18-75
LA-5691

Dr-1144

UC-20

Reporting Date: July 1974

Issued: December 1974

Semiempirical Cable Design

by

G. P. Boicourt

los alamos
scientific laboratory
of the University of California
LOS ALAMOS, NEW MEXICO 87544

UNITED STATES
ATOMIC ENERGY COMMISSION
CONTRACT W-7405-ENG. 36

Printed in the United States of America. Available from
National Technical Information Service
U.S. Department of Commerce
5285 Port Royal Road
Springfield, VA 22151
Price: Printed Copy \$4.00 Microfiche \$2.25

This report was prepared as an account of work sponsored by the United States Government. Neither the United States nor the United States Atomic Energy Commission, nor any of their employees, nor any of their contractors, subcontractors, or their employees, makes any warranty, express or implied, or assumes any legal liability or responsibility for the accuracy, completeness or usefulness of any information, apparatus, product or process disclosed, or represents that its use would not infringe privately owned rights.

-NOTICE-

This report was prepared as an account of work sponsored by the United States Government. Neither the United States nor the United States Atomic Energy Commission, nor any of their employees, nor any of their contractors, subcontractors, or their employees, makes any warranty, express or implied, or assumes any legal liability or responsibility for the accuracy, completeness or usefulness of any information, apparatus, product or process disclosed, or represents that its use would not infringe privately owned rights.

SEMIEMPIRICAL CABLE DESIGN

by

G. P. Boicourt

ABSTRACT

Coaxial cables are used extensively on Controlled Thermonuclear Research (CTR) at voltages much higher than manufacturers' ratings. The question of how such a cable should be designed is not easily answered. This report describes an approach to the problem that combines empirical knowledge and mathematical analysis. The result is an algorithm that will compute the optimal cable size for a cable having a given inductance, a given peak operating voltage, and a given maximum electric field strength in the dielectric.

Suggestions for future cable research and development are given.

I. INTRODUCTION

In Controlled Thermonuclear Research (CTR), coaxial cables are used extensively to transmit high voltages. In trigger systems or fast energy transfer systems, voltages may exceed by orders of magnitude the ratings the manufacturer is willing to put on the cable. Experiments have shown that these ratings are unrealistic and that, provided a few losses are acceptable, the cable will operate satisfactorily at the higher voltages. It is not easy to choose or design such cable because there are several conflicting criteria. Usually the inductance must be low, the size small, and the voltage capability high. We will first examine the problem of finding the minimum size cable for a given inductance, operating voltage, and maximum electric stress. Under general assumptions, we derive an algorithm that will compute the required cable dimensions. Plots obtained by using this procedure show how changes in the parameters affect the cable size.

The choice of the maximum electric field and the determination of the operating voltage is then discussed.

II. OPTIMAL CABLE CALCULATION

We assume that we know: L , the inductance required; E_m , the maximum allowed electric field in the dielectric; and V , the maximum operating voltage. We further assume that the cable is graded; that is, the cable is built with conducting screens outside the inner conductor and inside the outer conductor (Fig. 1). The use of such screens greatly improves the voltage-holding characteristics of the cable.¹ In addition, we assume that the maximum radial electric field is essentially equal to E_m . Because

$$E_m \leq \sqrt{(E_r \max)^2 + (E_z \max)^2},$$

Fig. 1. Screened coaxial cable.

this is equivalent to assuming that

$$(E_{z \max})^2 \ll (E_{r \max})^2.$$

The value of $E_{z \max}$ is determined from $L \frac{dI}{dt}$ in the cable. We are not usually interested in cable with inductances greater than 350×10^{-9} H/m and experience has shown that maximum currents greater than about 20×10^3 A are not practical in a single cable. If we further assume that this current rises in times greater than 10^{-8} s, we can estimate the size of $E_{z \max}$. We have

$$E_{z \max} < \frac{(3.5 \times 10^{-7})(2 \times 10^4)}{10^{-8}} = 7 \times 10^5 \text{ V/m}.$$

However, polyethylene can safely operate at fields well in excess of 36 MV/m (in 17/14 HR cable, $E_{r \max}$ is about 40 MV/m). Using the higher value of $E_{z \max}$ and the smaller value of $E_{r \max}$, we have

$$\sqrt{(E_{r \max})^2 + (E_{z \max})^2} = (1.296 + 0.49)^{1/2} \times 10^6 = 36.0068 \text{ MV/m},$$

which shows that the replacement of E_m by $(E_{r \max})$ produces negligible error.

