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Enhanced Radiation from Ionized Gases*

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H. Dreicer

Los Alamos Scientific Laboratory, University of California
Los Alamos, New Mexico

ABSTRACT

This paper deals with two problems whose solution requires a more detailed description of the interaction between the radiation field and the particles than is provided by the Boltzmann equation (collisionless or otherwise). The first of these is concerned with the saturation of the cyclotron radiation maser, and illustrates how cyclotron emission and absorption relaxes the electron momentum distribution. The second is the problem of deriving and evaluating the terms of the photon transport equation which govern the scattering of photons by electrons and plasmons when the radiation field is non-thermal and intense enough to enhance the scattering process.

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I. INTRODUCTION

The kinetic theory of ionized gases has been largely based upon two important simplified points of view. In historical order these are the discrete particle point of view, embodied in the Boltzmann collision integral, and the self consistent field point of view upon which the collisionless Boltzmann equation (the Vlasov equation) is based. Important advances have been made in recent years on the problem of bridging the gap between these two approaches, and in the development of their rigorous foundation. Nevertheless, the kinetic theory of an ionized gas is still far from being a closed subject. For example, a rigorous formulation which includes the radiation field, and leads to tractable equations is still lacking.

It is the purpose of this paper to present two problems whose solution requires a more detailed description of the interaction between the radiation field and the particles than is provided by the Boltzmann equation (collisionless or otherwise). For this purpose we introduce an electron-photon collision term. In Section II this collision term is applied to the problem of saturation in the cyclotron radiation maser. Section III deals with the theory of light scattering by an ionized gas which includes the scattering induced by the presence of intense light beams. Both of these problems can be classified as examples of enhanced radiation from ionized gases.

II. SATURATION OF THE CYCLOTRON RADIATION MASER

The existence of a negative absorption coefficient χ for radiation traversing a plasma, and interacting with its electrons has recently been shown theoretically by several authors.¹ From a quantum mechanical point of view the possibility of a negative χ is easily understood. The energy levels of an electron moving with the momentum component p_z along the magnetic field B are given by

$$\epsilon_n = \sqrt{(mc^2)^2 + 2mc^2(n + \frac{1}{2})\hbar\omega_B + p_z^2 c^2} \quad (1)$$

$$\omega_B = eB/m \quad ; \quad n = 0, 1, 2, \dots,$$

and this equation shows that for a constant p_z the density of energy levels increases as the orbital quantum number n increases. Since the radian frequency ω_{nl} emitted as a result of a transition between any two levels is given by $(\epsilon_n - \epsilon_l)/\hbar$ it follows that ω_{nl} decreases as n increases. Maser action is therefore possible at this frequency provided that the electron population in the upper state n appreciably exceeds the population in the lower state l . In the classical limit ($\hbar \rightarrow 0$; $n \rightarrow \infty$) we have the familiar formulas

$$\omega = \frac{(n-l)\omega_B}{1 - \beta_z \cos \theta} \cdot \frac{mc^2}{\epsilon} \quad ; \quad k = n-l = 1, 2, \dots \quad (2)$$

$$\varepsilon = \sqrt{(mc^2)^2 + (p_\perp^2 + p_z^2)c^2} \quad (3)$$

$$\beta_z = \frac{p_z c}{\varepsilon} ; \quad p = \sqrt{p_\perp^2 + p_z^2} \quad (4)$$

which show the decrease of ω due to the relativistic mass change, and the Doppler effect due to the electron's motion along B. The probability per unit time, unit frequency, and unit solid angle of emitting a photon of radian frequency ω and polarization α into the direction which subtends the angle θ with respect to the magnetic field is given by

$$P_1^\alpha(p, \omega, \theta) + P_3^\alpha(p, \omega, \theta) = \quad (5)$$

$$P_1^\alpha(p, \omega, \theta) \left[1 + \frac{4\pi^2 c^2}{\omega^2} N^\alpha(\omega, \theta) \right]$$

where

$N^\alpha(\omega, \theta)$ = number of photons of radian frequency ω , and propagation direction θ which cross a unit area per unit time, per unit frequency interval, and per unit solid angle.

