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DATA FILTER CHARACTERISTICS
AND EFFECTS

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ABSTRACT

The use of data filters in electronic circuits is normally considered undesirable because filtering irretrievably eliminates desirable as well as undesirable information. Filtering is necessary in some applications, however, to obtain any useful information from measurements made in high-noise and distorted environments and from FM tape-recording and carrier-frequency-type systems. This memorandum discusses related theory and special filter characteristics necessary for retaining the maximum useful information in a filtered signal, depending upon whether the desired data are periodic or transient in nature.

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DATA FILTER CHARACTERISTICS AND EFFECTS

The use of data filters in signal-conditioning circuits is normally considered undesirable since by definition an electronic filter discriminates against certain frequency spectrums and not against others. Filtering a data signal from an environmental test irretrievably destroys useful information which describes the test environment; however, no applied circuitry has an infinite frequency response, and sometimes data are so obscured that they cannot be analyzed without removing some complex frequency information, or noise. In some applications (such as tape recording, telemetry, and transducer-carrier systems), where either frequency or amplitude modulation is necessitated by other test considerations, filters are an absolute necessity in the detection and signal-recovery circuitry. A knowledge of generalized special filter characteristics theory and of the anticipated signal information should permit one to retain the maximum useful information in a filtered signal whether the desired data are periodic or transient in nature.

To understand the effects of a filter, it is first necessary to understand the related theory and the special filter characteristics resulting from that theory for at least the common types of filters. The following discussion will deal with generalized functions and the influence of the major variables on these functions, with use requirements as the determining criteria. Although the discussion will be limited in general to passive, low-pass, second-order filter networks, it should be understood that all mathematical expressions pertain equally well to all filters and that all remarks can be extended to apply to high-pass networks by merely reversing the so-called frequency pass band and stop band, or band pass and band rejection, by proper combination of a low-pass and high-pass network. Since active filters are combinations of passive filters with electronic impedance isolation stages between filters, no attempt will be made to discuss associated circuitry techniques except where absolutely necessary. It will be assumed that good associated instrumentation and circuitry techniques are practiced and understood, and that this associated circuitry can be represented by a linear Thévenin's equivalent circuit and a load impedance as shown in Figure 1.

Figure 1. Linear Thévenin's equivalent circuit with variable source and load matching resistances

Low-pass filters used for intentional frequency spectrum discrimination are basically an analog of a damped spring-mass system of any order from two up, either simple or complex, linear or non-linear. These filters can be described mathematically by four functions or figures of merit:

$$|Z_O| = |z(w, \xi, E)| \quad (1)$$

$$\theta_O = (w, \xi) \quad (2)$$

$$A = A_1 a(\xi, w, \theta_O) \quad (3)$$

$$\xi = Z(|Z_O|, R_S, R_1) \quad (4)$$

where $|Z_O|$ is the magnitude of the Characteristic Impedance Function, θ_O is the Output Phase Angle Function referred to the input, A is the Normalized Amplitude Transfer Function, ξ is the Damping Factor Ratio Function, w is the Fourier Series data frequencies, E is the reactive circuit elements, R_S is the variable matching source resistance, and R_1 the variable matching load resistance.

Normally only the circuit and filter designers will be concerned with the nominal value of $|Z_O|$; however, both the designer and user must be concerned with θ_O , A , and the resulting ξ . The nominal value of $|Z_O|$ is determined primarily by the reactive filter elements, E , which subsequently influence all other filter characteristics. The exact value of $|Z_O|$ is determined by the three factors w , ξ , and E . As will become evident later, the final value of $|Z_O|$ must be determined by performance criteria imposed by the intended use.

