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IDO-17017
April 1965

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THE CALCULATION OF GAMMA-RAY SHAPES
FOR SODIUM IODIDE SCINTILLATION SPECTROMETERS
NaI ---
COMPUTER PROGRAMS AND EXPERIMENTAL PROBLEMS

R. L. Heath, R. G. Helmer,
L. A. Schmittroth, and G. A. Cazier

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IDO-17017
AEC Research and Development Report
Physics
TID-4500 (39th Ed.)
Issued: April 1965

THE CALCULATION OF GAMMA-RAY SHAPES
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COMPUTER PROGRAMS AND EXPERIMENTAL PROBLEMS

BY

R. L. Heath, R. G. Helmer, L. A. Schmittroth, and G. A. Cazier

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Atomic Energy Division

Contract AT(10-1)-205

Idaho Operations Office

U. S. ATOMIC ENERGY COMMISSION

ACKNOWLEDGEMENT

The development of techniques described herein for the analysis of pulse-height spectra obtained with scintillation spectrometers has resulted from the combined efforts of a large number of people. These computer programs and methods have been perfected as a continuing effort over the past six years. The authors wish to acknowledge the assistance of E. C. Yates, J. E. Cline, C. W. Reich, W. W. Black, C. H. Hogg, L. D. McIsaac, W. J. Maeck, Wanda Hammer, and Cleone Pea in the measurement and analysis of the large volume of data required to define the response of a NaI scintillation detector. Assistance in the preparation of samples was obtained from S. D. Reeder, E. H. Turk, N. P. Alley, and S. F. Marsh. In the early stages of development, significant contribution to these programs was made by A. V. Grimaud and D. D. Metcalf. The data analysis programs discussed in Chapter VI were written by Marie Putnam, D. H. Gipson, and G. A. Jayne. Because of the magnitude of the data processing effort, the support of the computer operations staff under the direction of L. L. Marsden is acknowledged. For their assistance in drafting the many figures, we thank D. W. Ellison, G. Hammer, and D. V. Jenson.

ABSTRACT

A method is described for the calculation of pulse-height distributions which represent the response of a NaI scintillation spectrometer to monoenergetic photons. The method is based upon an interpolation between experimental spectra for sources emitting only one gamma ray. A detailed description of the experimental procedures used to accumulate input data is given. This discussion includes the analysis of photopeak shapes, detector efficiency, nonlinear relationship between energy and pulse-height for NaI, and a general consideration of problems incidental to the analysis of gamma-ray spectra. The interpolation method is described, and the computer program devised to carry out this procedure is outlined in considerable detail. The resulting calculated gamma-ray shapes are compared with the input shapes and other simple spectra. Examples of the use of calculated gamma-ray shapes for the analysis of complex spectra are presented. The computer programs described are written in FORTRAN IV for use on an IBM-7040 computer. The complete program listings are included together with flow charts. Listings of sample input to and output from the programs are given. In addition, computer programs are briefly described which are designed to gain-shift spectra, to fit modified-gaussian functions to photopeaks, to calculate coincidence sum spectra, and to do least-squares fits to complex spectra with calculated gamma-ray shapes and other components.

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FOR SODIUM IODIDE SCINTILLATION SPECTROMETERS

COMPUTER PROGRAMS AND EXPERIMENTAL PROBLEMS

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I. INTRODUCTION

In the study of nuclear decay schemes, gamma-ray spectra are measured with a scintillation spectrometer consisting of a NaI(Tl) crystal, amplifying system, and pulse-height analyzer. The resulting pulse-height spectrum represents the response of the system to the radiations emitted by the source. For the cases of interest here, these radiations include continuous electron or positron spectra and discrete photons. The primary information to be obtained from such pulse-height spectra is the energy and intensity of the monoenergetic photons. For such an analysis the response of the system must be known for photons of the energies which are present. If the shape of the pulse-height distribution representing the response of the system to monoenergetic gamma rays is known, the analysis becomes a mathematical problem of determining the set of energies and intensities that best represent the experimental spectrum.

This report is primarily concerned with the description of a computer program that has been written to calculate distributions which represent the response of a scintillation spectrometer to monoenergetic photons. A detailed description of the accumulation of the necessary experimental information is included along with discussions of the problems arising in the measurement and analysis of gamma-ray spectra. Examples of the use of calculated shapes in the analysis of complex gamma-ray spectra are presented.

In the past, pulse-height distributions for single gamma rays have usually been determined either by graphical interpolation between a small set of experimental spectra for such gamma rays, or by approximation of the desired distribution by the spectrum of a gamma ray of somewhat different energy. In the first case, a few experimental gamma-ray spectra would be plotted and overlaid. Then, for example, a smooth curve might

be drawn between the two spectra which lay on either side of the desired energy. The accuracy of the method depended to a considerable extent on the artistic ability of the experimenter. In the second case, a gamma-ray spectrum was frequently measured with the gain of the amplifying system altered so that the experimental photopeak fell at the desired position. As long as the spectral analysis was carried out by hand, the inaccuracy of these interpolation techniques did not place a significant limitation on the accuracy of the energies and intensities obtained. However, with the development of techniques for the use of digital computers for data analysis, these techniques are no longer satisfactory.

There are two principal methods which have been used for the calculation of pulse-height distributions to represent the response of the detector to monoenergetic gamma rays. The one, which at first would appear to be most satisfactory, is to calculate the response from a knowledge of the physical processes which take place. This has been done by Miller and Snow [1] and by Zerby and Moran [2] using Monte Carlo techniques. There are two limitations to the calculations, however, as they have been carried out. First, the calculations assume a source and a NaI(Tl) crystal in an otherwise empty space. Therefore, the results do not include any effects of radiations scattered from surrounding

[1] W. F. Miller and W. J. Snow, Monte Carlo Calculation of the Energy Loss Spectra for Gamma Rays in Sodium Iodide and Cesium Iodide, Argonne National Laboratory Report ANL-6318 (1961).

[2] C. D. Zerby and H. S. Moran, Calculation of the Pulse-height Response of NaI Scintillation Counters, Oak Ridge National Laboratory Report, ORNL-3169 (1962).

material and especially that scattered by material used to prevent electrons from reaching the detector. Hence, these calculated distributions may not suffice if an actual spectrum is to be analyzed. Also, since this method is based on Monte Carlo type calculations, it would become extremely time consuming to calculate with suitable statistical accuracy the large number of shapes that might be desired.

The second method, which is the basis of this study, is to apply the graphical interpolation method in a more systematic manner. The use of digital computer techniques has made this feasible. This method has the basic advantage that the scattered radiations are automatically taken into account. A disadvantage is that the calculated spectra are correct only for one detector in one experimental arrangement. Any other system will have different scattering properties and different photopeak widths. Another approach to the automation of the graphical interpolation method has been reported by Wapstra [3].

It is not obvious that a satisfactory scheme can be devised for graphical interpolation. It is known, however, that the probabilities for the various processes taking place in the detector all vary smoothly with the incident photon energy. It is this smoothly varying character which makes accurate interpolation possible.

Brief outlines of this interpolation method have been published previously [4].

Chapter II contains a discussion of the phenomena which determine the pulse-height spectrum of a NaI(Tl) spectrometer and the standard experimental arrangement used in this study. In Chapter III

the various types of information needed for input to the computer program are treated. The procedure for the generation of single gamma-ray distributions is described in Chapter IV. Chapter V contains comparisons of the calculated shapes with experimental spectra. Other computer programs and techniques which are important in the analysis of spectra are discussed in Chapter VI. The input and output for the shape generation program are described in Chapter VII. A group of appendices follows which include discussions of specific points involved in setting up the electronics system, in the interpretation of photopeak shapes, and in sine series and polynomial fits. Samples of the input and output data are given in the last two appendices along with the FORTRAN listings from the program.

The present program is compiled for use on an IBM-7040 computer. It consists of two programs. The first processes the set of experimental spectra while the second generates gamma-ray shapes. The original version, which did not include all of the features of the present program, was compiled four years ago for use on an IBM-650 computer.

The results presented in this report are from the first set of spectra processed by this program. With the complete program now available, it is hoped that it will be possible to discern and correct errors and inconsistencies in the input data. This procedure for improvement of the input data should be important since it is felt that for energies above 0.2 MeV the accuracy of the calculated distributions are limited by the quality of the input data and not by the methods used in the program.

[3] A. H. Wapstra and J. Oberski, "Computer Analysis of Coincident Gamma-ray Spectra", Proceedings of Symposium on Applications of Computers to Nuclear and Radiochemistry, USAEC Report NAS-NS-3107, p 137 (1962).

[4] R. L. Heath, "Recent Developments in Scintillation Spectrometry", IRE Transactions on Nuclear Science, NS-9, p 294 (1962); and Data Analysis Techniques for Gamma-ray Scintillation Spectrometry, USAEC Report IDO-16784 (1962).

II. DETECTOR RESPONSE AND EXPERIMENTAL CONDITIONS

1. RESPONSE OF SCINTILLATION DETECTOR AND COUNTING SYSTEM

1.1 Monoenergetic Photons

The pulse-height spectrum produced by a NaI(Tl) scintillation spectrometer for monoenergetic photons is a complicated function which varies smoothly with photon energy. As an example, Figure 1 shows the pulse-height spectra obtained for gamma rays of 0.060, 0.166, 0.279, 0.662, and 2.16 MeV. The prominent features of these spectra can be explained simply in terms of basic physical processes. Each spectrum has a photopeak, or full-energy peak, which is nearly symmetrical and is approximately Gaussian in shape. This photopeak is produced by events in which essentially all of the energy of the incident photon

is absorbed in the crystal. This includes those events in which the photon interacts by the photoelectric process. Also included are those events in which the photon undergoes a Compton scattering, with the secondary photon interacting in the crystal and dissipating its energy there. The relative importance of these two processes varies with gamma-ray energy and crystal size. (See Reference 5 for a discussion of the interaction of photons with matter.) The finite width of a photopeak results from the variation of the efficiency of converting electron kinetic energy to light in the crystal and the statistical uncertainty in the number

[5] C. M. Davisson, Alpha-, Beta-, and Gamma-ray Spectroscopy, Chapter II, edited by K. Siegbahn, North-Holland Publishing Co., Amsterdam (1965).

Fig. 1 Pulse-height spectra obtained with a 3 x 3 inch NaI scintillation detector for gamma rays of 0.060, 0.166, 0.279, 0.662, and 2.16 MeV,

of photoelectrons produced in the photo-multiplier.

The pulse height at the center of the photopeak is almost proportional to the energy of the gamma ray. (Strictly, the "center of the photopeak" is the center of a symmetrical distribution the shape of which is determined by the peak and high-energy side of the photopeak.) Thus, all of the information available in a complex spectrum can be determined from the positions and areas of the photopeaks.

For the 0.060-MeV gamma ray in Figure 1, the photopeak results almost completely from events in which the interaction is by the photoelectric process. The escape peak, which is the only other discernible feature, results from events in which an iodine K X-ray escapes from the crystal. The energy of this peak is 0.028 MeV lower than that of the photopeak. Since at this low energy most of the gamma rays interact very close to the front face of the crystal, this effect is particularly prominent.

At 0.166 MeV, the escape peak has become less pronounced. Events in which the primary photon interacts in the crystal by the Compton process and the secondary photon escapes from the crystal produce the Compton "edge" or peak at 0.065 MeV. Other photons will strike material outside the crystal and interact by the Compton process. The secondary photons from these events, that enter the detector, produce the "backscatter" peak which in this case is at about 0.100 MeV.

In the spectrum for the 0.279-MeV gamma ray, the escape peak is not perceptible and the Compton edge and the backscatter peak are at almost the same energy. Most of the counts at energies lower than the Compton edge are produced by Compton scattering in which the secondary photon escapes. The Compton edge represents the maximum energy which can be transferred to an electron in this scattering process. Most of the counts

at energies between the Compton edge and the photopeak are produced by either (a) multiple scattering in the crystal in which a low-energy photon escapes, or (b) Compton scattering in the beryllium absorber with the secondary photon scattered forward into the crystal.

The 0.662-MeV gamma-ray spectrum shows the distinct backscatter peak and Compton edge.

For gamma rays above about 1.3 MeV, three additional peaks are observed. These result from the interaction of the photons by pair production in either the crystal or the surrounding material. The resulting positrons annihilate and each produces two 0.511-MeV photons. If the positrons are produced in the crystal, one or both of the annihilation photons can escape. This is the cause of the pair-production escape peaks at $E_{\gamma}-0.511$ MeV and $E_{\gamma}-1.022$ MeV. The 0.511-MeV peak results from the detection of annihilation radiation produced in the surrounding material and not from primary interactions in the detector.

1.2 Electrons and Bremsstrahlung

Electrons, which are emitted in the decay of many radioactive nuclei, can penetrate the thin aluminum can surrounding the NaI(Tl) crystal. These electrons will produce undesired events in the crystal. These counts are generally eliminated by placing an absorber in front of the detector. This is illustrated in Figure 2.

However, the electrons which are stopped in the absorber will radiate a certain fraction of their energy in the form of photons. Some of these bremsstrahlung photons will be detected, as shown in Figure 3.

For a given electron energy, the fraction of the energy radiated increases as the square of the atomic number, Z , of the absorber. In order to minimize the number of bremsstrahlung photons,

Fig. 2 Effect of beta absorber on observed spectrum. Curve 1 was taken with no absorber, curve 2 with standard Be absorber, and curve 3 with sides of detector also shielded to prevent entry of scattered electrons.

Fig. 3 Pulse-height spectrum showing contribution from bremsstrahlung continuum arising from electrons being stopped in the Be absorber.

the absorber should be made of material of the lowest possible Z . Beryllium and polystyrene are often used.

The electron absorber will also have an effect on the gamma-ray spectrum that reaches the crystal since some photons will undergo Compton scattering in the absorber. For small-angle scattering, the secondary photons entering the crystal will have a lower energy than the incident gamma rays. This effect is illustrated in Figure 4.

Fig. 4 The pulse-height spectrum of Be^7 showing the contribution due to photons scattered within the beta absorber and from surrounding shielding material.

1.3 Coincidence Summing

A complication in the analysis of gamma-ray spectra occurs when the decay

of a radioactive element gives rise to two or more coincident gamma rays. In this case the pulse-height spectrum will include a distribution of pulses which results from the simultaneous detection of two such radiations. This "coincidence sum" spectrum will contain pulses of all amplitudes from zero to the sum of the maximum pulse heights obtainable from the individual gamma rays. As shown in Figure 5, the most prominent feature of this spectrum is the "sum peak" which results from detection of pulses from the two photopeaks. It is possible to calculate the shape of this summation spectrum. The intensity can also be calculated if sufficient information is available about the decay scheme. The details of these calculations are discussed in Section VI-3.

Fig. 5 Spectrum of Nb⁹⁴ indicating the coincidence sum spectrum.

1.4 Accidental Summing

Accidental summation occurs whenever two pulses overlap in time as in

Figure 6a. Since the pulses do not, by definition, come from coincident radiations, the time interval between the leading edges of the two pulses can take on any value. The combined pulse will then not have the same shape as a single pulse. As a result, the response of the analyzer to the combined pulse will depend on the details of the method by which it measures the pulse height. For the example in Figure 6a the measured pulse height might be either v_1 or $v_1 + v_2$ depending upon whether the analyzer turns off when $\frac{dv}{dt} = 0$ or stays "open" for a fixed time τ . If the analyzer measures the area of the pulse, some intermediate value might be recorded.

Fig. 6-a Amplifier output pulse for random coincidence of two events.

A few measurements have been made to determine the approximate spectral distribution and intensity for accidental sum spectra. An example is shown in Figure 6b. Since the intensity of the sum spectrum is proportional to the square of the counting rate, any analysis must take into account the actual counting rates and not just the relative rates. A discussion of the calculation of random

Fig. 6-b Spectrum of monoenergetic gamma ray showing shape and intensity of the random summation spectrum as a function of source strength.

summation spectra has been presented in a recent paper by Kennett et al. [6].

1.5 Scattering from Surrounding Material

Often it is necessary to surround a detector with some material in order to reduce the intensity of the background radiation reaching the detector. In the design of this shield there are three problems to consider. The first is the amount of material needed to reduce the background to a satisfactory level. In this case four inches of lead is generally sufficient. The second consideration is the effect of the material and the shield-detector configuration on the backscatter radiation that will be detected. A spectrum measured in three different shield configurations is shown in Figure 7. The

Fig. 7 Effect of detector shield configuration on scattered component of pulse-height spectrum obtained from monoenergetic gamma-ray source.

[6] T. J. Kennett, W. V. Prestwich, and G. L. Keech, Nuclear Instruments and Methods 29, 325 (1964).

reduction in scattering from lead as opposed to iron is due to the high photoelectric absorption cross section relative to the Compton cross section for the high Z material. As shown, the intensity of the backscatter radiation can also be reduced by increasing the size of the shield. The third problem is that of the detection of X-rays emitted from the shield as a result of the photoelectric interaction of gamma rays from the source. As shown in Figure 8, these X-rays can be effectively eliminated by lining the shield with a lower Z material which has a high probability for absorption of Pb X-rays. In this case 0.030 inch cadmium is used. An additional liner of 0.015 inch copper is used to reduce the intensity of cadmium X-rays resulting from fluorescence.

Fig. 8 Illustration showing Pb X-rays produced by photoelectric interaction in the detector shield and the effect of using critical absorption techniques to reduce their intensity.

1.6 Nonlinearity of System

There are two effects which cause the photopeak of a gamma ray to have

a pulse height which is not proportional to the photon energy. The more basic is the fact [7, 8] that the light output of a NaI(Tl) crystal is not proportional to the energy dissipated in the crystal. Secondary, the pulse-height analysis system used does not have a linear correspondence between the pulse amplitude and the pulse-height recorded. The results of an experimental determination of the pulse height versus energy relationship is given in Section III-3.

2. STANDARD EXPERIMENTAL CONDITIONS

2.1 Physical Arrangement

Since the response function of the detection system depends on the environment, it becomes very important that this environment be the same during the measurement of all of the spectra being used in the interpolation as well as during that of the spectrum to be analyzed. This necessitates the establishment of a "standard geometry". The term "geometry" includes the source-detector distance, absorber, shielding configuration, and the thickness of the source. Also, the gain of the electronic system must be the same for all data.

The standard physical arrangement of the detector is shown in Figure 9. The NaI(Tl) crystal is (3.000 ± 0.005) inches in diameter and (3.000 ± 0.005) inches high. The crystal is contained in an 0.005-inch-thick aluminum container and is mounted on a 8054 photomultiplier. The source is positioned 10.0 cm from the top face of the NaI(Tl) crystal. The standard electron

[7] D. Engelkemeir, Rev. Sci. Instr. **27**, 589 (1956).

[8] W. W. Managan, Applied Gamma-ray Spectrometry, edited by C. E. Crouthamel (Pergamon Press, New York) (1960).

Fig. 9 Standard laboratory detector arrangement used for experimental measurements.

absorber is a beryllium disk with an areal density of 1.18 g/cm^2 . This detector system is situated with the crystal approximately at the center of a $32 \times 32 \times 32$ inch lead shield which is used to reduce the background radiation.

Although the details of the standard geometry were determined somewhat arbitrarily, there are general arguments which can be made concerning each feature of the arrangement. The source-crystal distance was chosen such that the intensity of the real summing between two coincident gamma rays would be a few percent of the intensity of each gamma ray. This choice is thus a compromise between the desire to maximize the solid angle and to minimize the summation effects. (In the measured spectra, the intensity of

the backscatter radiation, including the 0.511-MeV peak, relative to the photopeak depends on this distance. Therefore, in any systematic study it is necessary to compare only those spectra measured at the same distance.)

All of the sources are prepared so as to minimize the amount of material in and around the source. The source material is placed between two layers of cellulose tape ($\approx 6 \text{ mg/cm}^2$) mounted on a piece of cardboard and placed on the source support shown in Figure 9. The material is confined to a circle whose diameter is less than $1/2$ inch and, where possible, less than $1/4$ inch. This arrangement is not possible for gaseous sources such as Ar-41. These sources are contained in thin walled quartz ampoules which were mounted in the standard position. A more detailed description of the standard configuration is given in Reference 9.

2.2 Electronics System

The electronics system is shown schematically in Figure 10. It includes a high voltage supply for the photomultiplier voltage, a preamplifier, double-differentiated amplifier (Type A-8), and a Nuclear Data model 130A multichannel analyzer. The features of the system which are of prime importance are its

- [9] R. L. Heath, Scintillation Spectrometry Gamma-ray Spectrum Catalogue, 2nd Edition, IDO-16880 (1964).

Fig. 10 Block diagram showing elements of laboratory gamma-ray scintillation spectrometer.

linearity and the stability of the gain and the zero of the pulse-height scale.

The gain of the amplifying system is usually adjusted by varying the high voltage on the photomultiplier. The alternative method would be to adjust the fine gain control of the amplifier. This is not done because it changes the operating point of the input tube and, thus, may alter the linearity of the amplifier. (The method used has one problem. It is possible that the resolution of some photomultiplier tubes may depend to a small extent on the photomultiplier high voltage.) The gain is usually adjusted so that the 0.6616-MeV gamma ray from Cs-137 falls about in channel 66.2 for a 10-keV scale or 132.3 for a 5-keV scale.

As is well known, the gain of a photomultiplier may depend on the current through it. This means it will vary as the counting rate of the source varies. The gain shift occurs rather rapidly in most phototubes and the system is stable after a few seconds. Therefore, the source is usually placed in front of the detector for a minute or so before taking data. This time can be shortened by maintaining another source producing the same photomultiplier current in position until the source to be measured is put in place.

To measure the linearity of the multi-channel analyzer and amplifier, the relationship between the amplitude of input pulses and the pulse height, determined by the analyzer, must be accurately determined. In this work, linearity measurements were based on a pulser which is accurate to better than 1 part in 10^5 . (See Appendix I for details.) The recorded pulse heights are always determined with the pulses being stored approximately equally in two channels. At

this point the recorded pulse height is determined to about 0.05 channel. (If the pulses are all in one channel, the pulse height is uncertain by about 0.2 channel.)

The linearity is determined by plotting the pulse amplitude (V) versus the recorded pulse height (PH). Small deviations can be accentuated by computing

the ratio $\frac{V_1 - V_2}{PH_1 - PH_2}$ which is constant

for a linear system for any pair of measurements. The results of these measurements for a typical system are shown in Figure 11. After an equipment failure, this result is rechecked to assure that the system has returned to its previous state.

The fact that the electronic system is very nearly linear makes it worthwhile considering the possible advantages of various methods of adjusting the zero control of the linear ramp voltage. It has been decided to set the zero such that a pulse of zero amplitude would be stored at -0.5 channel. The reasons for this choice are discussed in Appendix I.

Fig. 11 Plot of measured linearity of electronic system.

III. ACCUMULATION AND TREATMENT OF INPUT DATA

In this chapter, all of the experimental data which are needed as input to the first program are discussed. This includes the collection of experimental shapes for single gamma rays. Analytical expressions are also needed to represent, as a function of the gamma-ray energy, the photopeak parameters, photopeak efficiency, etc.

1. SPECTRA OF MONOENERGETIC GAMMA RAYS

In order to make the interpolation procedure as accurate as possible, it is desirable to have a set of single gamma-ray spectra which

- (1) span as large an energy range as possible,
- (2) are fairly uniformly distributed, and
- (3) are as accurate as possible.

The first two conditions require the availability of a variety of source production methods. However, a combination of those samples that are commercially available and those made by the (n, γ) reaction provides a good collection. The presence of unwanted radiations is the usual limitation on the accuracy of the data.

A list of the sources used to represent the detector response to monoenergetic gamma radiation is given in Table I along with the method of production, mode of decay, half-life, the energy of the gamma ray, and the percent of decays which result in the desired photons.

All of the spectra measured included a certain contribution from background

radiation. This was removed by accumulating a background spectrum for the same duration (i.e., same analyzer live time) and subtracting the latter spectrum from the former.

The problem of accidental-coincidence summing was reduced to a negligible level by keeping the source strengths relatively low. The range of count rates used in measuring the spectra was from 1000 to 4000 counts per second. As an example of the magnitude of this summing effect, a Cs-137 source producing a total of 4000 counts per second will yield one summation count per channel in the region of the photopeak for every 3000 counts per channel at the maximum of the photopeak. At 1000 counts per second, the summation spectrum will be reduced by a factor of four relative to the photopeak. No attempt was made to correct the measured spectra for this small effect.

In addition to the above effects which apply to all spectra, most of the sources have radiations, other than the desired gamma ray, which must be taken into account (viz., bremsstrahlung or X-rays). Table II is a list of these radiations for each source.

For those spectra containing an observable bremsstrahlung contribution, the shape of the bremsstrahlung spectrum was estimated by hand interpolation between the spectra shown in Figure 12. This bremsstrahlung shape was subtracted with various intensities until the low-energy portion of the residual spectrum had a reasonable shape.

X-rays were usually removed by use of a suitable absorber in place of a portion of the 1.18 g/cm^2 beryllium or in addition to the beryllium. The amount of absorber used in these cases is noted in Table II. For Cs-137, the spectrum was simply extrapolated to zero pulse

TABLE I
SOURCES USED TO REPRESENT THE
DETECTOR RESPONSE TO MONOENERGETIC GAMMA RADIATION
(Data from Reference 10)

Source	Production	Mode of Decay	Half-Life	Gamma-Ray Energy (MeV)	Photon Intensity (% of decays)
Te-125m	n, γ	IT	58 d	0.110	0.3
Cs-134m	n, γ	IT, β^-	2.9 h	0.127	14
Ce-141	n, γ	β^-	32 d	0.145	8
Sc-47	n, p	β^-	3.4 d	0.155	65
Ce-139	n, γ	EC	140 d	0.166	80
Ru-97	n, γ	EC	2.9 d	0.213	?
Hg-203	n, γ	β^-	47 d	0.279	88
Cr-51	n, γ	EC	28 d	0.321	9
In-115m	Cd-115 decay	IT, β^-	4.4 h	0.336	45
In-113	Sn-113 decay	IT	1.7 h	0.393	66
Be-7	p, n	EC	53 d	0.478	12
Sr-85	n, γ	EC	64 d	0.513	100
Y-91m	Fission	IT	50 m	0.558	95
Cs-137	Fission (10 yr old)	β^-	30 y	0.6616	90
Nb-95	Fission	β^-	35 d	0.764	100
Mn-54	p, n	EC	314 d	0.835	100
Nb-92	γ , n	EC	10 d	0.931	98
Zn-65	n, γ	EC, β^+	245 d	1.114	44
Ar-41	n, γ	β^-	1.8 h	1.290	99
V-52	n, γ	β^-	3.8 m	1.43	100
K-42	n, γ	β^-	12 h	1.51	18
Al-28	n, γ	β^-	2.3 m	1.780	100
K-38	n, γ	EC, β^+	8 m	2.16	100

[10] Nuclear Data Sheets, compiled by K. Way et al (Printing and Publishing Office, National Academy of Sciences -- National Research Council, Washington 25, D. C.).

TABLE II

LIST OF RADIATIONS TO BE REMOVED FROM EXPERIMENTAL SPECTRA

Source	E_γ	Bremsstrahlung Maximum Beta Energy (MeV)	X-Ray		γ Rays	
			Energy (MeV)	Absorber Used (g/cm ²)	Energy (MeV)	Method
Te-125m	0.110	---	0.027	Cd(0.44)	0.035	Cd absorber
Cs-134m	0.127	---	0.031	Cd(0.44)	0.010, 0.137, and Cs-134	[a]
Ce-141	0.145	---	0.036	Cd(0.44)	---	
Sc-47	0.153	---	---		---	
Ce-139	0.166	---	0.033	Cd(0.44)	---	
Ru-97	0.213	---				
Hg-203	0.279	---	0.081	[b]	---	
Cr-51	0.323	---	---		---	
In-115m	0.336	---	0.024	Cu(0.09)	---	
In-113m	0.393	---	0.024	Cu(0.09)	---	
Be-7	0.478	---	---		---	
Sr-85	0.513	---	---		---	
Y-91m	0.558	---	---		---	
Cs-137	0.662	---	0.032	None	---	
Nb-95	0.764	---	---		---	
Mn-54	0.835	---	---		---	
Nb-92	0.931	---	---		0.90, 1.8	Ignored
Zn-65	1.11	---	---		0.511	S. S. [d]
Ar-41	1.29	1.2[c]			---	
V-52	1.43	2.5[c]			---	
K-42	1.51	3.6[c]			0.32	S. G. S. [e]
Al-28	1.78	2.8[c]			---	
K-38	2.16	---	---		1.64	S. G. S. [e]

[a] The Cs-134 contribution was determined after the Cs-134m had decayed. It was then subtracted from the original spectrum. The 0.010-keV gamma ray was absorbed in the cadmium and beryllium. The weak 0.137-keV gamma ray was ignored.

[b] The low-energy portion was estimated by making it the same as that of a Cr-51 spectrum measured with its photopeak at the position of the Hg-203 photopeak.

[c] The bremsstrahlung spectra used in these subtractions were estimated from the data in Figure 12.

[d] S. S. is the same as S. G. S. except an experimental shape for the 0.511-MeV annihilation radiation was used.

[e] S. G. S. indicates that the spectrum corresponding to this gamma ray was generated from the results of a previous pass through this program. This distribution was subtracted until the spectrum was smooth in the region of this gamma ray.

Fig. 12 Bremsstrahlung spectra measured on a 3 x 3 NaI detector. Spectra are normalized to about 10^7 disintegrations of the source.

height in order to remove the X-ray. Weak gamma rays were removed by generating the necessary shape from a previous run of the program to be described and subtracting the interfering gamma ray until the residual spectrum had an acceptable shape.

The short half-lives of Al-28 (2.3 minutes) and V-52 (3.8 minutes) introduce

a special problem. Since the gain of a photomultiplier changes with anode current, it will change as the source decays. On the other hand, hot sources cannot be used due to the accidental-summation problem. Therefore, several sources were used, one of which was added at regular intervals so that the counting rate was constant within a factor of two throughout the measurement.

All of the spectra which required major modifications are shown in Figures 13 through 16 before and after the corrections.

After removing the effects of extraneous radiations, the spectra are shifted to the standard gain scale. To accomplish this, another spectrum is measured of each source together with Cs-137. The positions x_o^m and $x_o(\text{Cs})$ for the two photopeaks are then determined by fitting them with a Gaussian function as described in the next section. The position of the photopeak on the standard scale is then

$$x_o = x_o^m \left[\frac{66.162}{x_o(\text{Cs})} \right] \text{ for the 10-keV scale,}$$

and

$$= x_o^m \frac{132.324}{x_o(\text{Cs})} \text{ for the 5-keV scale.}$$

The single gamma shapes are then shifted so their photopeaks are at these channels. This was accomplished with the gain-shift routine described in Section VI-3.

Fig. 13 Gamma-ray spectra for input to shape generation program showing original data (open circles) and data points after removal of unwanted contributions (solid circles).

Fig. 14 Gamma-ray spectra for input to shape generation program showing original data (open circles) and data points after removal of unwanted contributions (solid circles).

Fig. 15 Gamma-ray spectra for input to shape generation program showing original data (open circles) and data points after removal of unwanted contributions (solid circles).

Fig. 16 Gamma-ray spectra for input to shape generation program showing original data (open circles) and data points after removal of unwanted contributions (solid circles). Data for K-38 were measured with a different detector and were corrected to the proper energy resolution (see Section V-3.2).

2. PHOTOPEAK SHAPES AND PARAMETERS

2.1 Functional Representation

In most studies it is generally assumed that the photopeak can be satisfactorily described by a Gaussian function

$$\bar{y}(x) = y_0 e^{-\frac{(x-x_0)^2}{b_0}} \quad (1)$$

where $\bar{y}$ is the calculated count in the pulse-height channel x , x_0 is the pulse height at the center of the distribution, y_0 is the number of counts per channel at x_0 , and the full-width of the peak at $y_0/2$ is given by $w_0 = \sqrt{2 \ln 2} \sqrt{b_0}$.

Figure 17 shows a fit to the photopeak of a 0.6616-MeV gamma ray with a Gaussian function. The solid experimental points were used in the determination of the parameters y_0 , x_0 , and b_0 by use of a least-squares fitting program. The

Fig. 17 Fit to experimental photopeak of Cs-137 (0.662 MeV) showing deviation from gaussian function.

residuals, or differences between the experimental and calculated counts, are plotted in the regions in which they are systematically positive.

The large positive residuals on the left ($x < x_0$) are to be expected. The excess of counts in this region can be caused by loss of energy by photons resulting from small angle Compton scattering as they pass through material in front of the detector, or by multiple processes which do not result in total energy loss in the crystal. These residuals are taken into account in the program by considering them as part of the Compton distribution.

In contrast, the large residuals for $x > x_0$ do not have a simple explanation. Also, they cannot be taken into account as part of the Compton distribution. Due to this problem, an investigation was undertaken to determine a more satisfactory function for the representation of photopeaks. The approach was completely empirical in that no attempt was made to deduce the physically correct form. Thus, these results may be valid only for a particular counting system. (Some data for another system and a discussion of possible causes of the non-gaussian shapes are presented in Appendix II.)

The functional form to be used to represent a photopeak should

- (1) approximate a Gaussian in shape,
- (2) contain a small number of parameters in addition to y_0, x_0 , and b_0 ,
- (3) go to zero more slowly than e^{-x^2} , and
- (4) be symmetric about x_0 .

The desirability of (1), (2), and (3) is obvious. The last condition is necessary because the calculation of relative gamma-ray intensities (or source emission rates) is usually based on the area of the

photopeak (see Section III-4). This area is defined as the area under a symmetric function which fits the high-energy side of the photopeak.

The function used in this study was chosen arbitrarily, but it does satisfy the above conditions. It is called a "modified Gaussian" and has the form

$$\bar{y}(x) = y_0 e^{-\frac{(x-x_0)^2}{b_0}} [1 + \alpha_1 |x-x_0|^{n_1} + \alpha_2 |x-x_0|^{n_2}] \quad (2)$$

where the powers n_1 and n_2 are integers and y_0 , x_0 , b_0 , α_1 , and α_2 are parameters that are to be determined for each peak. The absolute values are taken for odd powers in order to make the function symmetric about x_0 . The five parameters are calculated by use of a nonlinear least-squares fitting method [11]. A nonlinear fitting routine written to do this calculation on an IBM-7040 computer is described in Reference 12.

2.2 Determination of n_1 and n_2

In the initial study, a set of sixteen gamma-ray spectra was used as data. Each spectrum contains a photopeak resulting from a monoenergetic gamma ray having little spectral background under it. Exceptions are Na-24 and Y-88, which

have coincidence sum spectra. For the measurements of these spectra this effect was reduced by moving the source to 40 and 60 cm from the crystal, respectively. The remaining real sum spectrum was compared with calculated distributions for such spectra (see Chapter VI). The calculated shape was then normalized and subtracted.

A series of exploratory fits was made in order to determine what single power n_1 (with $\alpha_2 \equiv 0$), or what pair of powers n_1 and n_2 , would provide the best fits. This decision depends critically on how far down on the high-energy side of the photopeak a fit is attempted. Practical limits result from the presence of accidental summing and the fact that the background subtractions were carried out without computing the standard deviations associated with the net count. In light of these problems, it was, somewhat arbitrarily, decided to fit the Cs-137 peak down a factor of 500 from y_0 . That is, $y_0/y_{\min} \approx 500$ where $y_{\min}$ is the smallest count used. The same considerations led to the $y_0/y_{\min}$ values listed in column 4 of Table III. On the low-energy side of the peak the last channel included in the region of fit had a count of about $y_0/3$.

For these values of $y_0/y_{\min}$, the best choice for a single power ($\alpha_2 \equiv 0$) appeared to be the 8th. Powers 3 through 10 were tried. When two terms were used, the best combination was the 4th and 12th powers, respectively. The quality of the fits for these two functional forms is compared in Table III with the quality of gaussian fits. Here, the quantity $\emptyset = \frac{\chi^2}{n-m} = \frac{1}{n-m} \sum w_i (y_i - \bar{y}_i)^2$ is used as a measure of the quality of the fit. ($\emptyset$ decreases as the fit improves.) Here $\bar{y}_i$ and y_i are the calculated and experimental counts, $w_i = \frac{1}{\sigma_{y_i}^2} = \frac{1}{y_i}$ is the weighting factor, n is the number of data points used, m is the number of variable parameters (4 or 5), and χ^2 is the quantity

[11] See, for example, R. H. Moore and R. K. Zeigler, The Solution of the General Least-Squares Problem with Special Reference to High-Speed Computers, Los Alamos Scientific Laboratory Report LA-2367 (1960).

[12] Marie Putnam, R. G. Helmer, R. L. Heath, and D. H. Gipson, A Non-Linear Least-Squares Program for the Determination of Parameters of Photopeaks by the Use of a Modified Gaussian Function, IDO-17016 (1965).

TABLE III

COMPARISON OF THE QUALITY OF FITS FOR A GAUSSIAN
AND MODIFIED GAUSSIANS WITH ONE OR TWO α -TERMS
(These data are from 10-keV scale.)