$P_1^\alpha(\underline{p}, \omega, \theta)$, $P_3^\alpha(\underline{p}, \omega, \theta)$ = probability per unit time, per unit frequency and unit solid angle of spontaneous and induced emission respectively by an electron of momentum $\underline{p}$. The corresponding induced absorption probability P_2^α is given by

$$P_2^\alpha(\underline{p}, \omega, \theta) = \frac{\epsilon'}{\epsilon} P_1^\alpha(\underline{p}', \omega, \theta) \frac{4\pi^2 c^2}{\omega^2} N^\alpha. \quad (6)$$

The factor ϵ'/ϵ in eq. (6) is the ratio of statistical weights in two neighboring states, and arises from the fact that the number of free electron states which coalesce into a given quantum state n as the magnetic field is turned on depends upon n . After absorption of a photon the final electron momentum $\underline{p}'$ and energy ϵ' , entering into eq. (6), are given by the conservation laws

$$p'_z = p_z + \frac{\hbar \omega}{c} \cos \theta \quad (7)$$

$$\epsilon' = \epsilon + \hbar \omega.$$

The expression for the net absorption coefficient, $\kappa^\alpha(\omega, \theta)$ (i.e., induced absorption minus induced emission), for an electron momentum distribution $f(\underline{p})$ which is independent of position and isotropic in momentum space is given by

$$K^{\alpha}(\omega, \theta) =$$

$$- \frac{8\pi^3 c^2}{\omega^2} \int_0^{\infty} P_1^{\alpha}(p, \omega, \theta) \hbar \omega \frac{\epsilon}{c^2} \frac{\partial f}{\partial p} 4\pi p dp. \quad (8)$$

In the absence of line broadening effects, the spontaneous emission probability for an electron which is only slightly relativistic, and which is emitting the fundamental ($k = 1$) extraordinary wave is given by

$$P_1(p, \omega, \theta) = \frac{1}{\hbar \omega} \frac{e^2 \omega^2}{8\pi c} \left(\frac{p_{\perp} c}{\epsilon} \right)^2 \delta \left[\omega_B \frac{mc^2}{\epsilon} - \omega (1 - \beta_z \cos \theta) \right]. \quad (9)$$

Here δ denotes the Dirac delta function, and the plasma dielectric constant has been set equal to unity. Figures 1 and 2 show $K(\omega, \theta)$ (in relative units) as a function of ω_B/ω with the angle of propagation, θ , as a parameter. Equation (9), and the isotropic distribution

$$f(p) = n_0 A \exp \left[-\frac{(p-p_0)^2}{2mKT} \right] \quad (10)$$

with $KT/e = 10$ eV and $p_0/mc = 0.275$ were used in their evaluation.

For perpendicular propagation ($\theta = 90^\circ$) we find

$$K(\omega, \pi/2) = \frac{2\pi^2}{3} \frac{\omega_p^2}{\omega_0 c} \frac{mc^2}{KT} Q \quad (11)$$

where

$$Q = (mc)^3 A z^2 (z - z_0)(1 + z^2) \exp \left[-\frac{mc^2}{2KT} (z - z_0)^2 \right]$$

$$z = \sqrt{\left(\frac{\omega_0}{\omega}\right)^2 - 1} \quad ; \quad z_0 = p_0/(mc) \quad (12)$$

$$\omega_p^2 = 4\pi n_0 e^2 / m .$$

This expression shows that $\chi(\omega, \pi/2)$ is proportional to $\partial f / \partial p$ evaluated at

$$p = mc \sqrt{\left(\frac{\omega_0}{\omega}\right)^2 - 1} \quad (13)$$

Thus the minimum value of $\chi(\omega, \pi/2)$ (i.e., the maximum amplification) occurs at the frequency ω_0 which is emitted by electrons whose momentum p_0 corresponds to the value of the momentum at the lower inflection point of f . This is the point of maximum slope where

$$p_0 = p_0 - \sqrt{mKT} \quad (14)$$

Figures 1 and 2 indicate the importance of aligning the propagation direction perpendicular to $\underline{B}$. Even for small deviations from perpendicularity we find that a strong reduction in χ is produced by the linear Doppler effect.

