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FINAL SPECIFICATIONS FOR THE SEFOR EXPERIMENTAL PROGRAM

L.D. NOBLE
C.D. WILKINSON

U.S. ATOMIC ENERGY COMMISSION
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ADVANCED PRODUCTS OPERATION

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ABSTRACT

The SEFOR experimental program is comprised of a series of tests beginning with low power physics tests, proceeding through static and frequency response tests at various power levels up to full power, and culminating in a series of sub-prompt and super-prompt critical transients.

The work reported here describes an initial sequence of tests, detailed specifications of the number of tests, and the reactor operating conditions (such as power temperature, flow rates, and reactivity changes) for each test, as well as detailed specification of data reduction and interpretation procedures and estimated results for each of the tests.

1.0 INTRODUCTION

The Southwest Experimental Fast Oxide Reactor (SEFOR) is a 20 MW(t) fast spectrum reactor fueled with mixed $\text{PuO}_2\text{-UO}_2$ and cooled with sodium. SEFOR will have characteristics similar to large, soft spectrum fast breeder reactors fueled with mixed $\text{PuO}_2\text{-UO}_2$. Economic studies of these large power reactors indicate significant potential for producing low cost power. SEFOR will be used to obtain physics and engineering data at fuel compositions, temperatures, and crystal-line states characteristic of power reactor operating conditions. SEFOR is particularly designed for the systematic determination of the Doppler coefficient of reactivity at temperatures up to the vicinity of fuel melting.

The SEFOR Project consists of two major parts: the design and construction of the reactor and a related research and development program. Funds for the design and construction of the facility are being provided by the Southwest Atomic Energy Associates (a group of seventeen investor-owned utility companies located in the South and Southwest part of the United States), together with the Karlsruhe Laboratory of the Federal Republic of Germany, Euratom, and the General Electric Company.

The United States Atomic Energy Commission is supporting the research and development program. The R and D Program consists of two phases:

Phase I - Pre-Operational Research and Development

Phase II - Post Construction Research and Development

Work performed in Phase I of the program under Task 1.1, Specifications for the Reactor, and Experiment Equipment, has been reported in reference 1 and 2.

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This report describes work performed in Phase I of the program under Task 1.2, Final Specifications of the Experimental Program, in the period November, 1965 through April, 1967. The purpose of Task 1.2 was to conduct detailed analysis necessary for preparing, and to prepare, final specifications of the experimental program with known reactor and equipment specifications.

SECTION II

EXPERIMENTAL PROGRAM SUMMARY

2.1 GENERAL EXPERIMENTAL PROGRAM OBJECTIVES

The objectives of the SEFOR experimental program are to:

- a. Investigate the operating characteristics of sodium-cooled, mixed $\text{PuO}_2\text{-UO}_2$ fueled fast spectrum reactors.
- b. Investigate and measure the Doppler reactivity effect during normal reactor operating conditions.
- c. Investigate and measure the Doppler reactivity effect during abnormal reactor transient conditions, and in particular, to determine its relation to inherent reactor shutdown capability.

The Doppler reactivity effect is one of several significant reactivity feedback effects present in SEFOR, which in the general case, act simultaneously. Thus, to evaluate the Doppler effect quantitatively, it is necessary that the experimental program objectives include investigation and measurement of all other significant reactivity effects.

2.2 General Program Description

The experiments which have been planned for the first core of the SEFOR reactor are described in detail in the following sections. Similar tests, but fewer in number, will be performed in the second core. Earlier descriptions of the experimental program appear in references 1 to 5.

The tests have been broken down into the following four broad classifications: 1) zero power tests, 2) static tests, 3) oscillator tests (conventional and balanced), and coolant flow oscillations, and 4) transient tests. Except for the balanced oscillator tests and the sinusoidal coolant flow oscillations, all of the proposed experiments are the same, or similar, to tests which have been successfully performed on other reactors. The balanced oscillator tests are not essential to the successful execution of the SEFOR experimental program. However, they may provide additional, independent checks on the analysis of other experiments, and the balanced oscillator tests will be attempted on the SEFOR reactor.

1. The zero power tests are conducted at low (zero) power and include conventional tests such as the approach to critical, control rod calibration, power distribution measurements, and material worth measurements. In addition, it is planned that the isothermal temperature coefficient of reactivity will be measured by using an external (non-nuclear) heat source, and that the ratio of the prompt neutron lifetime to the delayed neutron fraction (ℓ/β) will be determined by noise analysis methods.

2. In the static tests, the reactor is operated at a steady-state condition at a number of different reactor conditions (i.e. at different power levels, flow rates, and coolant inlet temperatures). The planned test conditions were selected so that the "static" reactivity effects associated with 1) a change in power at constant average coolant temperature, 2) a change in average coolant temperature at constant power, and 3) a change in outlet temperature at constant power and average coolant temperature can be separately determined.

3. Two types of oscillator tests are planned. These are:
 A) Conventional (including zero power) oscillator tests in which a poison rod is oscillated at several frequencies and the reactor power and coolant temperatures are allowed to oscillate about their mean value. B) Balanced oscillator tests (of which there are two types⁽⁶⁾) in which a poison rod, the primary coolant flow rate, and the secondary coolant flow rate are simultaneously oscillated. The relative phase angle and amplitudes of the reactivity and flow oscillations are adjusted so that one of the following conditions is satisfied.

- a). The inlet and outlet core coolant temperatures remain constant. This is a first balanced oscillator test.
- b). The reactor power remains constant. This is called a second balanced oscillator test.

The primary and secondary coolant flow will be independently oscillated, with no attempt to "balance" the power or coolant temperature. These oscillations will serve to demonstrate reactor stability

to flow (temperature) oscillations and will be used to predict the relative phase angles and amplitudes of oscillations required for the balanced oscillator tests.

4. The transient tests include sub-prompt critical tests (rapid insertion of $< \$1.0$ positive reactivity) and super-prompt critical tests (rapid insertion of $> \$1.0$ positive reactivity). These tests will be performed at different initial power levels and will allow determination of the reactivity-energy coefficient over a range of fuel temperatures.

2.2.1 Initial Sequence of Tests

A sequence of tests that can be used during the first portion of the planned experimental program is outlined below. It is expected that the initial tests will be followed by the balanced oscillator tests and the remainder of the static tests which are described in this report. These would, in turn, be followed by the sub-prompt critical tests and finally by super-prompt critical tests.

The initial test sequence describe below provides:

A. A safe approach to full power..

Static and oscillator tests are performed at lower power before proceeding to higher power levels, and checks of previous tests are periodically performed in order to observe any changes (due to fuel pellet cracking, for example) that may occur as a result of operation at high power.

B. An overall understanding of the reactor at a reasonably early stage in the experimental program. Upon completion of the initial tests, the reactor will have been operated over essentially the complete range of power levels, flow rates, and coolant temperatures.

C. Information on "as fabricated" fuel.

Several tests are performed while maintaining the power level at or below 5 MW. The peak fuel temperature is low ($\leq 1200^{\circ}\text{F}$) during these tests and the fuel pellets remain essentially "as fabricated." After operation at high power, these tests may be repeated in order to observe any changes that occur.

D. Information for predicting the relative phase angles and amplitudes required for balanced oscillator tests.

This information is obtained from the flow and reactivity oscillations at 5 and 15 MW.

The outline of the initial test sequence is given in Table 1-1. The specified tests and the general procedure for each test are described in other sections of this report. The outline is not intended to serve as a detailed procedure or description of the exact sequence of tests, but rather to serve as a guide for accomplishing points A through D above.

TABLE 1-1

INITIAL TEST SEQUENCE

$\bar{T}_c$ (average coolant temperature) $\approx$ 760°F unless otherwise indicated

<u>POWER (MW)</u>	<u>TESTS</u>
0	Perform the zero power tests described in Section 3.0; Perform the zero power oscillator test ($\bar{T}_c \approx 400^\circ\text{F}$ to $\sim 700^\circ\text{F}$)
1 $\rightarrow$ 2	Obtain static equilibrium; Obtain a heat balance; Perform a conventional oscillator test
5	Obtain static equilibrium at 30, 50, and 100% flow; Obtain a heat balance; Measure the reflector temperature coefficient of reactivity; Perform conventional oscillator tests at 30, 50, and 100% flow; Perform primary and secondary coolant flow oscillations at 30% flow; Obtain static equilibrium at 3 coolant temperatures between $\sim 500^\circ\text{F}$ and 760°F
1	Check the previous 1(2) MW static and oscillator test results
5	Check a previous 5 MW static equilibrium condition
10	Obtain static equilibrium at 50 and 100% flow; Obtain a heat balance; Measure the reflector temperature coefficient of reactivity; Perform conventional oscillator tests at 50 and 100% flow
5	Check a previous 5 MW static and oscillator test result
10	Check a previous 10 MW static equilibrium condition
15	Obtain static equilibrium at 60 and 75% flow; Obtain a heat balance; Measure the reflector temperature coefficient of reactivity; Perform a conventional oscillator test at 75% flow; Perform reactivity and primary and secondary coolant flow oscillations at 60% flow
10	Check a previous 10 MW static and oscillator test result
15	Check a previous 15 MW static equilibrium condition
19	Obtain static equilibrium at 95% flow; Perform a conventional oscillator test at 95% flow
20	Obtain static equilibrium; Obtain a heat balance; Measure the reflector temperature coefficient of reactivity.
10 or 15	Check a previous static and oscillator test result

2.2.2 Power Calibration

It is expected that the reactor power level in SEFOR will be measured primarily by means of calibrated neutron flux monitors. These monitors will be calibrated by means of a reactor heat balance which relies on measurements of the coolant flow rate and the coolant temperature rise across the reactor. The estimated error in the absolute power calibration at high ($\geq 5 \rightarrow 10$ MW) power levels is $\sim \pm 5\%$, (e.g. references 1 and 2).

It has been assumed throughout this report that the flux monitors are calibrated in terms of the actual energy, including the equilibrium fission product heating, that is produced in the core. (See Appendix II for a discussion of the relation between the energy produced and the energy absorbed in various regions of the reactor.) The actual calibration used in the SEFOR experiments is unimportant. However, the relationships described in Appendix II should be kept in mind when the energy distribution in the reactor (e.g. the fraction of the energy that is absorbed in the fuel) is of interest.

2.2.3 Reproducibility

The expected reproducibility of the measurements are as follows:

Fine Reflectors

Reproducibility $\leq \pm 0.5\%$ per reflector

Coarse Reflectors

Reproducibility $\pm 0.5\%$ per reflector

Coolant Temperatures

Reproducibility $\pm 3^\circ\text{F}$ in the core temperature rise at full power (120°F core ΔT). The core outlet temperature measured by the instrumented fuel assemblies may be "noisy" due to mixing in the cross flow area.

Power

The flux monitors will accurately reproduce the same current for the same measured flux level. However, in order to account for the different time behavior of the "fission" heating and the fission product "decay" heating, a record should be kept of the operating time at each power level and the time required to change from one power level to another. These times can then be used with standard fission product decay heat curves to calculate the actual heat produced in the core. Examples of differences between indicated power levels (i.e. power levels inferred from neutron monitors which are calibrated as described in Section 2.2.2) and the actual power being produced in the core are shown in Table 1.2. In calculating the values shown in this table, it was assumed that the reactor was rapidly brought from a low power to the "initial" power level; that the indicated power level was held constant for two hours and that the reactor power was then rapidly changed to the new power, and the indicated power level was held constant for 10 minutes.

TABLE 1-2

EFFECT OF FISSION PRODUCT DECAY HEAT

<u>Indicated Initial</u> <u>Power Level After</u> <u>2 Hours Operation</u> (MW)	<u>Actual Initial</u> <u>Power Produced</u> <u>in Core After</u> <u>2 Hours Operation</u> (MW)	<u>Indicated New</u> <u>Power Level After</u> <u>10 Minutes Operation</u> (MW)	<u>Actual New</u> <u>Power Produced</u> <u>in Core After</u> <u>10 Minutes Operation</u> (MW)
6.00	5.92	4.00	3.97
4.00	3.95	6.00	5.90
1.00	0.99	10.00	9.77
10.00	9.87	1.00	1.09

Fuel Thermocouples

The thermocouple junction should accurately produce the same current for the same junction temperature. However, the relationship between the thermocouple junction temperature (fuel temperature) and power may change when the fuel temperature is raised above the sintering temperature ($\sim 3000^{\circ}\text{F}$). The change in this relationship above the sintering temperature depends on the fuel time-temperature history. It is expected that an equilibrium fuel structure in a fuel pellet can be generated within ~ 40 hours at fuel temperatures above $\sim 4000^{\circ}\text{F}$. The relationship between the thermocouple junction temperature and the measured power level in such a pellet should remain unchanged thereafter. Tests (e.g. reference 7 - Section 6.1.3.2)

indicate that after sintering occurs, the junction temperature for the same power level may decrease by 10% for temperatures in the range 4000 → 4500°F, and that the junction temperature may increase by 15% for temperatures in the 1500°F range.

Flow Meters

The expected reproducibility of the flow meters in the instrumented fuel assemblies is $\pm 5\%$ for flow rates above 25 gpm (full flow = 60 gpm) and $\pm 10\%$ for flow rates in the 10 → 25 gpm range. The expected reproducibility of the Venturi flow meter and the large electromagnetic flow meter in the primary coolant inlet line is of the order of $\pm 1\%$.

The actual reproducibility of the instrumentation and equipment in SEFOR should be checked by performing the same tests a number of times.

2.3 Individual Reactivity Effects

Analyses indicate that feedback reactivity changes in SEFOR will be primarily dependent on the Doppler (fuel temperature), core average coolant temperature, and the core structure temperature. Other feedback reactivity effects, believed to be of secondary importance include fuel axial expansion (fuel temperature), fuel-clad axial expansion, and reflector temperature effects.

Determination of the Doppler effect, and hence, the Doppler reactivity coefficient, is a primary objective of the SEFOR experimental program. Evaluation of a single feedback effect is complicated by the fact that most reactor experiments involve several feedback effects simultaneously. Thus, the SEFOR experimental program is designed to evaluate the composite reactivity feedback over the range of reactor conditions of interest in such a manner that the individual effects, and hence, the Doppler effect, may be separated or "backed out" of the experimental data. Separation of the various reactivity effects cannot be accomplished exactly because of practical experimental limitations and imperfect prediction of theoretical models for the various effects. Fortunately, in the case of SEFOR, different types of experiments (e.g. static, oscillator, and transient) can be used to obtain independent evaluations of some of the more important feedback effects.

Recommended models for each of the reactivity feedback effects are described in the following sections. It is expected that these feedback models will adequately describe SEFOR behavior so as to afford a good fit to the experimental data.

2.3.1 Doppler Effect

Calculations⁽⁷⁾ have shown that for SEFOR the Doppler effect, Δk_D , may be closely approximated by:

$$\Delta k_D = \frac{\int P^2(r) f(r) dV}{\int P^2(r) dV} \quad (2.3-1)$$

where: $P(r)$ is the local power density, and where the range of integration extends over the fuel volume.

$f(r)$ = function of the change in local fuel temperature, $T_f(r)$.

A local Doppler coefficient, Γ , may be defined such that:

$$f(r) = \int_{\Delta T(r)} \Gamma dT_f = \int_{\Delta T(r)} \left[\frac{dk}{dT} \right]_f dT_f = \Delta k_D(r) \quad (2.3-2)$$

The local Doppler coefficient may be represented as:

$$\Gamma = \frac{A}{T_f^m} \quad (2.3-3)$$

Substitution and integration over a change in fuel temperature from $T_{f1}(r)$ to $T_{f2}(r)$ gives:

$$\Delta k_D = \frac{A}{1-m} \frac{\int_{V_{Fuel}} P^2(r) \Delta [T_f(r)]^{1-m} dV}{\int_{V_{Fuel}} P^2(r) dV} \quad (m \neq 1) \quad (2.3-4a)$$

$$\equiv A F_m(\Delta T_c, \Delta P)$$

where: $F_m(\Delta T_c, \Delta P)$, [which can be calculated by the WEDOP code⁽¹²⁾], depends on the change in local fuel temperature (and thus on the change in average coolant temperature and power level) as well as the functional form (the value of m) that is assumed for the local Doppler effect.

If $m=1$, (2.3-4a) reduces to:

$$\begin{aligned} \Delta k_D &= \frac{A \int_{V_{Fuel}} P^2(r) \ln \frac{T_{f2}(r)}{T_{f1}} dV}{\int_{V_{Fuel}} P^2(r) dV} \quad (m=1) \quad (2.3-4b) \\ &= A F_1(\Delta T_c, \Delta P) \end{aligned}$$

It is also planned to represent the Doppler effect by a less complicated form that is more useful for reactor operating purposes. Calculations indicate that one of the following forms may be useful in this regard:

$$\Delta k_D = \frac{A}{1-m} \Delta [T_f^{1-m}] \quad (m \neq 1) \quad (2.3-5)$$

$$\Delta k_D = A \ln \frac{T_{f2}}{T_{f1}} = A \ln \left[1 + \frac{\Delta T_f}{T_{f1}} \right] \quad (m=1) \quad (2.3-6)$$

where: T_f = effective fuel temperature = $T_c + R_f P$

ΔT_f = change in effective fuel temperature = $\Delta T_c + R_f \Delta P$

and where:

T_c = average coolant temperature

R_f = change in effective fuel temperature with respect to power

For small changes in power and average coolant temperature about a reference condition (T_{co}, P_o),

$$\Delta k_D \approx C_D \Delta T_f = C_D \Delta T_c + \alpha_D \Delta P \quad (2.3-7)$$

where (for the simple model)

$$C_D \equiv \frac{A}{(T_{co} + R_f P_o)^m}$$

$$\alpha_D \equiv \frac{A R_f}{(T_{co} + R_f P_o)^m} \quad (2.3-8)$$

or (for the more detailed model)

$$C_D \equiv A \cdot \left. \frac{\partial F_m}{\partial T_c} \right|_{T_{co}, P_o} \quad (2.3-9)$$

$$\alpha_D \equiv A \cdot \left. \frac{\partial F_m}{\partial P} \right|_{T_{co}, P_o}$$

2.3.2 Fuel Axial Expansion

The fuel in SEFOR is made of "dished" pellets and the fuel axial expansion, therefore, depends upon the temperature of the fuel at the shoulder of the dish. The "shoulder temperature" can be expressed as a linear combination of the fuel surface temperature and the average fuel temperature. Thus, the reactivity effect ΔK_e due to the fuel axial expansion can be written as:

$$\Delta K_e = C_e \Delta T_e = C_e [a \Delta T_{fs} + (1-a) \Delta \bar{T}_f] \quad (2.3-10)$$

where:

C_e = fuel axial expansion reactivity coefficient ($\text{¢}/^\circ\text{F}$)

ΔT_e = change in fuel temperature at the inner edge of the shoulder ($^\circ\text{F}$)

a = 0.65 for an 0.875 inch diameter pellet with a 40 mil shoulder

ΔT_{fs} = change in fuel surface temperature ($^\circ\text{F}$)

$\Delta \bar{T}_f$ = change in average fuel temperature ($^\circ\text{F}$)

For a static change, the changes in fuel surface temperature and in average fuel temperature can be expressed in terms of changes in power and average coolant temperature by:

$$\Delta T_{fs} = K_s \Delta P + \Delta T_c \quad \text{and} \quad \Delta \bar{T}_f = K_{av} \Delta P + \Delta T_c \quad (2.3-11)$$

where:

ΔP = power change (MW)

ΔT_c = average coolant temperature change ($^{\circ}\text{F}$)

K_s = change in fuel surface temperature for a small power change at a constant average coolant temperature.

K_{av} = change in average fuel temperature for a small power change at a constant average coolant temperature.

The quantities K_s and K_{av} are related to the "effective" heat transfer coefficient, h_f (based on fuel surface area) between fuel surface and coolant, the "average" fuel conductivity k , the fuel pellet radius R , the fuel length L , and the number of fuel rods N , by the following expressions.

$$K_s = \frac{A_f}{2 \pi h_f R N L} = \frac{A_f R}{2 h_f V_f} \quad (2.3-12)$$

$$K_{av} = K_s + \frac{A_f}{8 \pi N L k} = K_s + A_f \frac{R^2}{8 k V_f}$$

where:

V_f = fuel volume

A_f = fraction of power which is absorbed in the fuel

The reactivity effect due to fuel axial expansion can thus be expressed as:

$$\Delta K_e = C_e [aK_s + (1-a)K_{av}] \Delta P + C_e \Delta T_c \quad (2.3-13)$$

2.3.3 Coolant Temperature Effect

It is planned to represent the average coolant temperature effect by:

$$\Delta k_c = C_c \Delta T_c \quad (2.3-14)$$

where the coefficient C_c includes the reactivity effects of sodium density changes in the core as well as temperature coefficients associated with the expansion of core structure; i.e. structure expansion effects should depend primarily on the average coolant temperature.

Calculations⁽⁸⁾ indicate that there will be a small (compared to average coolant temperature effect) reactivity effect associated with sodium density changes in the sodium between the core and the vessel. This effect should depend primarily on the inlet sodium temperature, T_i , and may be represented by:

$$\Delta K_i = C_i \Delta T_i \quad (2.3-15)$$

2.3.4 Fuel-Clad Axial Expansion

The temperature of the clad depends on the fuel surface temperature, T_{fs} , as well as the coolant temperature. The reactivity effect of the fuel-clad axial expansion may thus be represented by:

$$\Delta k_t = C_t \Delta T_t = C_t [b \Delta T_{fs} + (1-b) \Delta T_c] \quad (2.3-16)$$

where:

$b < 1$ depends on the fuel gap coefficient.

From equation (2.3-11), this can be written as:

$$\Delta K_t = b C_t K_s \Delta P + C_t \Delta T_c \quad (2.3-17)$$

for a static change in power and average coolant.

2.3.5 Reflector Temperature Effect

The change of reactivity, Δk_r , for a change in the average temperature of the reflector, should be of the form:

$$\Delta k_r = C_r \Delta T_r \quad (2.3-18)$$

where C_r is the reflector temperature coefficient.

2.4 REACTIVITY MODEL FOR STATIC TESTS

Using the expressions for the individual reactivity effects that are given in Section 2.3, the composite reactivity feedback may be written (using the simple logarithmic form of the Doppler feedback) as:

$$K_f = K_o + A \ln [T_c + R_f P] + C_p P + C_q T_c + C_i T_i + C_r T_r \quad (2.4-1)$$

where:

$$K_o \equiv -A \ln [T_{co} + R_f P_o] - C_p P_o - C_q T_{co} - C_i T_{io} - C_r T_{ro} \quad (2.4-2)$$

where the subscript (o) denotes a reference power or temperature, and where (in terms of the parameters of Section 2.3)

$$C_p \equiv C_e [aK_s + (1-a)K_{av}] + bC_t K_s \quad (2.4-3)$$

$$C_q = C_c + C_t + C_e \quad (2.4-4)$$

A change in reactivity between a measured condition and the reference condition can be written as:

$$\Delta K_f = A \ln \left[1 + \frac{\Delta T_c + R_f \Delta P}{T_{co} + R_f P_o} \right] + C_p \Delta P + C_q \Delta T_c + C_i \Delta T_i + C_r \Delta T_f \quad (2.4-5)$$

2.4.1 Change in Reflector Temperature

If the reflector temperature is changed while the power and coolant temperature are held constant, the change in reactivity between two steady-state conditions is given by:

$$\Delta K_f = C_r \Delta T_r \quad (2.4-6)$$

2.4.2 Inlet Temperature Effects

The inlet temperature reactivity coefficient may be measured by holding the power and average coolant temperature constant. In this case:

$$\Delta K = \Delta K_f - C_r \Delta T_r = C_i \Delta T_i \quad (2.4-7)$$

where ΔK is the measured reactivity change, corrected for any reactivity effect due to changes in the reflector temperature.

2.4.3 Tests at Constant Coolant Temperature

When the average coolant temperature is held constant, equation (2.4-5) reduces to:

$$\Delta K = \Delta K - C_i \Delta T_i - C_r \Delta T_r = A \ln \left[1 + \frac{R_f \Delta P}{T_{co} + R_f P_o} \right] + C_p \Delta P \quad (2.4-8)$$

where ΔK is the measured reactivity change, corrected for any reactivity effects due to change in inlet temperature or reflector temperature.

If the more detailed model for the Doppler effect is used (see Section 2.3.1):

$$\Delta K = A F_m (T_{co}, \Delta P) + C_p \Delta P \quad (2.4-9)$$

For small changes in power at a constant coolant temperature:

$$\Delta K \approx (\alpha_D + C_p) \Delta P \equiv \alpha_f \Delta P \quad (2.4-10)$$

where α_f is the power coefficient (defined by this equation) and where α_D (the Doppler power coefficient) is defined in Section 2.3.

2.4.4 Tests at Constant Power

When the power is held constant, the feedback can be written in the form:

$$\Delta K = A \ln \left[1 + \frac{\Delta T_c}{T_{co} + R_f P_o} \right] + C_q \Delta T_c \quad (2.4-11)$$

of if the more detailed Doppler model is used, as:

$$\Delta K = A F_m (\Delta T_c, P_o) + C_q \Delta T_c \quad (2.4-12)$$

where ΔK is defined as before.

For small changes in coolant temperature:

$$\Delta K \approx (C_D + C_q) \Delta T_c \quad (2.4-13)$$

where C_D (the Doppler temperature coefficient) is defined in Section 2.3.

The calculated⁽⁸⁾ reactivity coefficients for the static and oscillator reactivity models at 10 MW and 760°F coolant temperature are shown in Table 2-1. As indicated, it is expected that the dominant effects will be due to the power coefficient, the average coolant temperature coefficients, and the fuel temperature coefficient.

TABLE 2-1

CALCULATED COEFFICIENTS (10 MW)

<u>Reactivity Effect or Reactivity Coefficient</u>	<u>Temperature Coefficient ($\text{¢}/^{\circ}\text{F}$)</u>	<u>Power Coefficient ($\text{¢}/\text{MW}$)</u>
Doppler $A = -0.0085 \Delta k/k = -2.66\%$	$C_D = -0.145$	$\alpha_D = -8.12$
Power dependence of fuel and fuel clad expansion	--	$C_P = -0.48$
Fuel Temperature Coefficient	$C_f = -0.149$	--
Composite Power Coefficient	--	$\alpha_f = -8.60$
Composite Average Coolant Temperature Coefficient	$C_q = -0.450$	--
Inlet Temperature	$C_i = -0.029$	--
Reflector Temperature	$C_r = -0.04$	--

2.5 REACTIVITY MODEL FOR OSCILLATOR TESTS

In the derivation of the model for the oscillator tests, it is assumed that the amplitudes of the power, reactivity, temperature, and flow oscillations are sufficiently small, compared to their mean values, that second order terms can be neglected. With this assumption, the mathematical model for the reactor can be reduced to a system of linear equations with constant coefficients. The model for the Fourier transform of these equations is shown below. Results of conventional oscillator tests are analyzed to determine the overall power-reactivity frequency response. The block diagram below shows the analytical model.

Simple Analytical Model for Oscillator Tests

The nomenclature used in the block diagram is defined below.

- Δk_i = input reactivity (oscillator) (dollars)
- Δk_f = feedback reactivity (dollars)
- Δk_n = net reactivity (dollars)
- $\Delta P/P_o$ = fractional change in reactor power about mean power level P_o
- $G_o(j\omega)$ = zero power neutron kinetics frequency response
- $h(j\omega)$ = transform of feedback kernel ($\$/MW$)

Measured quantities are:

$G_o(j\omega)$ = measured at zero power (0.01 MW)

$$\frac{\Delta P}{P_o \Delta K_i} \equiv G_l(j\omega) = \frac{G_o}{1 - P_o G_o h}$$

Figure 2.5-1 shows a block diagram of a typical analytical model for SEFOR.^(1,2) For clarity, the model is shown for a "point reactor."

Actually, the reactivity effects are distributed over the whole reactor. Possible direct effects of inlet and outlet coolant temperature on Δk have been omitted. The fuel coefficient (C_f) includes both Doppler and fuel axial expansion effects.

The nomenclature used in Figure 2.5-1 is defined as follows:

$G_o(j\omega)$ = zero power frequency response of neutron kinetics

$F_{av}(j\omega)$ = average fuel temperature response to power oscillation

$F_s(j\omega)$ = average fuel surface temperature response to power oscillation (or average fuel temperature response to coolant temperature oscillation)

$F_n(j\omega)$ = average fuel surface temperature response to coolant temperature oscillation

$F_w(j\omega)$ = core temperature rise response to fuel surface temperature oscillation (essentially constant in the range 0.001 to 0.1 cps)

$F_l(j\omega)$ = core inlet temperature response to fuel surface temperature oscillation

$F_m(j\omega)$ = core structure temperature response to average sodium temperature oscillation

$F_r(j\omega)$ = reflector temperature response to core power oscillation (guide structure time constant ≈ 1.5 min.; segment time constant ≈ 18 min.)

Figure 2.5-1. EXPANDED ANALYTICAL MODEL FOR OSCILLATOR TESTS

The $F(j\omega)$ functions described above are normalized so that the zero frequency values are all 1.0. The "K" values associated with each $F(j\omega)$ will vary with power levels, coolant flow rates, and the method of plant control.

2.5.1 Mathematical Model for Oscillator Tests

As shown in Appendix I, the reactivity model for the oscillator tests can be approximated by:

$$\Delta K_f(i\omega) = \alpha_f F_{av}(i\omega) X_f(i\omega) \Delta P(i\omega) + [C_f F_s(i\omega) + C_q] \Delta T_c(i\omega) + C_i \Delta T_i(i\omega) \quad (2.5-1)$$

where:

- $X_f(i\omega)$ = frequency response of the ratio of the total power absorbed in the fuel to the "fission" power (normalized to unity at zero frequency)
- $\Delta P(i\omega)$ = "fission" power oscillation (proportional to neutron flux)
- $\Delta K_f(i\omega)$ = transform of the total feedback oscillation (corrected for any effects due to reflector temperature oscillation)

The function $X_f(i\omega)$ has been calculated by Storrer⁽⁹⁾, and is described in more detail in Appendix II.

The average coolant temperature oscillation, $\Delta T_c(i\omega)$, is defined in terms of the inlet temperature oscillation, $\Delta T_i(i\omega)$, and the outlet temperature oscillation, $\Delta T_o(i\omega)$, by the following:

$$\Delta T_c(i\omega) \equiv F_i(i\omega) \Delta T_i(i\omega) + [1 - F_i(i\omega)] \Delta T_o(i\omega) \quad (2.5-2)$$

where $F_i \sim \frac{1}{2}$ is a frequency dependent correction factor defined in Appendix III.

The reactivity coefficients, α_f , C_f , C_q , and C_i are defined in Section 2.3 (see also Appendix I).

The frequency response functions F_{av} and F_s are defined in Section 2.5. The frequency response functions, which are derived by Caldarola⁽⁶⁾, may be written as follows:

$$F_s = \frac{2J_1(x)/x}{J_0(x) + i\omega\gamma + 2J_1(x)/x} \quad (2.5-3)$$

$$F_{av} = \frac{1}{i\omega(\gamma + v^2/8)} [1 - F_s] \quad (2.5-4)$$

where $J_0(x)$ and $J_1(x)$ are Bessel functions of the first kind of order zero and order one, respectively.

The argument x is defined by: $x = i^{3/2} v \sqrt{\omega}$ (2.5-5)
where ω is the frequency of oscillation in radians per second, and $i = \sqrt{-1}$.

The parameters γ and v can be expressed in terms of fuel density, ρ , heat capacity, c , average conductivity k , fuel pellet radius R , and the effective heat transfer coefficient h_f as follows:

$$\gamma = \rho c R / 2 h_f \quad (2.5-6)$$

$$v^2 = \rho c R^2 / k \quad (2.5-7)$$

From the definitions of K_{av} and K_s given in Section 2.3.2, it follows that:

$$K_{av} = \frac{A_f}{\rho c V_f} (\gamma + v^2/8) \quad (2.5-8)$$

$$K_s = \frac{A_f \gamma}{\rho c V_f} \quad (2.5-9)$$

where A_f is the fraction of the energy which is absorbed in the fuel.

2.5.2 Energy Coefficient

The excess energy, ΔE , that is produced as a result of a sudden power transient is defined by:

$$\Delta E(t) = \int_0^t [P(u) - P_0] dy = \int_0^t \Delta P(u) du \quad (2.5-10)$$

where P_0 is the initial power level. Thus, the Fourier transform, $\Delta E(i\omega)$, of $\Delta E(t)$ is given by:

$$\Delta E(i\omega) = \Delta P(i\omega)/i\omega \quad (2.5-11)$$

Neglecting gamma and neutron heating in the coolant and structure, a reactivity energy coefficient γ_E (¢/MW-sec-produced) can be obtained from the oscillator tests. By definition:

$$\gamma_E \equiv \lim_{i\omega \rightarrow \infty} \frac{\Delta K_f(i\omega)}{\Delta E(i\omega)} = \lim_{i\omega \rightarrow \infty} i\omega \alpha_f F_{av}(i\omega) X_f(i\omega) \quad (2.5-12)$$

$$\gamma_E = \frac{\alpha_f}{(\gamma + \nu^2/8)} X_f(\infty) = 0.93 \alpha_f / (\gamma + \nu^2/8) \quad (2.5-13)$$

where the calculated value $X_f(\infty) = 0.93$ (see Appendix II) has been used.

Only the fraction A_f of the energy is absorbed in the fuel, and as shown in Appendix II, the actual excess energy absorbed in the fuel, ΔE_f , in the first few seconds during a transient is very nearly equal to $A_f X_f(\infty) \Delta E(t)$.

The composite power coefficient α_f is the sum of the Doppler power coefficient, α_D , and the coefficient, C_p , which represents the power dependence of the fuel and fuel-clad axial expansion effect. Thus, the Doppler energy coefficient, η ($\text{¢/MW-sec absorbed in the fuel}$), is given by:

$$\eta = \gamma_E \left(\frac{\alpha_D}{\alpha_f} \right) \left(\frac{\Delta E}{\Delta E_f} \right) \quad (2.5-14)$$

or in terms of "measured" and "known" quantities

$$\eta = \frac{(\alpha_f - C_p)}{A_f (\gamma + v^2/8)} \quad (2.5-15)$$

From previous definitions

$$\alpha_D = (\alpha_f - C_p) = C_D K_{av} = \frac{C_D A_f}{\rho c v_f} (\gamma + v^2/8) \quad (2.5-16)$$

where it is assumed (see Appendix I) that R_f of equation (2.3-8) and K_{av} of equation (2.5-8) are the same. Thus, the Doppler energy coefficient can also be written as:

$$\eta = \frac{C_D}{\rho c v_f} = \frac{A}{\rho c v_f T_{fo}^m} \quad (2.5-17)$$

where T_{fo}^m is the effective initial fuel temperature $T_{co} + R_f P_o$ raised to the m -th power (e.g. Section 2.3).