A series of curves illustrating the effect of w on the Characteristic Impedance, with the passive elements held constant, are shown in Figure 2. Particular study should be made of impedance changes in the pass band and of the slope of the change near the stop band since changes in $|Z_0|$ are complex and result in phase angle changes

Figure 2. Variations of $|Z_0|$ for a low pass m derived filter, invert β for high pass filter curves

The Output Phase Angle Function referred to the input, θ_o , is primarily dependent upon w and ξ and can produce any value between zero and π radians for a two-section filter at a given desired frequency, depending upon these two variables. Terms commonly used in describing the effect of the Phase Angle Function on the data signal are Envelope or Group Delay (τ) and, what is perhaps more descriptive, Real Time Delay (T). These terms are defined as:

$$T = \frac{\theta_o(\text{rad})}{w} \text{ seconds} \quad (5)$$

and

$$\tau = \frac{d\theta_o(\text{rad})}{dw} \text{ seconds} \quad (6)$$

where T is the Real Time Delay of the filter at a given frequency and τ is the instantaneous slope of the Phase Angle-Frequency Curve. Figure 3 illustrates the effect of ξ on this curve. It should be noted here that when τ equals a constant, or phase is proportional to frequency, there exists a constant Real Time Delay at all frequencies. Figure 4 illustrates diagrammatically the effect on a square transient type pulse, described by the first, third, and fifth harmonics, when the Fourier components are subjected to the same and different time delays. It is not difficult to see from this figure that in complex frequency information it is not unusual for the Phase Function to introduce more amplitude and pulse duration error than the Amplitude Transfer Function.

Figure 3. Approximate second-order phase characteristics for varying damping (ξ)

Figure 4. Effect of time delays on the first, third, and fifth Fourier components of a square transient single pulse; (a) ideal input, (b) output with all harmonics delayed equally in time, (c) distorted output due to varying time delays (phase distortion)

The Normalized Amplitude Transfer Function is, of course, a function of all passive equivalent circuit elements, but is primarily determined by the selected damping factor, frequency, and phase where peak amplitudes are of interest instead of rms values. In the common filter types it is possible to adjust ξ so that the Amplitude-Frequency Curve varies from a peaked "flat response" to a "Gaussian shape," and then to a near "constant slope" curve from DC to maximum attenuation.

The curves presented in Figure 5 illustrate the changes in the Normalized Amplitude Transfer Function-Frequency Curves with varying ξ . A comparison of the curves of Figures 3 and 5 shows that an approximately linear phase curve occurs when the amplitude curve is Gaussian, or bell shaped, and the damping factor ratio is approximately 0.7 of critical.

The filter system Damping Factor is a function of $|Z_0|$ and, more importantly, of the source and load resistances, R_s and R_l ; ξ is the single common influencing factor in the filter function Eqs. (1), (2), (3), and (4). The Damping Factor Ratio Function has the distinction of being the only filter parameter that is readily variable by virtue of R_s and R_l . Caution should be exerted, however, in varying ξ since it does affect the other equations. A study of the effects of ξ on each function in Figures 3 through 8 shows how these interrelated functions can influence test data.

Figure 5. Normalized Amplitude-Frequency Ratio Curve with varying damping for second order system

a. input pulse with duration equal to $5/f_0$

b. Maximally flat response, also see Figure 53

c. Maximally flat response with linear phase in pass band only

d. Gaussian response with linear phase to at least $2f_0$

Figure 6. Filter effects on actual test signals

Figure 7. Maximally-flat filter characteristics of existing 7300 instruments

Figure 8. Transient, Gaussian, filter characteristics of existing 7300 shock instruments

No matter what the filter input data origin (tape recorder, transducer-carrier system, shock test, telemetry, sine or random vibration test, etc.), the significant data information can be classified as:

1. periodic sinusoidal, where the fundamental frequency and amplitude are of prime concern;
2. single complex transient, where envelope and duration are of principal interest; or
3. periodic complex transient, where all data characteristics are important.

Selection of the optimum filter characteristics should be based on these three data categories with the realization that filter-type selection must be determined principally by the θ_0 , A , and ξ functions since little can be done by the user to change the nominal value of $|Z_0|$.