It is clear from these calculations that maser oscillation is possible in an electron gas whose velocity distribution is appropriately non-Maxwellian. However the calculations presented above must be extended in several directions before a realistic estimate of the condition for self-oscillation of a practical system can be made. Transit time broadening of the spectrum, as well as the effect of anisotropy in $f(p)$ upon χ must be investigated. When such calculations are carried out² we find that both effects can produce serious reduction in the amplification. A more interesting question concerns the manner

in which such a maser saturates. That saturation must eventually occur follows immediately from conservation of energy. The radiation field receives its energy from the electrons, and unless their energy is continuously replenished, the distribution function f must relax towards a form for which $\chi > 0$. This relaxation problem can not be handled by the Boltzmann equation, because it does not adequately include the effects of the radiation field upon the motion of electrons. Only the averaged electric and magnetic fields produced by charges and currents internal and external to the ionized gas are included, and these alone are not capable of relaxing the gas and the radiation field towards the Maxwellian and Planck distributions. To remedy this situation the effect of fluctuations in the radiation field upon the motion of the electrons must also be included. Our approach is to introduce the photon nature of the radiation field into the kinetic equation by deriving a collision term which describes the interactions between photons and electrons. Such a derivation results in³

$$\left(\frac{\partial f(\underline{p}, t)}{\partial t} \right)_c = \sum_{\alpha=1,2} \sum_{i=1}^3 \left\{ \int_{\Omega} \int_{\omega} P_i^{\alpha}(\underline{p}', \omega, \theta) f(\underline{p}') \frac{\epsilon'}{\epsilon} d\omega d\Omega \right. \quad (15)$$

$$\left. - f(\underline{p}) \int_{\Omega} \int_{\omega} P_i^{\alpha}(\underline{p}, \omega, \theta) d\omega d\Omega \right\}$$

$$d\Omega = 2\pi \sin\theta d\theta$$

where the P_i , defined by eqs. (5), (6), are functions of $N(\omega, \theta)$, the photon distribution. The primed energy and momentum, ϵ' and $\underline{p}'$, here refer to the values which these quantities must have before an emission or absorption event causes the electron to appear with the energy ϵ and momentum $\underline{p}$. The P_i are now assumed to be the emission and absorption probabilities for an electron moving through the plasma. Appendage of this collision term to the Boltzmann equation raises the question of whether or not interactions between electrons and the radiation field are duplicated when both the flow term $(e\underline{E} \cdot \partial f / \partial \underline{p})$ and the collision term in eq. (15) are used jointly. A rigorous justification of this procedure must of course await a more fundamental development of the kinetic theory. It seems reasonable to suppose, however, that their simultaneous use is justified in the limit of small particle densities when the collective interactions do not seriously alter the spontaneous emission probability from its vacuum value. Under such dilute plasma conditions the flow term, with f modified by electron-photon encounters, then describes the collective interactions between particles and the radiation field which lead to the dielectric coefficient. The collision term, with P_i appropriately corrected for dielectric effects, describes the random interactions between photons and particles which lead to relaxation of f . A clear analogy exists here to the dichotomy represented by the Boltzmann collision integral approach, and the Vlasov self consistent field approach used in the kinetic theory of charged particles interacting through their Coulomb field.

To treat the maser saturation problem we ignore the deviations of the dielectric coefficient from its vacuum value altogether. The photon distribution N is assumed to be independent of position, and its frequency and angular distribution are chosen as

$$N(\omega, \theta) = N_0 F(\omega) \delta(\cos \theta). \quad (16)$$

Furthermore, we suppose that the initial value of f gives rise to a negative χ over the entire range of frequencies covered by $F(\omega)$, and that χ is sufficiently small so that N can be taken as constant in time, for short periods. In principle the calculation could be made self consistent by recalculating χ during the time evolution of f , and also by recalculating N with the help of the photon transport equation. Such detailed calculations have not yet been carried out, but our approximate treatment while valid only for short relaxation times will turn out to indicate the nature of a more general solution.

