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ENCLOSURE (1) Multivariable Controller Design Process

Enclosure (1) describes the techniques used to design
multivariable controls for the NERVA engine.

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CONTROLLER DESIGN PROCESS

I INTRODUCTION

The present NERVA engine is a highly cross-coupled system in that all of the control inputs have a strong effect on all the controlled variables. This cross-coupled nature makes the design of controls on the basis of multiple loops, each loop using one control input to control one engine variable, an undesirable, iterative process. Multivariable control design, based on use of state variable feedback, provides a desirable, one pass design process. Inclusion of a digital computer within the control system provides a compact, flexible means of implementing multivariable control. The following sections present the techniques developed to automate the multivariable controller design process.

II SUMMARY

Techniques have been developed to produce a multivariable controller for the NERVA engine. These techniques emphasize automation of the procedure through use of digital computer programs wherever possible.

The design procedure has three main divisions, system modeling, controller design, and controller simulation and checkout. System models are produced from the reference model (E-1 CAM) which are linear and low order. These models are used to calculate the feedback matrix using a quadratic optimization technique. Results of engine controllability studies are used to design nominal actuator profiles and the corresponding state reference trajectories. State estimation is provided through use

of specific E-1 CAM equations. For design work to date, requirements for a supervisor function have been minimal. However, this function will assume greater importance when controls for different engine operating modes and regions are integrated. The controller elements, demand generator, feedback matrix, and state estimator are mechanized with the E-1 CAM on the hybrid computer and the performance of the controlled system is evaluated against control requirements. Information from the controller evaluation is fed back to modify and improve the controller design.

Digital Programs

The following digital programs are used in the design of the multi-variable controller.

<u>Program</u>	<u>Input</u>	<u>Output</u>
TAF	E-1 CAM Equations	Steady state data Transient data Frequency response data Linear model coefficients
Model	Linear model coefficients State variables to be retained	Reduced linear model coefficients
Quad Op	Reduced linear model coefficients State and control use penalty coefficients	Feedback gain matrix

III DISCUSSION

The multivariable controller design process may be divided into three subsections, system modeling, controller design, and controller

simulation and checkout. These subsections and their interrelationships are shown in more detail in Figure 1. The basis for controller design and evaluation is the engine system as described by a set of nonlinear, first order differential equations, which for the normal operating range is the E-1 CAM (Common Analog Model).

System Modeling

The E-1 CAM, while providing the basis for controller design, is not in itself in a form to be used directly in the design of a controller feedback matrix. A set of linear, first order differential equations in the form

$$\dot{\underline{X}} = \underline{A}\underline{X} + \underline{B}\underline{U}$$

where:

$\underline{X}$ = vector of state variables

$\underline{U}$ = input vector

$\underline{A}, \underline{B}$ = constant coefficient matrices

$$\dot{\underline{X}} = \frac{d}{dt}(\underline{X})$$

is required for the feedback matrix design. Furthermore, this linear model should be low order, containing equations for only those states desired for control and required for a good approximation to the low frequency characteristics of the full model.

A technique has been developed to obtain low order, linear models from the high order, nonlinear E-1 CAM as mechanized with the TAF (Time and Frequency) digital program. The technique, described more fully in Appendix A*, is based on dividing the states (variables defined by first

*Extracted from memorandum 4420:5051M, a System Simplification Technique to Obtain a Controls Analysis Model of the NERVA Engine, E. A. Parziale/
E. F. Dowling, 18 January 1971.

order differential equations) in the E-1 CAM into two groups. The first group ($\underline{X}_1$) contains those state variables dominated by low frequency dynamics and are retained as the state variables of the reduced order model. In general, these variables define the engine feedback measurements to be used for control. The second group ($\underline{X}_2$) contains states dominated by high frequency dynamics. These states are treated as algebraic variables and are eliminated from the reduced model. The steps in the procedure are:

1. Linearization of the full model to get

$$\dot{\underline{X}} = \underline{A}\underline{X} + \underline{B}\underline{U}$$

2. Rearranging and subdividing the full linear model

$$\begin{pmatrix} \dot{\underline{X}}_1 \\ \dot{\underline{X}}_2 \end{pmatrix} = \begin{pmatrix} A_{11} & A_{12} \\ A_{21} & A_{22} \end{pmatrix} \begin{pmatrix} \underline{X}_1 \\ \underline{X}_2 \end{pmatrix} + \begin{pmatrix} B_1 \\ B_2 \end{pmatrix} \underline{U}$$

3. Setting $\dot{\underline{X}}_2 = 0$ (treating the variables, $\underline{X}_2$, as algebraic) and solving for $\underline{X}_2$ in terms of $\underline{X}_1$ and $\underline{U}$

$$\underline{X}_2 = -A_{22}^{-1} (A_{21} \underline{X}_1 + B_2 \underline{U})$$

4. Eliminating $\underline{X}_2$ from the equations for $\dot{\underline{X}}_1$ to get the reduced model

$$\dot{\underline{X}}_1 = (A_{11} - A_{12} A_{22}^{-1} A_{21}) \underline{X}_1 + (B_1 - A_{12} A_{22}^{-1} B_2) \underline{U}$$

Advantages and disadvantages of producing reduced models in this fashion include:

Advantages

1. The dc gains between the inputs ($\underline{U}$) and the retained states ($\underline{X}_1$) exactly match those of the full model. This implies that controllers

designed on the reduced model will exhibit the same degree of low frequency stability and performance when used with the full model as when used with the reduced model.

2. No internal relationships between variables are arbitrarily eliminated. Non intuitive cross coupling effects present in the full model automatically will be retained in the reduced model as low frequency approximations.

Disadvantages

1. Since a unique set of variables to be retained in the reduced model does not exist, some degree of engineering judgment is needed in the selection of these variables. This does have the advantage, however, of allowing other factors, such as the feasibility or the reliability of a particular measurement, to affect the selection of the model variables.

2. The higher the bandwidth desired for the controlled system, the higher must be the frequency to which the reduced model frequency response matches that of the full model. Extending the bandwidth of model agreement requires the addition of more variables to the reduced model, and thus increases complexity.

Controller Design

For nominal engine control (non-malfunction) the controller consists of four main elements, demand generator, feedback matrix, state estimator and supervisor. This configuration is shown in Figure 2.

The demand generator provides reference trajectories for the feedback states and nominal actuator position demand profiles. The nominal

actuator demand profiles are chosen on the basis of engine controllability studies to provide the desired chamber condition sequencing while keeping engine operation within constraints. The state reference trajectories are calculated to be consistent with the nominal actuator demand profiles. Actuator trim signals are produced by the feedback matrix to minimize deviation of the engine states from the reference trajectories.

The key feature that distinguishes the multivariable controller from multiple loop control is the feedback matrix. The feedback matrix trims each actuator demand based on all state errors. This integrated control concept is ideally suited to control of a highly cross-coupled system, such as the NERVA engine. The feedback gain matrix is calculated using a digital program employing an optimal control design technique based on quadratic optimization. The constant gain matrix is calculated, as the asymptotic solution of the Matrix Riccati equation, to minimize deviation of state and control input variables from desired reference values. Input to the digital program consists of the coefficient matrices for the linear engine model and the weighting matrices which determine relative penalties associated with state and control input deviations. The program is more fully defined in Appendix B. *

The state estimator is designed to estimate, from available engine measurements, those variables desired for control but not directly measurable. Although techniques exist for estimating states from noisy measurement signals, to date, state estimation design has been approached more simply by implementing the applicable CAM equations to obtain unmeasurable states from engine measurements.

*Extracted from memorandum 4420:5068M, Quadratic Optimal Control Digital Computer Program, QUAD OP, D. L. Gray, 26 February 1971.

The supervisor monitors engine and controller state and modifies controller configuration and constants to be compatible with desired engine operation and present engine states. To date multivariable design effort has been concentrated in one region of engine operation (the throttle line) so that no supervisory function has been required. Extension of the design effort to all regions of engine operation will entail design of a suitable supervisor.

Controller Simulation and Checkout

The final steps in the controller design process are simulation of the controller in conjunction with the full, nonlinear engine model and evaluation of the controlled system performance. The hybrid computer provides a convenient, powerful means to simulate and evaluate the complete controller. Appendix C* presents in detail the simulation and checkout of a sixth order controller designed to control the engine model between design and throttle points. For this controller design the demand generator and feedback matrix were implemented on the digital portion of the hybrid computer, while the state estimator (two algebraic and one differential equation) were simulated on the analog portion. The flexibility and speed of the hybrid computer simulation allow parametric studies to be carried out to obtain feedback of design information. Primary feedback of information from the simulation is data to help determine the choice of penalty functions used in calculating the feedback matrix and to help shape the open loop actuator profiles.

*Extracted from memorandum 4420:5055M, Design and Simulation Results for the E-1 CAM Revision 5A Throttle Control Using State Variable Feedback, E. A. Parziale/R. F. Schenz, 19 January 1971.

FIGURE 1 CONTROLLER DESIGN PROCESS

FIGURE 2 MULTIVARIABLE CONTROLLER

A SYSTEM SIMPLIFICATION TECHNIQUE TO OBTAIN A CONTROLS ANALYSIS MODEL OF THE NERVA ENGINE

This technique is based on approximating those state variables whose major frequency content is above a band of interest as algebraic variables and reducing the equations to a model possessing the remaining states. The steps for carrying out the technique include: (1) choosing an appropriate set of state variables in terms of which the model is to be characterized, (2) finding from the linearized engine system the frequency responses from these states to their derivatives and from the inputs to the state derivatives, and (3) using the static components of the frequency responses to construct the "A" and "B" matrices of the simplified model.

The technique attempts to match input-output transfers as well as internal transfers for the simplified model and the corresponding states of the engine system. An exact match at DC is insured by nature of the technique and a good match over the frequencies of interest is obtained by properly selecting states which are representative of the system over these frequencies.

The present technique has been applied to the E-1 (Revision 5A) engine model at design point to produce a sixth order simplified model whose transfer functions match quite well with those of the E-1 CAM. Furthermore, a state variable feedback controller designed for the sixth order simplified model successfully controlled the 52nd order CAM along the throttling line, thus substantiating this technique's ability to provide a model with which design results may be transferred to CAM. The transfer function comparisons for the simplified model are also given herein.

DESCRIPTION OF SIMPLIFICATION TECHNIQUE

Given a linearized system in the form

$$\dot{x} = Ax + Bu$$

it is desired to find a lower order approximate model which matches the low frequency characteristics of the system. For controls analysis, it is the low frequencies up to some suitable cutoff point which are of primary interest. Such a simplified model may then be used for design of controls and the results transferred to control of the original system.

Let the state variables be divided into those which contribute primarily to the low frequency characteristics of the system and those whose frequency contents are generally above the band of interest. Now rearrange and partition the system according to subvector x_1 which contains the states of interest and subvector x_2 composed of the high frequency states

$$\begin{bmatrix} \dot{x}_1 \\ \dot{x}_2 \end{bmatrix} = \begin{bmatrix} A_{11} & A_{12} \\ A_{21} & A_{22} \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} + \begin{bmatrix} B_1 \\ B_2 \end{bmatrix} u$$

For the dynamic purposes of the simplified model, the states in x_2 may be considered algebraic variables relative to the states in x_1 . This approximation is equivalent to setting $\dot{x}_2 = \underline{0}$, so that

$$\underline{0} = A_{21} x_1 + A_{22} x_2 + B_2 u$$

solving for x_2

$$x_2 = A_{22}^{-1} \begin{bmatrix} -A_{21} x_1 & -B_2 u \end{bmatrix}$$

Substituting this into the partitioned system equations gives

$$\dot{x}_1 = A_{11} x_1 + A_{12} A_{22}^{-1} \begin{bmatrix} -A_{21} x_1 & -B_2 u \end{bmatrix} + B_1 u$$

or

$$\dot{x}_1 = \underbrace{\begin{bmatrix} A_{11} & -A_{12} A_{22}^{-1} A_{21} \end{bmatrix}}_{A^*} x_1 + \underbrace{\begin{bmatrix} B_1 & -A_{12} A_{22}^{-1} B_2 \end{bmatrix}}_{B^*} u$$

Thus a simplified model of order equal to the number of states in x_1 has been obtained which approximates the low frequency characteristics of the original system.

Two important features of the reduced model should be emphasized.

1. The dc relationships between the inputs and the selected states (x_1) exactly match those of the original model. Given:

$$\begin{bmatrix} \dot{x}_1 \\ \dot{x}_2 \end{bmatrix} = \begin{bmatrix} A_{11} & A_{12} \\ A_{21} & A_{22} \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} + \begin{bmatrix} B_1 \\ B_2 \end{bmatrix} u$$

At steady state $\dot{x}_1 = \dot{x}_2 = 0$, so that in eliminating x_2 from the equation set we get:

$$0 = \begin{bmatrix} A_{11} & -A_{12} A_{22}^{-1} A_{21} \end{bmatrix} x_1 + \begin{bmatrix} B_1 & -A_{12} A_{22}^{-1} B_2 \end{bmatrix} u$$

which is identical to the steady state relationship for the reduced model.