Isotope	E_γ	Number of Data Points Used	$\frac{y_o}{y_{min}}$	$\phi = \frac{\chi^2_{min}}{n-m}$		
				Gaussian	One α -Term $n_1 = 8$	Two α -Terms $n_1 = 4, n_2 = 12$
In-115m	0.336	10	1430	2.1	1.2	0.9
In-113m	0.393	10	820	8.4	1.3	1.2
Be-7	0.478	12	987	14	1.4	1.5
Sr-85	0.513	12	890	19	2.3	1.7
Y-91m	0.558	13	740	5.9	0.4	0.4
Cs-137	0.662	15	450	25	2.7	1.7
Nb-95	0.764	17	620	35	2.4	1.1
Mn-54	0.835	17	400	61	3.6	2.3
Nb-92	0.931	19	660	40	2.4	1.6
Zn-65	1.114	18	640	47	5.2	1.2
Ar-41	1.29	20	390	136	7.5	2.0
V-52	1.43	22	130	70	5.7	1.6
K-42	1.51	22	160	45	5.2	1.1
Al-28	1.78	25	150	36	4.9	1.0
Y-88	1.837	24	140	---	---	2.3
Na-24	2.754	30	90	43	6.4	1.5

which is known to statisticians as chi-squared [13].

It is clear that the addition of one α -term to the Gaussian decreases ϕ by an

- [13] R. H. Moore, "On the Meaning and Use of 'CHI-SQUARE' in Curve Fitting", Appendix II in Proceedings of Symposium on Applications of Computers to Nuclear and Radiochemistry, Edited by G. D. O'Kelley, NAS-NS Publication No. 3107 (1963).

order of magnitude above 0.4 MeV. The addition of the second α -term reduces this quantity by an additional factor of from 2 to 5 above 0.6 MeV. If these photopeaks were modified Gaussians with $n = 4$ and $n_2 = 12$ and the weights are properly chosen, ϕ should be 1 on the average [13]. Since it is not known what the correct function is, the final range of 0.4 to 2.3 for ϕ is considered satisfactory.

On the basis of the data in Table III it was decided to use $n_1 = 4$ and $n_2 = 12$ in the remainder of the study. The

contributions of each of the three terms,
 $y_0 e^{-\frac{(x-x_0)^2}{b_0}}$, $y_0 e^{-\frac{(x-x_0)^2}{b_0}} \alpha_1 (x-x_0)^4$,
 and $y_0 e^{-\frac{(x-x_0)^2}{b_0}} \alpha_2 (x-x_0)^{12}$, are
 shown in Figure 18 for photopeaks of 0.393,
 0.662, and 1.78 MeV.

2.3 Variation of α_1 , α_2 , and w_0 with Photopeak Position

The investigation of photopeak shapes was carried out on a set of experimental spectra. Before the resulting photopeak parameters can be used, they must be recalculated for the standard gain scale. On the standard scale, the results of the fits are given in Table IV and Figures 19 and 20. The errors quoted are the values from the least-squares fit and, therefore, do not include any estimate of the nonstatistical errors.

For use in the generation of spectra of monoenergetic gamma rays, it is necessary to interpolate the values of α_1 , α_2 , and w_0 at the desired value of x_0 . This can be done most efficiently by representing $\alpha_1(x_0)$, $\alpha_2(x_0)$, and $w_0(x_0)$ by analytical functions. For α_1 and α_2 a polynomial of the form

$$\log \alpha = a_0 + a_1(\log x_0) + a_2(\log x_0)^2 + a_3(\log x_0)^3 + \dots + a_n(\log x_0)^n \quad (3)$$

is used. For each α the data were fitted by the least-square method with polynomials of degrees 1 through 6. Then the lowest degree which gave a satisfactory fit was used. Due to the large errors in the low-energy data for α_1 , weighted least-squares fits were made to these data.

Fig. 18 Contribution of each of the three terms in the "modified-gaussian" expression to photopeaks of 0.393, 0.662, and 1.78 MeV.

The data for $w_0(x_0)$ were fitted with a polynomial in $\sqrt{x_0}$; that is,

$$w_0 = \alpha_0 + \alpha_1 \sqrt{x_0} + \alpha_2 (\sqrt{x_0})^2 + \dots + \alpha_n (\sqrt{x_0})^n \quad (4)$$

The resulting sets of coefficients are given in Table V. The solid curves in Figures 19 and 20 represent these functions.

TABLE IV
EXPERIMENTAL PHOTOPEAK PARAMETERS

Isotope	10-keV Scale				5-keV Scale			
	x_0	b_0	α_1	α_2	x_0	b_0	α_1	α_2
Te-125m	---	---	---	---	24.35	4.83 ± 0.04	$(0 \pm 4.0) \times 10^{-4}$	$(5.0 \pm 8.0) \times 10^{-10}$
Cs-134m	---	---	---	---	27.63	5.00 ± 0.04	$(3.0 \pm 2.0) \times 10^{-4}$	$(4.0 \pm 6.0) \times 10^{-11}$
Ce-141	---	---	---	---	31.62	6.6 ± 0.1	$(4.0 \pm 1.0) \times 10^{-4}$	$(1.9 \pm 0.3) \times 10^{-10}$
Sr-87	17.102	1.86 ± 0.02	$(2.3 \pm 1.0) \times 10^{-3}$	$(2.1 \pm 4.0) \times 10^{-8}$	34.20	6.7 ± 0.1	$(2.0 \pm 1.0) \times 10^{-4}$	$(9.0 \pm 1.0) \times 10^{-11}$
Ce-139	17.690	1.94 ± 0.03	$(3.5 \pm 2.2) \times 10^{-3}$	$(9.1 \pm 6.0) \times 10^{-10}$	35.42	7.2 ± 0.1	$(1.0 \pm 1.0) \times 10^{-4}$	$(1.0 \pm 0.2) \times 10^{-10}$
Ru-97	---	---	---	---	45.10	10.4 ± 0.1	$(4.0 \pm 3.0) \times 10^{-5}$	$(5.0 \pm 1.0) \times 10^{-12}$
Hg-203	28.856	4.46 ± 0.04	$(0 \pm 2.0) \times 10^{-4}$	$(9.7 \pm 2.4) \times 10^{-10}$	57.71	16.1 ± 0.1	$(1.0 \pm 0.9) \times 10^{-5}$	$(2.7 \pm 0.8) \times 10^{-13}$
Cr-51	32.883	5.3 ± 0.1	$(0 \pm 3.0) \times 10^{-4}$	$(3.9 \pm 1.7) \times 10^{-10}$	65.77	18.9 ± 0.1	$(0 \pm 1.0) \times 10^{-5}$	$(1.5 \pm 0.5) \times 10^{-13}$
In-115m	34.24	5.3 ± 0.1	$(0 \pm 1.5) \times 10^{-4}$	$(2.1 \pm 0.9) \times 10^{-10}$	68.95	19.2 ± 0.2	$(0.7 \pm 1.1) \times 10^{-5}$	$(5.7 \pm 4.1) \times 10^{-14}$
In-113m	39.937	6.2 ± 0.1	$(0.8 \pm 0.7) \times 10^{-4}$	$(2.3 \pm 0.3) \times 10^{-10}$	79.88	23.5 ± 0.2	$(0.3 \pm 0.6) \times 10^{-5}$	$(5.5 \pm 1.0) \times 10^{-14}$
Be-7	48.10	7.5 ± 0.1	$(2.1 \pm 0.5) \times 10^{-4}$	$(1.0 \pm 0.1) \times 10^{-10}$	96.95	28.9 ± 0.4	$(1.8 \pm 0.4) \times 10^{-5}$	$(2.5 \pm 0.3) \times 10^{-14}$
Sr-85	51.98	8.2 ± 0.1	$(3.4 \pm 0.5) \times 10^{-4}$	$(3.2 \pm 0.5) \times 10^{-11}$	103.97	31.4 ± 0.2	$(2.0 \pm 0.4) \times 10^{-5}$	$(1.7 \pm 0.2) \times 10^{-14}$
Y-91m	55.83	9.6 ± 0.1	$(1.0 \pm 0.6) \times 10^{-4}$	$(1.9 \pm 0.4) \times 10^{-11}$	112.37	35.5 ± 0.3	$(1.4 \pm 0.3) \times 10^{-5}$	$(1.1 \pm 0.1) \times 10^{-14}$
Cs-137	66.16	10.9 ± 0.1	$(2.0 \pm 0.5) \times 10^{-4}$	$(1.2 \pm 0.2) \times 10^{-11}$	132.32	42.4 ± 0.5	$(1.4 \pm 0.3) \times 10^{-5}$	$(3.0 \pm 0.6) \times 10^{-15}$
Nb-95	76.16	12.8 ± 0.1	$(2.1 \pm 0.2) \times 10^{-4}$	$(6.0 \pm 0.5) \times 10^{-12}$	152.18	49.8 ± 0.3	$(1.2 \pm 0.1) \times 10^{-5}$	$(2.0 \pm 0.1) \times 10^{-15}$
Mn-54	82.83	14.1 ± 0.1	$(2.0 \pm 0.7) \times 10^{-4}$	$(4.2 \pm 0.4) \times 10^{-12}$	165.69	54.1 ± 0.4	$(1.5 \pm 0.1) \times 10^{-5}$	$(1.2 \pm 0.1) \times 10^{-15}$
Nb-92	92.36	16.9 ± 0.2	$(1.4 \pm 0.3) \times 10^{-4}$	$(1.4 \pm 0.1) \times 10^{-12}$				
Zn-65	109.51	19.4 ± 0.2	$(1.7 \pm 0.3) \times 10^{-4}$	$(9.6 \pm 1.0) \times 10^{-13}$	219.30	75.8 ± 0.6	$(1.0 \pm 0.2) \times 10^{-5}$	$(2.2 \pm 0.3) \times 10^{-16}$
Ar-41	126.19	23.8 ± 0.3	$(1.1 \pm 0.1) \times 10^{-4}$	$(3.7 \pm 0.4) \times 10^{-13}$				
V-52	139.91	27.5 ± 0.3	$(9.1 \pm 0.9) \times 10^{-5}$	$(1.8 \pm 0.2) \times 10^{-13}$				
K-42	140.66	29.6 ± 0.4	$(8.1 \pm 0.8) \times 10^{-5}$	$(8.8 \pm 1.0) \times 10^{-14}$				
Al-28	172.31	35.1 ± 0.5	$(6.8 \pm 0.7) \times 10^{-5}$	$(3.7 \pm 0.5) \times 10^{-14}$				
Na-24	266.35	61.1 ± 1.0	$(3.0 \pm 0.3) \times 10^{-5}$	$(2.2 \pm 0.3) \times 10^{-15}$

Fig. 19 Photopeak widths (full-width-at-half-maximum) as a function of photopeak position for 5- and 10-keV energy scales.

Fig. 20 Plots of the variation of the photopeak parameters α_1 and α_2 for the 5- and 10-keV scales.

TABLE V

COEFFICIENTS IN POLYNOMIAL EXPRESSIONS FOR w_0 , α_1 , AND α_2

Coefficient	10-keV Scale			5-keV Scale		
	w_0	α_1	α_2	w_0	α_1	α_2
α_0	-1.4816	-11.536	14.652	5.282	-11.4868	- 740.8588
α_1	1.00850	11.495	-31.614	-2.372	7.6339	2079.0172
α_2	-0.027944	- 5.116	14.415	0.59365	- 2.5126	-2293.7513
α_3	0.0012692	0.6488	- 2.626	-0.042602	0.1864	1235.7398
α_4				0.0010922		- 327.1203
α_5						34.12756

The area of a photopeak is given by

$$N_p = \int_{-\infty}^{+\infty} y(x) dx$$

where

$$y(x) = y_0 e^{-\frac{(x-x_0)^2}{b_0}} [1 + \alpha_1 (x-x_0)^4 + \alpha_2 (x-x_0)^{12}] \quad (5)$$

and $b_0 = \frac{w_0^2}{4 \ln 2}$. Integrating gives

$$N_p = y_0 \sqrt{\pi b_0} [1 + \frac{3}{4} \alpha_1 b_0^2 + \frac{10,395}{64} \alpha_2 b_0^6]. \quad (6)$$

3. PULSE HEIGHT VERSUS ENERGY RELATIONSHIP

3.1 Nonlinear Response of Spectrometer

It has long been known that the observed pulse height of the photopeak of a gamma ray is approximately proportional to the energy of the gamma ray. However, it has been shown [7, 8] that the light output of a NaI(Tl) crystal is not exactly proportional to the energy absorbed in the crystal. This fact, together

with possible electronic nonlinearities, makes it necessary to establish empirically the relationship between pulse height and energy for any given spectrometer.

Measurements of the response of the electronic system used are described in the following paragraphs. That portion of the nonlinearity due to the analyzer has been noted in Section II-2.2 and is discussed in Appendix I.

3.2 Method of Measurement

3.21 Zero Adjustment. Generally, the position of the "zero" of the pulse-height scale of the multichannel analyzer would not be important. (This position is defined as the channel in which a pulse of zero amplitude would be stored.) However, if it is desirable to have a pulse height-energy relationship which is valid for all gain scales, the zero of the pulse-height scale must be adjusted to -0.5 channel. (See Appendix I for arguments.) Also in this investigation it was desired to make a comparison of the system with the results of Engelkemeir [7] and of Managan [8] for the nonlinearity of NaI(Tl). To make this comparison it is necessary to measure relative pulse heights for photopeaks from various energy gamma rays. This requires

the same zero setting for all measurements.

3.22 Pulse-height Measurements. A convention has been adapted such that all pulse heights are measured relative to that of the 0.66162-MeV gamma ray from Cs-137. (This is the convention used in References 7 and 8.) For these measurements, a series of radioactive sources emitting gamma rays with accurately known energies was chosen. The spectrum of each of these sources was then measured together with a Cs-137 source. See, for example, Figure 21. The source strengths were adjusted so that the Compton distribution from the higher energy gamma ray contributed less than 10 percent of the total count at the peak of the lower energy gamma ray.

The pulse heights of the photopeaks were determined by least-squares fits to the data with a gaussian function, as discussed in Section III-2. (The parameters α_1 and α_2 of the "modified Gaussian"

were set equal to zero so that the functional form used was a gaussian.) For the lower energy peak, the contribution from the underlying Compton distribution was approximated by a straight line which was added to the Gaussian.

The data were taken with the Cs-137 photopeak very close to one of its standard positions; that is, channel 66.16 (10 keV per channel scale) or 132.324 (5-keV scale). This was done so that any nonlinearities in the electronics system would be the same for all spectra on one scale.

For simplicity in comparing data from the 10- and 5-keV scales, a measure of pulse height which is independent of the gain is used. One "pulse-height unit" is defined as $\frac{1}{66.162}$ times the pulse height of the 0.66162-MeV gamma ray from Cs-137. Thus, one pulse-height unit (PHU) is equal to one channel on the 10-keV scale, but it is equal to two channels on the 5-keV scale.

Fig. 21 Energy calibration procedure showing measurement of Hg-203 and Lu-177 together with Cs-137. Peak channel positions indicated are results of least-squares fits of modified-gaussian shape to each photo-line. The two figures show calibration of relative peak positions for 0.279-MeV line in Hg-203 and the 0.1129- and 0.20836-MeV lines in Lu-177.

3.3 Pulse-Height-Energy Relationship

The results of the relative pulse-height measurements are listed in Table VI and shown in Figure 22 where the ratio of the energy (MeV) to the pulse height $\frac{E_\gamma}{\text{PHU}}$ is plotted against the pulse height and $\frac{\text{PHU}}{E_\gamma}$ is plotted as a function

of E_γ . These graphs contain both 5- and 10-keV data. The errors quoted in Table VI are the estimated standard deviations computed in the least-squares fit and do not include any estimate of the systematic errors involved. (A discussion of some aspects of the accuracy of the method is given in Appendix I).

The smooth curves in Figure 22 represent the functions

$$E(\text{PHU}) = -0.01387 + 0.01030 x - 5.439 \times 10^{-6} x^2 + 8.350 \times 10^{-8} x^3 - 3.701 \times 10^{-10} x^4 + 5.190 \times 10^{-13} x^5, \text{ and} \quad (7)$$

$$\text{PHU}(E) = 1.320 + 97.55E + 3.885E^2 - 6.453E^3 + 2.841E^4 - 0.3894E^5 \quad (8)$$

TABLE VI

RESULTS OF RELATIVE PULSE-HEIGHT MEASUREMENTS
USED TO DETERMINE PULSE-HEIGHT-ENERGY RELATIONSHIP

Source	Gamma-Ray Energy (MeV)	10-keV Scale				5-keV Scale			
		Pulse Heights		Pulse Height on Standard Scale	PH 100 E_γ	Pulse Heights		Pulse Height on Standard Scale	PH 200 E_γ
		Cs-137	Other			Cs-137	Other		
Am-241	0.05957					133.46 ± 0.03	13.40 ± 0.01	13.78 ± 0.01	1.157
Te-125m	0.110					132.86 ± 0.03	24.51 ± 0.01	24.35 ± 0.01	1.110
Lu-177	0.11297					133.25 ± 0.03	24.96 ± 0.01	24.78 ± 0.01	1.157
Cs-134m	0.1274					132.21 ± 0.04	27.62 ± 0.01	27.64 ± 0.01	1.177
Ce-141	0.145	66.16 ± 0.01	15.35 ± 0.02	15.35 ± 0.03	1.059	132.65 ± 0.03	31.19 ± 0.01	31.12 ± 0.01	1.074
Ce-139	0.1664					132.85 ± 0.03	35.56 ± 0.01	35.42 ± 0.01	1.066
Lu-177	0.20836	66.25 ± 0.01	21.77 ± 0.01	21.74 ± 0.01	1.048	133.25 ± 0.03	44.00 ± 0.01	43.69 ± 0.01	1.048
In-115m	0.33625	66.52 ± 0.01	34.45 ± 0.01	34.27 ± 0.01	1.020	132.77 ± 0.02	69.17 ± 0.02	68.94 ± 0.02	1.026
I-131	0.36447	66.70 ± 0.01	37.43 ± 0.01	37.14 ± 0.01	1.019	133.24 ± 0.03	75.17 ± 0.01	74.65 ± 0.02	1.024
In-113m	0.393	66.00 ± 0.01	39.89 ± 0.01	39.99 ± 0.01	1.013	132.93 ± 0.03	80.25 ± 0.01	79.88 ± 0.02	1.019
Be-7	0.478	66.75 ± 0.01	48.74 ± 0.01	48.31 ± 0.01	1.011	132.57 ± 0.03	97.11 ± 0.02	96.93 ± 0.02	1.014
Sr-85	0.515	66.61 ± 0.01	52.33 ± 0.01	51.98 ± 0.01	1.009	133.11 ± 0.03	104.59 ± 0.02	103.97 ± 0.02	1.009
Bi-207	0.5695	66.57 ± 0.01	57.65 ± 0.01	57.30 ± 0.01	1.006				
Cs-137	0.6616				≡1.000				≡1.000
Nb-95	0.7657	65.93 ± 0.01	75.95 ± 0.01	76.16 ± 0.01	0.995	132.82 ± 0.03	152.74 ± 0.03	152.18 ± 0.04	0.994
Mn-54	0.835	66.50 ± 0.01	83.17 ± 0.02	82.76 ± 0.02	0.991	134.02 ± 0.03	167.83 ± 0.03	165.70 ± 0.04	0.992
Y-88	0.8989	66.20 ± 0.01	88.98 ± 0.02	88.93 ± 0.02	0.989				
Nb-92	0.931	66.00 ± 0.01	92.13 ± 0.02	92.16 ± 0.03	0.992				
Bi-207	1.0637	66.57 ± 0.01	105.46 ± 0.02	104.82 ± 0.03	0.985				
Zn-65	1.114	66.20 ± 0.01	109.56 ± 0.02	109.51 ± 0.03	0.983	133.25 ± 0.03	219.52 ± 0.04	217.99 ± 0.07	0.978(a)
V-52	1.43	65.94 ± 0.01	139.44 ± 0.02	139.91 ± 0.03	0.978				
K-42	1.52	65.98 ± 0.01	148.24 ± 0.02	148.66 ± 0.03	0.978				
Al-28	1.780	66.271 ± 0.01	172.60 ± 0.03	172.31 ± 0.04	0.968				
Y-88	1.837	66.20 ± 0.01	178.11 ± 0.03	178.01 ± 0.04	0.969				
Na-24	2.7535	[a]		263.35 ± 0.03	0.956				

[a] Measured relative to 1.368-keV peak in same source using 0.977 for PH/100 E_γ at this energy.

Fig. 22 Results of pulse-height versus energy measurements for 5- and 10-keV scales. The curves represent the functions in Equations (7) and (8).

where the coefficients were determined by a least-squares fits to the data in Table VI. In Table VII a comparison is given of the values of these polynomials with the experimental data (columns 1 and 2). The energies in column 4 are from Equation (7) for the experimental pulse heights and the pulse heights in column 3 are from Equation (8) evaluated at the energies in column 1.

A comparison of these data with those in References '7 and '8 for the response of a 3 x 3 inch NaI(Tl) detector is shown in Figure 23. It is clear that the predominant contribution to the nonproportionality of the system is from the detector. (This verifies the electrical measurements of system linearity discussed in Appendix I.)

TABLE VII

COMPARISON OF VALUES FROM EQUATIONS (7) AND (8) WITH EXPERIMENTAL DATA

Experimental Data		Calculated Values	
E_γ	PHU	PHU(E)	E(PHU)
0.11297	12.362	12.381	0.11274
0.20836	21.742	21.761	0.20821
0.36447	37.136	37.125	0.36462
0.478	48.404	51.496	0.51059
0.5695	57.298	57.218	0.57030
0.66162	66.162	66.187	0.66132
0.7657	76.155	76.269	0.76449
0.8989	88.930	89.086	0.89727
1.0637	104.82	104.819	1.06372
1.114	109.606	109.598	1.11411
1.2736	124.895	124.700	1.27569
1.837	177.76	177.833	1.83620
2.7535	266.30	266.340	2.75365

Fig. 23 Comparisons of the pulse-height versus energy relationship determined for this complete spectrometer with the previous results for NaI detectors.

4. PHOTOPEAK EFFICIENCY

The principle objective in the analysis of a pulse-height distribution obtained with a NaI scintillation spectrometer is to obtain energies and intensities for the gamma rays emitted from the source. The usual method employed to obtain absolute source intensities is to calculate from a knowledge of the gamma-ray absorption cross section of NaI(Tl), the source-detector geometry, and other experimental parameters, the probability of detecting a photon of a given energy ϵ_t . For a given experimental arrangement, it is possible to derive exact expressions for the detection efficiency. Extensive calculations of detection efficiencies for cylindrical NaI detectors have been published by

Vegors, Marsden, and Heath [14]. For the detector size and source-detector geometry used it is possible to obtain a value for the total efficiency of the detector for a monoenergetic photon. If the source and detector were completely isolated from the outside world, then the emission rate from a monoenergetic source would be represented by the area under the resultant pulse-height spectrum divided by this calculated efficiency. In a practical case, however, the pulse-height

- [14] S. H. Vegors, Jr., L. L. Marsden, and R. L. Heath, Calculated Efficiencies of Cylindrical Radiation Detectors, USAEC Report IDO-16370 (1958).

spectrum will contain additional counts from photons scattered by the detector shield and other material in the vicinity of the detector. These scattered photons are not included in the calculated value for detection efficiency.

In the usual experimental arrangement the unwanted contribution to the pulse-height distribution results from Compton scattering of photons at rather large angles. These scattered photons are considerably reduced in energy from the primary gamma-ray and contribute only to that portion of the pulse-height spectrum due to Compton events in the crystal. Since the full-energy peak remains virtually free from any spurious contribution, its area is still proportional to the emission rate from the source. The quantity known as the peak-to-total ratio is then defined as the fraction of total events in the pulse-height spectrum, excluding all spurious scattered radiation, which appears in the full-energy peak (see Figure 24).

$$P = \frac{N_p}{N_t} = \text{peak to total ratio, where}$$

N_p = area of full-energy peak (symmetric about x_0 whose shape is determined by top and high-energy side of observed distribution)

N_t = area under pulse-height distribution for gamma ray minus all scattered photons

Experimentally, the peak-to-total ratio is obtained by measuring the response of a detector in a specified geometrical arrangement to a series of monoenergetic gamma-ray sources. To eliminate scattering from surrounding materials, such measurements are made with the source and detector mounted in a large volume shield (8 x 8 x 8 feet). In this geometry the resulting pulse height spectrum will contain almost no contribution from scattered gamma rays. Sources used for this purpose are mostly nuclides which decay by electron capture or are accompanied by only low-energy

beta rays. This allows the possibility of obtaining data with no beta absorbing material between source and detector, thus eliminating any distortion of the spectrum by small-angle Compton scattering.

The peak-to-total ratio is then obtained by fitting a "modified-Gaussian" shape to the photopeak, as described in Section III-2, to obtain the peak area. This is then related to the integrated area under the entire spectrum using the relationship above. For gamma-ray energies above 1.5 MeV, it is difficult to obtain sources which are free from bremsstrahlung and other complicating contributions to the spectrum. In these cases an alternative method is used. The emission rate of monoenergetic sources is first determined by use of

Fig. 24 Illustration of definition of peak-to-total ratio. This spectrum was measured without an electron absorber; this accounts for the fact that the valley is deeper than in the spectrum used in the shape generation program. Note that the backscatter peak is not included in the total area.

$4\pi\beta\text{-}\gamma$ coincidence techniques. The peak efficiency is then determined by measuring these calibrated sources with a NaI detector.

A list of nuclides used for the determination of the peak-to-total ratio as a function of gamma-ray energy is shown in Table VIII along with experimental results for a 3 x 3 inch NaI detector. Figure 25 shows a plot of these results.

In the usual experimental arrangement it is necessary to use absorbing material to eliminate beta particles from the detector. The calculation of emission rates must then include a correction factor A to account for the absorption of gamma rays by the beryllium beta absorber. Values of A (photopeak attenuation) as a function of gamma-ray energy are given in Reference 9. The emission

Fig. 25 Peak-to-total ratio as a function of the gamma-ray energy.

TABLE VIII

EXPERIMENTAL PEAK-TO-TOTAL RATIOS FOR 3 x 3 INCH NaI DETECTOR

Isotope	E_γ (MeV)	Point Source		
		10 cm Source Distance		3 cm Source Distance
		Integration	$4\pi\beta\text{-}\gamma$	Integration
Sc-47	0.155	0.960		0.962
Ce-139	0.166	0.950		
Cr-51	0.323	0.820		0.813
Au-198	0.4117		0.737	
Be-7	0.478	0.668		0.657
Cs-137	0.6616	0.536		0.532
Nb-95	0.766	0.500	0.504	
Mn-54	0.835	0.474		0.464
Zn-65	1.114	0.395		0.388
Co-60	1.382		0.357	
Al-28	1.78	0.290	0.295	
Y-88	1.837	0.280		
Na-24	2.753		0.225	
S-37	3.13		0.207

Fig. 26 Photopeak efficiency (including correction for beryllium absorber) as a function of energy. The curve represents the functions in Equations (9) and (10).

rate will then be equal to the observed photopeak area divided by $A\epsilon_p$, where $\epsilon_p = P_{et}$. In Figure 26 a plot of $A\epsilon_p$

is shown for a point source 10 cm from a 3 x 3 inch NaI detector. The curve represents the functions

$$A\epsilon_p = 0.00136 + 0.73157E - 7.8085E^2 + 39.468E^3 - 98.717E^4 + 96.75E^5 \quad (9)$$

for $E \leq 0.3$ MeV

$$A\epsilon_p = 0.000342 + 0.006841\left(\frac{1}{E}\right) - 0.000419\left(\frac{1}{E}\right)^2 + 0.000380\left(\frac{1}{E}\right)^3 - 0.000169\left(\frac{1}{E}\right)^4 + 0.0000188\left(\frac{1}{E}\right)^5 \quad (10)$$

for $E \geq 0.3$ MeV

which were obtained from least-square fits to experimental values. The accuracy of the ratio of any two points on the best curve through these data is believed to be about $\pm 1\frac{1}{2}$ percent. It should be noted that the data in Figure 26 and Equations (9) and (10) do not include the latest values of P reported in Reference 9, although the differences are quite small.

A series of spectra containing two gamma-ray transitions of equal intensity was analyzed in order to provide an independent check on the $A\epsilon_p$ values. In these spectra the two photopeaks are well resolved. Each peak was fitted with a modified-Gaussian function to determine its area. The results given in Table IX indicate very satisfactory agreement between measured intensities and those to be

TABLE IX
CHECK OF $A\epsilon_p$ RESULTS

Isotope	Transition Energy (MeV)	Line Intercept (counts)	$\frac{\chi^2}{n-m}$	Photopeak Area (counts)	$\frac{1}{A\epsilon_p}$	Ratio of Gamma-Ray Intensities	Value Expected from Decay Scheme
Y-90m	0.203	100	46	$(83.8 \pm 1.0) \times 10^3$	41.84	1.080	1.08[a]
	0.482	200	6	$(43.0 \pm 0.3) \times 10^3$	75.50	1.00	
Nb-94	0.702	80	2.3	$(201.5 \pm 0.4) \times 10^3$	102.0	1.00	
	0.871	110	0.8	$(168.2 \pm 0.2) \times 10^3$	125.0	1.023	1.000
Sc-46	0.887	100	5.7	$(239.3 \pm 0.6) \times 10^3$	125.3	1.00	
	1.119	100	1.7	$(195.1 \pm 0.3) \times 10^3$	155.1	1.009 ± 0.004	1.000
V-48	0.99	250	4.8	$(194.0 \pm 0.5) \times 10^3$	141.8	1.075 ± 0.005	1.000
	1.31	200	2.2	$(138.4 \pm 0.3) \times 10^3$	184.7	1.00	
Na-24	1.369	1900	3.9	$(296.3 \pm 5.4) \times 10^3$	192.8	1.00	
	2.754	0	1.7	$(165.5 \pm 0.5) \times 10^3$	347.2	1.005 ± 0.020	0.999

[a] Computed from $\alpha_t(482) = 0.11$ and $\alpha_t(203) = 0.03$ as given by R. L. Heath, J. E. Cline, C. W. Reich, E. C. Yates, and E. H. Turk, *Phys. Rev.* **123**, 903 (1961).

expected from a detailed knowledge of the decay scheme of each isotope.

The amount of annihilation radiation removed is computed relative to a generated shape corresponding to an 0.511-MeV gamma ray. The experimental intensity as a function of photon energy is shown in Figure 27. The solid curve represents the function

5. INTENSITY OF ANNIHILATION RADIATION

In the computer program for generation of gamma-ray shapes, it is convenient to treat the annihilation radiation as a separate effect. Therefore, the 0.511-MeV peak in any experimental spectrum is removed before the spectrum is used.

$$I_{511} = 3 \times 10^{-4} (\text{PHU} - 100) \quad (11)$$

where the pulse-height, PHU, is as defined above.

Fig. 27 Intensity of annihilation radiation, resulting from the pair production interaction in the surrounding material, as a function of the primary gamma-ray energy.

IV. METHOD OF INTERPOLATION

1. NORMALIZATION AND PHOTOPEAK SUBTRACTION

1.1 Normalization

In order to carry out the desired interpolation, it is necessary to normalize the experimental gamma-ray shapes in some manner. They could be adjusted to have the same photopeak height or equal photopeak areas. For use in spectral analysis problems, however, it is more convenient to normalize them to the same number of gamma rays emitted from the source.

In this program the input spectra are normalized to the emission of 10^7 photons from a source in the standard geometric arrangement. This is done by multiplying each count a spectrum by the factor

$$f = \frac{N_{\text{norm}}}{N}$$

where N is the area of the photopeak of the input spectrum and N_{norm} is the area of a photopeak that corresponds to

the emission of 10^7 photons. The area N is calculated from Equation (6) for values of w_0 , α_1 , and α_2 obtained from the polynomial expressions in Equations (3) and (4) for the pertinent value of x_0 , and y_0 obtained from a fit to the spectrum used. This value of N is calculated by another program and supplied as input. N_{norm} is given by

$$N_{\text{norm}} = 10^7 A \epsilon_p$$

where $A \epsilon_p$ is evaluated from Equation (9) or (10). Then

$$f = \frac{10^7 A \epsilon_p}{N} \quad (12)$$

After normalization of the input spectra, a three-dimensional surface can be constructed representing the response of a NaI(Tl) scintillation spectrometer to monoenergetic photons (see Figure 28).

1.2 Photopeak Subtraction

After an input spectrum has been normalized, the calculated photopeak distribution is subtracted. As previously described, the photopeak is represented by

$$y(x) = y_0 e^{-\left(\frac{x-x_0}{w_0/2 \sqrt{\ln 2}}\right)^2} \left[1 + \alpha_1 (x-x_0)^4 + \alpha_2 (x-x_0)^{12} \right] \quad (5)$$

The values used for x_0 , w_0 , α_1 , and α_2 are the same as those used in the intensity normalization while y_0 is obtained from the expression

$$y_0 = \frac{10^7 A \epsilon_p}{\sqrt{\frac{\pi}{4 \ln 2}} w_0 \left[1 + 3/4 \alpha_1 \left(\frac{w_0^2}{4 \ln 2}\right)^2 + \frac{10,395}{64} \alpha_2 \left(\frac{w_0^2}{4 \ln 2}\right)^6 \right]} \quad (13)$$

Fig. 28 Three-dimensional representation of the response of a NaI scintillation spectrometer to mono-energetic gamma rays.

2. STRETCHING AND SMOOTHING OF INPUT SPECTRA

2.1 A Simple Interpolation Scheme

Although the following procedure is not used for the interpolation, it is included because it illustrates the ideas involved in the actual program.

After intensity normalization and subtraction of the photopeaks, a group of residual, or Compton, distributions is obtained such as is shown in Figure 29. One possible method of interpolation would be by obtaining a set of polynomials, each of which represented the experimental counts in one channel as a function of the photopeak position x_0 . Figure 30 shows the points which would then have to be represented for channels 20, 40, 60, 80. It is evident that a high degree polynomial might be needed to fit these points since the function varies over a large range of intensity. A more serious problem is that as the value of

Fig. 29 Six of the normalized, residual spectra for gamma rays between 0.30 and 1.2 MeV.

Fig. 30 Plot of points in channels 20, 40, 60, and 80 of all eleven of the normalized, residual spectra between 0.3 and 1.2 MeV. These points represent the function which would provide the interpolation between input shapes, if the simple method were used.

the channel increases, the number of points defining the function decreases to such an extent that interpolation is not meaningful. The actual procedure used was devised in order to eliminate these two problems.

2.2 Segmentation

The complete set of input spectra are divided into three energy ranges and each range is processed separately. The high-energy range includes those spectra which exhibit features resulting from pair production (i.e., peaks at 0.511 MeV and $E_\gamma - 0.511$ MeV). The middle range extends down to where the back-scatter peak and Compton edge have about

the same energy. The low-energy range extends down as far as is feasible. In order to provide continuity between ranges in the interpolation, at least one spectrum is common to any two adjacent regions.

For simplicity, the description of the interpolation method will be limited to the middle-energy range and data on the 10-keV scale. Then the modifications needed to treat the other two regions will be discussed. No modifications are required to handle different gain scales.

In the middle-energy region the prominent features of any single gamma-ray spectrum are the photopeak (with energy E_γ and channel position X'_0), the Compton edge (energy E_C and position X'_C), and the backscatter peak (with E_b and X'_b). Given E_γ in MeV

$$E_b = \frac{E_\gamma}{1 + \frac{2}{0.511} E_\gamma} \quad (14)$$

and

$$E_C = E_\gamma - E_b. \quad (15)$$

The positions X'_0 , X'_C , and X'_b can then be calculated from the relationship in Equation (8). As input, the program requires X'_0 so E_γ is initially computed from Equation (7). The other energies are then computed for this value of E_γ . The values of the X'_j will not be integers. For use in the program these values are next rounded to the nearest integer, and are called X_0 , X_C , and X_b , hereafter.

For each spectrum the values of X_0 , X_C , and X_b are computed. After subtraction of the photopeak, a normalized spectrum consists of the three segments $1 \leq x \leq X_b$, $X_b < x \leq X_C$, $X_C < x \leq X_0$ where x is the channel or pulse height (see Figure 31).

2.3 Sine Series Fits to Segments

Each segment of each spectrum is fitted with the function

$$g(u) = a + b u + \sum_{k=1}^N b_k \sin \frac{k \pi u}{M-1} \quad (16)$$

where the variable u is 0 at one end of the segment and $u = M-1$ at the other end.

The decision to use a function of this form to fit the segments was motivated by the following:

- (1) The coefficients a , b , and b_k are easily computed with no trouble from roundoff error.
- (2) The experimental data can be faithfully reproduced by taking a large number of terms.
- (3) Any degree of smoothing can be achieved by limiting the number of terms.
- (4) Values of $g(u)$ can be computed easily at noninteger values of u .

In order to ensure a smooth "joint" where two segments abutt, four points on either side of the endpoints are used

Fig. 31 Segments and ends of segments for spectra between about 0.3 and 1.3 MeV.

in generating the fit to a segment. For example, to generate the coefficients a , b , and b_k for the segment X_b to X_c , the fitting limits are chosen as $X_b - 4$ and $X_c + 4$. Experience has shown that this procedure generates a smooth curve when the fitted spectrum is recomputed. The fitting limits used are listed in Table X.

The numerical procedure for computing a , b , and b_k is based on the assumption that M values $y_0, y_1, y_2, \dots, y_{M-1}$ are plotted on a segment from $u = 0$ to $u = M-1$ (see Figure 32).

Fig. 32 Sine series fit to M data points in one segment.