Equation (15) may be transformed by expanding $P_i(\underline{p}') f(\underline{p}')$ in powers of the momentum interchange. The result is a Fokker-Planck equation which, for the distribution in eq. (16) has the form

$$\begin{aligned} \frac{1}{a} \frac{\partial f(x, y, t)}{\partial t} = & \frac{\partial f}{\partial x} \left[\frac{F(\omega)}{x} - x \omega_0 \frac{\partial F(\omega)}{\partial \omega} \right] \\ & + F(\omega) \frac{\partial^2 f}{\partial x^2} \end{aligned} \quad (17)$$

to lowest order in x . Spontaneous emission is ignored in this equation and its symbols are defined as follows

$$x = p_x / (mc) \quad ; \quad y = p_z / (mc)$$

$$a = \frac{\pi}{2} \frac{e^2}{mc^2} \frac{N_0 \hbar \omega_B}{mc} \quad (18)$$

$$\omega = \frac{\omega_B}{\sqrt{1 + x^2 + y^2}}$$

It can be shown that the Fokker-Planck expansion, and therefore eq. (17), is valid only provided the band width $\Delta\omega$ over which $F(\omega)$ is appreciable satisfies $\Delta\omega/\omega_B \gg \hbar\omega_B/mc^2$. Otherwise the treatment must revert to eq. (15). The particular choice made for N in eq. (16) (i.e., $\theta = \pi/2$) insures that y , the normalized electron momentum along B enters eq. (17) as a parameter only. Essentially two different kinds of relaxation processes are described by eq. (17). The first, a diffusion in momentum space, is important where the $\partial^2 f / \partial x^2$ term dominates. The second, a flow in momentum space due to the dynamical friction exerted by the radiation field on the electrons, can be singled out where the $\partial f / \partial x$ term dominates.

To solve eq. (17) it is necessary to specify $F(\omega)$. If we choose the Gaussian distribution

$$F(\omega) = \frac{1}{\sqrt{\pi} \Delta\omega} \exp \left[- \left(\frac{\omega - \omega_f}{\Delta\omega} \right)^2 \right] \quad (19)$$

and take the initial distribution to be

$$f(x, y, 0) = A' \exp \left\{ -\frac{mc^2}{2KT} [(x-x_0)^2 + y^2] \right\} \quad (20)$$

then we may center $F(\omega)$ in frequency by choosing

$$\omega_f = \frac{\omega_B}{\sqrt{1 + x_l^2}} \quad (21)$$

where the lower inflection momentum x_l is equal to $(x_0 - \sqrt{KT/mc^2})$. The diffusion term vanishes at x_l , and can be ignored in eq. (17) provided we restrict our solution both to the neighborhood of x_l , and to the early times before $\partial^2 f / \partial x^2$ grows even near x_l . With these assumptions the method of characteristics leads to the solution pictured in fig. 3 where

$$\frac{\Delta f(x, y, \tau)}{f(x, y, 0)} \left(\frac{\omega_B}{\Delta \omega} \right) \sqrt{\frac{KT}{mc^2}} g \quad (22)$$

is given as a function of

$$(x - x_s) (\omega_B / \Delta \omega) g \quad (23)$$

with normalized time τ as a parameter. The quantities q , τ , and x_s are defined by

$$q = \frac{x_s}{(1 + x_s^2)^{3/2}} ; \tau = \frac{4 x_s}{\sqrt{\pi}} \frac{\omega_B^2}{(\Delta\omega)^3} q a t \quad (24)$$

$$\omega_f = \frac{\omega_B}{\sqrt{1 + x_s^2}} = \frac{\omega_B}{\sqrt{1 + x_s + y^2}} \quad (25)$$

From this graph $f(x, y, \tau)$ can be calculated with the help of

$$f(x, y, \tau) = f(x, y, 0) \left[1 - \frac{\Delta f(x, y, \tau)}{f(x, y, 0)} \right] \quad (26)$$

It should be clearly understood that fig. 3 shows the quantity in (22) on a vastly magnified scale in the neighborhood of x_s only. For early times such magnification is necessary, because only those electrons which emit radiation inside of the initial band-width $\Delta\omega$ around the frequency ω_f are affected by the relaxation process. Amplification of the radiation field is seen to occur at the net expense of the energy of the electrons on the high energy side of x_s . These electrons

move to momenta just below x_s , and as a result $\partial f / \partial x$ at x_s decreases with time. Simultaneously $\partial f / \partial x$ increases just above and below x_s , i.e., the point of maximum slope splits into two points of maximum slope. If we had recalculated χ as a function of time we would have found its minimum splitting into two corresponding minima, one shifting to higher and one to lower frequency. In time as $\partial f / \partial x$ tends to zero at x_s , χ would tend to zero at ω_f . The net effect of the saturation process is a spreading of the radiation field in frequency, and a relaxation of f towards equilibrium.

