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General Remarks.

This year was marked by the transition from theoretical research pertaining to mirror confined plasmas to research pertaining TOKAMAK type plasmas. This transition was spurred by personnel changes (Prof. Callen's leave of absence from MIT, Prof. McCune's sabbatical at Garching and Prof. Sigmar's joining the contract). Moreover, taking the recent developments of the CTR program and its ensuing needs into account the transition was decided upon at this time because

i) R. McCrory's thesis on the radial eigenmode problem of drift instabilities in mirror geometry was nearing completion and ii) Prof. McCune's longstanding experience in hydro- and magnetohydrodynamics and Prof. Sigmar's training in kinetic theory of toroidal plasmas under Prof. Coppi enabled a frictionless change over into broad areas of toroidal plasma research.

The work performed consists of three distinct pieces.

i) McCrory's thesis, a brief summary of which is found in enclosure 1.

(Full text available as AEC report COO-2168-4.) (100%, 9 mos.)

ii) Prof. McCune's work on the effect on diffusion of keeping the inertial terms in the MHD equations of toroidally confined plasmas, in collaboration with K.v. Hagenow at Garching (see enclosure 2). (15%, 12 mos.).

iii) Prof. Sigmar's work on neoclassical transport theory of TOKAMAK plasmas containing a second $Z > 1$ impurity ion species, in collaboration with J.F. Clarke, ORNL. (Enclosure 3). This work is presently being prepared as AEC Report COO-2168-6. (25%, 9 mos.)

Enclosure 1.

"Theory of Drift Universal and Drift Cone Modes of Collisionless Plasma in Cylindrical Geometry."[1]

(McCrory, McCune)

This work was concentrated on an analysis of the electrostatic drift-universal instability, and with particular relevance to mirror devices, the electrostatic drift-cone instability [2] , [3] . The method involved an analysis of the standard orbit-integral solution of the Vlasov equation in plasmas of cylindrical geometry with a gaussian equilibrium radial density distribution. In this analysis one is led, through Poisson's equations, to an integro-differential equation for the disturbance potential, which defines an eigenmode-eigenvalue problem. All finite-Larmor radius effects are included.

It has proved extremely useful to apply a Hankel transformation to the above integro-differential equation, leading to an integral equation of the Fredholm type in the transformed space ("k-space"). The kernel in the transformed space turns out to be sharply cut-off in the difference variable ($k-k'$), allowing one to reduce the transformed integral equation to a second-order differential equation in k-space. Beyond this point, it is possible to find solutions for the transformed potential by WKBJ methods. Consideration of the conditions for the proper behavior of the potential in k-space for small and large k distinguish between allowed and non-allowed solutions. Taking only the allowed solutions, one is able to derive a dispersion relation relevant to the type of mode being studied.

A feature of the results is that at threshold, determined by the equilibrium center-line density and parallel wave length, the mode is very much non-localized in ordinary space (r-space), at obvious variance with the "local assumption" so often used in slab-model analyses of modes of the drift type.

Despite this, it is shown, for the drift-universal mode, that the dispersion relation is extremely similar to that obtained from slab model analyses, employing the local assumption. Examples of actual radial mode structure near threshold are given [1].

In contrast to the case of the drift-universal mode, results for the $k_{\parallel} = 0$ drift-cone mode are very different from those predicted by slab-model analyses. In particular, it is shown [1] that for small to moderate azimuthal mode numbers ("l"), the critical density gradient below which the mode is stable (or non-existent) is much higher than that originally predicted by Rosenbluth, Post. The implication is that cylindrical plasmas of surprisingly small radius will be stable to the drift-cone mode -- at least at moderate l-numbers. If one assumes that drift-cone modes of large azimuthal mode number (large "l") have comparatively little effect on confinement, the new results may offer an explanation of the apparent stability of the 2-X and 2-X II machines against the drift-cone mode [4].

Conclusion

Including the complete radial eigenmode problem for the drift universal mode it is shown that nonlocalized radial perturbations exist but the dispersion relation remains basically unchanged. However, for the $k_{\parallel} = 0$ drift cone mode, for small to moderate azimuthal mode numbers, the critical plasma radius for stabilization is substantially smaller than that predicted from the "local" theory, consistent with experimental observations.

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2. R.F. Post, M.N. Rosenbluth, Phys. Fluids 9, 730 (1966).
3. A.B. Mikhailovski, Nucl. Fusion 5, 125 (1965).
4. F.H. Coengen, W.F. Cummins, W.E. Nexson, T.C. Simonen, UCRL-51208.