A Fourier Series fit to these data assumes that they are values of a periodic function $f(u)$. If a sine series is used, the period is $2L = 2(M-1)$ and the function is assumed to be odd, i.e., $f(-u) = -f(u) = f(2L-u) = -f(2L+u)$. Since y_0 and y_{M-1} are greater than zero, there would be, in general, discontinuities at $u = 0$ and $u = M-1$. If, however, the straight line through $(0, y_0)$ and $(M-1, y_{M-1})$ is subtracted, the resulting function can be defined to be odd, $\bar{f}(-x) = -\bar{f}(x)$, and still be continuous. Also, it is well known that if the function and its first derivative are continuous, its Fourier Series converges more rapidly, and does not exhibit the Gibbs phenomenon [15].

The line that passes through the two endpoints is given by $f(u) = y_0 + \left(\frac{y_{M-1} - y_0}{M-1} \right) u$.

If the $\bar{y}_i$ are defined as the values of the function $y(u)$ after subtraction of the straight line,

$$\bar{y}_i = y_i - y_0 - \left(\frac{y_{M-1} - y_0}{M-1} \right) u_i$$

for $u_i = i = 0, 1, 2, \dots, M-1$,

[15] C. O. Lanczos, Applied Analysis (Prentice Hall), Chapter IV (1955).

the coefficients b_k are determined to give the best fit to the y_i . The coefficients in Equation (16) are thus given by

TABLE X
SUMMARY OF SINE SERIES FITTING RANGES

Isotope	E_γ	X'_b	X'_c	X'_o	Middle-Energy 10-keV		
					$X_1 - (X_b + 4)$	$(X_b - 4) - (X_c + 4)$	$(X_c + 4) - X_o$
In-115m	0.336	14.6	19.6	34.3	1-19	10-24	15-34
In-113m	0.393	15.6	24.4	40.0	1-20	11-28	20-40
Be-7	0.478	16.7	31.6	48.3	1-21	12-36	27-48
Sr-85	0.513	17.1	34.8	51.9	1-21	13-39	30-52
Y-91m	0.558	17.6	38.6	56.2	1-22	13-43	34-56
Cs-137	0.662	18.4	47.7	66.2	1-22	14-52	43-66
Nb-95	0.764	19.1	57.1	76.2	1-23	15-61	53-76
Mn-54	0.835	19.5	63.2	82.7	1-24	15-67	59-83
Nb-92	0.931	20.0	72.3	92.3	1-24	16-76	68-92
Zn-65	1.114	20.7	88.9	109.7	1-25	16-93	84-110
A-41	1.29	21.2	104.9	126.2	1-25	17-109	100-126

$$\begin{aligned}
 a &= y_0, \\
 b &= \frac{y_{M-1} - y_0}{M-1}, \text{ and} \\
 b_k &= \frac{2}{M-1} \sum_{i=1}^{M-2} \bar{y}_i \sin \frac{k \pi u_i}{M-1} \\
 &\text{for } k = 1, 2, \dots, N. \quad (17)
 \end{aligned}$$

These trigonometric sums are computed by an algorithm due to Goetzel [16] which is described in Appendix III.

2.4 Stretching of Spectra

The main purpose of the sine series fitting procedure is to provide a method for a linear transformation, or stretching, of the pulse-height scale for each segment of each spectrum. The transformation parameters can then be chosen such that the prominent features of each spectrum will occur at the same positions on the new scale. This is done by construction of a new, stretched spectrum from the three sine series expressions. In particular, 51, 100, and 50 equally spaced points are evaluated in the three segments of each spectrum in the middle-energy region.

In the first segment, the first point is at $x = 1$ and the last is at $x = X_b$. In the second segment, the first point is at $x = X_b + \frac{X_c - X_b}{100}$ and the last is at $x = X_c$. The first point of the third region is at $x = X_c + \frac{X_o - X_c}{50}$ and the last is at $x = X_o$.

Let the variable z correspond to the pseudo-pulse height of the stretched spectra. (It should be clear that the value of the sine series at any point still represents the counts per unit pulse height $[N(z)\Delta x]$ and not the counts per unit Δz .) The variable z will have the range 1 to 201, and the features denoted by the X_j will now fall at the same value of z .

Figures 33 and 34 show overlays of the stretched spectra for this energy range.

2.5 Smoothing

Until now the number of terms N in the sine series, $g(u)$ in Equation (16), has not been specified. If N is about equal to the number of data points, M , the function $g(u)$ will follow the data points including the statistical fluctuations. Large oscillations would then be expected between the data points. On the other hand, if N is too small, $g(u)$ may omit essential structure of the data. Since no automatic method of terminating the series was discovered, a large number of fits was made initially to various segments of various spectra. It was empirically determined that satisfactory fits were obtained with N about equal to $\frac{M}{2}$. The results of a typical fit for this value of N are given in Table XI.

[16] Mathematical Methods for Digital Computers, edited by A. Ralston and H. S. Wilf (Wiley), Chapter 24 by G. Goertzel (1960).

Fig. 33 Plot of normalized, stretched, residual spectra for middle-energy range on 10-keV scale. (All counts-per-channel should be divided by two.)

Fig. 34 Plot of normalized, stretched, residual spectra for middle-energy range on 10-keV scale. (All counts-per-channel should be divided by two.)

TABLE XI
RESULTS OF A SINE
SERIES TO FIT TWENTY-ONE
CHANNELS WITH TEN TERMS

Sr-85

X_b to X_c

Channel	Normal Count	Calculation Count	Residual
1	4327	4327	0
2	4321	4318	3
3	4320	4324	- 4
4	4331	4331	0
5	4325	4318	7
6	4292	4296	- 4
7	4280	4293	-13
8	4331	4303	28
9	4283	4298	-15
10	4257	4278	-21
11	4316	4271	45
12	4240	4275	-35
13	4274	4268	6
14	4314	4296	18
15	4451	4478	-27
16	4858	4831	27
17	5134	5149	-15
18	5178	5189	-11
19	4989	4951	38
20	4626	4661	-35
21	4461	4461	0

3. POLYNOMIAL INTERPOLATION BETWEEN INPUT SPECTRA

The sine-series fitting procedure of the previous section produces a set of sections of the response surface which is to be calculated. These are sections for given values of the gamma-ray pulse height X_0 . The functional values represent $y(z)\Delta x$ where z is the stretched, or pseudo-, pulse height. From these sections it is desired to find analytical expressions for the complete surface. This is done by computing orthogonal polynomials to represent sections of the surface with

constant z ; that is, sections perpendicular to the z axis if $y(z;X_0)\Delta x$ is plotted as a function of X_0 and z .

Each polynomial can include only the data in one energy region. Therefore, the complete 10-keV response surface will consist of three sets of polynomials.

As an illustration, Figure 35 shows the data to which polynomial fits are to be made for $z = 20, 40, 60, 80, 100, 120, 140, 160$, and 180 . The polynomial coefficients are calculated by a least-squares technique which is described in Appendix III together with a discussion of the method of generating the polynomials that are orthogonal with respect to the given set of data points [17] (i.e., values of X_0).

Let P be the number of points available for any polynomial fit (i.e., the number of spectra in the particular region). The value of P is 11 in the middle-energy region. Since the polynomials are orthogonal with respect to the particular points used, all possible information is obtained by fitting with a polynomial of degree $P-1$ which has P coefficients. Then, the resulting function $h(X_0; z; P)$ can be evaluated for any degree less than $P-1$ by simply neglecting the higher coefficients. Generally, the degree should be $< (P-1)$ in order to provide some smoothing of the function and to reduce the possibility of oscillations between the points.

The 201 sets of coefficients for this region are stored on magnetic tape for use in the generation portion of the program. A list of the degree to be used in the evaluation of $h(X_0; z; P)$ for each value of z is included as part of the input of this program and is stored on the same magnetic tape. Also stored

[17] M. Ascher and B. E. Forsythe, "SWAC Experiments on the Use of Orthogonal Polynomials for Data Fitting", J. Assoc. for Comp. Mach. **29**, (January, 1958).

Fig. 35 Plots of points which are fitted with orthogonal polynomials for values of the pseudo-pulse height Z of 20, 40, 60, 80, 100, 120, 140, 160, and 180. (These points come from Figures 33 and 34 and the counts should likewise be divided by two.) Data are for 10-keV scale, middle-energy range.

on this tape are the coefficients for the polynomials that give w_0 , α_1 , α_2 , a_{ϵ_p} , $\text{PHU}(E)$, $E(\text{PHU})$, and the annihilation radiation intensity.

4. GENERATION OF INTERPOLATED SHAPES

4.1 Evaluation of Orthogonal Polynomials

From the coefficients of the orthogonal polynomials, pulse-height distributions for any desired gamma-ray energy within the range of the input spectra can be produced. The response surface is given by the above polynomials $h(X_0; z; P)$.

The user may request the generated shape for either a given energy or a given pulse height. In the former case, the gamma-ray energy is converted to pulse-height units by use of Equation (8). The polynomials are then evaluated at the desired value of X_0 .

4.2 Regeneration of Spectra

Evaluation of the orthogonal polynomials provides a Compton distribution $y(z, X_0)$ which contains more points, 201, than would appear in a normal spectrum.

These points are equally spaced in the pseudo-pulse height z and not in the pulse height x .

The transformation from z to x involves the length of the segment involved for the particular gamma-ray energy or pulse height requested. The values of X'_b and X'_c are computed from X'_0 by means of Equations (14) and (15). As before, these values are rounded to the nearest integer. For the energy region and segment in question, the fifty (or one hundred) points in the $y(z, X_0)$ curve will correspond to $X_{j+1} - X_j$ channels in the spectrum.

The channel index i then varies from X_j to X_{j+1} in the segment. Let Z_j and Z_{j+1} be the values of z at the ends of this segment. Then the value of z corresponding to x_i is given by

$$z_i = z(x_i) = Z_j + \frac{Z_{j+1} - Z_j}{x_{i+1} - x_j} \cdot (x_i - x_j) \quad (18)$$

In general, z_i will not be an integer. Let z_k and z_{k+1} be the integer values of z that include z_i . The desired value of y_i is then found by linear interpolation between $y(z_k)$ and $y(z_{k+1})$. This process is carried out for all the points $x_i \leq X_0$.

thus generating the complete Compton distribution as a function of x .

4.3 Addition of Photopeak

The photopeak is computed from Equation (5) where w_0 , α_1 , and α_2 are computed from Equations (3) and (4) for the appropriate value of x_0 . The value of y_0 is computed from Equation (13) which provides the desired intensity normalization. The computed photopeak is then added to the above Compton distribution.

4.4 Comments

The most serious limit on the accuracy of the results is expected to be the quality of the input spectra and the random and systematic discrepancies between these spectra. This situation can only be improved as methods of collecting and correcting the experimental data improve.

One problem with this method is that it provides no guarantee that the Compton distributions will stay normalized at energies between the input spectra. Since the photopeaks are always normalized, this may result in small errors in the peak-to-total ratios for the output spectra.

5. EXTENSION OF METHOD TO HIGH- AND LOW-ENERGY REGIONS

5.1 High-Energy Region

The method which has been described is based on the fact that the photopeak, Compton edge, and backscatter peak are the prominent features of the spectrum for a single gamma ray. In the region above 1.2 MeV, three other distinctive features of the spectrum must also be considered; namely, the two escape peaks at $E_\gamma - 0.511$ and $E_\gamma - 1.022$ MeV and the 0.511-MeV peak.

The annihilation radiation peak at 0.511 MeV is removed from the input

spectra before processing. This is done by subtracting a 0.511-MeV gamma-ray shape from the original data until the spectrum is flat in this region. (The proper shape to use in the subtraction would be different than this because the annihilation photons produced are not all of the same energy. Also, since most of the annihilation photons are produced in the Pb shield, many of them are scattered before they reach the crystal. However, no better shape is available.) The data from some of these subtractions were used (Section III-5) to determine the intensity of this peak as a function of the photon energy. In the generation of a new gamma-ray shape, the 0.511-MeV peak is added back into the spectrum after it has been otherwise completed. (The fact that the same shape is used in the subtraction before processing and addition after generation to some extent eliminates problems associated with use of an incorrect shape.)

The segmentation of the input spectra is done as shown in Figure 36. The energies of the two escape peaks are computed from the gamma-ray energies and then are converted to pulse height. In each of the five resulting regions the sine series fits are evaluated at fifty equally spaced points (fifty-one for the first region). The spectrum, without the photopeak, is then represented by 251 points. These "stretched" spectra are shown in Figure 37. The interpolation method is exactly the same as used in the middle-energy range. Some of the data points to be fitted with orthogonal polynomials are shown in Figure 38. The generation of a shape is exactly analogous to that described above, except for the addition of the 0.511-MeV peak.

5.2 Low-Energy Region

Below 0.28 MeV, the Compton edge is lower in energy than the backscatter peak; therefore, the method used in the middle-energy range is not valid. The backscatter peak also becomes unobservable while the iodine K X-ray escape peak appears.

Fig. 36 Segments of input spectra for those above about 1.2 MeV.

In this region three segments are used with the Compton edge, iodine escape peak, and photopeak being the ends of the intervals. The number of points evaluated in the three segments are 51, 100, and 50, respectively. The remaining analysis is exactly as in the middle-energy range. The "stretched" spectra for this region are shown in Figure 39.

Fig. 37 Plot of normalized, stretched, residual spectra for high-energy range on the 10-keV scale.

Fig. 38 Plots of points which are fitted with orthogonal polynomials for 10-keV scale, high-energy range for values of the pseudo-pulse height Z shown in the figure.

Fig. 39 Plot of normalized, stretched, residual spectra for low-energy range on the 5-keV scale.

V. RESULTS

1. COMPARISON OF SHAPES WITH EXPERIMENTAL SPECTRA

1.1 Input Spectra

To indicate the accuracy of the computed shapes, all experimental spectra used as input are compared with output spectra calculated for the same energies. The low- and middle-energy regions (5-keV scale) are shown in Figures 40-44. The middle-energy region (10-keV scale) is shown by Figures 45-47, and the high energy region (10-keV) by Figures 48 and 49. The normalized input spectra have been used to allow a comparison of both the spectral shape and the intensity normalization. As can be seen, there are regions which exhibit small but systematic differences

between input and output spectra. These may be due to small inconsistencies between input spectra which cause the polynomials to average the differences between spectra. Even in these regions the computed spectra generally are within one standard deviation of the input count in a given channel.

It is clear that the poorest agreement is in the low-energy regions of the 5-keV scale. This is expected since these spectra have the least amount of data to define details in the spectra. These problems would be even more serious on the 10-keV scale. Therefore, for analysis of 10-keV spectra, consideration should be given to the advantages of generating low-energy shapes on the 5-keV scale and then shifting the gain to the 10-keV scale.

Fig. 40 Comparison of normalized input spectra (solid circles) and shapes calculated at the same energies (open circles) for the low-energy region of the 5-keV scale.

Fig. 41 Comparison of normalized input spectra (solid circles) and shapes calculated at the same energies (open circles) for the low-energy region of the 5-keV scale.

Fig. 42 Comparison of normalized input spectra (solid circles) and shapes calculated by computer program (open circles) for the 5-keV scale, medium-energy region.

Fig. 43 Comparison of normalized input spectra (solid circles) and shapes calculated by computer program (open circles) for the 5-keV scale, medium-energy region.

Fig. 44 Comparison of normalized input spectra (solid circles) and shapes calculated by computer program (open circles) for the 5-keV scale, medium-energy region.

Fig. 45 Comparison of normalized input spectra (solid circles) and shapes calculated at the same energies (open circles) for the medium-energy region of the 10-keV scale.

Fig. 46 Comparison of normalized input spectra (solid circles) and shapes calculated at the same energied (open circles) for the Medium-energy region of the 10-keV scale.

47 A

47 B

47 C

Fig. 47 Comparison of normalized input spectra (solid circles) and shapes calculated at the same energies (open circles) for the medium-energy region of the 10-keV scale.

Fig. 48 Comparison of normalized input spectra (solid circles) and shapes calculated by computer program (open circles) for the 10-keV, high-energy region.

Fig. 49(a) Comparison of normalized input spectra (solid circles) and shapes calculated by computer program (open circles) for the 10-keV, high-energy region. The differences around channel 50 result from the fact that the annihilation-radiation peak has been removed from the normalized input, but is included in the output.

49 B

Fig. 49(b) Comparison of normalized input spectra (solid circles) and shapes calculated by computer program (open circles) for the 10-keV, high-energy region. The differences around channel 50 result from the fact that the annihilation-radiation peak has been removed from the normalized input, but is included in the output.

1.2 Fits to Spectra with Two Gamma Rays

Since the calculated shapes might be expected to be less accurate for energy values intermediate between those of the input spectra, a series of other comparisons were made. Figures 50 through 53 show such comparisons with experimental spectra including two or three prominent gamma rays. In this case there is no way to normalize the experimental spectra, so that each generated shape had to be multiplied by a scaling factor. These scaling factors, or gamma-ray intensities, were calculated by use of the SISYPHUS II least-squares fitting

Fig. 50 Results of a least-squares analysis of the Nb-94 spectrum to obtain gamma-ray intensities. The analysis used two calculated gamma rays and a coincidence sum spectrum. The calculated intensities of each constituent of the composite spectrum are shown to be dotted lines. Residuals are shown plotted at the bottom of the figure.

program described in Section VI-5. The sum spectra shown in the figures were calculated with the program discussed in Section VI-3, and their intensities were computed in the least-squares fit.

Fig. 51 Analysis of Sc-46 spectrum.

Fig. 52 Analysis of V-48 spectrum.

Fig. 53 Least-squares analysis of a Y-88 gamma-ray spectrum to obtain relative intensities.

At the bottom of each figure is shown the difference between the experimental and calculated counts in units of the estimated standard deviation for the experimental count (i.e., $\frac{y_i - \bar{y}_i}{\sigma_i}$). The quality of

the fit in each region and the generated shapes is indicated to some extent by the magnitude and sign distribution of these residuals. The overall quality of the fit is also indicated by the quantity $\chi^2/(n-m)$

where $\chi^2 = \sum_i \left(\frac{y_i - \bar{y}_i}{\sigma_i} \right)^2$, n is the

number of points in fit, and m is the number of variable intensities in the fit. In ideal cases [13] $\chi^2/(n-m)$ should be about 1. The values obtained for these fits are given in Table XII.

TABLE XII

TEST OF SPECTRAL NORMALIZATION METHODS USED IN PROGRAM
BY LEAST-SQUARES FITS TO SPECTRA WITH TWO GAMMA RAYS OF EQUAL INTENSITIES

Isotope	Gamma-Ray Energy (MeV)	Intensities		Channels Used	$\chi^2/(n-m)$
		Calculated ^[a]	Ratio ^[a]		
Nb-94	0.70	2.097 ± 0.006	1.013 ± 0.006	5-166	2.0
	0.86	2.124 ± 0.006			
	Sum	2.82 ± 0.05			
V-48	0.99	2.717 ± 0.011	1.082 ± 0.010	47-228	3.2
	2.16	0.062 ± 0.003			
	1.31	2.568 ± 0.011			
	0.51	2.302 ± 0.008			
Sc-46	0.887	3.060 ± 0.010	1.022 ± 0.007	25-207	2.6
	1.119	3.127 ± 0.010			
	Sum	4.11 ± 0.08			

[a] These error estimates are from the least-squares fitting program. Thus they do not include the uncertainty in the $A\epsilon_p$ values.

2. CHECKS OF NORMALIZATION

The data shown in Figures 50-52 can be used to check the normalization procedures of the program. Since the gamma-ray intensities, or scaling factors, were computed for the channel ranges indicated in the figures, this test is not the same as the check of $A\epsilon_p$ discussed in Section III-4. For these sources the two transitions present have the same intensity. For these energies, the corrections for internal conversion are all negligible, so that relative intensities of 1.00 are expected. The relative intensities from these fits are given in Table XII.

Except for V-48, the computed ratios agree with the expected value within 2%. This indicates that the photopeak efficiency data and the normalization methods used in the program are accurate to within this amount. This statement refers only to the ratio of two efficiencies since this method does not provide a check on absolute efficiency. The fit to an Y-88 spectrum shown in Figure 52 also resulted in an intensity ratio which agreed with the literature value to within 2%.

3. COMMENTS

3.1 Limitations on Quality

It is the feeling of the authors that above 0.2 MeV the accuracy of the generated shapes is limited mostly by the accuracy of the input information, that is, the input spectra and polynomial coefficients. The results shown in this report are from the first complete set of spectra processed with this program. (Previous reports [4] were concerned with smaller sets of spectra and with a simpler program which was used on an IBM-650 computer.) By a careful comparison of the input and generated spectra, it is believed that inconsistencies in the input can be found and corrected. Also, the representation of the photopeaks might

still be improved. The 201 or 251 orthogonal polynomials are currently all evaluated for the same degree. Careful study may indicate how this degree should vary with the pseudo-pulse height in order to improve the results.

On the other hand, some improvements in the program may be possible. One problem is in the sine series fit to the last segment. After subtraction of the photopeak, the data at the high-energy end of this segment have large oscillations and correspondingly large uncertainties. Since the sine series fit is unweighted, these oscillations may adversely effect the fit. Therefore, it might be advantageous to change to some type of weighted fit.

3.2 Correction of Generated Spectra for Change in Resolution

There are several problems which might be solved if a generated spectrum could be correctly adjusted to correspond to the response of a crystal resolution is different from that of the crystal on which the original data were obtained. The program might be applicable to spectra taken on different crystals.

First, consider the case of two detectors which have identical crystals mounted on photomultipliers with different resolution. It is presumed that the crystals have the same intrinsic resolution and that the effect of any photomultiplier is to smear (i.e., convolute) the spectrum from the crystal with a gaussian function.

It then follows that for a given gamma ray

$$w_1^2 = w_c^2 + w_{p1}^2 \text{ and } w_2^2 = w_c^2 + w_{p2}^2$$

where w_c is the effective width of the distribution coming from the crystal, w_{p1} and w_{p2} are the widths of the gaussian smearing functions for the two photomultipliers, and w_1 and w_2 are the widths of the observed photopeaks. Then, if $w_{p2}^2 > w_{p1}^2$,

$$\Delta w^2 = w_2^2 - w_1^2 = w_{p2}^2 - w_{p1}^2.$$

Now if a Gaussian of width w_{p1} is convoluted with a Gaussian of width Δw , the resulting function is a Gaussian of width w_{p2} . Therefore, to the extent that a photopeak is a Gaussian, the photopeaks of the poorer detector (i.e., the one with wider peaks) can be produced from those of the better detector by this convolution process. If it is now assumed that w_p^2 is proportional to the pulse height, a complete, approximate response curve could be produced for the poorer detector from that of the better one by use of

Fig. 54 Pulse-height spectra of Cs-137 obtained from two different standard laboratory 3 x 3 inch NaI detectors. Experimental spectra were gain-shifted to the same position and "smeared" to the same photopeak width (resolution).

a gaussian smearing function in which Δw^2 is proportional to the pulse height.

As an example, Cs-137 spectra from two "identical" crystals are compared in Figure 54 after both spectra were gain-shifted to the same location and smeared to give the same resolution. (These crystals were both packaged at this laboratory in identical cans.) If the two detectors are not identical, the results may not be as satisfactory. An example of such a comparison is shown in Figure 55.

Fig. 55 Pulse-height spectra from a standard laboratory detector and a commercial 3 x 3 inch NaI detector gain shifted to same position and "smeared" to same peak width. Observed difference in shape of spectra attributed to scattering from material used in the packaging of commercial detector. Reduction of X-ray intensity is due to absorption in can.

Here, one detector is a commercial package while the other crystal was packaged and mounted locally. Although the agreement is quite good, the generated shapes from one crystal might not be satisfactory for some purposes.

Another situation in which this method might be of use is where the resolution of a detector has deteriorated. If the

deterioration is due to the photomultiplier, the arguments presented above for identical crystals should apply, and it should be possible to correct the generated shapes. If the change is in the crystal, or in the optical coupling, it is not clear that these arguments are valid. In this case comparisons similar to that in Figure 55 could be made in order to determine how good the results would be.

VI. COMPUTER PROGRAMS AND PROCEDURES FOR ANALYSIS OF GAMMA-RAY SPECTRA

1. INTRODUCTION

It was pointed out in Section II-1 that there are contributions to complex pulse-height spectra in addition to the detector response to single gamma rays. These may result from coincidence summing, bremsstrahlung, and accidental summing. These effects must be taken into account in the analysis of most experimental spectra. Computer programs to change the gain of a spectrum and for the calculation of coincidence sum spectra are discussed in the following sections. Comments on the treatment of bremsstrahlung are given in Section VI-4. This is followed by a brief discussion of programs for the analysis of complex spectra by least-squares fitting techniques.

2. GAIN-SHIFT PROGRAM

The gain (keV/channel) of a counting system is normally adjusted by varying the photomultiplier high voltage (or amplifier gain) until the center of a photopeak is at the desired pulse height. For example, it is standard procedure to adjust the photomultiplier high voltage until the Cs-137 photopeak is in about channel 66.2 for the 10 keV/channel scale. This process is limited to an accuracy of about 0.2 channels, or 0.3 percent in this case. However, for the quantitative analysis of gamma-ray spectra, it has been found that it is often necessary to compare the gain of spectra to within 0.03 percent. Therefore, some method of changing the gain of a measured spectrum is needed.

The program discussed here is written in FORTRAN and compiled for use on an IBM-7040 computer. In addition to allowing the gain to be changed, it is possible to shift the zero of the pulse-height scale. Thus, a general linear transformation can be made between two pulse-height scales.

Let us assume that there is no zero shift. Then if g is the original gain (in keV/channel at the Cs-137 peak), the new gain g' is given by

$$g' = \frac{1}{r} g \quad (19)$$

where r is called the "gain-shift ratio". Or, if channel x' in the new spectrum corresponds to channel x in the original spectrum

$$x' = r x \quad (20)$$

If the independent variables x and x' were continuous it would be possible to compute the new count $y'(x')$ at x' as

$$y'(x') = \frac{1}{r} y(x) \quad (21)$$

The factor $\frac{1}{r}$ is introduced so that the area under the curve (i.e., the total number of counts) is conserved.

Since the original spectrum, $y(x)$, is represented by a set of discrete points the following method of interpolation is used. Given a channel x' (an integer), the corresponding pulse height x (not an integer in general) is computed from Equation (20). The three channels nearest to x are chosen. (That is, x is rounded to the nearest integer, x_j ; and x_{j-1} , x_j , and x_{j+1} are selected.) These three data points are fitted with the function $y = a + bx + cx^2$ to determine the parameters a , b , and c . Then $y(x)$ is calculated by evaluating this polynomial at x , and $y'(x')$ is computed from Equation (21).

Since there are three parameters and three data points, the function $a + bx + cx^2$ fits the points exactly. Thus, this procedure is not intended as a smoothing process. However, there may be some incidental smoothing of the data.

To change the zero as well as the gain of the pulse-height scale, Equation (20) becomes

$$x' = r(x - \delta) \quad (22)$$

where δ is the desired shift in the zero. Equation (21) remains unchanged.

Although this program is intended for use with small gain shifts (i.e., $0.9 < r < 1.1$), it can be used for larger shifts. As an extreme example, Figure 56 shows

Fig. 56 Illustration of results of gain-shift program. The closed circles represent a Cs-137 spectrum measured at 5-keV per channel. The open circles show a spectrum taken at 10-keV per channel and gain shifted to the 5-keV scale.

two Cs-137 spectra, one measured at 5 keV/channel and the other at 10 keV/

channel and shifted ($r = 2.0$) to a gain scale of 5 keV/channel. The agreement is considered good for such a large scale change. The effect of finite channel width is indicated by the increased width of the X-ray peak on the shifted 10-keV spectrum.

A second check on the program was made in conjunction with the modified-Gaussian photopeak fitting program. In this case the problem was to determine the distortion of the photopeak parameters (namely, y_0 , x_0 , b_0 , α_1 , and α_2) by a gain shift. For this purpose, a Cs-137 spectrum was measured and the photopeak parameters were determined by a least-squares fit. The spectrum was then gain-shifted by a factor of $r_1 = 0.958$. The latter spectrum was then shifted by a factor of $r_2 = 1.04384$. After these two shifts the photopeak was back to its original position ($r_1 r_2 = 0.999999$). The parameters of the last spectrum were determined by a second least-squares fit. This procedure was also carried out for spectra of Nb-95, Mn-54, and Zn-65. The results of the fits are given in Table XIII.

The excellent agreement between the sets of parameters indicates that such shifts (<5 percent) do not have an observable effect on the parameters of a photopeak. Since this portion of the spectrum has the greatest curvature, the distortion might be expected to be negligible in other regions of a pulse-height spectrum.

TABLE XIII
RESULTS OF LEAST-SQUARES FITS

	Cs-137		Nb-95	
	Before Shift	After Two Shifts	Before Shift	After Two Shifts
y_0	25,890 $\pm$ 120	25,881 $\pm$ 115	64,770 $\pm$ 170	64,764 $\pm$ 150
x_0	68.812 $\pm$ 0.009	68.812 $\pm$ 0.010	75.935 $\pm$ 0.006	75.935 $\pm$ 0.005
b_0	11.72 $\pm$ 0.16	11.74 $\pm$ 0.15	12.78 $\pm$ 0.10	12.78 $\pm$ 0.09
α_1	(2.10 $\pm$ 0.45) $\times 10^{-4}$	(2.07 $\pm$ 0.45) $\times 10^{-4}$	(1.93 $\pm$ 0.25) $\times 10^{-4}$	(1.95 $\pm$ 0.20) $\times 10^{-4}$
α_2	(7.7 $\pm$ 1.3) $\times 10^{-12}$	(7.5 $\pm$ 1.2) $\times 10^{-12}$	(6.5 $\pm$ 0.6) $\times 10^{-12}$	(6.2 $\pm$ 0.5) $\times 10^{-12}$
r_1	0.958		0.96	
r_2	1.04384		1.04167	

	Mn-54		Zn-65	
	Before Shift	After Two Shifts	Before Shift	After Two Shifts
y_0	66,757 $\pm$ 200	66,598 $\pm$ 200	28,551 $\pm$ 100	28,539 $\pm$ 90
x_0	82.605 $\pm$ 0.007	82.606 $\pm$ 0.007	111.373 $\pm$ 0.011	111.373 $\pm$ 0.010
b_0	14.11 $\pm$ 0.13	14.24 $\pm$ 0.13	20.20 $\pm$ 0.23	20.23 $\pm$ 0.21
α_1	(1.94 $\pm$ 0.25) $\times 10^{-4}$	(1.76 $\pm$ 0.22) $\times 10^{-4}$	(1.52 $\pm$ 0.15) $\times 10^{-4}$	(1.51 $\pm$ 0.14) $\times 10^{-4}$
α_2	(4.2 $\pm$ 0.4) $\times 10^{-12}$	(3.9 $\pm$ 0.3) $\times 10^{-12}$	(7.5 $\pm$ 0.8) $\times 10^{-13}$	(7.4 $\pm$ 0.7) $\times 10^{-13}$
r_1	0.97		0.98	
r_2	1.03093		1.02041	

3. PROGRAM TO CALCULATE COINCIDENCE SUM SPECTRA

An event is considered a real coincidence sum if two photons emitted in the decay of a single nucleus enter the NaI(Tl) crystal within a time very short compared to 10^{-6} seconds. Two such photons will produce a single pulse whose pulse shape is identical to that of a single gamma ray.

It is possible to calculate both the shape and magnitude of this summation spectrum if sufficient information about a decay scheme is available. Consider a simple cascade of two gamma rays. With the program previously described, it is possible to compute the detector response to the individual gamma rays. If these computed pulse-height distributions $N(x)$ are normalized to 10^7 photons emitted, then the values $10^{-7} N(x)$ represent the probability of detecting the gamma ray from any one decay as a

count in the channel x . Pulse-height spectra for the two coincident gamma rays may be designated as $N_1(x)$ and $N_2(x)$. The probability of detecting gamma 1 in channel a and gamma 2, from the same disintegration of the source, in channel b is the product $N_1(a)N_2(b) \times 10^{-14}$. For the emission of 10^7 photons of each gamma ray, $10^{-7} N_1(a)N_2(b)$ is the number of counts in the sum spectrum in channel $(a + b)$ from pulses in a and b . The total contribution to any channel c is then the sum of contributions from all pairs of channels for which $a + b = c$. This may be expressed in the following manner:

$$N_{12}(c) = \sum_{x=1}^c N_1(x) \cdot N_2(c-x) \cdot 10^{-7} \quad (23)$$

A program has been written which performs this summation for cascades of 2, 3, or 4 gamma rays, using as input data the calculated spectra from the

previously described spectrum generation program.

Two corrections are necessary to obtain the correct intensity for the sum spectrum relative to the cascade gamma rays. First, each pulse which appears in the sum spectrum represents a count missing from the pulse-height spectra of each of the summing gamma rays. Therefore, the individual gamma-ray distributions should each be reduced in area by the area of the sum spectrum.

Secondly, since both gamma rays must be emitted in the same direction, the above summation intensities should be corrected for the angular correlation relationship $W(\theta)$ between the two gamma rays. The above intensity should be multiplied by

$$\frac{\pi}{\theta_1} \frac{\int_0^{\theta_1} W(\theta) d\theta}{\int_0^\pi W(\theta) d\theta} \quad (24)$$

where θ_1 is the effective half-angle subtended by the crystal. If the solid angle subtended by the crystal is very small, this reduces to

$$\frac{\pi W(0)}{\int_0^\pi W(\theta) d\theta} \quad (25)$$

Figures 50-53 show examples of the results of this program. The shape of the coincidence sum spectrum is reproduced with considerable precision. Figure 57 shows a comparison between the experimental pulse-height spectrum of 42-day Pm-148 and the computed spectrum obtained using the programs described above. The computed spectrum was generated from data obtained at this laboratory [18] on the decay scheme of this nuclide. This is a complicated case involving 17 gamma rays with many double and triple cascades. With the exception of the two low-energy peaks, which were not included in the calculation, all points on the computed spectrum agree quite well with the experimental data.

[18] C. W. Reich, R. P. Schuman, J. R. Berreth, M. K. Brice, and R. L. Heath, Phys. Rev. 127, 192 (1962).

Fig. 57 Comparison of the experimental spectrum obtained from a source of 42-day Pm-148 with a spectrum calculated by use of the computer programs described herein.

4. ESTIMATION OF BREMSSTRAHLUNG SPECTRA

In order to investigate the spectral distribution of bremsstrahlung spectra, the series of spectra shown in Figure 12 (see Section III-1) was measured.

These spectra were normalized in the following manner. Thin sources of each isotope (except Kr-85) were made up. The disintegration rate was determined from the electron counting rate in a 4π beta counter. The sources were then placed between two aluminum foils which were thick enough to stop the electrons emitted. This produces a point source of bremsstrahlung with all electrons being stopped in the aluminum. The bremsstrahlung spectra were then measured in the standard geometric arrangement. The resulting spectra were normalized to 10^7 disintegrations. Krypton 85 was available

only as a gas. This sample was sealed in a thin-walled quartz ampoule and the gamma-ray spectrum was measured. The intensity of the 0.51-MeV gamma ray was used to normalize the bremsstrahlung spectrum which was obtained by subtracting the contribution of this gamma ray.

Currently, bremsstrahlung spectra are estimated by hand interpolation between those in Figure 12. Some work has been done which indicates that it will probably be possible to use a very simple computer program to interpolate between these shapes.

5. SPECTRUM ANALYSIS PROGRAMS

5.1 Introduction

As discussed in the introduction, the purpose of the shape generation program is to allow the analysis of complex spectra to be carried out as accurately as possible. The authors have available three different computer programs for the analysis of such spectra. These programs range from a fairly simple linear least-squares fitting program to a complex program which includes techniques for varying the energies of the gamma-ray shapes used. In the next three sections the basic features of these three programs are outlined along with descriptions of the interesting options provided for specific applications.

5.2 Linear Least-Squares Fitting Program

This program has been used for analyzing composite spectrum to obtain intensities of either individual gamma rays or isotopes (e.g., activation analysis problems).

Let y_i be the count in channel i of the spectrum to be analyzed and y_{ij} be the count in channel i of the component spectrum j . It is desired to determine intensities β_j , such that

$$y_i \approx \bar{y}_i = \sum_{j=1}^J \beta_j y_{ij} \quad (26)$$

where J is the number of component spectra and $\bar{y}_i$ is the calculated spectrum. The β_j are calculated so that the quantity

$$R^2 = \sum_i w_i (y_i - \bar{y}_i)^2 \quad (27)$$

is a minimum. The w_i are a set of weights and the sum is over the channels of interest. The weights are usually taken to be $w_i = \frac{1}{y_i}$. Since y depends linearly

on the β_j , the latter can be computed simply by solving a set of J linear, simultaneous equations.

The results of the fitting procedure include the intensities, β_j , an estimate of the uncertainties in the β_j , and an indication of the quality of the fit. The latter is derived from the minimum value of R^2 . In particular, under certain assumptions, $\frac{\chi^2}{n-m}$ should be about 1 where χ^2 is the minimum value of R^2 and $(n-m)$ is the number of degrees of freedom.