2.5.3 Conventional Oscillator Tests

In the conventional oscillator tests, the transform of the feedback kernel $h(i\omega) \equiv \Delta K_f(i\omega)/\Delta P(i\omega)$ is determined from the measurement of the zero power reactivity transfer function, G_o , and the at power-reactivity transfer function, G , (measured at a mean power level P), by means of the following expression:

$$h(i\omega) = \frac{1}{P} \left[\frac{1}{G_o(i\omega)} - \frac{1}{G(i\omega)} \right] \quad (2.5-18)$$

where:

$$G_o(i\omega) \equiv (\Delta P/P_o)/\Delta K_{in} \quad - \text{measured at zero power } \sim .01 \text{ MW.}$$

$$G(i\omega) \equiv (\Delta P/P)/\Delta K_{in} \quad - \text{measured at mean power } P$$

and where $\Delta K_{in}(i\omega)$ is the transform of the oscillating reactivity.

The model for $h(i\omega)$ (see the previous section) is given by:

$$h(i\omega) = \alpha_{f_{av}} F_{av}(i\omega) X_f(i\omega) + [C_{fS}(i\omega) + C_q] U(i\omega) + C_i U_i(i\omega) \quad (2.5-19)$$

where:

$$U(i\omega) \equiv \frac{\Delta T_c(i\omega)}{\Delta P(i\omega)} = F_i(i\omega) U_i(i\omega) + [1-F_i(i\omega)] U_o(i\omega) \quad (2.5-20)$$

$$U_i(i\omega) \equiv \frac{\Delta T_i(i\omega)}{\Delta P(i\omega)} \quad (2.5-21)$$

and:

$$U_o(i\omega) = \frac{\Delta T_o(i\omega)}{\Delta P(i\omega)} \quad (2.5-22)$$

The transfer functions G , U_i , and U_o are obtained from "measured" values of $\Delta P(i\omega)$, $\Delta K_{in}(i\omega)$, $\Delta T_i(i\omega)$, and $\Delta T_o(i\omega)$. The procedures for performing these calculations are outlined in Section 5.0.3. The transfer function between average coolant temperature and power, $U(i\omega)$, is then calculated from equation (2.5-20) using U_i , U_o , and calculated values of F_i . (F_i is defined in Appendix II.)

From the "measured" values of $h(i\omega)$, $U(i\omega)$, and $U_i(i\omega)$ and the calculated values of $X_f(i\omega)$, the parameters α_f , C_f , C_q , C_i , γ , and ν are determined in the manner described in Section 5.2.3.2.

The Doppler energy coefficient is calculated from equation (2.5-15) using "determined" values of α_f , γ , and ν , and using calculated values of C_p and A_f .

If it is assumed that:

$$F_{av} = F_s = \frac{1}{1 + i\omega\tau_f} \quad (2.5-23)$$

the two parameters γ and ν are replaced by the one parameter, τ_f . The Doppler energy coefficient is then obtained from equation (2.5-15) by replacing $\gamma + \nu^2/8$ by τ_f .

2.5.4 First Balanced Oscillator

In the first balanced oscillator test, the coolant flow rate and the reactivity are both oscillated. The relative phase angle and amplitudes of the two oscillations are adjusted to hold the inlet temperature and core temperature rise constant. For this experiment, the model for $h(i\omega)$ is given as before by equation (2.5-19). If a perfect balance is achieved, the latter two terms vanish, however, and the model for $h(i\omega)$ reduces to:

$$h(i\omega) = \alpha_f F_{av}(i\omega) X_f(i\omega) \quad (2.5-24)$$

Thus, for a perfect balance, only three parameters, γ , v , and α_f , need be determined in order to fit the model to the measured values of $h(i\omega)$ and calculated values of $X_f(i\omega)$.

In addition to power and reactivity oscillations, the fractional flow oscillation $\Delta V(i\omega)/V_o$ is measured during the first balanced oscillator test. This measurement allows one to determine the frequency response function F_s and thus provides an additional check on the model.

As shown in Appendix III, the ratio of the fractional flow oscillation to the fractional power oscillation can be written as:

$$\frac{\frac{\Delta V(i\omega)}{V_o}}{\Delta P(i\omega)/P} = a_c X_c + (1-a_c) F_s X_f - \frac{P}{\Delta\theta} [(1+b-bF_i)U_o - (1-bF_i)U_i] \quad (2.5-25)$$

where:

$\Delta V(i\omega)$ = transform of the flow oscillation

V_o = mean flow rate

$X_c(i\omega)$ = transform of the ratio of total power absorbed in the coolant and structure to the "fission" power absorbed in the coolant and structure (see Appendix II)

$\Delta\theta$ = steady-state core temperature rise at power P and mean flow rate V_o .

The function $b(i\omega)$ is a calculated function defined in Appendix III and the other quantities are as previously defined.

The latter term vanishes in the event of a perfect balance and equation (2.5-25) reduces to:

$$\frac{\Delta V/V_o}{\Delta P/P_o} = a_c X_c + (1-a_c) F_s X_f \quad (2.5-26)$$

Measured quantities are $\Delta V/V_o$ and $\Delta P/P_o$. Calculated quantities are X_c , a_c , and X_f . The two parameters to be determined are γ and ν (the parameters in the model for F_s).

It should be noted that it has been assumed that the frequency response function $F_s(i\omega)$ in the equation related to reactivity feedback (i.e. equation (2.5-1), (2.5-24), and (2.5-28) below) is the same as the function used in (2.5-25), (2.5-26), and (2.5-31) below).

This assumption is strictly valid only if the fuel "thermal impedance" is constant throughout the core, since the former equations involve an "effective fuel temperature for reactivity feedback" (an importance weighted temperature) while the latter equations involve an "effective fuel temperature for heat transfer" (an "average" temperature).

2.5.5 Second Balanced Oscillator Test

In the second balanced oscillator test, the coolant flow rate and the reactivity are both oscillated. The relative phase angle and amplitude of the two oscillations are adjusted to hold the power constant.

$$\text{Since: } \frac{\Delta P(i\omega)}{G_o(i\omega)} = P [\Delta K_{in}(i\omega) + \Delta K_f(i\omega)] \quad (2.5-27)$$

the transfer function between the reactivity oscillation, ΔK_{in} , and the average coolant temperature oscillation, ΔT_c , is given by:

$$\frac{\Delta K_{in}}{\Delta T_c} = -(C_f F_s + C_q) - C_i \frac{\Delta T_i}{\Delta T_c} + \left(\frac{1}{P G_o} - \alpha_f F_{av} X_f \right) \frac{\Delta P}{\Delta T_c} \quad (2.5-28)$$

where, as before, ΔT_c is defined in terms of the measure inlet and outlet temperature oscillations and the calculated factor F_i as follows:

$$\Delta T_c = F_i \Delta T_i + (1-F_i) \Delta T_o \quad (2.5-29)$$

The measured quantities are ΔK_{in} , ΔT_i , and ΔT_o . The calculated quantity is F_i and the parameters to be determined are C_f , C_q , C_i , γ , and ν .

2.5.6 Additional Measurements

In either the conventional oscillator test or the balanced oscillator tests an additional check on the model and on the parameters γ and ν can be obtained by analyzing the central fuel temperature response (measured by the instrumented fuel assemblies) to a power oscillation. The model for the measured transfer function is given by:⁽⁶⁾

$$\frac{\Delta T_{cf}(i\omega)}{\Delta P(i\omega)} = K_c F_c(i\omega) + G_c U(i\omega) \quad (2.5-31)$$

where K_c is the static temperature change in the center fuel temperature for a power change at constant coolant temperature and where $U(i\omega)$ is the response of the average coolant temperature to a power oscillation as defined in Section 2.3.3.

The models for the frequency response functions, F_c and G_c , can be defined in terms of the parameters γ and ν (see Section 2.3.2), and the frequency response function, F_s , (see Section 2.3.2) as follows:

$$G_c(i\omega) \equiv \frac{x F_s}{2J_1(x)} \quad (2.5-32)$$

and:

$$F_c(i\omega) = \frac{1}{i\omega(\gamma + \nu^2/4)} [1 - G_c] \quad (2.5-33)$$

and where x and $J_1(x)$ are defined in Section 2.3.2.

Measurements of $\Delta T_{cf}/\Delta P$ and U can be used with the above expression to obtain K_c and an independent check on γ and ν .

2.6 REACTIVITY MODEL FOR TRANSIENT TESTS

2.6.1 Doppler Effect

The transient tests involve rapid insertion of positive reactivity in a time which is very short compared to the fuel time constant and the fuel temperature rise during the transient occurs approximately adiabatically. The change in fuel temperature, ΔT_f , may thus be approximated by:

$$\Delta T_f = \frac{1}{\rho c V_f} \Delta E_f \quad (2.6-1)$$

where ρ is the fuel density, c is the fuel heat capacity, V_f is the fuel volume, and ΔE_f is the transient energy absorbed in the fuel.

From equations (2.3-6) and (2.6-1), the transient Doppler reactivity effect may thus be written as:

$$\Delta k_D = A \ln \left[1 + \frac{\Delta E_f}{\rho c V_f T_{fo}} \right] \quad (2.6-2)$$

where T_{fo} is the initial effective fuel temperature at the beginning of the transient. Using equation (2.5-17), with $n=1$, this can be written as:

$$\Delta k_D = A \ln \left[1 + \frac{n}{A} \Delta E_f \right] \quad (2.6-3)$$

or for $\frac{\eta}{A} \Delta E_f \ll 1$

$$\Delta k_D \approx \eta \Delta E_f \quad (2.6-4)$$

For very large transients ($\eta \Delta E_f / A$ not $\ll 1$) the expression given by (2.6-3) or a corresponding expression, if the logarithmic model is not applicable (see Section 2.3.1), can, in principle, be used to determine both A and η from analysis of transient data. For SEFOR, the quantity η/A is of the order of $0.001 \text{ (MW-sec)}^{-1}$ and the planned transients will produce an energy release of the order of 200 MW-sec. Thus, the logarithmic nature of the Doppler effect is not emphasized during the transient and it is expected that the Doppler feedback can be accurately represented by a second or third order polynomial in ΔE_f , even for the super-prompt critical transients. Such a representation may be written as:

$$\Delta k_D = \eta \Delta E_f \equiv (\eta_0 + \eta_1 \Delta E_f + \eta_2 \Delta E_f^2) \Delta E_f \quad (2.6-5)$$

2.6.2 Non-Doppler Feedback

In the transient tests, reactivity feedback due to increased fuel-clad and sodium-structure temperature, as well as reactivity feedback due to fuel axial expansion will occur along with the Doppler feedback. Most of the temperature rise in the sodium-structure temperature is due to neutron and prompt gamma heating while a portion of the fuel-clad axial expansion effect and the

fuel axial expansion effect will depend on the fuel temperature. It is expected that these non-Doppler effects will contribute only ~15% of the total reactivity feedback during the first 200 to 400 msec of the transient.

The model for the non-Doppler feedback, ΔK_o , is of the form

$$\Delta K_o(t) = C_e \Delta T_e(t) + C_t \Delta T_t(t) + C_c \Delta T_c(t)$$

where ΔT_e is the "fuel shoulder temperature," ΔT_t is the average clad temperature and ΔT_c is the average coolant temperature. The coefficients C_e , C_t , and C_c are the transient-fuel-expansion coefficient, the fuel-clad coefficient, and the coolant-structure coefficients, respectively. The time dependent behavior of ΔT_e , ΔT_t , and ΔT_c can be calculated for each transient by using a code such as ARGUS⁽¹⁰⁾ or FORE.⁽¹¹⁾ The non-Doppler feedback ΔK_o is then obtained using calculated and measured values of the reactivity coefficients.

2.7 EXPERIMENTAL DOMAIN

It is desired to investigate the validity of the reactivity model and evaluate individual reactivity feedback effects, particularly the Doppler effect, over a wide range of reactor operating conditions. Permissible equilibrium operating conditions at which test data may be obtained, are determined by plant operating characteristics and limitations and safety considerations. The range of permissible equilibrium operating conditions is called the experimental domain.

For purposes of planning the experimental program, the experimental domain was determined by evaluating the operating characteristics in the reactivity-power-average core coolant temperature domain and then superimposing reasonable operating and safety related limitations on the operating characteristics. The operating characteristics were determined from reactivity coefficients listed in the Task 8, Core Design Topical Report⁽⁸⁾ and the Doppler reactivity effect calculated using WEDOP.⁽¹²⁾ The following coefficients were used:

Doppler:	$T \frac{dk}{dT} = -.0085 \frac{\Delta k}{k}$
Sodium density out of core:	-.029 $^{\circ}\text{C}/^{\circ}\text{F}$ coolant
Sodium density in core:	-.121 $^{\circ}\text{C}/^{\circ}\text{F}$ coolant
Structure expansion:	-.233 $^{\circ}\text{C}/^{\circ}\text{F}$ coolant
BeO axial expansion:	-.006 $^{\circ}\text{C}/^{\circ}\text{F}$ coolant
Fuel-clad axial expansion:	-.078 $^{\circ}\text{C}/^{\circ}\text{F}$ coolant
	-.467 $^{\circ}\text{C}/^{\circ}\text{F}$ coolant
Fuel axial expansion:	-.0058 $^{\circ}\text{C}/^{\circ}\text{F}$ fuel

Reflector temperature effects were neglected. The resulting operating characteristics of SEFOR are shown in Figure 2.7-1, as the change in reactivity as a function of average coolant temperature for different power levels.

Figure 2.7-1. EXPERIMENTAL DOMAIN

SECTION III

ZERO POWER EXPERIMENTS

3.1 INITIAL LOADING TO CRITICAL

3.1.1 Objectives

The objectives are to determine the minimum critical number of fuel rods for the reactor mode in which:

1. all reflectors are up
2. core temperature $\leq 400^{\circ}\text{F}$
3. primary system contains sodium
4. nuclear power is zero

and to obtain preliminary estimates of reflector worth.

3.1.2 General Procedures

The standard method of loading fuel in which fuel is initially loaded at the center of the core structure followed by additions of fuel at the edge of the partially loaded core is recommended. Fuel should be added so as to maintain a cylindrical array of fuel that is concentric with the core structure. Although the loading increments cannot be determined before the actual test, it is expected that the minimum critical core can be achieved with 8 to 10 increments starting with an initial increment of 1/4 to 1/3 of the estimated minimum critical core.

Analysis indicates that the inherent source of neutrons (due to spontaneous fission and α -n reactions) in SEFOR fuel will not need to be supplemented by a startup source, provided that appropriate neutron detectors are placed in the core during the critical approach. The inherent source has been estimated to be of the order of 10^5 neutrons per second per fuel rod. At least two channels of instrumentation will be available for use during zero power tests, including incore neutron detectors, amplifiers, and pulse counters (scalers). Helium-3 rather than boron or fission detectors have been recommended because of their high sensitivity per unit volume, and the high temperature ($\sim 200^\circ\text{C}$) at which commercially available He^3 chambers can be operated.

It is recommended that the detectors be housed in a drywell to provide isolation from the sodium. It may be possible to use the FRED drywell for this purpose. It appears, however, that the FRED drywell will interfere with fuel handling in adjacent channels; thus a special drywell will likely be provided. To maximize detector sensitivity, it should be placed in the central channel near the estimated axial power peak.

Analysis indicates that the reactor source level neutron instrumentation will be on scale before the minimum critical core loading has been completed. Thus, it may be possible to remove the incore detectors after the critical loading has been completed. Near the end of the critical approach, the worth of the reflectors may be approximately determined using the rod drop technique.

3.1.3 Data Collection and Analysis

Count rate data may be obtained from the scalers provided for the critical approach. Fuel loading increments are usually determined from a plot of the inverse multiplication (or inverse count rate since the count rate and multiplication are proportionally related) versus the number of fuel rods. However, in the case of SEFOR, the neutron source will be proportional to the number of fuel rods, thus the multiplication should be proportionally related to the ratio of count rate to number of fuel rods. It is therefore recommended that the ratio, count rate/number of fuel rods, be plotted versus the number of fuel rods to analyze the critical approach loadings.

Reflector worth may be determined from rod drop data when the reactor is initially near critical by finding the prompt drop in power. If P_0 and P_1 are the powers before and after the prompt drop, respectively, then:

$$\frac{P_0}{P_1} = 1 + \left| \frac{\Delta k}{\sum \beta_i s_i} \right| = 1 + \left| \frac{\Delta k}{\beta} \right| \quad (3-1)$$

where:

s_i = effectiveness of neutrons of group i

β = effective delayed neutron fraction

This measurement of reflector worth is only expected to yield an approximate value because of adverse effects of the transient on the spacial distribution of flux.

Reflector worth may also be determined in terms of fuel rods if data is recorded during the critical approach with the reflectors down as well as in the up position.

3.1.4 Estimated Results

The calculated effective multiplication factor as a function of the percent of a full core that is loaded is shown in Figure 3.1-1 for all reflectors up and all reflectors down. The estimated minimum critical core is 79% (512 rods) of the full core.

The estimated worth of the reflectors at 79% of a full core is 3.5% $\Delta k/k$ (1.1\$). Or, in terms of additional peripheral fuel rods, 94 rods.

3.2 LOADING TO FULL SIZE CORE

3.2.1 Objective

The objective is to load additional fuel and poison rods to achieve a full size core that has enough excess reactivity to permit calibration of each fine control rod over its complete range of position.

3.2.2 General Procedure

This is similar to a continuation of the critical approach except that poison rods as well as fuel rods will be loaded so as to maintain the reactivity approximately constant. To achieve a somewhat

Figure 3.1-1. SEFOR CRITICAL APPROACH CENTER LOAD METHOD

uniform distribution of poison in the core, it will be necessary to replace, with poison rods, some of the fuel rods that were loaded in the critical approach. The final loading of the core should be adjusted so that the excess reactivity allows criticality to be achieved with one fine reflector completely down, and perhaps one other reflector down slightly.

Core temperature should be held constant but the level is not important for this step of the program. It will probably be convenient to maintain the same temperature level that is used during the approach to critical.

At various points, additional checks on the worth of the reflectors may be obtained by rod drop tests.

3.2.3 Data Collection and Analysis

Data collection and analysis is minimal for this step of the program. Changes in the core multiplication factor may be monitored continuously by using the low level neutron instrumentation and the initial startup detectors if they are still in the core.

3.2.4 Estimated Results

The worth of a B_4C poison rod at an average worth position in the core is $\$0.61$ relative to a fuel rod at that position. From Figure 3.1-1, a full core of fuel is estimated to have an excess k of $.05 \Delta k/k$ (15.6%) at $350^\circ F$. With 1.5% excess for rod calibration, the worth of poison rods should be approximately 14% at $350^\circ F$. Thus, if the loading to a full size core is carried out at $350^\circ F$, the core will contain an estimated 23 poison rods.

3.3 PRELIMINARY CALIBRATION OF NEUTRON INSTRUMENTATION

3.3.1 Objective

The objective is to perform an approximate calibration, in terms of reactor power level, for the reactor low level neutron instrumentation.

3.3.2 General Procedure

It is recommended that more than one method be used to perform a preliminary power calibration because:

- a. Several inexpensive methods are available which require little or no additional experimental time, and
- b. None of the methods is expected to independently yield an accurate power calibration.

Procedures for four methods are given below.

Method I: This method utilizes the incore Helium-3 chambers.

Power calibration of this incore instrumentation is obtained and used in turn to calibrate the low level reactor instruments. The procedure is to obtain values of the current output from the incore instruments to be calibrated, at several equilibrium power levels. The Helium-3 chambers should be located in the instrument drywell at an axial position near the flux peak. The chambers should be operated in the pulse mode.

Method II: This method is based on experimental determination of the output of the low level instrumentation at known values of the core multiplication factor, k , in the subcritical condition. Instrument calibration is then performed by using a pre-calculated relation between power and k .

The general procedure is to allow the core flux to achieve equilibrium at each of several subcritical conditions and record the low level instrumentation output and reflector positions.

An accurate knowledge of the source level and reactivity k is required for this method.

Method III: This method involves irradiation of fuel foils in SEFOR and subsequent counting of the activity of a particular fission product activity in the SEFOR counting system. It is necessary to calibrate the SEFOR counting system. The method may be applied in one or both of two different ways, depending on how the SEFOR counting system is calibrated. The SEFOR counting system may be calibrated in terms of power or terms of counter efficiency, for a particular isotope.

In the first case (method IIIa) a foil(s) (e.g. U-235 or U-238) is irradiated in a flux that is similar in energy distribution to that of SEFOR, and in a position of known flux level, for a specified time interval. The foil(s) is then counted at SEFOR for a specific fission product isotope (e.g. La-140 or Mo-99) thus allowing calibration of the SEFOR counting system in terms of counting rate of a specific isotope for known reactor conditions.

In the second case (method IIIb) an irradiated foil(s) (e.g. U-238 or U-235) is obtained that has had its activity determined in an absolutely calibrated counter so that the total activity of a particular fission product isotope (e.g., La-140 or Mo-99) is known. The foil(s) and the activity data are then used to calibrate the SEFOR counter in counts per disintegration for the particular isotope chosen.

To minimize the amount of time required during the experimental program to analyze the preliminary power calibration data, the SEFOR counting system should be calibrated before the actual reactor power calibration is performed.

For either method IIIa or IIIb, the irradiation at SEFOR is made with fuel foil(s) inserted in the central drywell or in a dummy fuel rod designed for foil irradiations. To minimize the irradiation time, the foil(s) should be located near the core flux peak. The estimated irradiation time necessary to achieve a desired foil activity can be determined from data used in method I or II of this section.

Method IV: As discussed in Section 3.11.3, the theoretical expression for the cross-power spectral density of the reactor neutron noise is:

$$\Phi_{xy}(\omega) = \bar{q}_x \bar{q}_y F \epsilon_x \epsilon_y \frac{\sqrt{\nu-1}}{\sqrt{\nu}} \frac{k^2}{\ell^2(\alpha^2 + \omega^2)} ,$$

for frequencies higher than the delayed neutron region (above 1 Hz). (Symbols are defined in Section 3.11.3.) If both the average chamber current and cross-power spectral density are measured, the reactor fission rate (F) can be obtained. Thus, the above equation can be solved for the fission rate.

$$F = \frac{\bar{i}_x \cdot \bar{i}_y}{\Phi_{xy}(\omega)} \cdot \frac{\frac{\nu(\nu-1)}{\bar{\nu}^2}}{\ell^2(\alpha^2 + \omega^2)}$$

where $\bar{i}_x = F \epsilon \bar{q}_x$, is the average detector current and where

$$\frac{\nu(\nu-1)}{\bar{\nu}^2} \approx 0.8$$

For the case where the reactor is very near delayed critical ($1-k \ll \beta$), and in the frequency range $\omega \ll \alpha$ where the power spectral density is independent of frequency, the expression for F is:

$$F = \frac{\bar{i}_x \cdot \bar{i}_y}{\Phi_{xy}(\omega)} \cdot \frac{\nu(\nu-1)}{\bar{\nu}^2 \beta^2} \quad \text{fissions/sec}$$

This method has been checked against the foil activation technique of determining reactor power, and the two methods were found to agree within 20% at power levels between 10 watts and 100 μ w in the STARK assembly.⁽¹⁾

3.3.3 Data Collection and Analysis

Method I: The following data are required: S, Incore chamber sensitivity, counts/sec-nv. If S is not known it may be determined approximately from the relation, $S = \Sigma_{n-p} V_c$

where:

V_c = active volume of chamber

Σ_{n-p} = n-p reaction macroscopic cross section for He-3, averaged over the calculated neutron spectrum at the chamber location. A value of 3.3 barns was obtained for σ_{n-p} for the neutron spectrum in the central channel that was calculated in core design problem C4HR1. This spectrum is given in Table 3-1.

Using the additional notation:

e, dead-time correction factor ($e \leq 1.0$)

R, ratio of flux at the He-3 chamber to the average flux in the full core

Σ_f , core volume and spectrum averaged fission cross section, cm^{-1}

V, total volume of core, cm^3

c, power per unit fission rate, $\frac{\text{watt}}{\text{fission/sec}}$

The He-3 chamber may then be calibrated at a specific position with the expression,

$$\frac{\Sigma_f V_c}{R S e} = \frac{\text{watt}}{\text{observed cts/sec}} \quad (3-2)$$

TABLE 3-1

Calculated Neutron Spectrum in FRED Channel (Case C4HR1)

<u>Group</u>	<u>E_{lower}</u>	<u>$\int \phi(E) dE$</u> <u>$\Sigma \int \phi(E) dE$</u>
1	3.7 MeV	.015
2	1.35	.0859
3	.825	.0610
4	.5	.108
5	.18	.229
6	67 keV	.200
7	25	.131
8	9.1	.0873
9	5.5	.022
10	3.3	.0128
11	2.0	.0063
12	1.0	.0239
13	502 eV	.0117
14	249	.0048
15	55.6	.0018
16	12.4	.0001
17	.625	---
18	--	---
		<hr/> 1.0006

Experimental data which relates the observed counts/sec from the incore instruments to the output of the low level neutron instrumentation can then be used with (3-2) to calibrate the low level instrumentation.

Method II: At each subcritical condition, the output of the low level instrumentation is related to k . By using the preliminary worths of the reflectors, as determined in 3.1 and 3.2, or by using the calculated worths from core design data, calibration of the low level instruments can be performed. The relation between power and reactivity may be formulated as follows.

The inherent source of neutrons (S_o) in SEFOR is almost totally due to spontaneous fission of Pu-240 and alpha-neutron reactions in oxygen from Pu-239 and Pu-240 alpha decay. Thus:

$$S_o = s_{s.f. \ 240} M_{240} + s_{\alpha-n \ 239} M_{239} + s_{\alpha-n \ 240} M_{240} \quad (3-3)$$

where:

$s_{s.f. \ 240}$ = specific spontaneous fission source
from Pu-240 = 10^6 n/sec-kg of Pu-240

$s_{\alpha-n \ 240}$ = specific α -n source from Pu-240 alphas
 $\approx 0.29 \times 10^6$ n/sec-kg of Pu-240

$s_{\alpha-n \ 239}$ = specific α -n source from Pu-239 alphas
 $\approx 0.078 \times 10^6$ n/sec-kg of Pu-239

M_{240} and M_{239} = mass of Pu-240 and Pu-239, respectively,
in the core, in kg.

Then for an equilibrium subcritical condition the power can be related to the multiplication factor by:

$$P = \frac{S_o}{1-k} \frac{k}{v} c \quad (3-4)$$

where:

c = power per unit fission rate, $\frac{\text{watt}}{\text{fission/sec}}$

v = average number of neutrons released per fission.

In Section 3.5.4, the value of $S_o C/v$ is estimated to be 0.001 watt- $\Delta k/k$, or 32 watt-cents.

Method IIIa: Consider the irradiation of a foil that contains N_s fissionable atoms, at position X_o in a reactor. For irradiation times short enough that we can neglect destruction of any of the atoms, the rate of formation, A , of a particular fission product isotoped (with yield Y) of interest is:

$$A = \int dE N_s \sigma_{fs}(E) \phi(E, X_o) Y \text{ nuclei/sec} \quad (3-5)$$

For energy and spatial separation of the flux ϕ

$$\phi(E, X) = F(E) \theta(X) \quad (3-6)$$

thus:

$$A = Y \theta(X_o) N_s \int dE \sigma_{fs}(E) F(E) \quad (3-7)$$

Then if foils of the same material are irradiated in SEFOR and another reactor, denoted by subscript 2, having similar flux spectrum $F(E)$, the ratio of activation rates in the two reactors is:

$$\frac{A_{\text{SEFOR}}}{A_2} = \frac{\theta(X_o)_{\text{SEFOR}} N_{s \text{ SEFOR}}}{\theta(X_o)_2 N_{s 2}} \quad (3-8)$$

The total power, P , generated in a reactor with volume V and number of fuel atoms N_f , may be represented by:

$$P = c \int dE \int dV \sigma_f(E) \frac{N_f}{V} F(E) \theta(X) \quad (3-9)$$

For SEFOR and another reactor (or critical assembly), with a similar flux spectrum and fuel,

$$\left(\frac{P}{N_f \bar{\theta}} \right)_{\text{SEFOR}} = \left(\frac{P}{N_f \bar{\theta}} \right)_2 \quad (3-10)$$

Since the flux at the sample location is related to the total power, equation (3-8) can be written as:

$$\frac{A_{\text{SEFOR}}}{A_2} = \frac{P_{\text{SEFOR}} N_{f2} N_{s \text{ SEFOR}}}{P_2 N_{f \text{ SEFOR}} N_{s2}} \left(\frac{\theta(X_o)}{\bar{\theta}} \right)_{\text{SEFOR}} \left(\frac{\bar{\theta}}{\theta(X_o)} \right)_2 \quad (3-11)$$

The foil activity at the time of counting is related to the activation rate by a function which depends on irradiation time, t_i , and decay time after irradiation, t_d . In particular, if we neglect absorption removal of the product isotope of interest, then the activity, $a(\text{dis/sec})$, at time, t_d , is:

$$a = A (1 - e^{-\lambda t_i}) e^{-\lambda t_d} \quad \text{dis/sec} \quad (3-12)$$

Thus, the SEFOR power can be found from equations (3-11) and (3-12):

$$P_{\text{SEFOR}} = \frac{P_2 N_{f\text{SEFOR}} N_{s2}}{N_{f2} N_{s\text{SEFOR}}} \left(\frac{\theta(X_o)}{\bar{\theta}} \right)_2 \left(\frac{\bar{\theta}}{\theta(X_o)} \right)_{\text{SEFOR}} \times \quad (3-13)$$

$$\frac{a_{\text{SEFOR}} [(1 - e^{-\lambda t_i}) e^{-\lambda t_d}]_2}{a_2 [(1 - e^{-\lambda t_i}) e^{-\lambda t_d}]_{\text{SEFOR}}}$$

If the type of fuel and the flux spectrum, $F(E)$, in reactor-2- is not similar to SEFOR, equation (3-10) is invalid, and the right-hand side of equation (3-13) must be multiplied by the following correction factor:

$$\left(\frac{\bar{\sigma}_{fs}}{\bar{\sigma}_f(\text{fuel})} \right)_2 \left(\frac{\bar{\sigma}_f(\text{fuel})}{\bar{\sigma}_{fs}} \right)_{\text{SEFOR}}$$

where the "bar" over the fission cross section denotes flux spectrum averaged cross sections.

Method IIIb: The power generation in SEFOR may be expressed as:

$$P = c \int dE \int dV \sigma_f(E) \frac{N_f}{V} \phi(E, X) \text{ watts} \quad (3-14)$$

Assuming that $\phi(E, X) = F(E)\theta(X)$ gives:

$$P = cN_f \int dE \sigma_f(E) F(E) \frac{\int dV \theta(X)}{V} = cN_f \bar{\sigma}_f \bar{\theta} \quad (3-15)$$

The rate of formation, A, of a particular fission product isotope in a fuel foil at position X_o is:

$$A = YN_s \int dE \sigma_{fs}(E) \phi(E, X_o) \text{ nuclei/sec} \quad (3-16)$$

The activity, a, of this fission product isotope (with decay constant λ) in the foil, after irradiation for time, t_i , and subsequent decay for time, t_d , is:

$$a = A(1 - e^{-\lambda t_i}) e^{-\lambda t_d} \text{ dis/sec} \quad (3-17)$$

Thus:

$$\begin{aligned} a &= YN_s (1 - e^{-\lambda t_i}) e^{-\lambda t_d} \int dE \sigma_{fs}(E) F(E)\theta(X_o) \\ a &= YN_s (1 - e^{-\lambda t_i}) e^{-\lambda t_d} \bar{\sigma}_{fs} \theta(X_o) \text{ dis/sec} \end{aligned} \quad (3-18)$$

Combining (3-18) and (3-15) we get:

$$a = \frac{YN_s P \bar{\sigma}_{fs} \theta(X_o)}{cN_f \bar{\sigma}_f \bar{\theta}} (1 - e^{-\lambda t_i}) e^{-\lambda t_d} \quad (3-19)$$

Then if the calibration factor of the SEFOR counting system for the fission product activity of interest is measured to be α counts/disintegration (so that $a_{\text{observed}} = \alpha\alpha$), the power level of SEFOR may be determined from:

$$P = \frac{(a_{\text{observed}}) c N_f \bar{\sigma}_f \bar{\theta}}{\alpha Y N_s \bar{\sigma}_{fs} (X_o) e^{-\lambda t_d} (1 - e^{-\lambda t_i})} \text{ watts} \quad (3-20)$$

3.3.4 Estimated Results

The calculated calibration of the low level neutron instrumentation is 3000 counts/sec-watt. This is based on values of the neutron flux in the location of the low level sensors, from the SEFOR shielding analysis and on an estimated sensor sensitivity of 2 counts/sec per equivalent thermal neutron/cm²-sec at the sensor. It is estimated that the calculated calibration may be error by up to a factor of five.

Based on data from noise experiments in the SEFOR loading of ZPR-III⁽²⁾ a calibration factor of 10⁶ counts/sec-watt is estimated for a Helium-3 chamber with the characteristics listed below, located near the center of the core.

Chamber length	~ 6 inches
Chamber diameter	~ 1 inch
Gas pressure	~10 atmospheres

3.4 PRELIMINARY ISOTHERMAL TEMPERATURE COEFFICIENT

3.4.1 Objective

The objective of this test is to obtain preliminary information concerning the isothermal temperature coefficient of the core. This information would be used when determining an appropriate fuel loading for normal operating temperatures and for preliminary correction of data from other zero power tests for undesired changes in core temperature.

3.4.2 General Procedure

Establish criticality at the zero power reference temperature of $\sim 400^{\circ}\text{F}$ with heat supplied to the coolant by the electrical heaters. Allow the system to achieve nuclear and thermal equilibrium at a convenient flux level. Using the electrical heaters raise the core temperature $\sim 50^{\circ}\text{F}$ while maintaining a constant flux level by raising a fine reflector(s). Allow the system to achieve thermal equilibrium at the higher temperature.

After recording appropriate data reverse the procedure by lowering the core temperature $\sim 50^{\circ}\text{F}$ while maintaining a constant flux level, and re-establish thermal equilibrium.

3.4.3 Data Collection and Analysis

System parameters should be recorded before and after (and possibly during) each of the two temperature changes. Recorded parameters should include the time, core inlet and outlet coolant temperatures, coolant flow, fine reflector position(s), selected fuel temperatures, and relative flux or power level. It may be necessary to record the parameters several times at each temperature level to ensure that an equilibrium condition has been established.

