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INFORMAL REPORT

Permissible Multipole Field Components in Quadrupole Magnets

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scientific laboratory

of the University of California

LOS ALAMOS, NEW MEXICO 87544

MASTER

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ABSTRACT

The rms multipole flux density sum useful for engineering specifications is derived from a set of permissible multipole potentials.

I. DEFINITIONS

The multipole field in the Riemann plane of Fig. 1a may be mapped into the upper half-plane of Fig. 1b by

$$w = \frac{z^n}{n} = \frac{r^n}{n} e^{in\phi} \quad (1)$$

The scalar potential is given from Fig. 1b

$$\phi_n = n \operatorname{IN} \left(\frac{v}{a} \right)^n \quad (2)$$

where we have from Eq. (1)

$$v = \frac{r^n}{n} \sin n\phi \quad (3)$$

Inserting Eq. (3) in Eq. (2) yields

$$\phi_n = \operatorname{IN} \left(\frac{r}{a} \right)^n \sin n\phi \quad (4a)$$

or

$$\phi_n = \frac{G_n a}{n} \left(\frac{r}{a} \right)^n \sin n\phi \quad (4b)$$

where the multipole gradient

$$G_n = \frac{n \operatorname{IN}}{a} \quad (4c)$$

Fig. 1a.

Fig. 1b.

By differentiating Eq. (4b) one obtains

$$H_n(r, \phi) = G_n \left(\frac{r}{a}\right)^{n-1} (\hat{r} \sin n\phi + \hat{\phi} \cos n\phi) \quad (5)$$

and

$$H_n(x, y) = G_n \left(\frac{r}{a}\right)^{n-1} [\hat{x} \sin(n-1)\phi + \hat{y} \cos(n-1)\phi]. \quad (6)$$

Alternatively, one may write the n^{th} term of the Taylor series in the circle $z = ae^{i\phi}$

$$F_n(z) = A_n + i \phi_n = \frac{z^n}{n!} \left(\frac{\partial^n F}{\partial z^n} \right)_{z=0} \quad (7)$$

where the derivatives

$$\frac{d^n F}{dz^n} = \frac{\partial^n \phi}{\partial x^{n-1} \partial y} + i \frac{\partial^n \phi}{\partial x^n} \quad (7a)$$

or

$$\frac{d^n F}{dz^n} = \frac{\partial^{n-1} H_y}{\partial x^{n-1}} + i \frac{\partial^{n-1} H_x}{\partial x^{n-1}}. \quad (7b)$$

The imaginary terms in Eqs. (7a) and (7b) vanish for the case of midplane symmetry, which shall be assumed in the following.

By inserting Eq. (7b) in Eq. (7) we get for the scalar potential

$$\phi_n(z) = \frac{a^n}{n!} \left(\frac{r}{a}\right)^n \frac{\partial^{n-1} H_y}{\partial x^{n-1}} \sin n\phi. \quad (8)$$

By comparing the Eqs. (4b) and (8), we see that

$$G_n = \frac{n! IN}{a} = \frac{a^{n-1}}{(n-1)!} \frac{\partial^{n-1} H_y}{\partial x^{n-1}} \quad (9a)$$

and

$$IN = \frac{a^n}{n!} \frac{\partial^{n-1} H_y}{\partial x^{n-1}}. \quad (9b)$$

Differentiating Eq. (8) yields

$$H_n = \frac{r^{n-1}}{(n-1)!} \frac{\partial^{n-1} H_y}{\partial x^{n-1}} (\hat{r} \sin n\phi + \hat{\phi} \cos n\phi). \quad (10)$$

Hence, one may write the multipole ratio

$$\frac{H_{n+1}}{H_2} = \frac{r^{n-1}}{n!} \left(\frac{\partial^n H_y}{\partial x^n} \right) \frac{\hat{r} \sin(n+1)\phi + \hat{\phi} \cos(n+1)\phi}{\hat{r} \sin 2\phi + \hat{\phi} \cos 2\phi}. \quad (11)$$

The ratio in parentheses is given by Eq. (14) of the Report, LA-5076-MS, i.e.,

$$\frac{\frac{\partial^n H_y}{\partial x^n}}{\frac{\partial H_y}{\partial x}} = \frac{(n+1)!}{2} \frac{C_{n+1}}{C_2}, \quad (12)$$

where the ratio C_{n+1}/C_2 is obtained from the permissible aberrations as shown in the report. By comparing the two Eqs. (11) and (12), we get

$$\frac{H_{n+1}}{H_2} = \frac{n+1}{2} r^{n-1} \frac{C_{n+1}}{C_2}. \quad (13)$$

II. PERMISSIBLE MULTIPOLE COMPONENTS FOR BEAM LINE D LA-5076-MS shows that for the set of permissible aberrations

$$A_x = 0.1 \text{ (cm)}; \quad A_{x'} = 0.03 \text{ (mr)}$$

$$A_y = 0.09 \text{ (cm)}; \quad A_{y'} = 0.2 \text{ (mr)}$$

observed at Target Station 1, the multipole potential ratios given in Table IVa of the report are allowable in the sense that each multipole potential produces this permissible set of aberrations. The multipole potential ratios are listed also in Table I of this report. Also given in Table I are the corresponding ratios of high order derivatives to quadrupole gradient and of multipole peak value to quadrupole flux density as evaluated at a radius of 1 in. and of 2 in.

In a real magnet all multipoles are represented to some extent. It is therefore practical to use the rms sum of the components as quality specification so that the inequality

TABLE I

n+1 (sextupole through 12-pole)	C(n+1)/C(2) from Appendix IVa, LA-5076-MS	$\frac{\partial^n H_y}{\partial x^n} / \frac{\partial H_y}{\partial x}$	$\left(\frac{H_{n+1}}{H_2}\right)_{r=1 \text{ in.}}$	$\left(\frac{H_{n+1}}{H_2}\right)_{r=2 \text{ in.}}$
3	$4.263 \cdot 10^{-5}$	$1.28 \cdot 10^{-4}$	$1.6 \cdot 10^{-4}$	$0.32 \cdot 10^{-3}$
4	$0.679 \cdot 10^{-5}$	$0.81 \cdot 10^{-4}$	$0.9 \cdot 10^{-4}$	$0.35 \cdot 10^{-3}$
5	$2.371 \cdot 10^{-5}$	$14.2 \cdot 10^{-4}$	$9.7 \cdot 10^{-4}$	$7.7 \cdot 10^{-3}$
6	$0.059 \cdot 10^{-5}$	$2.11 \cdot 10^{-4}$	$0.7 \cdot 10^{-4}$	$1.2 \cdot 10^{-3}$

$$\sqrt{\left(\frac{\% B_3}{0.032}\right)^2 + \left(\frac{\% B_4}{0.035}\right)^2 + \left(\frac{\% B_5}{0.77}\right)^2 + \left(\frac{\% B_6}{0.12}\right)^2} < 1 \quad (14)$$

be satisfied, according to the fifth column in Table I, i.e., at the radius $r = 2$ in. The $\% B_{(n+1)}$ - values in the Inequality (14) are the fields integrated over the axial range $-\infty$ to ∞ and expressed in percent of the quadrupole field integrated over the same range.