If we choose for the saturation time constant $\tau = 1$ we find

$$t = \frac{(\Delta\omega)^3}{2\sqrt{\pi} \omega_B^2 \left(\frac{e^2}{mc^2}\right) \frac{N_0 \hbar \omega_B}{mc} x_s q} \quad (27)$$

For example if $\Delta\omega = 10^{-6} \omega_B$; $N_0 \hbar \omega_B = 10^{-10}$ watts per cm^2 ; $B = 10^4$ gauss; and $x_s \cong 0.25$ then $t \cong 7 \times 10^{-8}$ sec. The relaxation time is extremely short, because it is necessary to alter the energy of only a small fraction of the electrons. With the inclusion of transit time or collisional broadening this time would become longer.

The calculation we have described centered around the lower inflection point of f . We can also carry the entire treatment over

to the neighborhood of the upper inflection point where the slope of f has its maximum negative value, and κ its maximum positive value. With appropriate change in the sign of Δf the solution shown in fig. 3 holds for this case also.

III. ENHANCED LIGHT SCATTERING

In this section we briefly present the theory of a new type of light scattering experiment which is based upon the enhancement of the scattering process by means of intense auxiliary light beams. The case of Compton scattering from individual electrons, is treated first in Section III A, and is followed in Section III B by a brief discussion of the modification introduced when the scattering takes place from plasma oscillations (plasmons).

A. Compton Scattering

Consider an ionized gas traversed by light of radian frequency ω_1 , in a direction given by the wave vector $\underline{k}_1$. If the wavelength is short compared to the Debye length then a portion of this radiation is scattered by the individual electrons of the ionized gas into other directions, $\underline{k}_2$, where it appears with a different frequency, ω_2 . The number of photons which appear, per unit time, in the radiation oscillator characterized by $\omega_2, \underline{k}_2$, is not simply proportional to the average number of photons n_1 populating the radiation oscillator $\omega_1, \underline{k}_1$, but rather to the expression $n_1(n_2 + 1)$, where n_2 is the average number of photons (of the proper polarization) populating the radiation oscillator $\omega_2, \underline{k}_2$. This result states that the probability

for a photon to be scattered out of the ω_1, k_1 oscillator into the ω_2, k_2 oscillator becomes enhanced whenever the ω_2, k_2 oscillator is already populated to some extent.⁴ Induced emission of cyclotron radiation, discussed in the previous section, is a much more familiar example of the same phenomenon which is also a consequence of the Bose statistics obeyed by photons. Important modifications in scattering experiments can be expected whenever the photon occupation number, n , is large compared to unity.

Highly efficient microwave oscillators, or light masers may be used to enhance scattering in a manner illustrated in fig. 4. To enhance the rate at which protons are scattered out of light beam 1 (or out of a mode of the Fabry-Perot resonator BB') it is necessary to irradiate the ionized gas with light beam 2 (or with the radiation in a mode of Fabry-Perot AA') which intersects beam 1 within the scattering medium. For masers in the visible part of the spectrum, the frequency ω_2 and direction k_2 are chosen so that $(\omega_2 - \omega_1)$ corresponds to the Doppler shift expected on the basis of the motion of a given group of electrons. In order to separate those photons which have been scattered out of beam 1 into beam 2, from those photons in beam 2 which traverse the ionized gas unscattered, it is necessary to identify, in some way, those photons which originate in beam 1. Amplitude or frequency modulation of beam 1 seems feasible.

In scattering experiments the primary interest lies in the modification of the radiation field by the ionized medium. For this

reason we shall give the transport equation for the photon distribution $N(\omega, \theta, \phi)$, defined in conjunction with eq. (5), in terms of an arbitrary electron momentum distribution $f(\underline{p})$ only. Following a derivation similar to the one which leads to eq. (15) we find, in the absence of spatial gradients

$$\frac{\partial \rho^\alpha(\omega, \theta, \phi)}{\partial t} = \sum_{\alpha} \int \int \frac{\omega_0 \epsilon_0}{\omega \epsilon} \sigma_0^\alpha(\underline{p}_0, \chi, \eta) \hbar \omega \left\{ N^\alpha(\omega', \theta', \phi') \left[1 + \frac{4\pi^2 c^2}{\omega'^2} N^\alpha(\omega, \theta, \phi) \right] \left(\frac{\omega}{\omega'} \right)^2 f(\underline{p}') \right. \quad (28)$$

$$\left. - N^\alpha(\omega, \theta, \phi) \left[1 + \frac{4\pi^2 c^2}{(\omega')^2} N(\omega', \theta', \phi') \right] f(\underline{p}) \right\} d^3 p d(\cos \chi) d\eta$$

$$\rho^\alpha(\omega, \theta, \phi) = \frac{1}{c} N^\alpha(\omega, \theta, \phi) \hbar \omega$$