Since the reflectors may not be accurately calibrated at this point in the experimental program it may be necessary to use the calculated reflector worth data, possibly modified by the results of reflector worths determined in Section 3.1, to determine a preliminary value for the isothermal temperature coefficient. If readings are taken of T_1 and T_2 , the core temperatures for conditions 1 and 2, respectively, the preliminary isothermal temperature coefficient may be calculated simply by dividing the change in reactivity by the change in isothermal temperature.

3.4.4 Estimated Results

The isothermal temperature coefficient may be estimated from the data given in Section 2.4. For example, at an isothermal temperature of 400°F, the coefficient is estimated to be $-.79\text{¢}/^\circ\text{F}$.

3.5 SOURCE LEVEL DETERMINATION

3.5.1 Objective

The objective of this experiment is to determine a power level such that the presence of the inherent neutron source will have a negligible effect on the results of zero power experiments (e.g., reflector calibrations) performed at or above that level.

3.5.2 General Procedure

The general procedure to be followed is to establish a series of steady-state conditions (at successively higher power levels) starting from a power level of less than ~ one watt. The position of a fine reflector is noted at each of the steady-state power levels. At higher power levels, the inherent neutron source becomes less significant relative to the source from multiplication, thus the steady-state k_{eff} asymptotically approaches unity with increasing power level. The deviation from criticality, at steady-state, can be made arbitrarily small.

Practically, the source effect is sufficiently small when the power is high enough so that the equilibrium change in reflector position for a decade change in power level is small compared to the minimum possible detectable change.

3.5.3 Data Collection and Analysis

At each steady-state condition, the power level, reflector, position, and core coolant temperature are determined. Data from the preliminary measurement of the isothermal temperature coefficient may be used to correct the reflector positions for changes in core temperature.

It is suggested that the data be analyzed by plotting the reflector position as a function of steady-state power level, or reciprocal steady-state power as a function of reflector position.

3.5.4 Estimated Results

The estimated inherent neutron source due to spontaneous fission of Pu-2340 and α -n reactions in oxide fuel from Pu-240 and Pu-239 decay is:

$$\begin{aligned}
 &1.3 \times 10^6 \text{ n/sec-kg Pu-240} \\
 &7.9 \times 10^4 \text{ n/sec-kg of Pu-239}
 \end{aligned}$$

The total source based on the fully loaded design core composition is estimated to be 7×10^7 n/sec. This results in an estimated source factor C in the equation $P = C k / (1 - k)$ of:

$$.001 \text{ watt} - \frac{\Delta k}{k}$$

or

$$32 \text{ watt-}\phi$$

Thus, to reduce the source reactivity effect to less than 0.1% would require a power level of more than 320 watts.

3.6 REFLECTOR CALIBRATION

3.6.1 Objective

To obtain the reactivity worth of each reflector segment as a function of its vertical position. Calibrations must be obtained with sufficient accuracy to allow accurate determination of the reactivity status of the reactor throughout the experimental program.

3.6.2 General Procedure

Each of the two fine reflectors should be calibrated differentially using the period technique and the eight course reflectors may be calibrated by comparison to one of the calibrated fine rods using the rod swap technique.

For period calibrations of the fine rods, reactivity increments of approximately 10¢ are recommended. Such increments will result in asymptotic periods of approximately 100 seconds. The waiting times for 1 and 5 percent accuracy are approximately 100 sec and 70 sec, respectively, if the reactor is initially critical. If the reactor is slightly subcritical in its initial state, these times may be considerably lengthened.⁽³⁾ Calibrations should be performed with an equilibrium core temperature of 350 to 400°F and also at approximately 760°F, the design full power temperature. The resulting data will indicate the degree of dependence of reflector worth on reactor temperature.

It may be advantageous to perform alternately positive and negative period measurements.

At this point in the experimental program, the core will have excess reactivity approximately equal to one reflector. The reflector calibrations will thus be representative of a 15 to 20 MW configuration. In order to perform reflector calibrations with several different reflector system configurations, it will be necessary to provide additional core reactivity. To determine the effect of different reflector system configurations, reflector calibrations should be repeated after fuel loading increments are added later in the program when the core loading for high power operation is completed. If the high power core loading is completed in three equal increments, the reflector configuration after the first, second and third increments will be representative of reflector configurations for approximately 10, 5, and 0 MW operation, respectively.

3.6.3 Data Collection and Analysis

Reflector positions and the relative reactor power as a function of time are the variables of primary interest and are obtained from the normal installed instrumentation. The reactor period may be determined directly from the slope of the log power versus time curve. The corresponding reactivity may then be determined in the usual manner using the reactor lifetime and the standard reactor period-reactivity relation.

Additionally, reactor parameters normally recorded for static tests should be recorded.

3.6.4 Expected Results

The total reflector system worth was calculated to be 11\$. The worth of one reflector is expected to be 1.6\$ with all other reflectors raised and 0.8\$ with all other reflectors lowered. The expected relative worth curve for the reflectors is shown in Figure 3.6.1.

3.7 MATERIAL WORTH MEASUREMENTS

3.7.1 Objective

To determine the reactivity worth of B_4C , steel and fuel rods at various positions in the core.

Figure 3.6-1. REFLECTOR WORTH VS. POSITION

3.7.2 General Procedure

In addition to fuel rods, the plant equipment includes steel rods and rods that contain B_4C . Both types of rods are physically capable of replacing fuel rods in the core structure. The worth, relative to sodium, of each of these three types of rods is to be determined for three radial core positions.

The reactor should be brought to nuclear and thermal equilibrium at "zero" power and a convenient core temperature. Appropriate steady-state parameters are recorded, including reflector positions, temperatures, flows and power level. At a selected core position, the fuel rod is removed and the reactivity effect is compensated by a change in fine reflector position. Similarly, the reactivity changes resulting from series placement of a steel rod and a B_4C rod at the same core position are compensated by changes of fine reflector position. Appropriate steady-state parameters are recorded before and after rod insertions and removals. After measurements with all types of rods are completed at a particular core position, return of the rod that originally occupied that position will afford a means of checking that the final reactivity status is identical with the original status.

Similar measurements are to be carried out for at least two additional core positions.

In preparing for these measurements, the system reactivity status should be adjusted so that reflector positions are appropriate for expected reactivity changes.

In the selection of core positions, appropriate consideration should be given to minimizing the effects of 1) asymmetrical flux distribution, for example, resulting from asymmetrical reflector patterns or asymmetrical core loading, and 2) local flux perturbations, for example, resulting from localized effects of the absence of regular fuel rods. It may be necessary to choose core positions that lie along a radius that is perpendicular to the radius on which compensating reflectors are located.

In instances where it may be necessary to use more than one rod of a given type appropriate normalization of the "identical" rods should be carried out for at least one core position.

3.7.3 Data Collection and Analysis

Primary data for these measurements comes from reflector positions before and after rod changes in the core. Associated reactivity changes may be determined from reflector reactivity-position calibrations. Recording of other steady-state parameters throughout these measurements will yield secondary data which can be used to perform appropriate corrections to the primary results. For example, corrections can be performed for changes in flow, temperatures and pressures that may occur during the measurements.

3.7.4 Estimated Results

The calculated worth of a regular fuel rod, relative to sodium, at the center of the core is 0.39\$. The average worth of a fuel (relative to sodium) for all core positions except for the outer row of rods is calculated to be 0.24\$.

Analogous values for a B_4C rod are 0.70\$ and 0.37\$.

The worth of a steel rod, relative to sodium, at the center of the core is estimated to be of the order of 0.05\$.

These estimates are for the condition where adjacent fuel positions are occupied by regular fuel rods. Also for these estimates, it was assumed that the reflector system was in the raised position.

3.8 FLOW AND PRESSURE COEFFICIENTS OF REACTIVITY

3.8.1 Objective

To determine the reactivity effects of changes in primary coolant flow rate and reactor vessel hydraulic pressure.

3.8.2 General Procedure

The reactor should be at nuclear and thermal equilibrium at "zero" power and at some convenient core temperature. The primary coolant flow should be changed in several increments from the minimum permissible value to full flow. Thermal equilibrium should be maintained and reactivity changes should be compensated by fine reflector position changes.

In a similar manner, the reactor vessel hydraulic pressure should be changed in several increments over its design range and any resulting reactivity effects compensated by fine reflector movement.

3.8.3 Data Collection and Analysis

Normal steady-state plant data should be recorded before and after each pressure and flow change. Fine reflector calibration data will be used to relate pressure and flow to reactivity. Changes in reactor parameters other than flow and pressure should be recorded and used to perform corrections to the reactivity data. After the data have been corrected, flow and pressure coefficients may be determined directly, or if necessary, may be determined by least square fitting of the data.

3.8.4 Expected Results

It is expected that the pressure and flow coefficients of reactivity will be very small, possibly negligible.

3.9 LOADING TO FULL POWER CORE

3.9.1 Objective

The objective to be achieved is to load the core to enable operation at design power and temperature with appropriate excess reactivity. A secondary objective is to obtain reflector calibrations at intermediate core loadings.

3.9.2 General Procedure

To achieve the objective it will be necessary to add additional fuel to the core by replacing selected non-fuel rods with fuel rods. It is likely that some rearrangement of remaining non-fuel rods will be appropriate. It is expected that excess reactivity of \$0.50 available at design power and temperature will provide both reasonable safety and experimental flexibility.

The appropriate core loading can be estimated from previously determined reactivity coefficients, material worths, reflector calibrations and the estimated power coefficient of reactivity. Core additions would be carried out under shutdown conditions. It is likely that due to safety considerations the final core arrangement would be achieved by using more than one loading increment.

At some intermediate core loadings the reflectors should be recalibrated following the general procedures given earlier. This will allow determination of the effects of reflector system arrangement on individual reflector calibrations. Reflector calibrations at intermediate core loadings would thus be representative of those for the final core arrangement operated at less than full power conditions.

3.9.3 Data Collection and Analysis

Reactivity data from each core loading increment should be utilized to predict the reactivity effect of the subsequent increment. Data collection and analysis associated with reflector calibrations will be similar to that discussed previously in the section on reflector calibration.

3.9.4 Expected Results

The design criticality factor of the fully fueled SEFOR core (648 fuel rods) is $k=1.035$ at 20 MW and 760°F average core coolant temperature.

The calculated average worth* of a boron carbide rod (relative to a fuel rod) is \$0.61, or 0.00195 $\Delta k/k$. Thus, the final core loading to achieve \$0.50 excess at design power and temperature is expected to consist of 631 fuel rods and 17 boron carbide rods. This assumes that the boron rods are loaded so that the average worth value is valid.

3.10 ACTIVATION MEASUREMENTS

3.10.1 Objective

To determine the absolute power density, axial power distribution, and fission ratios of U-238, U-235, and Pu-239.

3.10.2 General Procedure

The determination of absolute power density may be accomplished by using the method(s) of Section 3.3. Refer to that section for a discussion of the general procedures and data analysis.

The axial power distribution at a fuel rod position may be determined by foil activation and by fuel rod gamma scanning methods.

* Average worth excluding the outer ring of rod positions.

In the case of foil activation experiments, hollow foil holder rods are provided. During use, each of these rods will occupy a core fuel rod position. Foils of the composition that is selected may be placed in foil holder rods at the axial locations that are chosen for this experiment. Steel pellets may be inserted into the rods as a means of positioning the foils. UO_2 pellets may be placed above and below individual foils as a means of providing a reasonably representative activation neutron flux. After the foil holder rods have been filled and closed, they are placed in position in the core. The irradiation time is chosen on the basis of the type of foil material (U-235, U-238, Pu-239), the desired foil activity at the expected time of counting, and the estimated neutron flux in the region of the foils. After completion of the irradiation, the foil holder rods may be removed from the reactor and sodium cleaning may be performed as necessary.

After removing the foils from the rods, their activities may be determined using the counting and analysis equipment that is provided. Since only the power distribution is needed, only relative activities are needed. Activity of a particular fission product (e.g. La-140) or a sum of activities could be used in this experiment. The techniques and methods involved are standard and well documented.

It may be desirable to perform this experiment at one or more radial positions that are near to reflector rods that have been specially positioned so as to yield results pertaining to the effect of their position on the axial power distribution.

After SEFOR fuel rods have received sufficient burnup, the axial power distribution may be determined from the fission product gamma activity. The general procedure is to remove a selected fuel rod from the core and place it in the gamma scan facility. As the fuel rod is traversed in front of the beam slit, the activity (which is proportional to the local power density) is determined by using a single or multi-channel counting system. It is expected that either gross fission product activity or counting of a specific fission product (e.g. La-140) will prove suitable. If the multi-channel system is not available for use with the gamma scan facility and it is desired to perform analysis on the basis of individual isotopes, the fuel rod may be traversed repetitively accompanied by appropriate gate adjustment of the single channel system.

For determination of fission ratios of U-238, U-235, and Pu-239, standard foil activation methods may be used. Since fuel rods of different composition are available (UO₂, regular mixed oxide, and high Pu content rods) another method for determining these fission ratios may be used.

If foil activations are to be made this experiment could be combined with foil activations for power distribution by inserting a packet of foils, rather than a single foil, at appropriate locations in the foil holder rods. Standard techniques using the multi-channel analyzer for counting selected fission product activities will yield the desired fission ratios.

The second method for determining fission ratios requires that fuel rods of different compositions be placed in core positions having similar neutron fluxes for irradiation. Necessary cleanup would be performed to allow removal of the rods from the refueling cell to an area of lower background radiation. The **relative** activity of a selected

fission product (e.g. La-140) may be determined as a function of axial position along the rods using the multi-channel system and a shielded collimator. Care should be taken to limit the irradiation so that fuel rod radiation levels are appropriate for removal of the rods from the refueling cell and for close handling by personnel.

3.10.3 Data Collection and Analysis

Standard and well documented procedures pertaining to activity analysis of radioactive materials are applicable to the experiments discussed in this section. Normal techniques for correcting for background radiation, changes in counting efficiency and decay of isotopes to be counted may be used. Random experimental errors may be expected to occur. They should be considered quantitatively by incorporating standard techniques such as repetitive counting, chi-square testing and mean square deviation analysis into the detailed experimental procedures.

Analysis of data from fuel rods of different compositions, as part of the fission ratio experiments, involves certain mathematical relations that may not be well documented. Therefore, some of these relations are listed below.

Consider a case where a depleted UO_2 rod and a natural UO_2 rod have been irradiated in the same neutron flux and the fission product to be analyzed is La-140 which has a 1.6 MeV gamma emission.

Then:

$$C^D = C_{235}^D + C_{238}^D = K(a_{235}^D + a_{238}^D) \quad (3-21)$$

where:

Superscript D stands for depleted UO_2 rod

C = La-140 count rate

C_{235} = La-140 count rate for U-235 fission products

C_{238} = La-140 count rate for U-238 fission products

K = System counting efficiency for 1.6 MeV gammas

a_{235} = La-140 activity for U-235 fission products

a_{238} = La-140 activity for U-238 fission products

The La-140 activity for U-235 fission products is:

$$a_{235} = y_{235} N_{235} \frac{\int \sigma_{235}^f(E) \phi(E) dE}{\int dE} F \quad (3-22)$$

where:

y_{235} = La-140 fission yield from U-235

N_{235} = U-235 concentration

σ_{235}^f = U-235 fission cross section

ϕ = Neutron flux

F = Fraction of total La-140 remaining at counting time

and where:

$$N_{235} = \frac{A \rho_u f_{235} V}{235}$$

where:

A = Avagadro's number

ρ_u = Uranium density in fuel

f_{235} = Isotopic U-235 fraction of uranium

V = Fuel volume

Using similar notation for U-238 and combining with the above relations gives:

$$C^D = [K A \rho_u V \int \phi(E) dE F]^D \left[\frac{y_{235} \bar{\sigma}_{235}^f f_{235}^D}{235} + \frac{y_{238} \bar{\sigma}_{238}^f f_{238}^D}{238} \right] \quad (3-23)$$

A similar relation may be written for fuel rod that contains natural uranium which has been irradiated in the same flux (or if not, appropriate corrections performed). For this equation, the results would be corrected so that the constant term would be the same as that of equation (3-23) above. Then equation (3-23) and the analogous equation for natural uranium may be combined to yield the ratio of fission cross sections:

$$\frac{\bar{\sigma}_{235}^f}{\bar{\sigma}_{238}^f} = \frac{235}{238} \left(\frac{y_{238}}{y_{235}} \right) \left[\frac{C^N f_{238}^D - C^D f_{238}^N}{C^D f_{235}^N - C^N f_{235}^D} \right] \quad (3-24)$$

Similarly, by including in the experiment a regular SEFOR fuel rod and a high plutonium content fuel rod, the following equations may be formed:

$$\frac{b_1 \bar{\sigma}_{235}^f + b_2 \bar{\sigma}_{238}^f}{b_3 \bar{\sigma}_{239}^f + b_4 \bar{\sigma}_{240}^f} = 1 \quad (3-25)$$

$$\frac{c_1 \bar{\sigma}_{235}^f + c_2 \bar{\sigma}_{238}^f}{c_3 \bar{\sigma}_{239}^f + c_4 \bar{\sigma}_{240}^f} = 1 \quad (3-26)$$

$$\frac{d_1 \bar{\sigma}_{235}^f + d_2 \bar{\sigma}_{238}^f + d_2 \bar{\sigma}_{239}^f + d_4 \bar{\sigma}_{240}^f}{d_5 \bar{\sigma}_{235}^f + d_6 \bar{\sigma}_{238}^f + d_7 \bar{\sigma}_{239}^f + d_8 \bar{\sigma}_{240}^f} = 1 \quad (3-27)$$

where the constants a,b,c, and d are determined from known data.

Any three of equations (3-24) through (3-27) may be combined to evaluate the desired fission cross section ratios.

3.10.4 Expected Results

Calculated axial and radial power distributions for SEFOR appear in Reference (4). Calculated core average cross section ratios appear in Table 3-2. These may be compared with calculated and measured values for the SEFOR critical experiment in ZPR-III.⁽⁵⁾

TABLE 3-2

Core Average Cross Section Ratios

<u>Ratio</u>	<u>Calculated Value</u>
$\sigma_f(\text{U-235})/\sigma_f(\text{Pu-239})$	1.22
$\sigma_f(\text{Pu-240})/\sigma_f(\text{Pu-239})$	0.179
$\sigma_f(\text{U-238})/\sigma_f(\text{Pu-239})$	0.027
$\sigma_c(\text{U-238})/\sigma_f(\text{Pu-239})$	0.141

3.11 REACTOR POWER NOISE EXPERIMENT

3.11.1 Objective

To determine the ratio of the delayed neutron fraction to the neutron lifetime, β/λ .

3.11.2 General Procedure

The procedure follows closely that used in the noise measurements which were made in the SEFOR critical mockup in ZPR-3.⁽²⁾ In the ZPR-3 measurements, the values of λ/β obtained by the noise and pulsed neutron methods agreed within the experiment error.⁽⁶⁾

The power noise experiment should be conducted with the reactor critical, or slightly subcritical, if it should be necessary to limit the current from the neutron detectors below a certain level. The reactor should be maintained at nuclear and thermal equilibrium. Two neutron detection systems, each equipped with a Helium-3 detector, will be utilized to produce two independent reactor power noise signals. A special drywell placed in the central channel of the core should be used to house the two detectors and lead cables. The core temperature should be maintained below the detector design maximum ($\sim 400^{\circ}\text{F}$), or provisions for cooling the detectors should be utilized. Signal cables of appropriate design for the expected temperature should be used. It is planned that detector power supplies and signal amplifiers from the transient monitoring system will be utilized for the noise experiment. The noise signals will be recorded digitally by the equipment used for the transient neutron flux recording. To prevent aliasing of noise components that have frequencies greater than one-half of the digitizing frequency, it may be necessary to include high frequency cut-off filters in the noise detection and amplification system.

For the noise experiment, the detectors should be operated in the current mode. A supply voltage of approximately 300 volts should be satisfactory.

A separate power supply should be used in each detection system so that uncorrelated noise originating in the power supplies will be eliminated by the subsequent cross correlation analysis.

The two noise signals should be amplified so that they may be received by two transient channels of the data acquisition system. To allow analysis over the widest range of frequencies, the signals should be digitized and recorded at the maximum number of samples per second (10^4).

Either before or after recording the reactor noise signals, the system should be operated without the neutron detectors (or with dummy detectors) to determine the system induced noise. This data, after cross correlation analysis, can be used to correct the results from the reactor power noise analysis.

It is recommended that the Helium-3 detectors be placed side-by-side at the core midplane thereby eliminating possible effects of asymmetrical axial flux distribution.

3.11.3 Data Collection and Analysis

Figure 3.11-1 shows a schematic diagram of the experimental equipment used for the reactor noise measurements (similar to that used in the ZPR-3 measurements), and typical reactor operating conditions. The He^3 chamber current output and efficiency have been checked in the ZPR-3 work. The RC filter network at the amplifier input blocks the d.c. chamber current signal and allows only the fluctuating current signal to be amplified and recorded. The RC filter passes frequencies above 1 Hz with no significant effect. A SEFOR transient amplifier can be used for these measurements since these amplifiers have greater than a 16 kHz bandwidth with a 1000 pf input shunt capacitance. A voltage amplifier will be

Chamber-to-Amplifier Cable: Approximately 1000 pf capacitance

He³ Chambers: 1" Diameter x 6" Long; 10 Afm. He³; 400°F maximum operating temp; 10 pf capacitance; efficiency = 3×10^{-5} detections in chamber per fission in SEFOR core.

Current Levels: Reactor at 15 watts (2^c Subcritical).

d.c. - 5×10^{-8} amp

a.c. - 10^{-8} amp, rms

Figure 3.11-1. REACTOR POWER NOISE MEASUREMENT

needed to raise the SEFOR transient amplifier output to a level suitable for recording. Since the sampling rate is 10,000 samples per second per channel, it will be necessary to filter out the noise above 4 kHz, in order to avoid "aliasing" (contamination of low frequencies by frequencies above $1/2 \times$ sample frequency). The data is sampled one channel at a time in the pattern shown below:

The $33\frac{1}{3}$ microsecond delay between sampling the two channels is not expected to significantly affect the result, since the frequency of interest is in the 1 kHz region and the low pass filter will remove fluctuations at frequencies above 5 kHz.

There are several methods for analyzing the recorded data by digital computer. At present, it is planned to compute an estimate of the cross power spectral density (ϕ_{xy}) by directly computing the Fourier transform of each channel and then computing ϕ_{xy} , rather than first computing the cross correlation function. Thus, the function ϕ_{xy} will be estimated from:

$$|\tilde{\phi}_{xy}(\omega)| = \frac{1}{T} \sqrt{A^2 + B^2} \quad \frac{\text{amps}^2\text{-sec}}{\text{radian}}$$

where $\tilde{\phi}$ denotes the measured (estimated) value of $\phi_{xy}(\omega)$

$$A = X_r Y_r + X_i Y_i$$

$$B = X_r Y_i + Y_r X_i$$

$$X_r = \int_0^T x(t) \cos \omega t \, dt$$

$$X_i = \int_0^T x(t) \sin \omega t \, dt$$

$$Y_r = \int_0^T y(t) \cos \omega t \, dt$$

$$Y_i = \int_0^T y(t) \sin \omega t \, dt$$

where T is the data record length in seconds. Note that $\bar{x} = \bar{y} = 0$, since the d.c. currents have been blocked by the series capacitors. The Fast Fourier transform method⁽⁷⁾ appears to make this approach attractive because of shorter computer time than the alternative method described next.

Alternatively, the cross correlation function could first be computed:

$$\tilde{\phi}_{xy}(\tau) = \frac{1}{T} \int_0^T x(t) y(t+\tau) \, dt \quad \text{for } 0 < |\tau| < T$$

The function $\tilde{\phi}_{xy}(\omega)$ is then the Fourier transform of $\tilde{\phi}_{xy}(\tau)$:

$$\tilde{\phi}_{xy}(\omega) = \int_{-\infty}^{\infty} \tilde{\phi}_{xy}(\tau) e^{-j\omega\tau} \, d\tau$$

The function $\phi_{xy}(\omega)$ is desired rather than $\phi_{xy}(\tau)$ since the frequency domain information gives more physical insight in the event that unexpected results occur. It is estimated that 10 to 30 seconds of data recording will be adequate to achieve an uncertainty in ℓ/β of a few percent.⁽⁸⁾

Prior to the reactor noise measurements, it is necessary to determine the noise background by analyzing the recorded signals with the reactor shutdown. The "background" frequency spectrum thus obtained is used to correct the reactor noise spectrum. Also, it is necessary to determine the frequency response of the data channel so that, if necessary, corrections may be made to the reactor noise data. Of course, the ZPR-3 work indicates that no correction for instrumentation frequency response will be necessary but a check can be made at SEFOR by using a "white noise" neutron source. A white noise neutron source can be obtained by removing the He^3 chambers several feet from the reactor core and operating the reactor at a power level high enough to give a detectable signal at the chambers.

When the $\phi_{xy}(\omega)$ function has been determined experimentally, the value of ℓ/β is determined by fitting the data to the theoretical expression: (valid at frequencies above the delayed neutron region):

$$\phi_{xy}(\omega) = \bar{q}_x \bar{q}_y F \epsilon_x \epsilon_y \frac{\nu(\nu-1)}{\sqrt{2}} \frac{k^2}{\ell^2(\alpha^2 + \omega^2)}$$

where

$\bar{q}$ = average charge released per detected neutron

F = core fission rate

ϵ = detection in chamber per fission in core

ν = number of neutrons per fission

$$\frac{\nu(\nu-1)}{\bar{\nu}^2} \approx 0.8$$

k = multiplication constant

ℓ = prompt neutron lifetime

$$\alpha = \frac{1-k(1-\beta)}{\ell}$$

β = effective delayed neutron fraction

For values of $1-k \ll \beta$, the expression for $\phi_{xy}(\omega)$ reduces to:

$$\phi_{xy}(\omega) = \bar{q}_x \bar{q}_y F \varepsilon_x \varepsilon_y \frac{\nu(\nu-1)}{\bar{\nu}^2 \beta^2} \frac{(\beta/\ell)^2}{[(\beta/\ell)^2 + \omega^2]}$$

The value of ℓ/β is then found by fitting the shape of $\phi_{xy}(\omega)$ to the function $\frac{1}{(\beta/\ell)^2 + \omega^2}$ in the frequency range around $\omega = \beta/\ell$.

3.11.4 Expected Results

The expected value of ℓ/β is 2×10^{-4} sec, thus the break frequency should be 800 Hz.

SECTION IV

STATIC POWER TESTS

4.1 OBJECTIVE

The objective of the static tests conducted at power is to determine the reactivity effects associated with slow changes in reactor operating conditions over a wide range of power levels and coolant temperatures.

4.2 GENERAL PROCEDURE

As discussed in Section 2.4, the total reactivity feedback is a combination of several individual reactivity feedback effects. The reactivity effect of a change in the reflector temperature is easily measured experimentally. This may be accomplished while at equilibrium power by a series of measurements in which the reflector coolant flow is changed and the corresponding reactivity effect measured. This section of the report, therefore, is primarily concerned with the primary reactivity effects which are related to the power level and coolant temperature.

During the static tests, the reactor is operated in an equilibrium condition at a number of different power levels, flow rates and coolant temperatures. The tests can be performed by changing one of the reactor variables while others remain constant. The planned test conditions were selected so that the reactivity effects associated with 1) a change in power at constant average coolant temperature,

2) a change in average coolant temperature at constant power, and
 3) a change in inlet temperature at constant power and average coolant temperature can be separately determined. By proper sequencing of the variables to be changed, a significant range of power and temperature can be covered without a large amount of unnecessary overlap in the measurement sequence.

The change in reactivity, Δk_f , for a change between two equilibrium conditions may be written as:

$$\Delta k_f = A \ln \left[1 + \frac{\Delta T_c + R_f \Delta P}{T_{co} + R_f P_o} \right] + C_p \Delta P + C_q \Delta T_c + C_i \Delta T_i + C_r \Delta T_r \quad (4-1)$$

The general procedure for conducting the static tests is to allow the reactor to achieve equilibrium conditions and then record the values of all appropriate variables with the data acquisition system. After the variables are recorded certain variables are changed so that the reactor achieves other desired equilibrium conditions at which the reactor variables are again recorded. This procedure is repeated until equilibrium conditions have been achieved and the variables recorded for all desired reactor conditions.

The planned reactor equilibrium conditions for the static tests are listed in Table 4-1. The test points that lie along lines of constant power include different coolant flows, and hence, different inlet coolant temperatures, for the same average coolant temperature. If it is determined that the inlet coolant temperature reactivity effect is small, it may be appropriate to omit most of the tests that are listed for 25 and 40 percent coolant flow rates.

4.3 DATA COLLECTION AND ANALYSIS

Data collection and recording will be performed through use of the SEFOR data acquisition system. Data to be recorded includes primary coolant system temperatures, power level, primary coolant flow, reflector temperature and reflector positions. The reactivity change associated with a change between two equilibrium reactor conditions is evaluated from the reflector data. For changes between equilibrium conditions, where the reflector temperature is the only variable that changes, equation (4-1) reduces to:

$$\Delta k_f = C_r \Delta T_r \quad (4-2)$$

Similarly for static tests, in which the power and average coolant temperature do not change, we have:

$$\Delta k = C_i \Delta T_i \quad (4-3)$$

where the reactivity has been corrected for changes in reflector temperature, i.e.

$$\Delta k = \Delta k_f - C_r \Delta T_r \quad (4-4)$$

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TABLE 4-1

EQUILIBRIUM CONDITIONS FOR STATIC TESTS

Tests at constant average coolant temperature:

$$T_c = 600^\circ\text{F}$$

$$P = 2, 4, 6, 8, 10, 12, 14, 16 \text{ MW}$$

$$T_c = 760^\circ\text{F}$$

$$P = 2, 4, 6, 8, 10, 12, 14, 16, 18, 20 \text{ MW}$$

Tests at constant power:

$$P = 5 \text{ MW}$$

$$P = 10 \text{ MW}$$

T_c ($^\circ\text{F}$)	T_i ($^\circ\text{F}$)		T_c ($^\circ\text{F}$)	T_i ($^\circ\text{F}$)	
	(25% Flow)	(100% Flow)		(40% Flow)	(100% Flow)
500	440	485	520	--	490
545	485	530	565	490	535
590	530	575	610	535	580
635	575	620	655	580	625
680	620	665	700	625	670
725	665	710	745	670	715
770	710	755	790	715	760
815	755	800	835	760	805
860	800	845			

For static tests in which the average coolant temperature is held constant, we have:

$$\Delta k = A \ln \left[1 + \frac{R_f \Delta P}{T_{co} + R_f P_o} \right] + C_p \Delta P \quad (4-5)$$

where Δk has been corrected for reflector temperature and inlet coolant temperature effects.

For static tests in which the power is held constant, the feedback reactivity may be represented by 4-6 after correction for effects of changes in the reflector temperature and inlet coolant temperature.

$$\Delta k = A \ln \left[1 + \frac{\Delta T_c}{T_{co} + R_f P_o} \right] + C_q \Delta T_c \quad (4-6)$$

The contribution to the feedback reactivity from the Doppler effect is of primary interest in the static tests. It is expected to be the most complex contribution from the data analysis standpoint. It is recommended that models other than the simple model in equation 4-1 be considered for the Doppler reactivity contribution. Other relatively simple models for tests at constant average coolant temperatures are:

$$\Delta k_D = A \ln \left[1 + \frac{(a-b\Delta P)\Delta P}{T_{co} + R_f P_o} \right] \quad (4-7)$$

and:

$$\Delta k_D = A \ln \left[1 + \frac{a\Delta P}{(T_{co} + R_f P_o)(1+b\Delta P)} \right] \quad (4-8)$$

It is expected that least squares fitting methods will be used for reduction of the static test data to determine the best values for the reactivity coefficients. The general least squares curve fitting equations and methods to be used are presented in Appendix V.

The importance of the Doppler coefficient may require that analysis of the Doppler data be also performed using a more sophisticated model than those listed above.

It has been shown⁽²⁾ that the Doppler effect Δk_D may be closely approximated by spacial weighting of the local Doppler coefficient $\Gamma = A/T_f^m$ with the square of the local power density rather than the more exact flux flux-adjoint weighting. Thus, the spacially weighted Doppler reactivity effect may be represented as:

$$\Delta k_D = \frac{A}{1-m} \frac{\int_{\text{fuel}} P^2(r) \Delta[T_f(r)^{1-m}] dV}{\int_{\text{fuel}} P^2(r) dV} \quad (m \neq 1) \quad (4-9)$$

or:

$$\Delta k_D = \frac{A \int_{\text{fuel}} P^2(r) \ln \frac{T_{f2}(r)}{T_{f1}(r)} dV}{\int_{\text{fuel}} P^2(r) dV} \quad (m=1) \quad (4-10)$$

The compute code WEDOP⁽¹⁾ has been prepared for use in evaluating the parameters A and m based on corrected experimental reactivity and power density distribution data and fuel thermal-hydraulics parameters.

4.4 ESTIMATED RESULTS

Calculated temperature and power coefficients of reactivity at a power level of 10 MW are listed in Table 2-1.

The calculated Doppler constant A and exponent m are -0.0085 $\Delta k/k$ or -2.66\$ and approximately 1.0, respectively.

The estimated reactivity coefficients may be used to determine the operating characteristics under which the static tests will be performed. The calculated operating characteristics, excluding reflector and coolant flow reactivity effects are shown in Figure 4-1.

The calculated change in reactivity as a function of power, with a constant average coolant temperature of 760°F, is shown in Figure 4-2 for WEDOP results. In addition, the best least squares fitting of the non-spatially weighted Doppler model

$$\ln \left(1 + \frac{R_f P}{T_{co} + R_f P_o} \right)$$

is shown for purposes of comparison. The best fit coefficients of three non-spatially weighted models are listed in Table 4-2.

Figure 4-1. STATIC TEST POINTS

Figure 4-2. DOPPLER REACTIVITY VERSUS POWER

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TABLE 4-2.

CALCULATED COEFFICIENTS
NON-SPATIALLY WEIGHTED MODELS

<u>MODEL</u>	<u>COEFFICIENT</u>
$\ln \left(1 + \frac{R_f P}{T_{fo}} \right)$	$R_f = 65.1$
$\ln \left[1 + \frac{(a-bP)P}{T_{fo}} \right]$	$a = 66.44$ $b = 0.09$
$\ln \left[1 + \frac{aP}{T_{fo} (1+bP)} \right]$	$a = 66.36$ $b = 0.0016$

Calculations show that use of a model that does not include spatial weighting of the local Doppler effect may underestimate the Doppler effect by approximately ten percent (10%) relative to the result from the more detailed model used in WEDOP. Thus, if the simple model based on core average fuel temperatures is used in the data analysis, the resulting value of the Doppler constant may be in error about ten percent (10%).