The program includes several options which are of use in spectral analysis. Any β_j can be fixed at a particular value and it is possible to fix the ratio of pairs of β_j . Also, the gain-shift routine is included. This allows the shifting of all the input spectra to the same gain as they are read into the computer. Cycling provisions simplify the processing of large numbers of similar spectra. The principal limitations in the program are (a) that the gains or energies of the spectra are fixed during the fit, and (b) that all spectra, including generated shapes and sum spectra, must be in card form for input. A complete description of this program appears in Reference 19.

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- [19] R. G. Helmer, R. L. Heath, D. D. Metcalf, and G. A. Cazier, A Linear Least-Squares Fitting Program for the Analysis of Gamma-Ray Spectra Including a Gain-Shift Routine, U. S. Atomic Energy Commission Report IDO-17015 (1964).

5.3 Fitting Program with Variable Gain

A program called SHIFTY has been written which allows the gain of a spectrum, composite or component, to be varied, during the fitting process in order to find the best position as well as intensity. This program is used exclusively for isotopic analysis, but it could be used to determine gamma-ray intensities.

This program computes intensities by the same method as the previous program. In addition, the best gain scale for each component is determined by a "direct search" technique which operates as follows. Suppose a composite is being fitted with two components and both intensities and both gains are variable. The intensities β_1° and β_2° are computed for the spectra on the gain at which they are read in. R^2 is computed. With this intensity, β_1° , the first component is gain-shifted by a ratio $1 + \delta_{g1}$ (δ_{g1} given as input), and a new value of R^2 is computed. If the latter R^2 is smaller than the previous value, the new gain is retained, if not a ratio $1 - \delta_{g1}$ is tried. Next, the second component is shifted by a ratio of $1 + \delta_{g2}$ or possibly $1 - \delta_{g2}$. After the best gains among this set have been determined in this manner, a new set of intensities is calculated. This procedure continues until the shifts cease to have any significant affect on R^2 .

The many options of this program include:

- (1) obtaining input spectra from cards, magnetic tape, or core from a previous problem;
- (2) choice of weighting functions, w_i ;
- (3) fixing intensities and/or intensities ratios;
- (4) use of several sets of δ_{g_j} ;

(5) gains can be fixed or variable;

(6) half-life correction options.

5.4 Grandiose Fitting Program

The most recent of the spectrum fitting programs is called SISYPHUS [a] II. Like SHIFTY, it calculates intensities by the same linear, least-squares method and varies the positions of component spectra by the direct search method. However, in this program the positions can be varied by either the gain-shift routine or by generating new gamma-ray shapes. In either case, the search method is the same as that used in SHIFTY.

This program includes as subroutines the programs to generate shapes, to compute sum spectra, and to gain-shift spectra. This means that the input spectra can come from cards, from a magnetic tape spectrum file, from the shape generation program, from the sum spectrum program, or from the computer core from the previous problem. Options (2) through (5) from the previous section are also included. Although the main feature of this program is its versatility and ease of use (e.g., no cards to spill between programs), it can also handle large amounts of data. It can analyze up to 6000 channels of information distributed among up to 40 spectra.

An example of a fit made with this program is shown in Figure 58. This Pt-199 spectrum was fitted with 19 gamma-ray components, a bremsstrahlung shape, the Au-199 spectrum, and detector background. The number of gamma-ray components used in the fit and their approximate energies were obtained from analysis of NaI γ - γ coincidence

[a] In Greek mythology, Sisyphus is the shrewd and greedy king of Corinth who was doomed forever in Hades to roll uphill a heavy stone which always rolled down again.

Fig. 58 Results of a fit with SISYPHUS II to an experimental spectrum (circles) of 30-minute Pt-199 . The calculated composite spectrum and the component spectra are all shown as lines. The residuals are shown at the bottom of the plot.

spectra and internal conversion lines from an electron spectrograph. This is one of a series of five spectra which were measured during the decay of a source. One of these spectra was analyzed with the positions of the components allowed to vary. Then, with the components fixed in these positions, all five spectra were fitted. The values of $\chi^2/(n-m)$ ranged

from 1.2 to 1.8 for the five fits. The existence of these gamma rays and their approximate relative intensities agree with subsequent measurements with a lithium-ion-drifted, germanium detector.

Reports on this program and SHIFTY will be published when the programs have been more thoroughly checked.

VII. COMPUTER PROGRAM

1. PROGRAM DESCRIPTION

1.1 Introduction

The complete procedure described in Chapter IV is accomplished with two main programs and several subroutines. All programming was carried out in standard FORTRAN IV compiler language, except for three subroutines which were coded in MAP (7040 assembly language). A complete listing of these programs and their subroutines appear in the Appendix V together with flow charts. Sample inputs and outputs for the programs are listed in Appendix IV. Figure 59 shows the general flow of data through the two programs.

1.2 Program 1 -- GAMSPEC Input

This program consists of one main program and eight subroutines. The program accepts as input the monoenergetic

gamma-ray spectra and associated control information. The input data are intensity normalized, a normalized Gaussian is computed and subtracted, and trigonometric and polynomial fits performed on the remaining Compton distributions. The polynomial coefficients thus obtained are stored on a magnetic tape for use as input to the SHAPE program.

A detailed description of each card indicated in Figure 60 follows. Items "b through f" occur once for each energy range, and items "j through m" once for each spectrum. Items "g through i" are included only if the data are for the high-energy range. Following the name of each card is a list of the items included on the card and the format (F, E, I, with the normal FORTRAN IV interpretation) to which each item must correspond. All "I" formats must be right adjusted in their fields.

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Fig. 59 Flow of information through the GAMSPEC and SHAPE programs which make up the shape generation program.

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Fig. 60 Arrangement of input card deck for GAMSPEC program.

- (a) GAMSPEC (Program Deck). This may be the original FORTRAN source deck or the final binary deck, each with the proper monitor control cards.
- (b) Problem Title (1H0,12A6). Card columns 2 through 73 -- this title identifies the output for this energy range.
- (c) Date Card (IDATE,IRTPY,I) (10X,3I10)

<u>Symbol</u>	<u>Description</u>
IDATE (I10)	The current date as DD, MM, YY, day, month and year.
IRTPY (I10)	A four digit identification of the output coefficients. This is used later in the SHAPE program to request use of these coefficients.
I (I10)	An indicator to tell the program if one or more sets of generated coefficients are presently on the output tape being used. I = 0, yes and I $\neq$ 0, no.

- (d) Problem Control (NSETS,GAIN,JCAL,JOPT1) (10X,I10,F10.5,2I10)

<u>Symbol</u>	<u>Description</u>
NSETS (I10)	Number of gamma-ray spectra to be read in for this run.
GAIN (F10.5)	Gain scale (keV/channel).
JCAL (I10)	Energy range of the data. JCAL = 0 for low energy JCAL = 1 for middle energy JCAL = 2 for high energy
JOPT1 (I10)	Indicator of output options JOPT1 = 0 for regular output JOPT1 = 1 for special output

- (e) Conversion Coefficients (10X,5(1PE14.6)). Seven sets of coefficients must be included, at 2 cards per set, for a total of up to 10 coefficients per set. These must be arranged in the following order:

<u>Symbol</u>	<u>Coefficients</u>
PHUTE	Conversion of pulse-height units to energy in MeV (Eq. 7)
ETPHU	Conversion of energy in MeV to pulse-height units (Eq. 8)
EPSCØ	Photopeak efficiency, $A\epsilon_p$, coefficients (Eq. 9 & 10)
PEAKC	High-energy range, 0.511 spectrum intensity factors (Eq. 11)
WOCØ	Photopeak width (Eq. 4)

A1CØ Modified Gaussian α_1 value (Equation 3)

A2CØ Modified Gaussian α_2 value (Equation 3)

- (f) Degree of Fit Indicators (A5,I1,37I2). To evaluate each polynomial, a degree of fit indicator (201 for low and intermediate energy and 251 for high energy) is necessary. The first five card columns are any alphabetic identification. Card column 6 is a sequence number that is checked by the program. These quantities are stored on the output tape along with the coefficients for use in SHAPE.
- (g) Data Title Card (1H0,12A6). Each page of printed output pertaining to this spectrum is labeled with this information.
- (h) Data Parameter Card (XO,WO,YO,A1,A2,AREA,ISPUL) (2F10.5,4E10.5,I5)

<u>Symbol</u>	<u>Description</u>
XO (F10.5)	Position of photopeak in channels
WO (F10.5)	Full-width at half-maximum for photopeak in channels
YO (E10.5)	Height of photopeak in counts
A1 (E10.5)	Modified-gaussian parameter α_1
A2 (E10.5)	Modified-gaussian parameter α_2
AREA (E10.5)	Area under photopeak
ISPUL (I5)	Upper spectrum limit

- (i) 0.511-MeV Spectrum Data (I2,10F7.0). This is the 0.511-MeV spectral data to be used in the high-energy calculation. These are prepared in the ten channels per card form. These cards are included only with high-energy data.
- (j) Data Title Card (1H0,12A6). Each page of the printed output pertaining to this data is labeled with this information.
- (k) Data Parameter Card (XO,WO,YO,A1,A2,AREA,ISPUL) (2F10.5,4E10.5,I5). All the spectra within any one group must be ordered according to XO in increasing order (e.g., 66.16, 76.19, 82.74, . . .).

<u>Symbol</u>	<u>Description</u>
XO (F10.5)	Position of photopeak in channels
WO (F10.5)	Full-width at half-maximum for photopeak in channels
YO (E10.5)	Height of photopeak in counts
A1 (E10.5)	Modified-gaussian parameter α_1
A2 (E10.5)	Modified-gaussian parameter α_2

AREA (E10.5)

Area under photopeak

ISPUL (I5)

Upper spectrum limit

- (l) Gamma Spectrum (I2,10F7.0). These spectra are prepared in ten per card format. (See item 1.3 (c) below.)
- (m) Transfer Card (50 in col. 1,2). Each input spectrum must conclude with one of these. In addition, one must be placed at the physical end of the total input.

1.3 Special Considerations

(a) Multiple Groups of Input. If more than one energy-range is included, a blank card must be inserted between each group. (See Figure 61.)

Fig. 61 Card arrangement for processing more than one energy range at a time in the GAMSPEC program.

(b) Subsequent Problems. If coefficients already exist on the output tape, a "0" in card column 40 of the date card indicates this fact. The first group of data for each run must indicate this status. If coefficients are already present, they are copied from FORTRAN tape 4 to FORTRAN tape 8. A message to save both tapes should be given to the operator, one for a backup tape, the other for current operating tape.

(c) Standard 10/C Format. All input gamma spectra are prepared in the

following standard 10/C format: (I2,10 (IX.16)) where I2 is a card number starting with 00 and the first channel is assumed to be the time channel and is ignored. (See sample input.)

(d) GAMSPEC Output. The output includes a listing of all input data, normalized spectra, and Compton distributions after subtraction of a normalized, modified Gaussian. Next is a printout of the sine series fit with the original values, the calculated values, and the residuals for each segment. Last is a printout of the set of 201 or 251 calculated points describing the Compton distribution. This is then repeated for each spectrum read in. An output option can be specified on the problem control card which will limit each output to the problem control information and the original spectrum input.

1.4 Program 2 -- SHAPE Input

The SHAPE Program consists of one main program controlling five sub-routines. A control card is read which specifies which set of polynomial coefficients is to be used in the generation process. The magnetic tape containing the coefficients is then searched for the correct set. The resulting coefficients are then evaluated to produce the desired gamma spectra which are entered from the gamma-ray request cards as shown in Figure 62. The resulting spectra are punched on cards in a ten channel per card format as well as being printed.

The gamma-ray shapes may be requested in channel number as well as energy units. The control card preceding

the gamma-ray request cards must specify which "mode" is being used.

The following is a detailed description of each card indicated in Figure 62.

Fig. 62 Arrangement of input cards and tape for SHAPE program.

(a) SHAPE (Program Deck). The program deck may be the original source deck or the final binary deck, each with its proper monitor control cards.

(b) Control Card (IR,I1) (2I10)

<u>Symbol</u>	<u>Description</u>
IR(I10)	The record identification stored on the tape, identifying the desired coefficients to be used. (Same as IRTYP in item 3 of GAMSPEC Input.)
I1(I10)	The indicator to the computer specifying if the gamma ray is requested in channel number (I1=1) or in energy (I1=2).

(c) Data Cards (IC,X) (I1,9X,F10.5)

<u>Symbol</u>	<u>Description</u>
IC (I1)	Zero or blank on all but the last card. IC = 1 on the last card.
X (F10.5)	The requested gamma ray in channel number or energy; one on each card.

1.5 Special Considerations

(a) Restrictions. Any number of sets of input composed of a control card and several data cards may be loaded at one time. No ordering between the sets is necessary.

A maximum of 20 shapes can be produced in any one set; this may be

enlarged by changing the size of the dimension area. Between each such set of data, the coefficient tape is rewound to load point.

(b) Output Data. The gamma-ray spectra produced are printed on the system output tape and punched on cards in the standard 10/C format.

2. GLOSSARY OF SUBROUTINE NAMES

FORTRAN NAME	FUNCTION OR USE
CNVRT	Used for converting from energy to pulse-height units, from pulse-height units to energy, and calculating photopeak parameters.
DATIN	Loads data into memory.
DATØUT	Arranges data for output on the system output tape.
DØNE	Rewinds and unloads output coefficient tape (MAP subroutine).
ØRPFIT	Fits a specific range of data with orthogonal polynomials.
PAGE	Numbers each page of printed output; restores to next page.
REGEN	Converts generated gamma shape on pseudo pulse-height scale to shape on integer channel scale.
SHAPE	Generates a detector response gamma shape from stored coefficients.
SSFIT	Fits a specific range of data to a Fourier sine series.
TYPEW	Types message to operator on on-line type-writer. (MAP subroutine.)
ZERØ	Converts from binary to BCD with leading zeros left in. (MAP subroutine.)

3. GLOSSARY OF VARIABLE NAMES

FORTRAN NAME ^[a]	EXPLANATION, USE
A1CØ(10), A2CØ(10)	Modified-gaussian coefficients.
ALPHA(18),BETA(17)	Numbers chosen to make the orthogonality relation hold.
AREA	Area of photopeak.
D(251),B(251)	Temporary storage used in the sine series fit subroutine.
DAT(10),IDAT(10)	Used for input and output subroutines.
EPSCØ(10)	Coefficients used for interpolating, photopeak efficiency.
ETPHU(10)	Energy to pulse-height conversion coefficients.
GAIN	A number indicating the gain in keV per channel.
IDATE	Date the coefficients were generated.
IRTYP	Identification of current set of coefficients for the generation of gamma shapes of a particular energy range.
ISPUL	Number of channels describing current spectrum.
IZ(37)	Intermediate storage.
JCAL	Control for calculation type.
JØPT1	Output option indicator.

[a] Numbers in parentheses indicate the amount of storage reserved. If none are indicated (1) is assumed.

GLOSSARY OF VARIABLE NAMES (cont.)

FORTRAN NAME	EXPLANATION, USE
KP(251)	Order indicators for evaluation of orthogonal polynomials.
NSETS	Number of input spectra.
P(17,20)	Used in orthogonal polynomial subroutine.
PEAKC(10)	Coefficients used for obtaining the intensity of 0.511-MeV annihilation radiation.
PHUTE(10)	Pulse height to energy conversion coefficients.
PTITL(12)	Problem title area.
S(251,18)	Used in orthogonal polynomial subroutine.
SP(65)	0.511-MeV annihilation radiation shape.
SPECT (251)	Current input data.
TITLE(12)	Identification of input spectra.
W(18)	Photopeak width parameter.
WO	Width of photopeak in channel number.
WOCØ(10)	Modified-gaussian coefficients.
X(29)	Location of photopeak of all input spectra.
XO	Position of photopeak in channel number.
YO	Height of photopeak in counts.
ZDATA(251,20)	Calculated dense set of points describing the Compton distribution of each input spectra.

APPENDIX I

LINEARITY AND ZERO OF ELECTRONIC SYSTEM

1. LINEARITY MEASUREMENTS

An adequate test of the linearity of the electronic system (preamplifier, amplifier, and analyzer) requires a pulse generator with the following properties:

- (1) output pulse shape simulating pulses coming from the photo-multiplier,
- (2) sufficient stability to carry out tests without distortion, and
- (3) linearity much better than the system to be measured, or a method of measuring the magnitude of the output pulse to 1 part in 10^4 .

In order to meet these requirements, a special pulse generator was constructed. (For details of the circuit see Reference 20). The decay constant of the output pulse is variable over a wide range so that it can be matched with the normal input to the preamplifier. A useful degree of stability is obtained by use of a regulated high-voltage power supply as a voltage reference. This voltage is then attenuated to about 50 mV for use in the pulser. The pulse is formed in the conventional manner by a mercury relay operated at 60 pulses per second. The pulse amplitude is controlled by varying the voltage on a capacitor which is discharged by the relay. Pulse amplitude is determined by varying this charging voltage with a precision decade potentiometer

which provides voltage control accurate to 1 part in 10^5 (manufacturer's specification). This method is then based on the assumption that the amplitude of the pulse is proportional to the charging voltage.

With this pulser, the "profiles" of several consecutive channels were measured. This measurement can be done in two different ways. First, the input pulse amplitude can be varied in small increments and the relative number of counts observed in the one, two or three channels in which they fall. Second, the linear ramp "zero" control of the analyzer can be varied in small increments for a fixed pulse amplitude and the pulse distribution observed. If this control is linear, the latter method should yield the same results as the former. The second method can also be used to calibrate this zero control in units of channels as well as checking its linearity if it is assumed that the channels are of equal width.

A typical channel "profile" is shown in Figure 63. These measurements indicate that more than 90 percent of the input pulses are stored in one channel over an amplitude range of about $2/3$ of a channel. Most of this uncertainty is due to amplifier and pulser noise. The RMS noise level at the amplifier output in this system is equivalent to about 0.3 channels. Over the remainder of the channel the pulses are stored in two channels. The edge of a channel is defined as the point at which equal numbers of pulses are stored in the two channels.

The linearity of the electronic system measured in this manner is shown in Figure 11 of Section II.

[20] D. F. Crouch and R. L. Heath, Routine Testing and Calibration Procedures for Multichannel Pulse Analyzers and Gamma-Ray Spectrometers, IDO-16923 (November 1963).

Fig. 63 Profile of channels of a multichannel analyzer. The counts in each of three adjacent channels are plotted as a function of the input pulse amplitude.

2. ZERO OF PULSE-HEIGHT SCALE

2.1 Arguments for Unique Scale

The convention adopted for the position of zero of the pulse-height scale is that a pulse of zero amplitude should correspond to -0.5 channel. The reasons for this choice are discussed below.

Let $n(V)dV$ be the number of pulses arriving at the analyzer input per unit time that have amplitudes between V and $V + dV$. Then the number of pulses stored in channel i will be

$$N_i = \int_{V_i}^{V_i + \Delta V} n(V) dV$$

where V_i is the amplitude corresponding to the lower edge of the channel and ΔV

is the channel width. (The approximation is made that the channel profiles are non-overlapping and have 100 percent efficiency within the channel.) If $n(V)$ varies approximately linearly between V_i and $V_i + \Delta V$, then

$$N_i \approx N(V_i + \frac{1}{2}\Delta V)\Delta V$$

That is, the observed count is approximately equal to the count at the center of the amplitude range times the width of the channel.

This means that the observed count should be treated (plotted or labeled for data processing) as a measure of the counting rate at the pulse height corresponding to the center of the channel. It is customary to label the first data channel as 1, etc. It then follows that the first channel should span the range from $1/2 \Delta V$ to $3/2 \Delta V$, the second from $3/2 \Delta V$ to $5/2 \Delta V$, etc. Thus, the pulses from 0 to $1/2 \Delta V$ are discarded. This is usually no real loss since these pulses are mostly noise pulses.

If the analyzing system is set up in this manner and is linear, it is possible to measure relative pulse amplitudes. For example, consider two pulses with amplitudes V_1 and V_2 such that $2V_1 = V_2$. For simplicity, let us further assume that V_1 falls at the upper edge of channel C_1 ; that is, $V_1 = (C_1 + 1/2)\Delta V$ where C_1 is an integer. This pulse height is called $C_1 + 1/2$ since the counts will be distributed equally between channels C_1 and $C_1 + 1$. The second pulse will be at the pulse height

$$\frac{V_2}{\Delta V} = \frac{2V_1}{\Delta V} = \frac{2(C_1 + 1/2)\Delta V}{\Delta V} = 2C_1 + 1,$$

or, at the center of channel $C_2 = 2C_1 + 1$. This pulse height is usually called $2C_1 + 1$. The ratio of the observed pulse heights is then 2, as desired.

For comparison, consider a system set up so that the zero of the pulse-height scale is at the lower edge of the first data channel. In this case,

$$N_1 = \int_0^{\Delta V} n(V) dV \approx n(1/2\Delta V)\Delta V$$

and

$$N_{i+1} = \int_{i\Delta V}^{(i+1)\Delta V} n(V) dV \approx n(i\Delta V + 1/2\Delta V)\Delta V.$$

Now the same pulse V_1 falls in the center of channel $C_1 + 1$ and its pulse height is called $C_1 + 1$. The pulse $V_2 = 2V_1$ will fall at the upper edge of channel $2C_1 + 1$, and its pulse height is called $2C_1 + 3/2$. (That is, the larger pulses are equally distributed in $2C_1 + 1$ and $2C_1 + 2$.) The ratio of the observed pulse heights is now

$$\frac{2C_1 + 3/2}{C_1 + 1} = 2 - \frac{1}{2(C_1 + 1)}.$$

This differs from the ratio of the amplitudes.

The adjustment of the zero of the pulse-height scale to -0.5 channel also makes the relative amplitude of two pulses independent of the magnitude of their amplitudes (i.e., the amplifier gain or photomultiplier high voltage) if the circuitry is linear. This fact can be demonstrated by an argument which is a simple extension of that in the preceding paragraphs.

2.2 Method of Adjustment

The method of adjustment of the zero of the pulse-height scale follows from this zero position. The pulse amplitude corresponding to the upper edge of channel 19 (equal counts in channels 19 and 20) is $19.5 \Delta V$ on the desired scale. Sim-

ilarly, amplitude $199.5 \Delta V$ corresponds to an equal number of counts in channels 199 and 200.

The procedure of setting the zero is to vary the amplitude of the pulse from the pulser until the counts fall equally in channels 199 and 200. Then the pulse amplitude is reduced by a factor of $\frac{19.5}{199.5}$, and the analyzer zero control is varied until this pulse is stored equally in channels 19 and 20. Since the change in zero control will change the pulse height for the first pulse, this process must be repeated until the two pulses (ratio $\frac{19.5}{199.5}$)

are both stored in the proper channels without any zero adjustment.

If the ramp voltage is exactly linear, the system will now be adjusted to the desired condition. Any nonlinearity will produce deviations throughout the pulse-height scale and, in particular, the zero will not be at exactly -0.5 channel.

2.3 Effect of Zero Shift on Relative Pulse-Height Measurements

In Section III-3 there is a discussion of the measurements of the pulse heights of various photopeaks relative to that of the photopeak of Cs-137. In this regard it is of interest to determine the error in these values that would result from a shift, or improper adjustment, of the zero of the pulse-height scale. (This result will also be pertinent to the question of the ultimate accuracy of an energy measurement by use of the data in Figure 22.) It is assumed throughout that the electronic system is exactly linear.

Let

P = pulse height of a photopeak on the proper scale (channels)

P_m = pulse height of a photopeak on the actual scale (channels)

P^C = pulse height of Cs-137 photopeak on the proper scale (channels)

P_m^c = pulse height of Cs-137 photopeak on the actual scale (channels)

δ = the difference between the zero settings of the two scales (in channels)

The quantity $\frac{P}{P_c}$ is desired, while $\frac{P_m}{P_m^c}$ is measured. If the gains on the two scales are identical, the following is obtained:

$$P_m = P + \delta \text{ and } P_m^c = P_m^c + \delta .$$

It follows that

$$\frac{P}{P_c} = \frac{P_m - \delta}{P_m^c - \delta} = \frac{P_m (1 - \frac{\delta}{P_m})}{P_m^c (1 - \frac{\delta}{P_m^c})} \approx \frac{P_m}{P_m^c} (1 - \frac{\delta}{P_m}) (1 + \frac{\delta}{P_m^c}) \sim \frac{P_m}{P_m^c} (1 - \frac{\delta}{P_m} + \frac{\delta}{P_m^c})$$

where the relation $\delta \ll P_m$ has been used. For one of the standard gain scales, the Cs-137 peak is in channel 66.16. For this value of P_m and $\delta = 0.1$ channel, the percentage error in using $\frac{P}{P_c} = \frac{P_m}{P_m^c}$ is plotted as a function of $P_m^c \rightarrow P_m$ in Figure 64.

Fig. 64 Percentage error in relative pulse-height measurement as a function of the pulse amplitude. Plot is for zero shift of 0.1 channels and with Cs-137 photopeak as reference and in channel 66.

APPENDIX II

POSSIBLE CAUSES OF NON-GAUSSIAN PHOTOPEAKS

The results of Section III-2 indicate that, at least for this detector, the photopeak shapes deviate systematically from a gaussian shape. Two possible causes are examined here. Of course, this deviation could be due to "improper" operation of one or more components of the system. If this is the case, then the photopeak shapes are significant only for this detection system in its present state of operation. On the other hand, if the deviations are due to basic processes in the detection system, the results are pertinent to other systems. In this regard, some data from another detector are presented here.

1. FINITE RESOLUTION

Any pulse-height spectrum which enters a pulse-height analyzer will be distorted by the finite width of the analyzer channels (i.e., a continuous function is approximated by a histogram). The question then arises as to whether the observed effect is due to the finite channel widths.

To check this, a Gaussian function was integrated numerically over equal ranges of the independent variable. This is equivalent to the effect of an analyzer with non-overlapping channels. This is not strictly true for pulse-height analyzers, but is a good approximation (see Appendix I). The width of the resulting peak was approximately that of a Cs-137 photopeak. The fit was excellent when these data were fitted with a gaussian function. (The residuals were all less than $y_0/10^4$.) This indicates that the finite channel widths do not produce a deviation from a Gaussian that is even of the same order of magnitude as the one observed.

2. NONLINEARITY OF NaI(Tl)

It has been shown [7, 8, 21] that the total light output of a NaI(Tl) crystal is not proportional to the energy dissipated in the crystal by the incident photons. It has also been pointed out [22] that this nonproportional response can be explained by postulating that the amount of light produced by a moving electron is not exactly proportional to its kinetic energy. The effect is such that above 20 keV the light output per keV of electron kinetic energy decreases with increasing kinetic energy. (It is assumed throughout that the pulse height in the analyzing system is proportional to the light output.) The pulse height of the photopeak of a 1.0-MeV gamma ray is then less than twice that of an 0.5-MeV gamma ray.

As discussed by Zerby et al [22] this nonproportional response will, in general, have an effect on the photopeak shape. At "low" energies, the pulses in the photopeak are essentially all due to events in which the absorption is by the photoelectric effect. The electrons are, therefore, monoenergetic and all produce the same amount of light. If it is assumed the photomultiplier produces a broadening which is an exact Gaussian, the low-energy peaks should be very nearly Gaussian in shape. For higher energies, an appreciable fraction of the gamma rays will undergo

[21] P. Iredale, "The Non-Proportional Response of NaI(Tl) Crystals to γ Rays", Nucl. Instr. and Methods, 11 336 (1961).

[22] C. D. Zerby, A. Meyer, and R. B. Murray, "Intrinsic Line Broadening in NaI(Tl) Gamma-Ray Spectrometers", Nucl. Instr. and Methods, 12, 115 (1961).

Compton scattering. The pulse-height, or light, spectrum will then consist of a monoenergetic group from the photoelectric effect plus a distribution from the Compton process. The latter events, which produce electrons with a lower average kinetic energy, will produce a larger pulse height. This effect is illustrated in Figure 65 which is taken from the paper of Zerby *et al* (their Figure 3) [22]. Here the number of events is plotted as a function of the light intensity

or pulse height. These data are from a Monte Carlo calculation for 662-keV gamma rays incident on a 2.5-inch diameter by 2-inch high NaI(Tl) crystal.

In Figure 66 is shown the same pulse-height distribution after it has been convoluted with a Gaussian (photomultiplier smearing). The width of this Gaussian was chosen to produce a reasonably close fit to the complete experimental Cs-137 photopeak. Both spectra were fitted with modified Gaussians. The results were

	$\frac{x_0}{\text{b}_0}$	$\frac{\alpha_1}{\alpha_2}$
Experimental Cs-137:	142.76 ± 0.02	48.7 ± 0.6
Zerby's data smeared:	142.93 ± 0.03	53.2 ± 1.0

Within the computed errors, after smearing, Zerby's data are consistent with a Gaussian photopeak shape. Therefore, these results do not support the

premise that the nonproportionality effect produces the observed deviations from a Gaussian shape.

Fig. 65 Calculated light intensity spectrum in the total absorption peak of 662-keV gamma rays incident on a 2.5-inch-diameter by 2-inch-high NaI(Tl) crystal.

Fig. 66 Comparison of experimental Cs-137 photopeak with data of Figure 65 smeared with a Gaussian function.

3. COMPARISON OF PHOTOPEAK SHAPES WITH THOSE OF OTHER DETECTOR SYSTEMS

Since the results of the last section are inconclusive as to the origin of the deviation from a gaussian shape, a set of spectra was collected on another detection system. If this deviation does result from basic processes in the NaI(Tl) crystal, it should appear in this system.

This system has components (3 x 3 inch crystal, A-8 amplifier, and Nuclear Data ND-130 pulse-height analyzer) similar to those in the system used for the original data. However, the NaI(Tl) crystal was a commercial package (Harshaw Integral-Line). The resulting data for α_1 and α_2 are shown in Figure 67. For comparison, the solid curves from Figure 20 are also shown. In general, for the same energy, the new values of α are larger than those in Figure 20, while the corresponding values of b_0 are 15 percent smaller than those in Figure 19. The similarity of Figures 67 and 20 are thus consistent with, but do not prove, the hypothesis that this effect is the result of basic processes in the crystal.

Fig. 67 Plot of α_1 and α_2 data from another scintillation spectrometer. For comparison, the curves from Figure 20 for the 10-keV scale are shown.

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APPENDIX III

DETAILS OF SINE SERIES AND ORTHOGONAL POLYNOMIAL FITTING PROCEDURES

1. SINE SERIES

As discussed in Section IV-2, the data in any one segment of a spectrum are fitted with the function

$$g(u) = a + bu + \sum_{k=1}^N b_k \sin \frac{k \pi u}{M-1}$$

where u varies from 0 to $M-1$. If the data points are given by y_i , then

$$a = y_0,$$

$$b = \frac{y_{M-1} - y_0}{M-1}, \text{ and}$$

$$b_k = \frac{2}{M-1} \sum_{i=0}^{M-1} \bar{y}_i \sin \frac{\pi k}{M-1} u_i$$

where $\bar{y}_i = y_i - (a + bu_i)$ and $u_i = i = 0, 1, 2, \dots, M-1$.

Let $\theta_k = \frac{\pi k}{M-1}$. Since $u_0 = 0, u_1 = 1, \dots, u_{M-1} = M-1$ and $\sin 0 = \sin \pi k = 0$, the sum for b_k can be written

$$b_k = \frac{2}{M-1} \sum_{j=1}^{M-2} Z_j \sin j \theta_k, \text{ for } k = 1, 2, \dots, N, \text{ where } Z_j = \bar{y}_j.$$

Let U_{n+1} be defined by

$$U_{n+1} \sin \theta_k = \sum_{j=n+1}^{M-2} Z_j \sin(j-n) \theta_k.$$

Since $\sin 0 = 0$, it is possible to have

$$U_{n+1} \sin \theta_k = \sum_{j=n}^{M-2} Z_j \sin(j-n) \theta_k.$$

A recursion relation for the U is as follows:

$$U_n = Z_n + 2 \cos \theta_k U_{n+1} - U_{n+2}.$$

To see this form

$$\begin{aligned} & (Z_n + 2 \cos \theta_k U_{n+1} - U_{n+2}) \sin \theta_k \\ &= Z_n \sin \theta_k + \sum_{j=n+1}^{M-2} Z_j 2 \cos \theta_k \sin(j-n) \theta_k \\ & \quad - \sum_{j=n+2}^{M-2} Z_j \sin(j-n-1) \theta_k \\ &= Z_n \sin \theta_k + \sum_{j=n+1}^{M-2} Z_j \left[2 \cos \theta_k \sin(j-n) \theta_k \right. \\ & \quad \left. - \sin(j-n-1) \theta_k \right] \\ &= \sum_{j=n}^{M-2} Z_j \sin(j-n+1) \theta_k = U_n \sin \theta_k \end{aligned}$$

where $2 \cos \theta_k \sin(j-n) \theta_k = \sin(j-n+1) \theta_k + \sin(j-n-1) \theta_k$ has been used.

Now, from the equation defining U_{n+1} , it follows that $U_M = 0, U_{M-1} = 0$ and

$$U_1 \sin \theta_k = \sum_{j=1}^{M-2} Z_j \sin j \theta_k.$$

Multiplying both sides of this by $\frac{2}{M-1}$, gives

$$b_k = \frac{2}{M-1} U_1 \sin \theta_k.$$

The prescription for computing b_k is shown in detail in the accompanying flow charts, Appendix IV, page 95. Briefly, with $U_M = 0$ and $U_{M-1} = 0$, U_{M-2} is obtained from the recursion relation.

$$U_{M-2} = Z_{M-2} + 2 \cos \theta_k \cdot 0 - 0 = Z_{M-2}$$

and, in general,

$$U_n = Z_n + 2 \cos \theta_k U_{n+1} - U_{n+2}$$

until $n = 1$, then $b_k = (2/M-1) U_1 \sin \theta_k$. The advantage is that no use is made of the sine and cosine routine, except to get $\cos \theta_k$ and $\sin \theta_k$, which results in a significant saving of computer time.

2. FITTING BY ORTHOGONAL POLYNOMIALS

The discussion here closely follows that in Forsythe [18], but is included for the sake of completeness.

The "data points" involved here are the set of values $y(x_0; z)$ computed from the sine series representations of the input spectra in an individual segment. If there are m input spectra, then each set will have m "data points," $f_1, \dots, f_m$. To keep the notation simple, let $x_1, \dots, x_m$ denote the pulse heights of the photopeaks of the input spectra. Finally let $p_i(x)$ be a polynomial of degree i . The problem then is to find coefficients $S_0, S_1, \dots, S_n$, such that

$$\sum_{k=1}^m \left[f_k - \sum_{i=0}^n S_i p_i(x_k) \right]^2 = \text{minimum}$$

The standard procedure of differentiating with respect to the S_i leads to the $n+1$ equations (called the normal equations):

$$\begin{aligned} w_{00} S_0 + \dots + w_{on} S_n &= \omega_0 \\ \vdots & \\ w_{n0} S_0 + \dots + w_{nn} S_n &= \omega_n \end{aligned}$$

where

$$w_{ij} = \sum_{k=1}^m p_i(x_k) p_j(x_k); \quad \omega_i = \sum_{k=1}^m f_k p_i(x_k)$$

One now requires that the polynomials p_i , $i = 1, \dots, n$ be orthogonal with respect to summation over the points x_k , i.e., that

$$w_{ij} = \sum_{k=1}^m p_i(x_k) p_j(x_k) = 0 \text{ for } i \neq j$$

The normal equations then take the form

$$w_{00} S_0 = \omega_0$$

$$w_{11} S_1 = \omega_1$$

$$w_{nn} S_n = \omega_n$$

In other words, the normal equations take the form of a set of linear equations with a diagonal coefficient matrix. The important thing is that there is no matrix inversion necessary to compute the coefficients S_i . They are given by the simple formula

$$S_i = \frac{\omega_i}{w_{ii}} = \frac{\sum_{k=1}^m f_k p_i(x_k)}{\sum_{k=1}^m p_i(x_k) p_i(x_k)}$$

The computation of the orthogonal polynomials is by means of a recursion relation:

$$p_0(x) = 1$$

$$p_1(x) = x p_0(x) - \alpha_1 p_0(x)$$

$$p_2(x) = x p_1(x) - \alpha_2 p_1(x) - \beta_1 p_0(x)$$

$$\vdots$$

$$p_{i+1}(x) = x p_i(x) - \alpha_{i+1} p_i(x) - \beta_i p_{i-1}(x)$$

where the α_i and β_i are certain constants chosen so that the resulting polynomials are orthogonal. They are given by the formulas

$$\beta_i = \frac{\sum_{k=1}^m [p_i(x_k)]^2}{\sum_{k=1}^m [p_{i-1}(x_k)]^2}$$

$$\alpha_{i+1} = \frac{\sum_{k=1}^m x_k [p_k(x_k)]^2}{\sum_{k=1}^m [p_i(x_k)]^2}$$

The α_i and β_i can be computed recursively along with the p_i . Note also that the p_i depend on the values of the x_i so that they are only orthogonal for a particular set of input spectra.

The computer program stores on magnetic tape the 50 or 100 sets of values of S_i and the $m-1$ values of α_i and β_i for the representation of any one segment of the response surface.

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APPENDIX IV

LISTINGS OF PROGRAM INPUT AND OUTPUT

In the following four sections, listings are given for samples of the input card decks along with copies of the printed output for both programs.

1. INPUT DATA FOR GAMSPEC

This listing shows the complete input for the middle-energy range on the 10-keV scale and low-energy range on the 5-keV scale.