2. Since the reduced model configuration is constrained only as to order, none of the relationships between the selected states (x_1) existing in the original model are arbitrarily eliminated. This feature guarantees that the cross coupling effects present in the original model will also exist in the reduced model (albeit as low frequency approximations). Dependence on intuition in arriving at the reduced model configuration is then limited to choosing the best set of states (x_1).

As an example for the simplification technique, consider the third order linear system given by

$$\begin{bmatrix} \dot{x}_1 \\ \dot{x}_2 \\ \dot{x}_3 \end{bmatrix} = \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ -200 & -230 & -31 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} + \begin{bmatrix} 1.06 \\ -2.1 \\ 22 \end{bmatrix} u$$

Investigation of the frequency responses shows x_3 to have a somewhat higher frequency content than x_1 or x_2 , so that a simplified model will be derived in terms of x_1 and x_2 . From the above partitioned system the needed submatrices are

$$A_{11} = \begin{bmatrix} 0 & 1 \\ 0 & 0 \end{bmatrix} \quad A_{12} = \begin{bmatrix} 0 \\ 1 \end{bmatrix}$$

$$A_{21} = \begin{bmatrix} -200 & -230 \end{bmatrix} \quad A_{22} = -31$$

$$B_1 = \begin{bmatrix} 1.06 \\ -2.1 \end{bmatrix} \quad B_2 = 22$$

Substitution gives the reduced system matrices A^* and B^*

$$A^* = A_{11} - A_{12} A_{22}^{-1} A_{21} = \begin{bmatrix} 0 & 1 \\ -6.45 & -7.42 \end{bmatrix}$$

$$B^* = B_1 - A_{12} A_{22}^{-1} B_2 = \begin{bmatrix} 1.06 \\ -1.39 \end{bmatrix}$$

so that the second order simplified model is

$$\begin{bmatrix} \dot{x}_1 \\ \dot{x}_2 \end{bmatrix} = \begin{bmatrix} 0 & 1 \\ -6.45 & -7.42 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} + \begin{bmatrix} 1.06 \\ -1.39 \end{bmatrix} u$$

To indicate the frequency response comparison between this simplified model and the third order system, consider the transfer

$\frac{x_2(s)}{u(s)}$. For the original system, this transfer is

$$\frac{x_2(s)}{u(s)}_{3rd} = \frac{-1.06 \left(\frac{s}{8.2} + 1 \right) \left(\frac{s}{12.32} + 1 \right)}{\left(\frac{s}{1} + 1 \right) \left(\frac{s}{10} + 1 \right) \left(\frac{s}{20} + 1 \right)}$$

while for the simplified model it is

$$\frac{x_2(s)}{u(s)}_{2nd} = \frac{-1.06 \left(\frac{s}{4.92} + 1 \right)}{\left(\frac{s}{6.41} + 1 \right) \left(\frac{s}{1.01} + 1 \right)}$$

Figure 1 shows the bode plot comparison. As guaranteed by the technique an exact match exists at DC. Also, as may be expected there is deviation between the two transfers at high frequencies.

APPLICATION TO E-1 CAM (REV 5A) AT DESIGN

The present technique was applied to the E-1 CAM (Rev 5A) linearized at design point to reduce the 52nd order model to a sixth order simplified model. While the matrix algebra approach to carrying

x_2/u 3RD
 x_2/u 2ND

out the technique shown in the previous section could have been taken, it was found that with only slight modification the TAF program could more directly perform this simplification.

First a set of states is chosen in terms of which the model is to be characterized. This is done on the basis of engineering judgment. States should be selected which contain the low frequencies of interest and it is very important to choose states which are dispersed and separated from one another in the physical system. As far as the number of states (which is the order of the simplified model) is concerned, this is very much a function of the system under consideration but generally speaking the fewer the number of states retained for the simplified model the larger the difference from the original system characteristics.

For the E-1 CAM (Rev. 5A) at design it was found that a six state simplified model was sufficient to characterize the low frequency characteristics of the system. These states were CI3, the third neutron precursor group, HCl, the enthalpy of core section 1 material temperature, P195, support stem inlet pressure, R9, reflector exit density, T15M, measured chamber temperature, and P15M, measured chamber pressure.

The technique is implemented in the TAF program by setting the selected states at their steady-state levels and producing frequency responses from each selected state to the derivative of each selected state and from each input to the derivative of each selected state. In making these frequency responses the states which are not to be retained in the simplified model are not constrained to their steady-state levels. In effect, the nonretained states are left in a closed-loop configuration

while the selected states are held fixed at steady state. Recalling the basic assumption of this technique, the states not retained are considered algebraic variables, it is desired to incorporate only their static effects in the A and B matrices of the simplified model. This is accomplished by simply using the DC components in the above frequency responses as the coefficients in the A and B matrices. The DC components of transfers from states to state derivatives become the coefficients comprising the A matrix while the DC components of transfers from inputs to state derivatives become the B matrix coefficients.

A very useful byproduct of this approach to carrying out the technique is that inspection of these frequency responses indicates how much the nonretained states affect the low frequency dynamics of the simplified model. For instance, if a transfer function from one state to the derivative of another state had constant gain from DC through the frequencies of interest then the nonretained states in this path exhibit dynamics above those of interest in the simplified model. On the other hand, if a transfer function had significant dynamics in the low frequencies this would imply that a dynamically important state (or states) was neglected in the model and should therefore be added.

As an example, using this approach, consider the third order system given previously. Figure 2 shows its state diagram with x_1 and x_2 fixed at their steady-state levels and x_3 , the state to be discarded, left closed. The required transfer functions are found to be:

FIGURE 2

STATE DIAGRAM OF THIRD ORDER SYSTEM
SYSTEM SIMPLIFICATION EXAMPLE

$$\frac{SX_1(s)}{X_1(s)} = 0$$

$$\frac{SX_1(s)}{X_2(s)} = 1$$

$$\frac{SX_2(s)}{X_1(s)} = \frac{-200}{s + 31}$$

$$\frac{SX_2(s)}{X_2(s)} = \frac{-230}{s + 31}$$

$$\frac{SX_1(s)}{U(s)} = 1.06$$

$$\frac{SX_2(s)}{U(s)} = -2.1 + \frac{22}{s + 31}$$

using the DC components of the above to form the A and B matrix coefficients

$$a_{11} = 0$$

$$a_{12} = 1$$

$$a_{21} = -6.45$$

$$a_{22} = -7.42$$

$$b_1 = 1.06$$

$$b_2 = -1.39$$

so that the simplified model becomes

$$\begin{bmatrix} \dot{x}_1 \\ \dot{x}_2 \end{bmatrix} = \begin{bmatrix} 0 & 1 \\ -6.45 & -7.42 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} + \begin{bmatrix} 1.06 \\ -1.39 \end{bmatrix} u$$

as obtained using the matrix algebra approach.

In the case of applying the simplification technique to the E-1 CAM (Rev. 5A) equations linearized at design point, the TAF program was used to generate transfer functions according to the approach just described. Again, the states were CI3, HCl1, Pl95, R9, Tl5M and Pl5M while the inputs were θ_{BCV} and θ_{SSCV} (θ_D was assumed fixed). The DC components were taken from these, and a sixth order simplified model was obtained. This model is shown in Figure 3 with one additional

FIGURE 3

6th Order Controls Analysis Model
of E-1 CAM Revision 5A at Design Point

$$\dot{x} = A*x + B*u$$

	CI3	HC1	P195	R9	T15M	P15M			θ_{BCV}	θ_{SSCV}			
$\begin{bmatrix} \dot{CI3} \\ \dot{HC1} \\ \dot{P195} \\ \dot{R9} \\ \dot{T15M} \\ \dot{P15M} \end{bmatrix}$							$=$	$\begin{bmatrix} -1.64 & -10.43 & 20.69 & -6589. & 0. & 0. \\ .00033 & -1.314 & .5975 & -536.2 & 0. & 0. \\ .04744 & -3.916 & -3.41 & 5715. & 0. & 0. \\ -.00002 & -.00022 & -.00193 & -24.21 & 0. & 0. \\ .0008 & .359 & 1.382 & -1171. & -.3366 & 0. \\ 0. & .039 & .413 & 113.14 & 0. & -1.00 \end{bmatrix}$	$\begin{bmatrix} CI3 \\ HC1 \\ P195 \\ R9 \\ T15M \\ P15M \end{bmatrix}$	$+$	$\begin{bmatrix} 183.7 & 14.6 \\ 5.1 & .5547 \\ -141.3 & 204.9 \\ -.415 & -.053 \\ 12.42 & 1.381 \\ 3.58 & .147 \end{bmatrix}$	$\begin{bmatrix} \theta_{BCV} \\ \theta_{SSCV} \end{bmatrix}$	

modification. A better high frequency match with the E-1 CAM for P15M and T15M versus θ_{BCV} and θ_{SSCV} transfer functions was obtained by arbitrarily reducing the bandwidths of the P15 and T15 transducers. Since these transducer outputs are decoupled from the other model variables, the bandwidth reductions could be made to produce a better overall match without disrupting internal model relationships.

TRANSFER FUNCTION COMPARISON BETWEEN E-1 CAM AND SIMPLIFIED MODEL

To compare the response characteristics between the simplified model and CAM, closed loop frequency responses from θ_{BCV} and θ_{SSCV} to each of the six selected states were generated for the two models. Figures 4 through 15 show the frequency response comparisons.

In simplifying the 52nd order CAM to a six-state model, it is apparent that some deviations in frequency response do exist, especially at higher frequencies. In general, however, the low frequency characteristics do match. Since a tradeoff exists between increasing the number of states to improve the high frequency match and minimizing the number of states to limit the complexity of the state feedback controller, this simplified model was considered satisfactory for controls analysis. This was substantiated by the fact that a state variable feedback controller designed for the sixth order simplified model successfully controlled the 52nd order CAM along the throttling line.*

* Quadratic optimization was the design technique used in designing a state variable feedback controller for this sixth order model. The results of the design technique and E-1 CAM throttling simulation will be published elsewhere.

PI5M / θ_{BCV} 52ND

PI5M / θ_{BCV} 6TH

W (RAD / SEC) FIGURE 5

P195/ θ BCV 52ND

P195/ θ BCV 6TH

FIGURE 6

R9/θ_{BCV} 6TH ----

(52nd Order Data Not Available)

HCl / θ_{BCV} 52ND _____

HCl / θ_{BCV} 6TH -----

FIGURE 8

CI3 / θ_{BCV} 52ND

CI3 / θ_{BCV} 6TH

ND. 340-L410 DIETZGEN GRAPH PAPER
SEMI-LOGARITHMIC
4 CYCLES X 10 DIVISIONS PER INCH

EUGENE DIETZGEN CO.
MADE IN U. S. A.

T15 / θ_{SSCV} 52ND

T15 / θ_{SSCV} 6TH

FIGURE 10

NO. 340-L410 DIETZGEN GRAPH PAPER
SEMI-LOGARITHMIC
4 CYCLES X 10 DIVISIONS PER INCH

EUGENE DIETZGEN CO.
MADE IN U. S. A.

PI5M / θ_{SSCV} 52ND ———

PI5M / θ_{SSCV} 6TH ———

W (RAD / SEC) FIGURE 11

PI95 / θ_{SSCV}

52ND

PI95 / θ_{SSCV}

6TH

FIGURE 12

HCI / θ_{SSCV} 52ND ———

HCI / θ_{SSCV} 6TH - - - - -

FIGURE 14

CI3/ θ_{SSCV} 52ND _____

CI3/ θ_{SSCV} 6TH - - - - -

W (RAD/SEC) FIGURE 15

DESCRIPTION AND USE OF THE QUADRATIC OPTIMAL CONTROL
DIGITAL COMPUTER PROGRAM, QUAD OP

INTRODUCTION

Quadratic Optimal Control is proving to be one of the most useful of all the results of modern control theory. This has led to a search for better and simpler ways to do the required computations. So far, the most promising method seems to be one first written by A. G. J. MacFarlane (Reference 1). A version of this method was developed at McDonnell Douglas Corporation by several people under NASA Contract NAS 8-18126 (Reference 3). The core of this program has been updated by us, including a more flexible way of inserting the data, and is now used for designing feedback controllers.