SECTION V

OSCILLATOR TESTS

5.0 DISCUSSION

Two kinds of oscillator tests are planned. These are:

1. Conventional (included zero power) oscillator tests in which a poison rod is oscillated at several frequencies and the reactor power and coolant temperatures are allowed to oscillate about their mean values.
2. Balanced oscillator tests (of which there are two types) in which a poison rod, the primary coolant flow rate, and the secondary coolant flow rate are simultaneously oscillated. The relative phase angle and amplitudes of the reactivity and flow oscillations are adjusted so that one of the following conditions is satisfied:
 - a. The inlet and outlet core coolant temperatures remain constant. This is called a first balanced oscillator test.
 - b. The reactor power remains constant. This is called a second balanced oscillator test.

The first and second balanced oscillator tests are discussed in detail by Caldarola⁽¹⁾ and Russell.⁽²⁾

5.0.1 Objectives

The overall objectives of the oscillator tests are:

1. To demonstrate reactor stability to reactivity and to coolant flow (temperature) oscillations at different mean flow rates and at different power levels up to 20 MW.

2. To provide information for the determination of the Doppler and coolant reactivity coefficients at different fuel temperatures.

3. To provide information for the determination of the frequency dependence of the fuel temperature response to a power or coolant temperature oscillation.

The above information will provide an independent check of results obtained from the static tests, and will provide for the determination of the fuel energy coefficient.

5.0.2 General Procedure

In order to obtain the above information, the oscillator tests will be performed at a number of different mean power levels, and at a number of different mean primary coolant flow rates at specified mean coolant temperatures. In addition, the oscillator rod and coolant flow rate will be oscillated at a number of different frequencies at each mean power level and mean flow rate. The range of frequency oscillation for the oscillator rod is ~ 0.00125 to ~ 0.5 cps, and the range of frequency oscillation for the coolant flow rate is ~ 0.00125 to 0.03 cps. The frequency range of interest is low (~ 0.002 to 0.5 cps) because the fuel time constant for the one-inch diameter SEFOR fuel rods is ~ 35 sec.⁽³⁾

All of the oscillations performed at a specified mean power level, flow rate, and coolant temperature will be considered as one oscillator test.

It is expected that a power oscillation in the range ± 3 to $\pm 5\%$ will provide a high signal to noise ratio, and that the magnitude of the oscillation will not be so large as to invalidate the results of the analysis. If the noise level is very small, it may be possible to lower the amplitude of the power oscillation. Some tests may be performed in which the amplitude of the power oscillation is increased to $\sim \pm 10\%$ in order to investigate the non-linear characteristics of the system, or, in the first balanced oscillator tests, to increase the ratio of fuel to coolant reactivity effects.

The general procedure for each oscillator test is outlined below:

1. Allow reactor to reach static equilibrium at the specified power, flow, and coolant temperature conditions.
2. Record background "noise" for each channel while system is at steady-state. The noise can be recorded at several different frequencies by oscillating the poison rod at zero amplitude in the "oscillator test mode" of operation. High frequency noise may also be determined from data recorded in the "transient mode" of operation and low frequency noise may be determined from data recorded in the "static test mode" of operation. It may be possible to perform this step at another time - during the static tests for example. It may not be necessary to repeat this step for each different test.

3. Oscillate the reactivity rod and coolant flow at the desired amplitude and frequency. The variations in the mean power level, and mean coolant flow rate, should be maintained below 1/2% and variations in mean system temperatures should be maintained below $\sim 0.5^{\circ}\text{F}$ for several successive cycles of oscillation. This may require some initial adjustment of the fine reflector control or of the coolant flow rates.

Recommended frequencies for each test are listed in Section 5.1 through 5.4. It is suggested that the amplitude of the reactivity oscillation be adjusted so that the expected power oscillation is $\sim \pm 3$ to $\sim \pm 5\%$. The reactivity-power transfer functions shown in Section 5.1 through 5.4 can be used to estimate the desired amplitude of the reactivity oscillation. The amplitude of the flow oscillation is expected to be $\leq \sim \pm 5\%$, but will be adjusted to achieve the desired balance (constant power or constant coolant temperature). The desired amplitudes and frequencies may change as a result of analysis of the data.

4. Allow initial starting transients, including any initial adjustments required, to die out, and record data to determine the amplitude and phase angle of the power, reactivity, flow, and temperature oscillations. The estimated number of cycles for which data will be recorded are given for each frequency and each test in Section 5.1 through 5.4.

5. Change to a new frequency of oscillation, while maintaining constant mean test conditions, and repeat steps 3 and 4 until the frequency range of interest has been traversed.

In order to keep the power oscillation in the approximate range of ± 3 to $\pm 5\%$, it may be necessary to change the amplitude of the oscillator rod when the frequency of oscillation is changed. In the zero power oscillator tests, for example, (see Figure 5.1.3), a reactivity oscillation of $\pm 5\text{c}$ at 0.5 cps will cause a $\pm 5\%$ power oscillation, whereas a reactivity oscillation of only $\pm 0.5\text{c}$ at ~ 0.0016 cps will cause a $\pm 5\%$ power oscillation.

In order to ensure that all the data from a given oscillator test are properly normalized, whenever the amplitude of the oscillator rod is changed, steps 3 and 4 above should be performed, at the same frequency, using both the new and the old amplitudes. Depending upon the "spread" in the data, it may be necessary to repeat steps 3 and 4 two or three times with each amplitude.

It is suggested that the first frequency of oscillation for each new oscillator test be 0.5 cps and that the frequency spectrum be traversed by successively decreasing the frequency of oscillation.

5.0.3 Data Collection and Analysis

When the data acquisition system at the SEFOR site is placed in the oscillator test mode of operation, the power, flow and temperature signals are sampled every $1/32$ of the period of the oscillator rod. The signal which causes the computer to sample the various data channels is provided by the oscillator rod mechanism. Thus, the oscillator mechanism must be used whenever the oscillator test mode of operation is used. The "noise spectrum" and the effects of inde-

pendent flow oscillations can be obtained, however, by oscillating the reactivity rod at zero amplitude.

In addition to the data recorded by the data acquisition system, the range in which the flux monitors are operating, the reflector positions, and the amplitude, frequency, and mean position of the oscillating poison rod must be recorded.

It is expected that the analysis of data from a typical oscillator test will include the following steps.

1. Convert the oscillating output signals from the various channels into engineering units. This step can be performed by the data acquisition system at the SEFOR site.

2. Obtain the fundamental frequency components of the oscillating power, flow, reactivity, or temperature signal by evaluating the following integrals:

$$R = \frac{2\omega}{\pi} \sum_{n=1}^N q_n \int_{(n-1)\pi/\omega}^{n\pi/\omega} f(t) \sin \omega t \, dt, \quad N \geq 2 \quad (5.0-1)$$

$$I = \frac{2\omega}{\pi} \sum_{n=1}^N q_n \int_{(n-1)\pi/\omega}^{n\pi/\omega} f(t) \cos \omega t \, dt, \quad N \geq 2 \quad (5.0-2)$$

The mean value, M , of the signal is obtained from the following integral:

$$M = \frac{\omega}{N\pi} \int_0^{N\pi/\omega} f(t) dt \quad \begin{matrix} N \geq 2 \\ (N \text{ even}) \end{matrix} \quad (5.0-3)$$

If the signal contains drift which can be represented by a polynomial in time, Persson⁽⁴⁾ has shown that the contribution of drift to the fundamental components R and I can be eliminated by choosing the weight functions q_n to be:

$$q_n = \frac{1}{2^{N-1}} \frac{(N-1)!}{(N-n)! (n-1)!} \quad (5.0-4)$$

If the signal $f(t)$ contains no drift, either the above choice of q_n or the conventional choice

$$q_n = \frac{1}{N} \quad (N \text{ even}) \quad (5.0-5)$$

will yield the same values (within the uncertainties due to noise) for the coefficients R and I .

A computer program has been specified which will allow either choice for the weight functions q_n . A comparison of the use of the weight functions and the effect of discrete sampling is described in Appendix IV.

If desired, the higher harmonics in the test data can also be determined. The m -th coefficient R_m is determined by replacing $\sin \omega t$ in (5.0-1) by $\sin m\omega t$ and the m -th coefficient I_m by replacing $\cos \omega t$ in (5.0-2) by $\cos m\omega t$. Either weight function (5.0-4) or (5.0-5) may be used if M is odd, but only the conventional weight function $q_n = \frac{1}{N}$ should be used if m is even, since for m even the weight functions given by (5.0-4) will give erroneous results.

3. The "noise" frequency spectrum is determined by performing the above analysis, at several frequencies ω , on the output signals when neither the reactivity nor the flow is being oscillated. It is assumed that the "noise" frequency spectrum does not change during the test, and that a "noise" correction can be applied to the values of R and I that are determined from the oscillator test data.

4. The standard error in the fundamental components R and I can be estimated by performing the same test a number of times. The "spread" in the data can be estimated by determining R and I from a number of different cycles of data that are obtained during the same test.

5. Calculate the transfer function between two desired signals $f(t)$ and $g(t)$ by using the coefficients obtained in step 2. Once the quantities R_f and I_f have been determined for channel f and the corresponding quantities determined for channel g , the complex quantities f and g can be defined by:

$$f = R_f + iI_f$$

$$g = R_g + iI_g$$

where $i \equiv \sqrt{-1}$. The complex transfer function $T(i\omega) = R_T + iI_T$ is then determined from

$$T = \frac{f}{g} \quad (5.0-6)$$

Based on the assumption that the standard errors σ_f in R_f and δ_f in I_f are random and uncorrelated, and that the standard errors in g are likewise uncorrelated, the standard error σ_T in R_T and δ_T in I_T is calculated from the following expression.

$$\sigma_T^2 = R^2\left(\frac{T}{f}\right) \sigma_f^2 + I^2\left(\frac{T}{f}\right) \delta_f^2 + R^2(T/g) \sigma_g^2 + I^2(T/g) \delta_g^2 \quad (5.0-7)$$

$$\delta_T^2 = I^2\left(\frac{T}{f}\right) \sigma_f^2 + R^2\left(\frac{T}{f}\right) \delta_f^2 + I^2(T/g) \sigma_g^2 + R^2(T/g) \delta_g^2 \quad (5.0-8)$$

where $R(x)$ denotes the real part of X and $I(x)$ denotes the imaginary part of X .

The amplitude A_f and phase angle ϕ_f of the complex function f are calculated from the following relations:

$$A_f = \sqrt{R_f^2 + I_f^2}$$

$$\phi_f = \tan^{-1} I_f/R_f$$

The standard errors v_f in A_f and μ_f in ϕ_f are calculated from:

$$v_f^2 = \frac{1}{A_f^2} [R_f^2 \sigma_f^2 + I_f^2 \delta_f^2]$$

$$\mu_f^2 = \frac{1}{A_f^4} [I_f^2 \sigma_f^2 + R_f^2 \delta_f^2]$$

The amplitude A_T , and phase angle ϕ_T of the transfer function T are given by:

$$A_T = A_f / A_g$$

$$\phi_T = \phi_f - \phi_g$$

and the uncertainties v_T and μ_T are calculated from the following expressions:

$$\frac{v_T^2}{A_T^2} = \left(\frac{v_f}{A_f} \right)^2 + \left(\frac{v_g}{A_g} \right)^2$$

$$\mu_T^2 = \mu_f^2 + \mu_g^2$$

6. In the conventional and first balanced oscillator tests, the transform, $h(i\omega)$, of the feedback kernel is calculated from the inverse, z_o , of the zero power-reactivity transfer function, G_o , and from the inverse, z , of the at power-reactivity transfer function, G , by means of the following expression:

$$h(i\omega) = \frac{1}{P} (z_o - z) = \frac{1}{P} \left(\frac{1}{G_o} - \frac{1}{G} \right) \quad (5.0-9)$$

where P is the power level at which z is determined.

The transfer function z can be written as:

$$Z = \frac{\Delta K M}{\Delta P} = M T \equiv M \left(\frac{R_f + i I_f}{R_g + i I_g} \right)$$

where M is the mean value of the power signal, and T is defined by equation (5.0-6) when $f = \Delta k$, and $g = \Delta P$.

The standard errors σ_z in R_z and δ_z in I_z are thus calculated from:

$$\sigma_z^2 = R_z^2 \left(\frac{\sigma_M}{M} \right)^2 + M^2 \sigma_T^2$$

$$\delta_z^2 = I_z^2 \left(\frac{\sigma_M}{M} \right)^2 + M^2 \sigma_T^2$$

where σ_T^2 and δ_T^2 are given by equations (5.0-7) and (5.0-8). The quantity σ_M^2 is the random error in the mean signal from the flux detectors and does not include bias errors due to the uncertainty in absolute power level.

The standard deviations in R_h and I_h are calculated from:

$$\sigma_h^2 = \frac{1}{p^2} [\sigma_{zo}^2 + \sigma_z^2] + R_h^2 (\sigma_p/p)^2$$

$$\delta_h^2 = \frac{1}{p^2} [\delta_{zo}^2 + \delta_z^2] + I_h^2 (\sigma_p/p)^2$$

where σ_p is the random error in the mean power level P.

7. In the conventional, or the first balanced, oscillator tests, the product $Ph/z_o = PhG_o$ can be plotted to obtain information concerning reactor stability. The reactor would be linear unstable (if h did not depend upon the power) at a power level where $PhG_o = 1.0$. Regardless of whether or not $PhG_o = 1.0$, the phase margin, and gain margin (if defined) can be determined from the conventional, or first balanced, oscillator tests. Reactor stability, gain and phase margins are discussed in several references (5,6,7) and the discussions need not be repeated here.

8. Fit $h(i\omega)$, or transfer functions such as the one between coolant temperature and input reactivity, to a model in order to obtain reactivity coefficients and frequency response functions. The models for the various frequency response functions and transfer functions are described in Section 2.5.

9. In addition to the above, it may be desirable to check for "consistency" between the real and imaginary parts and/or the amplitude and phase of transfer functions calculated in step 5 above, or to check for "consistency" in the data obtained in step 2.

It can be shown (8) that if the Fourier transform, $F(\omega)$, of a causal function is analytic and bounded in the lower half ω plane, including the real axis, the real part, $R(\omega)$, and imaginary part, $I(\omega)$, of $F(\omega)$ are related, to within an arbitrary additive constant, by the following expressions:

$$R(\omega) = \frac{-1}{\pi} P \int_{-\infty}^{\infty} \frac{I(u)}{u-\omega} du \quad (5.0-10)$$

$$I(\omega) = \frac{1}{\pi} P \int_{-\infty}^{\infty} \frac{R(u)}{u-\omega} du \quad (5.0-11)$$

where P denotes the principal value.

If the system is minimum phase, (i.e., if the logarithm of the amplitude $A(\omega)$, and the phase angle $\phi(\omega)$ are analytic in the lower half ω plane including the real axis) the following relations are valid:

$$\ln A(\omega) = \frac{-1}{\pi} P \int_{-\infty}^{\infty} \frac{\phi(u)}{u-\omega} du$$

and

$$\phi(\omega) = \frac{1}{\pi} P \int_{-\infty}^{\infty} \frac{\ln A(u)}{u - \omega} du$$

$$= \frac{1}{\pi} P \int_{-\infty}^{\infty} \frac{d[\ln A(\omega e^{\lambda})]}{d\lambda} \ln \coth \left| \frac{\lambda}{2} \right| d\lambda$$

Thus, for example, the experimental values of $I(\omega)$ can be used in equation (5.0-10) to calculate values of $R(\omega)$ which can then be compared with the measured values of R . The calculated and measured values of $R(\omega)$ should agree, within the limits of the experimental measurements and accuracy of the numerical integration.

In addition, the consistency of the amplitude and phase data can be checked in order to determine whether or not the system is minimum phase. The real and imaginary components should always be "consistent". The amplitude and phase data will not be consistent, however, if the system is non-minimum phase, e.g. if the transfer function in question contains a pure delay of the form $e^{-i\omega t}$.

5.1 ZERO POWER OSCILLATOR TEST

A zero power oscillator test is a conventional oscillator test which is performed at a power level which is sufficiently low that changes in power level on the order of 10% cause an indiscernible change in fuel or coolant temperature.

For the zero power oscillator tests, a poison rod is successively oscillated (sinusoidally) at several different frequencies. The reactor

power is allowed to oscillate with amplitudes up to 10% of the mean power level. The mean power level and the coolant temperature are held constant during the test.

5.1.1 Objective

The primary objective of the zero power oscillator test is to determine the power-reactivity transfer function for the critical reactor at a power level which is sufficiently low that reactivity feedback effects can be neglected. The reactor should be critical, and feedback effects will be small for power levels of ~ 10 kW. The absolute coolant temperature is unimportant as long as it is held constant during the test. The effects of coolant temperature drift are discussed in Section 5.1.2.

A possible secondary objective (the determination of the fraction of fissions caused by U-238) is discussed in Section 5.1.3.

5.1.2 General Procedure

In general, the procedure for the zero power oscillator test is the same as for all the oscillator tests and is described in Section 5.0.2. Only the oscillating power and reactivity (oscillator rod position) signals are of primary interest, however, and other signals, such as coolant and fuel temperatures or primary and secondary coolant flow rates, need be observed only to the extent necessary to ensure that they remain constant during the test.

The frequencies of oscillation for the zero power tests are the same as for the conventional oscillator test and it is suggested that 5 frequencies per decade be used over the frequency range of interest (~ 0.002 to 0.5 cps). A possible sequence of frequencies and the number of cycles of data (after the initial transients have died out) are given in Table 5-2.

Determination of the zero power transfer function at lower frequencies may be difficult due to the reactivity effect that results from a small slow drift in the core temperature. At zero power, the change in reactivity as a result of a core temperature change is $\sim 0.6\text{¢}/^\circ\text{F}$. For a linear drift of $"A"^\circ\text{F}/\text{sec}$, the power increases (if initially critical) approximately as $\exp(0.3At^2/T)$ where $T \sim 1000$ and t is the time in sec. Thus, a drift of only 1°F per 10 minutes will cause an increase in the mean power level of $\sim 38\%$ during one of oscillation at a frequency of 0.00125 cps.

5.1.3 Data Collection and Analysis

The data will be collected and analyzed as described in steps 1 through 5 of Section 5.0.3 to determine the zero power transfer function $G_o = \Delta P/P_o / \Delta K$, and to estimate the standard errors in the real and imaginary parts of G_o .

The calculated value of the zero power transfer function at 0.5 cps can be used to obtain a "dynamic" measurement of the oscillator rod worth as follows. Since the calculated amplitude of the zero power transfer function at 0.5 cps is 1.055, the rod worth, in cents, is 1.055 times the percent power oscillation at 0.5 cps. The "dynamic" measurement of the worth can be compared with the "static" worth that is measured when the oscillator rod is moved the same distance in a static test.

In addition to the above, it may be possible to determine the delayed neutron fraction β and the fraction of fission neutrons, q , which are produced by U-238. The procedure for this analysis is described below.

The inverse, z_o , of the zero power transfer function may be written as follows:

$$z_o = \frac{1}{G_o} = i\omega \frac{\ell}{\beta} + \frac{1}{\beta} \sum_{m=1}^M \beta_m q_m \sum_{n=1}^N a_{mn} e_{mn} / (i\omega + \lambda_{mn})$$

where $i = \sqrt{-1}$; ω is the frequency of oscillation; M is the number of fissionable isotopes; N is the number of delayed neutron groups; β_m is the delayed neutron fraction of isotope M ; ℓ/β is the ratio of the prompt neutron lifetime to the delayed fraction; a_{mn} , e_{mn} , and λ_{mn} are the relative abundance, the effectiveness (relative to prompt neutrons), and the decay constants, respectively, for the delayed neutrons in group n from isotope m ; q_m is the fraction of the fission neutrons which are produced by isotope m ; and the delayed neutron fraction, β , is defined by:

$$\beta = \sum_{m=1}^M \beta_m q_m \sum_{n=1}^N a_{mn} e_{mn}$$

The delayed neutron parameters, β_m , a_{mn} , and λ_{mn} are assumed to be known, and equal to the values given by Keepin.⁽⁹⁾ The fractions of fission neutrons produced have been calculated for SEFOR by Reynolds and Stewart.⁽¹⁰⁾ The calculated values of the e_{mn} for SEFOR are constant, to within 1%, and equal to 0.91.⁽¹¹⁾ The values of ℓ/β in SEFOR will be determined by noise analysis (see Section 3.11). The calculated value of β is 0.0032.⁽¹⁰⁾

In SEFOR, ~97.5% of the fissions occur in the U-238 and the Pu-239.⁽¹⁰⁾ Thus, the inverse of the zero power transfer function may be approximated by the following expression:

$$z_o = i\omega \left[\frac{\ell}{\beta} + \frac{\epsilon\beta_1}{\beta} (1-f) \sum_{n=1}^N \frac{a_{n1}}{i\omega + \lambda_{n1}} + \frac{\epsilon\beta_2 f}{\beta} \sum_{n=1}^N \frac{a_{n2}}{i\omega + \lambda_{n2}} \right] \quad (5.1-1)$$

where the subscripts 1 and 2 refer to the delayed neutron parameters for Pu-239 and for U-238, respectively, where ϵ is the effectiveness (0.91), and where f is the fraction of fission neutrons produced by U-238. The approximate expression above over predicts the calculated fraction of fission neutrons produced by U-238 by ~0.4% and over predicts the fraction of fission neutrons produced by Pu-239, $(1-f)$, by ~2.1%.

The simplified model for z_o can be used, along with the curve fitting procedure described in Appendix V, to determine the value of f for which the sum of the weighted squares of the residuals for the real and imaginary parts of z_o are a minimum.

For this case, the least squares procedure outlined in Appendix V is particularly simple and the resultant equations are given below:

Let ω_m ($m=1,2,\dots,M$) be the frequencies at which the real part, R_m , and the imaginary part, I_m , of the inverse, $z_o(i\omega)$, of the zero power transfer function are measured. Let σ_{Rm} and σ_{Im} be the estimated standard errors in R_m and I_m , respectively. Let σ_o be an arbitrary constant and define:

$$g_j = \frac{\epsilon\beta_j}{\beta} \quad (j=1,2)$$

$$R_{mj} = \omega_m^2 \sum_{n=1}^N \left[\frac{a_{nj}}{\lambda_{nj}^2 + \omega_m^2} \right] \quad (j=1,2)$$

$$I_{mj} = \omega_m \sum_{n=1}^N \left[\frac{a_{nj} \lambda_{nj}}{\lambda_{nj}^2 + \omega_m^2} \right] \quad (j=1,2)$$

where $j=1$ refers to the Pu-239 parameters and $j=2$ refers to the U-238 parameters, and where the estimated value of β is used in the calculation of g_1 and g_2 .

The sum of the weighted squares of the residuals in the real and imaginary parts of z_0 can be written as follows:

$$S = \sum_{m=1}^M \frac{\sigma_o^2}{\sigma_{Rm}^2} \left[R_m - g_1 R_{m1} - f (g_2 R_{m2} - g_1 R_{m1}) \right]^2 \quad (5.1-2)$$

$$+ \sum_{m=1}^M \frac{\sigma_o^2}{\sigma_{Im}^2} \left[I_m - \omega_m \frac{\ell}{\beta} - g_1 I_{m1} - f (g_2 I_{m2} - g_1 I_{m1}) \right]^2$$

Minimizing S with respect to f , one obtains the following equation:

$$f = \frac{1}{Q} \sum_{m=1}^M \left\{ \frac{\sigma_o^2}{\sigma_{Rm}^2} [R_m - g_1 R_{m1}] [g_2 R_{m2} - g_1 R_{m1}] + \frac{\sigma_o^2}{\sigma_{Im}^2} [I_m - \omega_m \frac{\ell}{\beta} - g_1 I_{m1}] [g_2 I_{m2} - g_1 I_{m1}] \right\} \quad (5.1-3)$$

where:

$$Q = \sum_{m=1}^M \left\{ \frac{\sigma_o^2}{\sigma_{Rm}^2} [g_2 R_{m2} - g_1 R_{m1}]^2 + \frac{\sigma_o^2}{\sigma_{Im}^2} [g_2 I_{m2} - g_1 I_{m1}]^2 \right\} \quad (5.1-4)$$

The estimated standard error, σ_f , in f is then obtained from the following equation:

$$\sigma_f^2 = \frac{1}{Q} \frac{S}{2M-1}$$

where the value of f obtained from (5.1-3) is used in (5.1-2) to calculate S .

The value of f obtained above can then be used to calculate β from the following expression:

$$\beta = \epsilon [\beta_1(1-f) + \beta_2 f]$$

If the value of β obtained from this expression does not agree with the value that was used in the initial calculation of g_1 and g_2 , the new value of β can be used to recalculate the functions g_1 and g_2 and the above procedure for the determination of f can be repeated.

5.1.4 Expected Results

The amplitude and phase of the zero power transfer function, G_o , is shown in Figure 5.1-1 and 5.1-2 for different fractions of fissions in U-238. The calculated effective f for the simplified model, equation (5.1-1) is 0.108, and the calculated value of β is 0.0031.

The expected amplitude and phase of the zero power transfer function is shown in Figure 5.1-3 and 5.1-4, for cases in which the reactor is subcritical.

Figure 5.1-1. AMPLITUDE FOR PURE Pu-239, SEFOR, AND PURE U-238
ZERO POWER TRANSFER FUNCTION

Figure 5.1-2. PHASE ANGLE FOR PUEE Pu-239, SEFOR, AND U-238 ZERO POWER TRANSFER FUNCTIONS

Figure 5.1-3. AMPLITUDE OF SUBCRITICAL
ZERO POWER TRANSFER FUNCTION

Figure 5.1-4. PHASE ANGLE OF SUBCRITICAL
ZERO POWER TRANSFER FUNCTION

5.2 CONVENTIONAL OSCILLATOR TESTS

In the conventional oscillator tests, a poison rod is successively oscillated (sinusoidally) at several different frequencies. The reactor power and system temperatures are allowed to oscillate with amplitudes up to approximately 10% of their mean values. Although the mean power level and mean coolant temperature should not deviate (within $\sim 0.5\%$ or 0.5°F) from their mean values during the test, no attempt is made to dampen either the power or the coolant temperature oscillation by means of a coolant flow oscillation, i.e. the coolant flow rate is held constant.

5.2.1 Objective

The objectives of the conventional oscillator tests and their analysis are:

1. to demonstrate reactor stability at different flow rates and at different power levels up to 20 MW.
2. to determine the Doppler and coolant reactivity coefficient.
3. to determine the frequency dependence of the fuel temperature response to a power oscillation.

5.2.2 General Procedure and Test Conditions

In general, the procedure for the conventional oscillator test is the same as for all the oscillator tests and is outlined in Section 5.0.3.

The signals of primary interest are:

1. The reactor power (neutron flux)
2. The oscillator rod (reactivity) signal
3. The inlet and outlet coolant temperature signals from the instrumented fuel assemblies.

4. The fuel temperature signals from the instrumented fuel assemblies.

In addition, the flow rate, absolute mean power level, system temperatures, and reflector positions should be recorded.

The test conditions for the conventional oscillator tests are given in Table 5-1.

The number of frequencies, the sequence of frequencies, and the number of cycles of data to be collected at each frequency depend to a great extent upon the observed results; e.g. the appearance of an unexpected "resonance" may influence the number of frequencies; or the amount of drift or "noise" may influence the number of cycles of collected data.

Although, as pointed out above, it may prove to be inapplicable, a possible sequence of frequencies for each conventional oscillator test, along with the tentative number of cycles of data (after the initial transients have died out) for each frequency are shown in Table 5-2. With this sequence, the test could be performed using the frequencies indicated by Case A (5 frequencies per decade) and then repeated (as a check) using the frequencies indicated by Case B ($\sqrt{2}$ -1/2 frequencies per decade). Any additional checks could also be performed using the frequencies indicated by Case B.

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TABLE 5-1

CONVENTIONAL OSCILLATOR TESTS

Average Coolant Temperature = 760°F

<u>Power</u> <u>(MW)</u>	<u>Flow</u> <u>(%)</u>	<u>Core ΔT</u> <u>(°F)</u>
1	50	12
5	100	30
	50	60
	30	100
10	100	60
	50	120
15	75	120
19	95	120

TABLE 5-2

POSSIBLE FREQUENCIES FOR CONVENTIONAL OSCILLATOR TESTS

<u>Frequencies (cycles/sec)</u>	<u>Number of cycles</u>	<u>Minutes for Specified Cycles</u>	
		<u>Case A</u>	<u>Case B</u>
0.50	15	1	1
0.25	15	1	
0.16	15	2	
0.10	15	3	3
0.063	15	4	
0.040	15	6	
0.025	15	10	10
0.016	10	11	
0.010	10	17	17
0.0063	7	19	
0.0040	5	21	21
0.0025	3	20	
0.0016	3	32	32

5.2.3 Data Collection and Analysis5.2.3.1 Discussion

The general procedure for data collection and analysis are described in Section 5.0.3 and these procedures can be used, where applicable, in the conventional oscillator tests.

As in the zero power oscillator tests (see Section 5.0.3), the "dynamic" measurement of the oscillator rod worth can be compared with the "static" measurement of the rod worth. The calculated amplitude of

the power transfer function at 20 MW is 1.048, if the fraction of heat which is generated in the coolant is 0.035 or 1.056, if no heat is generated in the coolant. The difference between the amplitude of the at-power-transfer-function and the amplitude of the zero-power-transfer-function (1.055) is approximately linear with power between 0 and 20 MW at a frequency of 0.5 cps.

It is planned that the reactivity coefficients and parameters in the fuel frequency response functions will be determined for each of the oscillator tests. It is expected that the Doppler reactivity coefficients and the frequency response parameters will change as the mean power or coolant temperature is changed.

The data from each oscillator test will be treated as described in Section 5.0.3, in order to provide "measured" transfer functions and estimated standard errors in the transfer functions at several different frequencies. The transfer functions, and the estimated standard errors, will then be used to obtain the reactivity coefficients and fuel frequency response parameters by means of a least square fitting of the models to the "measured" data.

A derivation of general least squares fitting equations based on the derivation described by Deming⁽¹²⁾ is shown in Appendix V. The fitting equation for the conventional oscillator tests is described in more detail below.

The mathematical model for the conventional oscillator tests is given in Section 2.5. In terms of this model, the "condition equations" can be written as follows:

$$L(i\omega) = h(i\omega) - \alpha_f F_{av}(i\omega) X_f(i\omega) - [C_f F_s(i\omega) + C_q] U(i\omega) - C_i U_i(i\omega) = 0$$

This expression represents two equations since it is desired that both the real and imaginary part of L be equal to zero.

Thus, the condition equations can be written as: $\theta \equiv \text{Real}(L) = 0$ and $\phi \equiv \text{Imaginary}(L) = 0$.

The "measured" quantities are $h(i\omega)$, $U(i\omega)$, and $U_i(i\omega)$. The term $X_f(i\omega)$ is a known calculated quantity (see Appendix II), and $F_{av}(i\omega)$ and $F_s(i\omega)$ are expressed in terms of two parameters γ and ν as shown in Section 2.5.

In the following discussion, it will be assumed that there are six unknown parameters to be determined. These parameters are α_f , γ , ν , C_f , C_q , and C_i . In case it is desired to treat any of these parameters as a known quantity, the equations shown below may be modified in a manner to be described at the end of this section.

In the fitting of the model to the conventional oscillator test data, a total of $6 \times N$ (where N is the total number of frequencies of oscillation) data points are used. These are the real and imaginary components of h , of U , and of U_i at each of the frequencies of oscillation. Let these data points be stored in a matrix, X , and let the elements $x(m,n)$ of the X matrix be defined so that at each frequency, ω_n , the measured values can be written as:

$$\begin{aligned} h(i\omega_n) &= x(1,n) + i x(2,n) \\ U(i\omega_n) &= x(3,n) + i x(4,n) \\ U_i(i\omega_n) &= x(5,n) + i x(6,n) \end{aligned}$$

i.e. $x(1,n)$ is the real part of the measured value of h at frequency ω_n , $x(2,n)$ is the imaginary part of measured value of h at frequency ω_n , $x(3,n)$ is the real part of the measured value of U at frequency ω_n , etc.

In addition to the data points themselves, the standard errors and the coefficients of correlation in each of these data points is used. Let the standard error in $x(m,n)$ be denoted by $\sigma(m,n)$, and let the coefficient of correlation between $x(j,n)$ and $x(k,n)$ be denoted by $r_n(j,k)$. Define, using the notation of Appendix V, the elements of a $\bar{P}_n$ matrix by:

$$P_n(j,k) = \sigma(j,n) \sigma(k,n) r_n(j,k) / \sigma_0^2$$

where σ_0^2 is an arbitrary constant.

Let the elements $a(j)$ ($j=1,2,\dots,6$) be defined by:

$$\begin{aligned} a(1) &= \alpha_f \\ a(2) &= \gamma \\ a(3) &= v \\ a(4) &= C_f \\ a(5) &= C_q \\ a(6) &= C_i \end{aligned}$$

In order to use the procedure outlined in Appendix V, initial estimates of the six parameters are needed. Let the initial estimates of the parameters $a(j)$ be denoted by $a_0(j)$.

The only additional quantities which are required are the partial derivatives of L with respect to the measured quantities $x(m,n)$, and the partial derivatives with respect to the parameters $a(j)$. All of these derivatives can be easily calculated, except for the partial derivatives with respect to γ and v . These derivatives can be written as follows:

$$\frac{\partial L}{\partial \gamma} = -\alpha_f X_f \frac{\partial F_{av}}{\partial \gamma} - C_f U \frac{\partial F_s}{\partial \gamma}$$

$$\frac{\partial L}{\partial v} = -\alpha_f X_f \frac{\partial F_{av}}{\partial v} - C_f U \frac{\partial F_s}{\partial v}$$

where:

$$\frac{\partial F_s}{\partial \gamma} = -i\omega F_s^2$$

$$\begin{aligned} \frac{\partial F_s}{\partial v} &= -\frac{1}{v} \left[\frac{i\omega v^2}{2} F_s^2 + 2(F_s - G_s)G_s \right] \\ &= -\frac{2i\omega}{v} \left[\frac{v^2}{4} F_s^2 + \gamma F_s G_s - (\gamma + \frac{v^2}{8}) F_{av} G_s \right] \end{aligned}$$

$$\frac{\partial F_{av}}{\partial \gamma} = \frac{F_s^2 - F_{av}}{\gamma + v^2/8}$$

$$\frac{\partial F_{av}}{\partial v} = \frac{\frac{v}{4} (2F_s^2 - F_{av} - F_{av} G_s) + \frac{2\gamma}{v} G_s (F_s - F_{av})}{\gamma + v^2/8}$$

where G_s is defined by: $G_s \equiv 1 - i\omega \gamma F_s$

The expression for F_{av} and F_s are given in Section 2.5.