When the significant data are periodic sinusoidal (that is, when the fundamental sinusoidal frequency and amplitude are of prime interest), the user would like to have a filter Normalized Transfer Function with zero slope in the pass band and an infinite negative slope at the cutoff frequency (ω_0). The cutoff frequency is the frequency at which the transition from pass band to stop band is considered to start, and the Phase Angle Function equals 90 degrees in a second-order system.

In actual practice it is possible to select a cutoff frequency which permits sufficient attenuation of harmonics and noise. At the same time, it is possible to adjust the Damping Factor so that the Transfer Function Curve is flat within ± 1 to 2 percent to at least $0.8 \omega_0$ (Figure 5). The Phase Angle Function is normally of no concern when sinusoidal information is sought unless time correlation studies are to be conducted between two or more sources (e.g., mechanical impedance studies). If time correlation is desired, the shape of the Phase Angle-Frequency Curve is not important, but all channels being correlated must have the same Phase Function or, stated differently, must have equal instantaneous Real Time Delays. For this type of data a maximally flat filter is normally used. Typical curves for this type of filter are shown in Figure 7.

When the significant data are single complex transient data, retention of the input signal envelope shape is of utmost importance. The envelope amplitude, duration, rise and fall times, and area as well as the relative zero voltage crossover times are of particular interest. It is obvious from Figures 4 and 5 (where $0.3 \leq \xi \leq 3$) that a nonlinear Phase Angle Function-Frequency Curve can cause considerably more error than significant changes in flatness of the Transfer Function-Frequency Curve.

Theoretically it is possible to have a +78.2-percent amplitude error in a square wave composed of only the first, third, and fifth harmonics if their relative Real Time Delay is such that all first peaks of these Fourier Series components are additive. For envelopes whose Fourier Series is composed of high-amplitude, high-harmonic

components, it is important to maintain a linear Phase-Frequency Curve to at least twice the filter cutoff frequency. Mathematically what has been said is that:

$$T = \frac{\theta_0}{w} = k_1, \quad 0 \leq w \leq 2w_0 \quad (7)$$

or

$$\tau = \frac{d\theta_0}{dw} = k_2 \quad (8)$$

where k_1 and k_2 are constants. Both of these expressions result in the same relative positioning of all significant Fourier components on the output as they were at the input.

If phase linearizing is accomplished by Damping Factor adjustment only, caution should be exercised to ensure that the filter cutoff frequency is not lowered too far and that too long a system time constant is not generated; otherwise, rise and fall times will be increased, and the pulse duration and zero voltage crossover points will be changed. A good "rule of thumb" for adjusting ξ is to allow one cycle of overshoot on a near-infinite slope input step function. This adjustment will limit the peak overshoot to less than 5 percent of the step function amplitude and will result in a ξ of approximately 0.7. Figure 6 illustrates the effect on an actual signal of a transient (Gaussian), and maximally flat filter.

When the significant data are periodic complex transient data, the necessary features of both periodic and transient filters have to be combined to obtain optimum filtered results. There will necessarily have to be tradeoffs made in pass band flatness, cutoff frequency, and phase linearity, with the choice being made by the user and depending upon what information is most desirable. Typical curves for maximally flat and Gaussian filters are shown in Figures 7 and 8.

In summary, then, before any instrumentation system is used in data retrieval, particularly one consisting of filter networks with designed cutoff frequencies, the user should determine the characteristics of the anticipated signal information and the desired circuit filter characteristics and effects. On the basis of the three general data information categories, the user should decide which of the two basic filter types he needs, Gaussian or maximally flat response. With this criteria he should then select his specific instrumentation (Figures 7 and 8). For periodic type data--or for random data if rms values are desired--a maximally flat filter characteristic should be selected with a cutoff frequency and stop band attenuation characteristics such that undesired harmonics are eliminated. If single pulse transient-type information is of interest, a transient or Gaussian filter network should be selected such that a proper phase function is realized, and relative harmonic time displacement is maintained from input to output.*

*For information on transient circuitry quality tests, see "Mechanical Shock Tests: Instrumentation Fidelity Check," Sandia Corporation Standard Environment Test Methods, SC-4452C(M).