THEORETICAL BACKGROUND

The program begins with a set of linear, time invariant differential equations in the state determined form, with a state vector $x(t)$.

$$\dot{x}(t) = Ax(t) + Bu(t) \quad (1)$$

To improve the dynamic response, the control vector $u(t)$ is allowed to be any linear, time invariant function of the state.

$$u(t) = -Gx(t) \quad (2)$$

In order to choose a feedback matrix G which is best in some sense, a penalty function J is defined. It will penalize deviations from the desired state trajectory to be followed and will balance this with a penalty on use of control power. The result is a controller which balances the size of state deviations against the amount of control used. For our purposes, the penalty function can be reduced to the quadratic regulator integral function.

$$J = 1/2 \int_0^{\infty} \left[x^T(t) Q x(t) + u^T(t) R u(t) \right] dt \quad (3)$$

Q and R are the penalty matrices (usually diagonal). Choosing the diagonal elements q_i and r_j is the important practical step for the control designer and is discussed in the next section. Here it suffices to note that q_i penalizes deviations in the i^{th} state $x_i(t)$, and r_j penalizes use of the j^{th} controller $u_j(t)$. The function is quadratic, which means that the square of the deviation is penalized; large deviations are penalized more heavily. Also the time interval is infinite, which leads to a time invariant feedback matrix G.

This penalty function is chosen because it leads to reasonable results from an engineering point of view, and also because closed form solutions are available for the feedback matrix G. This last feature greatly simplifies the solution of the optimal control.

~~The solution used is the following:~~

1. Form the matrix Hamiltonian H.

$$H = \begin{bmatrix} -A & | & B^T R^{-1} B \\ \hline Q & | & A^T \end{bmatrix} \quad (4)$$

2. This is really a Wiener (Reference 5) problem in Spectral Factorization. Let the eigenvalues be λ_i . Half of these will have positive real parts, and we denote the set of positive eigenvalues by λ_{i+} . Let the corresponding set of eigenvectors be P_+ . Finally, we partition the eigenvectors P_+ into two square matrices.

$$P_+ = \begin{bmatrix} P_{1+} \\ P_{2+} \end{bmatrix} \quad (5)$$

Then the steady state matrix of influence functions K is given by

$$P_{1+} K = P_{2+} \quad (6)$$

and the feedback matrix G is

$$G = R^{-1} B^T K \quad (7)$$

Some very efficient and accurate computer subroutines are available for solving equations (4) through (7), based on the original Douglas programming (Reference 3). The computations involved in finding eigenvalues, finding eigenvectors, and solving sets of linear algebraic equations are classical ones and have been treated in great detail elsewhere (e.g. Wilkinson, Reference 6).

There are several possible checks on the accuracy of the solution. Perhaps the most direct check arises from a matrix differential equation (the Riccati equation) which the matrix K must satisfy.

$$\dot{K} = -KA - A^T K - Q + KBR^{-1} B^T K \quad (8)$$

In steady state the $\dot{K}$ term should be zero. This term is computed and printed out under the title "RICSS2 CHECK SOLUTION P-DOT". The smaller this matrix in comparison with the matrix Riccati solution matrix, the more accurate the solution is.

PRACTICAL USE

Assuming the plant model has been decided (A and B matrices fixed), the users job is to select the q_i and r_j weighting factors on the state deviations $x_i(t)$ and control usage $u_j(t)$.

We use a three step method which is outlined below. There are many other possible ways of choosing the weighting factors, but this way seems clear, is straightforward, and works well in practice.

1. Choose an initial set of weighting factors based preferably on maximum allowable deviations x_n for each state and maximum magnitude u_m for each control variable. The initial weighting factors are then given by

$$\begin{aligned} q_i &= (x_{i \text{ max}})^{-2} \\ r_j &= (u_{j \text{ max}})^{-2} \end{aligned} \tag{9}$$

Where maximum values are not known, they can often be estimated. We estimated maximum allowable deviations as 10% of the steady state values for most of the variables. Any values can be chosen, as long as $r_j > 0$ and $q_i \geq 0$.

2. Let the initial weighting factors be Q_o and R_o . That is

$$\begin{aligned} Q_o &= \begin{bmatrix} (x_1 \text{ max})^{-2} & & 0 \\ & \ddots & \\ 0 & & (x_n \text{ max})^{-2} \end{bmatrix} \\ R_o &= \begin{bmatrix} (u_1 \text{ max})^{-2} & & 0 \\ & \ddots & \\ 0 & & (u_m \text{ max})^{-2} \end{bmatrix} \end{aligned}$$

These initial weighting factors may be very approximate, and the next step is to experiment to see about how big to make R in order to use about the right amount of control power. To do this, make several runs using different values of a parameter k, setting

$$\begin{aligned} Q &= Q_o \\ R &= kR_o \end{aligned} \tag{10}$$

The resulting controllers should be evaluated on a simulation of the system to decide on a desirable level for the parameter k . Raising k lowers the use of control, usually by about the square root of the factor raised. That is, multiplying k by four should reduce control gains G by about a factor of two.

3. Some individual adjustment of weighting factors may need to be done to satisfy engineering requirements or just to fine tune the controller. Raising the weighting factor on a state variable will lower the deviations of that variable, usually by about the square root of the weighting factor change. That is, multiplying q_i by 100 should lower the deviations in x_i to about one tenth of their former value. Thus a limit on the deviations of a particular state (or control) variable (or even its derivative) can be satisfied by adjusting the weighting factors.

Final adjustments should be made in tests using the hardware; the physical plant controlled by the actual controllers.

DATA FORMAT

Data input is composed of the four matrices A , B , Q and R . A and B are put in just once, but as many different Q and R matrices can be added as desired, to make a series of runs. Zero elements can be left out of the input.

The input format is

3X, 2I4, E12.6, 2I4, E12.6, 2I4, E12.6, I4

The two integers are I and J , and the next number is $A(I,J) = A_{ij}$. The final integer is used only to signal the end of a matrix. Different integers are used to end the matrices, according to the table below.

End of A matrix;	Integer is	N = dimension of A matrix
B	M =	no. of controllers
		= columns in B matrix
Q		1
R		2

After the first Q and R matrices, only changes in these matrices need be made for successive cases. Thus a card with nothing on it except a 1 in column 67 means "No change in the Q matrix for this case".

Two sample inputs are attached. The first is a 6th order plant with two controllers and one case run. The second is a 6th order plant with four controllers. Two integral trims have been added to make the total plant 8th order, and 10 different cases are run, with various changes in the Q and R matrices. The output from the first sample input is also attached.

REFERENCES

1. MacFarlane, A. G. J., "An Eigenvector Solution of the Optimal Linear Regulator Problem", Journal of Electronics & Control, Vol. 14, No. 6, June 1963.
2. Potter, James, "Matrix Quadratic Solutions", SIAM Journal on Applied Math., Vol. 14, No. 3, May 1966.
3. Vaughan, D. and Blackburn, T., et. al., "Continuation of the Study of Bending Feedback Control System", Vols I & II, Douglas Aircraft Reports DAC 62178 and DAC 62179, February 1968.
4. Technical Session #5 on "Computational Problems in High Order State Space Analysis", 1968 Joint Automatic Control Conference, U. of Michigan, June 1968.
5. Wiener, N., Extrapolation, Interpolation, and Smoothing of Stationary Time Series, M.I.T. Press, 1949 (first published as a classified report to the National Defense Research Committee in wartime).
6. Wilkinson, J. H., The Algebraic Eigenvalue Problem, Clarendon Press, Oxford, 1965.

LSAGD

DATAGE

QUADRATIC OPTIMIZATION PROGRAM, CASE NO. 1

W/ELIM
V-TRAN

THE A MATRIX IS

-0.164000D 01	-0.104300D 02	0.206900D 02	-0.658900D 04	0.0	0.0
0.330000D-03	-0.131400D 01	0.597500D 00	-0.536200D 03	0.0	0.0
0.474400D-01	-0.391600D 01	-0.341000D 01	0.571500D 04	0.0	0.0
-0.200000D-04	-0.220000D-03	-0.193000D-02	-0.242100D 02	0.0	0.0
0.800000D-03	0.359000D 00	0.138200D 01	-0.117100D 04	-0.336600D 00	0.0
0.0	0.390000D-01	0.413000D 00	0.113140D 03	0.0	-0.100000D 01

THE B MATRIX IS

0.183700D 03	0.146000D 02
0.510000D 01	0.554700D 00
-0.141300D 03	0.204900D 03
-0.415000D 00	-0.530000D-01
0.124200D 02	0.138100D 01
0.358000D 01	0.147000D 00

THE Q MATRIX IS

0.246000D-07	0.0	0.0	0.0	0.0	0.0
0.0	0.116000D-04	0.0	0.0	0.0	0.0
0.0	0.0	0.753000D-06	0.0	0.0	0.0
0.0	0.0	0.0	0.194000D 01	0.0	0.0
0.0	0.0	0.0	0.0	0.554000D-03	0.0
0.0	0.0	0.0	0.0	0.0	0.494000D-02

THE R MATRIX IS

0.100000D 00	0.0
0.0	0.100000D 00

APPENDIX B

HAMILTONIAN MATRIX

0.164000D 01	0.104300D 02	-0.206900D 02	0.658900D 04	0.0	0.0	0.339589D 06	0.944969D 04
-0.330000D-03	0.131400D 01	-0.597500D 00	0.536200D 03	0.0	0.0	0.944969D 04	0.263177D 03
-0.474400D-01	0.391600D 01	0.341000D 01	-0.571500D 04	0.0	0.0	-0.229653D 06	-0.606972D 04
0.200000D-04	0.220000D-03	0.193000D-02	0.242100D 02	0.0	0.0	-0.770093D 03	-0.214590D 02
-0.600000D-03	-0.359000D 00	-0.138200D 01	0.117100D 04	0.336600D 00	0.0	0.230172D 05	0.641080D 03
0.0	-0.390000D-01	-0.413000D 00	-0.113140D 03	0.0	0.100000D 01	0.659792D 04	0.183375D 03
0.246000D-07	0.0	0.0	0.0	0.0	0.0	-0.164000D 01	0.330000D-03
0.0	0.116000D-04	0.0	0.0	0.0	0.0	-0.104300D 02	-0.131400D 01
0.0	0.0	0.753000D-06	0.0	0.0	0.0	0.206900D 02	0.597500D 00
0.0	0.0	0.0	0.194000D 01	0.0	0.0	-0.658900D 04	-0.536200D 03
0.0	0.0	0.0	0.0	0.554000D-03	0.0	0.0	0.0
0.0	0.0	0.0	0.0	0.0	0.494000D-02	0.0	0.0

-0.229653D 06	-0.770093D 03	0.230172D 05	0.659792D 04
-0.606972D 04	-0.214590D 02	0.641080D 03	0.183395D 03
0.619497D 06	0.477798D 03	-0.147198D 05	-0.475734D 04
0.477798D 03	0.175034D 01	-0.522749D 02	-0.149349D 02
-0.147198D 05	-0.522749D 02	0.156164D 04	0.446666D 03
-0.475734D 04	-0.149349D 02	0.446666D 03	0.128380D 03
0.474400D-01	-0.200000D-04	0.800000D-03	0.0
-0.391600D 01	-0.220000D-03	0.359000D 00	0.390000D-01
-0.341000D 01	-0.193000D-02	0.138200D 01	0.413000D 00
0.571500D 04	-0.242100D 02	-0.117100D 04	0.113140D 03
0.0	0.0	-0.336600D 00	0.0
0.0	0.0	0.0	-0.100000D 01

POLYNOMIAL COEFFICIENTS

12 ROOTS

INPUT	OUTPUT
0.100000000D 01	0.100000000D 01
-0.60790716D-14	-0.71054274D-14
-0.58437074D 03	-0.58437074D 03
-0.24011866D-10	-0.22907898D-10
0.15250430D 05	0.15250430D 05
0.39181749D-08	0.33007784D-08
-0.96153117D 06	-0.96153117D 06
0.20102847D-07	0.20576408D-07
0.58850338D 07	0.58850338D 07
0.83541807D-07	0.85841748D-07
-0.12462691D 08	-0.12462691D 08
-0.10178467D-06	-0.10471558D-06
0.86821814D 07	0.86821814D 07

ROOTS

	REAL	IMAG
1	0.23668064D 02	0.0
2	0.49218559D 01	-0.39219301D 01
3	0.49218559D 01	0.39219301D 01
4	0.15803351D 01	-0.14857870D 00
5	0.15803351D 01	0.14857870D 00
6	0.12475794D 01	0.0
7	-0.12475794D 01	0.0
8	-0.15803351D 01	-0.14857870D 00
9	-0.15803351D 01	0.14857870D 00
10	-0.49218559D 01	-0.39219301D 01
11	-0.49218559D 01	0.39219301D 01
12	-0.23668064D 02	0.0