The voltage that the cable dielectric is required to hold is given by

$$V = \int_{r_i}^{r_o} E_r dr,$$

where r_o is the outer radius of the cable dielectric, r_i is the inner radius of the cable dielectric, and E_r is the radial electric field. In a cylindrical geometry,

$$E_r = \frac{a}{r}$$

so that

$$V = a \ln \frac{r_o}{r_i}. \quad (1)$$

Here, a is a constant to be determined. In this we have neglected any voltage drop in the conducting screen and to prove that this is justified, we show that the skin depth in the screen material is large compared to the transverse screen dimension. Measurement shows the resistivity of the screen is

$$\rho \approx 1.7 \times 10^{-2} \text{ } \Omega \cdot \text{m}.$$

The skin depth is given by

$$s = \sqrt{\frac{4 \pi \rho}{\mu_0 f}} = \sqrt{\frac{\rho}{f}} \times 10^7.$$

Oscillation frequencies of less than 0.5×10^8 /s are expected, therefore,

$$s \approx 0.058 \text{ m}.$$

Because s is at least 50 times greater than the screen thickness, even at very high frequencies, no substantial voltage will appear across the screen.

With these assumptions, the maximum radial electric field in the cable is given by

$$E_m = \frac{a}{r_i}. \quad (2)$$

The inductance of a coaxial cable is given by

$$L = 2 \times 10^{-7} \ln \frac{R_o}{R_i}, \quad (3)$$

where R_o and R_i are the geometric mean radii of the inside and outside conductors. We assume that the current is distributed uniformly across the area of the conductors, which is usually the case when braided conductors are used. If the conductors are thick, then corrections for skin depths should be made. In the following discussion we will assume that this geometric mean distance is at the middle radius of the conductor.

Let s_i and s_o be the thickness of the inner and outer screens, respectively. Let b_i and b_o be the thickness of the inner and outer conductors. Then

$$R_o = r_o + s_o + b_o/2, \quad (4)$$

and

$$R_i = r_i - s_i - b_i/2. \quad (5)$$

Using these in Eq. (3), we find

$$r_o = (r_i - s_i - b_i/2)e^{L/2 \times 10^{-7}} - s_o - b_o/2. \quad (6)$$

We need a second expression relating r_o and r_i . By themselves, E_m , V , and L do not uniquely determine r_o and r_i . However, we must also have

$$\frac{a}{r_i} \geq E_m,$$

which, using Eq. (1), yields

$$V \geq E_m r_i \ln \frac{r_0}{r_i}. \quad (7)$$

We want to solve Eqs. (6) and (7) subject to the condition that the cable diameter be minimum. This is equivalent to r_0 being as small as possible. But Eq. (6) implies that if r_0 is minimal, so is r_i . When r_i is minimal, the maximum electric field will be attained and we can assume equality in Eq. (7).

$$V = E_m r_i \ln \frac{r_0}{r_i}. \quad (8)$$

Equations (6) and (8) can be combined to give

$$\begin{aligned} V/E_m &= r_i \ln \frac{1}{r_i} \left[r_i e^{L/2 \times 10^{-7}} \right. \\ &\quad \left. - (s_i + b_i/2) e^{L/2 \times 10^{-7}} - (s_0 + b_0/2) \right] \\ &= r_i \ln \left(e^{L/2 \times 10^{-7}} - K/r_i \right), \end{aligned} \quad (9)$$

where

$$K = (s_i + b_i/2) e^{L/2 \times 10^{-7}} + (s_0 + b_0/2).$$

We can solve Eq. (9) for r_i by iteration. Using the Newton-Raphson method, we arrive at the following iterative equations.