The symbols in this equation have the following meaning:

ω = radian frequency of photon

θ, ϕ = polar and azimuthal angles which define the propagation direction of the photon with respect to a stationary laboratory frame

σ_0^α = differential scattering cross-section

χ, η = polar and azimuthal scattering angles specified in the center of "mass" frame

$\underline{p}, \epsilon$ = electron momentum and total energy

All primed quantities refer to the value of the quantity after the scattering event. The subscript "zero" refers to the value of a quantity in the center of "mass" frame in which the electron and photon momenta are equal in magnitude and oppositely directed.

It is noteworthy that the enhanced scattering terms in eq. (28) cancel each other unless the recoil experienced by the electron when it scatters a photon is taken into account. This requirement holds even though the frequency shift produced by the scattering encounter may be primarily due to the motion of the electron (i.e., Doppler effect) rather than due to the momentum exchanged between electron and photon. It is for this reason that we have entitled this section "Compton" rather than Thomson scattering.

The terms in eq. (28) will here be evaluated for a very special case only. We assume that the radiation field is occupied by photons of two definite frequencies, ω_1 and ω_2 , which are propagating into two definite directions θ_1, ϕ_1 , and θ_2, ϕ_2 respectively. For the electron momentum distribution we chose the cylindrically symmetric displaced Maxwellian distribution

$$f(p_x, p_y) = n_0 A'' \exp \left[-\frac{E_x}{2} (x - x_0)^2 - \frac{E_y}{2} (y - y_0)^2 \right] \quad (29)$$

where

$$E_{\perp} = mc^2/(KT) \quad ; \quad E_z = mc^2/(KT)$$

$$x = p_{\perp}/(mc) \quad ; \quad y = p_z/(mc)$$

This distribution is frequently useful in discussing an ionized gas located in a magnetic field. Provided our treatment is restricted to photon energies which are small compared to mc^2 we can use the Thomson cross section

$$\sigma_0 = \frac{1}{2} \left(\frac{e^2}{mc^2} \right)^2 (1 + \cos^2 \chi) \quad (30)$$

in the center of "mass" frame. This formula applies to a light beam which is unpolarized in the center of mass frame. To first order in the ratio of electron speed to light speed, (v/c) , the light beam is then also unpolarized in the stationary laboratory frame. Equation (28) can finally be reduced to the form

$$\begin{aligned}
\frac{\partial \rho(\omega_1, \theta_1=0)}{\partial t} = & \\
& - \frac{8\pi}{3} \left(\frac{e^2}{mc^2} \right)^2 n_0 \left[N(\omega_1, \theta_1) \hbar \omega_1 \right] \left\{ 1 + \frac{4\pi^2 c^2}{\omega_2^2} \frac{N_2 \hbar \omega_2}{mc^2} S \right\} \quad (31) \\
& + \frac{1}{4} \left(\frac{e^2}{mc^2} \right)^2 n_0 \frac{N_2 \hbar \omega_2}{\omega_2} R
\end{aligned}$$

where

$$\begin{aligned}
R = & \frac{1 + \cos^2 \theta_2}{1 - \cos \theta_2} (mc)^3 A'' \exp[-J] \left\{ \int_0^{2\pi} \frac{d\beta}{G} \right. \\
& \left. - \sqrt{\pi} \int_0^{2\pi} \frac{\sqrt{K}}{G} [1 - \mathcal{E}_2(\sqrt{K})] \exp(K) d\beta \right\} \quad (32)
\end{aligned}$$

$$S = \frac{3}{16\pi} (1 + \cos^2 \theta_2) (mc)^3 A'' \exp[-J]$$

$$\begin{aligned}
& \left\{ (E_\perp - E_z) \left[\frac{1}{2} \int_0^{2\pi} \frac{\sqrt{K}}{G^{3/2}} b d\beta - \int_0^{2\pi} L(\beta) b \exp(K) d\beta \right] \right. \quad (33) \\
& \left. + \int_0^{2\pi} M(\beta) d\beta - \sqrt{\pi} \int_0^{2\pi} \sqrt{K} M(\beta) [1 - \mathcal{E}_2(\sqrt{K})] \exp(K) d\beta \right\}.
\end{aligned}$$