THE NORMALIZED CLOSED LOOP EIGENVECTORS ARE

LENGTH	PHASE	ANGLE	LENGTH	PHASE	ANGLE	LENGTH	PHASE	ANGLE
--------	-------	-------	--------	-------	-------	--------	-------	-------

0.895219D 00	0.0		0.975303D 00	0.0		0.975303D 00	0.0	
0.489473D-01	0.0		0.281633D-01	0.783365D 01		0.281633D-01	-0.783365D 01	
0.430763D 00	0.180000D 03		0.211203D 00	-0.131380D 03		0.211203D 00	0.131380D 03	
0.156021D-02	0.0		0.421516D-04	0.835730D 02		0.421516D-04	-0.835730D 02	
0.103091D 00	0.0		0.560782D-01	0.157715D 02		0.560782D-01	-0.157715D 02	
0.103033D-04	0.180000D 03		0.154635D-01	0.457723D 01		0.154635D-01	-0.457723D 01	

0.999923D 00	0.0		0.999142D 00	0.0		0.999923D 00	0.0	
0.111262D-01	-0.112439D 03		0.345329D-01	0.180000D 03		0.111262D-01	0.112439D 03	
0.350032D-02	-0.175215D 03		0.970215D-02	0.0		0.350032D-02	0.175215D 03	
0.624801D-05	-0.153928D 03		0.998467D-05	0.0		0.624801D-05	0.153928D 03	
0.365485D-02	-0.159383D 03		0.190910D-01	0.0		0.365485D-02	0.159383D 03	
0.214166D-02	0.260631D 02		0.796247D-02	0.180000D 03		0.214166D-02	-0.260631D 02	

RICSS2 CHECK SOLTUION, P-DOT

0.161620D-19	-0.235664D-20	0.631801D-20	-0.106070D-17	-0.375871D-20	0.102570D-19
-0.347185D-17	-0.137299D-18	0.668150D-18	0.384350D-16	0.267451D-18	-0.853809D-18
0.130999D-17	-0.332844D-18	-0.173112D-17	0.120943D-15	-0.250383D-17	-0.233781D-18
-0.963085D-15	-0.884472D-16	-0.620326D-15	0.788675D-13	-0.290642D-14	0.350414D-15
0.591079D-17	-0.114577D-17	0.124372D-17	0.192087D-15	-0.175717D-17	0.416740D-18
-0.962131D-17	-0.139803D-17	-0.163139D-17	0.400721D-15	-0.135729D-16	0.0

THE MATRIX RICATTI SOLUTION

0.883590D-08	-0.983366D-07	0.793815D-07	0.161233D-05	0.288255D-06	-0.375155D-06
-0.983366D-07	0.962925D-05	-0.296337D-05	-0.741544D-03	-0.450030D-05	-0.151610D-04
0.793815D-07	-0.296337D-05	0.868341D-05	0.633248D-03	0.284864D-04	0.302742D-04
0.161233D-05	-0.741544D-03	0.633248D-03	0.545287D 00	-0.509580D-02	0.242410D-01
0.288255D-06	-0.450030D-05	0.284864D-04	-0.509580D-02	0.218841D-03	-0.229761D-03
-0.375155D-06	-0.151610D-04	0.302742D-04	0.242410D-01	-0.229761D-03	0.173300D-02

THE FEEDBACK GAIN MATRIX

-0.852701D-04	0.647340D-02	-0.102811D-01	-0.295765D 01	0.150879D-03	-0.111335D 00
0.165972D-03	-0.572431D-02	0.178897D-01	0.969906D 00	0.637710D-01	0.484198D-01

APPENDIX A

Column

	7	11	23	27	31	43	47	51	63	67
1	1	1-1.64000D 00	1	2-1.04300D+01	1	3 2.06900D 01				
2	1	4-5.58900D 03	2	1 3.30000D-04	2	2-1.31400D+00				
3	2	3+5.97500D-01	2	4-5.36200D+02	3	1+4.74400D-02				
4	3	2-3.91600D+00	3	3-3.41000D+00	3	4+5.71500D+03				
5	4	1-2.00000D-05	4	2-2.20000D-04	4	3-1.93000D-03				
6	4	4-2.42100D+01	5	1+8.00000D-04	5	2+3.59000D-01				
7	5	3+1.38200D+00	5	4-1.17100D+03	5	5-3.36600D-01				
8	6	2+3.90000D-02	6	3+4.13000D-01	6	4+1.13140D+02				
9	6	6-1.00000D+00								6
	1	1 -.18370D+03	1	2 -.14600D+02	2	1 .51000D+01				
	2	2 .55470D 00	3	1 -.14130D+03	3	2 .20490D+03				
	4	1 -.41500D 00	4	2 -.53000D-01	5	1 .12420D+02				
	5	2 .13810D+01	6	1 .35800D+01	6	2 .14700D 00				2
	1	1 .24600D-07	2	2 .11600D-04	3	3 .75300D-06				
	4	4 .19400D+01	5	5 .55400D-03	6	6 .49400D-02				1
	1	1 .1 D 00	2	2 .1 D 00						2

Input #1

1	1	-0.1640E 01	2	1	0.4200E-03	3	1	0.4827E-01		
4	1	-0.1800E-04	5	1	0.1000E-02	6	1	0.2200E-03		
1	2	-0.1067E 02	2	2	-0.1321E 01	3	2	-0.4002E 01		
4	2	-0.1640E-03	5	2	0.3430E 00	6	2	0.3430E-01		
1	3	0.2033E 02	2	3	0.5862E 00	3	3	-0.3535E 01		
4	3	-0.2000E-02	5	3	0.1357E 01	6	3	0.4070E 00		
1	4	-0.5725E 04	2	4	-0.5108E 03	3	4	0.5640E 04		
4	4	-0.2436E 02	5	4	-0.1111E 04	6	4	0.1295E 03		
1	5	0.0	2	5	0.0	3	5	0.0		
4	5	0.0	5	5	-0.3366E 00	6	5	0.0		
1	6	0.0	2	6	0.0	3	6	0.0		
4	6	0.0	5	6	0.0	6	6	-0.1000E 01		
7	5	.1 E 01	8	6	.1 E 01					8
1	1	0.1475E 03	2	1	0.4079E 01	3	1	-0.1247E 03		
4	1	-0.3445E 00	5	1	0.9960E 01	6	1	0.2877E 01		
1	2	0.1266E 02	2	2	0.4046E 00	3	2	0.4694E 02		
4	2	-0.1776E-01	5	2	0.9654E 00	6	2	0.2100E 00		
1	3	.1011 E 02	2	3	.3887 E 00	3	3	.1514 E 03		
4	3	-.381 E-01	5	3	.971 E 00	6	3	.964 E-01		
1	4	-0.1107E 01	2	4	-0.4341E-01	3	4	-0.1802E 01		
4	4	0.5641E-02	5	4	-0.1090E 00	6	4	-0.9650E-02		4
1	1	.246 D-07	2	2	.116 D-04	3	3	.753 D-06		
4	4	.194 D+01	5	5	.554 D-03	6	6	.494 D-02		
7	7	.2 D-04	8	8	.494 D-03					
1	1	.1 D 00	2	2	.1 D+10	3	3	.1 D 00		1
4	4	.1 D 00								2
5	5	.2 D-02								1
										2
5	5	.5 D-02								1
										2
5	5	.2 D-01								1
										2
1	1	.1 D-06	5	5	.554 D-03					1
										2
1	1	.5 D-06								1
										2
7	7	.1 D-03	8	8	.1 D-03	1	1	.246 D-07		1
										2
1	1	.5 D-01	3	3	.5 D-01	4	4	.5 D-01		1
										2
1	1	.25 D-01	3	3	.25 D-01	4	4	.25 D-01		1
										2
1	1	.1 D-01	3	3	.1 D-01	4	4	.1 D-01		1
										2

Input #2

CONTROLLER SIMULATION AND EVALUATIONMODEL DEVELOPMENT

In order to determine a low order controls analysis model of the E-1 CAM, a system simplification technique was developed*. The technique produces a low frequency simplified model of the engine whose input-output and internal transfer functions approximate those of the engine over the frequencies of interest. The simplified model is characterized in terms of selected state variables determined to be important in representing low frequency system dynamics.

The TAF program was modified to carry out the simplification technique and a sixth order model of the E-1 CAM (Rev. 5A) equations linearized at design point was produced. The six state variables in the model were CI3 - the third neutron precursor group, HCl - enthalpy of core section 1 material, P195 - support stem inlet pressure, R9 - reflector exit density, T15M - measured chamber temperature, and P15M - measured chamber pressure. The inputs were θ_{BCV} - bypass control valve position, and θ_{SSCV} - support system control valve (θ_D - drum position was held fixed).

Figure 1 gives this sixth order simplified model in the state variable format, $\dot{x} = Ax + Bu$. In order to compare the response characteristics between CAM and this model, frequency responses were obtained from the inputs to each of the selected state variables for both models. Appendix A contains these frequency response comparisons.

*A detailed description of this system simplification technique is given in Appendix A.

6TH ORDER CONTROLS ANALYSIS MODEL
OF E1 CAM REVISION 5A AT DESIGN POINT

-2-

$$\dot{X} = A^* X + B^* u$$

$$\begin{bmatrix} \dot{CI3} \\ \dot{HC1} \\ \dot{PI95} \\ \dot{R9} \\ \dot{T15M} \\ \dot{P15M} \end{bmatrix} = \begin{bmatrix} CI3 & HC1 & PI95 & R9 & T15M & P15M \\ -1.64 & -10.43 & 20.69 & -6589. & 0. & 0. \\ .00033 & -1.314 & .5975 & -536.2 & 0. & 0. \\ .04744 & -3.916 & -3.41 & 5715. & 0. & 0. \\ -.00002 & -.00022 & -.00193 & -24.21 & 0. & 0. \\ .0008 & .359 & 1.382 & -1171. & -.3366 & 0. \\ 0. & .039 & .413 & 113.14 & 0. & -1.00 \end{bmatrix} \begin{bmatrix} CI3 \\ HC1 \\ PI95 \\ R9 \\ T15M \\ P15M \end{bmatrix} + \begin{bmatrix} \Theta_{BCV} & \Theta_{SSCV} \\ 183.7 & 14.6 \\ 5.1 & .5547 \\ -141.3 & 204.9 \\ -.415 & -.053 \\ 12.42 & 1.381 \\ 3.58 & .147 \end{bmatrix} \begin{bmatrix} \Theta_{BCV} \\ \Theta_{SSCV} \end{bmatrix}$$

FIGURE 1

CONTROL CONFIGURATION

Figure 2 shows the control configuration assumed for the design of a state variable feedback control system for the E-1 CAM (Rev. 5A) along the throttling line. This is a specific case and limited version of the general NERVA control system block diagram shown previously (3/70) and includes only the essential functions of demand generation, state estimation, and control. The other control system functions such as malfunction detection, adaption, and data processing will be incorporated in the near future.

The demand generator provides the state reference vector which in the case of a throttling maneuver is a linear interpolation of the steady-state values of the state variables between design point and throttle point. The chamber temperature and pressure reference variables are merely the demands: 4250°R for temperature, and a 50 psi/sec ramp from 450 to 270 for pressure. The need for static reference levels of the states arises from the fact that the control law was designed on the basis of a linearized engine model and should, therefore, operate on only the differential state variables. By subtracting the static reference levels from the total state feedback vector (static plus dynamic), the necessary differential state vector is obtained. Since the control law operates on the differential state vector to produce a differential or dynamic control vector, it is necessary to also add an input reference vector since the nonlinear engine model, not a linearized model, is being controlled. The demand generator provides this input reference vector which, again, is merely a linear interpolation of the steady-state values of the input actuator positions between design point and throttle point.

FIGURE 2
CONTROL SYSTEM CONFIGURATION
FOR DIGITAL CONTROL OF E1 CAM (5A)
USING STATE FEEDBACK

The state variable feedback control law consists of a matrix of feedback gains which multiplies the differential state vector to give the dynamic control vector. The control law was determined using an optimal control design technique in which the design criterion is to minimize a quadratic performance index. This technique will be described in another section.

In the general case of controlling the engine over the entire map and throughout lifetime, this matrix of gains would be modified by a supervisory function. However, for the present situation of control along the throttle line, it was found that a set of fixed gains was quite satisfactory.

The steady-state trim controller indicated in Figure 2 operates on the temperature and pressure errors to null these in the steady-state. If the engine equations were modeled precisely and were stationary throughout life, there would be no need for the trim controller. Since this is not the case, such a control function is necessary. For the particular situation of throttle control, this consists of a pair of digital integrators with appropriate gain constants. In the general case of control over the entire map and throughout life the steady-state trim controller would be composed of additional logic to compensate for static error in all situations.