$$\begin{aligned} (r_i)_{n+1} &= (r_i)_n \\ &+ \frac{\left(V/E_m - (r_i)_n \ln \left[e^{L/2 \times 10^{-7}} - K/(r_i)_n \right] \right)}{\left(\ln \left[e^{L/2 \times 10^{-7}} - K/(r_i)_n \right] + K / \left[(r_i)_n e^{L/2 \times 10^{-7}} - K \right] \right)}, \end{aligned}$$

and

$$(r_i)_0 = \frac{V/E_m}{\ln \left(\frac{V/E_m + s_i + b_i}{s_i + b_i} \right)}. \quad (10)$$

We will return to the solution of Eq. (1) in the final section.

III. CHOICE OF OPERATING VOLTAGE AND MAXIMUM FIELD STRENGTH

The peak operating voltage V used in the derivation of Eq. (9) is the maximum potential that will

appear across the cable; it is not the dc operating voltage. The determination of V must take into consideration transient pulses produced in the cable or in components connected to it and propagated in the cable. Direct measurement of such voltages is usually not possible. One way to determine V is by computation. For instance, the NET-2 computer program can be used.^{2,3} The cable is treated as a lumped constant line of more than five sections. The peak voltage appearing is a good estimate of V although voltages calculated in this way are usually a few percentage points too high.

The determination of E_m is a more complicated situation. The maximum electric field at which the polyethylene can operate satisfactorily is in part determined by how carefully it was extruded. This implies that some knowledge of the manufacturers' past performance would be helpful in deciding which E_m to specify. However, given manufacturing uniformity, a lot can be said about the rational determination of E_m .

Some knowledge about the failure behavior of high voltage polyethylene cable is required. When the raw failure data for such cable are plotted on Weibull coordinates, a knee appears at some percentage of failures and at some number of shots. This characteristic occurs in ac cable,⁴ high-voltage pulse cable,¹ and recently in dc cable. At this knee, the failure rate changes from an increasing function of the number of operations to a decreasing function. The position of this knee can be used to determine E_m . As E_m is increased, the percentage of cables failing before the knee is reached should increase, whereas the number of operations required should decrease. Therefore, if pretesting cannot be done and if the losses must be kept low, E_m must be kept low. However, if pretesting can be done and if one is willing to lose a large percentage of cables before installation, then E_m can be chosen fairly large. The latter situation is the usual one in much CTR work. Because space is limited, the size of the cable must be small. This forces large E_m , but by accepting 5 to 15% preinstallation testing losses, the actual machine can be made reliable with respect to cable failures.

These considerations determine E_m qualitatively. At present there is no accurate way to determine E_m .

TABLE I

POSITION OF WEIBULL PLOT KNEE FOR CERTAIN CABLES

Cable	Mfgr	Losses to Knee (%)	E_m (MV/m)	V (kv)
17/14 HR	Plastoid	~ 4	39.5	110
17/14 HR	Plastoid	~ 10-20	42.0-47.0	125-145
17 DS	Chester	~ 10	43.0	135
17 DS	Plastoid	~ 25	43.0	135

quantitatively. A research program is needed that would determine the position of the knee as a function of E_m and V. The preceding table gives some information obtained from related experiments.

The losses in Table I are cable losses and not losses per unit length. The termination regions of the cable account for a disproportionate fraction of the total if the losses are analyzed on a per foot basis. Probably this occurs because termination faults have a different distribution than the bulk of the cable. One object of a research program would be to untangle these two distributions. The reason for the large difference between Chester 17 DS and Plastoid 17 DS is not known but could be due to the use of polyethylene from different manufacturers or to different extrusion techniques. A research program probably could clarify this. An investigation to determine how the knee position on the Weibull plot varies with the density of the polyethylene would also be of great interest.

From experience with present cable manufacturers, a choice of E_m at or below 40 MV/m should lead to reasonably low losses and fairly reliable cable. Higher choices of E_m are acceptable only if greater initial cable losses can be tolerated.