INTERMEDIATE ENERGY SHAPE GENERATION JUNE 1964 10 KEV

DATE CARD	062364	1010	1		
PROB. CONTL	11	10.0	1	1	
PHUTE	1	-0.138650E-01	+0.102967E-01	-0.543930E-05	+0.835000E-07 -0.370103E-09
PHUTE	2	+0.519010E-12			
ETPHU	1	+0.132050E+01	+0.975486E+02	+0.388550E+01	-0.645330E+01 +0.284116E+01
ETPHU	2	-0.389410E-00			
EPSCO HI	1	+0.341900E-03	+0.684080E-02	-0.419070E-03	+0.380230E-03 -0.168692E-03
EPSCO HI	2	+0.188240E-04			
PEAKC HI	1	0	+0.550000E-04		
PEAKC HI	2				
WOCO	10	-0.148160E+01	+0.100850E+01	-0.279440E-01	+0.126920E-02
WOCO	10	2			
AICO	10	-0.115390E+02	+0.114954E+02	-0.511630E+01	+0.648800E+00
AICO	10	2			
A2CO	10	+0.146523E+02	-0.316138E+02	+0.144150E+02	-0.262630E+01
A2CO	10	2			
KPFIT1	5	5	5	5	5
KPFIT2	5	5	5	5	5
KPFIT3	5	5	5	5	5
KPFIT4	5	5	5	5	5
KPFIT5	5	5	5	5	5
KPFIT6	5	5	5	5	5

IN-115M SPECTRUM 10 KEV SHAPE SEPT. 5

	34.263	3.7187	0.35600+5	0.2517-03	0.3911-09	0.1417+06	40	
0	0	2600	2520	2415	2345	2254	2100	2007 1898 1794IN-115
1	1775	1719	1763	1792	2129	2429	2445	2358 2383 1909IN-115
2	1511	1173	997	914	890	878	901	994 1118 1374IN-115
3	2092	5238	13431	25916	35193	32338	19942	8239 2190 338IN-115
4	45	0	0	0	0	0	0	0 0IN-115

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IN-113M SPECTRUM 10 KEV SHAPE SEPT. 5

	40.016	4.1011	0.50291+5	0.2611-03	0.1595-09	0.2214+06	46	
0	0	4390	4220	4100	3950	3840	3750	3680 3524 3383IN-113
1	3352	3245	3246	3285	3262	3447	3858	3759 3432 3452IN-113
2	3597	3002	4046	3598	2760	2080	1609	1423 1287 1237IN-113
3	1266	1227	1310	1417	1767	2392	5037	12488 26322 42561IN-113
4	49963	43370	26900	11972	3837	1031	229	0 0IN-113

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BE-7 SPECTRUM 10 KEV SHAPE SEPT. 5

	40.310	4.8042	0.81409+5	0.2580-03	0.5639-10	0.4340+06	56	
0	0	7280	7260	7244	7298	7307	7222	7141 7182 7238BE-7
1	7031	7039	7008	7029	7276	7327	7811	8478 8327 7728BE-7
2	7146	7224	7226	7186	7029	6977	7271	7439 7816 7826BE-7
3	7064	5984	4471	3342	2694	2328	2221	2044 1970 2027BE-7
4	2152	2396	2841	4327	9304	21453	43169	69315 86164 82107BE-7
5	59808	33610	14575	5027	1595	482	189	0 0 0BE-7

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SR-85 SPECTRUM 10 KEV SHAPE SEPT. 5

	51.904	4.8083	0.93676+5	0.2530-03	0.3835-10	0.4868+06	61	
0	0	8250	8240	8237	8258	8247	8183	8161 8258 8166SR-85
1	8117	8229	8084	8150	8226	8488	9263	9789 9874 9513SR-85
2	8821	8506	8338	8430	8361	8218	8118	8151 8165 8485SR-85
3	8762	9021	8975	8345	7226	5534	4178	3147 2611 2328SR-85
4	2161	2078	2142	2222	2372	2713	3654	7058 15842 34451SR-85
5	60394	84582	93455	81435	55011	30159	13203	4826 1612 490SR-85
6	152	19	0	0	0	0	0	0 0 0SR-85

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Y-91M SPECTRUM 10 KEV SHAPE SEPT. 5

56.172	5.0416	0.22218+5	0.2453-03	0.2519-10	0.1214+06	63	
0	0	2040	2040	2043	2035	2040	2061 2043 2056 2057Y-91
1	2005	1987	1980	2003	2048	2079	2146 2256 2371 2324Y-91
2	2216	2138	2106	2100	2102	2104	2091 2098 2072 2064Y-91
3	2065	2036	2012	2109	2187	2163	2072 1810 1375 997Y-91
4	787	681	610	575	554	563	576 612 674 846Y-91
5	1104	1593	3554	7713	13509	19343	22183 20486 15532 9702Y-91
6	4924	2092	770	202	60	0	0 0 0 0Y-91

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CS-137 SPECTRUM 10 KEV SHAPE SEPT. 5

66.162	5.5557	0.55771+5	0.2236-03	0.1065-10	0.3378+06	76	
0	0	6200	6180	6140	6120	6100	6120 6080 6070 6050CS-137
1	6076	5997	6022	6046	6283	6184	6112 6452 6992 7444CS-137
2	7247	6768	6437	6257	6365	6481	6395 6489 6392 6485CS-137
3	6294	6328	6129	6154	6239	6094	6193 6066 6207 6280CS-137
4	6347	6570	6589	6657	6944	6545	6039 5089 4152 3101CS-137
5	2410	1856	1686	1467	1351	1349	1300 1422 1593 2117CS-137
6	3216	6062	12301	22693	35860	49838	55646 52084 41101 27158CS-137
7	14978	7293	2983	1209	487	205	104 0 0 0CS-137

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NB-95 SPECTRUM 10 KEV SHAPE SEPT. 5

76.188	6.0362	0.64888+5	0.2004-03	0.5080-11	0.4295+06	86	
0	0	7320	7300	7330	7309	7307	7244 7317 7371 7206NB-95
1	7152	7193	7181	7170	7429	7385	7457 7886 8157 8911NB-95
2	9294	9027	8463	7978	7849	7975	7904 8027 8050 7925NB-95
3	7892	8019	7819	8142	7821	7835	7781 7904 7964 7752NB-95
4	8047	8164	7981	8049	7911	8173	8195 8177 8396 8377NB-95
5	8623	8784	8989	8889	8666	8211	7010 5731 4707 3644NB-95
6	2908	2363	2021	1813	1634	1606	1605 1760 2179 2934NB-95
7	4768	9394	16963	29889	44916	58103	64600 61702 50140 35583NB-95
8	21764	11715	5724	2619	1167	545	254 0 0 0NB-95

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MN-54 SPECTRUM 10 KEV SHAPE SEPT. 5

82.745	6.3352	0.66649+5	0.1857-03	0.3285-11	0.4648+06	95	
0	0	7600	7600	7608	7562	7516	7494 7549 7526 7517MN-54
1	7565	7600	7478	7590	7796	7788	7838 8289 8581 9226MN-54
2	9879	9562	9040	8441	8350	8452	8414 8367 8409 8287MN-54
3	8266	8408	8311	8268	8260	8238	8231 8184 8249 8381MN-54
4	8406	8431	8514	8537	8359	8579	8554 8772 8838 8718MN-54
5	8598	8931	8920	9005	8926	9320	9330 9747 9692 9816MN-54
6	9240	8958	7713	6536	5256	4180	3306 2802 2233 2052MN-54
7	1784	1741	1755	1854	2195	2881	4455 8023 14911 25408MN-54
8	39717	54521	63817	65826	59654	47033	32085 19926 10767 5542MN-54
9	2635	1340	632	353	174	80	0 0 0 0MN-54

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NB-92 SPECTRUM 10 KEV SHAPE SEPT. 5

92.346	6.7556	0.37122+5	0.1658-03	0.1828-11	0.2775+06	105	
0	0	4440	4440	4424	4445	4477	4502 4515 4525 4527NB-92
1	4601	4632	4570	4686	4800	4744	4771 4983 5363 5747NB-92
2	6017	5934	5516	5059	5017	5009	5026 5042 5014 5095NB-92
3	4980	4964	4945	4969	4862	4940	4919 4852 4865 4812NB-92
4	4858	4936	4956	4909	4910	4874	4989 4999 5251 5236NB-92
5	5286	5407	5317	5407	5376	5421	5254 5338 5496 5515NB-92
6	5587	5598	5699	5656	5825	5910	6018 6063 5851 5697NB-92
7	5213	4774	4036	3331	2730	2237	1849 1487 1305 1196NB-92
8	1087	1074	1126	1355	1697	2499	4320 7505 12700 19402NB-92
9	27094	33447	36625	36382	31380	24652	17198 10976 6331 3417NB-92
10	1806	861	466	236	151	40	0 0 0 0NB-92

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[illegible][illegible][illegible]

24.359	3.5616	0.23470+5	0.5815-05	0.8411-08	0.9116+05	31				
0	0	125	115	120	150	245	259	172	112	94TE-125
1	139	188	298	448	610	935	1411	1788	1657	1411TE-125
2	1612	2966	7609	16360	22542	21506	13953	5611	1425	225TE-125
3	43	17	0	0	0	0	0	0	0	0TE-125

CS-134 SPECTRUM 5 KEV SHAPE SEPT. 5

27.627	3.8620	0.94960+5	0.7154-05	0.1724-08	0.3931+06	34	
0	0	500	470	470	485	520	990 1280 950 495CS-134
1	405	381	359	361	510	678	1032 1529 2041 2887CS-134
2	3733	4150	3901	4436	9088	25042	56113 88064 92114 65290CS-134
3	31480	10508	2524	459	50	0	0 0 0 OCS-134

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CE-141 SPECTRUM 5 KEV SHAPE SEPT. 5

31.651	4.2345	0.19150+6	0.8716-05	0.2837-09	0.8665+06	39	
0	0	1400	1350	1302	1327	1539	1995 2558 3243 3402CE-141
1	3070	2215	1550	1186	1146	1322	1632 1960 2319 3058CE-141
2	3896	4824	6708	8742	10432	11303	12060 16796 34284 72340CE-141
3	128076	178739	187860	146735	85028	36980	12892 3873 1187 371CE-141

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SC-47-2 SPECTRUM 5 KEV SHAPE SEPT. 5

34.212	4.4706	0.10530+6	0.9641-05	0.1022-09	0.4940+06	43	
0	0	1200	1100	1066	953	1110	1079 1133 1149 1316SC-472
1	1488	1885	1927	1597	1228	1108	1096 1154 1386 1544SC-472
2	1907	2152	2321	2395	2608	3309	4170 4967 5307 6883SC-472
3	11982	25617	51076	82790	103909	95801	66545 34469 13623 3857SC-472
4	807	168	90	73	17	33	0 0 0 OSC-472

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SC-47 SPECTRUM 5 KEV SHAPE SEPT. 5

34.218	4.4712	0.22170+6	0.9643-05	0.1020-09	0.1058+07	42	
0	0	2950	2800	2729	2583	2494	2391 2401 2460 2674SC-47
1	2990	3562	3991	3510	2596	2029	2126 2451 2608 3040SC-47
2	3731	4390	4755	4873	5346	6674	8680 10096 11221 13132SC-47
3	21979	50661	107600	179225	220643	201621	138856 72718 29254 9109SC-47
4	2308	318	85	0	0	0	0 0 0 OSC-47

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CE-139 SPECTRUM 5 KEV SHAPE SEPT. 5

35.425	4.5818	0.76370+5	0.1006-04	0.6535-10	0.3733+06	44	
0	0	1050	1001	918	863	823	793 779 781 827CE-139
1	912	1027	1241	1362	1109	841	678 664 679 807CE-139
2	938	1203	1394	1387	1410	1536	1981 2537 2999 3368CE-139
3	4057	7153	15941	33792	57362	74298	73300 54083 30208 12914CE-139
4	4059	921	305	107	56	27	16 12 0 OCE-139

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RU-97 SPECTRUM 5 KEV SHAPE SEPT. 5

45.098	5.4437	0.77110+5	0.1287-04	0.3905-11	0.4477+06	55	
0	0	1700	1650	1580	1510	1460	1390 1350 1300 1250RU-97
1	1240	1185	1184	1245	1377	1672	1859 2103 2308 2371RU-97
2	2190	1793	1648	1664	1909	2067	1925 1738 1733 1750RU-97
3	1790	1840	1900	2020	2193	2419	2787 3082 3697 4876RU-97
4	7889	15750	30603	50333	68942	77234	70917 54503 34154 18049RU-97
5	7809	2830	841	300	83	36	0 0 0 ORU-97

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HG-203 SPECTRUM 5 KEV SHAPE SEPT. 5

57.715	6.4823	0.75830+5	0.1523-04	0.4170-12	0.5251+06	71	
0	0	3200	3120	2998	2963	2791	2730 2645 2627 2556HG-203
1	2483	2507	2551	2409	2381	2308	2338 2273 2255 2269HG-203
2	2361	2460	2620	2852	3214	3601	3952 4089 3760 3226HG-203
3	2622	2096	1902	1815	1776	1788	1817 1911 1807 1900HG-203
4	1844	1932	1897	2015	2069	2217	2368 2535 2829 3098HG-203
5	4123	6410	10919	19383	32226	48140	62818 73492 75641 68568HG-203
6	54680	38829	24262	13514	6609	2951	1127 414 157 67HG-203
7	37	25	0	0	0	0	0 0 0 OHG-203

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IN-115 SPECTRUM 5 KEV SHAPE SEPT. 5

68.949	7.3184	0.25620+5	0.1636-04	0.1224-12	0.2007+06	81	
0	0	1450	1410	1400	1390	1380	1350 1340 1320 13005IN115
1	1290	1280	1260	1250	1240	1230	1210 1195 1180 11705IN115
2	1155	1159	1140	1140	1140	1120	1130 1160 1209 13405IN115
3	1487	1563	1600	1635	1592	1527	1504 1504 1382 12435IN115
4	1069	942	757	680	612	594	592 549 556 5725IN115
5	534	571	569	584	608	627	659 771 801 9685IN115
6	1231	1560	2580	4404	7311	11382	16288 20821 24628 258325IN115
7	23995	20597	15660	10898	6792	3867	1948 906 357 1375IN115
8	73	19	15	12	0	0	0 0 0 05IN115

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2. PRINTED OUTPUT OF GAMSPEC

The portion shown here includes the first two and last two spectra of the middle-energy range on the 10-keV scale and first one and last one of the low-energy range of the 5-keV scale.

The results of the orthogonal polynomial fits are stored on magnetic tape for use in SHAPE. These results are not listed herein.

GAMSPEC1- GAMMA SHAPE GENERATION PROGRAM

INTERMEDIATE ENERGY SHAPE GENERATION JUNE 1964 10 KEV

PROBLEM CONTROL

A. DATE = 62364
 B. TYPE = 1010
 C. NUMBER INPUT SPECTRA USED = 11
 D. GAIN SCALE = 10.00
 E. CALCULATION TYPE = 1
 F. DIAGNOSTIC OPTION = 1

PAGE 2

CONVERSION POLYNOMIAL COEFFICIENTS

PULSE HEIGHT TO ENERGY
 -1.386500E-02 1.029670E-02 -5.439300E-06 8.350000E-08 -3.701030E-10
 5.190100E-13 -0. -0. -0. -0.

ENERGY TO PULSE HEIGHT
 1.320500E 00 9.754860E 01 3.885500E 00 -6.453300E 00 2.841160E 00
 -3.894100E-01 -0. -0. -0. -0.

EPSILON COEFFICIENTS
 3.419000E-04 6.840800E-03 -4.190700E-04 3.802300E-04 -1.686920E-04
 1.882400E-05 -0. -0. -0. -0.

0.511PEAK NORMALIZATION
 0. 5.500000E-05 -0. -0. -0.
 -0. -0. -0. -0. -0.

PHOTO PEAK WIDTH COEFFICIENTS
 -1.481600E 00 1.008500E 00 -2.794400E-02 1.269200E-03 -0.
 -0. -0. -0. -0. -0.

MODIFIED GAUSSIAN A1 COEFFICIENTS
 -1.153900E 01 1.149540E 01 -5.116300E 00 6.488000E-01 -0.
 -0. -0. -0. -0. -0.

MODIFIED GAUSSIAN A2 COEFFICIENTS
 1.465230E 01 -3.161380E 01 1.441500E 01 -2.626300E 00 -0.
 -0. -0. -0. -0. -0.

ORTHOGONAL POLYNOMIAL ORDER INDICATORS

KPFIT1 5
 KPFIT2 5
 KPFIT3 5
 KPFIT4 5
 KPFIT5
 KPFIT6 5

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IN-115M SPECTRUM 10 KEV SHAPE SEPT. 5

DATA CONTROL

A. LOCATION OF PHOTO PEAK = 34.26300 CHANNELS
 B. WIDTH OF PHOTO PEAK = 3.71870 CHANNELS
 C. HEIGHT OF PHOTO PEAK = 0.35600E 05COUNTS
 D. AREA UNDER PHOTO PEAK = 0.14170E 06COUNTS
 E. ALPHA 1 = 0.25170E-03
 F. ALPHA 2 = 0.39110E-09

G. SPECTRA UPPER LIMIT = 40
H. GAMMA RAY ENERGY = 0.33542MEV.
I. ABSORP. X PHOTO EFF. =0.18194E-01
J. NORM. FACTOR =0.12840E 01

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ORIGINAL INPUT SPECTRUM

IN-115M SPECTRUM 10 KEV SHAPE SEPT. 5

	0	1	2	3	4	5	6	7	8	9
0	0	2600	2520	2415	2345	2254	2100	2007	1898	1794
1	1775	1719	1763	1792	2129	2429	2445	2358	2383	1909
2	1511	1173	997	914	890	878	901	994	1118	1374
3	2092	5238	13431	25916	35193	32338	19942	8239	2190	338
4	45	0	0	0	0	0	0	0	0	0

PAGE 5

NORMALIZED SPECTRUM

IN-115M SPECTRUM 10 KEV SHAPE SEPT. 5

	0	1	2	3	4	5	6	7	8	9
0	0	3338	3235	3100	3010	2894	2696	2576	2437	2303
1	2279	2207	2263	2300	2733	3118	3139	3027	3059	2451
2	1940	1506	1280	1173	1142	1127	1156	1276	1435	1764
3	2686	6725	17245	33275	45187	41521	25605	10578	2811	433
4	57	0	0	0	0	0	0	0	0	0

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COMPTON DISTRIBUTION

IN-115M SPECTRUM 10 KEV SHAPE SEPT. 5

	0	1	2	3	4	5	6	7	8	9
0	0	3338	3235	3100	3010	2894	2696	2576	2437	2303
1	2279	2207	2263	2300	2733	3118	3139	3027	3059	2451
2	1940	1506	1280	1173	1142	1127	1154	1264	1386	1521
3	1374	1161	765	56	106	525	585	254	-112	-164
4	-52	-25	-5	0	0	0	0	0	0	0

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IN-115M SPECTRUM 10 KEV SHAPE SEPT. 5

SIN SERIES FIT ERROR ANALYSIS (9 HARMONICS)

K12= 1.7424E 00 B01= 1.0000E 00 BD2= 1.9531E 01

CHANNELS FIT 1 TO 20

CHN	ORIGINAL	CALCULATED	RESIDUAL
1	0.33384E 04	0.33384E 04	-0.
2	0.32357E 04	0.32441E 04	-0.84144E 01
3	0.31008E 04	0.31155E 04	-0.14688E 02
4	0.30110E 04	0.29858E 04	0.25132E 02
5	0.28941E 04	0.28791E 04	0.14966E 02

6	0.26964E 04	0.27461E 04	-0.49715E 02
7	0.25770E 04	0.25604E 04	0.16608E 02
8	0.24370E 04	0.24001E 04	0.36956E 02
9	0.23035E 04	0.23318E 04	-0.28355E 02
10	0.22791E 04	0.22926E 04	-0.13509E 02
11	0.22072E 04	0.22127E 04	-0.55131E 01
12	0.22637E 04	0.21897E 04	0.73934E 02
13	0.23009E 04	0.23798E 04	-0.78919E 02
14	0.27336E 04	0.27413E 04	-0.76574E 01
15	0.31188E 04	0.30481E 04	0.70678E 02
16	0.31394E 04	0.31594E 04	-0.20037E 02
17	0.30277E 04	0.31120E 04	-0.84347E 02
18	0.30598E 04	0.29305E 04	0.12926E 03
19	0.24511E 04	0.25376E 04	-0.86412E 02
20	0.19401E 04	0.19401E 04	0.76294E-04

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IN-115M SPECTRUM 10 KEV SHAPE SEPT. 5

SIN SERIES FIT ERROR ANALYSIS (6 HARMONICS)

K12= 2.4652E 00 BDI= 1.1531E 01 BD2= 2.3993E 01

CHANNELS FIT 11 TO 24

CHN	ORIGINAL	CALCULATED	RESIDUAL
11	0.22072E 04	0.22072E 04	-0.
12	0.22637E 04	0.21710E 04	0.92661E 02
13	0.23009E 04	0.23778E 04	-0.76864E 02
14	0.27336E 04	0.27538E 04	-0.20164E 02
15	0.31188E 04	0.30466E 04	0.72231E 02
16	0.31394E 04	0.31514E 04	-0.11999E 02
17	0.30277E 04	0.31193E 04	-0.91652E 02
18	0.30598E 04	0.29344E 04	0.12532E 03
19	0.24511E 04	0.25167E 04	-0.65553E 02
20	0.19401E 04	0.19485E 04	-0.83903E 01
21	0.15061E 04	0.14842E 04	0.21900E 02
22	0.12801E 04	0.12683E 04	0.11835E 02
23	0.11736E 04	0.12010E 04	-0.27382E 02
24	0.11428E 04	0.11428E 04	-0.15259E-04

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IN-115M SPECTRUM 10 KEV SHAPE SEPT. 5

SIN SERIES FIT ERROR ANALYSIS (9 HARMONICS)

K12= 4.7224E 01 BDI= 1.5993E 01 BD2= 3.4263E 01

CHANNELS FIT 15 TO 34

CHN	ORIGINAL	CALCULATED	RESIDUAL
15	0.31188E 04	0.31188E 04	-0.
16	0.31394E 04	0.30964E 04	0.42918E 02
17	0.30277E 04	0.31057E 04	-0.78010E 02
18	0.30598E 04	0.29676E 04	0.92167E 02
19	0.24511E 04	0.25211E 04	-0.70007E 02
20	0.19401E 04	0.19226E 04	0.17550E 02
21	0.15061E 04	0.14837E 04	0.22402E 02
22	0.12801E 04	0.12917E 04	-0.11512E 02
23	0.11736E 04	0.11984E 04	-0.24832E 02
24	0.11428E 04	0.11188E 04	0.23904E 02
25	0.11271E 04	0.11099E 04	0.17180E 02
26	0.11547E 04	0.11847E 04	-0.29984E 02
27	0.12645E 04	0.12732E 04	-0.86806E 01
28	0.13861E 04	0.13653E 04	0.20794E 02
29	0.15217E 04	0.14767E 04	0.44926E 02
30	0.13742E 04	0.14689E 04	-0.94636E 02
31	0.11616E 04	0.11522E 04	0.94636E 01
32	0.76545E 03	0.62000E 03	0.14545E 03
33	0.56448E 02	0.22441E 03	-0.16796E 03
34	0.10690E 03	0.10690E 03	-0.44823E-04

NORMALIZED SPECTRUM

IN-113M SPECTRUM 10 KEV SHAPE SEPT. 5

	0	1	2	3	4	5	6	7	8	9
0	0	3213	3089	3001	2891	2810	2745	2693	2579	2476
1	2453	2375	2376	2404	2387	2523	2824	2751	2512	2526
2	2633	2841	2961	2633	2020	1522	1177	1041	942	905
3	926	898	958	1037	1293	1750	3687	9141	19267	31154
4	36573	31747	19690	8763	2808	754	167	0	0	0

COMPTON DISTRIBUTION

IN-113M SPECTRUM 10 KEV SHAPE SEPT. 5

	0	1	2	3	4	5	6	7	8	9
0	0	3213	3089	3001	2891	2810	2745	2693	2579	2476
1	2453	2375	2376	2404	2387	2523	2824	2751	2512	2526
2	2633	2841	2961	2633	2020	1522	1177	1041	942	905
3	926	895	946	994	1132	1049	926	744	348	93
4	-238	357	373	104	-64	19	-0	-44	-12	-2

IN-113M SPECTRUM 10 KEV SHAPE SEPT. 5

SIN SERIES FIT ERROR ANALYSIS (10 HARMONICS)

K12= 1.2840E 00 BDI= 1.0000E 00 BD2= 2.0509E 01

CHANNELS FIT 1 TO 21

CHN	ORIGINAL	CALCULATED	RESIDUAL
1	0.32135E 04	0.32135E 04	-0.
2	0.30891E 04	0.30843E 04	0.40069E 01
3	0.30012E 04	0.29932E 04	0.79893E 01
4	0.28914E 04	0.29086E 04	-0.17197E 02
5	0.28109E 04	0.28086E 04	0.23272E 01
6	0.27450E 04	0.27338E 04	0.11241E 02
7	0.26938E 04	0.26882E 04	0.56196E 01
8	0.25796E 04	0.26040E 04	-0.24395E 02
9	0.24764E 04	0.24777E 04	-0.13226E 01
10	0.24537E 04	0.24064E 04	0.47242E 02
11	0.23754E 04	0.24149E 04	-0.39580E 02
12	0.23761E 04	0.24019E 04	-0.25786E 02
13	0.24046E 04	0.23464E 04	0.58233E 02
14	0.23878E 04	0.23905E 04	-0.27189E 01
15	0.25232E 04	0.25921E 04	-0.68902E 02
16	0.28241E 04	0.27669E 04	0.57139E 02
17	0.27516E 04	0.27318E 04	0.19804E 02
18	0.25122E 04	0.25700E 04	-0.57732E 02
19	0.25269E 04	0.25042E 04	0.22649E 02
20	0.26330E 04	0.26199E 04	0.13078E 02
21	0.28416E 04	0.28416E 04	-0.91553E-04

IN-113M SPECTRUM 10 KEV SHAPE SEPT. 5

SIN SERIES FIT ERROR ANALYSIS (8 HARMONICS)

K12= 1.6530E 00 BDI= 1.2509E 01 BD2= 2.8771E 01

CHANNELS FIT 12 TO 29

CHN	ORIGINAL	CALCULATED	RESIDUAL
12	0.23761E 04	0.23761E 04	-0.
13	0.24046E 04	0.23458E 04	0.58854E 02
14	0.23878E 04	0.24106E 04	-0.22827E 02
15	0.25232E 04	0.25845E 04	-0.61305E 02
16	0.28241E 04	0.27499E 04	0.74194E 02
17	0.27516E 04	0.27478E 04	0.38199E 01
18	0.25122E 04	0.25830E 04	-0.70805E 02
19	0.25269E 04	0.24769E 04	0.49998E 02
20	0.26330E 04	0.26167E 04	0.16330E 02
21	0.28416E 04	0.28848E 04	-0.43124E 02
22	0.29617E 04	0.29466E 04	0.15083E 02
23	0.26338E 04	0.26162E 04	0.17568E 02
24	0.20203E 04	0.20414E 04	-0.21035E 02
25	0.15226E 04	0.15127E 04	0.98581E 01
26	0.11778E 04	0.11854E 04	-0.75751E 01
27	0.10416E 04	0.10286E 04	0.13047E 02
28	0.94209E 03	0.95388E 03	-0.11792E 02
29	0.90549E 03	0.90549E 03	-0.22888E-04

IN-113M SPECTRUM 10 KEV SHAPE SEPT. 5

SIN SERIES FIT ERROR ANALYSIS (10 HARMONICS)

K12= 2.8578E 00 BDI= 2.0771E 01 BD2= 4.0016E 01

CHANNELS FIT 20 TO 40

CHN	ORIGINAL	CALCULATED	RESIDUAL
20	0.26330E 04	0.26330E 04	-0.
21	0.28416E 04	0.28933E 04	-0.51631E 02
22	0.29617E 04	0.29259E 04	0.35775E 02
23	0.26338E 04	0.26151E 04	0.18706E 02
24	0.20203E 04	0.20575E 04	-0.37174E 02
25	0.15226E 04	0.15116E 04	0.10925E 02
26	0.11778E 04	0.11733E 04	0.44923E 01
27	0.10416E 04	0.10295E 04	0.12116E 02
28	0.94209E 03	0.96139E 03	-0.19294E 02
29	0.90549E 03	0.91268E 03	-0.71931E 01
30	0.92632E 03	0.89626E 03	0.30055E 02
31	0.89545E 03	0.91222E 03	-0.16768E 02
32	0.94663E 03	0.94810E 03	-0.14624E 01
33	0.99428E 03	0.10114E 04	-0.17121E 02
34	0.11330E 04	0.10844E 04	0.48598E 02
35	0.10496E 04	0.10855E 04	-0.35962E 02
36	0.92643E 03	0.94557E 03	-0.19140E 02
37	0.74476E 03	0.68943E 03	0.55326E 02
38	0.34865E 03	0.39017E 03	-0.41519E 02
39	0.93316E 02	0.81101E 02	0.12216E 02
40	-0.23854E 03	-0.23854E 03	0.36240E-04

DENSE SET OF CALCULATED POINTS DESCRIBING THE COMPTON DISTRIBUTION

IN-113M SPECTRUM 10 KEV SHAPE SEPT. 5

	0	1	2	3	4	5	6	7	8	9
0	0	3213	3170	3129	3092	3059	3030	3004	2979	2954
1	2927	2898	2867	2836	2805	2777	2754	2735	2720	2707
2	2693	2676	2653	2623	2587	2548	2507	2470	2439	2418
3	2406	2404	2407	2413	2417	2416	2408	2393	2374	2355
4	2343	2343	2359	2394	2446	2510	2582	2652	2712	2756
5	2778	2776	2774	2771	2767	2762	2755	2747	2737	2727
6	2715	2702	2689	2675	2660	2645	2629	2614	2598	2583
7	2568	2554	2541	2528	2517	2506	2497	2490	2484	2480
8	2477	2476	2478	2481	2486	2493	2501	2512	2525	2539
9	2555	2572	2591	2611	2632	2654	2677	2700	2723	2747
10	2770	2793	2816	2838	2858	2878	2896	2912	2927	2939
11	2950	2958	2963	2966	2967	2964	2959	2951	2940	2926
12	2909	2889	2867	2842	2814	2783	2751	2715	2678	2639
13	2598	2555	2510	2465	2418	2370	2321	2272	2223	2173
14	2123	2074	2025	1976	1928	1880	1834	1788	1744	1700
15	1658	1618	1477	1354	1254	1175	1115	1071	1038	1012
16	991	971	954	937	923	911	902	897	896	898
17	902	908	917	926	938	952	969	989	1011	1035
18	1059	1079	1094	1100	1094	1075	1041	994	934	863
19	784	698	609	518	426	333	239	144	49	-47
20	-145	-243	0	0	0	0	0	0	0	0

ZN-65 SPECTRUM 10 KEV SHAPE SEPT. 5

DATA CONTROL

A. LOCATION OF PHOTO PEAK = 109.65500 CHANNELS
 B. WIDTH OF PHOTO PEAK = 7.47220 CHANNELS
 C. HEIGHT OF PHOTO PEAK = 0.29128E 05COUNTS
 D. AREA UNDER PHOTO PEAK = 0.24300E 06COUNTS
 E. ALPHA 1 = 0.13510E-03
 F. ALPHA 2 = 0.71290E-12
 G. SPECTRA UPPER LIMIT = 124
 H. GAMMA RAY ENERGY = 1.11463MEV.
 I. ABSORP. X PHOTO EFF. = 0.63181E-02
 J. NORM. FACTOR = 0.26000E-00

ORIGINAL INPUT SPECTRUM

ZN-65 SPECTRUM 10 KEV SHAPE SEPT. 5

	0	1	2	3	4	5	6	7	8	9
0	0	3750	3750	3740	3721	3706	3671	3790	3710	3711
1	3729	3736	3732	3825	3829	3822	3770	3811	3981	4396
2	4756	4909	4637	4241	3981	4063	4010	4077	4038	4060
3	3995	4144	4090	3986	3950	3867	3973	3892	3835	3895
4	3821	3869	3845	3789	3831	3812	3866	3854	3889	3917
5	3976	3896	3957	3954	4060	4005	4003	4097	4160	4105
6	4234	4186	4243	4299	4428	4266	4432	4556	4536	4429
7	4560	4584	4555	4751	4814	4913	4912	4966	4992	5059
8	5045	5154	5295	5170	5375	5163	4940	4497	3977	3442
9	2896	2353	1990	1692	1352	1164	1106	1045	1017	1056
10	1228	1603	2271	3798	6431	10107	15094	20587	25442	28446
11	28901	26456	22155	16671	11591	7564	4666	2740	1606	882
12	476	303	173	127	75	0	0	0	0	0

NORMALIZED SPECTRUM

ZN-65 SPECTRUM 10 KEV SHAPE SEPT. 5

	0	1	2	3	4	5	6	7	8	9
0	0	975	975	972	967	963	954	985	964	964
1	969	971	970	994	995	993	980	990	1035	1142
2	1236	1276	1205	1102	1035	1056	1042	1060	1049	1055
3	1038	1077	1063	1036	1027	1005	1032	1011	977	1012
4	993	1005	999	985	996	991	1005	1002	1011	1018
5	1033	1012	1028	1028	1055	1041	1040	1065	1081	1067
6	1100	1088	1103	1117	1151	1109	1152	1184	1179	1151
7	1185	1191	1184	1235	1251	1277	1277	1291	1297	1315
8	1311	1340	1376	1344	1397	1342	1284	1169	1034	894
9	752	611	517	439	351	302	287	271	264	274
10	319	416	590	987	1672	2627	3924	5352	6615	7396
11	7514	6878	5760	4334	3013	1966	1213	712	417	229
12	123	78	44	33	19	0	0	0	0	0

COMPTON DISTRIBUTION

ZN-65 SPECTRUM 10 KEV SHAPE SEPT. 5

	0	1	2	3	4	5	6	7	8	9
0	0	975	975	972	967	963	954	985	964	964
1	969	971	970	994	995	993	980	990	1035	1142
2	1236	1276	1205	1102	1035	1056	1042	1060	1049	1055
3	1038	1077	1063	1036	1027	1005	1032	1011	997	1012
4	993	1005	999	985	996	991	1005	1002	1011	1018
5	1033	1012	1028	1028	1055	1041	1040	1065	1081	1067
6	1100	1088	1103	1117	1151	1109	1152	1184	1179	1151
7	1185	1191	1184	1235	1251	1277	1277	1291	1297	1315
8	1311	1340	1376	1344	1397	1342	1284	1169	1034	894
9	752	611	516	437	345	288	261	227	193	159
10	123	70	-25	-79	-90	-118	-70	-19	-1	-17
11	-14	-47	-26	-83	-95	-69	-40	-20	3	-3
12	-11	-3	-6	2	2	-8	-3	0	0	0

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ZN-65 SPECTRUM 10 KEV SHAPE SEPT. 5

SIN SERIES FIT ERROR ANALYSIS (12 HARMONICS)

K12= 4.2372E-01 BDI= 1.0000E 00 BD2= 2.5712E 01

CHANNELS FIT 1 TO 26

CHN	ORIGINAL	CALCULATED	RESIDUAL
1	0.97501E 03	0.97501E 03	-0.
2	0.97501E 03	0.97475E 03	0.26702E-00
3	0.97241E 03	0.97276E 03	-0.34751E-00
4	0.96747E 03	0.96715E 03	0.32127E-00
5	0.96357E 03	0.96150E 03	0.20742E 01
6	0.95447E 03	0.96326E 03	-0.87899E 01
7	0.98542E 03	0.97122E 03	0.14199E 02
8	0.96461E 03	0.97399E 03	-0.93758E 01
9	0.96487E 03	0.96749E 03	-0.26136E 01
10	0.96955E 03	0.96281E 03	0.67494E 01
11	0.97137E 03	0.96915E 03	0.22272E 01
12	0.97033E 03	0.98065E 03	-0.10313E 02
13	0.99452E 03	0.98867E 03	0.58493E 01
14	0.99556E 03	0.99343E 03	0.21257E 01
15	0.99374E 03	0.99406E 03	-0.32281E-00
16	0.98022E 03	0.98628E 03	-0.60690E 01
17	0.99088E 03	0.98638E 03	0.44943E 01
18	0.10351E 04	0.10341E 04	0.94606E 00
19	0.11430E 04	0.11406E 04	0.23727E 01
20	0.12366E 04	0.12464E 04	-0.98270E 01
21	0.12764E 04	0.12703E 04	0.60189E 01
22	0.12056E 04	0.12006E 04	0.50113E 01
23	0.11027E 04	0.11052E 04	-0.25220E 01
24	0.10351E 04	0.10492E 04	-0.14167E 02
25	0.10564E 04	0.10374E 04	0.19009E 02
26	0.10426E 04	0.10426E 04	0.30518E-04

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ZN-65 SPECTRUM 10 KEV SHAPE SEPT. 5

SIN SERIES FIT ERROR ANALYSIS (38 HARMONICS)

K12= 6.9836E-01 BDI= 1.7712E 01 BD2= 9.3841E 01

CHANNELS FIT 17 TO 94

CHN	ORIGINAL	CALCULATED	RESIDUAL
17	0.99088E 03	0.99088E 03	-0.
18	0.10351E 04	0.10392E 04	-0.41318E 01
19	0.11430E 04	0.11362E 04	0.67402E 01