The feedback gain matrix used for the E-1 throttle run was designed on the sixth order simplified model of the 52nd order E-1 CAM linearized at design point. The details of the model simplification technique have been described previously. Recalling the six states

characterizing this model: CI3, HCl, P195, R9, T15M, P15M, it is apparent that some of these states are not measurable variables.

STATE ESTIMATION

The state estimator serves to estimate inaccessible states from system variables which can be measured. One such state is CI3, the third neutron precursor group, which can be estimated from a measurement of nuclear power using the dynamic equation relating the two. CI3 may be obtained from a measurement of nuclear power via the difference equation:

$$CI3(N) = \begin{bmatrix} 1 & -1.65 \Delta T \end{bmatrix} CI3(N-1) + \begin{bmatrix} 110.3 \Delta T \end{bmatrix} S_N(N-1)$$

where the initial estimate of CI3 is obtained through the steady state equilibrium equation, $CI3(0) = 66.855 S_N(0)$.

Another state which must be estimated is HCl, the enthalpy of core section 1 material temperature. HCl is uniquely related to ϕ_{C1} through the enthalpy function of the core material, so that this state may be algebraically estimated using:

<u>HCl (BTU/LB)</u>	<u>ϕ_{C1} ($^{\circ}R$)</u>
0.	0.
106.	1000
203.	1500
316.	2000
433.	2500
685.	3500
1094.	5000
1379.	6000

Finally, R9, hydrogen density at the reflector exit, can be estimated using measurements of T9 and P9 and the gas law relating the variables. Along the throttling line the ideal gas law was found to be suitable so that R9 may be estimated with the algebraic equation:

$$R9 = \frac{P9}{5.32 T9}$$

CONTROL LAW DESIGN TECHNIQUE

The feedback gain matrix which operates on the differential state vector to provide dynamic control was determined using an optimal control design approach. Since the design criterion in the particular approach used is to minimize a quadratic performance index along some trajectory in state space, the approach is termed quadratic optimization.

Given a set of linear time invariant differential equations (the "plant")

$$\dot{x} = Ax + Bu$$

where x is the "state" of the plant and u is the "control" vector, the procedure consists of:

1. Choosing a set of weighting numbers q_i and r_i for the penalty function (quadratic performance index)

$$J = \frac{1}{2} \int_0^{\infty} [x' Qx + U' Ru] dt$$

where the weighting matrices were chosen here to be

$$Q = \begin{bmatrix} q_1 & & 0 \\ & q_2 & \\ 0 & & \ddots \\ & & & q_n \end{bmatrix} \quad \text{and} \quad R = \begin{bmatrix} r_1 & & 0 \\ & r_2 & \\ 0 & & \ddots \\ & & & r_m \end{bmatrix}$$

In the above, q_i is the penalty per unit of error in the i th state variable x_i and r_i is the penalty per unit use of the i th control variable u_i (where the dynamic control vector is given by $u = -Hx$ and G is the feedback gain matrix to be found).

2. The computer program then finds the feedback gain matrix G which minimizes the penalty function J .

3. The design procedure is now to pick the weighting numbers to produce a "good" controller, satisfying the design requirements. Various rules and aides are available for carrying this out. Some of these rules of thumb are:

a. Raising the weighting number q_i lowers the excursions of the state variable x_i .

b. Raising the weighting number r_i lowers the use of the control variable u_i .

c. By changing one number, or one fixed combination of numbers at a time a root locus similar to the one used in classical control is produced. This makes available some of the design feel and rules of classical control.

The above procedure was employed with the sixth order simplified model of CAM at design point. Several gain matrices were generated each with different weighting coefficients. In general, it was found that the resulting feedback gain matrices of the sixth order design were satisfactory in controlling the E-1 CAM. In fact, it was found that a fixed feedback gain matrix based on the design carried out on the simplified design point model was quite adequate for control of the nonlinear E-1 CAM along the whole throttle line. The simulation results for these designs are given in the next section.

A modification to this design approach was also tried in which a pair of digital integrators (one for temperature error and one for pressure error) were included as additional states to incorporate into the design determination of the steady state trim controller gains. These results are also shown in the succeeding section.

SIMULATION RESULTS

a. TAF Transient Runs

Two five-second transient runs were made on the E-1 CAM Revision 5A TAF simulation. One was a 50 psi/sec ramp in chamber pressure

from design point to the throttle point. The other was the transient from throttle point to design point. The chamber temperature and pressure transients for the two runs are shown in Figures 3.1 and 3.2. The dashed lines indicate the tolerances as shown in S102, December 1970.

b. Hybrid Simulation Runs

Three different state variable controllers were checked on the nonlinear hybrid simulation of E-1 CAM Revision 6.1. The pump discharge valve was maintained at 100% in order to make the model more closely resemble the 5A revision for which the controllers were designed. The rate limit on the SSCV was increased from $\pm 10\%/sec$ to $\pm 14\%/sec$ for the controller evaluation runs. However, once the PDCV is integrated into the controller, the required SSCV rate will be less than $14\%/sec$.

The results of the hybrid simulation tests of the controllers are recorded in hybrid run numbers 010971-101 and -102, also 0011471-101 through -109. The results are summarized below.

Controller C5A6D.1
Description State Feedback control without integral trim
Performance See record runs 010971-101E

	Max T15M Error	Max P15M Error
Design to Throttle	-40°R	-12 psi
Throttle to Design	-90°R	+16 psi

Controller C5A8D.1
Description State Feedback control with integral trim
Performance See record runs 011471-104E

	Max T15M Error	Max P15M Error
Design to Throttle	-50°R	- 9 psi
Throttle to Design	-120°R	+10 psi

EUGENE DIETZGEN CO.
MADE IN U. S. A.

NO. 340-10 1/4 DIETZGEN GRAPH PAPER
10 X 10 PER HALF INCH

THRUST CHAMBER PRESSURE, PSIA

THRUST CHAMBER TEMPERATURE, °R

FIGURE 2 1

EUGENE DIETZEN CO.
MADE IN U. S. A.

NO. 340-10% DIETZEN GRAPH PAPER
10 X 10 PER HALF INCH

THRUST CHAMBER TEMPERATURE, °R

THRUST CHAMBER PRESSURE, PSIA

TIME IN SECONDS

FIGURE 3.2

Controller C5A8D.2

Description State Feedback control with integral trim - penalty coefficients on T15M and P15M made larger than on C5A8D.1 causing higher feedback gains on T15M and P15M. However, this resulted in somewhat oscillatory behavior whereas C5A8D.1 was damped.

Performance See record runs 011471-107E

	Max T15M	Max P15M
Design to Throttle	+70°R	-8 psi
Throttle to Design	-140°R	+8 psi

Figures 3.3a, 3.3b, and 3.3c contain the output of the controller design program for C5A6D.1. Figures 3.4a - c and 3.5a - c show the output for controllers C5A8D.1 and C5A8D.2 respectively. In the Figure C printout for each controller's design is given the feedback gain matrix which, as mentioned previously, multiplies the differential state vector to produce the dynamic control vector. The feedback gain matrix is printed out in such a way that the control vector is given by

$$\begin{bmatrix} \underline{u} \\ \theta_{BCV_D} \\ \theta_{SSCV_D} \end{bmatrix} = - \begin{bmatrix} \underline{H} \\ h_{11} & h_{12} & h_{13} & h_{14} & h_{15} & h_{16} \\ h_{21} & h_{22} & h_{23} & h_{24} & h_{25} & h_{26} \end{bmatrix} \begin{bmatrix} \underline{x} \\ CI3 \\ HC1 \\ P195 \\ R9 \\ T15M \\ P15M \end{bmatrix}$$

THE A MATRIX IS

-0.164000D 01	-0.104300D 02	0.206900D 02	-0.658900D 04	0.0	0.0
0.330000D-03	-0.131400D 01	0.597500D 00	-0.535200D 03	0.0	0.0
0.474400D-01	-0.391600D 01	-0.341000D 01	0.571500D 04	0.0	0.0
-0.200000D-04	-0.220000D-03	-0.193000D-02	-0.242100D 02	0.0	0.0
0.800000D-03	0.359000D 00	0.134200D 01	-0.117100D 04	-0.336600D 00	0.0
0.0	-0.398000D-01	0.413000D-00	0.113140D-03	0.0	-0.100000D-01

THE B MATRIX IS

0.183700D 03	0.146000D 02
0.510000D 01	0.554700D 00
-0.141300D 03	0.204900D 03
-0.415000D-00	-0.530000D-01
0.124200D 02	0.138100D 01
0.358000D 01	0.147000D 00

THE Q MATRIX IS

0.246000D-07	0.0	0.0	0.0	0.0	0.0
0.0	0.116000D-04	0.0	0.0	0.0	0.0
0.0	0.0	0.753000D-06	0.0	0.0	0.0
0.0	0.0	0.0	0.194000D 01	0.0	0.0
0.0	0.0	0.0	0.0	0.554000D-03	0.0
0.0	0.0	0.0	0.0	0.0	0.494000D-02

THE R MATRIX IS

0.100000D 00	0.0
0.0	0.100000D 00

FIGURE 3.3a

HAMILTONIAN MATRIX

0.1640000 01	0.1040000 02	-0.2000000 02	0.0000000 04	0.0	0.0	0.3300000 06	0.0440000 04
-0.1400000-03	0.1310000 01	-0.5000000 00	0.1000000 03	0.0	0.0	0.0440000 04	0.0440000 03
-0.4740000-01	0.3000000 01	0.3000000 01	-0.1000000 04	0.0	0.0	-0.2200000 06	-0.0000000 04
0.2000000-04	0.2000000-03	0.1000000-02	0.2000000 02	0.0	0.0	-0.7000000 03	-0.2140000 02
-0.0000000-03	-0.3500000 00	-0.1000000 01	0.1100000 04	0.3000000 00	0.0	0.2000000 05	0.0440000 03
0.0	-0.3000000-01	-0.1000000 00	-0.1000000 04	0.0	0.1000000 01	0.0500000 04	0.1000000 03
0.2400000-07	0.0	0.0	0.0	0.0	0.0	-0.1600000 01	0.3000000-03
0.0	0.1000000-04	0.0	0.0	0.0	0.0	-0.1000000 02	-0.1000000 01
0.0	0.0	0.7000000-05	0.0	0.0	0.0	0.0600000 02	0.5000000 00
0.0	0.0	0.0	0.1000000 01	0.0	0.0	-0.0500000 04	-0.5000000 03
0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0
0.0	0.0	0.0	0.0	0.0	0.4000000-02	0.0	0.0

-0.2200000 06	-0.7000000 03	0.2000000 05	0.0500000 04
-0.0000000 04	-0.2140000 02	0.0440000 03	0.1000000 03
0.0440000 03	0.0440000 04	-0.2200000 06	-0.0000000 04
0.4740000 01	0.1000000 04	-0.2000000 02	-0.1000000 03
-0.1400000-03	-0.3500000 00	-0.1000000 01	0.1100000 04
0.1000000-04	-0.3000000-01	-0.1000000 00	-0.1000000 04
0.2400000-07	0.0	0.0	0.0
0.0	0.1000000-04	0.0	0.0
0.0	0.0	0.7000000-05	0.0
0.0	0.0	0.0	0.1000000 01
0.0	0.0	0.0	0.0
0.0	0.0	0.0	0.4000000-02
0.0	0.0	0.0	0.0

POLYNOMIAL COEFFICIENTS

12 ROOTS

INPUT	OUTPUT
0.100000000 01	0.100000000 01
-0.607007160-14	-0.710542740-14
-0.584370740 03	-0.584370740 03
-0.240118000-10	-0.229078980-10
0.152504300 05	0.152504300 05
0.301817400-03	0.300077840-03
-0.961531170 06	-0.961531170 06
0.201028470-07	0.200764080-07
0.508503380 07	0.508503380 07
0.335419070-07	0.335417480-07
-0.124626910 08	-0.124626910 08
-0.101764070-06	-0.104715560-06
0.868218140 07	0.868218140 07

ROOTS

	REAL	IMAG
1	0.124757940 01	0.0
2	-0.124757940 01	0.0
3	0.158033510-01	-0.148578700-00
4	0.158033510 01	0.148578700 00
5	-0.158033510 01	-0.148578700 00
6	-0.158033510 01	0.148578700 00
7	0.492185590-01	-0.392193010 01
8	0.492185590 01	0.392193010 01
9	-0.492185590 01	-0.392193010 01
10	-0.492185590 01	0.392193010 01
11	-0.236680640-02	0.0
12	0.236680640 02	0.0

