This talk arises from the understanding gained in a series of recent experiments and their analyses in terms of the DWBA. It may also be considered an extension of earlier theoretical work [1-5]. I have relied principally on the results of early experiments performed at Brookhaven [6] and Copenhagen [7] and confirmed in a host of recent experiments at these institutions as well as at Argonne [8], Pittsburgh [9], and in France [10] and Germany [11]. I am grateful to my colleagues at Brookhaven and especially to P. D. Bond, J. D. Garrett and A. J. Baltz for helping me prepare this manuscript.

When I was originally asked to speak at Nashville it was suggested I talk about optical potentials in elastic scattering and in inelastic processes induced by heavy projectiles. After some discussion it was decided I could consider optical potentials and transfer reactions. At some risk of overlapping with other speakers here, I would like to concentrate on the transfer reactions or really on matters which apply to any of the direct reactions induced by heavy ions. One might, in fact, refer to any process in which the projectile and target come out in more or less one piece, only slightly altered in mass, charge, energy, etc., as quasi-elastic. There is no need to distinguish too much between elastic, inelastic and transfer of a few nucleons. What will not be considered are the more violent reactions which to a large extent destroy the nature of the entrance channel. Direct-reaction theory permits the latter reactions to play only an indirect role, through the determination of the imaginary part of the optical potential. It is, in fact, a basic tenet of the direct reaction single-step mechanism that higher order processes enter only in the optical distortion of the elastic wave functions. It may be taken for granted, although I will make only slight reference to this, that elastic data is always employed to aid in defining permissible optical potentials. This constraint is taken far more seriously in the heavy-ion induced reactions than ever it was in (d,p), (p,t), etc. Transfer reactions in the heavy-ion situation are particularly sensitive to the nature of these potentials and are then a principle source of information on the average ion-target interaction. Inevitably we hope to find exotic aspects of the interaction between complex nuclei. A good starting point though may be the average interaction at grazing and larger separation, with the highly peripheral direct reactions speaking to just this issue. Before proceeding let me introduce a caveat on the concept of optical potential between ion and target. We are discussing a monopole potential in the separation between center of mass of two reasonably large nuclei, of the form

\[ U(r) = V_{\text{nuclear}}(r) + V_{\text{Coulomb}}(r) + iW(r). \]

At separations involving strong overlap of nuclear densities it will be difficult to sustain the idea of a potential. At larger, grazing separations appreciable distortion of nuclear shapes must be present. Nevertheless hiding some of these aspects in the imaginary (strongly absorptive) parts of the potential allows us to make considerable progress.

What I wish to discuss specifically is a rather general picture of the direct reaction which has emerged recently, and containing a certain element of surprise. Had more attention been paid to earlier theories of Ford and Wheeler [12], Smirnitskii [1], Frahn and Venter [2], Dar [3], the degree of

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surprise might have been lessened. It is, however, the force of recent data that has led to the present picture, defining and sharpening the theoretical issues. In question is a transition in the heavy-ion direct reactions from smooth classical to oscillating quantum mechanical shapes. The surprise is in retrospect associated with the appearance of interference or diffractive behaviour at an extremely low energy where one thought the strong Coulomb field between massive projectile and reasonably heavy target cast a classical hue over the transfer process. The key element in the low threshold for this behaviour is an unexpectedly low absorption at grazing separations, a surface transparency for the projectile. The diffraction is not signalled by a low value of the Sommerfeld parameter \( \eta = Z_1 Z_2 e^2/\hbar v \), occurring for values as high as \( \eta = 18 \) and at energies as little as 10 MeV above the barrier. One result of these developments will be an added degree of richness for heavy-ion induced direct reactions, a richness which permits structure information to be extracted from what were formerly considered uninteresting distributions.