20	0.12366E 04	0.12430E 04	-0.64455E 01
21	0.12764E 04	0.12735E 04	0.28358E 01
22	0.12056E 04	0.12044E 04	0.12239E 01
23	0.11027E 04	0.11025E 04	0.19261E-00
24	0.10351E 04	0.10438E 04	-0.86963E 01
25	0.10564E 04	0.10404E 04	0.15972E 02
26	0.10426E 04	0.10551E 04	-0.12476E 02
27	0.10600E 04	0.10577E 04	0.22977E 01
28	0.10499E 04	0.10497E 04	0.20374E-00
29	0.10556E 04	0.10466E 04	0.90219E 01
30	0.10387E 04	0.10553E 04	-0.16589E 02
31	0.10775E 04	0.10662E 04	0.11243E 02
32	0.10634E 04	0.10631E 04	0.36342E-00
33	0.10364E 04	0.10423E 04	-0.59434E 01
34	0.10270E 04	0.10206E 04	0.64152E 01
35	0.10054E 04	0.10150E 04	-0.95856E 01
36	0.10330E 04	0.10203E 04	0.12681E 02
37	0.10119E 04	0.10179E 04	-0.59867E 01
38	0.99712E 03	0.10051E 04	-0.79984E 01
39	0.10127E 04	0.99777E 03	0.14948E 02
40	0.99348E 03	0.10025E 04	-0.90557E 01
41	0.10060E 04	0.10057E 04	0.28594E-00
42	0.99972E 03	0.99718E 03	0.25340E 01
43	0.98516E 03	0.98789E 03	-0.27377E 01
44	0.99608E 03	0.99047E 03	0.56073E 01
45	0.99114E 03	0.99873E 03	-0.75937E 01
46	0.10052E 04	0.10010E 04	0.41867E 01
47	0.10021E 04	0.10015E 04	0.54106E 00
48	0.10112E 04	0.10109E 04	0.28103E-00
49	0.10184E 04	0.10237E 04	-0.52491E 01
50	0.10338E 04	0.10258E 04	0.79286E 01
51	0.10130E 04	0.10198E 04	-0.68525E 01
52	0.10288E 04	0.10223E 04	0.64887E 01
53	0.10281E 04	0.10366E 04	-0.85196E 01
54	0.10556E 04	0.10467E 04	0.89253E 01
55	0.10413E 04	0.10449E 04	-0.36156E 01
56	0.10408E 04	0.10451E 04	-0.42981E 01
57	0.10652E 04	0.10593E 04	0.59003E 01
58	0.10816E 04	0.10776E 04	0.40032E 01
59	0.10673E 04	0.10843E 04	-0.16944E 02
60	0.11009E 04	0.10835E 04	0.17379E 02
61	0.10884E 04	0.10915E 04	-0.31100E 01
62	0.11032E 04	0.11096E 04	-0.63786E 01
63	0.11178E 04	0.11230E 04	-0.52725E 01
64	0.11513E 04	0.11265E 04	0.24808E 02
65	0.11092E 04	0.11329E 04	-0.23707E 02
66	0.11523E 04	0.11515E 04	0.78886E 00
67	0.11846E 04	0.11717E 04	0.12845E 02
68	0.11794E 04	0.11779E 04	0.14770E 01
69	0.11516E 04	0.11723E 04	-0.20700E 02
70	0.11856E 04	0.11709E 04	0.14717E 02
71	0.11919E 04	0.11828E 04	0.90811E 01
72	0.11843E 04	0.12034E 04	-0.19126E 02
73	0.12353E 04	0.12268E 04	0.84503E 01
74	0.12517E 04	0.12512E 04	0.44138E-00
75	0.12774E 04	0.12729E 04	0.44686E 01
76	0.12771E 04	0.12855E 04	-0.83839E 01
77	0.12912E 04	0.12897E 04	0.14944E 01
78	0.12979E 04	0.12953E 04	0.26219E 01
79	0.13154E 04	0.13087E 04	0.66244E 01
80	0.13117E 04	0.13252E 04	-0.13516E 02
81	0.13401E 04	0.13396E 04	0.49872E-00
82	0.13767E 04	0.13550E 04	0.21688E 02
83	0.13442E 04	0.13730E 04	-0.28769E 02
84	0.13975E 04	0.13790E 04	0.18533E 02
85	0.13424E 04	0.13508E 04	-0.83905E 01
86	0.12844E 04	0.12789E 04	0.55336E 01
87	0.11692E 04	0.11709E 04	-0.16246E 01
88	0.10340E 04	0.10385E 04	-0.44999E 01
89	0.89493E 03	0.89215E 03	0.27833E 01
90	0.75297E 03	0.74670E 03	0.62735E 01
91	0.61179E 03	0.61995E 03	-0.81558E 01
92	0.51645E 03	0.51784E 03	-0.13903E 01
93	0.43727E 03	0.43013E 03	0.71398E 01
94	0.34510E 03	0.34510E 03	0.41962E-04

ZN-65 SPECTRUM 10 KEV SHAPE SEPT. 5

SIN SERIES FIT ERROR ANALYSIS (12 HARMONICS)

K12=-1.2408E 01 BDI= 8.5841E 01 BD2= 1.0965E 02

CHANNELS FIT 85 TO 110

CHN	ORIGINAL	CALCULATED	RESIDUAL
85	0.13424E 04	0.13424E 04	-0.
86	0.12844E 04	0.12736E 04	0.10844E 02
87	0.11692E 04	0.11749E 04	-0.56839E 01
88	0.10340E 04	0.10404E 04	-0.63896E 01
89	0.89493E 03	0.88925E 03	0.56827E 01
90	0.75297E 03	0.74613E 03	0.68448E 01
91	0.61179E 03	0.62277E 03	-0.10981E 02
92	0.51645E 03	0.51725E 03	-0.79716E 00
93	0.43727E 03	0.42594E 03	0.11335E 02
94	0.34510E 03	0.35082E 03	-0.57179E 01
95	0.28899E 03	0.29499E 03	-0.59979E 01
96	0.26185E 03	0.25575E 03	0.60943E 01
97	0.22777E 03	0.22534E 03	0.24374E 01
98	0.19341E 03	0.19651E 03	-0.30924E 01
99	0.15946E 03	0.16387E 03	-0.44125E 01
100	0.12394E 03	0.12077E 03	0.31698E 01
101	0.70939E 02	0.61062E 02	0.98771E 01
102	-0.25459E 02	-0.11312E 02	-0.14147E 02
103	-0.79299E 02	-0.76838E 02	-0.24608E 01
104	-0.90380E 02	-0.11075E 03	0.20367E 02
105	-0.11823E 03	-0.10205E 03	-0.15383E 02
106	-0.70680E 02	-0.66765E 02	-0.39150E 01
107	-0.19793E 02	-0.28591E 02	0.87985E 01
108	-0.19674E 01	-0.71882E 01	0.52207E 01
109	-0.17866E 02	-0.52102E 01	-0.12656E 02
110	-0.14404E 02	-0.14404E 02	-0.60797E-05

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DENSE SET OF CALCULATED POINTS DESCRIBING THE COMPTON DISTRIBUTION

ZN-65 SPECTRUM 10 KEV SHAPE SEPT. 5

	0	1	2	3	4	5	6	7	8	9
0	0	975	974	974	974	973	972	970	967	965
1	962	961	961	963	966	969	972	974	973	971
2	968	965	963	962	964	968	973	978	982	986
3	988	990	992	994	994	993	991	987	983	983
4	989	1005	1032	1071	1117	1167	1214	1250	1270	1273
5	1257	1227	1163	1096	1052	1037	1042	1052	1058	1056
6	1051	1046	1047	1053	1062	1067	1063	1051	1035	1020
7	1014	1016	1020	1020	1013	1004	998	998	1003	1005
8	1002	995	988	987	992	997	1000	1000	1001	1005
9	1014	1023	1026	1024	1019	1019	1026	1037	1045	1046
10	1044	1043	1049	1062	1074	1082	1084	1083	1086	1095
11	1107	1118	1124	1126	1129	1137	1151	1166	1175	1177
12	1174	1170	1171	1178	1191	1206	1222	1238	1255	1270
13	1281	1286	1289	1292	1298	1307	1319	1329	1339	1349
14	1361	1373	1379	1373	1347	1300	1235	1156	1066	969
15	868	769	715	665	619	575	534	496	459	425
16	393	364	338	315	295	278	263	249	237	226
17	214	203	191	179	165	150	133	113	90	65
18	37	8	-20	-48	-72	-91	-105	-111	-111	-105
19	-93	-79	-63	-46	-32	-20	-11	-6	-4	-4
20	-6	-10	0	0	0	0	0	0	0	0

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A-41 SPECTRUM 10 KEV SHAPE SEPT. 5

DATA CONTROL

A. LOCATION OF PHOTO PEAK = 126.16500 CHANNELS

B. WIDTH OF PHOTO PEAK = 8.11930 CHANNELS
 C. HEIGHT OF PHOTO PEAK =0.57187E 05COUNTS
 D. AREA UNDER PHOTO PEAK =0.52230E 06COUNTS
 E. ALPHA 1 =0.11190E-03
 F. ALPHA 2 =0.32050E-12
 G. SPECTRA UPPER LIMIT = 146
 H. GAMMA RAY ENERGY = 1.28914MEV.
 I. ABSORP. X PHOTO EFF. =0.55179E-02
 J. NORM. FACTOR =0.10565E-00

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ORIGINAL INPUT SPECTRUM

A-41 SPECTRUM 10 KEV SHAPE SEPT. 5

	0	1	2	3	4	5	6	7	8	9
0	0	8240	8240	8236	8231	8225	8220	8215	8210	8204
1	8199	8193	8192	8197	8202	8206	8229	8470	8882	9312
2	10034	11132	11245	10664	9753	9062	8946	8876	8663	8479
3	8295	8281	8172	8179	8150	8118	8116	8150	7943	7981
4	8073	8222	7970	7989	8016	8055	8070	7982	7937	7899
5	7996	8086	8168	8053	8100	8012	7975	8043	8049	8066
6	8215	8290	8283	8318	8521	8540	8481	8402	8627	8742
7	8764	8862	8928	8960	9167	9301	9368	9482	9587	9821
8	9987	9953	10007	10014	10066	10341	10387	10620	10476	10639
9	10828	10863	10995	11199	11248	11544	11542	11940	12003	11885
10	11802	11575	10997	10114	9245	8133	7062	6121	5097	4299
11	3518	3047	2954	2552	2460	2593	2981	3790	5062	8137
12	12540	19105	28053	38100	47521	54255	57034	55247	49797	41190
13	31394	22598	15397	9932	6147	3745	2342	1422	909	600
14	401	289	222	169	130	94	60	0	0	0

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NORMALIZED SPECTRUM

A-41 SPECTRUM 10 KEV SHAPE SEPT. 5

	0	1	2	3	4	5	6	7	8	9
0	0	870	870	870	869	868	868	867	867	866
1	866	865	865	865	866	866	869	894	938	983
2	1060	1176	1187	1126	1030	957	945	937	915	895
3	876	874	863	864	861	857	857	861	839	843
4	852	868	841	844	846	850	852	843	838	834
5	844	854	862	850	855	846	842	849	850	852
6	867	875	875	878	900	902	895	887	911	923
7	925	936	943	946	968	982	989	1001	1012	1037
8	1055	1051	1057	1057	1063	1092	1097	1121	1106	1123
9	1143	1147	1161	1183	1188	1219	1219	1261	1268	1255
10	1246	1222	1161	1068	976	859	746	646	538	454
11	371	321	312	269	259	273	314	400	534	859
12	1324	2018	2963	4025	5020	5731	6025	5836	5260	4351
13	3316	2387	1626	1049	649	395	247	150	96	63
14	42	30	23	17	13	9	6	0	0	0

COMPTON DISTRIBUTION

A-41 SPECTRUM 10 KEV SHAPE SEPT. 5

	0	1	2	3	4	5	6	7	8	9
0	0	870	870	870	869	868	868	867	867	866
1	866	865	865	865	866	866	869	894	938	983
2	1060	1176	1187	1126	1030	957	945	937	915	895
3	876	874	863	864	861	857	857	861	839	843
4	852	868	841	844	846	850	852	843	838	834
5	844	854	862	850	855	846	842	849	850	852
6	867	875	875	878	900	902	895	887	911	923
7	925	936	943	946	968	982	989	1001	1012	1037
8	1055	1051	1057	1057	1063	1092	1097	1121	1106	1123
9	1143	1147	1161	1183	1188	1219	1219	1261	1268	1255
10	1246	1222	1161	1068	976	859	746	646	536	448
11	360	301	277	215	178	148	112	64	-23	-47
12	-95	-105	-47	16	47	24	-9	-30	10	11
13	-16	-11	-4	-7	-8	-2	8	3	2	1
14	2	5	9	10	10	9	6	0	0	0

A-41 SPECTRUM 10 KEV SHAPE SEPT. 5

SIN SERIES FIT ERROR ANALYSIS (12 HARMONICS)

K12= 4.0584E-01 BDI= 1.0000E 00 BD2= 2.6241E 01

CHANNELS FIT 1 TO 26

CHN	ORIGINAL	CALCULATED	RESIDUAL -
1	0.87052E 03	0.87052E 03	-0.
2	0.87052E 03	0.86982E 03	0.70438E 00
3	0.87010E 03	0.87001E 03	0.94734E-01
4	0.86957E 03	0.87024E 03	-0.66762E 00
5	0.86894E 03	0.86923E 03	-0.29461E-00
6	0.86841E 03	0.86767E 03	0.74292E 00
7	0.86788E 03	0.86741E 03	0.47061E-00
8	0.86735E 03	0.86817E 03	-0.82004E 00
9	0.86672E 03	0.86748E 03	-0.76040E 00
10	0.86619E 03	0.86514E 03	0.10552E 01
11	0.86556E 03	0.86451E 03	0.10485E 01
12	0.86545E 03	0.86677E 03	-0.13205E 01
13	0.86598E 03	0.86763E 03	-0.16460E 01
14	0.86651E 03	0.86455E 03	0.19623E 01
15	0.86693E 03	0.86394E 03	0.29906E 01
16	0.86936E 03	0.87443E 03	-0.50669E 01
17	0.89482E 03	0.89665E 03	-0.18323E 01
18	0.93835E 03	0.93004E 03	0.83124E 01
19	0.98378E 03	0.98561E 03	-0.18310E 01
20	0.10601E 04	0.10719E 04	-0.11877E 02
21	0.11761E 04	0.11611E 04	0.14953E 02
22	0.11860E 04	0.11920E 04	-0.40048E 01
23	0.11266E 04	0.11321E 04	-0.55031E 01
24	0.10304E 04	0.10266E 04	0.37732E 01
25	0.95736E 03	0.95648E 03	0.88261E 00
26	0.94511E 03	0.94511E 03	0.76294E-05

A-41 SPECTRUM 10 KEV SHAPE SEPT. 5

SIN SERIES FIT ERROR ANALYSIS (46 HARMONICS)

K12= 4.1331E-01 BDI= 1.8241E 01 BD2= 1.0998E 02

CHANNELS FIT 18 TO 110

CHN	ORIGINAL	CALCULATED	RESIDUAL
18	0.93835E 03	0.93835E 03	-0.
19	0.98378E 03	0.97956E 03	0.42149E 01
20	0.10601E 04	0.10681E 04	-0.80195E 01
21	0.11761E 04	0.11669E 04	0.91582E 01
22	0.11880E 04	0.11939E 04	-0.59111E 01
23	0.11266E 04	0.11265E 04	0.14146E-00
24	0.10304E 04	0.10262E 04	0.41673E 01
25	0.95736E 03	0.96197E 03	-0.46106E 01
26	0.94511E 03	0.94321E 03	0.19027E 01
27	0.93771E 03	0.93630E 03	0.14116E 01
28	0.91521E 03	0.91799E 03	-0.27792E 01
29	0.89577E 03	0.89385E 03	0.19222E 01
30	0.87633E 03	0.87773E 03	-0.13949E 01
31	0.87485E 03	0.87170E 03	0.31562E 01
32	0.86334E 03	0.86795E 03	-0.46118E 01
33	0.86408E 03	0.86211E 03	0.19666E 01
34	0.86101E 03	0.85835E 03	0.26604E 01
35	0.85763E 03	0.86007E 03	-0.24378E 01
36	0.85742E 03	0.86100E 03	-0.35777E 01
37	0.86101E 03	0.85350E 03	0.75168E 01
38	0.83915E 03	0.84323E 03	-0.40797E 01
39	0.84316E 03	0.84371E 03	-0.54770E 00
40	0.85288E 03	0.85478E 03	-0.18998E 01
41	0.86862E 03	0.86008E 03	0.85419E 01
42	0.84200E 03	0.85133E 03	-0.93362E 01
43	0.84401E 03	0.84136E 03	0.26460E 01
44	0.84686E 03	0.84393E 03	0.29316E 01
45	0.85098E 03	0.85293E 03	-0.19538E 01
46	0.85256E 03	0.85330E 03	-0.73994E 00
47	0.84327E 03	0.84360E 03	-0.33274E-00
48	0.83851E 03	0.83554E 03	0.29703E 01
49	0.83450E 03	0.83684E 03	-0.23400E 01
50	0.84475E 03	0.84502E 03	-0.27328E-00
51	0.85425E 03	0.85417E 03	0.84709E-01
52	0.86292E 03	0.85924E 03	0.36774E 01
53	0.85077E 03	0.85728E 03	-0.65131E 01
54	0.85573E 03	0.85082E 03	0.49147E 01
55	0.84644E 03	0.84640E 03	0.33752E-01
56	0.84253E 03	0.84624E 03	-0.37136E 01
57	0.84971E 03	0.84698E 03	0.27259E 01
58	0.85034E 03	0.84871E 03	0.16355E 01
59	0.85214E 03	0.85591E 03	-0.37732E 01
60	0.86788E 03	0.86688E 03	0.99783E 00
61	0.87581E 03	0.87347E 03	0.23377E 01
62	0.87507E 03	0.87556E 03	-0.49144E-00
63	0.87876E 03	0.88277E 03	-0.40099E 01
64	0.90021E 03	0.89604E 03	0.41650E 01
65	0.90222E 03	0.90228E 03	-0.68451E-01
66	0.89598E 03	0.89621E 03	-0.22190E-00
67	0.88764E 03	0.89258E 03	-0.49400E 01
68	0.91141E 03	0.90467E 03	0.67390E 01
69	0.92356E 03	0.92346E 03	0.92850E-01
70	0.92588E 03	0.93251E 03	-0.66305E 01
71	0.93623E 03	0.93290E 03	0.33317E 01
72	0.94321E 03	0.93829E 03	0.49165E 01
73	0.94659E 03	0.95270E 03	-0.61112E 01
74	0.96846E 03	0.96818E 03	0.27393E-00
75	0.98261E 03	0.97980E 03	0.28123E 01
76	0.98969E 03	0.99007E 03	-0.37971E-00
77	0.10017E 04	0.10018E 04	-0.47287E-01
78	0.10128E 04	0.10162E 04	-0.33869E 01
79	0.10375E 04	0.10343E 04	0.32201E 01

80	0.10551E 04	0.10517E 04	0.33812E 01
81	0.10515E 04	0.10588E 04	-0.72723E 01
82	0.10572E 04	0.10547E 04	0.24832E 01
83	0.10579E 04	0.10538E 04	0.41333E 01
84	0.10634E 04	0.10678E 04	-0.44068E 01
85	0.10925E 04	0.10896E 04	0.28694E 01
86	0.10973E 04	0.11042E 04	-0.68765E 01
87	0.11220E 04	0.11097E 04	0.12216E 02
88	0.11067E 04	0.11154E 04	-0.86902E 01
89	0.11240E 04	0.11256E 04	-0.16100E 01
90	0.11439E 04	0.11373E 04	0.66542E 01
91	0.11476E 04	0.11498E 04	-0.21806E 01
92	0.11616E 04	0.11646E 04	-0.30518E 01
93	0.11831E 04	0.11796E 04	0.35783E 01
94	0.11883E 04	0.11929E 04	-0.45710E 01
95	0.12196E 04	0.12094E 04	0.10180E 02
96	0.12194E 04	0.12326E 04	-0.13201E 02
97	0.12614E 04	0.12547E 04	0.67161E 01
98	0.12681E 04	0.12646E 04	0.34192E 01
99	0.12556E 04	0.12611E 04	-0.54691E 01
100	0.12468E 04	0.12475E 04	-0.71375E 00
101	0.12229E 04	0.12184E 04	0.44140E 01
102	0.11618E 04	0.11616E 04	0.15970E-00
103	0.10685E 04	0.10750E 04	-0.64830E 01
104	0.97670E 03	0.97025E 03	0.64497E 01
105	0.85922E 03	0.86016E 03	-0.94406E 00
106	0.74607E 03	0.74994E 03	-0.38675E 01
107	0.64666E 03	0.64178E 03	0.48758E 01
108	0.53605E 03	0.53974E 03	-0.36900E 01
109	0.44864E 03	0.44673E 03	0.19159E 01
110	0.36038E 03	0.36038E 03	0.30518E-04

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A-41 SPECTRUM 10 KEV SHAPE SEPT. 5

SIN SERIES FIT ERROR ANALYSIS (12 HARMONICS)

K12=-1.2597E 01 BDI= 1.0198E 02 BD2= 1.2616E 02

CHANNELS FIT 101 TO 126

CHN	ORIGINAL	CALCULATED	RESIDUAL
101	0.12229E 04	0.12229E 04	-0.
102	0.11618E 04	0.11588E 04	0.29784E 01
103	0.10685E 04	0.10751E 04	-0.66288E 01
104	0.97670E 03	0.97069E 03	0.60088E 01
105	0.85922E 03	0.85906E 03	0.15980E-00
106	0.74607E 03	0.75036E 03	-0.42902E 01
107	0.64666E 03	0.64394E 03	0.27234E 01
108	0.53605E 03	0.53862E 03	-0.25609E 01
109	0.44864E 03	0.44213E 03	0.65157E 01
110	0.36038E 03	0.36498E 03	-0.45995E 01
111	0.30122E 03	0.30835E 03	-0.71252E 01
112	0.27755E 03	0.26335E 03	0.14204E 02
113	0.21578E 03	0.22178E 03	-0.59954E 01
114	0.17829E 03	0.18263E 03	-0.43355E 01
115	0.14808E 03	0.14661E 03	0.14743E 01
116	0.11252E 03	0.10841E 03	0.41080E 01
117	0.64564E 02	0.59038E 02	0.55264E 01
118	-0.23489E 02	-0.29198E 01	-0.20570E 02
119	-0.47452E 02	-0.63706E 02	0.16254E 02
120	-0.95441E 02	-0.99731E 02	0.42901E 01
121	-0.10586E 03	-0.93366E 02	-0.12492E 02
122	-0.47106E 02	-0.47953E 02	0.84625E 00
123	0.16170E 02	0.97729E 01	0.63975E 01
124	0.47591E 02	0.44231E 02	0.33601E 01
125	0.24278E 02	0.34650E 02	-0.10373E 02
126	-0.92505E 01	-0.92505E 01	-0.15616E-04

DENSE SET OF CALCULATED POINTS DESCRIBING THE COMPTON DISTRIBUTION

A-41 SPECTRUM 10 KEV SHAPE SEPT. 5

	0	1	2	3	4	5	6	7	8	9
0	0	870	870	869	869	869	870	870	870	870
1	869	868	868	867	867	867	867	868	868	868
2	867	866	865	864	864	864	865	866	867	867
3	866	865	863	863	864	866	872	879	889	900
4	913	929	949	974	1005	1042	1083	1123	1158	1183
5	1193	1185	1118	1033	972	947	940	932	915	895
6	880	873	870	866	861	858	859	861	858	849
7	842	843	852	859	857	847	840	844	851	854
8	848	839	835	837	843	851	857	859	856	850
9	846	846	846	847	849	856	866	872	874	877
10	886	897	902	898	892	896	910	925	932	932
11	935	944	957	970	979	988	997	1008	1022	1037
12	1051	1058	1056	1052	1057	1072	1090	1103	1108	1112
13	1119	1128	1138	1148	1160	1173	1185	1196	1211	1230
14	1250	1262	1264	1258	1245	1220	1176	1109	1027	936
15	844	752	709	666	623	580	538	498	459	424
16	392	363	339	317	297	279	261	244	228	211
17	195	180	165	151	137	121	105	86	65	42
18	17	-8	-34	-58	-78	-93	-101	-102	-95	-82
19	-63	-40	-16	6	25	39	45	44	36	21
20	2	-17	0	0	0	0	0	0	0	0

PROBLEM CONTROL

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A. DATE                      = 62364
B. TYPE                      =2005
C. NUMBER INPUT SPECTRA USED = 9
D. GAIN SCALE                = 5.00
E. CALCULATION TYPE          = 0
F. DIAGNOSTIC OPTION         = 1

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PAGE 2

CONVERSION POLYNOMIAL COEFFICIENTS

[illegible]

PAGE 3

TE-125 SPECTRUM 5 KEV SHAPE SEPT. 5

DATA CONTROL

A. LOCATION OF PHOTO PEAK	= 24.35900 CHANNELS
B. WIDTH OF PHOTO PEAK	= 3.56160 CHANNELS
C. HEIGHT OF PHOTO PEAK	=0.23470E 05COUNTS
D. AREA UNDER PHOTO PEAK	=0.91160E 05COUNTS
E. ALPHA 1	=0.58150E-05
F. ALPHA 2	=0.84110E-08
G. SPECTRA UPPER LIMIT	= 31
H. GAMMA RAY ENERGY	= 0.11088MEV.
I. ABSORP. X PHOTO EFF.	=0.26979E-01
J. NORM. FACTOR	=0.29595E 01

ORIGINAL INPUT SPECTRUM

TE-125 SPECTRUM 5 KEV SHAPE SEPT. 5

	0	1	2	3	4	5	6	7	8	9
0	0	125	115	120	150	245	259	172	112	94
1	139	188	298	448	610	935	1411	1788	1657	1411
2	1612	2966	7609	16360	22542	21506	13953	5611	1425	225
3	43	17	0	0	0	0	0	0	0	0

NORMALIZED SPECTRUM

TE-125 SPECTRUM 5 KEV SHAPE SEPT. 5

	0	1	2	3	4	5	6	7	8	9
0	0	369	340	355	443	725	766	509	331	278
1	411	556	881	1325	1805	2767	4175	5291	4903	4175
2	4770	8777	22518	48416	66712	63646	41293	16605	4217	665
3	127	50	0	0	0	0	0	0	0	0

COMPTON DISTRIBUTION

TE-125 SPECTRUM 5 KEV SHAPE SEPT. 5

	0	1	2	3	4	5	6	7	8	9
0	0	369	340	355	443	725	766	509	331	278
1	411	556	881	1325	1805	2767	4159	5184	4523	3428
2	3244	2773	1928	2027	-817	153	2734	1463	207	-489
3	-517	-234	-66	0	0	0	0	0	0	0

TE-125 SPECTRUM 5 KEV SHAPE SEPT. 5

SIN SERIES FIT ERROR ANALYSIS (5 HARMONICS)

K12= 7.8465E-01 BDI= 1.0000E 00 BD2= 1.1196E 01

CHANNELS FIT 1 TO 11

CHN	ORIGINAL	CALCULATED	RESIDUAL
1	0.36993E 03	0.36993E 03	-0.
2	0.34034E 03	0.33941E 03	0.92690E 00
3	0.35514E 03	0.34108E 03	0.14053E 02
4	0.44392E 03	0.47578E 03	-0.31859E 02
5	0.72507E 03	0.69984E 03	0.25231E 02
6	0.76650E 03	0.75881E 03	0.76926E 01
7	0.50903E 03	0.54429E 03	0.35259E 02
8	0.33146E 03	0.30133E 03	0.30134E 02
9	0.27819E 03	0.28178E 03	-0.35905E 01
10	0.41137E 03	0.42171E 03	-0.10344E 02
11	0.55638E 03	0.55638E 03	-0.

TE-125 SPECTRUM 5 KEV SHAPE SEPT. 5

SIN SERIES FIT ERROR ANALYSIS (6 HARMONICS)

K12= 2.9556E 00 BDI= 7.1960E 00 BD2= 1.9359E 01

CHANNELS FIT 7 TO 19

CHN	ORIGINAL	CALCULATED	RESIDUAL
7	0.50903E 03	0.50903E 03	-0.
8	0.33146E 03	0.37404E 03	-0.42582E 02
9	0.27819E 03	0.27894E 03	-0.74629E 00
10	0.41137E 03	0.34165E 03	0.69713E 02
11	0.55638E 03	0.60389E 03	-0.47505E 02
12	0.88192E 03	0.93009E 03	-0.48171E 02
13	0.13258E 04	0.12491E 04	0.76701E 02
14	0.18053E 04	0.17948E 04	0.10515E 02
15	0.27671E 04	0.28430E 04	-0.75869E 02
16	0.41600E 04	0.41731E 04	-0.13127E 02
17	0.51844E 04	0.50133E 04	0.17110E 03
18	0.45232E 04	0.47073E 04	-0.18411E 03
19	0.34289E 04	0.34289E 04	-0.61035E-04

TE-125 SPECTRUM 5 KEV SHAPE SEPT. 5

SIN SERIES FIT ERROR ANALYSIS (4 HARMONICS)

K12= 3.9462E 01 BDI= 1.5359E 01 BD2= 2.4359E 01

CHANNELS FIT 15 TO 24

CHN	ORIGINAL	CALCULATED	RESIDUAL
15	0.27671E 04	0.27671E 04	-0.
16	0.41600E 04	0.42705E 04	-0.11053E 03
17	0.51844E 04	0.49843E 04	0.20010E 03
18	0.45232E 04	0.46203E 04	-0.97161E 02
19	0.34289E 04	0.36425E 04	-0.21361E 03
20	0.32447E 04	0.20981E 04	0.34656E 03
21	0.27731E 04	0.27373E 04	0.35843E 02
22	0.19280E 04	0.25381E 04	-0.61003E 03
23	0.20273E 04	0.13739E 04	0.65341E 03
24	-0.81706E 03	-0.81706E 03	0.24414E-03

DENSE SET OF CALCULATED POINTS DESCRIBING THE COMPTON DISTRIBUTION

TE-125 SPECTRUM 5 KEV SHAPE SEPT. 5

	0	1	2	3	4	5	6	7	8	9
0	0	369	365	360	355	349	344	339	335	332
1	330	330	333	339	349	363	382	405	433	465
2	500	538	576	615	652	687	717	741	759	769
3	771	765	750	727	696	659	617	572	525	478
4	432	390	352	320	294	275	264	259	262	270
5	284	303	275	277	280	284	290	298	307	318
6	330	343	358	375	393	412	432	453	476	499
7	524	549	575	601	627	654	681	709	736	763
8	790	817	843	869	895	921	946	971	996	1021
9	1046	1071	1096	1121	1147	1174	1202	1231	1261	1293
10	1326	1362	1399	1439	1482	1528	1576	1628	1683	1742
11	1804	1870	1939	2013	2090	2171	2255	2343	2435	2530
12	2628	2728	2832	2938	3045	3154	3265	3376	3488	3599
13	3710	3819	3928	4034	4137	4237	4334	4426	4514	4597
14	4674	4745	4809	4867	4917	4960	4995	5023	5042	5052
15	5054	5048	4925	4862	4780	4683	4571	4448	4315	4176
16	4032	3887	3743	3604	3470	3344	3228	3124	3033	2954
17	2890	2838	2799	2771	2753	2743	2739	2737	2735	2730
18	2718	2698	2664	2616	2549	2462	2354	2221	2064	1882
19	1676	1445	1192	918	625	317	-3	-334	-670	-1007
20	-1342	-1672	0	0	0	0	0	0	0	0

IN-115 SPECTRUM 5 KEV SHAPE SEPT. 5

DATA CONTROL

A. LOCATION OF PHOTO PEAK = 68.94900 CHANNELS
 B. WIDTH OF PHOTO PEAK = 7.31840 CHANNELS
 C. HEIGHT OF PHOTO PEAK = 0.25620E 05COUNTS
 D. AREA UNDER PHOTO PEAK = 0.20070E 06COUNTS
 E. ALPHA 1 = 0.16360E-04
 F. ALPHA 2 = 0.12240E-12
 G. SPECTRA UPPER LIMIT = 81
 H. GAMMA RAY ENERGY = 0.33757MEV.
 I. ABSORP. X PHOTO EFF. = 0.18931E-01
 J. NORM. FACTOR = 0.94325E 00

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ORIGINAL INPUT SPECTRUM

IN-115 SPECTRUM 5 KEV SHAPE SEPT. 5

	0	1	2	3	4	5	6	7	8	9
0	0	1450	1410	1400	1390	1380	1350	1340	1320	1300
1	1290	1280	1260	1250	1240	1230	1210	1195	1180	1170
2	1155	1159	1140	1140	1140	1120	1130	1160	1209	1340
3	1487	1563	1600	1635	1592	1527	1504	1504	1382	1243
4	1069	942	757	680	612	594	592	549	556	572
5	534	571	569	584	608	627	659	771	801	968
6	1231	1560	2580	4404	7311	11382	16288	20821	24628	25832
7	23995	20597	15660	10898	6792	3867	1948	906	357	137
8	73	19	15	12	0	0	0	0	0	0

NORMALIZED SPECTRUM

IN-115 SPECTRUM 5 KEV SHAPE SEPT. 5

	0	1	2	3	4	5	6	7	8	9
0	0	1367	1329	1320	1311	1301	1273	1263	1245	1226
1	1216	1207	1188	1179	1169	1160	1141	1127	1113	1103
2	1089	1093	1075	1075	1075	1056	1065	1094	1140	1263
3	1402	1474	1509	1542	1501	1440	1418	1418	1303	1172
4	1008	888	714	641	577	560	558	517	524	539
5	503	538	536	550	573	591	621	727	755	913
6	1161	1471	2433	4154	6896	10736	15363	19639	23230	24366
7	22633	19428	14771	10279	6406	3647	1837	854	336	129
8	68	17	15	12	0	0	0	0	0	0

COMPTON DISTRIBUTION

IN-115 SPECTRUM 5 KEV SHAPE SEPT. 5

	0	1	2	3	4	5	6	7	8	9
0	0	1367	1329	1320	1311	1301	1273	1263	1245	1226
1	1216	1207	1188	1179	1169	1160	1141	1127	1113	1103
2	1089	1093	1075	1075	1075	1056	1065	1094	1140	1263
3	1402	1474	1509	1542	1501	1440	1418	1418	1303	1172
4	1008	888	714	641	577	560	558	517	524	539
5	503	538	536	549	569	582	604	691	677	729
6	726	486	370	205	28	-86	-61	-217	164	203
7	-190	-14	-175	-99	-113	-64	-83	-54	-62	-38
8	-2	-14	-0	4	-3	-1	0	0	0	0

IN-115 SPECTRUM 5 KEV SHAPE SEPT. 5

SIN SERIES FIT ERROR ANALYSIS (20 HARMONICS)

K(2= 1.5826E-01 BDI= 1.0000E 00 BD2= 4.2330E 01

CHANNELS FIT 1 TO 42

CHN	ORIGINAL	CALCULATED	RESIDUAL
1	0.13677E 04	0.13677E 04	-0.
2	0.13300E 04	0.13327E 04	-0.27060E 01
3	0.13205E 04	0.13173E 04	0.32792E 01
4	0.13111E 04	0.13130E 04	-0.10601E 01
5	0.13017E 04	0.13000E 04	0.16492E 01
6	0.12734E 04	0.12776E 04	-0.42361E 01
7	0.12640E 04	0.12587E 04	0.52584E 01
8	0.12451E 04	0.12456E 04	-0.54839E 00
9	0.12262E 04	0.12309E 04	-0.46915E 01
10	0.12168E 04	0.12146E 04	0.22144E 01
11	0.12074E 04	0.12027E 04	0.46696E 01
12	0.11805E 04	0.11934E 04	-0.40036E 01
13	0.11791E 04	0.11806E 04	-0.15085E 01
14	0.11696E 04	0.11668E 04	0.28763E 01
15	0.11602E 04	0.11570E 04	0.31479E 01
16	0.11413E 04	0.11464E 04	-0.50840E 01
17	0.11272E 04	0.11282E 04	-0.10182E 01
18	0.11130E 04	0.11094E 04	0.36707E 01
19	0.11036E 04	0.11007E 04	0.29345E 01
20	0.10895E 04	0.10976E 04	-0.81061E 01
21	0.10932E 04	0.10885E 04	0.47493E 01
22	0.10753E 04	0.10765E 04	-0.12294E 01
23	0.10753E 04	0.10730E 04	0.22628E 01
24	0.10753E 04	0.10740E 04	0.13148E 01
25	0.10564E 04	0.10664E 04	-0.10005E 02

26	0.10659E 04	0.10591E 04	0.68102E 01
27	0.10942E 04	0.10832E 04	0.11008E 02
28	0.11404E 04	0.11573E 04	-0.16873E 02
29	0.12640E 04	0.12671E 04	-0.31512E 01
30	0.14026E 04	0.13830E 04	0.19652E 02
31	0.14743E 04	0.14782E 04	-0.38970E 01
32	0.15092E 04	0.15301E 04	-0.20859E 02
33	0.15422E 04	0.15277E 04	0.14534E 02
34	0.15017E 04	0.14898E 04	0.11844E 02
35	0.14403E 04	0.14536E 04	-0.13246E 02
36	0.14186E 04	0.14316E 04	-0.12965E 02
37	0.14186E 04	0.13949E 04	0.23794E 02
38	0.13036E 04	0.13095E 04	-0.59018E 01
39	0.11725E 04	0.11761E 04	-0.36563E 01
40	0.10083E 04	0.10215E 04	-0.13205E 02
41	0.88854E 03	0.86624E 03	0.22303E 02
42	0.71404E 03	0.71404E 03	-0.30518E-04

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IN-115 SPECTRUM 5 KEV SHAPE SEPT. 5

SIN SERIES FIT ERROR ANALYSIS (13 HARMONICS)

K12= 1.5845E 00 BDI= 3.8330E 01 BD2= 6.3949E 01

CHANNELS FIT 38 TO 64

CHN	ORIGINAL	CALCULATED	RESIDUAL
38	0.13036E 04	0.13036E 04	-0.
39	0.11725E 04	0.11681E 04	0.43670E 01
40	0.10083E 04	0.10217E 04	-0.13355E 02
41	0.88854E 03	0.86862E 03	0.19923E 02
42	0.71404E 03	0.73068E 03	-0.16642E 02
43	0.64141E 03	0.63259E 03	0.88171E 01
44	0.57727E 03	0.58150E 03	-0.42312E 01
45	0.56029E 03	0.56104E 03	-0.74510E 00
46	0.55840E 03	0.54784E 03	0.10562E 02
47	0.51784E 03	0.53278E 03	-0.14936E 02
48	0.52445E 03	0.52236E 03	0.20898E 01
49	0.53954E 03	0.52183E 03	0.17707E 02
50	0.50370E 03	0.52592E 03	-0.22228E 02
51	0.53860E 03	0.52904E 03	0.95555E 01
52	0.53611E 03	0.53521E 03	0.89904E 00
53	0.54923E 03	0.54985E 03	-0.61640E 00
54	0.56960E 03	0.56873E 03	0.87008E 00
55	0.58299E 03	0.58660E 03	-0.36072E 01
56	0.60444E 03	0.61231E 03	-0.78753E 01
57	0.69192E 03	0.65894E 03	0.32978E 02
58	0.67761E 03	0.71409E 03	-0.36481E 02
59	0.72962E 03	0.73332E 03	-0.36986E 01
60	0.72605E 03	0.67486E 03	0.51192E 02
61	0.48682E 03	0.53813E 03	-0.51306E 02
62	0.37066E 03	0.36200E 03	0.86586E 01
63	0.20531E 03	0.18787E 03	0.17441E 02
64	0.28377E 02	0.28377E 02	0.18597E-04

IN-115 SPECTRUM 5 KEV SHAPE SEPT. 5

SIN SERIES FIT ERROR ANALYSIS (5 HARMONICS)

K12=-2.5491E 01 BDI= 5.9949E 01 BD2= 6.8949E 01

CHANNELS FIT 59 TO 69

CHN	ORIGINAL	CALCULATED	RESIDUAL
59	0.72962E 03	0.72962E 03	-0.
60	0.72605E 03	0.68523E 03	0.40821E 02
61	0.48682E 03	0.53957E 03	-0.52741E 02
62	0.37066E 03	0.34419E 03	0.26474E 02
63	0.20531E 03	0.18944E 03	0.15868E 02
64	0.28377E 02	0.59401E 02	-0.31024E 02
65	-0.86519E 02	-0.76802E 02	-0.97169E 01
66	-0.61331E 02	-0.14734E 03	0.86013E 02
67	-0.21732E 03	-0.82618E 02	-0.13471E 03
68	0.16479E 03	0.62097E 02	0.10269E 03
69	0.20322E 03	0.20322E 03	-0.22888E-04

DENSE SET OF CALCULATED POINTS DESCRIBING THE COMPTON DISTRIBUTION

IN-115 SPECTRUM 5 KEV SHAPE SEPT. 5

	0	1	2	3	4	5	6	7	8	9
0	0	1367	1338	1321	1315	1311	1301	1284	1267	1254
1	1244	1233	1219	1208	1200	1193	1183	1172	1162	1155
2	1147	1133	1117	1105	1100	1098	1092	1082	1074	1073
3	1074	1070	1062	1060	1081	1133	1212	1304	1394	1469
4	1519	1535	1520	1488	1458	1439	1421	1383	1312	1211
5	1093	969	937	904	871	839	808	778	750	723
6	698	676	656	638	622	609	598	588	581	575
7	570	566	562	559	557	554	551	548	545	542
8	538	535	532	529	526	524	522	521	521	520
9	521	521	522	523	524	525	526	526	527	528
10	528	529	530	531	533	535	537	540	543	547
11	551	555	559	563	567	571	575	579	583	587
12	591	596	602	608	616	625	634	645	657	669
13	682	694	706	716	725	731	734	734	730	727
14	710	694	674	650	623	592	559	524	487	449
15	410	371	328	303	280	258	237	217	198	180
16	162	144	126	108	90	71	52	33	13	-5
17	-25	-44	-62	-80	-96	-110	-123	-133	-140	-145
18	-147	-146	-142	-135	-126	-114	-100	-83	-66	-47
19	-26	-6	14	36	57	78	98	119	138	158
20	177	196	0	0	0	0	0	0	0	0

3. INPUT DATA FOR SHAPE

A sample of the cards needed to generate shapes is shown here. In addition, SHAPE uses an output tape from GAMSPEC.

