Theory of Anomalous Parity States in $g_{9/2}$ -shell Nuclei

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Abstract

It is shown that the observed features of the anomalous parity states in $g_{9/2}$ -shell odd-mass nuclei can be qualitatively explained if core-quasi-particle coupling interactions of the type $-\lambda \sum_{\xi} J_{\xi} \cdot j_{qp}$ and $-\chi \sum_{\xi} Q_{\xi} \cdot Q_{qp}$ are taken into account.

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Understanding of the low-lying spectra of odd-mass nuclei in the intermediate mass region of $A = 60-110$ has not progressed as rapidly over the years as that in other regions of the periodic table. The simplest approach¹ has considered the weak coupling of an extra nucleon to the 2^+ vibrational excitation of the core.² The highly successful model involving quasiparticle-phonon coupling³ has unfortunately not worked too well in this mass region, with a particular lack of success in explaining those $g_{9/2}$ nuclei with $41 \leq N$ or $Z \leq 49$. For example, the behavior of the low-lying $5/2^+$ and $7/2^+$ "anomalous parity" states is impossible to describe on this basis. The purpose of the present paper is to show that the trends observed in recent experiments⁴⁻⁷ for the $g_{9/2}$ anomalous parity states can be understood in terms of an extension of the quasiparticle-phonon core coupling model.

According to the shell model, the $g_{9/2}$ neutron and proton odd mass nuclei should have a $9/2^+$ quasiparticle state as either the ground or as a low-lying excited level and this is borne out by the data. The problem of the anomalous parity states started with the observation of a low-energy $7/2^+$ level in these nuclei which can not be a $g_{7/2}$ quasiparticle state as this state should appear a few MeV higher in energy. In several nuclei, a second $7/2^+$ state has been found with $E_{ex} \approx 500$ KeV. An even more complicated behavior is observed for the $5/2^+$ level which occurs at low excitation energies at the beginning of the $g_{9/2}$ shell and moves towards higher excitation energies as the shell fills. Finally, it should be noted that higher angular momentum states have not been found so far at excitation energies below 600 KeV, which is a crucial point for the discussion below.

The first quasiparticle-phonon coupling calculations³ (QPC) were completely unable to explain these phenomena. These calculations were then improved by the inclusion of backward (quasi-hole-phonon) coupling⁸ (EQPC) and by the extension of this latter approach to include the effect of

the static moment of the quadrupole phonon.⁹ The improved calculations predict four low-energy (200-600 KeV) phonon levels of spin and parity $5/2^+$, $7/2^+$, $11/2^+$, and $13/2^+$, with the $9/2^+$ level of the multiplet pushed up from the interaction with the $9/2^+$ quasiparticle. These low-energy high spin states have been searched for experimentally, so far without success. The levels of ^{111}Ag were populated⁵ by β -decay of ^{111m}Pd ($J^\pi = 11/2^-$), and those of ^{73}Ge and ^{99}Tc by Coulomb excitation,^{6,7} all with negative results insofar as low-lying $11/2^+$ or $13/2^+$ states are concerned. These three spectra are summarized in Fig. 1 along with the predictions of EQPC for the positive parity states and the extension of EQPC to include the quadrupole moment. The QPC prediction would have given all five levels of the multiplet at about 900 KeV.¹⁰ The inclusion of the E2 static moment can help explain the movement of the $5/2^+$ level with shell filling. However, it still cannot predict the $5/2^+$ state at low energies as is often observed. Also, at least one high spin state is always predicted to be at low energy, in disagreement with the experimental data.

It is suggested that many of the theoretical difficulties can be overcome by the addition of a dipole-dipole interaction term $-\lambda \mathbf{J}_c \cdot \mathbf{j}_{qp}$ to supplement the static moment interaction term (which is equivalent to adding a term of the form $-\chi \mathbf{Q}_c \cdot \mathbf{Q}_{qp}$, where the subscripts c and qp refer to core phonon and quasiparticle respectively). In effect we are postulating the quasiparticle extension of the core-coupling interaction of Thankappan and True,¹¹

$$H_{int} = -\lambda \mathbf{J}_c \cdot \mathbf{j}_{qp} - \chi \mathbf{Q}_c \cdot \mathbf{Q}_{qp} \quad (1)$$

For the single j -shell case, the quasiparticle ~~wave functions~~ operators

$$\begin{aligned}
 (Q_{8p})_{\mu} &= \frac{1}{\sqrt{5}} \langle j \| Q \| j \rangle \sum_{m, m'} (-i)^{j-m'} (jm j-m' | 2 \mu) (1-2V_j^2) a_{jm}^{\dagger} a_{j m'} \\
 &\quad + (\text{other terms}) \\
 (j_{8p})_{\mu} &= \frac{1}{\sqrt{3}} \langle j \| \vec{r} \| j \rangle \sum_{m, m'} (-i)^{j-m'} (jm j-m' | 1 \mu) a_{jm}^{\dagger} a_{j m'}.
 \end{aligned} \tag{2}$$