5.2.3.2 Procedure for Fitting of Data

Using the nomenclature of Appendix V, and the previously defined quantities, the quantities ξ_{kn} and ψ_{kn} are calculated from the following:

$$\xi_{kn} + i \psi_{kn} = \frac{\partial L(i\omega_n)}{\partial x_{kn}}$$

where the right hand side is evaluated using measured coordinates and estimated parameters.

The values for the weights σ_{fn} , σ_{gn} , and $R_n \sigma_{fn} \sigma_{gn}$ of Appendix V are then calculated from equations (10), (18), and (19) in the appendix, where the elements of the $\bar{P}_n$ matrix have been defined above.

The quantities F_{kn} and G_{kn} are calculated from the following:

$$F_{kn} + i G_{kn} = \frac{\partial L(i\omega_n)}{\partial a_k}$$

where the right hand side is evaluated using measured coordinates and estimated parameters.

The initial values f_{no} and g_{no} are calculated from:

$$f_{no} + i g_{no} = L(i\omega_n)$$

where the right hand side is evaluated using measured coordinates and estimated parameters.

The $\bar{T}$ matrix and the C vector are calculated from equations (24) and (25) in Appendix V, and the $\bar{T}$ matrix is then inverted to obtain the $\bar{Q}$ matrix. The estimate of the values of the parameters is then given by (27) in Appendix V.

The value of S is obtained from equation (20), (21), and (22) of Appendix V and the estimated standard errors and coefficients of correlation are obtained from equations (28), (29), and (30) of Appendix V.

Values for the calculated curve, $h_c(i\omega)$, of the feedback kernel $h(i\omega)$ can be obtained from the following:

$$h_c(i\omega_n) = X(1,n) + i X(2,n) - f_n - i g_n \quad (5.2-1)$$

where (from Appendix V - equations (21) and (22)):

$$f_n = f_{no} - \sum_{\ell=1}^L F_{\ell n} (a_{\ell o} - a_{\ell})$$

and

$$g_n = g_{no} - \sum_{\ell=1}^L G_{\ell n} (a_{\ell o} - a_{\ell})$$

where L = the number of parameters.

The estimated standard error in the real part of h (due to uncertainties in the parameters) at frequency ω_n is given by:

$$\sigma = \left[\sum_{n=1}^L \sum_{k=1}^L F_{jn} F_{kn} Q_{jk} \right]^{1/2}$$

The estimated standard error in the imaginary part of h is given by:

$$\sigma = \left[\sum_{j=1}^L \sum_{k=1}^L G_{jn} G_{kn} Q_{jk} \right]^{1/2}$$

The calculated value of $h(i\omega_n)$ which corresponds to the measured value of $h(i\omega_n)$, i.e., the "adjusted value of the coordinate (see equation (15) and (15-a) of Appendix V) is given by:

$$h_m(i\omega_n) = X(1,n) - \lambda_n \sum_{j=1}^K P_n(1,j) \xi_{jn} - v_n \sum_{j=1}^K P_n(1,j) \psi_{jn}$$

$$+ i \left\{ X(2,n) - \lambda_n \sum_{j=1}^K P_n(2,j) \xi_{jn} - v_n \sum_{j=1}^K P_n(2,j) \psi_{jn} \right\}$$

where K = the number of coordinates and where λ_n and v_n are given by equations (16) and (17) of Appendix V, and the f_n and g_n which are used in these equations are defined above.

Although the value of $h_m(i\omega_n)$, (see above), should lie on the calculated curve for $h(i\omega_n)$, it will not, in general, coincide with the value $h_c(i\omega_n)$ are given by equation (5.2-1), but rather will lie on the curve drawn between the points $h_c(i\omega_n)$.

The Doppler energy coefficient, η , is obtained from the following expression:

$$\eta = \frac{a(1) - C_p}{(a(2) + a^2(3)/8)A_f}$$

where C_p is the calculated power dependent portion of the fuel and fuel clad axial expansion effects (see Section 2.4).

The square of the estimated standard error, σ_η in η , is calculated from the following:

$$\sigma_\eta^2 = \frac{\sigma_C^2}{[a(2) + a^2(3)/8]^2} \left[\left(\frac{1}{A_f} \right)^2 Q_{11} + \eta^2 Q_{22} + \eta^2 \frac{a^2(3)}{16} Q_{33} - 2\eta Q_{12}/A_f - \eta a(3) Q_{13}/2A_f + \eta^2 \frac{a(3)}{2} Q_{23} + \sigma_C^2 \frac{1}{\sigma^2 A_f^2} \right]$$

where σ_C^2 is the estimated standard error in C_p , and the $Q(i,j)$ are elements of the $\bar{Q}$ matrix.

In the event that it is desired to treat one of the parameters, say $a(\ell)$, as known, the F_n and G_n vectors are reduced to 5 element vectors by removing the ℓ -th element (this reduces T and Q to 5×5 matrices, and Y and C to a 5 element vector).

The term $\sigma^2(\ell)F_{\ell n}^2$ is added to σ_{fn}^2 , the term $\sigma^2(\ell)G_{\ell n}^2$ is added to σ_{gn}^2 , and the term $\sigma^2(\ell)F_{\ell n}G_{\ell n}$ is added to $R_n\sigma_{fn}\sigma_{gn}$, where $\sigma(\ell)$ is the estimated standard error in $a(\ell)$. The calculation then proceeds as before.

5.2.3.3 Effects of Biased Errors

The standard errors in the reactivity coefficients which have been discussed in the previous sections include only the "random" errors in the measurements. Uncertainties in instrument calibration (i.e. absolute power, flow, or temperature) have little effect on the frequency response parameters. These uncertainties do affect the reactivity coefficients, however, and must be included in an estimate of the over-all uncertainty in the coefficients.

If it is assumed that there is equal probability that the true mean value lies above or below the calibrated value, and that the over-all uncertainty in a parameter which depends on more than one measurement can be approximated by propagating the biased errors as if they were random, the uncertainty in the reactivity coefficients can be calculated as follows.

Using the notation of the previous section, the over-all uncertainty $\bar{\sigma}_1$ in the fuel power coefficient, a_1 , is obtained from the following:

$$\left(\frac{\bar{\sigma}_1}{a_1}\right)^2 = \left(\frac{\sigma_1}{a_1}\right)^2 + \left(\frac{\sigma_p}{p_o}\right)^2 + \left(\frac{\sigma_k}{k_o}\right)^2$$

where σ_1 is the standard random error in a_1 , (σ_p/p_o) is the fractional error in the absolute power level, and σ_k/k_o is the fractional error in the absolute reactivity oscillation.

The over-all uncertainties $\bar{\sigma}_j$ ($j=4,5,6$) in the temperature coefficients a_j ($j=4,5,6$) defined in the previous section are obtained from the following:

$$\left(\frac{\bar{\sigma}_j}{a_j}\right)^2 = \left(\frac{\sigma_j}{a_j}\right)^2 + \left(\frac{\sigma_T}{T_o}\right)^2 + \left(\frac{\sigma_k}{k_o}\right)^2 \quad (j=4,5,6)$$

where σ_j is the random error described in the previous section, where (σ_k/k_o) is defined above, and where (σ_T/T_o) is the fractional error in the absolute coolant temperature measurement.

Since the Doppler energy coefficient, η , is determined from the calculated value of the fuel power coefficient, a_1 , the over-all uncertainty, $\bar{\sigma}_\eta$ in η must include the uncertainties in the absolute power and reactivity measurements. Thus,

$$\left(\frac{\bar{\sigma}_\eta}{\eta}\right)^2 = \left(\frac{\sigma_\eta}{\eta}\right)^2 + \left(\frac{\sigma_P}{P_o}\right)^2 + \left(\frac{\sigma_k}{k_o}\right)^2$$

where σ_η and η are defined in previous sections and where (σ_P/P_o) and (σ_k/k_o) are defined above.

5.2.4 Expected Results

The SEFOR plant has a regulating device which (when in use) automatically adjusted the air flow rate in the air blast heat exchanger in order to keep the cold leg inlet temperature of the intermediate heat exchanger (IHX) constant. In order to determine the effect of this device on the reactivity feedback during a conventional oscillator test, the power-reactivity transfer function was calculated with automatic control (constant IHX secondary inlet temperature) and without automatic

control (constant air flow). The reactivity feedback model described in Section 2.5.1 and the model for the coolant temperature response that is described in Appendix III were used. The results are shown in the following figures.

The automatic control "smooths out" the power-reactivity transfer function at 5 MW and 30% flow, (see Figures 5.2-1, 5.2-2, and 5.2-11, 5.2-12), and reduces* the coolant temperature feedback at very low frequencies (0.001 rad/sec). However, the relative magnitude of coolant and fuel temperature feedback in the frequency range of interest (~ 0.01 to ~ 0.2 rad/sec) are comparable in the two cases (i.e., the amplitude and phase of the power-reactivity transfer functions are similar over the frequency range ~ 0.01 to ~ 0.2 rad/sec), and it is expected that the Doppler effect will be determined with the same accuracy in the conventional oscillator tests regardless of whether or not the IHX cold leg inlet temperature is held constant. The effect of different Doppler and coolant coefficients upon the power-reactivity transfer functions is illustrated in Figures 5.2-1 through 5.2-10, where the coolant and Doppler coefficients are simultaneously increased and decreased by 20%. The effect of a change in the Doppler coefficient alone is more clearly illustrated in Section 5.4.4.

* This may be observed by noting that the power-reactivity transfer functions with constant IHX secondary inlet temperature have larger amplitudes and smaller phase angles than the comparable transfer functions with constant flow at 0.001 rad/sec. This is the same trend that occurs when the coolant reactivity effect is zero - see Section 5.4.4.

Figure 5.2-1. CONVENTIONAL OSCILLATOR TESTS AT CONSTANT AIR FLOW; AMPLITUDE OF POWER REACTIVITY TRANSFER FUNCTION AT 5 MW, 30% FLOW

Figure 5.2-2. CONVENTIONAL OSCILLATOR TESTS AT CONSTANT AIR FLOW; PHASE OF POWER REACTIVITY TRANSFER FUNCTION AT 5 MW, 30% FLOW

Figure 5.2-3. CONVENTIONAL OSCILLATOR TESTS AT CONSTANT AIR FLOW;
AMPLITUDE OF POWER REACTIVITY TRANSFER FUNCTION AT 5 MW, 100% FLOW

Figure 5.2-4. CONVENTIONAL OSCILLATOR TESTS AT CONSTANT AIR FLOW; PHASE OF POWER REACTOR TRANSFER FUNCTION AT 5 MW, 100% FLOW

Figure 5.2-5. CONVENTIONAL OSCILLATOR TESTS AT CONSTANT AIR FLOW; AMPLITUDE OF POWER REACTIVITY TRANSFER FUNCTION AT 10 MW, 50% FLOW

Figure 5.2-6. CONVENTIONAL OSCILLATOR TESTS AT CONSTANT AIR FLOW; PHASE OF POWER REACTIVITY TRANSFER FUNCTION AT 10 MW, 50% FLOW

Figure 5.2-7. CONVENTIONAL OSCILLATOR TESTS AT CONSTANT AIR FLOW;
AMPLITUDE OF POWER REACTIVITY TRANSFER FUNCTION AT
10 MW, 100% FLOW

Figure 5.2-8. CONVENTIONAL OSCILLATOR TESTS AT CONSTANT AIR FLOW;
 PHASE OF POWER REACTIVITY TRANSFER FUNCTION AT
 10 MW, 100% FLOW

Figure 5.2-9. CONVENTIONAL OSCILLATOR TESTS AT CONSTANT AIR FLOW;
AMPLITUDE OF POWER REACTIVITY TRANSFER FUNCTION AT
20 MW, 100% FLOW

Figure 5.2-10.

CONVENTIONAL OSCILLATOR TESTS AT CONSTANT AIR FLOW;
 PHASE OF POWER REACTIVITY TRANSFER FUNCTION AT
 20 MW, 100% FLOW

Figure 5.2-11. CONVENTIONAL OSCILLATOR TESTS AT CONSTANT IHX SECONDARY INLET TEMPERATURE; AMPLITUDE OF POWER REACTIVITY TRANSFER FUNCTION AT 5 MW

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Figure 5.2-12.

CONVENTIONAL OSCILLATOR TESTS AT CONSTANT IHX SECONDARY INLET TEMPERATURE; PHASE ANGLE OF POWER REACTIVITY TRANSFER FUNCTION AT 5 MW

Figure 5.2-13. CONVENTIONAL OSCILLATOR TESTS AT CONSTANT IHX SECONDARY INLET TEMPERATURE; AMPLITUDE OF POWER REACTIVITY TRANSFER FUNCTION AT 10 MW

Figure 5.2-14. CONVENTIONAL OSCILLATOR TESTS AT CONSTANT IHX SECONDARY INLET TEMPERATURE; PHASE ANGLE OF POWER REACTIVITY TRANSFER FUNCTION AT 10 MW

Figure 5.2-15. CONVENTIONAL OSCILLATOR TESTS AT CONSTANT IHX SECONDARY INLET TEMPERATURE; AMPLITUDE OF POWER REACTIVITY TRANSFER FUNCTION AT 20 MW, 100% FLOW

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Figure 5.2-16. CONVENTIONAL OSCILLATOR TESTS AT CONSTANT IHX SECONDARY INLET TEMPERATURE; PHASE ANGLE OF POWER REACTIVITY TRANSFER FUNCTION AT 20 MW, 100% FLOW

5.3 FLOW AND REACTIVITY OSCILLATIONS

5.3.1 Objective

The coolant flow rate in the primary loop, the coolant flow rate in the secondary loop, and the reactivity (poison rod) will be separately oscillated at a few frequencies at the mean power levels, flow rates, and coolant temperatures which will be used in the balanced oscillator tests. The purpose of these oscillations is to provide information which will be used to predict the relative phase angles and amplitudes required for the balanced oscillator tests. Since a number of the conventional and balanced oscillator tests are performed with the same mean reactor conditions, additional reactivity oscillations will not be required at these test conditions.

The flow oscillations will also serve to demonstrate reactor response to flow (temperature) oscillations.

5.3.2 General Procedure and Test Conditions

The general procedure is outlined in Section 5.0.2. The signals of primary interest are:

1. The reactor power
2. The oscillator rod (reactivity) signal - if oscillated
3. The primary coolant flow signals from the instrumented fuel assemblies and from the magnetic flow meter in the primary system - if oscillated
4. The secondary coolant flow signals, which can be recorded on one channel of the 8-channel oscillograph (these signals are not recorded in the data acquisition system) - if oscillated
5. The inlet and outlet coolant temperature signals from the instrumented fuel assemblies.

The test conditions for the flow and reactivity oscillations correspond to the test conditions for the first and second balanced oscillator tests and are shown in Table 5-3. The primary flow, and the secondary flow should be separately oscillated at these conditions. The reactivity should not be oscillated at those test conditions which are the same as the conventional oscillator test conditions. These are indicated in Table 5-3.

The four suggested frequencies of oscillation are 0.025, 0.010, 0.004, and 0.0016 cps. In order to correctly normalize the worth of the poison rod it may be desirable to oscillate the rod at the additional frequency of 0.5 cps.

TABLE 5-3

FLOW AND REACTIVITY OSCILLATIONS

<u>Power (MW)</u>	<u>Flow (%)</u>	<u>Average Coolant Temperature (°F)</u>
5*	30	760
10	40	560, 760
15	60	760
19	75	760

* Flow Oscillations Only - the reactivity oscillations are obtained from the conventional oscillator test.

5.3.3 Data Collection and Analysis

The data will be collected and analyzed as described in steps 1 through 3 of Section 5.0.3 to determine nine transfer functions; three from the reactivity oscillation at constant flow rate, three from the primary flow oscillation at constant reactivity and secondary flow rate, and three from the secondary flow oscillation at constant reactivity and primary flow rate.

The three transfer functions to be determined from the reactivity oscillation are the response of the fractional power oscillator ($\Delta P/P_o$); the response of the core inlet temperature (ΔT_i); and the response of the core outlet temperature (ΔT_o) to the reactivity oscillation ΔK . Let these transfer functions be denoted by G_1 , G_2 , and G_3 , respectively.

The three transfer functions to be determined from the primary coolant flow oscillation are the response of $\Delta P/P_o$, ΔT_i , and ΔT_o to the primary fractional flow oscillation $\Delta V_1/V_{1o}$. Let these transfer functions be denoted by F_1 , F_2 , and F_3 , respectively.

The transfer functions to be determined from the secondary coolant flow oscillations are the response of $\Delta P/P_o$, ΔT_i , and ΔT_o to the secondary fractional flow oscillation $\Delta V_2/V_{2o}$. Let these transfer functions be denoted by R_1 , R_2 , and R_3 , respectively.

The functions F_j , G_j , and R_j ($j=1,2,3$) will be determined at 3 or 4 frequencies (e.g., Section 5.3.2). Their values at other frequencies can be estimated from a plot of the measured values.

In the first balanced oscillator test the inlet and outlet temperatures are to be held constant. Therefore, the following two complex

equations must be satisfied.

$$\Delta T_1 = G_2 \Delta K + F_2 (\Delta V_1/V_{10}) + R_2 (\Delta V_2/V_{20}) = 0$$

$$\Delta T_0 = G_3 \Delta K + F_3 (\Delta V_1/V_{10}) + R_3 (\Delta V_2/V_{20}) = 0$$

Since the G, F, and R functions are known, the relative phase angle and amplitudes of the reactivity and flow oscillations required to achieve a balance can be estimated from the above equations. There is not a unique solution to these equations, however, the solutions should be chosen so that the resultant amplitudes of the power oscillation (which can be estimated for a known reactivity oscillation by means of the curves shown in Section 5.4.4) and the flow oscillations are in the range of ~2 to 10% of the mean power level and flow rates. Analog simulation of the first balanced oscillator test⁽¹³⁾ indicates that such solutions are possible.

In the second balanced oscillator test the power level is to be held constant. Therefore, the following equation must be satisfied.

$$\Delta P/P_0 = G_1 \Delta K + F_1 (\Delta V_1/V_{10}) + F_2 (\Delta V_2/V_{20}) = 0$$

Considerations similar to those for the first balanced oscillator test apply to the solutions of this equation. Specifically, the amplitude of the flow oscillations should be in the range of ~2 to 10% of the mean flow rate, and the coolant temperature oscillations should be $\lesssim \pm 15^\circ\text{F}$.

5.4 BALANCED OSCILLATOR TESTS

In the balanced oscillator tests the reactivity (poison rod) and the primary and secondary coolant flow rates are oscillated in a sinusoidal manner.

In the first balanced oscillator test the relative phase angles and amplitudes of the reactivity and flow oscillations are adjusted until the core inlet and outlet coolant temperatures remain constant. The flow and reactivity oscillations then remain unchanged as several cycles of data are recorded at the balanced condition.

In the second balanced oscillator test the relative phase angles and amplitudes of the reactivity and flow oscillations are adjusted until the reactor power (neutron flux) remains constant. The flow and reactivity oscillations then remain unchanged as several cycles of data are recorded at the balanced condition.

5.4.1 Objective

Successful performance and analysis of first and second balanced oscillator tests which are performed at the same mean reactor power, flow, and temperature conditions will provide values for the fuel temperature coefficient, the fuel power coefficient, the coolant-structure temperature coefficient, the fuel "time constants," and the reactivity-energy coefficient. By performing these tests at various power levels while maintaining a constant coolant temperature, and then performing the tests at different coolant temperatures while maintaining a constant power level, the dependence of the reactivity

coefficients and frequency response parameters upon power and coolant temperature can be determined. The power and coolant coefficients can be compared with the coefficients obtained from the static and conventional oscillator tests, and in addition, the frequency response parameters can be compared with the results obtained from the conventional oscillator tests.

5.4.2 General Procedure and Test Conditions

A general procedure for all the oscillator tests is outlined in Section 5.0.2. This procedure can be used, where applicable, in the balanced oscillator tests.

Initial estimates of the reactivity and flow oscillations that are required for the balanced conditions will be available from the flow and reactivity oscillations described in Section 5.3 as well as from any previous balanced oscillator tests. Minor adjustments in reactivity and flow will undoubtedly be required in order to obtain a balanced condition. After initial transients have died out and the balance has been achieved, within the limits described below, the data will be recorded.

In the first balanced oscillator tests the amplitude of the core inlet and outlet temperature oscillations should be held below $\pm 0.5^\circ\text{F}$ while the amplitude of the power oscillation is ± 0.5 MW.

In the second balanced oscillator tests the amplitude of the power oscillation should be held below ± 0.2 MW while the amplitude of the core coolant temperature oscillation is ± 10 to 15°F .

If the oscillations of the "balanced variable" is maintained within the above limits; the coolant temperature contribution to the reactivity feedback in the first balanced oscillator test will be $\leq 10\%$ of the contribution caused by the power oscillation; and in the second balanced oscillator test the contribution to the reactivity feedback that is caused by the power oscillation will be $\leq 10\%$ of the contribution caused by the coolant temperature oscillation.

The signals of primary interest are:

1. The reactor power (neutron flux) signal
2. The oscillator rod (reactivity) signal
3. The inlet and outlet coolant temperature signals from the instrumented fuel assemblies
4. The primary and secondary coolant flow rates
5. The fuel temperature signals from the instrumented fuel assemblies

In addition, the absolute flow rates, power level, system temperature and reflector positions should be recorded.

The suggested test conditions for the balanced oscillator tests are shown in Table 5-4. Both a first and a second balanced oscillator test are to be performed at each of these reactor conditions. Low flow rates are recommended since this will decrease the effect of a non-zero temperature oscillation in the first balanced oscillator test (see the equation in Section 5.4.3.2).

TABLE 5-4

BALANCED OSCILLATOR TESTS

<u>Power (MW)</u>	<u>Flow (%)</u>	<u>Average Coolant Temperature (°F)</u>
5	30	760
10	40	560, 760
15	60	760
19	75	760

A list of possible frequencies for the balanced oscillator tests are shown in Table 5-5. The tests can be performed using the frequencies listed under Case A and then repeated (as a check) using the frequencies indicated by Case B. The poison rod is oscillated at 0.5 cps in order to ensure proper normalization of the rod worth.

TABLE 5-5

POSSIBLE FREQUENCIES FOR BALANCED OSCILLATOR TESTS

<u>Frequency (cycles/sec)</u>	<u>Number of Cycles (after balance is achieved)</u>	<u>Minutes for Specified Cycles</u>	
		<u>Case A</u>	<u>Case B</u>
0.50 (rod only no balance)	15	1	1
0.025	10	7	7
0.016	10	10	
0.010	5	9	9
0.0063	5	13	
0.0040	3	13	13
0.0025	3	20	
0.0016	3	32	32

5.4.3 Data Collection and Analysis

The general procedures for data collection and analysis are described in Section 5.0.3, and these procedures can be used, where applicable, in the balanced oscillator tests.

As in the zero power and conventional oscillator tests (see Section 5.0.3 and 5.1.3.1) the dynamic measurement of the oscillator rod worth can be compared with the "static" measurement of the oscillator rod worth.

5.4.3.1 First Balanced Oscillator Feedback

The mathematical model for the feedback in the first balanced oscillator test is given in Section 2.5.4. In terms of this model, the condition equations can be written as follows:

$$L(i\omega) = [h(i\omega) - \{C_f F_s(i\omega) + C_q\} U(i\omega) - C_i U_i(i\omega)] - \alpha_f F_{av}(i\omega) X_f(i\omega) = 0$$

As before, this represents two equations: $\theta = \text{REAL}(L) = 0$ and $\phi = \text{Imaginary}(L) = 0$. These are the same condition equations as are used for the conventional oscillator test; however, in the first balanced oscillator test, the term $U(i\omega)$ and $U_i(i\omega)$ should be nearly zero and one seeks parameters α_f , γ , and ν such that $\alpha_f F_{av}(i\omega) X_f(i\omega)$ is equal to the "measured" term in the square brackets.

For the analysis of this experiment, it is necessary to assume that C_f , C_q , C_i , and $F_s(i\omega)$, in addition to $h(i\omega)$, $U(i\omega)$, and $U_i(i\omega)$ are known "measured" quantities. It is only when U and U_i are nearly zero (and their standard errors are small), that the values of α_f , γ , and ν , become independent of the value and standard errors assumed for C_f , C_q , C_i , and $F_s(i\omega)$. Since the value and standard error of F_s depends on the values of γ and ν , it may be necessary to "re-estimate" F_s after each iteration on γ and ν .

In order to treat the term in the square brackets as a measured quantity, and to retain the procedure outlined in Section 5.4.3 and in Appendix V, define $x(j,n)$ for $j=1,\dots,6$, as in Section 5.4.3. Re-define the parameters γ and ν , as they appear in F_s , not in F_{av} , to be η and ρ , (i.e. $F_s(i\omega, \gamma, \nu) \rightarrow F_s(i\omega, \eta, \rho)$). Define additional "measured coordinates" $x(k,n)$ for $k=7,8,\dots,11$, to be:

$$\begin{aligned} x(7,n) &= C_f & n &= 1,2,\dots,N \\ x(8,n) &= \eta & n &= 1,2,\dots,N \\ x(9,n) &= \rho & n &= 1,2,\dots,N \\ x(10,n) &= C_q & n &= 1,2,\dots,N \\ x(11,n) &= C_i & n &= 1,2,\dots,N \end{aligned}$$

Define the elements of the $\bar{P}_n$ matrix as in Section 5.2.3.

Let the elements $a(j)$ ($j=1,2,3$) be defined by:

$$\begin{aligned} a(1) &= \alpha_f \\ a(2) &= \gamma \\ a(3) &= \nu \end{aligned}$$

The analysis from this point on is described in Section 5.2.3.

5.4.3.2 First Balanced Oscillator - Flow Measurements

The mathematical model for the ratio of the relative flow oscillation to the relative power oscillation is given in Section 2.5.4. In terms of the model, the condition equations can be written as follows:

$$\begin{aligned} L(i\omega) &= \frac{\Delta V(i\omega)/V_o}{\Delta P(i\omega)/P} - a_c X_c(i\omega) + \frac{V_o}{K_n V_f} \{ (1 + b - bF_i) U_o - (1 - bF_i) U_i \} \\ &\quad - (1 - a_c) X_f(i\omega) F_s(i\omega) \end{aligned}$$

$$\theta = \text{Real } (L) = 0$$

$$\phi = \text{Imaginary } (L) = 0$$

The quantities a_c , $X_f(i\omega)$, $b(i\omega)$, and $F_1(i\omega)$ are calculated quantities which are defined in Appendices II and III. The quantity K_n is the ratio of the steady-state core temperature rise at full flow, V_f , to the total power, P , and V_o is the mean flow rate.

Since the procedure for the analysis of the tests have been outlined in Appendix V, and examples of the procedure have been given in previous sections, only the definition of the measured coordinates and the parameters to be determined will be given here.

The measured quantities are the 3 complex functions $(\Delta V/V_o)/(\Delta P/P_o)$, U_o , and U_1 , and the real quantity V_o . Thus, there are a total of 7 coordinates, the real and imaginary components of the complex functions and the coordinates V_o . The coordinates of the complex functions depend upon the n frequency points ω_n , but V_o is the same for all frequencies in a given test. The standard errors in the coordinates of the complex functions, likewise, depend upon the frequency ω_n , but the standard error in V_o is independent of n .

5.4.3.3 Second Balanced Oscillator Tests

The condition equations for the analysis of the second balanced oscillator test can be written (see Section 2.5.5) as follows:

$$L(i\omega) = \left[\frac{\Delta k_{im}}{\Delta T_c} - \left(\frac{1}{RG_o} - \alpha_{fi} F_{aw} X_{fi} \right) \frac{\Delta P}{\Delta T_c} \right] + (C_{fs} F_{qi} + C_{fi}) + C_{fi} \frac{\Delta T_i}{\Delta T_c} = 0$$

This represents two equations: $\theta = \text{Real}(L) = 0$, and $\phi = \text{Imaginary} = 0$.

In the analysis of the second balanced oscillator tests, one seeks parameters C_f , γ , ν , C_q , and C_i such that $(C_f F_s + C_q) + C_i \Delta T_i / \Delta T_c$ is the negative of the "measured" term in the square brackets above. Therefore, it is necessary to assume that α_f and F_{av} , in addition to $\Delta k_{in} / \Delta T_c$, PG_o , $\Delta P / \Delta T_c$, and $\Delta T_i / \Delta T_c$ are known "measured" quantities. It is only when $\Delta P / \Delta T_c$ is nearly zero (i.e. when a near "balanced" condition is attained) that the value of C_f , γ , ν , C_q , and C_i become independent of the values and standard errors assumed for α_f and the parameters γ and ν as they appear in F_{av} .

The procedure for the analysis is discussed in Appendix V and examples for similar cases are outlined in Sections 5.2.3 and 5.4.3.1. The quantities to be treated as measured variables and the quantities to be treated as unknown parameters have been defined above.

If so desired, the function F_s , as determined from the analysis of the first balanced oscillator flow measurements, may be treated as a "measured" function (note the comment at the end of Section 2.5.4). If this is the case, the parameters γ and ν in F_s are treated as measured coordinates rather than unknown parameters.

5.4.3.4 Fuel Thermocouple Response

The condition equation for the analysis of the response of the central fuel temperature to a power oscillation can be written (see Section 2.5.6) as:

$$L(i\omega) = \frac{\Delta T_{cf}(i\omega)}{\Delta P(i\omega)} - K_c F_c(i\omega) X_f(i\omega) - G_c(i\omega) U(i\omega) = 0$$

The "measured" coordinates are the real and imaginary components of $\Delta T_{cf}/\Delta P$, and U . The parameters to be determined are K_c , and the two parameters in F_c and G_c , γ and ν . The term X_f is calculated as described in Appendix II. and F_c and G_c are described in Section 2.5.6.

The procedure and examples for the least squares fitting have been described in Section 5.2.3 and in Appendix V. All that remains is to define the partial derivations of L with respect to the two parameters γ and ν . These derivations are defined below:

$$\frac{\partial L}{\partial \gamma} = -K_c X_f \frac{\partial F_c}{\partial \gamma} - U \frac{\partial G_c}{\partial \gamma}$$

$$\frac{\partial L}{\partial \nu} = -K_c X_f \frac{\partial F_c}{\partial \nu} - U \frac{\partial G_c}{\partial \nu}$$

The partial derivaties introduced above can be defined as follows:

$$\frac{\partial F_c}{\partial \gamma} = \frac{F_s G_c - F_c}{\gamma + \nu^2/4}$$

$$\frac{\partial G_c}{\partial \gamma} = -i\omega F_s G_c$$

$$\frac{\partial F_c}{\partial \nu} = \frac{\frac{2\gamma}{\nu} G_c (G_s - F_s) - \frac{\nu}{2} (F_c - G_c F_s)}{\gamma + \nu^2/4}$$

$$\frac{\partial G_c}{\partial \nu} = -i\omega \frac{\nu}{2} G_c F_s + 2 i\omega \frac{\gamma}{\nu} G_c (F_s - G_s)$$

5.4.4 Expected Results

The calculated power-reactivity transfer functions for the first balanced oscillator tests are shown in Figures 5.4-1 through 5.4-6 for cases in which a perfect balance (zero coolant temperature oscillation) is attained. The power-reactivity transfer function for the first balanced oscillator tests is independent of the flow rate, provided a perfect balance is attained, however, the effect of non-zero power oscillations on the determination of F_s is decreased with decreasing flow rate (see Section 5.4.3.2) and the low flow rates of Table 5.4-1 are recommended.

The calculated reactivity-average coolant temperature transfer function for the second balanced oscillator tests are shown in Figures 5.4-7 and 5.4-8. Note that the value of the amplitude at very low frequencies is the sum of the fuel and coolant temperature coefficients, while the value of the amplitude at very high frequencies is the coolant coefficient only.

The effect of a $\pm 20\%$ change in the Doppler coefficient on the first balanced oscillator tests is shown explicitly in Figures 5.4-1 to 5.4-6. The effect of a change in the Doppler or coolant coefficient in the results of the second balanced oscillator test (figures 5.4-7 and 5.4-8) is not shown explicitly. The effect of such changes can be readily visualized, however, since an increase in the coolant coefficient causes a constant increase in amplitude at low and high frequencies with little change in phase, while an increase in the Doppler coefficient causes an increase in the magnitude of the phase angle and an increase in the difference between the low and high frequency amplitude.

Figure 5.4-1. FIRST BALANCED OSCILLATOR TESTS; AMPLITUDE OF POWER REACTIVITY TRANSFER FUNCTION AT 5 MW

Figure 5.4-2. FIRST BALANCED OSCILLATOR TESTS; PHASE OF POWER REACTIVITY TRANSFER FUNCTION AT 5 MW

Figure 5.4-3. FIRST BALANCED OSCILLATOR TESTS; AMPLITUDE OF POWER REACTIVITY TRANSFER FUNCTION AT 10 MW

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Figure 5.4-4. FIRST BALANCED OSCILLATOR TESTS; PHASE ANGLE OF POWER REACTIVITY TRANSFER FUNCTION AT 10 MW

Figure 5.4-5. FIRST BALANCED OSCILLATOR TESTS; AMPLITUDE OF POWER REACTIVITY TRANSFER FUNCTION AT 20 MW

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Figure 5.4-6. FIRST BALANCED OSCILLATOR TESTS; PHASE OF POWER REACTIVITY TRANSFER FUNCTION AT 20 MW

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Figure 5:4-7. SECOND BALANCED OSCILLATOR TESTS; AMPLITUDE OF REACTIVITY TO AVERAGE COOLANT TEMPERATURE TRANSFER FUNCTION

Figure 5.4-8. SECOND BALANCED OSCILLATOR TESTS; AMELITUDE OF REACTIVITY TO AVERAGE COOLANT TEMPERATURE TRANSFER FUNCTION

SECTION VI

TRANSIENT TESTS

6.0.1 Objective

The primary objectives of the transient tests are to demonstrate the prompt Doppler reactivity feedback and to determine the Doppler energy coefficient. In order to meet these objectives, both sub-prompt critical (positive reactivity insertion of less than one dollar) and super-prompt critical (positive reactivity insertion of greater than one dollar) tests will be performed. It is expected that sub-prompt critical transients will be performed at various power levels up to ~ 15 MW and that super-prompt critical tests will be performed at various power levels up to ~ 10 MW in order to determine the dependence of the Doppler energy coefficient upon the initial fuel temperature as well as the dependence upon the amount of energy released during a transient.