IV. DEPENDENCE OF THE OPTIMAL DESIGN ON L , E_m , AND V

Equation (10) has been programmed for solution on a digital computer. Iteration is stopped when the relative difference between $(r_i)_n$ and $(r_i)_{n+1}$ is less than 10^{-6} . When r_i is determined, r_o is determined from Eq. (6), and all other cable design values are easily found in an obvious way.

It is of interest to know how the outside diameter of the optimal cable varies as L , E_m , and V are varied. To this end we choose fixed values of the inner and outer conductor and screen thickness,

as well as fixed jacket thickness, and calculate the cable diameter for various values of L , E_m , and V. Specifically, we choose

$$s_i = 0.000508 \text{ m},$$

$$s_o = 0.000508 \text{ m},$$

$$b_i = 0.0006504 \text{ m},$$

$$b_o = 0.0006504 \text{ m},$$

and

$$\text{jacket thickness} = 0.0009398 \text{ m}.$$

Representative results are given in the families of curves displayed in Fig. 2 through 15. Figures 2 through 7 plot diameter vs inductance for families of E_m values. Figures 8 through 11 show plots of diameter vs inductance for families of constant V. Note that minima occur in these two sets of curves. Because the cost of the cable should be proportional to the volume of material used, these minima should determine the cable design when inductance is unimportant. Figures 12 through 15 display cable diameter vs V for families of constant inductance. These curves are almost, but not quite, straight lines.

ACKNOWLEDGMENTS

I am grateful to E. L. Kemp for suggesting the study.

Fig. 2. Cable diameter vs inductance - curves of constant E_m . $V = 100 \text{ kV}$.

REFERENCES

1. G. P. Boicourt and E. L. Kemp, "The Development of Reliable, High-Voltage, Low-Inductance Cable for Scyllac," in "Proceedings of Symposium on Engineering Problems of Fusion Research," Los Alamos, New Mexico, April 8-11, 1969, Los Alamos Scientific Laboratory report LA-4250 (January 1970).
2. Allen F. Malmberg, "Users Manual NET-2 Network Analysis Program," AD-752600, Braddock, Dunn and McDonald, Inc., El Paso, Texas (October 1973).
3. G. P. Boicourt, "Application of the NET-1 Network Analysis Program to Distributed Circuits," in "Proceedings of Symposium on Engineering Problems of Fusion Research," Los Alamos, New Mexico, April 8-11, 1969, Los Alamos Scientific Laboratory report LA-4250 (January 1970).
4. R. H. Kreuger, "Endurance Tests with Polyethylene Cables," in Proceedings 22nd Session Conference International des Grands Réseaux Electriques a Haute Tension (CIGRE), UNESCO, Paris, August 1968, Group 21, Paper No. 02.

Fig. 3. Cable diameter vs inductance - curves of constant E_m . $V = 120$ kV.

Fig. 4. Cable diameter vs inductance - curves of constant E_m . $V = 140$ kV.

Fig. 5. Cable diameter vs inductance - curves of constant E_m . $V = 200$ kV.

Fig. 6. Cable diameter vs inductance - curves of constant E_m . $V = 300$ kV.

Fig. 7. Cable diameter vs inductance - curves of constant E_m . $V = 380$ kV.

Fig. 8. Cable diameter vs inductance - curves of constant V. $E_m = 36$ MV/m.

Fig. 9. Cable diameter vs inductance - curves of constant V . $E_m = 44$ MV/m.

Fig. 10. Cable diameter vs inductance - curves of constant V . $E_m = 52$ MV/m.

Fig. 11. Cable diameter vs inductance - curves of constant V . $E_m = 60$ MV/m.

Fig. 12. Cable diameter vs V - curves of constant inductance. $E_m = 36$ MV/m.

Fig. 13. Cable diameter vs V - curves of constant inductance. $E_m = 44$ MV/m.

Fig. 14. Cable diameter vs V - curves of constant inductance. $E_m = 52$ MV/m.

Fig. 15. Cable diameter vs V - curves of constant inductance. $E_m = 60$ MV/m.