SAMPLE INPUT TO SHAPE GENERATION PROGRAM PART II

2005

0.166
0.204
0.221
0.245
0.253
0.269
0.284
0.292

1

4010

1

177.76
127.00
140.
170.
200.

1

4. OUTPUT FROM SHAPE

Several calculated gamma-ray spectra are shown for the 10-keV scale. Photopeak positions are given in the last six columns.

0	000000	000697	000644	000616	000603	000586	000578	000589	000599	000615	35.19
1	000692	000845	000972	000947	000777	000596	000521	000540	000615	000706	35.19
2	000822	000973	001106	001170	001213	001378	001745	002209	002544	002800	35.19
3	003446	006226	013710	027052	042586	051869	047790	033670	018177	007528	35.19
4	002415	000645	000187	000073	000028	000008	000000	000000	000000	000000	35.19

0	000000	000892	000850	000806	000773	000754	000730	000694	000663	000639	42.66
1	000598	000554	000565	000655	000805	000980	001144	001229	001175	001008	42.66
2	000847	000815	000896	000982	001000	000971	000973	001035	001097	001109	42.66
3	001115	001175	001292	001461	001709	002007	002331	003019	005217	010750	42.66
4	020391	032040	040916	042383	035782	024676	013908	006413	002429	000771	42.66
5	000222	000070	000026	000010	000003	000000	000000	000000	000000	000000	42.66

0	000000	000891	000874	000839	000796	000763	000736	000701	000667	000651	46.01
1	000645	000632	000620	000642	000715	000817	000917	001025	001144	001226	46.01
2	001202	001065	000908	000854	000935	001031	001044	000960	000857	000819	46.01
3	000847	000873	000870	000892	000976	001074	001147	001257	001475	001806	46.01
4	002494	004366	008708	016551	026824	035847	039510	036012	027475	017488	46.01
5	009294	004131	001545	000498	000151	000050	000019	000007	000002	000000	46.01

0	000000	000918	000919	000901	000858	000809	000774	000740	000703	000680	50.74
1	000680	000688	000701	000721	000751	000799	000871	000939	000976	001003	50.74
2	001064	001163	001244	001229	001099	000931	000831	000880	000969	001001	50.74
3	000932	000790	000652	000580	000573	000576	000570	000598	000686	000784	50.74
4	000815	000799	000854	001090	001563	002959	005842	010986	018643	027234	50.74
5	033536	034670	030726	023253	015036	008316	003940	001606	000572	000187	50.74
6	000063	000024	000009	000003	000000	000000	000000	000000	000000	000000	50.74

0	000000	000954	000954	000937	000897	000847	000809	000778	000742	000715	52.31
1	000713	000720	000727	000744	000772	000807	000858	000923	000974	000997	52.31
2	001021	001088	001194	001277	001262	001128	000948	000826	000841	000914	52.31
3	000951	000905	000787	000651	000561	000534	000535	000532	000542	000606	52.31
4	000706	000768	000761	000757	000881	001255	002093	004174	007998	014078	52.31
5	021938	029354	033413	032278	026963	019367	011971	006375	002931	001171	52.31
6	000415	000138	000048	000019	000008	000003	000000	000000	000000	000000	52.31

0	000000	001086	001069	001048	001012	000963	000924	000899	000872	000841	55.46
1	000831	000837	000835	000823	000825	000842	000859	000874	000909	000954	55.46
2	000989	001018	001079	001185	001314	001406	001386	001240	001025	000847	55.46
3	000774	000784	000805	000799	000759	000694	000629	000589	000581	000585	55.46
4	000590	000612	000668	000746	000803	000826	000857	000993	001386	002296	55.46
5	004331	007980	013464	020208	026575	030452	030186	026118	019665	012893	55.46
6	007369	003678	001609	000624	000222	000078	000030	000012	000005	000002	55.46

0	000000	001273	001230	001195	001160	001115	001077	001060	001046	001022	58.42
1	001004	001006	001010	000984	000941	000914	000905	000889	000862	000850	58.42
2	000872	000915	000973	001063	001194	001358	001510	001600	001556	001386	58.42
3	001137	000911	000762	000684	000649	000645	000670	000711	000745	000761	58.42
4	000768	000780	000797	000815	000839	000873	000924	000995	001086	001208	58.42
5	001423	001912	003046	005163	008925	014268	020167	025236	028196	027911	58.42
6	024249	018512	012428	007345	003829	001766	000727	000274	000100	000039	58.42
7	000016	000007	000003	000001	000000	000000	000000	000000	000000	000000	58.42

0	000000	001386	001327	001283	001247	001205	001169	001154	001148	001132	59.99
1	001112	001111	001116	001096	001042	000984	000950	000923	000878	000829	59.99
2	000809	000828	000883	000968	001097	001268	001459	001621	001710	001666	59.99
3	001497	001237	000983	000800	000663	000582	000558	000598	000688	000788	59.99
4	000858	000895	000916	000938	000964	000984	000999	001022	001082	001195	59.99
5	001352	001553	001893	002652	004199	006921	011398	016922	022093	025850	59.99
6	027449	025754	021328	015597	010083	005769	002928	001323	000539	000204	59.99
7	000077	000031	000014	000006	000002	000000	000000	000000	000000	000000	59.99

0	000000	000872	000872	000871	000872	000871	000870	000870	000871	000871	127.00
1	000868	000866	000867	000868	000865	000863	000875	000899	000931	000985	127.00
2	001069	001157	001184	001123	001040	000972	000944	000933	000933	000907	127.00
3	000885	000872	000869	000865	000857	000853	000857	000857	000849	000841	127.00
4	000844	000852	000853	000845	000840	000843	000848	000848	000848	000852	127.00
5	000863	000877	000885	000880	000866	000852	000845	000843	000843	000846	127.00
6	000855	000864	000868	000870	000878	000887	000890	000887	000889	000901	127.00
7	000917	000925	000928	000935	000950	000968	000981	000990	001003	001023	127.00
8	001041	001050	001052	001052	001055	001067	001084	001098	001105	001111	127.00
9	001121	001135	001149	001160	001172	001188	001207	001229	001248	001260	127.00

10	001257	001237	001199	001138	001050	000944	000833	000725	000622	000523	127.00
11	000438	000371	000325	000293	000273	000267	000282	000327	000421	000600	127.00
12	000916	001426	002170	003119	004161	005112	005761	005962	005735	005068	127.00
13	004143	003153	002251	001521	000981	000609	000368	000221	000137	000088	127.00
14	000058	000038	000023	000013	000006	000002	000000	000000	000000	000000	127.00

0	000000	000766	000768	000765	000762	000763	000765	000764	000765	000766	140.00
1	000764	000758	000757	000759	000756	000752	000763	000788	000811	000833	140.00
2	000878	000942	000975	000957	000892	000838	000825	000822	000823	000822	140.00
3	000807	000793	000785	000779	000782	000775	000762	000753	000755	000769	140.00
4	000792	000802	000798	000777	000762	000759	000759	000761	000770	000788	140.00
5	000808	000816	000808	000794	000783	000773	000762	000753	000754	000761	140.00
6	000762	000757	000754	000761	000770	000773	000771	000769	000769	000773	140.00
7	000783	000797	000805	000808	000810	000816	000823	000826	000831	000840	140.00
8	000848	000851	000850	000853	000858	000866	000879	000899	000921	000944	140.00
9	000970	001002	001033	001044	001043	001040	001038	001033	001028	001027	140.00
10	001030	001035	001042	001054	001068	001079	001089	001100	001112	001125	140.00
11	001142	001158	001163	001150	001124	001084	001020	000931	000832	000742	140.00
12	000662	000586	000507	000438	000388	000352	000323	000308	000312	000336	140.00
13	000381	000475	000664	000988	001475	002126	002904	003719	004441	004936	140.00
14	005106	004923	004411	003686	002889	002140	001508	001020	000665	000422	140.00
15	000265	000168	000110	000074	000051	000034	000021	000012	000006	000003	140.00

0	000000	000589	000594	000593	000590	000587	000585	000582	000580	000582	170.00
1	000583	000582	000583	000586	000587	000588	000594	000609	000623	000634	170.00
2	000657	000707	000765	000787	000745	000684	000642	000625	000619	000615	170.00
3	000615	000617	000616	000614	000613	000611	000608	000604	000599	000594	170.00
4	000588	000587	000590	000592	000590	000589	000589	000594	000609	000635	170.00
5	000662	000670	000655	000633	000614	000597	000579	000567	000563	000563	170.00
6	000562	000563	000566	000569	000571	000575	000580	000589	000612	000653	170.00
7	000699	000727	000723	000693	000660	000636	000626	000623	000618	000609	170.00
8	000604	000605	000606	000607	000611	000622	000635	000638	000632	000625	170.00
9	000625	000631	000640	000650	000659	000663	000660	000654	000652	000658	170.00
10	000667	000672	000669	000665	000670	000685	000698	000703	000704	000707	170.00
11	000712	000716	000722	000731	000741	000748	000759	000784	000822	000867	170.00
12	000916	000968	001009	001026	001033	001033	001018	000987	000955	000935	170.00

13	000933	000936	000931	000921	000919	000930	000947	000959	000969	000982	170.00
14	000992	000990	000975	000958	000942	000915	000869	000809	000744	000670	170.00
15	000589	000518	000456	000404	000358	000319	000292	000286	000298	000326	170.00
16	000383	000496	000692	000991	001390	001871	002396	002908	003346	003649	170.00
17	003766	003628	003327	002887	002380	001875	001420	001040	000740	000514	170.00
18	000350	000236	000160	000111	000079	000057	000041	000029	000019	000011	170.00
19	000006	000003	000000	000000	000000	000000	000000	000000	000000	000000	170.00

0	000000	000559	000564	000564	000561	000558	000554	000551	000549	000550	177.76
1	000551	000552	000554	000557	000560	000563	000571	000584	000596	000607	177.76
2	000631	000680	000739	000767	000732	000678	000629	000606	000604	000607	177.76
3	000607	000608	000609	000608	000604	000600	000599	000597	000592	000586	177.76
4	000579	000573	000566	000560	000561	000566	000571	000577	000590	000615	177.76
5	000645	000666	000666	000646	000614	000581	000552	000535	000533	000544	177.76
6	000550	000548	000544	000541	000542	000543	000543	000544	000546	000549	177.76
7	000551	000553	000554	000557	000563	000584	000621	000668	000708	000727	177.76
8	000718	000686	000652	000625	000613	000608	000598	000585	000578	000578	177.76
9	000581	000582	000588	000603	000617	000620	000613	000604	000604	000613	177.76
10	000625	000635	000641	000643	000640	000635	000633	000640	000648	000646	177.76
11	000635	000630	000641	000659	000671	000674	000675	000677	000678	000682	177.76
12	000692	000708	000720	000728	000742	000772	000816	000867	000921	000972	177.76
13	001008	001025	001035	001032	001008	000969	000935	000920	000920	000918	177.76
14	000906	000892	000891	000907	000923	000934	000945	000957	000961	000949	177.76
15	000931	000917	000901	000872	000829	000776	000710	000625	000547	000479	177.76
16	000423	000373	000327	000291	000272	000272	000288	000325	000398	000533	177.76
17	000749	001056	001451	001909	002390	002842	003212	003448	003471	003332	177.76
18	003030	002618	002157	001705	001299	000960	000691	000487	000336	000230	177.76
19	000158	000111	000080	000058	000043	000030	000020	000013	000007	000004	177.76
20	000002	000000	000000	000000	000000	000000	000000	000000	000000	000000	177.76

0	000000	000512	000512	000512	000511	000507	000504	000503	000502	000499	200.00
1	000496	000497	000501	000508	000516	000527	000543	000560	000576	000593	200.00
2	000614	000642	000669	000682	000666	000635	000605	000579	000569	000566	200.00
3	000569	000571	000572	000570	000568	000565	000563	000561	000560	000559	200.00
4	000558	000555	000551	000547	000540	000534	000529	000533	000548	000576	200.00
5	000613	000639	000640	000612	000574	000542	000522	000512	000509	000507	200.00

6	000508	000507	000507	000506	000507	000507	000507	000505	000500	000492	200.00
7	000485	000485	000488	000494	000498	000500	000502	000503	000504	000503	200.00
8	000502	000500	000499	000499	000501	000503	000506	000509	000511	000512	200.00
9	000510	000507	000504	000506	000510	000523	000544	000570	000607	000645	200.00
10	000683	000701	000708	000695	000672	000643	000617	000599	000581	000561	200.00
11	000547	000543	000544	000545	000549	000558	000571	000579	000578	000573	200.00
12	000570	000576	000585	000592	000593	000594	000595	000596	000595	000597	200.00
13	000598	000593	000582	000578	000587	000600	000606	000607	000607	000607	200.00
14	000607	000611	000626	000646	000664	000679	000698	000732	000782	000842	200.00
15	000904	000960	001005	001035	001052	001047	001018	000974	000934	000908	200.00
16	000894	000881	000865	000853	000853	000864	000875	000882	000888	000890	200.00
17	000881	000861	000839	000818	000792	000759	000719	000671	000609	000534	200.00
18	000464	000408	000361	000318	000280	000253	000242	000245	000263	000307	200.00
19	000387	000516	000700	000947	001255	001610	001982	002333	002625	002822	200.00
20	002893	002808	002616	002330	001990	001637	001303	001008	000762	000564	200.00
21	000410	000294	000209	000149	000107	000079	000059	000045	000033	000024	200.00
22	000016	000010	000006	000003	000002	000000	000000	000000	000000	000000	200.00

END-OF-DATA ENCOUNTERED ON SYSTEM INPUT FILE.

APPENDIX V

PROGRAM FLOW CHARTS AND FORTRAN LISTINGS

The flow charts for these two programs are in Section 1 and the complete FORTRAN listings are in Section 2. See Section VII-3 for the uses of the subroutines and a table of variable names.

1. PROGRAM FLOW CHARTS

1.1 GAMSPEC -- Main Program

1.2 GAMSPEC -- Subroutines

1.21 DATIN Subroutine

1.22 DATOUT Subroutine

PP Co-B-6479

1.23 SSFIT Subroutine

PP Co-B-6478

1.24 ORPFIT Subroutine

1.25 PAGE Subroutine

1.26 CNVRT Subroutine

1.27 DONE Subroutine

1.28 TYPEW Subroutine

1.3 SHAPE Main Program

1.31 Main

1.4 SHAPE -- Subroutines

1.41 SHAPE Subroutine

1.42 REGEN Subroutine

1.43 CNVRT Subroutine

Same as that in GAMSPEC

1.44 TYPEW Subroutine

Same as that in GAMSPEC

1.45 ZERO Subroutine

2. SOURCE PROGRAM LISTINGS

2.1 GAMSPEC

```

C
C      IBM 7040 GAMMA SHAPE GENERATION PROGRAM IN FORTRAN 4 PART I
C
C      SPECIFICATION STATEMENTS
C
COMMON ZDATA(251,20),S(251,18),P(17,20),SPECT(251),IZ(37),X(20), 002
1  TITLE(12),DAT(10),IDAT(10),W(18),ALFA(18),BETA(17),D(251), 003
2  B(251),PHUTE(10),ETPHU(10),EPSCO(10),PEAKC(10),PTITL(12), 004
3  KP(251),WOCO(10),ALCO(10),A2CO(10),SP(65),JCAL,GAIN 005
C 006
1  SWT1=0 007
C 008
C      REWIND OUTPUT COEFFICIENT TAPE 009
C 010
C      REWIND 8 011
C 012
C      INITILIZE 013
C 014
7  IPAGE=0 015
   ISETS=0 016
   CALL PAGE(IPAGE) 017
C 018
C      WRITE OUT PROGRAM IDENTIFICATION 019
C 020
C      WRITE( 6,500) 021
C      READ( 5,501)PTITL 022
C 023
C      READ DATE CARD 024
C 025
C      READ(5,502)IDATE,IRTP,I 026
C 027
C      POSITION OUTPUT COEFFICIENT TAPE 028
C      1. I=0 (AT LEAST ONE SET OF COEFFICIENTS ON TAPE) 029
C      2. I=1 (NO COEFFICIENTS ON TAPE YET) 030
C      IF I=0 READ COEFFICIENTS FROM OLD TAPE AND WRITE ONTO NEW TAPE 031
C 032
C      IF(SWT1)2,11,2 033
11 IF(I)2,81,2 034
81 CALL TYPEW(40H MOUNT INDICATED COEFFICIENT TAPE ON U04,40,0) 035
   CALL TYPEW(24H PRESS START TO CONTINUE,24,1) 036
   REWIND 4 037
3  READ(4)IRTY1,JCAL,IDATE,K,N,GAIN 038
   WRITE(8)IRTY1,JCAL,IDATE,K,N,GAIN 039
   READ(4)PHUTE,ETPHU,EPSCO,PEAKC,WOCO,ALCO,A2CO 040
   WRITE(8)PHUTE,ETPHU,EPSCO,PEAKC,WOCO,ALCO,A2CO 041
   READ(4)KP 042
   WRITE(8)KP 043
   READ(4)SP,ALFA,BETA 044
   WRITE(8)SP,ALFA,BETA 045
   READ(4)S 046
   WRITE(8)S 047
   READ(4)IR1,IR2,ID 048
   IF(IR1-7777)80,12,12 049
80 WRITE(8)IR1,IR2,ID 050
   GO TO 3 051
12 WRITE(8)IRTY1,IRTY1,IDATE 052
C 053
C      READ PROBLEM CONTROL 054
C 055
2  READ(5,503)NSETS,GAIN,JCAL,JOPT1 056
   K=NSETS-2 057
C 058
C      WRITE PROBLEM CONTROL 059
C 060
C      WRITE( 6,504)PTITL,IDATE,IRTP,NSETS,GAIN,JCAL,JOPT1 061
C      CALL PAGE(IPAGE) 062
C      IF(JCAL-1)8,8,10 063
8  N=201 064
   GO TO 30 065
10 N=251 066
C 067

```

C	READ COEFFICIENTS	068
C		069
30	WRITE(6,551)	070
	READ(5,507)(PHUTE(I),I=1,5)	071
	READ(5,507)(PHUTE(I),I=6,10)	072
	WRITE(6,507)(PHUTE(I),I=1,5)	073
	WRITE(6,507)(PHUTE(I),I=6,10)	074
	WRITE(6,552)	075
	READ(5,507)(ETPHU(I),I=1,5)	076
	READ(5,507)(ETPHU(I),I=6,10)	077
	WRITE(6,507)(ETPHU(I),I=1,5)	078
	WRITE(6,507)(ETPHU(I),I=6,10)	079
	WRITE(6,553)	080
	READ(5,507)(EPSCO(I),I=1,5)	081
	READ(5,507)(EPSCO(I),I=6,10)	082
	WRITE(6,507)(EPSCO(I),I=1,5)	083
	WRITE(6,507)(EPSCO(I),I=6,10)	084
	WRITE(6,554)	085
	READ(5,507)(PEAKC(I),I=1,5)	086
	READ(5,507)(PEAKC(I),I=6,10)	087
	WRITE(6,507)(PEAKC(I),I=1,5)	088
	WRITE(6,507)(PEAKC(I),I=6,10)	089
	WRITE(6,555)	090
	READ(5,507)(WOCO(I),I=1,5)	091
	READ(5,507)(WOCO(I),I=6,10)	092
	WRITE(6,507)(WOCO(I),I=1,5)	093
	WRITE(6,507)(WOCO(I),I=6,10)	094
	WRITE(6,556)	095
	READ(5,507)(A1CO(I),I=1,5)	096
	READ(5,507)(A1CO(I),I=6,10)	097
	WRITE(6,507)(A1CO(I),I=1,5)	098
	WRITE(6,507)(A1CO(I),I=6,10)	099
	WRITE(6,557)	100
	READ(5,507)(A2CO(I),I=1,5)	101
	READ(5,507)(A2CO(I),I=6,10)	102
	WRITE(6,507)(A2CO(I),I=1,5)	103
	WRITE(6,507)(A2CO(I),I=6,10)	104
	GO TO 60	105
9	WRITE(6,508)	106
	CALL EXIT	107
C		108
C	READ KP RECORDS	109
C		110
60	WRITE(6,558)	111
	I1=0	112
	I3=0	113
66	I4=0	114
	READ(5,509)AKP,I2,(IZ(I),I=1,37)	115
	WRITE(6,559)AKP,I2,(IZ(I),I=1,37)	116
61	I1=I1+1	117
	IF(I1-I2)9,62,9	118
62	I3=I3+1	119
	I4=I4+1	120
	KP(I3)=IZ(I4)	121
	IF(I3-N)63,64,63	122
63	IF(I4-37)62,66,62	123
64	DO 65 I=1,65	124
65	SPECT(I)=0.	125
C		126
C	IF HIGH ENERGY, READ .511 PEAK DATA	127

C	IF(JCAL-1)67,67,68	128
68	READ(5,501)(TITLE(I),I=1,12)	129
	READ(5,510)X0,W0,Y0,A1,A2,AREA,ISPUL	130
	CALL DATIN	132
	DO 70 I=1,65	133
70	SP(I)=SPECT(I)	134
	CALL PAGE(IPAGE)	135
	WRITE(6,512)X0,W0,Y0,ISPUL	
C		137
C	WRITE OUT 51.1 PEAK DATA	138
C		139
	WRITE(6,525)	140
	CALL DATOUT(ISPUL)	141
67	DO 69 I=1,256	142
69	SPECT(I)=0.0	
C		144
C	READ INPUT DATA	145
C		146
	ISETS=ISETS+1	147
	CALL PAGE(IPAGE)	148
	READ(5,501)(TITLE(I),I=1,12)	149
	READ(5,510)X0,W0,Y0,A1,A2,AREA,ISPUL	
	B0=(W0*W0)/(4.*0.69314718)	151
	PH=(X0*GAIN)/10.	152
	CALL CNVRT(PH,EN,1)	153
	CALL CNVRT(EN,EPS,3)	154
	X(ISETS)=X0	155
C		156
C	COMPUTE NORMALIZATION FACTOR	157
C		158
	GAMNO=AREA/EPS	159
	FNORM=1.0E7/GAMNO	160
	CALL DATIN	161
	WRITE(6,501)(TITLE(I),I=1,12)	162
C		163
C	WRITE OUT DATA CONTROL	164
C		165
	WRITE(6,513)X0,W0,Y0,AREA,A1,A2,ISPUL,EN,EPS,FNORM	
C		167
C	WRITE OUT INPUT SPECTRUM	168
C		169
	CALL PAGE(IPAGE)	170
	WRITE(6,514)	171
	CALL DATOUT(ISPUL)	172
C		173
C	NORMALIZE DATA	174
C		175
	DO 17 I=1,ISPUL	176
17	SPECT(I)=SPECT(I)*FNORM	177
	IF(JOPT1)18,19,18	178
C		179
C	WRITE OUT NORMALIZED DATA	180
C		181
18	CALL PAGE(IPAGE)	182
	WRITE(6,523)	183
	CALL DATOUT(ISPUL)	184
C		185
C	CALCULATE NORMALIZED GAUSSIAN AND SUBTRACT FROM DATA	186
C		187
19	R=SQRT(B0*6.0*2.3025851)	
	MINX=X0-R	189
	MAXX=X0+R	190
	X1=MINX	191
	DO 20 I=MINX,MAXX	192
	BAS=X1-X0	193
	BA2=BAS*BAS	194
	BA4=BA2*BA2	195
	BA12=BA4*BA4*BA4	196
	GAUS=(Y0*EXP(-BA2/B0))*(1.+A1*BA4+A2*BA12)	197
	SPECT(I)=SPECT(I)-FNORM*GAUS	198
20	X1=X1+1.0	199
	IF(JOPT1)21,22,21	200

C		201
C	WRITE OUT COMPTON DISTRIBUTION	202
C		203
	21 CALL PAGE(IPAGE)	204
	WRITE (6,524)	205
	CALL DATOUT(MAXX)	206
C		207
C	CALCULATE PARAMETERS OF COMPTON	208
C		209
	22 IF(JCAL-1)23,24,25	210
C		211
C		212
C	LOW ENERGY CALCULATION PART 1	213
C		214
	23 E1=EN/(1.+(2.*EN)/0.511)	215
	EC=EN-E1	216
	CALL CNVRT(EC,PHC,2)	217
	XC=PHC*10./GAIN	218
	XE=X0-(3.5*10./GAIN)	219
	BD1=1.0	220
	BD2=XC+2.	221
	NP=50	222
	DT=(XC-1.)/50.	223
	TI=0.0	224
	ZDATA(1,ISETS)=SPECT(1)	225
	IP=2	226
	CALL SSFIT(BD1,BD2,NP,DT,TI,IP,ISETS,FNORM,JOPT1,IPAGE)	227
C		228
C	LOW ENERGY CALCULATION PART 2	229
C		230
	BD1=XC-2.	231
	BD2=XE+2.	232
	NP=100	233
	DT=(XE-XC)/100.	234
	N1=BD1	235
	FN1=N1	236
	TI=XC-FN1	237
	CALL SSFIT(BD1,BD2,NP,DT,TI,IP,ISETS,FNORM,JOPT1,IPAGE)	238
C		239
C	LOW ENERGY CALCULATION PART 3	240
C		241
	BD1=XE-2.	242
	BD2=X0	243
	NP=50	244
	DT=(X0-XE)/50.	245
	N1=BD1	246
	FN1=N1	247
	TI=XE-FN1	248
	CALL SSFIT(BD1,BD2,NP,DT,TI,IP,ISETS,FNORM,JOPT1,IPAGE)	249
	GO TO 50	250
C		251
C	INTERMEDIATE ENERGY CALCULATION PART 1	252
C		253
	24 EB=EN/(1.+(2.*EN)/0.511)	254
	CALL CNVRT(EB,PHB,2)	255
	XB=(PHB*10.)/GAIN	256
	EC=EN-EB	257
	CALL CNVRT(EC,PHC,2)	258
	XC=(PHC*10.)/GAIN	259

	BD1=1.0	260
	BD2=XB+4.0	261
	NP=50	262
	DT=(XB-1.0)/50.0	263
	TI=0.0	264
	ZDATA(1,ISETS)=SPECT(1)	265
	IP=2	266
	CALL SSFIT(BD1,BD2,NP,DT,TI,IP,ISETS,FNORM,JOPT1,IPAGE)	267
C		268
C	INTERMEDIATE ENERGY CALCULATION PART 2	269
C		270
	BD1=XB-4.0	271
	BD2=XC+4.0	272
	NP=100	273
	DT=(XC-XB)/100.0	274
	N1=BD1	275
	FN1=N1	276
	TI=XB-FN1	277
	CALL SSFIT(BD1,BD2,NP,DT,TI,IP,ISETS,FNORM,JOPT1,IPAGE)	278
C		279
C	INTERMEDIATE ENERGY CALCULATION PART 3	280
C		281
	BD1=XC-4.0	282
	BD2=X0	283
	NP=50	284
	DT=(X0-XC)/50.	285
	N1=BD1	286
	FN1=N1	287
	TI=XC-FN1	288
	CALL SSFIT(BD1,BD2,NP,DT,TI,IP,ISETS,FNORM,JOPT1,IPAGE)	289
	GO TO 50	290
C		291
C	HIGH ENERGY CALCULATION PART 1	292
C		293
25	EB=EN/(1.+(2.*EN)/0.511)	294
	CALL CNVRT(EB,PHB,2)	295
	XB=(PHB*10.)/GAIN	296
	EC=EN-EB	297
	CALL CNVRT(EC,PHC,2)	298
	XC=(PHC*10.)/GAIN	299
	E1=EN-1.022	300
	CALL CNVRT(E1,PH1,2)	301
	X1=(PH1*10.)/GAIN	302
	CALL CNVRT(E2,PH2,2)	303
	X2=(PH2*10.)/GAIN	304
	BD1=1.0	305
	BD2=XB+4.0	306
	NP=50	307
	DT=(XB-1.0)/50.0	308
	TI=0.0	309
	ZDATA(1,ISETS)=SPECT(1)	310
	IP=2	311
	CALL SSFIT(BD1,BD2,NP,DT,TI,IP,ISETS,FNORM,JOPT1,IPAGE)	312
C		313
C	HIGH ENERGY CALCULATION PART 2	314
C		315
	BD1=XB-4.0	316
	BD2=X1+4.0	317
	NP=50	318
	DT=(X1-XB)/50.0	319
	N1=BD1	320
	FN1=N1	321
	TI=XB-FN1	322
	CALL SSFIT(BD1,BD2,NP,DT,TI,IP,ISETS,FNORM,JOPT1,IPAGE)	323
C		324
C	HIGH ENERGY CALCULATION PART 3	325
C		326
	BD1=X1-4.0	327
	BD2=X2+4.0	328
	NP=50	329
	DT=(X2-X1)/50.0	330
	N1=BD1	331
		332

	FN1=N1	333
	TI=X1-FN1	334
	CALL SSFIT(BD1,BD2,NP,DT,TI,IP,ISETS, FNORM,JOPT1,IPAGE)	335
C		336
C	HIGH ENERGY CALCULATION PART 4	337
C		338
	BD1=X2-4.0	339
	BD2=XC+4.0	340
	NP=50	341
	DT=(XC-X2)/50.0	342
	N1=BD1	343
	FN1=N1	344
	TI=X2-FN1	345
	CALL SSFIT(BD1,BD2,NP,DT,TI,IP,ISETS, FNORM,JOPT1,IPAGE)	346
C		347
C	HIGH ENERGY CALCULATION PART 5	348
C		349
	BD1=XC-4.0	350
	BD2=X0	351
	NP=50	352
	DT=(X0-XC)/50.0	353
	N1=BD1	354
	FN1=N1	355
	TI=XC-FN1	356
	CALL SSFIT(BD1,BD2,NP,DT,TI,IP,ISETS, FNORM,JOPT1,IPAGE)	357
C		358
C		359
C		360
C	WRITE OUT A DENSE SET OF CALCULATED POINTS DESCRIBING THE COMPTON	361
C	DISTRIBUTION	362
C		363
	50 IF(JOPT1)52,51,52	364
	52 DO 56 I=1,N	365
	56 SPECT(I)=ZDATA(I,ISETS)	366
	CALL PAGE (IPAGE)	367
	WRITE (6,522)	368
	CALL DATOUT(N)	369
	51 IF(ISETS-NSETS)67,58,58	370
	58 K=NSETS-2	371
	CALL ORPFIT(NSETS,K,N)	372
C		373
C	WRITE OUT COEFFICIENTS	374
C		375
	WRITE(8)IRTP,JCAL,IDATE,K,N,GAIN	376
	WRITE(8)PHUTE,ETPHU,EPSCO,PEAKC,WOCO,AICO,AZCO	377
	WRITE(8)KP	378
	WRITE(8)SP,ALFA,BETA	379
	WRITE(8)S	380
C		381
C	READ TRANSFER CARD	382
C	1.BLANK (MORE DATA FOLLOWING)	383
C	2.1 (END OF DATA CLOSE OUT FILE)	384
C		385
	READ(5,550)IA	386
	IF(IA)5,6,5	387
	5 I=9999	388
	WRITE(8)I,I,IDATE	389
	END FILE 8	390
	CALL DONE	391
	CALL EXIT	392
	6 SWT1=1	393
	WRITE(8)IRTP,IRTP,IDATE	394
	GO TO 7	395
C		
C	FORMAT STATEMENTS	
C		
	500 FORMAT(1H 15X,40HGAMSPEC1- GAMMA SHAPE GENERATION PROGRAM////)	396
	501 FORMAT(1H012A6)	397
	502 FORMAT(10X,3I10)	398
	503 FORMAT(10X,I10,F10.5,2I10)	399
	504 FORMAT(1H012A6//13X,15HPROBLEM CONTROL//16X,30HA. DATE	400
	1 =16//16X,30HB. TYPE =14//16X,30HC.	401
	2 NUMBER INPUT SPECTRA USED =13//16X30HD. GAIN SCALE	402

