

$$E = \frac{I(I + 1)}{2\mathcal{I}_0} \{ 1 - x + 4x^2 - 24x^3 + \dots \} \quad (7)$$

where $x = CI(I + 1)/\mathcal{I}_0^2$. All powers of $I(I + 1)$ enter into the spectrum, although their coefficients are uniquely determined in terms of only two parameters, C and $\mathcal{I}_0$.

To see how well Eq. (6) fits the observed spectra, C and $\mathcal{I}_0$ are determined by a least-squares fit to the experimental energy levels^{1), 8)}. Equation (6) is then solved for each value of I , and the resulting energy eigenvalues are compared with experiment. Results of this program for a few representative nuclei are shown in Tables I and II.

It can be seen that Eq. (6) represents the experimental spectra satisfactorily. The R. M. S. deviations given in the last column of Table II compare favorably with the accuracy of the experimental energies ($\pm 0.3\%$).

Of course, perturbation theory is not expected to hold when the correction terms are as large as in the present situation ($\sim 50\%$). Arguments demanding self-consistency, however, require that Eqs. (3) and (5) (and therefore all following equations) be valid to order ω^4 even when perturbation theory does not apply. In this latter case, $\mathcal{I}_0$ and C are no longer given by Eqs. (2) and (4). Since these parameters are determined empirically here, the results of this paper are in no way dependent upon the validity

of perturbation theory. A more complete discussion of the present work is now in preparation.

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FOOTNOTES

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TABLE I. Parameters Used in Solving Eq. (6)

Nucleus	$\mathcal{I}_0 (10^{-2} \text{ KeV}^{-1})$	$C (10^{-8} \text{ KeV}^{-3})$
W ¹⁷⁴	2.596	9.470
W ¹⁷⁶	2.689	7.768
Hf ¹⁶⁶	1.734	9.909
Hf ¹⁶⁸	2.328	9.554
Hf ¹⁷⁰	2.894	11.647
Hf ¹⁷²	3.116	7.050
Yb ¹⁶⁴	2.369	8.385
Yb ¹⁶⁶	2.897	6.541

TABLE II. Rotational Energies (KeV)

Nucleus		$J = 2^+$	4^+	6^+	8^+	10^+	12^+	14^+	16^+	RMS% Deviation
W^{174}	Expt.	111.9	355.0	704.2	1137	1635	2186			0.30
	Theory	112.2	354.9	701.8	1133	1634	2196			
W^{176}	Expt.	108.7	348.5	699.4	1140	1648	2206			0.46
	Theory	109.1	348.7	696.4	1133	1645	2223			
Hf^{166}	Expt.	158.7	470.7	897.6	1407	1971	2565			0.59
	Theory	158.9	472.7	893.5	1396	1965	2591			
Hf^{168}	Expt.	123.9	385.0	756.1	1212	1734	2304			0.26
	Theory	123.8	386.2	755.0	1208	1731	2313			
Hf^{170}	Expt.	100.0	320.6	641.1	1041	1503	2013	2564	3147	0.76
	Theory	101.0	320.7	636.6	1031	1491	2008	2575	3186	
Hf^{172}	Expt.	94.5	307.9	627.0	1036	1519	2063	2651		0.46
	Theory	95.0	308.1	624.7	1030	1513	2062	2672		
Yb^{164}	Expt.	122.5	384.0	758.0	1219	1748				0.14
	Theory	122.4	384.8	757.0	1217	1750				
Yb^{166}	Expt.	101.8	329.7	667.1	1097	1604	2172			0.20
	Theory	102.0	329.7	665.9	1094	1602	2179			