Enclosure 2.

Effects of Plasma Inertia on Toroidal Confinement (McCune)

The problem of resistive plasma losses in toroidal geometry was first attacked by Pfirsch and Schlüter [1] who found that toroidal effects lead to an enhancement of classical resistive diffusion. The model used the classical, macroscopic, resistive MHD equations with Ohm's law with a scalar resistivity, and made use of the low- β approximation with inertial terms omitted. Several self-consistent (finite- β) calculations for axisymmetric toroidal discharges [2,3,4] have shown that the Pfirsch-Schlüter result is valid there also, provided once again that inertial effects are omitted. Mathematically, the Pfirsch-Schlüter result implies, as pointed out by Grad and Hogan [2], that the "cylindrical limit" of a torus can be very different (or even non-existent) depending on how the rotational transform is related to the toroidal aspect ratio.

Stringer [5] noted that inertial and resistive effects tend to make the low β toroidal plasma unstable as an inertial poloidal rotation would tend to increase until it reached the critical speed, (the sound speed multiplied by the ratio of the poloidal to toroidal field strengths).

Inertial effects were also considered in the low- β limit by Zehrfeld and Green [6,7]. These authors did not use an aspect ratio expansion. They found that rotation tends to enhance plasma diffusion over the Pfirsch-Schlüter value.

Hazeltine, Lee, and Rosenbluth [8] extended the work of Zehrfeld and Green and predicted that the rotational speed is stabilized at the critical speed by a weak shock. The same authors found a rotational instability for a self-consistent Tokamak model [9].

A finite- β two-fluid model was used by Haines [10] to study inertial effects and rotation. Finite Larmor radius effects were included. Inertial effects were given a certain order in terms of an aspect ratio expansion.

Haines found again increased diffusion and a tendency for the plasma to rotate.

Greene, Johnson, Weimer, and Winsor [11] treated the low- β , one-fluid model in a systematic expansion by employing an ordering scheme in which the resistivity is assumed to be third-order in the inverse aspect ratio. By considering the time-dependent problem with small time-derivatives, they derived a dispersion relation which again revealed a rotational instability. More recent work indicates that the rotation may stabilize at other than the critical speed.

Hagenow and McCune [12] have developed a scheme allowing the analysis of the self-consistent (finite- β) one fluid equations with inertia and small but finite resistivity. In this method, the resistivity is a small free parameter, and in contrast to Ref. [11], need not be ordered in powers of the inverse aspect ratio. Hagenow and McCune have shown that whether or not a plasma will rotate at all depends on the order of magnitude of the rotational transform, i.e., on how it is related to the toroidal aspect ratio. In this sense, the question of rotation of the plasma is as sensitive to the way the "cylindrical" limit is taken as is diffusion itself. When rotation is possible, enhanced diffusion is bound to occur. A special steady-state solution, with rotation, has been found [1] which is analogous to the no-source solution of Grad and Hogan [2]. It is found that rotation reduces the poloidal β still further.

The prediction of a rotational instability in a finite- β Tokamak [9] raises the question of whether steady-state solutions with finite rotation exist for certain intermediate states between zero rotation and the critical speed. The answer may depend on the nature of the sources (the rate of introduction of angular momentum) or it may be dependent on the plasma behavior at the boundaries.

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2. H. Grad and J. Hogan, Phys. Rev. Letters, 24, 1337 (1970).
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Enclosure 3

Neoclassical Transport Theory of Multispecies Plasmas

(Sigmar, Clarke)

The transport theory of plasmas consisting of electrons, protons and high Z-impurity ions in toroidal confinement is developed, in the "banana" regime of small collision frequencies. The drift kinetic equations are solved explicitly for the particle distribution functions to lowest order in the inverse aspect ratio, using the operator for like collisions given by Rosenbluth, Hazeltine and Hinton,^[1] and a related operator for collisions of heavy ions on protons derived by a mass ratio expansion of the Fokker Planck operator.

Quantitative expressions for the radial particle- and heat fluxes and the electric (bootstrap) current are obtained from the appropriate moments of these distributions functions, as a function of the impurity concentration. An expression for the radial electric field as a function of the density- and temperature gradients and the impurity concentration is derived in the proton restframe in the limit of small impurity concentration, and in the impurity ion restframe in the opposite limit. The modification of the Ware effect and the electrical resistivity due to impurity ions is also given. The transport coefficients reduce to those of Rosenbluth, Hazeltine and Hinton in the impurity free limit.

1. Physics of Fluids, 15, 116 (1972).