THE NORMALIZED CLOSED LOOP EIGENVECTORS ARE

LENGTH	PHASE-ANGLE	LENGTH	PHASE-ANGLE	LENGTH	PHASE-ANGLE	LENGTH	PHASE-ANGLE
--------	-------------	--------	-------------	--------	-------------	--------	-------------

0.9991420 00	0.0	0.9999230 00	0.0	0.9999230 00	0.0	0.9753030 00	0.0
0.3453290-01	0.1800000-03	0.1112620-01	-0.128640 03	0.1112620-01	-0.128640 03	0.2816370-01	0.1740370 03
0.9702150-02	0.0	0.3000320-02	-0.1643600 03	0.3590320-02	0.1643600 03	0.2112030 00	0.3482260 02
0.9384670-05	0.0	0.6246010-05	0.1743530 03	0.6246010-05	-0.1743530 03	0.4215160-04	-0.1102240 03
0.1903100-01	0.0	0.3054350-02	0.1700000 03	0.3054350-02	-0.1700000 03	0.5607820-01	-0.1780250 03
0.7962470-02	0.1800000 03	0.2141660-02	-0.5638260 01	0.2141660-02	0.5638260 01	0.1546350-01	0.1707800 03
0.9753030 00	0.0	0.8932190 00	0.0	0.8932190 00	0.0		
0.2816370-01	-0.1740370 03	0.4044730-01	0.0	0.4044730-01	0.0		
0.2112030 00	-0.3482260 02	0.4076700 00	0.1800000 03	0.4076700 00	0.1800000 03		
0.4215160-04	0.1102240 03	0.1560210-02	0.0	0.1560210-02	0.0		
0.5607820-01	-0.1780250 03	0.1030910 00	0.0	0.1030910 00	0.0		
0.1546350-01	-0.1707800 03	0.1030910-04	0.1800000 03	0.1030910-04	0.1800000 03		

FIGURE 3.3b

0.8833900-05	-0.9833660-07	0.7738150-07	0.1612330-05	0.2882550-06	-0.3751550-06
-0.5167880-24	0.1327340-21	-0.1985230-21	-0.6437450-19	0.2117580-21	-0.1905820-20
-0.7033980-07	0.9627250-05	-0.2363370-05	-0.7415440-03	-0.4003900-05	-0.1516100-04
-0.7692760-22	0.2329440-20	-0.1122320-19	-0.2674620-18	-0.1016440-19	-0.1253610-18
0.7738150-07	-0.2963370-05	0.8833900-05	0.6342480-03	0.2882550-06	0.3027420-04
-0.2638940-23	-0.6862140-21	0.5191610-21	0.352510-18	-0.741100-20	0.2710510-19
0.1612330-05	-0.7415440-03	0.6332430-03	0.5432370-00	-0.5093800-02	0.2424100-01
0.7673430-20	-0.1626300-18	0.4379910-18	0.4163340-16	-0.2183410-03	-0.6671530-17
0.2882550-06	-0.4500300-05	0.2882550-06	-0.5093800-02	0.2183410-03	-0.2297610-03
0.4500300-05	-0.1626300-18	-0.1270550-20	0.1746720-17	-0.3197120-20	0.2032680-19
-0.3751550-06	-0.1516100-04	0.3027420-04	0.2424100-01	-0.2297610-03	0.1733000-02
0.1861160-21	-0.1905820-20	0.2371630-19	-0.4553650-17	0.4404570-19	-0.8131520-19

RICSS2 CHECK SOLUTION, P=001

0.7779310-21	0.2346130-19	0.7704770-19	0.3327630-16	0.1612870-18	0.2770990-18
0.5952310-19	0.1660280-17	0.5380830-17	0.2391700-14	0.9244830-17	0.1750650-16
0.8174200-20	-0.3517300-18	0.6336380-18	-0.3670020-16	0.2668610-17	0.2642740-17
-0.1100080-17	-0.1141330-15	-0.1067020-15	-0.1042920-12	0.1553000-15	-0.4666410-15
0.9916450-20	-0.5513520-18	0.6915150-18	-0.9143020-16	0.1847990-17	0.6996490-17
-0.1007700-18	-0.6321410-17	-0.1101140-16	-0.6845980-14	-0.1118050-18	-0.4640390-16

THE MATRIX RICATTI SOLUTION

0.8833900-05	-0.9833660-07	0.7738150-07	0.1612330-05	0.2882550-06	-0.3751550-06
-0.5167880-24	0.1327340-21	-0.1985230-21	-0.6437450-19	0.2117580-21	-0.1905820-20
0.7738150-07	-0.2963370-05	0.8833900-05	0.6342480-03	0.2882550-06	0.3027420-04
0.1612330-05	-0.7415440-03	0.6332430-03	0.5432370-00	-0.5093800-02	0.2424100-01
0.2882550-06	-0.4500300-05	0.2882550-06	-0.5093800-02	0.2183410-03	-0.2297610-03
-0.3751550-06	-0.1516100-04	0.3027420-04	0.2424100-01	-0.2297610-03	0.1733000-02

THE FEEDBACK GAIN MATRIX

-0.4527010-04	-0.6473400-02	-0.1028110-01	-0.2957650-01	0.1504790-03	-0.1113350-00
0.1659720-03	-0.5724310-02	0.1788970-01	0.9639060-00	0.6377100-01	0.4841980-01

POLYNOMIAL COEFFICIENTS

6 ROOTS

INPUT	OUTPUT
0.100000000-01	0.100000000-01
0.379200250-02	0.379200250-02
0.426778790-03	0.426778790-03
0.235875770-04	0.235875770-04
0.599299960-04	0.599299960-04
0.631499910-04	0.631499910-04
0.294655420-04	0.294655420-04

ROOTS

	REAL	IMAG
1	-0.124757940-01	0.0
2	-0.158033510-01	-0.144578700-00
3	-0.158033510-01	0.143578700-00
4	-0.432185590-01	-0.392193010-01
5	-0.432185590-01	0.392193010-01
6	-0.236680640-02	0.0

LENGTH	PHASE ANGLE	THE NORMALIZED CLOSED LOOP EIGENVEC DRS ARE	LENGTH	PHASE ANGLE	LENGTH	PHASE ANGLE
0.9991420-00	0.0	0.9997230-00	0.0	0.9994230-00	0.0	0.9753030-00
0.3453270-01	0.0	0.1112620-01	0.8803940-02	0.1111620-01	-0.8803940-02	0.2816330-01
0.9702150-02	0.1800000-03	0.3590320-02	0.1508150-03	0.3500320-02	-0.1508150-03	0.2112030-01
0.9984670-05	0.1800000-03	0.6243010-05	0.1275290-03	0.6243010-05	-0.1275290-03	-0.2633200-02
0.1204100-01	0.1800000-03	0.3654830-02	0.1349840-03	0.3654830-02	-0.1349840-03	-0.1713790-03
0.7962470-02	0.0	0.2141660-02	-0.5046260-02	0.2141660-02	0.5046260-02	0.5607820-01
						0.1208200-03
						0.1546350-01
						0.1096260-03

0.9753030-00	0.0	0.8752190-00	0.0
0.2816330-01	-0.1128820-03	0.4894730-01	0.0
0.2112030-00	0.2633200-02	0.4307630-00	0.1800000-03
0.4215160-04	0.1713790-03	0.1500210-02	0.0
0.5607820-01	-0.1208200-03	0.1030910-00	0.0
0.1546350-01	-0.1096260-03	0.1930330-04	0.1800000-03

THE COMPLEX DETERMINANT OF THE MODAL MATRIX IS -0.2349250-01 -0.369450-17

THE CLOSED LOOP FUNDAMENTAL MATRIX IS

0.1903560-00	0.7318360-18	-0.3055080-01	0.2027790-16	-0.2941150-00	-0.3816390-16	0.2048230-02	0.5551120-13
0.4892450-04	-0.1306430-19	0.2978000-04	-0.1122400-16	-0.2941150-00	-0.2403660-17	-0.1017070-01	-0.2024560-14

FIGURE 3.3c

THE A MATRIX IS

-0.164000D 01	-0.104300D 02	0.206900D 02	-0.658900D 04	0.0	0.0	0.0	0.0
0.330000D-03	-0.131400D 01	0.597500D 00	-0.536200D 03	0.0	0.0	0.0	0.0
0.474400D-01	-0.391600D 01	-0.341000D 01	0.571500D 04	0.0	0.0	0.0	0.0
-0.200000D-04	-0.220000D-03	-0.193000D-02	-0.242100D 02	0.0	0.0	0.0	0.0
0.800000D-03	0.359000D 00	0.138200D 01	-0.117100D 04	-0.336600D 00	0.0	0.0	0.0
0.0	0.390000D-01	0.413000D 00	0.113140D 03	0.0	-0.100000D 01	0.0	0.0
0.0	0.0	0.0	0.0	0.100000D 01	0.0	0.0	0.0
0.0	0.0	0.0	0.0	0.0	0.100000D 01	0.0	0.0

THE B MATRIX IS

0.183700D 03	0.146000D 02
0.510000D 01	0.554700D 00
-0.141300D 03	0.204900D 03
-0.415000D 00	-0.530000D-01
0.124200D 02	0.138100D 01
0.358000D 01	0.147000D 00
0.0	0.0
0.0	0.0

THE Q MATRIX IS

0.246000D-07	0.0	0.0	0.0	0.0	0.0	0.0	0.0
0.0	0.116000D-04	0.0	0.0	0.0	0.0	0.0	0.0
0.0	0.0	0.753000D-06	0.0	0.0	0.0	0.0	0.0
0.0	0.0	0.0	0.194000D 01	0.0	0.0	0.0	0.0
0.0	0.0	0.0	0.0	0.554000D-03	0.0	0.0	0.0
0.0	0.0	0.0	0.0	0.0	0.494000D-02	0.0	0.0
0.0	0.0	0.0	0.0	0.0	0.0	0.554000D-05	0.0
0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.494000D-03

THE R MATRIX IS

0.100000D 00	0.0
0.0	0.100000D 00

FIGURE 3.4a

HAMILTONIAN MATRIX

C5A8D.1

$$R = \begin{bmatrix} .1 & \\ & .1 \end{bmatrix}$$

0.164000D 01	0.104300D 02	-.206900D 02	0.658900D 04	0.0	0.0	0.0	0.0
-.330000D-03	0.131400D 01	-.597500D 00	0.536200D 03	0.0	0.0	0.0	0.0
-.474400D-01	0.391600D 01	0.341000D 01	-.571500D 04	0.0	0.0	0.0	0.0
0.220000D-04	0.220000D-03	0.143000D-02	0.242100D 02	0.0	0.0	0.0	0.0
-.800000D-03	-.359000D 00	-.138200D 01	0.117100D 04	0.336000D 00	0.0	0.0	0.0
0.0	-.390000D-01	-.413000D 00	-.113140D 03	0.0	0.100000D 01	0.0	0.0
0.0	0.0	0.0	0.0	-.100000D 01	0.0	0.0	0.0
0.0	0.0	0.0	0.0	0.0	-.100000D 01	0.0	0.0
0.246000D-07	0.0	0.0	0.0	0.0	0.0	0.0	0.0
0.0	0.116000D-04	0.0	0.0	0.0	0.0	0.0	0.0
0.0	0.0	0.753000D-06	0.0	0.0	0.0	0.0	0.0
0.0	0.0	0.0	0.194000D 01	0.0	0.0	0.0	0.0
0.0	0.0	0.0	0.0	0.554000D-03	0.0	0.0	0.0
0.0	0.0	0.0	0.0	0.0	0.494000D-02	0.0	0.0
0.0	0.0	0.0	0.0	0.0	0.0	0.554000D-05	0.0
0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.494000D-03

0.339589D 06	0.944969D 04	-.229653D 06	-.770093D 03	0.230172D 05	0.659792D 04	0.0	0.0
0.944969D 04	0.263177D 03	-.606972D 04	-.214590D 02	0.641080D 03	0.183395D 03	0.0	0.0
-.229653D 06	-.606972D 04	0.619497D 06	0.477798D 03	-.147198D 05	-.475734D 04	0.0	0.0
-.770093D 03	-.214590D 02	0.477798D 03	0.175034D 01	-.522749D 02	-.149349D 02	0.0	0.0
0.230172D 05	0.641080D 03	-.147198D 05	-.522749D 02	0.156164D 04	0.446666D 03	0.0	0.0
0.659792D 04	0.183395D 03	-.475734D 04	-.149349D 02	0.446666D 03	0.128380D 03	0.0	0.0
0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0
0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0
-.164000D 01	0.330000D-03	0.474400D-01	-.200000D-04	0.200000D-03	0.0	0.0	0.0
-.104300D 02	-.131400D 01	-.391600D 01	-.220000D-03	0.359000D 00	0.390000D-01	0.0	0.0
0.206900D 02	0.597500D 00	-.341000D 01	-.193000D-02	0.138200D 01	0.413000D 00	0.0	0.0
0.658900D 04	-.536200D 03	0.571500D 04	-.242100D 02	-.117100D 04	0.113140D 03	0.0	0.0
0.0	0.0	0.0	0.0	-.336000D 00	0.0	0.100000D 01	0.0
0.0	0.0	0.0	0.0	0.0	-.100000D 01	0.0	0.100000D 01
0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0
0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0