The symbols in eqs. (31), (32), (33) have the following meaning:

n_o = average electron density

$N_2 \hbar \omega_2 = \iint N(\omega_2, \theta, \phi) \hbar \omega_2 \sin \theta d\theta d\phi$ = power carried across unit area by photons of frequency ω_2 .

$$K = \frac{[E_z b(a-y_o) - E_\perp x_o]^2}{4G}$$

$$G = \frac{1}{2} (E_\perp + E_z b^2)$$

$$J = \frac{1}{2} [E_\perp x_o^2 + E_z (a-y_o)^2]$$

$$a = \frac{\omega_1 - \omega_2}{\omega_1 (1 - \cos \theta_2)}$$

$$b = \frac{\sin \theta_2 \cos \beta}{1 - \cos \theta_2}$$

$$M(\beta) = \frac{E_z (a-y_o) + E_\perp b x_o}{2G}$$

$$L(\beta) = \frac{\sqrt{\pi} (K + \frac{1}{2}) [1 - E_2(\sqrt{K})]}{2G^{3/2}}$$

$$\mathcal{E}_2(\sqrt{K}) = \frac{2}{\sqrt{\pi}} \int_0^{\sqrt{K}} \exp(-t^2) dt$$

In the evaluation leading to eq. (31) beam 1 was aligned along p_z , i.e., $\theta_1 = 0$, and only the dominant terms in (v/c) were retained. The first term in the curly brackets in eq. (31) gives the result of the conventional scattering experiment in which the energy scattered spontaneously out of beam 1 is proportional to the specific intensity, $N(\omega_1, \theta_1) \hbar \omega_1$, of this beam. The second term in the bracket represents the enhanced scattering term which gives the difference between the amount of power scattered out of beam 1 into beam 2, and the amount of power scattered out of beam 2 back into beam 1. As is to be expected this term is proportional to $[N(\omega_1, \theta_1) \hbar \omega_1] [N_2 \hbar \omega_2]$. The last term in eq. (31) represents the energy scattered spontaneously out of beam 2 and into beam 1. It is proportional to the power per unit area (not the specific intensity) carried in beam 2. The integrals appearing in eqs. (32), (33) can be carried out analytically for certain limiting cases only. In general they must be evaluated numerically. It is possible to obtain information about $f(p)$, by arranging for the enhancement term to be large, and varying ω_2, θ_2 for a given ω_1, θ_1 .

B. Plasmon-Photon Scattering

When the light wavelength exceeds the Debye length we deal with scattering by plasmons (and phonons).⁵ Again a photon transport equation has been derived. It differs from eq. (28) in that the plasmons (and phonons) obey Bose statistics, and their spontaneous emission must be accounted for. In short, the power reaching the detector in a double-light beam experiment is now proportional to

$$n_1(1+n_2)g - n_2(1+n_1)(g+1) \quad (34)$$

where g is the average number of plasmons per plasma oscillator. The expression in (34) indicates that a plasmon can be emitted even when the electron density is initially perfectly uniform, i.e., $g = 0$, provided photons traverse the ionized gas.

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FIGURE CAPTIONS

- Fig. 1. Net absorption coefficient, in relative units, as a function of ω_b/ω , with the angle of propagation, θ , as parameter.
- Fig. 2. Net absorption coefficient, in relative units, as a function of ω_b/ω , with the angle of propagation θ , as parameter.
- Fig. 3. Generalized graph giving deviation of the electron momentum distribution from its initial value as a function of normalized momentum, x , with the normalized time τ as a parameter. The distribution $\bar{f}$ can be constructed, for any y , from this graph.
- Fig. 4. Schematic view of an enhanced scattering experiment showing two Fabry-Perot resonators fed by external lasers.

Figure 1.

Figure 2.

Figure 3.

Figure 4.