Let me quickly show some of the data that suggested featureless bell-shaped angular distributions were not universal. The next few slides and transparencies show the reactions \( ^{58,60,62,64}_{\text{Ni}}(^{180,160},^{160,140}) \) g.s. \([5]\), \(^{28}_{\text{Si}}(^{160,140})^{28}_{\text{Si}} \) g.s. \([6]\), \(^{48}_{\text{Ca}}(^{160,140})^{48}_{\text{Ca}} \) (g.s. and 3.08 MeV) \([5]\) \(^{40}_{\text{Ca}}(^{13}_{\text{C}},^{12}_{\text{C}})^{40}_{\text{Ca}} \) \([5]\), \(^{48}_{\text{Ca}}(^{160,140}) \) \([9]\) performed at various energies at Brookhaven, Copenhagen, and Argonne. A very low energy reaction showing strong oscillations is: \(^{40}_{\text{Ca}}(^{13}_{\text{C}},^{14}_{\text{N}})^{39}_{\text{K}} \) at 40 MeV. I will not discuss these in detail but pass on to a theoretical analysis of a reaction with non-vanishing angular momentum transfer \([13]\). It is necessary to consider non-vanishing (LM) if a complete understanding of the process is to be obtained and in addition the magnetic (M) state population is I believe one of the most interesting features of the reaction. L-dependence which reappears in heavy ion processes because of the diffraction and is seen in the data I showed, is one aspect of M-populations. Another aspect is the frequent domination of the differential cross-section by the \( M = \pm L \) partial cross-section. In fact the \( |M| = L \) "rule" is probably necessary for the existence of L-dependence. It should be emphasized that \( LM \neq 0 \) has not been properly covered in heavy-ion situations before.

A specific reaction to consider can be

\[
(A+n)+B \rightarrow A+(B+n) \tag{1}
\]

which is described in configuration space by the amplitude

\[
T_{if} = \int d\xi_1 \int d\xi_2 \chi^{(\pm)}_f (k_x \xi_1) \phi_{j_1j_2}^{m_1m_2} (\xi_1 \xi_2) \chi^{(+)}_i (k_x \xi_2) \tag{2}
\]

with \( \chi^{(\pm)}_f \), \( \chi^{(+)}_i \) distorted waves in the exit and entrance channels, and \( \phi_{j_1j_2}^{m_1m_2} \) the form factor for transfer from a bound state \( (j_1m_1) \) in the projectile to \( (j_2m_2) \) in the target. A transferred angular momentum is defined through \( L = j_2 - j_1 \). Introducing partial wave expansions \( \{\ell_1, 0\} \) and \( \{\ell_2, m_2\} \) with quantisation axis along the incoming beam leads to an expansion of \( T_{if} \) in terms of multipoles \([14]\)

\[
\beta^{LM} = \left(\frac{4\pi}{k_1 k_2}\right)^{3/2} \sum_{\ell_1 \ell_2} \sum_{M} \langle \ell_1 \ell_2 M | L \rangle \langle 0 | \ell_2 M \rangle \langle 0 | \ell_1 0 \rangle \tag{3}
\]

with the additional vector relation

\[
L = \ell_1 - \ell_2 \tag{4}
\]

The radial integrals are given by
\[ I_{1,2} = \int \int u(k_{\xi}, r_{\xi}) F_{L}(r_{\xi}) u(k_{\xi}, r_{\xi}) \, dr_{\xi} \, dr_{\xi} \]  

in a full finite range calculation and by

\[ I_{1,2} = \left[ \frac{2k_{\xi}^{2} + 1}{4\pi} \right] \left( -1 \right)^{l_{\xi} - \frac{1}{2}} \int I_{l_{\xi} \to l_{\xi} + \frac{1}{2}} \, \right] \int \int u_{f}(r_{1}) F_{L}(r_{1}) u_{1}(r_{1}) \, dr_{1} \]  

in a no recoil limit (employing a scaling \( r_{f} = \alpha r_{f} \)). I will work with the latter relation (6) and hence consider only normal parity transfer, \( l_{\xi} \neq l_{\eta} \), \( \ell_{\xi} \).  