POLYNOMIAL COEFFICIENTS

16 ROOTS

INPUT	OUTPUT
0.10000000D 01	0.10000000D 01
-.0.28851568D-14	-.0.35527137D-14
-.0.58437074D 03	-.0.58437074D 03
-.0.22511234D-11	-.0.21032065D-11
0.15250502D 05	0.15250502D 05
-.0.26616911D-08	-.0.26302587D-08
-.0.96167665D 06	-.0.96167665D 06
0.12416279D-07	0.12747478D-07
0.59313290D 07	0.59313290D 07
-.0.83056442D-07	-.0.89072273D-07
-.0.12793489D 08	-.0.12793489D 08
0.12328828D-06	0.13168028D-06
0.95035845D 07	0.95035845D 07
-.0.10779589D-07	-.0.11439624D-07
-.0.67522574D 06	-.0.67522574D 06
0.40043644D-09	0.40114756D-09
0.55983493D 04	0.55983493D 04

ROOTS

REAL	IMAG
1 0.26191124D 00	0.0
2 -.0.26191124D 00	0.0
3 0.97781729D-01	0.0
4 -.0.97781729D-01	0.0
5 -.0.12412817D 01	0.0
6 0.12412817D 01	0.0
7 -.0.15781695D 01	-.0.14456558D 00
8 -.0.15781695D 01	0.14456558D 00
9 0.15781695D 01	-.0.14456558D 00
10 0.15781695D 01	0.14456558D 00
11 -.0.49200241D 01	-.0.39228976D 01
12 -.0.49200241D 01	0.39228976D 01
13 0.49200241D 01	-.0.39228976D 01
14 0.49200241D 01	0.39228976D 01
15 -.0.23668066D 02	0.0
16 0.23668066D 02	0.0

THE NORMALIZED CLOSED LOOP EIGENVECTORS ARE

LENGTH PHASE ANGLE LENGTH PHASE ANGLE LENGTH PHASE ANGLE

FIGURE 3.4b

0.925325D 00	0.0	0.450472D-01	0.180000D 03	0.989932D 00	0.0	0.799921D 00	0.0
0.218812D-03	0.180000D 03	0.745526D-02	0.180000D 03	0.308255D-01	0.180000D 03	0.110180D-01	0.640483D 02
0.158820D 00	0.180000D 03	0.422818D-02	0.0	0.103630D-01	0.0	0.350739D-02	0.125267D 03
0.108631D-03	0.180000D 03	0.139339D-04	0.0	0.107698D-04	0.0	0.616812D-05	0.105099D 03
0.492051D-01	0.180000D 03	0.972143D-01	0.180000D 03	0.200749D-01	0.0	0.353576D-02	0.110973D 03
0.723433D-01	0.180000D 03	0.416719D-03	0.0	0.895964D-02	0.180000D 03	0.216895D-02	-750688D 02
0.187869D 00	0.0	0.994197D 00	0.0	0.161727D-01	0.180000D 03	0.223107D-02	-742605D 02
0.276213D 00	0.0	0.426173D-02	0.180000D 03	0.721806D-02	0.0	0.136862D-02	0.996973D 02

0.999921D 00	0.0	0.975262D 00	0.0	0.975262D 00	0.0	0.895211D 00	0.0
0.110180D-01	-640483D 02	0.281617D-01	0.161055D 03	0.211617D-01	-161055D 03	0.489468D-01	0.0
0.350739D-02	-125867D 03	0.211191D 00	0.218603D 02	0.211191D 00	-218603D 02	0.430758D 00	0.180000D 03
0.616812D-05	-105099D 03	0.421343D-04	-123165D 03	0.421343D-04	0.123165D 03	0.156019D-02	0.0
0.353576D-02	-110973D 03	0.560772D-01	0.168997D 03	0.560772D-01	-168997D 03	0.103090D 00	0.0
0.216895D-02	0.750688D 02	0.154643D-01	0.157798D 03	0.154643D-01	-157798D 03	0.103044D-04	0.180000D 03
0.223107D-02	0.742605D 02	0.891172D-02	-495696D 02	0.891172D-02	0.495696D 02	0.435565D-02	0.180000D 03
0.136862D-02	-996973D 02	0.245756D-02	-607680D 02	0.245756D-02	0.607680D 02	0.435370D-06	0.0

DETERMINANT IS-0.2617979D 03 0.1776357D-13 RANK IS -1

0.896170D-08	-964498D-07	0.798689D-07	-233563D-05	0.350021D-06	-625885D-06	0.471322D-07	-407118D-06
0.310193D-24	0.0	-794093D-22	-677626D-20	0.211758D-21	0.338813D-20	0.529396D-22	0.847033D-21
-964498D-07	0.969879D-05	-305402D-05	-823277D-03	-371241D-05	-206469D-04	-483337D-06	-107781D-04
-148893D-22	0.148231D-20	-129173D-19	0.216840D-18	-101644D-19	0.203288D-18	-296462D-20	-677626D-20
0.798689D-07	-305402D-05	0.893438D-05	0.674433D-03	0.290849D-04	0.337028D-04	0.317271D-05	0.107856D-04
-233563D-05	-145584D-21	0.939677D-21	0.704743D-18	-467044D-20	0.542101D-19	-648510D-21	0.196935D-19
0.169407D-20	-823277D-03	0.674433D-03	0.685278D 00	-707350D-02	0.333227D-01	-920723D-03	0.162243D-01
0.350021D-06	-108420D-18	0.813152D-18	0.555112D-16	-271051D-18	-433681D-17	0.216840D-18	0.346945D-17
-661744D-23	0.105879D-20	0.290849D-04	-700735D-02	0.250596D-03	-350122D-03	0.295078D-04	-193079D-03
-625885D-06	-206469D-04	0.337028D-04	0.0	0.101644D-19	-677626D-20	0.423516D-21	-237169D-19
-158819D-21	-529396D-20	0.406576D-19	0.333227D-01	-350122D-03	0.232524D-02	-492171D-04	0.107958D-02
0.471322D-07	-483337D-06	0.317271D-05	-346945D-17	-338813D-20	-799599D-18	0.135525D-19	0.169407D-19
-201625D-23	0.926442D-22	-502926D-21	-920723D-03	0.295078D-04	-492171D-04	0.594232D-04	-320699D-04
-407118D-06	-107781D-04	0.107856D-04	-542101D-19	0.847033D-21	0.0	-145584D-21	-232934D-20
0.272970D-22	-847033D-21	0.110114D-19	0.162243D-01	-193079D-03	0.107958D-02	-320699D-04	0.216743D-02
			-813152D-18	0.141878D-19	-145690D-18	0.402341D-20	0.0

RICSS2 CHECK SOLTUION, P-DOT

-661131D-22	-264698D-22	-473974D-20	-933557D-17	0.127584D-19	-590805D-19	0.151622D-20	-258345D-19
0.248923D-19	-550836D-19	0.155231D-17	0.267028D-14	0.326277D-17	0.553621D-17	0.372694D-18	0.155854D-17
-578530D-20	0.263811D-18	-823951D-18	-112323D-15	-235920D-17	-431119D-17	-196723D-18	-163308D-17
-300676D-17	0.668411D-16	-259016D-15	-209749D-12	-668573D-15	-144416D-14	-612574D-16	-517815D-15
0.105449D-19	0.329615D-18	0.503985D-19	0.125518D-14	0.129088D-17	-812813D-17	0.281215D-18	-471628D-17
-162630D-18	0.246148D-17	-134295D-16	-109244D-13	-446420D-16	-313877D-16	-467223D-17	-509575D-17
0.106200D-20	0.291168D-19	0.188068D-19	0.143735D-15	0.128749D-18	-674238D-18	-177348D-20	-365706D-18
-446810D-19	0.886632D-18	-419980D-17	-285969D-14	-121092D-16	-156125D-16	-120787D-17	-249366D-17

THE MATRIX RICATTI SOLUTION

0.896170D-08	-964498D-07	0.798689D-07	-233563D-05	0.350021D-06	-625885D-06	0.471322D-07	-407118D-06
-964498D-07	0.969879D-05	-305402D-05	-823277D-03	-371241D-05	-206469D-04	-483337D-06	-107781D-04
0.798689D-07	-305402D-05	0.893438D-05	0.674433D-03	0.290849D-04	0.337028D-04	0.317271D-05	0.107856D-04
-233563D-05	-823277D-03	0.674433D-03	0.685278D 00	-707350D-02	0.333227D-01	-920723D-03	0.162243D-01
0.350021D-06	-371241D-05	0.290849D-04	-700735D-02	0.250596D-03	-350122D-03	0.295078D-04	-193079D-03
-625885D-06	-206469D-04	0.337028D-04	0.333227D-01	-350122D-03	0.232524D-02	-492171D-04	0.107958D-02
0.471322D-07	-483337D-06	0.317271D-05	-920723D-03	0.295078D-04	-492171D-04	0.594232D-04	-320699D-04
-407118D-06	-107781D-04	0.107856D-04	0.162243D-01	-193079D-03	0.107958D-02	-320699D-04	0.216743D-02

THE FEEDBACK GAIN MATRIX

-705523D-04	0.684915D-02	-106133D-01	-352052D 01	0.702581D-02	-148356D 00	0.130279D-02	-692001D-01
0.169576D-03	-586325D-02	0.183950D-01	0.966021D 00	0.662955D-01	0.497731D-01	0.732821D-02	0.123023D-01

C5A8D.1

FIGURE 3.4c

THE A MATRIX IS

-0.1640000 01	-0.1043000 02	0.2069000 02	-0.6589000 04	0.0	0.0	0.0	0.0
0.3300000-03	-0.1314000 01	0.5975000 00	-0.5362000 03	0.0	0.0	0.0	0.0
0.4744000-01	-0.3916000 01	-0.3410000 01	0.5715000 04	0.0	0.0	0.0	0.0
-0.2000000-04	-0.2200000-03	-0.1930000-02	-0.2421000 02	0.0	0.0	0.0	0.0
0.8000000-03	0.3590000 00	0.1382000 01	-0.1171000 04	-0.3360000 00	0.0	0.0	0.0
0.0	0.3900000-01	0.4130000 00	0.1131400 03	0.0	-0.1000000 01	0.0	0.0
0.0	0.0	0.0	0.0	0.1000000 01	0.0	0.0	0.0
0.0	0.0	0.0	0.0	0.0	0.1000000 01	0.0	0.0

THE B MATRIX IS

0.1837000 03	0.1460000 02
0.5100000 01	0.5547000 00
-0.1413000 03	0.2049000 03
-0.4150000 00	-0.5300000-01
0.1242000 02	0.1381000 01
0.3580000 01	0.1470000 00
0.0	0.0
0.0	0.0

THE Q MATRIX IS

0.2460000-07	0.0	0.0	0.0	0.0	0.0	0.0	0.0
0.0	0.1160000-04	0.0	0.0	0.0	0.0	0.0	0.0
0.0	0.0	0.7530000-06	0.0	0.0	0.0	0.0	0.0
0.0	0.0	0.0	0.1940000 01	0.0	0.0	0.0	0.0
0.0	0.0	0.0	0.0	0.5540000-03	0.0	0.0	0.0
0.0	0.0	0.0	0.0	0.0	0.4940000-02	0.0	0.0
0.0	0.0	0.0	0.0	0.0	0.0	0.5540000-04	0.0
0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.4940000-02