The reduced matrix elements were calculated following the notation of de Shalit and Talmi.¹² The expectation value of H_{int} for the quasiparticle-phonon states under consideration is then easily evaluated as,

$$\begin{aligned}
 E_{int} &= -\frac{\lambda}{2} [j(j+1) - 30.75] - \chi \left[\{j(j+1) - 30.75\}^2 \right. \\
 &\quad \left. + \{j(j+1) - 30.75\} - 198 \right] \left[\frac{-5.4 b^2}{3578.3} \langle 2 \| Q \| 2 \rangle (1-2V_{3/2}^2) \right].
 \end{aligned} \tag{3}$$

In this expression, $\langle 2 \| Q \| 2 \rangle = 1.32 Q_{exp}$, b^2 is the usual harmonic oscillator wave function parameter and is given as $0.01A^{1/3}$ barns, and $\chi b^4 = 240/A^{5/3}$ MeV.¹³

The dipole-dipole interaction above has been utilized before,^{1,11} but no microscopic explanation has been previously given. The origin of this term for intermediate mass nuclei can be understood as follows. The existence of strong ground state correlations in the even core suggests the explicit coupling of quasiparticle amplitudes $\langle \psi_a | a_a^{\dagger} | \Psi_v \rangle$ with quasihole amplitudes^{††} $\langle \psi_a | a_{\bar{a}} | \Psi_v \rangle$ (i.e., coupling to backward going amplitudes.⁸) This coupling can be treated self-consistently through the equation of motion method with these matrix elements of the core states which are large (there are only a small number of these) giving rise to the self-consistent potentials.¹⁴ In this way it is possible to generate the large static quadrupole moment of the

^{††} The symbol ψ is used to denote states of the odd-mass nucleus while Ψ designates those of the even core nucleus.

2^+ phonon.¹⁵ An important consequence of this mixing of quasiparticle and quasihole amplitudes based on using a finite number of core states is that angular momentum conservation is violated. An example of this is found in BCS theory where the total number of particles is not conserved. Introducing an effective interaction $-\sum_c \lambda_c J_c \cdot J_{qp}$, where λ_c is defined for each core state separately, permits conserving angular momentum on the average.

The determination of λ_c will involve a very detailed self-consistent calculation. We will avoid this by making an approximate evaluation in the so-called quasirotational limit. In this limit, the only large matrix elements of the core occur for the quasirotational band states with spin and parity $0^+, 2^+, 4^+, \dots$, etc. By using a perturbative (QRPA) solution of the equations and the Elliott specification for the commutator of the quadrupole operator,¹⁶ it can be shown that the energy of these core states depends on the spin, J , as

$$E(J) = \frac{\omega_0}{2} J + G_J J(J+1), \quad (4)$$

where $G_J = -\frac{\lambda_2}{2} + \frac{15}{32} \frac{\lambda_4}{J}$. The parameters λ and b have the same meaning as described above and ω_0 is the QRPA solution for the energy of the 2^+ phonon. If λ_2 is constant for a given band, then a plot of $E(J)/J$ as a function of J yields a straight line with λ_2 obtainable from the slope. Naturally, at least the first three states of the quasirotational band must be known.

The behavior of the various phonon levels can now be investigated using the interaction given by eq. 1. In the numerical calculations, λ was chosen as -0.022 MeV. This was obtained from the experimental data for ^{110}Cd for which the $0^+, 2^+, 4^+$, and 6^+ levels are known. While this value of λ is justified only for ^{111}Ag , we have used it to indicate the trends to be expected. The $-\lambda \sum_c Q_c \cdot Q_{qp}$ term has been previously shown to originate from the $1/2 \sum Q \cdot Q$ force in a microscopic theory.⁹ This justifies identifying λ with the quadru-

pole force strength as noted above. The results of the numerical calculations for the proton $g_{9/2}$ region are presented in Fig. 2a as a function of the occupation probability, $Vg_{9/2}^2$. An experimental quadrupole moment of -0.6 barns was assumed¹⁴ and an A_{av} of 105 was used. The experimental data for ^{99}Tc , ^{105}Rh , and ^{111}Ag are presented as corresponding to $Vg_{9/2}^2 = 0.3, 0.5$, and 0.7 respectively. In Fig. 2b we present the results of a similar calculation for the neutron $g_{9/2}$ region. An A_{av} of 78 and $Q_{exp} = -0.4$ barns were assumed. The experimental data for ^{73}Ge , ^{77}Se and ^{89}Se are presented as corresponding to $Vg_{9/2}^2 = 0.2, 0.4, 0.6$, and 0.8 .