3	=F6.2//16X30HE. CALCULATION TYPE	=I2//16X,30HF. DIAGNOST	403
4	IC OPTION	=I2)	404
507	FORMAT(10X,5(1PE14.6))		405
508	FORMAT(25HOERROR IN INPUT CALL EXIT)		406
509	FORMAT(A5,I1,37I2)		407
510	FORMAT(2F10.5,4E10.5,I5)		
512	FORMAT(1HO	13X,12HDATA CONTROL//16X,31HA. LOCATION OF PHOTO	409
1	PEAK	=F10.5,9H CHANNELS//16X,31HB. WIDTH OF PHOTO PEAK	410
2	=F10.5,9H CHANNELS//16X,31HC. HEIGHT OF PHOTO PEAK	=E10.5,7	411
3	H COUNTS//16X,31HD. SPECTRUM UPPER LIMIT	=15)	412
513	FORMAT(1HO	13X,12HDATA CONTROL//16X,31HA. LOCATION OF PHOTO	413
1	PEAK	=F10.5,9H CHANNELS//16X,31HB. WIDTH OF PHOTO PEAK	414
2	=F10.5,9H CHANNELS//16X,31HC. HEIGHT OF PHOTO PEAK	=E11.5,6	415
3	H COUNTS//16X,31HD. AREA UNDER PHOTO PEAK	=E11.5,6HCOUNTS//16X	416
4	,31HE. ALPHA 1	= E11.5//16X,31HF. ALPHA 2	
5	= E11.5//16X,31HG. SPECTRA UPPER LIMIT	= 15//	
6	16X,31HH. GAMMA RAY ENERGY	= F10.5,4HMEV.//16X,31HI. ABS	
7	ORP. X PHOTO EFF.	= E11.5//16X,31HJ. NORM. FACTOR	
8	=E11.5)		
514	FORMAT(1H 23X,23HORIGINAL INPUT SPECTRUM)		422
522	FORMAT(7OH	DENSE SET OF CALCULATED POINTS DESCRIBING THE COMPT	423
	ION DISTRIBUTION)		424
523	FORMAT(51HO	NORMALIZED SPECTRUM)	425
524	FORMAT(46HO	COMPTON DISTRIBUTION)	426
525	FORMAT(44HO	51.1 PEAK DATA)	427
550	FORMAT(I4)		428
551	FORMAT(1HO///,35H CONVERSION POLYNOMIAL COEFFICIENTS,// 5X,		429
1	22HPULSE HEIGHT TO ENERGY)		430
552	FORMAT(27HO	ENERGY TO PULSE HEIGHT)	431
553	FORMAT(25HO	EPSILON COEFFICIENTS)	432
554	FORMAT(28HO	0.511PEAK NORMALIZATION)	433
555	FORMAT(34H	PHOTO PEAK WIDTH COEFFICIENTS)	434
556	FORMAT(38H	MODIFIED GAUSSIAN A1 COEFFICIENTS)	435
557	FORMAT(38H	MODIFIED GAUSSIAN A2 COEFFICIENTS)	436
558	FORMAT(39H ORTHOGONAL POLYNOMIAL ORDER INDICATORS)		437
559	FORMAT(1H A5,I1,37I3)		438
	END		439
C			443
C	SUBROUTINE FOR READING IN DATA		444
C	DATA FORMAT I2,10F7.0,I8		445
C			446
	SUBROUTINE DATIN		447
C			448
	COMMON ZDATA(251,20),S(251,18),P(17,20),SPECT(251),IZ(37),X(20),		449
1	TITLE(12),DAT(10),IDAT(10),W(18),ALFA(18),BETA(17),D(251),		450
2	B(251),PHUTE(10),ETPHU(10),EPSCO(10),PEAKC(10),PTITL(12),		451
3	KP(251),WOCO(10),A1CO(10),A2CO(10),SP(65),JCAL,GAIN		452
C			453
C			454
	NCDS=0		455
78	READ(5,500)IC,(DAT(I),I=1,10)		456
	IF(IC-50)69,311,69		457
69	IF(NCDS)71,72,71		458
72	DO 73 K=1,9		459
73	SPECT(K)=DAT(K+1)		460
	NCDS=NCDS+1		461
	GO TO 78		462
71	DO 79 J=1,10		463
	JS=10*NCDS-1+J		464
79	SPECT(JS)=DAT(J)		465
	NCDS=NCDS+1		466
	GO TO 78		467
311	RETURN		468
500	FORMAT(I2,10F7		469
	END		470

C	OUTPUT SUBROUTINE FOR PRINTING OUT SPECTRA AT VARIOUS PHASES OF	472
C	THE COMPUTATION	473
C		474
C		475
	SUBROUTINE DATOUT(MAX)	476
C		477
	COMMON ZDATA(251,20),S(251,18),P(17,20),SPECT(251),IZ(37),X(20),	478
1	TITLE(12),DAT(10),IDAT(10),W(18),ALFA(18),BETA(17),D(251),	479
2	B(251),PHUTE(10),ETPHU(10),EPSCO(10),PEAKC(10),PTITL(12),	480
3	KP(251),WOCO(10),ALCO(10),A2CO(10),SP(65),JCAL,GAIN	481
C		482
	WRITE (6,517)(TITLE(I),I=1,12)	483
	WRITE (6,518)	484
C		485
	JS=0	486
	NCDS=0	487
1	DO 2 I=1,10	488
2	IDAT(I)=0	489
	IF(NCDS)3,4,3	490
4	DO 5 I=1,9	491
5	IDAT(I+1)=SPECT(I)	492
9	WRITE (6,519)NCDS,(IDAT(I),I=1,10)	493
	IF(JS=MAX)6,7,7	494
3	DO 8 I=1,10	495
	JS=10*NCDS-1+I	496
8	IDAT(I)=SPECT(JS)	497
	GO TO 9	498
6	NCDS=NCDS+1	499
	GO TO 1	500
7	RETURN	501
517	FORMAT(1H012A6)	502
518	FORMAT(1H06X1H06X1H16X1H26X1H36X1H46X1H56X1H66X1H76X1H86X1H9)	503
519	FORMAT(1H 12,10(1X,I6))	504
	END	505

C		507
C		508
C	SIN SERIES FIT SUBROUTINE	509
C	DATA BETWEEN THE LIMITS SPECIFIED ARE FITTED AND SMOOTHED	510
C	BY THE USE OF FOURIER SINE SERIES	511
C		512
C	SIN SERIES FIT ARGUMENT DEFINITION	513
C		514
C	BD1=LEFT BOUNDARY	515
C	BD2=RIGHT BOUNDARY	516
C	NP =NUMBER OF POINTS REQUIRED IN REGION	517
C	DT =INCREMENT	518
C	IT =INITIAL CALCULATED POINT	519
C	IP =STORAGE INDEX	520
C	ISET=SPECTRUM INDEX	521
C		522
	SUBROUTINE SSFIT(BD1,BD2,NP,DT,TI,IP,ISET,FNORM,JOPT1,IPAGE)	523
C		524
	COMMON ZDATA(251,20),S(251,18),P(17,20),SPECT(251),IZ(37),X(20),	525
1	TITLE(12),DAT(10),IDAT(10),W(18),ALFA(18),BETA(17),D(251),	526
2	B(251),PHUTE(10),ETPHU(10),EPSCO(10),PEAKC(10),PTITL(12),	527
3	KP(251),WOCO(10),ALCO(10),A2CO(10),SP(65),JCAL,GAIN	528
C		529
	PI=3.1415927	530
	N1=BD1	531
	FN1=N1	532
	N2=BD2+0.5	533
	FN2=N2	534
	NK=(FN2-FN1)/2.0	535
	M=N2-N1	536
	EL=M	537
	N=M+1	538
	TDEL=2.0/EL	539
	SLY=(SPECT(N2)-SPECT(N1))/EL	540

	A=SPECT(N1)	541
	DYB=A	542
	I1=N1+1	543
	I2=N2-1	544
	DO 1 K=I1,I2	545
	DYB=DYB+SLY	546
1	D(K)=SPECT(K)-DYB	547
	TH=PI/EL	548
	C1=COS(TH)	549
	S1=SIN(TH)	550
	C=C1	551
	Z=S1	
	I=1	553
2	U2=0	554
	U1=0	555
	K=N-2	556
	L=N2-1	557
3	U0=2.0*U1*C-U2+D(L)	558
	U2=U1	559
	U1=U0	560
	L=L-1	561
	K=K-1	562
	IF(K)3,5,3	563
5	B(I)=Z*U1*TDEL	564
	IF(NK-I)7,7,6	565
6	I=I+1	566
	Q=C*C1-Z*S1	
	Z=Z*C1+C*S1	
	C=Q	569
	GO TO 2	570
C		571
C	CALCULATE A DENSE SET OF POINTS DESCRIBING THE COMPTON DISTRIBUTION	572
C		573
7	I=1	574
23	T=T1+DT	575
	TH1=T*PI/EL	576
	C=COS(TH1)	577
	Z=SIN(TH1)	
	TH=PI*DT/EL	579
	C1=COS(TH)	580
	S1=SIN(TH)	581
8	U2=0	582
	U1=0	583
	IXB=NK	584
9	U0=2.0*U1*C-U2+B(IXB)	585
	U2=U1	586
	U1=U0	587
	IXB=IXB-1	588
	IF(IXB)9,10,9	589
10	GO TO(20,21),I	590
20	ZDATA(IP,ISET)=T*SLY+A+Z*U1	
22	IP=IP+1	592
	NP=NP+1	593
	IF(NP)11,12,11	594
11	Q=C1*C-S1*Z	
	Z=C1*Z+S1*C	
	C=Q	597
	T=T+DT	598
	GO TO 8	599
21	D(IP)=T*SLY+A+Z*U1	
	GO TO 22	601
C		602
C	ERROR ANALYSIS OF SIN SERIES FIT	603
C		604
12	GO TO(13,14),I	605
13	IF(JOPT)15,16,15	606
		607
15	K=IP	608
	IP=1	609
	DT=1.0	610
	T1=-1.0	611
	I=2	612

	NP=N	613
	GO TO 23	614
C		615
14	IP=K	616
	SUM=0.0	617
	J=N1	618
	DO 17 K=1,N	619
	B(K)=SPECT(J)-D(K)	620
	R2=(B(K)*B(K))/SPECT(J)	621
	SUM=SUM+R2	622
	J=J+1	623
17	CONTINUE	624
	DFREE=N-NK-2	625
	EK=SUM/(DFREE*FNORM)	626
	L=0	627
C		628
C	PRINT OUT ERROR ANALYSIS	629
C		630
	CALL PAGE(IPAGE)	631
	WRITE (6,504)(TITLE(I),I=1,12)	632
	L=L+1	633
	WRITE (6,515)NK,EK,BD1,BD2,N1,N2	634
	L=L+7	635
	J=N1	636
	DO 18 I=1,N	637
	IF(L-50)60,61,60	638
60	WRITE (6,516)J,SPECT(J),D(I),B(I)	639
	L=L+1	640
	GO TO 62	641
61	CALL PAGE(IPAGE)	642
	WRITE (6,525)N1,N2	643
	L=4	644
	GO TO 60	645
62	J=J+1	646
18	CONTINUE	647
16	RETURN	648
504	FORMAT(1H012A6)	649
515	FORMAT(51H0	650
	11H HARMONICS)//20H	651
	2BD2=1PE11.4//44H	652
	3TO 13//61H	653
	4SIDUAL)	654
516	FORMAT(19H	655
525	FORMAT(44H0	656
	13//61H	657
	2L)	658
	END	659
C		661
C	ORTHOGONAL POLYNOMIAL FITTING SUBROUTINE	662
C		663
	SUBROUTINE ORPFIT(M,K,N)	664
C		665
C	N=NUMBER OF SETS OF DATA(N=1 TO 251)	666
C	M=NUMBER OF POINTS PER SET(M=1 TO 20)	667
C	K=DEGREE OF FIT(K=1 TO 18)	668
C		669
	COMMON ZDATA(251,20),S(251,18),P(17,20),SPECT(251),IZ(37),X(20),	670
1	TITLE(12),DAT(10),IDAT(10),W(18),ALFA(18),BETA(17),D(251),	671
2	B(251),PHUTE(10),ETPHU(10),EPSCO(10),PEAKC(10),PTITL(12),	672
3	KP(251),WOCO(10),ALCO(10),A2CO(10),SP(65),JCAL,GAIN	673
C		674
C		675
C	COMPUTE W(0,0),ALFA(1),P(1,XMU),(MU=1,2,...,M)	676
C		677
C		678
	W(1)=M	679
	SUM=0.0	680
	DO 1 MU=1,M	681
1	SUM=SUM+X (MU)	682
		683

	ALFA(1)=SUM/W(1)	684
	DO 2 MU=1,M	685
2	P(1,MU)=X (MU)-ALFA(1)	686
C		687
C	COMPUTE ALFA(2), W(1,1), BETA(1), AND P(2,XMU), (MU=1,2,...,M), IF	688
C	K IS GREATER THAN ONE	689
C		690
	IF(K-1)3,4,3	691
3	SUM1=0.0	692
	SUM2=0.0	693
	DO 5 MU=1,M	694
	T1=P(1,MU)**2	695
	SUM1=SUM1+T1	696
5	SUM2=SUM2+X (MU)*T1	697
	ALFA(2)=SUM2/SUM1	698
	W(2)=SUM1	699
	BETA(1)= W(2)/W(1)	700
	DO 6 MU=1,M	701
6	P(2,MU)=X (MU)*P(1,MU)-ALFA(2)*P(1,MU)-BETA(1)	702
C		703
C	TEST FOR K=2	704
C		705
8	IF(K-2)7,4,7	706
C		707
C	COMPUTE ALFA(I+1),BETA(I),W(I,I),P(I+1,XMU),	708
C	(I=2,3,4,...,K-1),(MU=1,2,...,M) IF K IS GREATER THAN 2	709
C		710
7	L=K-1	711
	DO 9 I=2,L	712
	ALFA(I+1)=0.0	713
	W(I+1)=0.0	714
	DO 11 MU=1,M	715
	T1=P(I,MU)**2	716
	ALFA(I+1)=ALFA(I+1)+X (MU)*T1	717
11	W(I+1)=W(I+1)+T1	718
	BETA(I)=W(I+1)/W(I)	719
	ALFA(I+1)=ALFA(I+1)/W(I+1)	720
	DO 12 MU=1,M	721
12	P(I+1,MU)=X (MU)*P(I,MU)-ALFA(I+1)*P(I,MU)-BETA(I)*P(I-1,MU)	722
9	CONTINUE	723
C		724
C	COMPUTE W(K+1,K+1)	725
C		726
4	W(K+1)=0.0	727
	DO 13 MU=1,M	728
13	W(K+1)=W(K+1)+P(K,MU)**2	729
C		730
C	COMPUTE DELTA,OMEGA,S(J,L)	731
C	(J=1,2,...,N),(L=1,2,...,K+1)	732
C		733
29	DO 15 J=1,N	734
	DEL=0.0	735
	SUM=0.0	736
	DO 14 MU=1,M	737
	DEL=DEL+ZDATA(J,MU)**2	738
14	SUM=SUM+ZDATA(J,MU)	739
	S(J,1)=SUM/W(1)	740
	DO 16 I=1,K	741
	WI=0.0	742
	DO 17 J1=1,M	743
17	WI=WI+ZDATA(J,J1)*P(I,J1)	744
	S(J,I+1)=WI/W(I+1)	745
16	CONTINUE	746
15	CONTINUE	747
	K2=K+1	748
	RETURN	749
	END	750

C		752
C	OUTPUT SUBROUTINE FOR NUMBERING PAGES	753
	SUBROUTINE PAGE(IPAGE)	754
	IPAGE=IPAGE+1	755
	WRITE (6,524)IPAGE	756
	RETURN	757
524	FORMAT(1H1,67X,5HPAGE 12)	758
	END	759
C		761
C	SUBROUTINE FOR EVALUATING POLYNOMIALS	762
C		763
	SUBROUTINE CNVRT(V,ANS,N)	764
	COMMON ZDATA(251,20),S(251,18),P(17,20),SPECT(251),IZ(37),X(20),	765
1	TITLE(12),DAT(10),IDAT(10),W(18),ALFA(18),BETA(17),D(251),	766
2	B(251),PHUTE(10),ETPHU(10),EPSCO(10),PEAKC(10),PTITL(12),	767
3	KP(251),WOCO(10),A1CO(10),A2CO(10),SP(65),JCAL,GAIN	768
C		769
	K=10	770
	K1=K-1	771
	GO TO(1,2,3,4,5,6,7),N	772
1	ANS=PHUTE(K)	773
	DO 15 I=1,K1	774
	J=K-I	775
15	ANS=PHUTE(J)+V*ANS	776
	RETURN	777
2	ANS=ETPHU(K)	778
	DO 8 I=1,K1	779
	J=K-I	780
8	ANS=ETPHU(J)+V*ANS	781
	RETURN	782
3	ANS=EPSCO(K)	783
	IF(JCAL)20,20,21	784
20	V1=V	785
	GO TO 22	786
21	V1=1./V	787
22	DO 9 I=1,K1	788
	J=K-I	789
9	ANS=EPSCO(J)+V1*ANS	790
	RETURN	791
4	Y=Y-100.*10./GAIN	792
	ANS=PEAKC(K)	793
	DO 10 I=1,K1	794
	J=K-I	795
10	ANS=PEAKC(J)+Y*ANS	796
5	Y=SQRT(V)	797
	ANS=WOCO(K)	798
	DO 11 I=1,K1	799
	J=K-I	800
11	ANS=WOCO(J)+Y*ANS	801
	RETURN	802
6	Y=ALOG(V)/2.302585	803
	ANS=A1CO(K)	804
	DO 12 I=1,K1	805
	J=K-I	806
12	ANS=A1CO(J)+Y*ANS	807
	ANS=EXP(2.302585*ANS)	808
	RETURN	809
7	Y=ALOG(V)/2.302585	810
	ANS=A2CO(K)	811
	DO 13 I=1,K1	812
	J=K-I	813
13	ANS=A2CO(J)+Y*ANS	814
	ANS=EXP(2.302585*ANS)	815
	RETURN	816
	END	817

*			819
*	TAPE REWIND AND UNLOAD SUBROUTINE		820
*			821
DONE	ENTRY DONE		822
	TRA **	GO TO S.IOOP TO	823
	TSX S.IOOP,4	REWIND TAPE	824
	IOSKP RUN		825
	PZE S.SU05		826
	CAL* S.SU05		827
	TSX S.XOVA,4		828
	SLW FIRST+1		829
	TSX S.XPRT,4	TYPE OUT MESSAGE ON TYPEWRITER	830
	PZE FIRST,,78		831
	TRA* DONE		832
RUN	MZE ,,-1		833
FIRST	BCI 9,TAPE	IS REWOUND AND UNLOADED REMOVE WRITE RING	834
	BCI 4,AND LABEL AS INDICATED		835
	END		836
**			838
**	7040 MAP SUBROUTINE THAT IS CALLED BY FORTRAN TO		839
**	PRINT A MESSAGE OUT ON THE ON LINE TYPEWRITER. G A CAZIER		840
**	CALLING SEQUENCE		841
**	CALL TYPEW(XXH ANY TEXT UP TO 54 CHARACTERS,XX,OPTION)		842
**	WHERE 1. XX IS NUMBER OF CHARACTERS		843
**	2. OPTION IS STOP OR GO OPTION		844
**	IF OPTION=0 PRINT MESSAGE AND GO ON		845
**	IF OPTION IS NOT ZERO PAUSE AFTER		846
**			847
	ENTRY TYPEW		848
TYPEW	PZE **		849
	SXA SAV4,4		850
	LAC TYPEW,4		851
	CLA 2,4		852
	STA CON		853
	CLA* 3,4		854
	ALS 18		855
	STD CON		856
	TSX S.XPRT,4		857
CON	PZE **,,**		858
	LAC TYPEW,4		859
	CLA* 4,4		860
	TZE SAV4		861
	TSX S.XPSE,4		862
SAV4	AXT **,4		863
	TRA* TYPEW		864
	END		865

2.2 SHAPE

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C      IBM 7040 SHAPE GENERATION PROGRAM WRITTEN IN FORTRAN 4    PART II
C
C      SPECIFICATION STATEMENTS
C
COMMON S(251,18),P(17,20),SPECT(251,20),ALFA(18),BETA(17),X(20),
1 W(251),GAIN,PHUTE(10),ETPHU(10),EPSCO(10),PEAKC(10),KP(251),
2 WOCO(10),A1CO(10),A2CO(10),SP(65),ISPUL(20),DAT(10),JCAL
005
006
007
C      TYWRITER COMMENTS TO MACHINE OPERATOR
C
CALL TYPEW(44H MOUNT COEFFICIENT TAPE ON U04 HIGH DENSITY,44,1)
CALL TYPEW(24H PRESS START TO CONTINUE,24,1)
009
30 READ(5,501)IR,I1
010
M=0
011
5 M=M+1
012
READ(5,502)IC,X(M)
013
IF(IC)6,5,6
014
6 CALL SHAPE(M,I1,IR)
015
IF(IR-7777)19,31,31
016
31 WRITE(6,507)
017
GO TO 30
018
19 DO 25 J=1,M
019
N=ISPUL(J)
020
XO=X(J)
021
JS=0
022
NCDS=0
023
IC=0
024
21 DO 20 I=1,10
025
20 DAT(I)=0
026
IF(NCDS)9,10,9
027
10 DO 8 K=1,9
028
IOUT=SPECT(K,J)
029
CALL ZERO(IOUT,OUT)
030
8 DAT(K+1)=OUT
031
C
C      OUTPUT GENERATED GAMMA SHAPES
C
WRITE(6,506)
032
23 PUNCH 504,IC,( DAT(I),I=1,10),XO
033
WRITE(6,505)IC,( DAT(I),I=1,10),XO
034
IF(JS-N)24,25,25
035
9 DO 11 K=1,10
036
JS=10*NCDS-1+K
037
IOUT=SPECT(JS,J)
038
CALL ZERO(IOUT,OUT)
039
11 DAT(K)=OUT
040
IC=IC+1
041
GO TO 23
042
24 NCDS=NCDS+1
043
GO TO 21
044
25 CONTINUE
045
GO TO 30
046
C
C      FORMAT STATEMENTS
C
501 FORMAT(2110)
047
502 FORMAT(11,9X,F10.5)
048
504 FORMAT(12,10(1X,A6),F8.2)
049
505 FORMAT(1H0,12,10(1X,A6),F8.2)
050
506 FORMAT(1H1)
051
507 FORMAT(43H0 NO COEFFICIENTS FOUND CORRESPONDING TO IR)
052
END
053
C
C      SUBROUTINE SHAPE(M,I1,IR)
055
C      GAMMA SHAPE REGENERATION SUBROUTINE
056
057
058

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C	IN THE CALL I1=1 (CHANEL NUMBER),I1=2 (ENERGY)	
C	COMMON S(251,18),P(17,20),SPECT(251,20),ALFA(18),BETA(17),X(20),	060
	1 W(251),GAIN,PHUTE(10),ETPHU(10),EPSCO(10),PEAKC(10),KP(251),	061
	2 WOCO(10),ALCO(10),A2CO(10),SP(65),ISPUL(20),DAT(10),JCAL	062
C		063
	REWIND 4	064
	3 READ(4)IRTP,JCAL,IDATE,K,N,GAIN	065
	READ(4)PHUTE,ETPHU,EPSCO,PEAKC,WOCO,ALCO,A2CO	066
	READ(4)KP	067
	READ(4)SP,ALFA,BETA	068
	READ(4)S	069
	IF(IRTP-IR)1,2,1	070
	1 READ(4)IR1,IR1,IDATE	071
	IF(IR1-7777)3,4,4	072
	4 IR=7777	073
	RETURN	074
	2 IF(I1-1)5,46,5	075
	5 DO 7 I=1,M	076
	Y=X(I)	077
	CALL CNVRT(Y,PH,2)	078
	7 X(I)=PH*10./GAIN	079
	GO TO 46	080
C		081
C	CALCULATE P(I,MU),(I=1,2,...K,MU=1,2,...M)	082
C		083
	46 DO 8 MU=1,M	084
	8 P(1,MU)=X(MU)-ALFA(1)	085
	IF(K-1)9,10,9	086
	9 DO 11 MU=1,M	087
	11 P(2,MU)=(X(MU)-ALFA(2))*P(1,MU)-BETA(1)	088
	IF(K-2)12,10,12	089
	12 DO 13 I=3,K	090
	DO 13 MU=1,M	091
	13 P(I,MU)=(X(MU)-ALFA(I))*P(I-1,MU)-BETA(I-1)*P(I-2,MU)	092
C		093
C	CALCULATE COMPTON DISTRIBUTION	094
C		095
	10 DO 14 J=1,M	096
	CX=X(J)	097
	DO 31 I=1,251	098
	SPECT(I,J)=0	099
	31 W(I)=0	100
	DO 15 I=1,N	101
	IK=KP(I)	102
	K2=IK+1	103
	SPECT(I,J)=S(I,1)	104
	DO 16 L=2,K2	105
	16 SPECT(I,J)=SPECT(I,J)+S(I,L)*P(L-1,J)	106
	15 CONTINUE	107
C		108
C	CALCULATE THE PARAMETERS OF THE GAUSSIAN	109
C		110
	CALL CNVRT(CX,W0,5)	111
	CALL CNVRT(CX,A1,6)	112
	CALL CNVRT(CX,A2,7)	113
	PH=(CX*GAIN)/10.	114
	CALL CNVRT(PH,E,1)	115
	CALL CNVRT(E,EP,3)	116
	SRTBO=W0/1.6651092	117
	A=A1	118
	AA=A2	119
	B0=SRTBO*SRTBO	120
	CM1=1.0	121
	35 CM1=CM1+2.0	122
	A=A*CM1	123
	IF(CM1-3.)35,36,36	124
	36 CM2=1.0	125
	67 CM2=CM2+2.0	126
	AA=AA*CM2	127
	IF(CM2-11.)67,39,39	128
	39 FR=B0/2.0	129
		130

	FR2=FR*FR	131
	FR6=FR2*FR2*FR2	132
	AOY0=SRTB0*1.7724539*(1.0+A*FR2+AA*FR6)	133
	Y0=(1.0E7*EPS)/AOY0	134
	R=SQRT (B0*6.*2.3025851)	135
	MINX=CX-R	136
	MAXX=CX+R	137
C		138
C	REGENERATE COMPTON TO CHANNEL NUMBER	139
C		140
	22 IF(JCAL-1)23,24,25	141
	LOW ENERGY CALCULATION PART 1	142
C		143
C		144
	23 E1=E/(1.+(2.*E)/0.511)	145
	EC=E-E1	146
	CALL CNVRT(EC,PHC,2)	147
	XC=PHC*10./GAIN	148
	XE=CX-(3.5*10./GAIN)	149
	PN=50.	150
	C=1.0	151
	BD1=1.0	152
	BD2=XC	153
	D1=0	154
	CALL REGEN(PN,C,BD1,BD2,D1,J)	155
C		156
C	LOW ENERGY CALCULATION PART 2	157
C		158
	PN=100.	159
	BD1=XC	160
	BD2=XE	161
	D1=50.	162
	CALL REGEN(PN,C,BD1,BD2,D1,J)	163
C		164
C	LOW ENERGY CALCULATION PART 3	165
C		166
	PN=50.	167
	BD1=XE	168
	BD2=CX	169
	D1=150.	170
	CALL REGEN(PN,C,BD1,BD2,D1,J)	171
	GO TO 50	172
C		173
C	INTERMEDIATE ENERGY CALCULATION PART 1	174
C		175
	24 EB=E/(1.+(2.*E)/0.511)	176
	CALL CNVRT(EB,PHB,2)	177
	XB=PHB*10./GAIN	178
	EC=E-EB	179
	CALL CNVRT(EC,PHC,2)	180
	XC=PHC*10./GAIN	181
	PN=50.	182
	BD1=1.0	183
	BD2=XB	184
	D1=0	185
	C=1.0	186
	CALL REGEN(PN,C,BD1,BD2,D1,J)	187
C		188
C	INTERMEDIATE ENERGY CALCULATION PART 2	189
C		190
	PN=100.	191
	BD1=XB	192
	BD2=XC	193
	D1=50.	194
	CALL REGEN(PN,C,BD1,BD2,D1,J)	195
C		196
C	INTERMEDIATE ENERGY CALCULATION PART 3	197
C		198
	PN=50.	199
	BD1=XC	200
	BD2=CX	201
	D1=150	202

	CALL REGEN(PN,C,BD1,BD2,D1,J)	203
	GO TO 50	204
C		205
C	HIGH ENERGY CALCULATION PART 1	206
C		207
25	EB=E/(1.+(2.*E)/0.511)	208
	CALL CNVRT(EB,PHB,2)	209
	XB=PHB*10./GAIN	210
	EC=E-EB	211
	CALL CNVRT(EC,PHC,2)	212
	XC=PHC*10./GAIN	213
	E1=E-1.022	214
	CALL CNVRT(E1,PH1,2)	215
	X1=PH1*10./GAIN	216
	E2=E-0.511	217
	CALL CNVRT(E2,PH2,2)	218
	X2=PH2*10./GAIN	219
	PN=50.	220
	BD1=1.0	221
	BD2=XB	222
	C=1.0	223
	D1=0	224
	CALL REGEN(PN,C,BD1,BD2,D1,J)	225
C		226
C	HIGH ENERGY CALCULATION PART 2	227
C		228
	PN=50.	229
	BD1=XB	230
	BD2=X1	231
	D1=50.	232
	CALL REGEN(PN,C,BD1,BD2,D1,J)	233
C		234
C	HIGH ENERGY CALCULATION PART 3	235
C		236
	PN=50.	237
	BD1=X1	238
	BD2=X2	239
	D1=100.	240
	CALL REGEN(PN,C,BD1,BD2,D1,J)	241
C		242
C	HIGH ENERGY CALCULATION PART 4	243
C		244
	PN=50.	245
	BD1=X2	246
	BD2=XC	247
	D1=150.	248
	CALL REGEN(PN,C,BD1,BD2,D1,J)	249
C		250
C	HIGH ENERGY CALCULATION PART 5	251
C		252
	PN=50.	253
	BD1=XC	254
	BD2=CX	255
	D1=200.	256
	CALL REGEN(PN,C,BD1,BD2,D1,J)	257
	GO TO 50	258
C		259
C	CALCULATE A GAUSSIAN AND ADD TO THE COMPTON	260
C		261
50	X1=MINX	262
	DO 26 I=MINX,MAXX	263
	BAS=X1-CX	264
	BA2=BAS*BAS	265
	BA4=BA2*BA2	266
	BA12=BA4*BA4*BA4	267
	GAUS=(Y0*EXP(-BA2/BO))*(1.+A1*BA4+A2*BA12)	268
	W(I)=W(I)+GAUS	269
	X1=X1+1.	270
26	CONTINUE	271
	IF(N-251)37,40,37	272
40	CALL CNVRT(PH,CM,4)	273
	DO 48 I=1,65	274

48	W(I)=W(I)+CM*SP(I)	275
37	DO 38 I=1,251	276
38	SPECT(I,J)=0	277
	ISPUL(J)=MAXX	278
	DO 32 I=1,MAXX	279
32	SPECT(I,J)=W(I)	280
14	CONTINUE	281
	RETURN	282
	END	283
C		285
C	REGENERATION SUBROUTINE	286
C		287
	SUBROUTINE REGEN(PN,C,BD1,BD2,D1,J)	288
	COMMON S(251,18),P(17,20),SPECT(251,20),ALFA(18),BETA(17),X(20),	289
1	W(251),GAIN,PHUTE(10),ETPHU(10),EPSCO(10),PEAKC(10),KP(251),	290
2	WOCO(10),AICO(10),A2CO(10),SP(65),ISPUL(20),DAT(10),JCAL	291
C		292
3	Z=(C-BD1)/(BD2-BD1)	293
	IF(Z=1.)1,1,2	294
1	ZP=Z*PN+1.+D1	295
	IC=C	296
	I=ZP	297
	FI=I	298
	FR=ZP-FI	299
	W(IC)=SPECT(I,J)+FR*(SPECT(I+1,J)-SPECT(I,J))	300
	C=C+1.	301
	GO TO 3	302
2	RETURN	303
	END	304
C		306
C	SUBROUTINE FOR EVALUATING POLYNOMIALS	307
C		308
	SUBROUTINE CNVRT(V,ANS,N)	309
	COMMON S(251,18),P(17,20),SPECT(251,20),ALFA(18),BETA(17),X(20),	310
1	W(251),GAIN,PHUTE(10),ETPHU(10),EPSCO(10),PEAKC(10),KP(251),	311
2	WOCO(10),AICO(10),A2CO(10),SP(65),ISPUL(20),DAT(10),JCAL	312
C		313
	K=10	314
	K1=K-1	315
	GO TO(1,2,3,4,5,6,7),N	316
1	ANS=PHUTE(K)	317
	DO 15 I=1,K1	318
	J=K-1	319
15	ANS=PHUTE(J)+V*ANS	320
	RETURN	321
2	ANS=ETPHU(K)	322
	DO 8 I=1,K1	323
	J=K-1	324
8	ANS=ETPHU(J)+V*ANS	325
	RETURN	326
3	ANS=EPSCO(K)	327
	IF(JCAL)20,20,21	328
20	V1=V	329
	GO TO 22	330
21	V1=1./V	331
22	DO 9 I=1,K1	332
	J=K-1	333
9	ANS=EPSCO(J)+V1*ANS	334
	RETURN	335
4	Y=V-100.*10./GAIN	336
	ANS=PEAKC(K)	337
	DO 10 I=1,K1	338
	J=K-1	339
10	ANS=PEAKC(J)+Y*ANS	340

	RETURN	341
5	Y=SQRT(V)	342
	ANS=WOCO(K)	343
	DO 11 I=1,K1	344
	J=K-I	345
11	ANS=WOCO(J)+Y*ANS	346
	RETURN	347
6	Y=ALOG(V)/2.302585	348
	ANS=A1CO(K)	349
	DO 12 I=1,K1	350
	J=K-I	351
12	ANS=A1CO(J)+Y*ANS	352
	ANS=EXP(2.302585*ANS)	353
	RETURN	354
7	Y=ALOG(V)/2.302585	355
	ANS=A2CO(K)	356
	DO 13 I=1,K1	357
	J=K-I	358
13	ANS=A2CO(J)+Y*ANS	359
	ANS=EXP(2.302585*ANS)	360
	RETURN	361
	END	362

**		364
**	7040 MAP SUBROUTINE THAT IS CALLED BY FORTRAN TO	365
**	PRINT A MESSAGE OUT ON THE ON LINE TYPEWRITER. G A CAZIER	366
**	CALLING SEQUENCE	367
**	CALL TYPEW(XXH ANY TEXT UP TO 54 CHARACTERS,XX,OPTION)	368
**	WHERE 1. XX IS NUMBER OF CHARACTERS	369
**	2. OPTION IS STOP OR GO OPTION	370
**	IF OPTION=0 PRINT MESSAGE AND GO ON	371
**	IF OPTION IS NOT ZERO PAUSE AFTER	372
**		373
	ENTRY TYPEW	374
TYPEW	PZE **	375
	SXA SAV4,4	376
	LAC TYPEW,4	377
	CLA 2,4	378
	STA CON	379
	CLA* 3,4	380
	ALS 18	381
	STD CON	382
	TSX S.XPRT,4	383
CON	PZE **,**	384
	LAC TYPEW,4	385
	CLA* 4,4	386
	TZE SAV4	387
	TSX S.XPSE,4	388
SAV4	AXT **,4	389
	TRA* TYPEW	390
	END	391

**	SUBROUTINE TO CONVERT NUMBER IN IR FROM BINARY TO BCD AND	
**	RETURN WITH THE RESULTS IN AIR	
**	ALL LEADING ZEROS WILL REMAIN	
**	JANUARY 1964 G.A.CAZIER	
**	INPUT NUMBER MUST BE LESS THAN 10**6, SIGN MUST BE +	CONV1395
**		CONV1396
**	CALLING SEQUENCE FROM FORTRAN IV	CONV1397
**	CALL ZERO(IR,AIR)	
**	TSL ZERO	CONV1399
**	TXI	CONV1400
**	BCI	CONV1401
**	PZE IR	CONV1402
**	PZE AIR	CONV1403
		CONV1404

ZERO	ENTRY	ZERO		CONV1405
	PZE	**		CONV1406
	SXA	SAV4,4		407
	LAC	ZERO,4		CONV1408
	CLA*	2,4	GET BINARY NUMBER.....	CONV1409
	LRS	35	PUT NUMBER INTO MQ.....	CONV1410
	AXT	25797,4	SET XR FOR CHAR 5	CONV1411
LOOP	SXD	SAC,4		CONV2412
	DVP	=10		CONV2413
SAC	SAC	ANSW,,**	STORE REMAINDER IN CURRENT	CONV2414
	ZAC			CONV2415
	TXI	**1,4,-1	CHARACTER.....	CONV2416
	TXH	LOOP,4,25791	NEXT CHARACTER .GO BACK.....	CONV2417
	CLA	ANSW		CONV2418
	LAC	ZERO,4	STORE RESULTS IN SECOND ARG.	CONV2419
	STO*	3,4	OF FORTRAN STATEMENT.....	CONV2420
SAV4	AXT	** ,4	RESTORE XR4.....	CONV2421
	TRA*	ZERO	RETURN.....	CONV2422
ANSW	BSS	1		423
	END			CONV2424

**PHILLIPS
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