THE R MATRIX IS

0.1000000 00	0.0
0.0	0.1000000 00

FIGURE 3.5a

HAMILTONIAN MATRIX

C5A8D.2

$$R = \begin{bmatrix} 1 & \\ & 1 \end{bmatrix}$$

0.164000D 01	0.104300D 02	-0.206900D 02	0.658900D 04	0.0	0.0	0.0	0.0
-0.330000D 03	0.131400D 01	-0.597500D 00	0.536200D 03	0.0	0.0	0.0	0.0
-0.474400D 01	0.391600D 01	0.341000D 01	-0.571500D 04	0.0	0.0	0.0	0.0
0.200000D 04	0.220000D 03	0.193000D 02	0.242100D 02	0.0	0.0	0.0	0.0
-0.800000D 03	-0.359000D 00	-0.138200D 01	0.117100D 04	0.336600D 00	0.0	0.0	0.0
0.0	-0.390000D 01	-0.413000D 00	-0.113140D 03	0.0	0.100000D 01	0.0	0.0
0.0	0.0	0.0	0.0	-0.100000D 01	0.0	0.0	0.0
0.0	0.0	0.0	0.0	0.0	-0.100000D 01	0.0	0.0
0.246000D 07	0.0	0.0	0.0	0.0	0.0	0.0	0.0
0.0	0.116000D 04	0.0	0.0	0.0	0.0	0.0	0.0
0.0	0.0	0.753000D 06	0.0	0.0	0.0	0.0	0.0
0.0	0.0	0.0	0.194000D 01	0.0	0.0	0.0	0.0
0.0	0.0	0.0	0.0	0.554000D 03	0.0	0.0	0.0
0.0	0.0	0.0	0.0	0.0	0.494000D 02	0.0	0.0
0.0	0.0	0.0	0.0	0.0	0.0	0.554000D 04	0.0
0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.494000D 02

0.339589D 06	0.944969D 04	-0.229653D 06	-0.770093D 03	0.230172D 05	0.659792D 04	0.0	0.0
0.944969D 04	0.263177D 03	-0.606972D 04	-0.214590D 02	0.641080D 03	0.183395D 03	0.0	0.0
-0.229653D 06	-0.606972D 04	0.619497D 06	0.477798D 03	-0.147198D 05	-0.475734D 04	0.0	0.0
-0.770093D 03	-0.214590D 02	0.477798D 03	-0.175034D 01	-0.522749D 02	-0.149349D 02	0.0	0.0
0.230172D 05	0.641080D 03	-0.147198D 05	-0.522749D 02	0.156164D 04	0.446666D 03	0.0	0.0
0.659792D 04	0.183395D 03	-0.475734D 04	-0.149349D 02	0.446666D 03	0.128380D 03	0.0	0.0
0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0
0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0
-0.164000D 01	0.330000D 03	0.474400D 01	-0.200000D 04	0.800000D 03	0.0	0.0	0.0
-0.104300D 02	-0.131400D 01	-0.391600D 01	-0.220000D 03	0.359000D 00	0.390000D 01	0.0	0.0
0.206900D 02	0.597500D 00	-0.341000D 01	-0.193000D 02	0.138200D 01	0.413000D 00	0.0	0.0
-0.658900D 04	-0.536200D 03	0.571500D 04	-0.242100D 02	-0.117100D 04	0.113140D 03	0.0	0.0
0.0	0.0	0.0	0.0	-0.336600D 00	0.0	0.100000D 01	0.0
0.0	0.0	0.0	0.0	0.0	-0.100000D 01	0.0	0.100000D 01
0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0
0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0

POLYNOMIAL COEFFICIENTS

16 RCOTS

INPUT	OUTPUT
0.100000000D 01	0.100000000D 01
0.122359020D 13	0.106581410D 13
-0.584370740D 03	-0.584370740D 03
-0.239141810D 10	-0.240447660D 10
0.152511510D 05	0.152511510D 05
0.181221020D 08	0.181807990D 08
-0.962985950D 06	-0.962985950D 06
-0.553014980D 07	-0.529835230D 07
0.634798570D 07	0.634798570D 07
0.202544860D 06	0.190178980D 06
-0.157706780D 08	-0.157706780D 08
0.257870040D 06	0.235682820D 06
0.169821120D 08	0.169821120D 08
0.100846520D 06	0.919098970D 07
-0.717093690D 07	-0.717093690D 07
-0.482439320D 09	-0.365616870D 09
0.559834930D 06	0.559834930D 06

ROOTS

REAL	IMAG
1 0.100000000D 01	0.0
2 -0.100000000D 01	0.0
3 0.104407910D 01	0.0
4 -0.104407910D 01	0.0
5 -0.315260830D 00	0.0
6 0.315260830D 00	0.0
7 0.155676970D 01	-0.875675180D 01
8 0.155676970D 01	0.875675180D 01
9 -0.155676970D 01	-0.875675180D 01
10 -0.155676970D 01	0.875675180D 01
11 0.490356530D 01	-0.393182040D 01
12 0.490356530D 01	0.393182040D 01
13 -0.490356530D 01	-0.393182040D 01
14 -0.490356530D 01	0.393182040D 01
15 -0.236680810D 02	0.0
16 0.236680810D 02	0.0

THE NORMALIZED CLOSED LOOP EIGENVECTORS ARE

LENGTH PHASE ANGLE LENGTH PHASE ANGLE LENGTH PHASE ANGLE

FIGURE 3.5b

0.14710 01	0.180000 03	0.41774 00	0.0
0.14710 02	0.180000 03	0.41774 01	0.0
0.14710 03	0.180000 03	0.41774 02	0.0
0.14710 04	0.180000 03	0.41774 03	0.0
0.14710 05	0.180000 03	0.41774 04	0.0
0.14710 06	0.180000 03	0.41774 05	0.0
0.14710 07	0.180000 03	0.41774 06	0.0
0.14710 08	0.180000 03	0.41774 07	0.0
0.14710 09	0.180000 03	0.41774 08	0.0
0.14710 10	0.180000 03	0.41774 09	0.0
0.14710 11	0.180000 03	0.41774 10	0.0
0.14710 12	0.180000 03	0.41774 11	0.0
0.14710 13	0.180000 03	0.41774 12	0.0
0.14710 14	0.180000 03	0.41774 13	0.0
0.14710 15	0.180000 03	0.41774 14	0.0
0.14710 16	0.180000 03	0.41774 15	0.0
0.14710 17	0.180000 03	0.41774 16	0.0
0.14710 18	0.180000 03	0.41774 17	0.0
0.14710 19	0.180000 03	0.41774 18	0.0
0.14710 20	0.180000 03	0.41774 19	0.0

0.975265D 00	0.0	0.975265D 00	0.0	0.975265D 00	0.0	0.895211C 00	0.0
0.281586D-01	0.0	0.281586D-01	0.0	0.281586D-01	0.0	0.489468C-01	0.0
0.211173D 00	0.0	0.211173D 00	0.0	0.211173D 00	0.0	0.430759C 00	0.180000C 03
0.419975D-04	0.0	0.419975D-04	0.0	0.419975D-04	0.0	0.156019C-02	0.0
0.560898D-01	0.0	0.560898D-01	0.0	0.560898D-01	0.0	0.103090C 00	0.0
0.154773D-01	0.0	0.154773D-01	0.0	0.154773D-01	0.0	0.103147C-04	0.180000C 03
0.892405D-02	0.0	0.892405D-02	0.0	0.892405D-02	0.0	0.435565C-02	0.180000C 03
0.246249D-02	0.0	0.246249D-02	0.0	0.246249D-02	0.0	0.435808C-06	0.0

DETERMINANT IS 0.6713279D 01 0.7993606C-14 RANK IS -1

0.911696D-08	-0.942989D-07	0.818866D-07	-0.760861D-05	0.439465D-06	-0.956429D-06	0.142051D-06	-0.956429D-06
0.129247D-23	0.224993D-21	0.291168D-21	0.230193D-18	-0.148231D-20	0.207523D-19	-0.123084D-20	0.766565D-19
-0.942989D-07	0.979667D-05	-0.319905D-05	-0.949291D-03	-0.249103D-05	-0.291354D-04	-0.809665D-06	-0.291354D-04
-0.132349D-22	0.302814D-19	0.561159D-20	0.186483D-16	-0.117660D-18	0.982558D-18	-0.847033D-19	0.612574D-17
0.818866D-07	-0.319905D-05	0.952779D-05	0.768519D-03	0.364283D-04	0.420540D-04	0.970366D-05	0.420540D-04
-0.703104D-23	-0.198523D-21	-0.330872D-20	-0.146367D-17	-0.315113D-20	-0.108420D-18	0.338813C-20	-0.332037C-18
-0.760861D-05	-0.949291D-03	0.768519D-03	0.927935D 00	-0.102104D-01	0.492699D-01	-0.330902D-02	0.492699D-01
-0.423516D-20	-0.135525D-17	-0.704731D-18	-0.735523D-15	0.423381D-17	-0.398986D-16	0.260209D-17	-0.279290D-15
0.439865D-06	-0.249103D-05	0.304683D-04	-0.102804D-01	0.307141D-03	-0.555076D-03	0.983941D-04	-0.555076D-03
0.198523D-22	0.232934D-20	-0.719978D-20	-0.466207D-17	0.406676D-19	-0.403188D-18	0.152466C-19	-0.755533C-18
-0.956429D-06	-0.291354D-04	0.420540D-04	0.492699D-01	-0.555076D-03	0.338732D-02	-0.179272D-03	0.338732D-02
0.579026D-22	-0.870326D-19	-0.376930D-19	-0.607695D-16	0.484303D-18	-0.359142D-17	0.249663C-18	-0.196241C-16
0.142051D-06	-0.809665D-06	0.970366D-05	-0.330902D-02	0.983941D-04	-0.179272D-03	0.206691C-03	-0.179272D-03
0.909890D-23	-0.190582D-20	-0.230552D-20	-0.238524D-17	0.184230D-19	-0.159242D-18	0.825857D-20	-0.223617C-18
-0.956429D-06	-0.291354D-04	0.420540D-04	0.492699D-01	-0.555076D-03	0.338732D-02	-0.179272D-03	0.832732D-02
0.794093D-22	-0.849151D-19	-0.387518D-19	-0.629921D-16	0.504532D-18	-0.371678D-17	0.254745D-18	-0.196241C-16

RICSS2 CHECK SOLUTION, P-DOT

-0.260875D-21	-0.453791D-20	0.516161D-21	0.184844D-17	-0.122489D-18	0.525796D-18	-0.397386D-19	0.126680D-17
0.610989D-19	0.619538D-18	0.444417D-17	0.326193D-14	0.150744D-16	0.586824D-17	0.517029D-17	-0.818573D-17
-0.628657D-20	0.821622D-19	-0.424787D-18	0.716116D-16	-0.280770D-17	0.383198D-17	-0.849998C-18	0.131324C-16
-0.880914D-18	0.673290D-16	-0.297776D-15	-0.282385D-12	0.853176D-15	-0.861877D-14	0.235597C-15	-0.172787C-13
-0.365018D-19	-0.454857D-18	0.427963D-18	0.250456D-14	-0.244521D-16	0.109884D-15	-0.739290D-17	0.241584D-15
-0.914796D-19	0.366596D-17	-0.201289D-16	-0.152335D-13	0.187567D-16	-0.444143D-15	0.494328D-17	-0.863892D-15
-0.115814D-19	-0.127783D-18	0.134890D-18	0.775801D-15	-0.778393D-17	0.342642D-16	-0.198439D-17	0.763854C-16
-0.132442D-18	0.276133D-17	-0.192886D-16	-0.136245D-13	-0.518345D-17	-0.324393D-15	-0.276133D-17	-0.552130C-15

THE MATRIX RICATTI SOLUTION

0.911696D-08	-0.942989D-07	0.818866D-07	-0.760861D-05	0.439465D-06	-0.956429D-06	0.142051D-06	-0.956429D-06
-0.942989D-07	0.979667D-05	-0.319905D-05	-0.949291D-03	-0.249103D-05	-0.291354D-04	-0.809665D-06	-0.291354D-04
0.818866D-07	-0.319905D-05	0.952779D-05	0.768519D-03	0.364283D-04	0.420540D-04	0.970366D-05	0.420540D-04
-0.760861D-05	-0.949291D-03	0.768519D-03	0.927935D 00	-0.102104D-01	0.492699D-01	-0.330902D-02	0.492699D-01
0.439865D-06	-0.249103D-05	0.304683D-04	-0.102804D-01	0.307141D-03	-0.555076D-03	0.983941D-04	-0.555076D-03
-0.956429D-06	-0.291354D-04	0.420540D-04	0.492699D-01	-0.555076D-03	0.338732D-02	-0.179272D-03	0.338732D-02
0.142051D-06	-0.809665D-06	0.970366D-05	-0.330902D-02	0.983941D-04	-0.179272D-03	0.206691D-03	-0.179272D-03
-0.956429D-06	-0.291354D-04	0.420540D-04	0.492699D-01	-0.555076D-03	0.338732D-02	-0.179272D-03	0.832732D-02

THE FEEDBACK GAIN MATRIX

-0.518004D-04	0.743378D-02	-0.113751D-01	-0.451221D 01	0.186054D-01	-0.214810D 00	0.604344D-02	-0.214810D 00
0.177295D-03	-0.608838D-02	0.195919D-01	0.100697D 01	0.713085D-01	0.570680D-01	0.227481D-01	0.570680D-01

C5A8D.2

FIGURE 3.5C