6.0.2 Description of Tests

In the transient tests, it may be necessary to experimentally verify, before each test or series of tests, that the desired poison slug is attached to the Fast Reactor Excurison Device (FRED) and that the available excess reactivity in the core (i.e. the remaining amount of reflector which can be raised) has not changed since a previous test. The worth of the FRED slug can be verified by the "rod swap" technique. The available excess reactivity can be determined, for example, by placing the reflectors in a prescribed configuration and comparing the resultant power level with a previous

power level obtained for the same configuration; or by comparing reflector positions at a prescribed power level with previous positions for the same power level.

Once the FRED slug worth, available excess reactivity, and the operation of the FRED and the data acquisition system have been verified, the reactor is maintained at the specified test conditions of power, flow, and coolant temperature for several minutes in order to ensure that static equilibrium is attained. The FRED is then fired while the data acquisition system is in the transient mode of operation. The reactor will be "scrammed" automatically a few hundred milliseconds after the initiation of the transient.

6.0.3 Data Collection and Analysis

The power (flux), fuel temperatures, and coolant temperature changes, as well as the time dependent FRED position, will be recorded by the data acquisition system. In addition, the following should be recorded in the log book:

1. Range setting on logarithmic amplifiers
2. Range setting on operational neutron monitors
3. Reflector positions
4. Poison slug worth
5. Main coolant venturi flowmeter
6. Pressure settings of the FRED
7. Reflector temperatures

As in the oscillator tests, the transient analysis is based on the assumption that the "point reactor" kinetic equation is valid and that the delayed neutron parameters and λ/β (the ratio of the prompt neutron lifetime to the delayed neutron fraction) are known.

In order to determine whether there are any changes in the spatial neutron flux shape during a transient, the ramp ejection of poison slugs worth $\sim 0.95\%$ were simulated with the VARI-QUIR computer code.⁽¹⁾ The results⁽²⁾ indicated that the maximum change in the spatial power shape during a transient is approximately 1% in a region immediately adjacent to the poison slug. During a simulated transient, the ratio of the high energy (≥ 0.5 MeV) neutron flux to the average power level changed by $\lesssim 0.2\%$ at a point corresponding to the U-238 fission chamber location in SEFOR. Any spatial changes in gamma flux at the SEFOR gamma detector location should be small, since the fraction of the gamma flux arising from thermal neutron capture is small. Thus, spatial effects should not invalidate the "point reactor" model.

A typical analysis of the transient test data will include the following steps:

1. The transient data, such as the logarithm of the power, the coolant temperature rise, and the fuel temperature rise is plotted in order to check the adequacy of the data. The plotting can be performed by the data acquisition system.

2. The total transient energy release is calculated and compared with predictions. The total energy release can be calculated in each of the following ways:

2a. The transient power data from the U-238 chambers can be combined with the data from the gamma chambers to provide a composite power trace. If the detectors are calibrated in the manner described in Section 2.2.2, the time integral of the power is the total excess energy which will eventually be produced (after the fission products produced by the excursion have decayed) in the core as a result of the transient. The actually energy absorbed in the fuel, up to time t , using the numbers described in Appendix II, is 88% of this value.

2b. Although the fuel thermocouples in the instrumented fuel assemblies will not demonstrate the time dependent behavior of the fuel temperature, they should provide a measure of the maximum fuel temperature that results during the transient. This measurement can be used, along with the fuel heat capacity data, to calculate the energy released to the fuel.

2c. The coolant flow rate and the time dependent coolant temperature measurements from the instrumented fuel assemblies can also be used to obtain a measure of the total energy produced. The peak coolant temperature will occur after the reactor has "scrammed" and the integration time of the coolant temperature response must be long enough to allow the heat to diffuse from the fuel.

3. The transient power data is "smoothed" and used to calculate the net reactivity from the "point kinetic" equation. The SPORT computer program⁽³⁾ can be used for this purpose. SPORT also calculates the energy release (the integral of the power) as a function of time and provides a smoothed power trace.

4. The measured FRED position vs time and the measured (from the static tests) FRED worth vs position are used to calculate the time dependent FRED worth. This time dependent reactivity should agree, within experimental uncertainties, with the early portion of the net reactivity calculated by the SPORT program (particularly during the sub-prompt critical transients), since the feedback reactivity during the first portion of the transient is small. Calculated curves of the FRED reactivity as a function of time and of position are shown in Appendix VII.

5. Calculate the time dependent feedback reactivity by subtracting the FRED reactivity from the net reactivity obtained in 3 above.

6. Calculate the non-Doppler feedback and subtract from the total feedback in order to obtain the Doppler feedback. The calculation of the non-Doppler feedback will be based upon calculated reactivity coefficients for the fuel and the fuel-clad axial expansion effects and upon the measured coolant-structure reactivity coefficients. In order to use these reactivity coefficients, the fuel, clad, coolant, and structure temperatures during the transients must also be calculated. The temperature

changes can be estimated either by using the actual power trace in a code such as ARGUS⁽⁴⁾, which calculates the various temperature changes, or by using the temperature changes (for the specific transient under consideration) which are predicted by a code such as FORE⁽⁵⁾.

7. Using best estimates of the standard errors in the Doppler reactivity and the energy absorbed in the fuel, the Doppler energy coefficient is calculated using the least squares procedures outlined in Section 6.1.3 and 6.2.3.

Although the uncertainties in the non-Doppler feedback may be large, calculations indicate that the Doppler effect during a transient is $\sim 80\%$ of the total feedback, and the uncertainty in the Doppler energy coefficient is expected to be considerably less than the uncertainties in the non-Doppler effects. An estimate of possible expected errors in the Doppler energy coefficient obtained from the analysis of transient tests is described in Appendix VIII.

6.1 SUB-PROMPT CRITICAL TRANSIENTS

6.1.1 Description of Tests

The sub-prompt critical transient tests involve rapid insertion of less than 1.0\$ positive reactivity (i.e. rapid ejection of poison slugs worth $< 1.0\%$), in time intervals of the order of 100 msec.

The initial fuel temperature rise is rapid so that heat conduction in the fuel is negligible (insofar as the fuel temperature is concerned) during the first 1 to 2 seconds (i.e. for times small compared to the ~ 30 sec fuel time constant).

The sub-prompt critical transient tests of primary interest are those in which the energy released during the transient causes several cents of reactivity feedback. Such transients require initial power levels in the megawatt range and reactivity insertions of the order of 95¢. Other transients, at lower initial power levels, and with smaller slug worths, will be run however, in order to check out the operation of the FRED, the transient power detector system, the data acquisition system, and the data analysis codes.

The sub-prompt critical transients of primary interest can result in peak power levels of 10 to 30 times the initial power and a maximum temperature rise in the average fuel of 100 to 200°F at rates up to 700°F/sec. At full coolant flow, the maximum clad and sodium temperature rises are approximately 20°F and 10°F, respectively. The clad, structure, and sodium temperature rise during the first few tenths of a second are due primarily to direct gamma and neutron heating.

6.1.2 Test Conditions

For purposes of experimental program planning, it was assumed that the sub-prompt critical transient tests shown in Tables 6-1 and 6-2 will be performed. The actual number of tests which will be performed at each power level, and with each slug size, will depend upon the results of the experiments; however, the tests shown in Tables 6-1 and 6-2 may provide an indication of the types of

tests and total number of tests which will be performed.

The tests shown in Table 6-1 are performed primarily for the purpose of gaining familiarity with the system. The tests shown in Table 6-2 will provide information for the determination of the Doppler energy coefficient. These tests will provide energy coefficients at three power levels between low and high power. The initial conditions for the tests correspond to reactor conditions at which oscillator tests are performed in order to facilitate comparison with the results of the oscillator tests.

TABLE 6-1

SUB-PROMPT CRITICAL FAMILIARIZATION TESTS

<u>Power (MW)</u>	<u>Slug Worth (dollars)</u>	<u>Number of Tests</u>
0.1	0.40	2
	0.70	2
	0.85	2
	0.93 → 0.98	3
10	0.40	2
	0.70	2
	0.85	2

TABLE 6-2

SUB-PROMPT CRITICAL TRANSIENTS

<u>Power (MW)</u>	<u>Slug Worth (dollars)</u>	<u>Number of Tests</u>
5	0.93 → 0.98	3
10	0.93 → 0.98	3
~15	0.93 → 0.98	3

6.1.3 Data Collection and Analysis

The general data collection and analysis procedure for the sub-prompt critical tests are outlined in Section 6.0.3.

In the sub-prompt critical tests most of the reactivity feedback and energy release occurs after the FRED slug has been completely ejected. This effect can be utilized in the following manner. Let the time of complete FRED ejection be denoted by t_o and let the FRED reactivity be denoted by $\Delta k_F(t)$. Then, for times $t > t_o$ the only change, $\delta k_n(t) \equiv \Delta k_n(t) - \Delta k_n(t_o)$ in the net reactivity, $\Delta k_n \equiv \Delta k_F + \Delta k_f$, will be due to changes in the feedback reactivity Δk_f , because $\Delta k_F(t) = \Delta k_F(t_o)$ for $t > t_o$. Thus, analysis of $\delta k_n(t)$, rather than $\Delta k_n(t)$, eliminates the large error in Δk_f ($0 \leq |\Delta k_f| \leq 10\%$) which could occur as a result of a small error in Δk_F ($0 \leq \Delta k_F \leq 95\%$).

The model for the Doppler reactivity feedback, $\Delta k_D(t)$, as a function of the energy absorbed in the fuel, $\Delta E_f(t)$, is given in Section 2.6 as:

$$\Delta k_D(t) = \eta \Delta E_f(t) \quad (6.1-1)$$

or as:

$$\Delta k_D(t) - \Delta k_D(t_0) = \eta [\Delta E_f(t) - \Delta E_f(t_0)] \quad (6.1-2)$$

where t_0 is the time of complete FRED ejection. The Doppler energy coefficient η is a function of the initial power level (i.e. the initial fuel temperature) and may depend to some extent upon the energy release (i.e. a better representation for Δk_D may be a quadratic in ΔE_f).

In the above model, the "measured", (see discussion is 6.0.3, step no. 7), quantities are Δk_D and ΔE_f and the parameter to be determined is η . Let k_n and E_n denote the "measured" value of Δk_D and ΔE_f , respectively, at time t_n , and let σ_{k_n} and σ_{E_n} denote the standard errors in k_n and E_n , respectively. If equation (6.1-2) is used, let k_n denote $\Delta k_D(t) - \Delta k_D(t_0)$ and let E_n denote $\Delta E_f(t) - \Delta E_f(t_0)$. Then the Doppler energy coefficient η , is obtained from the following expression:

$$\eta = \frac{1}{Q} \sum_{n=1}^N \frac{W_n E_n k_n}{E_n^2} \quad (6.1-3)$$

where the weight, W_n , is defined by:

$$W_n \equiv \frac{\sigma_0^2}{\sigma_{kn}^2 + \eta_0^2 \sigma_{En}^2}$$

where σ_0 is an arbitrary constant; where η_0 is the initial estimate of η ; and where Q is defined by:

$$Q = \sum_{n=1}^N W_n E_n^2$$

The sum, S , of the weighted squares of the residuals is given by:

$$S = \sum_{n=1}^N W_n (k_n - \eta E_n)^2$$

and the estimated standard error, σ_η , in η is given by:

$$\sigma_\eta^2 = \frac{S}{N-1}$$

Once a value of η is obtained from (6.1-3), this value can be used as the initial value η_0 for a new iteration.

The overall uncertainty $\bar{\sigma}_\eta$ in η , which includes the effect of "biased errors" such as uncertainties in the absolute power level, and thus in the absolute energy release, can be estimated from the following expression:

$$\frac{\bar{\sigma}_\eta^2}{\eta^2} = \frac{\sigma_\eta^2}{\eta^2} + \frac{\sigma_P^2}{P^2}$$

where $\frac{\sigma_P}{P}$ is the fractional uncertainty in the absolute power, or energy, calibration.

6.1.4 Expected Results

The FORE⁽⁵⁾ computer code was used to calculate the reactor transient behavior, and the associated reactivity feedbacks, for several of the sub-prompt critical transients listed in Table 6.1-1 and 6.1-2. It was assumed in all cases that the FRED is completely ejected in ~ 0.14 seconds.

The calculated transient for two typical sub-prompt critical familiarization tests are shown in Figures 6.1-1 and 6.1-2. For comparison, the results of a \$0.96 transient at an initial power level of 0.1 MW are shown in Figure 6.1-3 for a case in which the Doppler $T \frac{dk}{dT}$ is only -0.004 and all other reactivity coefficients are zero. These curves show, as pointed out above, that the reactivity feedback is particularly small for transients beginning at low initial power levels. Thus, little or no information

concerning the Doppler energy coefficient will be obtained from the tests listed in Table 6.1-1. However, these tests should provide a means for gaining familiarity with the operating characteristics of the FRED.

Typical sub-prompt critical transients which will be used to obtain information concerning the Doppler energy coefficient are shown in Figures 6.1-4 through 6.1-6. For comparison, the transient behavior that would occur as a result of the same FRED ejection if the Doppler $T \frac{dk}{dT}$ is only -0.004 and all other reactivity coefficients are zero, is shown in Figures 6.1-7 through 6.1-9.

As Figures 6.1-4 through 6.1-6 (or 6.1-7 through 6.1-9) indicate, most of the reactivity feedback occurs after the FRED is completely ejected (see Section 6.1.3 for the significance of this effect upon the determination of the reactivity feedback).

Figure 6.1-1 \$0.96 TRANSIENT AT 0.1 MW FOR CALCULATED COEFFICIENTS

Figure 6.1-2. \$0.85 TRANSIENT AT 10 MW FOR CALCULATED COEFFICIENTS

Figure 6.1-3 \$0.96 TRANSIENT AT 0.1 MW FOR $-.004$ DOPPLER

Figure 6.1-4 \$0.96 TRANSIENT AT 5 MW FOR CALCULATED COEFFICIENTS

Figure 6.1-5 \$0.96 TRANSIENT AT 10 MW FOR CALCULATED COEFFICIENTS

Figure 6.1-6 \$0.96 TRANSIENT AT 15 MW FOR CALCULATED COEFFICIENTS

Figure 6.1-7 \$0.96 TRANSIENT AT 5 MW FOR $-.004$ DOPPLER

Figure 6.11-83 50.96% TRANSIENT AT 10 MW FOR -0.004 DOPPLER

Figure 6.1-9 \$0.96 TRANSIENT AT 15 MW FOR $-.004$ DOPPLER

6.2 SUPER-PROMPT CRITICAL TRANSIENTS

6.2.1 Description of Tests

The super-prompt critical tests involve rapid insertion of more than 1.0\$ positive reactivity. Maximum planned transients of this type will result in peak power levels of 10^4 to 10^5 times the initial power level and a maximum temperature rise in the average fuel of $\sim 400^\circ\text{F}$ at maximum rates of $\sim 10,000^\circ\text{F}/\text{sec}$. The reactivity effects due to clad, structure, and sodium temperature rise may be 15 to 25% of the fuel reactivity effect.

Tests using reactivity insertions in the range of 1.0\$ to 1.3\$ in approximately 0.1 second are of primary interest, since these result in significant fuel temperature rise independent of heat conduction effects. An error analysis (see Appendix VIII) indicates that little increase in accuracy can be obtained by using reactivity insertions $> 1.3\%$.

6.2.2 Test Conditions

For the super-prompt critical transients, two different sets of test conditions are listed (Tables 6-3 and 6-4). The set that will be used will depend upon the Doppler coefficient that is determined from the static and oscillator tests. The actual number of tests shown in Tables 6-3 and 6-4 are listed for program planning purposes.

Either sets of tests (Table 6-3 or Table 6-4) will serve to demonstrate the ability of the Doppler effect to turn around a prompt critical excursion under temperature conditions approaching those of a large power reactor. Since static, oscillator, and sub-prompt critical tests will have already been performed at the initial power and temperature conditions of these tests, a direct comparison of results is afforded.

TABLE 6-3

SUPER-PROMPT CRITICAL TRANSIENTS(Doppler. $T \frac{dk}{dT} = -0.004$)

<u>Initial Power</u> (MW)	<u>Slug Worth</u> (\$)	<u>Number of Tests</u>
0.1	1.05	1
	1.1	2
	1.15	1
	1.2	3*
5	1.05	1
	1.1	1
	1.15	2
	1.2	1

* Repeat and rate change

TABLE 6-4

SUPER-PROMPT CRITICAL TRANSIENTS(Doppler $T \frac{dk}{dT} = -0.008$)

<u>Initial Power</u> (MW)	<u>Slug Worth</u> (\$)	<u>Number of Tests</u>
0.1	1.1	2
	1.2	1
	1.3	3*
5	1.1	1
	1.2	2
	1.3	1
10	1.1	1
	1.2	2
	1.3	1

* Repeat and rate change

6.2.3 Data Collection and Analysis

The general data collection and analysis procedures for the super-prompt critical tests are outlined in Section 6.0.3, and the analysis is similar to the analysis of the sub-prompt critical transients. Unlike the sub-prompt critical transients, however, a significant portion of the reactivity feedback occurs while the poison FRED slug is being ejected and the time dependent FRED worth needs to be accurately known.

Calculations indicate that the Doppler reactivity feedback can be well represented by a polynomial in the energy absorbed in the fuel. The model for the Doppler reactivity is discussed in Section 2.6.1 and is reproduced here for convenience.

$$\Delta k_D = (\eta_0 + \eta_1 \Delta E_f + \eta_2 \Delta E_f^2) \Delta E_f$$

The "measured" quantities are Δk_D and ΔE_f (see Section 6.0.3) and the parameters to be determined are η_0 , η_1 , and η_2 .

Let k_n and E_n denote the "measured" values of Δk_D and ΔE_f , respectively, at time, t_n , and let σ_{kn} and σ_{En} denote the standard errors in k_n and E_n , respectively. In addition, using the notation of Appendix V, let the elements, a_j ($j=1,2,3$) of a column vector, A , correspond to the parameters η_j ($j=0,1,2$) and let the vector, A_0 , denote the initial estimate of the vector A .

For the above model, equation (24) of Appendix V reduces to:

$$T_{jk} = - \sum_{n=1}^N \frac{\sigma_o^2}{\sigma_\eta^2} E_n^{j+k}$$

where (see equations (9) and (10) of Appendix V), for uncorrelated errors in k_n and E_n , σ_η^2 in the above equation is defined by:

$$\sigma_\eta^2 = \sigma_{kn}^2 + [a_{10} + 2a_{20} E_n + 3a_{30} E_n^2]^2 \sigma_{En}^2$$

Since the "condition equation" is linear in the unknown parameters a_j , the equation for the estimated values of the a_j can be written as:

$$\underline{A} = \bar{Q} \underline{C}^1 \quad (6.2-1)$$

where $\bar{Q} = (\bar{T})^{-1}$ and where the elements C_j^1 of $\underline{C}^1$ are defined by:

$$C_j^1 = \sum_{n=1}^N \frac{\sigma_o^2}{\sigma_\eta^2} k_n E_n^j$$

The value of the sum, S , of the weighted squares of the residuals is given by:

$$S = \sum_{n=1}^N \frac{\sigma_o^2}{\sigma_\eta^2} [k_n - \sum_{j=1}^3 a_j E_n^j]^2$$

where the a_j are obtained from (6.2-1).

The estimated standard error σ_j in a_j is given by:

$$\sigma_j^2 = Q_{jj} \frac{S}{N-3}$$

and the confidence bands for the reactivity feedback can be obtained by the procedure described in Appendix V.

6.2.4 Expected Results

The time dependence of the power, the rise in average fuel temperature, and of the reactivity feedback are shown in Figures 6.2-1 through 6.2-6 for the \$1.1 and \$1.3 transients listed in Table 6.2-2.

For comparison, the effect of an -0.004 Doppler $T \frac{dk}{dT}$, with other reactivity feedback coefficients set to zero, on the \$1.3 transients is shown in Figures 6.2-7 to 6.2-9. The latter three transients are not a part of the experimental program (see Table 6.2-1 for the transients that are planned for the case of an -0.004 Doppler $T \frac{dk}{dT}$) and the results are included solely for the purpose of demonstrating the effect of decreased reactivity feedback.

As the figures indicate, a significant portion of the reactivity feedback occurs during the time that the FRED is in motion. Knowledge of the FRED slug worth and the time dependent FRED behavior (as obtained from the sub-prompt critical familiarization tests and the early portion of all the transients) will allow the feedback reactivity to be calculated in the manner described in Section 6.0.3.

Figure 6.2-1. \$1:10 TRANSIENT AT 0.1 MW FOR CALCULATED COEFFICIENTS

Figure 6.2-2 \$1.10 TRANSIENT AT 5 MW FOR CALCULATED COEFFICIENTS

Figure 6.2-3 1.10 TRANSIENT AT 10 MW FOR CALCULATED COEFFICIENTS

Figure 6.2-4. \$1.30 TRANSIENT AT 0.1 MW FOR CALCULATED COEFFICIENTS

Figure 6.2-5. \$1.30 TRANSIENT AT 5 MW FOR CALCULATED COEFFICIENTS

Figure 6.2-6 \$1.30 TRANSIENT AT 10 MW FOR CALCULATED COEFFICIENTS

Figure 6.2-7. \$1.30 TRANSIENT AT 0.1 MW FOR $-.004$ DOPPLER

Figure 6.2-8. \$1.30 TRANSIENT AT 5 MW FOR $-.004$ DOPPLER

Figure 6.2-9 \$1.30 TRANSIENT AT 10 MW FOR $-.004$ DOPPLER

APPENDIX I

SIMPLIFICATION OF OSCILLATOR REACTIVITY MODEL

The expanded analytical model shown in Figure 2.5-1, can be simplified by expressing the reactivity feedback effects in terms of power, inlet and outlet temperature oscillations. The simplification can be facilitated by taking the Fourier transform of the composite reactivity model (for small changes in temperature and power) that is described in Section 2.3.

The transformed equation may be written as:

$$\Delta K_f(j\omega) = C_f \Delta T_f(j\omega) + C_{fs} \Delta T_{fs}(j\omega) + C_b \Delta T_c(j\omega) + C_i \Delta T_i(j\omega) \quad (1)$$

where it has been assumed that F_m (see Figure 2.5-1) = 1.0 and where

$\Delta K_f(j\omega)$ = transform of total feedback oscillation (corrected for any effects due to reflector temperature oscillations).

$\Delta T_f(j\omega)$ = transform of "effective" fuel temperature oscillation (assumed to be the same as the "average" fuel temperature).

$\Delta T_{fs}(j\omega)$ = transform of fuel surface temperature oscillation.

$\Delta T_c(j\omega)$ = transform of average coolant temperature oscillation.

$\Delta T_i(j\omega)$ = transform of inlet temperature oscillation.

and where (in terms of the nomenclature of Section 2.3):

$$C_f \equiv C_D + C_e (1-a) \quad (2)$$

$$C_{fs} \equiv aC_e + b C_t \quad (3)$$

$$C_b \equiv C_c + C_t (1-b) \quad (4)$$

The frequency response of the fuel surface temperature and of the average fuel temperature to power and to coolant temperature oscillations can be written as:

$$\Delta T_f(j\omega) = K_{av} F_{av}(j\omega) X_f(j\omega) \Delta P(j\omega) + F_s(j\omega) \Delta T_c(j\omega) \quad (5)$$

$$\Delta T_{fs}(j\omega) = K_s F_s X_f(j\omega) \Delta P(j\omega) + G_s(j\omega) \Delta T_c(j\omega) \quad (6)$$

where K_{av} and K_s are defined in Section 2.3, F_{av} and F_s are defined in Section 2.5 and where:

$G_s(j\omega)$ = fuel surface temperature response to coolant temperature oscillation

$X_f(j\omega)$ = frequency response of the ratio of the total power absorbed in the fuel to the "fission" power (normalized to unity at zero frequency)

$\Delta P(j\omega)$ = "fission" power oscillation (proportional to neutron flux)

The function $X_f(j\omega)$ has been calculated by Storrer,⁽¹⁾ and is described in more detail in Appendix II.

In terms of power and coolant temperature oscillations, the reactivity feedback becomes:

$$\Delta K_f = (C_{f_{av}} K_{f_{av}} + C_{fs} K_{fs}) X_f \Delta P + [C_{fs} + C_a - C_{fs}(1-G_s)] \Delta T_c + C_{i1} \Delta T_{i1} \quad (7)$$

where the average coolant temperature coefficient C_a is defined by:

$$C_a \equiv C_b + C_{fs} = C_c + C_t + aC_e = C_q - (1-a) C_e \approx C_q \quad (8)$$

Calculations for SEFOR indicate that at a frequency of 0.1 cps an error of $\sim 1.5\%$ amplitude and $\sim 0.7^\circ$ in phase angle is introduced into the quantity $C_{fs} + C_a - C_{fs}(1-G_s)$ if it is assumed that $G_s = 1.0$. The error at lower frequencies is even smaller. Since the uncertainty in $\Delta K_f(j\omega)$ is already large at a frequency of 0.1 cps (see Appendix VI for example), the assumption that $G_s(j\omega) = 1.0$ introduces little (if any) loss in the accuracy of the mathematical description of the feedback.

At a frequency of 0.1 cps, the calculated difference between $C_{f_{av}} K_{f_{av}} + C_{fs} K_{fs}$ and $(C_{f_{av}} + C_{fs}) K_{fs}$ is $\sim 2.2\%$ in amplitude and $\sim 2.0^\circ$ in phase angle. The difference is even smaller at lower frequencies.

In view of the above observation, the feedback for the oscillator tests should be adequately approximated by:

$$\Delta K_f = \alpha_f F_{av} X_f \Delta P + [C_f F_s + C_q] \Delta T_c + C_i \Delta T_i \quad (9)$$

where, as in the model for the static tests, the power coefficient, α_f , is defined by:

$$\alpha_f \equiv C_f K_{av} + C_{fs} K_s = \alpha_D + C_p \quad (10)$$

(note that this implies $K_{av} = R_f$ of Section 2.3.1), and where it has been assumed that $C_a = C_q$, i.e. that the average coolant coefficient for the static tests is the same as the average coolant coefficient for the oscillator tests.

Although the average coolant temperature oscillation cannot be measured, it is shown in Appendix III, that it can be expressed in terms of the measurable inlet, ΔT_i , and outlet, ΔT_o , oscillations as:

$$\Delta T_c(j\omega) = F_i(j\omega) \Delta T_i(j\omega) + F_o(j\omega) \Delta T_o(j\omega) \quad (12)$$

For low frequencies $F_i \approx F_o \approx 1/2$. At higher frequencies, particularly at low flow rates in the primary coolant system, F_o and F_i deviate from $1/2$. It is expected, however, that F_i and F_o can be adequately represented by the approximations described in Appendix III.

APPENDIX II

ENERGY ABSORPTION IN REACTOR

II.1 REACTOR ENERGY BALANCE

An energy balance for SEFOR has been calculated by Reynolds.⁽²⁾ The energy balance is reproduced, in modified form, in Table II-1. The "fission" energy absorption includes all energy absorption which is directly proportional to the neutron flux, while the "decay" energy absorption includes all heating effects caused by the radioactive decay of fission products. The decay energy shown in the table was calculated for an equilibrium fission product concentration.

If the neutron flux monitors are normalized to 20.0 MW when 20.0 MW is produced in the core (see Table II-1), the absorption rates can be expressed in terms of fractions of the measured power. These fractions are listed in Table II-2 for the materials within the core.

II.2 RESPONSE OF ENERGY ABSORPTION TO A POWER CHANGE

The total power which is absorbed in the fuel depends on the fission power and upon the decay power. If it is assumed that the flux monitors (which are proportional to the fission power) are normalized to the total steady-state power produced, the power generated in the fuel, P_f , may be written, using the values in Table II-2, as follows.

$$P_f(t) = A_f \left[0.932 P(t) + 0.068 \int_0^{\infty} k(u) P(t-u) du \right] \quad (1)$$

TABLE II-1

REACTOR ENERGY BALANCE

Basis: 20 MW Produced in Core

<u>Region</u>	<u>Energy Absorption Rate (MW)</u>		
	<u>"Fission"</u>	<u>Decay</u>	<u>Total</u>
Fuel	17.58	1.28	18.86
Na, fuel-clad & structure	0.51	0.19	0.70
BeO & B ₄ C	<u>0.11</u>	<u>0.02</u>	<u>0.13</u>
TOTAL IN CORE	18.20	1.49	19.69
Inside Vessel (Non-core)			0.42
TOTAL IN VESSEL			20.11
Outside Vessel			0.43
TOTAL ABSORPTION			20.54

TABLE II-2

ENERGY ABSORPTION FRACTIONS

Basis: 20 MW Produced in Core

<u>Region</u>	<u>Fraction of Energy Absorbed</u>	<u>Fraction of Absorbed Energy that is proportional to fission energy</u>	<u>Fraction of Absorbed Energy that is proportional to fission product decay</u>
Fuel	0.943	0.932	0.068
Na, Clad & Structure	0.035	0.73	0.27
BeO & B ₄ C	0.0065	0.85	0.15

where $A_f = 0.943$ is the fraction of the energy absorbed in the fuel, $P(t)$ is the power indication from the neutron flux monitors, and $k(t-u)$ is the decay kernel.

The kernel $k(u)$ is normalized so that:

$$\int_0^{\infty} k(u) du = \int_{-\infty}^{\infty} k(u) du = 1.0 \quad (2)$$

where the fact that $k(u)$ is a casual function, i.e. $k(u) \equiv 0$ for $u < 0$, has been used.

For steady-state operation at a constant power, P , equation (1) reduces to:

$$P_f = A_f P = 0.943P \quad (3)$$

in agreement with the values in Table II-2.

In a similar manner, the total power, P_c , generated in the coolant, clad, and structure can be written as:

$$P_d(t) = A_c [0.73P(t) + 0.27 \int_0^{\infty} k(u) P(t-u) du] \quad (4)$$

where $A_c = 0.035$ is the fraction of energy absorbed in coolant, clad, and structure, and where it has been assumed that the decay kernel for energy absorption in the coolant, clad, and structure is the same as for the energy absorption in the fuel.

For an oscillator test analysis, the transforms of the power in the fuel and in the coolant are needed. The transforms can be written as:

$$\Delta P_f(i\omega) = A_f X_f(i\omega) \Delta P(i\omega) \quad (5)$$

and:

$$\Delta P_c(i\omega) = A_c X_c(i\omega) \Delta P(i\omega) \quad (6)$$

where $\Delta P_f(i\omega)$, $\Delta P_c(i\omega)$, and $\Delta P(i\omega)$ are the transforms of the oscillations of the power absorbed in the fuel, the power absorbed in the clad, coolant, and structure, and in the recorded or produced power, respectively.

The quantities $X_f(i\omega)$ and $X_c(i\omega)$ are defined by:

$$X_f(i\omega) \equiv 0.932 + 0.068 f(i\omega) \quad (7)$$

$$X_c(i\omega) \equiv 0.73 + 0.27 f(i\omega)$$

where $f(i\omega)$ is the transform of the kernel $k(t)$.

The function $f(i\omega)$ can be expressed in terms of a function $T(i\omega)$ which has been calculated by Storrer⁽¹⁾. In terms of Storrer's $T(i\omega)$, the function $f(i\omega)$ is defined as follows:

$$f(i\omega) = \frac{T(i\omega) - 1.0}{T(0) - 1.0} \quad (8)$$

From this definition, $f(0) = 1.0$, in agreement with equation (2), and $f(\infty) = 0$. Thus,

$$X_f(0) = X_c(0) = 1.0 \quad (9)$$

$$X_f(\infty) = 0.932 \quad (10)$$

and:

$$X_c(\infty) = 0.73 \quad (11)$$

During a rapid transient, the decay power does not change appreciably during the first few seconds and the change in the fuel power can be approximated by:

$$\Delta P_f(t) \approx A_f .932 \Delta P(t) = A_f X_f(\infty) \Delta P(t) \quad (12)$$

The excess energy absorbed in the fuel $\Delta E_f(t)$ during the first few seconds after the transient has been initiated can thus be approximated by:

$$\Delta E_f \approx A_f X_f(\infty) \int_0^t \Delta P(u) du = A_f X_f(\infty) \Delta E(t) \quad (13)$$

where $\Delta E(t)$ is the total excess energy produced.

In order to obtain a more accurate calculation of the energy released into the fuel during a transient an explicit time dependent form of the decay kernel $k(t)$ can be used. One such representation is given by Storrer⁽¹⁾ and another is described in reference (3) and is incorporated into the computer code FORE-II.⁽⁴⁾

For times, t , up to 10 seconds, the approximation given by (13) agrees quite well (within 0.5%) with the results obtained using the kernel described in reference (3).

APPENDIX III

OSCILLATING COOLANT TEMPERATURE RESPONSE

III.1 COOLANT TEMPERATURE IN THE CORE

For small sinusoidal oscillations in power, flow and coolant temperature, the Fourier transform of the heat balance within a differential length of coolant channel can be approximated by:

$$\begin{aligned} \frac{i\omega\tau_c}{V_r} T(x, i\omega) = & \frac{a_c K_n X_c(i\omega)}{V_r} P(i\omega) q(x) \\ & + \frac{K_n(1-a_c)}{K_s V_r} [T_s(x, i\omega) - T(x, i\omega)] \\ & - \frac{d}{dx} T(x, i\omega) - \frac{v(i\omega)}{V_o} \frac{d\theta(x)}{dx} \end{aligned} \quad (1)$$

where $i = \sqrt{-1}$, ω is the frequency of oscillation in radians per second, and where:

τ_c = core flow time at full flow

V_r = relative flow rate

a_c = fraction of heat generated in coolant

K_n = static change in core temperature rise (at full flow)
for a static change in power

- K_s = static change in fuel surface for a power change at constant coolant temperature
 V_o = mean flow rate = Product of V_r and full flow rate
 $T(x, i\omega)$ = transform of the coolant temperature oscillation
 $P(i\omega)$ = transform of the prompt power oscillation (proportional to flux)
 $X_c(i\omega)$ = transform of ratio of total power generated in coolant (proportional to prompt power plus delayed gammas due to fission product decay) to the prompt power.
 $T_s(x, i\omega)$ = transform of the fuel surface temperature oscillation
 $V(i\omega)$ = transform of the flow oscillation
 $\theta(x)$ = static coolant temperature at axial position x and $\partial\theta/\partial x = K_n P_o q/V_r$ where P_o is the mean power level
 x = normalized axial position
 $q(x)$ = axial power distribution (assumed to be the same in coolant and fuel) normalized so that:

$$\int_0^1 q(x) dx = 1.0$$

According to reference (5), T_s can be written, after slight modification, as:

$$T_s(x, i\omega) = K_s F_s(i\omega) X_f(i\omega) P(i\omega) q(x) + [1 - i\omega \gamma F_s(i\omega)] T(x, i\omega)$$

where $F_s(i\omega)$, and γ are defined in Section 2.5, and $X_f(i\omega)$ is the transform of the ratio of the total power generated in the fuel to the prompt power generated in the fuel. (See Appendix II)

Equation (1) can thus be written as:

$$\frac{\partial T(x, i\omega)}{\partial x} + b(i\omega)T(x, i\omega) = S(i\omega)q(x) \quad (2)$$

where:

$$b(i\omega) \equiv \frac{i\omega}{V_r} (\tau_c + \gamma K_n (1-a_c) F_s / K_s)$$

and:

$$S(i\omega) \equiv \frac{K P}{V_r} \{ [a_c X_c(i\omega) + (1-a_c) F_s(i\omega) X_f(i\omega)] \frac{P(i\omega)}{P_o} - V(i\omega)/V_o \}$$

The solution of equation (2) is:

$$T = T_1 e^{-bx} + S \int_0^x q(z) dz \exp[-b(x-z)] \quad (3)$$

where $T_1 \equiv T(0, i\omega)$.