Note that both the movement of the $5/2^+$ level with shell filling and its appearance in some nuclei at low excitation energy is explained. The energy of the $7/2^+$ state is nearly flat as a function of the $g_{9/2}$ occupation probability, and seems to be in between the two experimental curves. The higher spin levels are now predicted to occur at higher energies, in accordance with the data available. The prediction for the $9/2^+$ state is expected to be inadequate since it should be pushed up in energy due to the repulsive effect of the interaction with the $9/2^+$ quasiparticle state.

In order to further test the theoretical suggestions made in this paper, calculations have been made for several specific cases. The calculation for ^{73}Ge with $Q_{exp} = -0.25$ barns and $\lambda = -0.025$ MeV has been shown⁷ to predict the correct experimental features. ^{108}Pd also has known $0^+, 2^+, 4^+$, and 6^+ levels from which a value of $\lambda = -0.008$ is extracted. The spectrum of ^{107}Ag and ^{109}Ag calculated with this value of λ is much the same as before except that the $5/2^+$ levels occur at higher energy, in agreement with the experimental situation for these two nuclei. (Because of the smallness of λ ,

the $11/2^+$ level will be at a lower energy.) Slight differences in λ and Q_{exp} can easily account for the differences between the spectra of these Ag isotopes. As a final check we consider the case of ^{63}Cu . Only the $0^+, 2^+$, and 4^+ states are known in ^{62}Ni and ^{64}Ni , yielding $\lambda = +0.070$ MeV. This has the same sign as assumed in Ref. 10, but would imply that the strengths assumed in this latter calculation for the dipole-dipole and quadrupole-quadrupole force were overestimated.

The experimental data for the $g_{9/2}$ -shell nuclei often reveal at least two $7/2^+$ states. We suggest that the lower of these is to be identified with the phonon states described above. The other $7/2^+$ state usually lies considerably higher in energy with $E_{\text{ex}} \sim 500$ KeV. Kisslinger¹⁷ and more recently Marumori et al.¹⁸ have suggested that $7/2^+$, low-energy states in this mass region may be due to the (j-1)-seniority three state of the $(g_{9/2})^3$ configuration. This could be the explanation of the higher $7/2^+$ state.

To conclude, we have shown that the phonon-quasiparticle coupling scheme can give an adequate explanation of the anomalous parity states in $g_{9/2}$ nuclei provided that a core-coupling interaction of the form given in eq. 1 is added. The dipole-dipole interaction is shown to be a natural consequence of demanding angular momentum conservation on the average. A simple approximation has been suggested to permit the dipole-dipole interaction strength parameter, λ , to be extracted from the experimental data of the neighboring even core nucleus. This interaction provides a simple explanation for the behavior of the $5/2^+$ level as the $g_{9/2}$ shell fills, explains the presence of the lowest $7/2^+$ level, and provides a rationale as to why the high spin $11/2^+$, and $13/2^+$ levels of the phonon multiplet are not usually found below 600-KeV excitation energy. As more data on the quasirotational states of the even core nuclei in this region becomes available, this model can be checked further.

For more detailed calculations, however, it must be cautioned that the above treatment was to first order only in the odd-mass nuclear wave functions. For a detailed comparison between theory and experiment, there are several obvious improvements which can be made. These include the effect of forward⁸ and backward⁹ quasiparticle-phonon coupling, backward coupling through the static phonon quadrupole moment, coupling to $g_{7/2}$ and $2d_{5/2}$ quasiparticles, and a self-consistent treatment.

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Figure Captions

Fig. 1. Comparison of the low-energy positive parity states of ^{73}Ge , ^{99}Tc , and ^{111}Ag with two different theoretical predictions. Further discussion and references for the theoretical spectra are given in the text. The location of the $9/2^+$ states in ^{77}Se and ^{79}Se is not presently known; the best estimates are shown.

Fig. 2. Results of the calculation of the energy of $g_{9/2}$ nuclei as a function of the occupation probability, $Vg_{9/2}^2$. A core phonon energy of 600 KeV was assumed for both the neutron and proton cases. A value of $\lambda = -0.022$ was chosen as described in the text. While the ^{73}Ge spectra are plotted for $Vg_{9/2}^2 = 0.2$, a value of 0.3 is probably more realistic. Occupation probabilities less than 0.2 are unlikely in this region.