The average coolant temperature, which is defined by:

$$\bar{T}(i\omega) = \int_0^1 T(x, i\omega) dx$$

can be written as:

$$\bar{T} = T_1 (1 - e^{-b})/b + \frac{S}{b} \int_0^1 q(z) dz [1 - \exp(bz - b)] \quad (4)$$

The average temperature can also be written in terms of the inlet temperature oscillation T_i and the outlet temperature oscillation T_o as follows:

$$\bar{T} = F_i T_i + F_o T_o \quad (5)$$

where $F_i \equiv (I-1)/bI$, and $F_o \equiv (e^b - 1)/bI$, and where

$$I \equiv \int_0^1 q(z) e^{bz} dz$$

For $|b| \ll 1$, the expression for F_i and F_o can be approximated by:

$$F_o \approx 1 - F_i \quad (6)$$

$$F_i \approx \frac{1}{2} (1 - b\alpha) \quad (7)$$

where it has been assumed that $q(x)$ is symmetric about $x = \frac{1}{2}$ and where the parameter α is defined:

$$\alpha \equiv \frac{1}{2} - \int_0^1 x^2 q(x) dx$$

For a parabolic power distribution $\alpha = (3+2r)/30$, where r is the axial peaking factor.

Values of the functions F_i and F_o have been calculated for several cases using SEFOR parameters and an assumed parabolic power distribution. These are shown in Figures III-1 and III-2. At low flow rates, the approximate deviation in amplitude from 0.5 and the deviation in phase from zero degrees varies inversely as the relative flow rate. The approximate expressions for F_i and F_o (equations (6) and (7)) are compared with the more accurate expressions in Figures III-3 and III-4.

It is seen from Figure III-1 and III-2, that the functions F_o and F_i deviate even at full flow from the zero frequency value of 0.5 for frequencies above 0.01 rad/sec (0.0016 cps). Even at low flow rates (30%), however, the approximate expression agrees well with the more accurate expression up to frequencies of ~ 0.6 rad/sec (0.1 cps). As the expected feedback is nearly zero above 0.1 cps, the approximate expressions, (6) and (7), can be used to calculate the functions F_i and F_o . The "measured" average coolant temperature is then calculated from (5) using measured values of inlet and outlet coolant temperature oscillations.

A mathematical expression for the ratio of the coolant flow oscillation to the power oscillation is needed for the analysis of the first balanced oscillator test data. This expression (obtained by integrating (2) over x) can be written as follows:

$$\begin{aligned} \frac{\frac{V(i\omega)}{V_o}}{\frac{P(i\omega)}{P_o}} &= a_c X_c + (1-a_c) F_s X_f - \frac{P_o}{\Delta\theta} \left[\frac{T_o - T_i + b\bar{T}}{P} \right] \\ &= a_c X_c + (1-a_c) F_s X_f - \frac{P_o}{\Delta\theta} \left[\frac{T_o}{P} (1+b-bF_i) - \frac{T_i}{P} (1-bF_i) \right] \end{aligned} \quad (8)$$

where $\Delta\theta$, the steady-state core temperature rise is defined by $\Delta\theta = K_n P_o / V_r$.

Figure III-1. AMPLITUDE OF THE COOLANT TEMPERATURE FUNCTIONS F_i AND F_o FOR SEFOR AT 100% FLOW

Figure III-2

PHASE ANGLE OF THE COOLANT TEMPERATURE FUNCTIONS
 F_i AND F_o FOR SEFOR AT 100% FLOW

Figure III-3. ERROR IN AMPLITUDE OF APPROXIMATE EXPRESSIONS
FOR F_i AND F_o IN SEFOR

Figure III-4. ERROR IN PHASE ANGLE OF APPROXIMATE EXPRESSIONS FOR F_i AND F_o IN SEFOR

Figure III-5. MODEL FOR FREQUENCY RESPONSE OF AVERAGE COOLANT TEMPERATURE TO A POWER OSCILLATION

III.2 COOLANT TEMPERATURES IN EXTERNAL LOOP

Heat Exchangers

In a conventional oscillator test no attempt is made to hold the coolant temperatures constant. The response of the power to a reactivity oscillation will thus be influenced by the frequency response of the entire coolant system.

In order to obtain an estimate of the frequency response of the power-reactivity transfer function for the conventional oscillator tests, the model for the SEFOR coolant system that is described by Russell⁽⁶⁾ was used. This model assumes counter-flow heat exchangers rather than the actual tube and shell heat exchangers. The model was modified, as described below, to allow for a continuous axial temperature distribution through the heat exchangers, and to allow the system to be analyzed directly in the frequency rather than the time domain.

For small sinusoidal oscillations in flow and temperature, the Fourier transform of the heat balance within a differential length of a counter-flow heat exchanger can be written as:

$$C_1 \frac{\partial T_1}{\partial x}(x, i\omega) = (1 + i\omega\tau_1) T_1(x, i\omega) - \frac{C_1 \Delta V_1(i\omega)}{V_{10}} \frac{\partial \theta_1(x)}{\partial x} - T_3(x, i\omega) \quad (9)$$

$$C_2 \frac{\partial T_2}{\partial x}(x, i\omega) = - (1 + i\omega\tau_2) T_2(x, i\omega) + T_3(x, i\omega) - \frac{C_2 \Delta V_2(i\omega)}{V_{20}} \frac{\partial \theta_2(x)}{\partial x} \quad (10)$$

$$T_3(x, i\omega) = \frac{a T_1(x, i\omega) + (1-a) T_2(x, i\omega)}{1 + i\omega\tau_3} \quad (11)$$

where $i = \sqrt{-1}$, ω is the frequency of oscillation in cycles per second, x is the dimensionless length and where:

$T_1(x, i\omega)$ = transform of coolant temperature oscillation
on the primary side

$T_2(x, i\omega)$ = transform of coolant temperature oscillation
on the secondary side

$T_3(x, i\omega)$ = transform of the temperature oscillation in
the heat exchanger wall between the primary
and secondary coolant

$\Delta V_j(i\omega)$ = transform of the fractional coolant temperature
oscillation on the primary ($j=1$) and secondary
($j=2$) side

$\theta_j(x)$ = steady-state coolant temperature distribution
on the primary ($j=1$) and secondary ($j=2$) side

τ_j/C_j = coolant flow time through the heat exchanger
in the primary ($j=1$) and secondary ($j=2$) side

a = fraction of steady-state wall temperature that
depends on the steady-state coolant temperature
on the primary side

τ_j = time constant for the decay of a coolant temper-
ature perturbation at "zero" flow on the primary
($j=1$) and secondary ($j=2$) side

The solutions for $\frac{\partial \theta_1}{\partial x}$ and $\frac{\partial \theta_2}{\partial x}$ can be written as:

$$\frac{\partial \theta_1}{\partial x} = g_1 e^{px}$$

and

$$\frac{\partial \theta_2}{\partial x} = g_2 e^{px}$$

where $p = (1-a)/c_1 - a/c_2$; where

$$g_1 = \frac{1-a}{c_1} (\theta_{10} - \theta_{2i})$$

where $g_2 = a c_1 g_1 / c_2 (1-a)$, and where θ_{10} is the steady-state outlet temperature on the primary side, and θ_{2i} is the steady-state inlet temperature on the secondary side.

Defining:

$$\mu_1 = \frac{1}{c_1} (1 + i\omega\tau_1) - \frac{a}{c_1(1 + i\omega\tau_3)} \quad (12)$$

$$\mu_2 = -\frac{1}{c_2} (1 + i\omega\tau_2) + \frac{(1-a)}{c_2(1 + i\omega\tau_3)} \quad (13)$$

$$\alpha_1 = \frac{(1-a)}{c_1(1 + i\omega\tau_3)}$$

$$\alpha_2 = \frac{a}{c_2(1 + i\omega\tau_3)}$$

equations (9) and (10) can be written as:

$$\frac{\partial T_1}{\partial x} = \mu_1 T_1 - \alpha_1 T_2 - \frac{\Delta V_1}{V_{10}} g_1 e^{px}$$

$$\frac{\partial T_2}{\partial x} = \mu_2 T_2 + \alpha_a T_1 - \frac{\Delta V_2}{V_{20}} g_2 e^{px}$$

The roots, S_1 and S_2 , of the characteristic equation are:

$$S_1(i\omega) = \frac{\mu_1 + \mu_2}{2} + \frac{1}{2} \sqrt{(\mu_1 - \mu_2)^2 - 4 \alpha_1 \alpha_2}$$

$$S_2(i) = \frac{\mu_1 + \mu_2}{2} - \frac{1}{2} \sqrt{(\mu_1 - \mu_2)^2 - 4 \alpha_1 \alpha_2}$$

The transform of the outlet coolant temperature oscillation on the primary side, T_{po} , and on the secondary side, T_{so} , can thus be written in terms of the transform of the inlet temperature oscillations on the primary side, T_{pi} , the secondary side, T_{si} , and in terms of the coolant flow oscillations as follows:

$$T_{po} = F_1 T_{pi} + F_2 T_{si} + G_1 \frac{\Delta V_1}{V_{10}} + G_2 \frac{\Delta V_2}{V_{20}}$$

$$T_{so} = F_3 T_{pi} + F_4 T_{si} + G_3 \frac{\Delta V_1}{V_{10}} + G_4 \frac{\Delta V_2}{V_{20}}$$

where:

$$F_1 = (S_1 - S_2)/D \quad (14)$$

$$F_2 = \alpha_1(e^{S_1} - e^{S_2})/D \quad (15)$$

$$F_3 = \alpha_2(e^{S_1} - e^{S_2})/D$$

$$F_4 = (S_1 - S_2) e^{S_1} e^{S_2}/D$$

$$G_1 = \frac{g_1}{D} \left[\frac{(S_1 - \mu_2)}{(P - S_1)} (e^P - e^{S_1}) - \frac{(S_2 - \mu_2)}{(P - S_2)} (e^P - e^{S_2}) \right]$$

$$G_2 = \frac{-\alpha_1 g_2}{D} \left[\frac{e^P - e^{S_1}}{P - S_1} - \frac{e^P - e^{S_2}}{P - S_2} \right]$$

$$G_3 = \frac{-\alpha_2 g_1}{D} \left[e^{S_2} \frac{(e^P - e^{S_1})}{P - S_1} - e^{S_1} \frac{(e^P - e^{S_2})}{P - S_2} \right]$$

$$G_4 = \frac{g_2}{D} \left[(S_2 - \mu_2) e^{S_2} \frac{(e^P - e^{S_1})}{P - S_1} - (S_1 - \mu_2) \frac{e^{S_1} (e^P - e^{S_2})}{P - S_2} \right]$$

where: $D = e^{S_1}(S_1 - \mu_2) - e^{S_2}(S_2 - \mu_2)$

The block diagram for a portion of the modified form of Russell's⁽⁶⁾ model is shown in Figure III-5. In this diagram, it is assumed that the inlet temperature of the air blast coolant and all flow rates are constant. The frequency response ($\Delta \bar{T}/\Delta P$) of the average coolant temperature to a power oscillation that is obtained from the model shown in Figure III-5 was used in the feedback equations described in Section 2.5 to calculate the power-reactivity transfer function for the conventional oscillator test ($\Delta V_1 = \Delta V_2 = 0$) shown in Section 5.2.4. The various response functions shown in the diagram are defined below.

The response function F_1 , F_2 , F_3 , and F_4 are defined above and refer to the primary and secondary side of the intermediate heat exchanger (IHX). The response function F_6 is defined in the same manner as F_1 , except that the parameters for the primary and secondary sides of the air blast cooler (ABC) are used in equation (14). The functions F_5 and F_7 are of the form $e^{-i\omega\tau}$ and represent pure delay time. The function F_8 represents the delay between the core outlet and the inlet on the primary side of the IHX, as well as the coolant mixing that occurs between the core outlet and the vessel outlet, and is of the form $e^{-i\omega\tau}/(1+i\omega\lambda)$. The function F_9 corresponds to F_8 , and represents similar processes between the outlet on the primary side of the IHX and the core inlet. The response function F_b describes the frequency response of that portion of the coolant which bypasses the core and is of the form $B\exp(-i\omega\tau_c/v_r)$ where B is the fraction of coolant which bypasses the core, τ_c is the core flow time at full flow and v_r is the relative flow rate in the primary system. The function $F_{nb} = 1-B$. The functions $\bar{F}$, F_p , F_i , and F_c are obtained from equations (3) and (4) and are defined as follows:

$$\bar{F} = \frac{K_N}{bv_r} [a_c X_c + (1-a_c) F_s X_f] \int_0^1 q(z) dz [1 - \exp \{-b(1-z)\}]$$

$$F_p = \frac{K_N}{v_r} [a_c X_c + (1-a_c) F_s X_f] \int_0^1 q(z) dz \exp \{-b(1-z)\}$$

$$F_i = (1 - e^{-b})/b$$

$$F_c = e^{-b}$$

where the nomenclature of equations (3) and (4) are used.

The response of the average coolant temperature to a power oscillation at constant primary and secondary flow can be written as:

$$\Delta \bar{T} = [\bar{F} + F_1 F_8 F_9 R F_{nb} F_p / \{1 - F_8 F_9 R (F_b + F_{nb} F_c)\}] \Delta P$$

where R, the response of the primary outlet to a primary inlet oscillation is:

$$R = F_1 + F_2 F_3 F_5 F_6 F_7 / (1 - F_4 F_5 F_6 F_7)$$

APPENDIX IV

METHODS FOR DETERMINING FREQUENCY COMPONENTS AND
EFFECTS OF FINITE DIGITIZING INTERVALS (ALIASING)IV.1 COMPARISON OF TWO METHODS

A method for eliminating the effects of drift (aperiodic terms) on the determination of the frequency components in an oscillator test has been described by Persson.⁽⁷⁾ In this procedure, the detector signal is multiplied by an input signal (e.g. the reactivity oscillation) and the product is multiplied by an appropriate weighting function and integrated over successive half cycles. The weighting function is constant within a given half cycle and the ratio of the weighting function from the first half cycle to the last is given by the binomial coefficients. In the "conventional" method for determining the components, the weight function is constant and does not vary from half cycle to half cycle.

Both of these methods were analyzed to determine how well they reject unwanted periodic signals which have a frequency at non-integer multiples of the input signal frequency. It was found that Persson's method has, in general, better rejection of unwanted periodic signals than the "conventional" method. In addition, the amplitude of noise with a constant frequency spectrum decreases as 2^{1-N} for Persson's method, but only as N^{-1} for the conventional method, where N is the number of consecutive half cycles of data which have been treated.

If the instrument signal, $f(t)$, is written in terms of its inverse Fourier transform as:

$$f(t) = \int_{-\infty}^{\infty} A(\omega) \cos \omega t d\omega + \int_{-\infty}^{\infty} B(\omega) \sin \omega t d\omega \quad (1)$$

The quantities which will be determined by the analysis are:

$$F_1(\omega_0) = \frac{2\omega_0}{\pi} \sum_{n=1}^N P_n \int_{\frac{\pi}{\omega_0}}^{\frac{n\pi}{\omega_0}} f(t) \cos \omega_0 t dt \quad (2)$$

and

$$F_2(\omega_0) = \frac{2\omega_0}{\pi} \sum_{n=1}^N P_n \int_{\frac{\pi}{\omega_0}}^{\frac{n\pi}{\omega_0}} f(t) \sin \omega_0 t dt \quad (3)$$

where the P_n are defined by:

$$P_n = \frac{1}{N} \quad n = 1, 2, \dots, N \text{ (conventional method) (N even)}$$

$$P_n = \frac{q_n}{2^{N-1}} \quad n = 1, 2, \dots, N \text{ (drift elimination method)}$$

and the q_n are the binomial coefficients (i.e. $q_n = \frac{(N-1)!}{(N-n)!(n-1)!}$)

Interchanging the order of integration gives:

Conventional Method

$$F_1 = \int_{-\infty}^{\infty} d\omega [A(\omega) \{f(\omega) + f(-\omega)\} + B(\omega) \{-g(\omega) + g(-\omega)\}]$$

$$F_2 = \int_{-\infty}^{\infty} d\omega [A(\omega)\{g(\omega) + g(-\omega)\} + B(\omega)\{f(\omega) - f(-\omega)\}]$$

where:
$$f(\omega) = \frac{\sin \frac{N\pi}{\omega_0} (\omega_0 - \omega)}{\frac{N\pi}{\omega_0} (\omega_0 - \omega)}$$

$$g(\omega) = \frac{\sin^2 \frac{N\pi}{2\omega_0} (\omega_0 - \omega)}{\frac{N\pi}{2\omega_0} (\omega_0 - \omega)}$$

Drift Elimination Method

$$F_1 = \int_{-\infty}^{\infty} d\omega [A(\omega)\{h(\omega) + h(-\omega)\} + B(\omega)\{-r(\omega) + r(-\omega)\}] \quad (6)$$

$$F_2 = \int_{-\infty}^{\infty} d\omega [A(\omega)\{r(\omega) + r(-\omega)\} + B(\omega)\{h(\omega) - h(-\omega)\}] \quad (7)$$

where:

$$h(\omega) = \frac{\sin \frac{\pi}{2\omega_0} (\omega_0 - \omega)}{\frac{\pi(\omega_0 - \omega)}{2\omega_0}} \cos^{N-1} \frac{\pi(\omega_0 - \omega)}{2\omega_0} \cos \frac{N\pi}{2\omega_0} (\omega_0 - \omega)$$

$$r(\omega) = \frac{\sin \frac{\pi}{2\omega_0} (\omega_0 - \omega)}{\frac{\pi(\omega_0 - \omega)}{2\omega_0}} \cos^{N-1} \frac{\pi(\omega_0 - \omega)}{2\omega_0} \sin \frac{N\pi}{2\omega_0} (\omega_0 - \omega)$$

Note that

$$\lim_{N \rightarrow \infty} f(\omega) = \lim_{N \rightarrow \infty} h(\omega) = f(\omega_0) = h(\omega_0) = 1 \quad \omega = \omega_0$$

$$= 0 \quad \omega \neq \omega_0$$

and $\lim_{N \rightarrow \infty} g(\omega) = \lim_{N \rightarrow \infty} r(\omega) = 0$

Thus, in the limit, both the conventional and drift "filter" approach an ideal filter.

If $A(\omega)$ and $B(\omega)$ are constant, one can perform the indicated integration to obtain:

Conventional Filter

$$F_1 = A \frac{\omega_0}{N\pi}$$

$$F_2 = 0$$

Drift Filter

$$F_1 = A \frac{\omega_0}{\pi 2^{N-1}}$$

$$F_2 = 0$$

It is seen that the amplitude of noise with a constant frequency spectrum decreases faster (with increasing N) for the drift filter, than for the conventional filter.

Figure IV-1 $f(\omega)$ and $h(\omega)$ when $N=6$

Figure IV-2 $g(\omega)$ and $r(\omega)$ when $N=6$

If $f(t)$ can be presented by $a (\cos \omega_0 t + b \sin \omega_0 t)$, which will be the case for the "desired" component of the power, or temperature signal, $A(\omega) = a \delta(\omega - \omega_0)$ and $B(\omega) = b \delta(\omega - \omega_0)$. In this case $F_1 = a$ and $F_2 = b$ for both the conventional and drift filter.

Plots of the functions $f(\omega)$, $h(\omega)$, $g(\omega)$, and $r(\omega)$ are shown in Figures IV-1 and IV-2.

IV.2 EFFECTS OF FINITE DIGITIZING INTERVALS (ALIASING)

When a continuous signal is digitized at time intervals, Δt , any component of the signal which has a frequency greater than half the digitizing frequency ($f_s = 1/\Delta t$) will appear, in the digitized data, as a component with frequency less than half the digitizing interval. This effect is commonly called aliasing^(8,9) and cannot be eliminated once the data is digitized. An example of this effect is shown below.

High frequency components can be eliminated from the sampled data either by passing the continuous signal through an appropriate filter prior to sampling, or by sampling at a frequency which is at least twice the frequency of any undesired component in the signal. If it is assumed that all the high frequency components in SEFOR are part of the "noise", the low frequency noise spectrum (which is obtained by analyzing the signals with the same digitizing interval when the oscillator rod is oscillated at zero amplitude (see Section 5.0.3) will include the high frequency components, and the desired low frequency components of the signals can be obtained by subtracting the low frequency "noise". Note that if the sampling frequency that is used to sample the "noise" is different than the sampling frequency which is used when the desired signal is present, the effects of aliasing must be calculated.

To provide an example of the effect of aliasing, it will be assumed that $B(\omega)$ in (1) is zero and only the "drift elimination" method described above will be considered. These assumptions do not alter the qualitative results obtained below. With these assumptions, the quantity F_1 of (6) can be written as:

$$F_1 = \frac{2\omega_0}{\pi 2^{N-1}} \sum_{n=1}^N q_n \int_{(n-1)\frac{\pi}{\omega_0}}^{\frac{n\pi}{\omega_0}} f(t) \cos \omega_0 t \, dt \quad (8)$$

$$= \int_{-\infty}^{\infty} A(\omega) H(\omega) \, d\omega$$

where

$$H(\omega) = \frac{2\omega_0}{\pi 2^{N-1}} \sum_{n=1}^N q_n \int_{(n-1)\frac{\pi}{\omega_0}}^{\frac{n\pi}{\omega_0}} \cos \omega t \cos \omega_0 t \, dt \quad (9)$$

Assume for simplicity that the signal $f(t)$ is digitized at time intervals $\Delta t = \pi/\omega_0 K$ where K is an integer. If the actual sampling does not produce an integer number of samples per cycle, the necessary correction to the results obtained below will be very small if the number of samples per cycle is large. In addition to this assumption, let the integral in (9) be approximated by the trapezoidal rule (the exact method of numerical integration does not significantly affect the results).

Rewriting (9) in terms of exponentials, using the relation:

$$\sum_{k=1}^K e^{iky} = e^{i(K+1)y/2} \frac{\sin \frac{Ky}{2}}{\sin y/2} \quad (10)$$

and noting that since the q_n are the binomial coefficients the following relation applies:

$$\begin{aligned} \sum_{n=1}^N q_n e^{i2ny} &= e^{i2y} [1 + e^{i2y}]^{N-1} \\ &= e^{iy(N+1)} 2^{N-1} \cos^{N-1} y \end{aligned} \quad (11)$$

one obtains

$$H(\omega) = S(\omega) + S(-\omega) \quad (12)$$

$$\text{where} \quad S(\omega) = h(\omega)R(\omega) \quad (13)$$

and where $h(\omega)$ is defined above.

The function $R(\omega)$ is defined by:

$$R(\omega) = \frac{\Delta t}{2} (\omega_0 - \omega) \cot \frac{\Delta t}{2} (\omega_0 - \omega) = \frac{\pi}{2\omega_0 K} (\omega_0 - \omega) \cot \frac{\pi}{2\omega_0 K} (\omega_0 - \omega) \quad (14)$$

$S(\omega)$ is a periodic function of $(\omega_0 - \omega)$ with period equal to the sampling interval Δt , i.e. the frequency is equal to the sampling frequency $f_s = 1/\Delta t$. If the sampling interval is much less than the integration time then $S(\omega)$, in the intervals

$$f \equiv \frac{\omega}{2\pi} = f_0 \pm \frac{l f_s}{2} \quad (l = 0, 1, 2, \dots)$$

looks very nearly the same as $h(\omega)$ in the interval $f = f_0 \pm f_s/2$. In the limit as N (i.e. the integration time) $\rightarrow \infty$, $S(\omega)$ and $h(\omega)$ are identical in this interval, however $S(\omega)$ is periodic and $h(\omega)$ is not, since as $N \rightarrow \infty$, $h(\omega)$ approaches a Kronecker delta function at $\omega = \omega_0$ ($f = f_0$).

Since $S(\omega) = 1$ when $\omega = \omega_0 + 2\pi l f_s$ ($l = 0, \pm 1, \pm 2, \dots$) and $S(-\omega) = 1$ when $\omega = \omega_0 + 2\pi j f_s$ ($j = 0, \pm 1, \pm 2, \dots$),

$$H(\omega_0) = 1 \text{ if } f_s/f_0 > 2$$

$$H(\omega_0) = 2 \text{ if } f_s/f_0 = 2/l \quad (l = 1, 2, \dots)$$

In other words, the fundamental component of the signal ($\omega = \omega_0$) is obtained exactly, in the above approximation, if the number of samples/cycle (f_s/f_0) is > 2 . If the number of samples/cycle is < 2 erroneous results may be obtained.

In addition, it is seen that $H(\omega) = 1$ when $\omega = \pm \omega_0 \pm 2\pi L f_s$ where L takes on all integer values such that $\omega \geq 0$. For example, if 32 samples per cycle are recorded $f_s = 16 \omega_0/\pi$ and all components of the signal with frequency $f > 17 f_0$ appear in $H(\omega)$ and those frequency $f = f_0$, and $f = f_0(32L \pm 1)$ are undamped.

Thus for oscillations at 0.001 cycles/sec, all frequencies $> .017$ cycles/sec appear in $H(\omega)$ and thus if $F_1(\omega_0)$ (i.e. in the measurement of the transfer function) and those components at 60.001 cps and 59.999 cps, in addition to many others, are undamped.

APPENDIX V

LEAST SQUARES CURVE FITTING

INTRODUCTION

In this section, the general form of the "fitting" equation is presented. The derivation shown below is based on the derivation described by Deming⁽¹⁰⁾ and much of the notation is the same, however, cases with "correlation" and with two "condition equations" are treated.

The purpose of the least squares fitting procedure is to determine values for the parameters in some mathematical model so that the sum of the weighted square of the residuals (the differences between measured "coordinates" and coordinates that are calculated by the model), is a minimum. As will be shown below, this is, in general, an iterative procedure, since initial estimates of the parameters are used to obtain new estimates, which can then be used as initial estimates for yet another iteration, and so on. If the initial estimates are reasonably close to the correct values, the procedure should converge quite rapidly.

LEAST SQUARES CURVE FITTING

It will be assumed in the following discussion that the measured values of the coordinates are randomly distributed about the "true or mean coordinates," and that the "mean" coordinates satisfy the condition equation when the true values of the parameters are used in the mathematical model. See reference (11) for a discussion of the problem if the assumed "condition equation" is not the "true" equation.

Let x_{kn} denote the observed value of the k-th coordinate at point "n" in the coordinate space, and let there be a total of K coordinates and a total of N points at which observations are made. The number x_{kn} may perhaps be the mean of a number of measurements of coordinate "k" at point "n", and each x_{kn} may thus have an associated standard error σ_{kn} . Since the measurements may be correlated, even at each point "n", there may also be coefficients of correlation, $(r_{jk})_n$, between the x_{jn} and x_{kn} coordinates.

Define the residual V_{kn} as:

$$V_{kn} = x_{kn}(\text{observed}) - x_{kn}(\text{calculated})$$

and define a set of N column vectors, V_n , so that the k-th element of V_n is V_{kn} .

The sum of the weighted squares of the residuals, S, can then be written as:

$$S = \sum_{n=1}^N S_n = \sum_{n=1}^N V_n^T \bar{M}_n V_n = \sum_{n=1}^N \sum_{k=1}^K \sum_{j=1}^K V_{kn} (M_{kn})_n V_{jn}$$

where V_n^T is the transpose of V_n and where $\bar{M}_n$ is a K x K symmetric matrix whose elements, $(M_{kj})_n$, are the weights. The elements of the $\bar{M}_n$ matrices can be expressed in terms of the standard errors σ_{kn} and the correlation coefficients, $(r_{jk})_n$. These elements will be discussed in more detail below.

The expression for S that is given above will first be rewritten in terms of the measured coordinates and the "true" parameters, rather

than measured coordinates and calculated coordinates, and the new expression for S will then be used to determine estimated values for these "true" parameters.*

In order to transform the expression for S, the following procedure, which is a slight modification of the derivation described by Deming⁽¹⁰⁾, is used.

Let the calculated coordinates be subjected to n condition equations.

$$\theta_n (x_{1n}, x_{2n}, \dots, x_{Kn}, a_1, a_2, \dots, a_L) = 0 \quad (n=1, 2, \dots, N)$$

where the a_ℓ ($\ell=1, 2, \dots, L$) are the parameters, independent of k and n, which are to be chosen so that S is a minimum and the condition equations are satisfied.

As an example of a condition equation: assume that coordinates x_{1n} and x_{2n} are the measured values at points $n=1, 2, \dots, N$ and that the "true coordinates" (i.e. the calculated coordinates for the true values of parameters, a_1, a_2, a_3) are z_{1n} and z_{2n} . Further assume that $z_1 = a_1 + a_2 z_2 + a_3 z_2^2$. Then the condition equation can be written as:

$$\theta_n = z_1 - a_1 - a_2 z_{2n} - a_3 z_{2n}^2 = 0 \quad (n=1, 2, \dots, N)$$

* It may be argued that the form of the transformed expression for S (see equation 11) could be written down "a priori" when only one condition equation is used. The form of the transformed equation for S when two condition equations are used was not so "obvious", however, and the transformation described below were used to obtain the result given in equation (20).

Neglecting higher order terms, the result obtained by expanding the condition equation θ_n in a Taylor's series about the measured values of the coordinates, can be written as:

$$\theta_n = f_n - \mathbf{V}_n^T \boldsymbol{\xi}_n = 0 \quad (1)$$

where f_n is θ_n evaluated at the measured values of the coordinates, and where $\boldsymbol{\xi}_n$ is a column vector with elements ξ_{kn} defined by:

$$\xi_{kn} = \frac{\partial \theta_n}{\partial x_{kn} \text{ (calculated)}} = \frac{\partial f_n}{\partial x_{kn} \text{ (measured)}}$$

where the partial derivatives are evaluated at the measured values of the coordinates.

For the example given above, $\xi_{1n} = 1$ and $\xi_{2n} = -a_2 - 2a_3 x_{2n}$, while $\mathbf{V}_{kn} = \mathbf{x}_{kn} - \mathbf{z}_{kn}$.

If S is a minimum, the variation of S is zero, and the variation δS can be written (noting that $\bar{\mathbf{M}}_n$ is symmetric) as:

$$\frac{\delta S}{2} = \sum_{n=1}^N (\delta \mathbf{V})_n^T \bar{\mathbf{M}}_n \mathbf{V}_n = 0 \quad (2)$$

where the symbol δ denotes a variation and where $\delta \mathbf{V}_n$ is a column vector with elements δV_{kn} .

Along with Deming,⁽¹⁰⁾ we note that the variations δV_{kn} are not arbitrary, but must be chosen so that the condition equations are satisfied.

Taking the variation of (1), with the parameters a_l fixed at their true values, and multiplying by an arbitrary non-zero scalar parameter λ_n , one obtains:

$$-\lambda_n (\delta V)_n^T \xi_n = 0 \quad (3)$$

The result of summing (3) over all n and adding to (2) is:

$$\sum_{n=1}^N (\delta V)_n^T (\bar{M}_n V_n - \lambda_n \xi_n) = 0 \quad (4)$$

There are KN variation in (4), but only $N(K-1)$ of these are independent because of (3). However, the λ_n are arbitrary and can be chosen so that N of the coefficients of the variations in (4) vanish. Since the remaining $N(K-1)$ variations are independent, all coefficients in (4) must vanish. Thus:

$$V_n = \lambda_n \bar{P}_n \xi_n \quad (5)$$

where $\bar{P}_n$ is the inverse of $\bar{M}_n$.

One finds, upon substitution of (5) into (1), that $\lambda_n = W_n F_n$ where the scalar W_n is defined by:

$$\frac{1}{W_n} = \xi_n^T \bar{P}_n \xi_n \quad (6)$$

Thus:

$$v_n = W_n f_n \bar{P}_n \xi_n = W_n f_n (\bar{M}_n)^{-1} \xi_n \quad (7)$$

and, finally, the sum of the weighted squares of the residuals becomes:

$$S = \sum_{n=1}^N W_n f_n^2 \quad (8)$$

or, since $\theta_n = 0$

$$S = \sum_{n=1}^N W_n (f_n - \theta_n)^2$$

Thus, S is the sum of the weighted squares of the deviations of an "observed", (if the true parameters a_ℓ were known) quantity f_n about its true mean value θ_n .

The weights, W_n , should be inversely proportional to the standard error in each observation n . Thus, if we define the elements $(P_{jk})_n$ of $\bar{P}_n$, (the inverse of $\bar{M}_n$) by equation (9):

$$(P_{jk})_n = \frac{1}{\sigma_o^2} (r_{jk})_n \sigma_{jn} \sigma_{kn} \quad (9)$$

where σ_o^2 is an arbitrary constant, and where $(r_{jk})_n$ is the coefficient of correlation between x_{jn} and x_{kn} , the weights W_n are as follows:

$$W_n = \frac{\sigma_o^2}{\sigma_{fn}^2}$$

where the square of σ_{fn} , the standard error in f_n , is given by:

$$\begin{aligned}\sigma_{fn}^2 &= \sum_{j=1}^K \sum_{k=1}^K \xi_{jn} \xi_{kn} (r_{jk})_n \sigma_{jn} \sigma_{kn} \\ &= \sigma_o^2 \xi_n^T \bar{P}_n \xi_n\end{aligned}\quad (10)$$

where $r_{jj} = 1$

Thus:

$$S = \sum \frac{\sigma_o^2}{\sigma_{fn}^2} f_n^2 \quad (11)$$

In the event that it is desired that the data simultaneously satisfy two condition equations:

$$\begin{aligned}\theta_n (x_{1n}, x_{2n}, \dots, x_{Kn}, a_1, a_2, \dots, a_L) &= 0 \\ \phi_n (x_{1n}, x_{2n}, \dots, x_{Kn}, a_1, a_2, \dots, a_L) &= 0\end{aligned}\quad (12)$$

the above procedure can be modified in the following manner:

Expand ϕ_n in a Taylors series (neglecting higher order terms) to give:

$$\phi_n = g_n - V_n^T \psi_n = 0 \quad (13)$$

where g_n is ϕ_n evaluated at the measured coordinates and the elements ψ_{kn} of ψ_n are analogous to the ξ_{kn} of ξ_n .

The variation of ϕ_n (multiplied by an arbitrary parameter v_n) is:

$$-v_n (\delta V)_n^T \psi_n = 0 \quad (14)$$

Summing (14) over all n , adding to (4) and using the argument given following equation (4), one finds that:

$$v_n = \lambda_n \bar{P}_n \xi_n + v_n \bar{P}_n \psi_n \quad (15)$$

Using (15) in (1) and (13), one obtains the following:

$$\lambda_n = \frac{\sigma_o^2}{[1 - R_n^2]} \left[\frac{f_n}{\sigma_{fn}^2} - \frac{R_n g_n}{\sigma_{fn} \sigma_{gn}} \right] \quad (16)$$

and:

$$v_n = \frac{\sigma_o^2}{[1 - R_n^2]} \left[\frac{g_n}{\sigma_{gn}^2} - \frac{R_n f_n}{\sigma_{fn} \sigma_{gn}} \right] \quad (17)$$

Thus, the calculated coordinate, x_{kn} (calculated) = x_{kn} (measured) - V_{kn} , can be written as:

$$x_{kn} \text{ (calculated)} = x_{kn} \text{ (measured)} \quad (15-a)$$

$$- \frac{1}{[1 - R_n^2]} \sum_{j=1}^K (r_{jk})_n \sigma_{jn} \sigma_{kn} \left\{ \frac{f_n \xi_{jn}}{\sigma_{fn}^2} - \frac{R_n (\xi_{jn} g_n + \psi_{jn} f_n)}{\sigma_{fn} \sigma_{gn}} + \frac{g_n \psi_{jn}}{\sigma_{gn}^2} \right\}$$

where σ_{fn}^2 is defined by (10), where σ_{gn}^2 , the square of the standard error in g_n is defined by (18):

$$\sigma_{gn}^2 = \sigma_o^2 \psi_n^T \bar{P}_n \psi_n \quad (18)$$

where R_n , the coefficient of correlation between f_n and g_n is defined by (19).

$$R_n = \frac{\sigma_o^2}{\sigma_{fn} \sigma_{gn}} \psi_n^T \bar{P}_n \xi_n = \frac{\sigma_o^2}{\sigma_{fn} \sigma_{gn}} \xi_n^T \bar{P}_n \psi_n \quad (19)$$

The sum of the weighted squares of the deviations is then given by (20).

$$S = \sum_{n=1}^N \frac{\sigma_o^2}{[1 - R_n]} \left[\frac{f_n^2}{\sigma_{fn}^2} - \frac{2R_n f_n g_n}{\sigma_{fn} \sigma_{gn}} + \frac{g_n^2}{\sigma_{gn}^2} \right] \quad (20)$$

$$= \sum_{n=1}^N \frac{\sigma_o^2}{2} \left[\frac{\left(\frac{f_n}{\sigma_{fn}} - \frac{g_n}{\sigma_{gn}} \right)^2}{1 - R_n} + \frac{\left(\frac{f_n}{\sigma_{fn}} + \frac{g_n}{\sigma_{gn}} \right)^2}{1 + R_n} \right]$$

The sum of the weighted squares of the residuals has thus been transformed to an expression (20) involving measured coordinates and the unknown parameters a_ℓ .

The values of the parameters, a_ℓ , which make S in (20) a minimum can be found by an iterative procedure. In order to retain linear equations for the iterations, the f_n and g_n in (20) are expanded in a Taylors series about estimated values $a_{\ell 0}$, of the parameters a_ℓ , and only the first two terms in the expansion are retained. Thus:

$$f_n \approx f_{no} - F_n^T \{A_o - A\} = f_{no} - F_n^T Y \quad (21)$$

and:

$$g_n \approx g_{no} - G_n^T \{A_o - A\} = g_{no} - G_n^T Y \quad (22)$$

where $Y = A_o - A$. The elements y_ℓ of the vector Y are equal to $a_{\ell 0} - a_\ell$. The elements $F_{\ell n}$ of F_n and $G_{\ell n}$ of G_n are defined by:

$$F_{\ell n} = \frac{f_n}{\partial a_\ell}$$

and:

$$G_{\ell n} = \frac{g_n}{\partial a_\ell}$$

where the partial derivatives are evaluated using the measured coordinates and the estimated values of the parameters. If the weights $\sigma_o^2 / \sigma_{fn}^2 (1-R_n^2)$, etc. are evaluated using measured coordinates and estimated parameters, and if the partial derivatives of S with respect to the y_ℓ are set equal to zero, the resultant equation for $\underline{Y}$ can be written as:

$$\bar{T} \underline{Y} = \underline{C} \quad (23)$$

where $\bar{T}$ is an $L \times L$ symmetric matrix with elements T_{jk} defined by:

$$T_{jk} = \sum_{n=1}^N \frac{\sigma_o^2}{[1-R_n^2]} \left[\frac{F_{jn} F_{kn}}{\sigma_{fn}^2} - \frac{2R_n}{\sigma_{fn} \sigma_{gn}} F_{jn} G_{kn} + \frac{G_{jn} G_{kn}}{\sigma_{gn}^2} \right] \quad (24)$$

and where $\underline{C}$ is a column vector with elements C_j defined by:

$$C_j = \sum_{n=1}^N \frac{\sigma_o^2}{[1-R_n^2]} \left[\frac{f_{no} F_{jn}}{\sigma_{fn}^2} - \frac{R_n (f_{no} G_{jn} + g_{no} F_{jn})}{\sigma_{fn} \sigma_{gn}} + \frac{g_{no} G_{jn}}{\sigma_{gn}^2} \right] \quad (25)$$

Thus, $\bar{T}$ and $\underline{C}$ are known and $\underline{Y}$, as obtained from (24), is given by:

$$\underline{Y} = (\bar{T})^{-1} \underline{C} = \bar{Q} \underline{C} \quad (26)$$

where $\bar{Q} = (\bar{T})^{-1}$.

The new estimates of the parameters a_ℓ are obtained from (27).

$$\underline{A} = \underline{A}_0 - \underline{Y} = \underline{A}_0 - \bar{Q} \underline{C} \quad (27)$$

and the value of S can be calculated, once $\underline{Y}$ is known, from equations (20), (21), and (22).

The square of the estimated standard error σ_ℓ in a_ℓ is given by (28)

$$\sigma_\ell^2 = Q_{\ell\ell} \sigma^2 \quad (28)$$

where the $Q_{\ell\ell}$ are diagonal elements of $\bar{Q}$ and where $\sigma^2 = \frac{S}{\mu}$, where μ is the number of degrees of freedom (i.e. the number of data points N times the number of condition equations (1 or 2) minus the number of parameters L).

The off-diagonal elements of the $\bar{Q}$ matrix are related to the coefficients of correlation between parameters a_j and parameter a_k . In general, (since $r_{jj} = 1$),

$$\sigma_j \sigma_k r_{jk} = Q_{jk} \sigma^2 = Q_{jk} \frac{S}{\mu} \quad (29)$$

The standard error σ_z in a function $z = Z(x_1, x_2, \dots, x_K, a_1, a_2, \dots, a_L)$ is thus given by:

$$\sigma_z^2 = \sigma^2 \sum_{j=1}^L \sum_{k=1}^L \frac{\partial Z}{\partial a_j} \frac{\partial Z}{\partial a_k} Q_{jk}$$

In the particular case that the condition equation θ_n is of the form:

$$\theta_n = z_n - Z_n(x_{1n}, x_{2n}, \dots, x_{Kn}, a_1, a_2, \dots, a_L) = 0$$

the standard error in the calculated curve for z at point n is obtained from:

$$\sigma_{zn}^2 = \sigma^2 \sum_{j=1}^L \sum_{k=1}^L F_{jn} F_{kn} Q_{jk} \quad (30)$$

Error bands, $Z_n \pm \sigma_{zn}$, can be plotted to show the standard error in z above and below the calculated curve (see Deming⁽¹⁰⁾ pg. 228 for example). A confidence band is obtained if σ is replaced by σT , when "T" is obtained from Students "t" distribution. The confidence band, rather than the error band, should be used if the number of degrees of freedom is small.

If one of the a_ℓ (say a_m) is changed by an amount Δa_m , is held constant at the new value, and the quantity S is minimized with respect to the other $a_\ell (\ell \neq m)$, the new values will differ from the old values by an amount Δa_ℓ which is approximately given by the following:

$$\Delta a_\ell = \frac{Q_{\ell m}}{Q_{mm}} \Delta a_m$$

The corresponding change in S is approximately given by:

$$\Delta S = (\Delta a_m)^2 / Q_{mm}$$

Explicit examples of condition equations for the analysis of oscillator, static, and transient data are given in the text of this report.

APPENDIX VI

ESTIMATED ERRORS IN FEEDBACK FOR OSCILLATOR TESTS

Estimated errors in the real and imaginary parts of the reactivity feedback for a conventional oscillator test were obtained by using the calculated SEFOR transfer function (e.g. Section 5.2.4). It was assumed that the measured quantities are G , the at-power transfer function; G_o , the zero power transfer function; ΔT_c , the average coolant temperature oscillation; and $\Delta P/P_o$, the fractional power oscillation.

Using the notation of Section 2.5, the product of the power level and the transform of the feedback kernel can be written as:

$$H \equiv Ph = \left(\frac{1}{G_o} - \frac{1}{G} \right) = \alpha_f F_{av} P X_f + [C_s F_s + C_q] \frac{\Delta T_c}{(\Delta P/P)} \quad (1)$$

where the reactivity effect due to inlet temperature oscillations has been neglected.

The inverse, σ_H^2 , of the weighting functions for a least squares fitting of the above model to the data (i.e. the "effective standard error") is calculated from:

$$\sigma_H^2 = \left[\frac{\sigma_{Go}^2}{|G_o|^4} + \frac{\sigma_G^2}{|G|^4} \right] + |C_s F_s + C_q|^2 \left[\frac{\sigma_T^2}{|\frac{\Delta P}{P}|^2} + \left| \frac{\Delta T_c}{\Delta P/P} \right|^2 \frac{\sigma_P^2}{|\frac{\Delta P}{P}|^2} \right] \quad (2)$$

where it has been assumed that:

$$\begin{aligned}
 \sigma_{Go} &= \text{standard error in real part of } G_o \\
 &= \text{standard error in imaginary part of } G_o \\
 \sigma_G &= \text{standard error in real part of } G \\
 &= \text{standard error in imaginary part of } G \\
 \sigma_T &= \text{standard error in real part of } \Delta T_c \\
 &= \text{standard error in imaginary part of } \Delta T_c \\
 \sigma_P &= \text{standard error in real part of } \Delta P/P \\
 &= \text{standard error in imaginary part of } \Delta P/P
 \end{aligned}$$

Thus, σ_H^2 is the inverse of the weighting function for a fit of both the real and imaginary parts of equation (1) to the data.

The calculated percent deviation in the real and imaginary parts of $H(|\sigma_H/R_H|$ and $|\sigma_H/I_H|)$ are shown in Figures VI-1 and VI-2, for the following set of assumptions: It was assumed that the error $|\sigma_{Go}/G_o|$ in the zero power transfer function, $|\sigma_G/G|$ in the at-power transfer function and, $|\sigma_P/\Delta P/P|$ in the fractional power oscillation were all equal to 1.0%. The assumed error for σ_T was $\pm 0.125^\circ\text{F}$ (this is a minimum error, since it represents the estimated resolution of the data acquisition system, but does not include random, or other, errors in thermocouple signals). A 5% power oscillation $\Delta P/P$ was also assumed.

At low frequencies, the contribution of the error, σ_T^2 , in ΔT_c to the total squared error σ_H^2 is $\sim 85\%$ at 5 MW and $\sim 50\%$ at 20 MW. Since the squared error σ_H^2 contains the term $\sigma_T^2/|\Delta P/P|^2$, the error in σ_H can be reduced, other things remaining constant, by increasing the fractional power oscillation from 5% to 10%.

At higher frequencies (approching 0.1 cps), the percent error in σ_H becomes more dependent on the errors in G and G_0 and less dependent on the error in the coolant oscillation and the fractional power oscillation (i.e., $G_0 \rightarrow G$ and $h \rightarrow 0$ as $\omega \rightarrow \infty$).

Figure VI-1. ESTIMATED DEVIATION IN REAL PART OF THE FEEDBACK (CONVENTIONAL OSCILLATOR TESTS)

Figure VI-2. ESTIMATED DEVIATION IN IMAGINARY PART OF THE FEEDBACK (CONVENTIONAL OSCILLATOR TESTS)

APPENDIX VII

TIME DEPENDENT REACTIVITY WORTH OF A REFLECTOR AND
THE FAST REACTIVITY EXCURSION DEVICE (FRED)

Calculated curves for the approximate time dependent behavior of reflector motion and of the FRED are shown below. Figure VII-1 illustrates the estimated⁽¹²⁾ reflector position as a function of time during the first portion of its travel. Figure VII-2 approximates the reflector reactivity-position curve for a two-segment core as one \$1.3 reflector is lowered from the completely up position; and was obtained from a renormalization of the data in reference (13). The actual curve for a given SEFOR reflector may be somewhat different because of "shadowing" effects of the reflectors and because (1) in the ZPR-3 mockup of SEFOR, the reflector was removed from the bottom rather than the top, and (2) in the ZPR-3 experiment, the reflector extended one-inch above the core while in SEFOR, the reflector will extend 4-inches above the core.

Let the FRED slug length (12-inches) be denoted by L , the extrapolated core half height by H , and the distance that the center of the slug is displaced above the core center by z . The FRED worth at any position z is given by perturbation theory as:

$$k(z) = \int_{z-L/2}^{z+L/2} \phi^2(x) dx \quad z+L/2 \leq H$$

$$k(z) = \int_{z-L/2}^H \phi^2(x) dx \quad H-L/2 \leq z \leq H+L/2$$

$$k(z) = 0 \quad z-L/2 \geq H$$

where $\phi^2(z)$ is the worth of a small sample at location z .

For the two-segment core $\phi^2(z)$ was obtained from a fit of Figure 13 of reference (13). The Pu axial reactivity traverse was used since no boron traverse was available for the two-segment core. The fit is given below and the normalized reactivity is shown in Figure VII-4.

$$\phi^2(z) = \sin^2 \frac{\pi}{2} \left(\frac{25+z}{22} \right) \quad -25 \leq z \leq 5$$

$$\phi^2(z) = .638 + .035 (7-z) \quad 5 \leq z \leq 7$$

$$\phi^2(z) = .668 \sin^2 \frac{\pi}{2} \left(\frac{26-z}{22} \right) \quad 7 \leq z \leq 26$$

The estimated⁽¹⁴⁾ FRED displacement as a function of the time shown in Figure VII-5 was used to generate the FRED reactivity vs time curve shown in Figure VII-6 and VII-7.

VII-1A

Figure VII-1. ESTIMATED REFLECTOR POSITION VS TIME

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Figure VII-2. REFLECTOR WORTH VS POSITION FOR 2-SEGMENT CORE

Figure VII-3. ESTIMATED REFLECTOR WORTH (\$1.3 TOTAL) AS A
FUNCTION OF TIME

Figure VII-4.

SLUG WORTH VS. POSITION - NORMALIZED TO 100c AT FLUX PEAK
(-3.0 INCHES)

Figure VII-5.

FRED DISPLACEMENT VS. TIME

Figure VII-6.

TIME DEPENDENT FRED WORTH FOR 20-INCH
TRAVEL TIME OF 95 MSEC

Figure VII-7. TIME DEPENDENT FRED WORTH FOR 20-INCH TRAVEL TIME OF 65 MSEC

APPENDIX VIII

ESTIMATED ERRORS IN DOPPLER ENERGY COEFFICIENTOBTAINED FROM SEFOR TRANSIENT TESTS

Estimates were made of the expected errors in the Doppler energy coefficient for transient experiments in SEFOR, where the slug is completely ejected in ~90 msec. The errors were calculated for various FRED slug sizes with and without reflector motion prior to FRED ejection. The FORE⁽¹⁵⁾ code was used to generate the feedback effects during the various transients, and the errors in the Doppler energy coefficient were calculated by assigning assumed uncertainties to the FRED worth, fuel expansion effect, etc.

The percent error in the Doppler energy coefficient is shown in Figure VIII-1 as a function of FRED slug size (with and without reflector motion prior to FRED ejection) for two different assumptions (see below) about the magnitude of the uncertainties in the various reactivity effects. Although the actual numbers may change as a result of a more thorough analysis, two trends are indicated by Figure VIII-1. These are: (1) the error is very large for FRED slug sizes $\lesssim 1.3$ when reflector motion is initiated prior to FRED ejection, and (2) little increase in accuracy is obtained by using slug sizes $\gtrsim 1.3$ when there is no reflector motion.

Using the assumptions described below, the values of the various reactivity effects were taken from several FORE runs and the error in the Doppler energy coefficient was calculated at various time intervals during a transient. The calculated error reached a minimum during the first one or two hundred milliseconds after FRED had been completely ejected and did not vary greatly during this time interval. An average

value of the error during a time interval of ≈ 100 msec after complete FRED ejection is shown as a calculated point in Figure VIII-1. Some explicit examples of the calculations are shown at the end of this Appendix.

The primary reason that the error in the Doppler energy coefficient is so large with reflector motion and slug sizes $\lesssim \$1.3$ is not because the magnitude of the errors in FRED, etc., change significantly, but rather that the magnitude of the Doppler feedback itself is decreased from that of the case with no reflector motion. As an example: in the case of a $\$1.1$ FRED slug and no reflector motion, the Doppler 10 msec after complete FRED ejection is 14.5% while the corresponding Doppler feedback in the case of reflector motion is only 4.3% . For slug sizes $\gtrsim \$1.3$ the FRED overrides the reflector and the Doppler feedback approaches the no-reflector-motion case. The effect of reflector motion upon the power, and thus the fuel temperature and Doppler effect, is illustrated in Figure VIII-2 for a $\$1.10$ FRED slug.

VIII.1 REACTIVITY EFFECTS

All of the results presented here are based upon the case in which the reactor is initially at 5 MW and the system has the following reactivity coefficients.

$$\text{Doppler } T \frac{dk}{dT} = -.008 \equiv \alpha$$

$$\text{Fuel axial expansion} = -1.32 \times 10^{-6} \Delta k/^{\circ}\text{F} \equiv \alpha_e$$

$$\text{Clad expansion} = -2.44 \times 10^{-6} \Delta k/^{\circ}\text{F} \equiv \alpha_c$$

$$\text{Structure and sodium} = -5.0 \times 10^{-6} \Delta k/^{\circ}\text{F} \equiv \alpha_s$$

Figure VIII-1. ERROR IN ENERGY COEFFICIENT VS. FRED WORTH

Figure VIII-2. REACTOR POWER WITH AND WITHOUT REFLECTOR MOTION, COMPLETE FRED EJECTION IN 90 MSEC

The fuel axial expansion coefficient corresponds to about twice the value of the axial expansion effect recently reported in reference (16). The structure and sodium coefficient is approximately one-half of the presently calculated reactivity effect in SEFOR. Any increase in the ratio of the extraneous effects to the Doppler effect would increase (with the same relative uncertainty in the extraneous effects) the uncertainty in the Doppler energy coefficient. A higher initial power would tend to increase the error in the Doppler energy coefficient (lower initial power $\rightarrow$ decrease in error) since the Doppler effect is reduced, relative to other effects, with increasing fuel temperature. Uncertainty in the proper errors to be assumed was accounted for to some extent by the use of the "pessimistic" and "optimistic" assumptions described below. In addition to the above values, the time dependent FRED and reflector curves shown in Appendix VII, were used.

For simplicity, it was assumed that the Doppler reactivity effect (Δk_D) during a transient can be described by a linear dependence upon the change in energy (ΔE), i.e.

$$\Delta k_D = \gamma \Delta E$$

It was also assumed that all errors, including biases, are uncorrelated and can be propagated as random errors.

The fractional random error in the Doppler energy coefficient (γ) is given by:

$$\frac{\sigma_\gamma}{\gamma} = \sqrt{\left(\frac{\sigma_D}{\Delta k_D}\right)^2 + \left(\frac{\sigma_E}{\Delta E}\right)^2} \quad (1)$$

where σ_γ denotes the error in γ , σ_D the error in Δk_D , etc.

The Doppler reactivity effect (Δk_D) can be written as:

$$\Delta k_D = \Delta k_N - \Delta k_F - \Delta k_r - \Delta k_e - \Delta k_o \quad (2)$$

where

Δk_N = the net change in reactivity (calculated in SEFOR experiments by inversion of the kinetic equation)

Δk_F = FRED reactivity

Δk_r = reflector reactivity

Δk_e = fuel axial expansion effect

Δk_o = clad + coolant and structure effect (lumped together in the error analyses, but not in the FORE calculations)

The random error in the Doppler effect is thus given by:

$$\sigma_D = \sqrt{\sigma_N^2 + \sigma_F^2 + \sigma_r^2 + \sigma_e^2 + \sigma_o^2} \quad (3)$$

for the super-prompt tests. Rather than looking at the change in reactivity from time zero (the time when FRED or reflector is fired), it was assumed for the sub-prompt tests that the data analysis is performed by looking at the difference between the reactivity at a time $t_o + \Delta t$ and the reactivity at time t_o , where t_o is the time at which complete FRED ejection occurs. In this case, $\sigma_F = 0$. The latter procedure can also be applied to the super-prompt tests; however, there is

some question as to its usefulness, since it yielded smaller errors than those shown in Figure VIII-1, when the "optimistic" assumptions were used, but larger errors when the "pessimistic" assumptions were used.

To first order, the error in the net reactivity during a transient should be proportional to $|\Delta k_N - 1|$, where Δk_N is in dollars, and the fractional error in FRED and in the reflector worth should be constant. In addition, it was also assumed that the fractional error in the expansion effect, clad + sodium and structure, and change in energy release are constant.

VIII.2 "PESSIMISTIC" ASSUMPTIONS

For the two-segment SEFOR core, the fractional error in the fuel expansion effect is about 0.35. Using the same fractional uncertainty, 0.35, and twice the expansion effect should yield a pessimistic absolute value of the fuel expansion error. For the other reactivity effects (clad + sodium and structure) a fractional error of 1.0 was assumed. This is larger than a $\pm 100\%$ uncertainty in each, since these reactivity effects were summed before propagating the error. The uncertainty in Δk_N was assumed to be 10% of $|\Delta k_N - 1|$, (the expected error is $< 5\%$) and a fractional error of 0.05 was assumed for the change in energy. The latter represents $\sim 5\%$ bias error and ~ 0 random error.

It was assumed that the FRED and one reflector can be calibrated with less than 5% error. The FRED calibration was assumed to be slightly better than the reflector calibration, since the reflector strength may be more sensitive to the position of the other reflectors. Should the error in FRED exceed $\sim 5\%$, the qualitative conclusions given above, and the qualitative behavior of the curves in Figure VIII-1 would remain unchanged.

The form and magnitude of the various errors are listed below.

$$\sigma_e / \Delta k_e = 0.35 \equiv \mu_e$$

$$\sigma_o / \Delta k_o = 1.0 \equiv \mu_o$$

$$\sigma_N = 0.10 |\Delta k_N - 1| \equiv \mu_N |\Delta k_N - 1|$$

$$\sigma_F / \Delta k_F = 0.03 \equiv \mu_F$$

$$\sigma_r / \Delta k_r = 0.04 \equiv \mu_r$$

$$\sigma_E / \Delta E = 0.05 \equiv \mu_E$$

These assumptions were used with the appropriate numbers obtained from the FORE output to calculate the numbers shown in Figure VIII-1.

VIII.3 "OPTIMISTIC" ASSUMPTIONS

For this case, it was assumed that the magnitude of all the errors listed above was decreased by a factor of 2, except for the energy release and the FRED worth. The error in the energy was unchanged and the error in the FRED worth which was decreased to 2%. The assumed form and magnitude are listed below.

$$\sigma_e / \Delta k_e = 0.175$$

$$\sigma_r / \Delta k_r = 0.02$$

$$\sigma_o / \Delta k_o = 0.50$$

$$\sigma_E / \Delta E = 0.05$$

$$\sigma_N = 0.05 |\Delta k_N - 1|$$

$$\sigma_F / \Delta k_F = 0.02$$

VIII.4 SAMPLE CALCULATIONS

Listed below are the FORE output at a time ≈ 60 msec after FRED has been completely ejected for cases in which a \$1.1 slug was fired with and without reflector motion.

TABLE VIII-1

Case S-150 (\$1.10 slug - no reflector motion - FRED out at $t = 90$ msec)

t (msec)	Δk_D (ϕ)	Δk_e (ϕ)	Δk_o (ϕ)	Δk_F (ϕ)	$1-\Delta k_N$ (ϕ)	Δk_r (ϕ)
154.0	18.82	4.32	1.51	110.0	14.65	0.0

TABLE VIII-2

Case S-158 (\$1.10 slug - reflector motion 50 msec before FRED - FRED out at $t = 140$ msec)

t (msec)	Δk_D (ϕ)	Δk_e (ϕ)	Δk_o (ϕ)	Δk_F (ϕ)	$1-\Delta k_N$ (ϕ)	Δk_r (ϕ)
197.5	6.23	1.39	.46	110.0	20.33	22.25

TABLE VIII-3

Calculated Errors

Case #	σ_e (¢)	σ_o (¢)	σ_F (¢)	σ_N (¢)	σ_r (¢)
S-150	0.76	0.76	2.20	0.73	0.0
S-158	0.24	0.23	2.20	1.01	0.45

Since σ_D is the square root of the sum of the squares of the above numbers, one obtains

$$\sigma_D = \sqrt{6.53} = 2.56¢ \text{ (S-150)}$$

$$\sigma_D = \sqrt{6.17} = 2.48¢ \text{ (S-158)}$$

Since

$$\frac{\sigma_Y}{Y} = \sqrt{\left(\frac{\sigma_D}{\Delta k_D}\right)^2 + \left(\frac{\sigma_E}{\Delta E}\right)^2}$$

one obtains:

$$\frac{\sigma_Y}{Y} = \sqrt{\left(\frac{2.56}{18.82}\right)^2 + (.05)^2} = 0.145 \text{ (S-150)}$$

$$\frac{\sigma_Y}{Y} = \sqrt{\left(\frac{2.48}{6.23}\right)^2 + (.05)^2} = 0.407 \text{ (S-158)}$$

VIII.5 ASYMPTOTIC BEHAVIOR OF ERRORS

The primary reason that the errors in Figure VIII-1 do not decrease significantly for slug sizes $\lambda \geq 1.3$ is the following:

During a time interval ≈ 100 msec after FRED has been completely ejected FORE results for a super-prompt transient show that the rise in clad temperature is $\approx 10\%$ of the rise in fuel temperature, and that the rise in sodium and structure temperature is $\approx 6\%$ of the fuel temperature rise. In addition, the total feedback reactivity, in cents, can be approximated, very roughly, by $\Delta k_F - 85$, where Δk_F is the FRED worth ($\lambda \approx 110$) in cents. If the Doppler effect is approximated by a linear coefficient A/T_0 , where A is the Doppler $T dk/dT$ and T_0 is the initial fuel temperature, then all feedback effects are proportional to the fuel temperature rise and can thus be written in terms of the Doppler effect. Using the above assumptions, the square of the fractional error in the Doppler energy coefficient can be approximated by:

$$\begin{aligned} \left(\frac{\sigma_Y}{Y}\right)^2 &\approx \left(\frac{\sigma_E}{E}\right)^2 + \left(\frac{T_0}{A}\right)^2 [(\mu_e \alpha_e)^2 + (0.1\mu_c \alpha_c)^2 + (0.06\mu_s \alpha_s)^2] \\ &+ \left[1 + \frac{T_0}{A} (\alpha_e + .1\alpha_c + .06\alpha_s)\right]^2 \left[\mu_N^2 \left(\frac{15}{\Delta k_F - 85}\right)^2 + \mu_F^2 \left(\frac{\Delta k_F}{\Delta k_F - 85}\right)^2\right] \end{aligned}$$

where the μ 's are the fractional uncertainties in the various effects (see "Pessimistic" Assumptions), and the α 's are the various reactivity coefficients (see Reactivity Effects). For the coefficients and assumptions used here (lumping of clad, coolant and structure errors) the above expression reduces to the following for the "pessimistic" values of the μ 's.

$$\frac{\sigma_Y}{Y} \approx \sqrt{.01823 + 1.78 \left[.01 \left(\frac{15}{\Delta k_F - 85} \right)^2 + \mu_F^2 \left(\frac{\Delta k_F}{\Delta k_F - 85} \right)^2 \right]}$$

This can be used to generate the following table.

$\Delta k_F (\epsilon)$	110	130	150	170	∞
$\sigma_Y / Y (\mu_F = .03)$	.236	.183	.167	.159	.141
$\sigma_Y / Y (\mu_F = .06)$	.385	.272	.231	.211	.157

Although the above numbers do not agree exactly with the curves in Figure VIII-1, they do indicate the trend that occurs with increasing slug size.

APPENDIX IX

EFFECT OF DELAYED Be PHOTONEUTRONS ON SEFOR KINETICS

IX.1 INTRODUCTION

Because of the $\text{Be}^9 (\gamma, n) \text{Be}^8$ reaction, photoneutrons will be produced in SEFOR. Those photoneutrons, which appear as a result of radioactive decay of fission products, or as a result of beta decay of activated core materials, will appear in the kinetics equations as extra groups of delayed neutrons. The effect of these extra groups on the kinetic behavior of the SEFOR reactor is described below.

IX.2 RESULTS AND CONCLUSIONS

In SEFOR (mass fraction of Be in core = 0.0089) the delayed photoneutron fraction (β_p) is $\approx 2 \times 10^{-6}$, while the total effective delayed neutron fraction ($\bar{\beta}$) is 3.2×10^{-3} . Thus, the contribution of the photoneutrons to $\bar{\beta}$ is approximately $(100)(2 \times 10^{-6}) / (3.2 \times 10^{-3}) \approx 0.06\%$. Although some of the delayed photoneutron groups have long half lives, the difference between the zero power transfer function at 0.00125 cps, with and without photoneutrons, is only about 0.2% in amplitude and 0.2 degrees in phase. The difference is even less at higher frequencies, and in any event is probably less than the uncertainties in the experiments.

Since the delayed photoneutron effects are small in SEFOR, they can be neglected except for the following exception: Because of their long half lives, continued operation at high power will generate a source

of neutrons which, if the reactor is subsequently operated at low power, will slowly decay away. Since the effective source strength effects the criticality of the system, experiments at low power should not be immediately preceded by continued operation at high power (e.g., Discussion).

IX.3 DISCUSSION

By fitting the photoneutron data of Bernstein, et al,⁽¹⁷⁾ the value of the delayed photoneutron fraction (β_p) obtained by Keepin⁽¹⁸⁾ is 1.5175×10^{-4} for a system in which all the gamma rays that produce photoneutrons are absorbed in Be. If it is assumed that the mass attenuation coefficient is constant for all materials, then the delayed photoneutron fraction for SEFOR (mass fraction of Be = 0.0089) is $(1.5175 \times 10^{-4})(0.0089) = 1.35 \times 10^{-6}$. The preceding calculation neglects gamma leakage from the core and gamma rays from activated core materials. Both effects are small.

In experiments performed in reactors containing mass fractions of Be between ~ 0.6 and 0.9 Krashin, et al,⁽¹⁹⁾ obtained a value for the delayed photoneutron fraction of 1.48×10^{-4} . Krashin does not state whether this value of β_p is normalized to an infinite amount of Be or whether it was the value obtained for some specific reactor configuration. Assuming (conservatively) that Krashin's value is for the reactor with the smallest mass fraction of Be, one obtains $\beta_p(\text{SEFOR}) = (1.48 \times 10^{-4})(0.0089)/0.6 = 2.2 \times 10^{-6}$.

Since the effective delayed neutron fraction ($\bar{\beta}$) for SEFOR is 3.2×10^{-3} , the contribution of β_p to $\bar{\beta}$ is only 0.06%. In order to determine the effect of the SEFOR delayed photoneutrons, the zero

power transfer function was computed at several frequencies between 0.00125 and 0.5 cps. Keepin's⁽¹⁸⁾ delayed neutron parameters for Pu-239, U-238 and Be photoneutrons (normalized to value of $\beta_p/\bar{\beta} = 6.1 \times 10^{-4}$) were used. The maximum difference in the transfer function with, and without delayed photoneutrons was $\sim 0.2\%$ in amplitude and 0.2 degrees in phase at 0.00125 cps.

In order to give an explicit example of the effect of the delayed photoneutrons, assume for simplicity that the reactor was operated at very low power for a long period of time, then operated at a high power P_1 for time T , and shut down to power P_0 at time $t = 0$. The reactivity (in dollars) required to hold the power constant at P_0 is (in the absence of a source) then given by

$$k(t) = - \sum_i \frac{\beta_i}{\bar{\beta}} e^{-\lambda_i t} \left[\frac{P_1}{P_0} (1 - e^{-\lambda_i T}) - 1 \right]$$

where t is the time that has elapsed since the reactor was instantaneously shut down to power P_0 , and the index i runs over all precursor groups (including photoneutrons).

After the first 10 minutes or so all the fission product delayed neutrons attain their new equilibrium values corresponding to the power P_0 . However, the delayed photoneutrons, because of their long half lives do not achieve equilibrium within 10 minutes and $k(t)$ will be negative (i.e. the reactor will be subcritical) for quite some time.

Using the same delayed neutron parameters that were used to calculate the SEFOR transfer function (see above) the following table can be constructed for the case $P_1 = 20$ MW, $P_0 = 100$ kw. If $P_0 = 10$ kw, the reactivity values listed below must be multiplied by a factor of 10.

t	k(t) (cents)	
	T = 10 hrs.	T = ∞
10 min	-4.9	-5.4
30 min	-3.1	-3.5
1 hr	-2.2	-2.6
2 hr	-1.6	-2.0
3 hr	-1.3	-1.6
4 hr	-1.0	-1.3
8 hr	-0.44	-0.64
24 hr	-0.036	-0.13

As the table indicates, the reactor will be subcritical for an extended length of time. In order to avoid the complication of having to know the exact delayed photoneutron parameters, a waiting period of from 8 to 24 hours after operation at high power should elapse before any oscillator experiments or rod calibrations are performed in the 10 kw to 100 kw power range.

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