An essentially semi-classical expression for \( E_{\ell M} \) may be obtained from certain assumed behaviour for the radial integrals and an asymptotic form for \( V_{M}^{F}(\theta_{\eta}, 0) \). The angular range for \( \theta_{\eta} \) may be roughly divided into two regions

\[ \theta_{\eta} \geq (2)\frac{1}{2} \frac{M}{l_{\eta}} \quad \text{and} \quad \theta_{\eta} \leq (2)\frac{1}{2} \frac{M}{l_{\eta}} \]  

The latter region contains the most forward potentially L-dependent, "(d,p)" peak. This may be considered separately and aside from the necessary factor \( (\sin \theta_{\eta})^{M} \) adds nothing new to the M-population. In the former region one may use [16]

\[ V_{M}^{F}(\theta_{\eta}, 0) = [2\pi (\sin \theta_{\eta})^{rac{1}{2}}]^{-1} \left\{ i\left[ (\ell_{f} + S_{f})\theta_{\eta} = \frac{\pi}{4} - \frac{\pi M}{2} \right] + \text{c.c.} \right\} \]  

The essential physics of the reaction is contained in the radial integrals which are parametrized by

\[ I_{L_{1}, L_{f}} \approx e^{i(6_{f} + S_{f})} \exp \left\{ -\frac{(6_{1} - 6_{f})^2}{2} - \frac{(S_{1} - S_{f})^2}{2} - \frac{(\Delta - \Delta_{0})^2}{2} \right\} \]  

where \( \Delta = \ell_{f} - \ell_{1} \), \( \Delta_{0} = l_{f} - l_{1} \).

Since the reaction takes place at reasonable large nuclear separation (6_{f} + S_{f}) can be related to elastic phase shifts. I limit these phases to a linear relation in the region of \( 6_{1}, S_{1}, 6_{f} + S_{f} = \frac{1}{2} \psi \), \( 6_{f} + S_{f} = \frac{1}{2} \psi \), \( \psi_{1} \), \( \psi_{2} \), \( V_{1}, V_{2} \) interpretable as classical deflection functions [12,17].

The peak values \( 6_{1}^{0}, S_{1}^{0} \) should be near the "orbitting" angular momenta determined from classical relations

\[ V_{f}(r) + \frac{\hbar^2}{2M_{1}} \frac{6_{1}^{0} (6_{1}^{0} + 1)}{r^2} = \frac{k_{1}^2}{2M_{1}} \]

and

\[ \frac{\partial}{\partial r} \left[ V_{1}(r) + \frac{\hbar^2}{2M_{1}} \frac{6_{1}^{0} (6_{1}^{0} + 1)}{r^2} \right] = 0 \]  

Actual resonances in the quantum mechanical situation if permitted by the level of absorption at these radii probably occur for slightly lower \( S \)-values and hence generally play no direct role in the transfer. It will be interesting to speculate on situations in which these resonances might be important [15]. For \( 6_{1}(\xi) < 6_{1}(\xi) \) the peak in the radial wave function will occur within a region
of strong absorption, while for \( \ell_1 > \ell_1^0 \) a decreasing form factor \( f_l \) or \( F_l \) will reduce the transition probability. The additional \( \ell \)-localization in (5) arises from the beating of (oscillating) initial and final wave functions against each other. This \( \ell \)-dependence is crucial and has been previously passed over. The radial integral behavior in (5) is confirmed in DWBA calculations though the use of a Gaussian form is not strictly accurate. The above asymmetry, which I ignore for simplicity, ought to be present. In addition the linear phase relation breaks down somewhat permitting a generally shallow minimum to occur, usually at the lower end of the \( \varepsilon \)-window. This might be the nuclear rainbow and along with possible resonances should be further investigated. The level of absorption seems to, however, keep the effect on a "linear phase" assumption reasonably small. Figures 1 and 2 show examples of phase and magnitude variation. The analysis presented here can be altered to accommodate non-linearity as recognized by Friedman, McVoy and Shuy [18]. The arguments presented hereafter are valid in the limits \( \ell^0_1, \ell^0_f > 1 \), which might be used to distinguish "heavy" from "light"-ion induced transfers.

One final ingredient in our calculations necessary for performing the important \( \ell \)-sum in (3), a sum absent when \( L = 0 \), may be generalized from arguments advanced by Austern and Blair [4] for a similar situation arising in light-ion reactions. These authors suggest for \( \ell_1, \ell_f \) large

\[
\sum \ell_1 \ell_1^L \ell_1^M \ell_1^M \ell_1^M = \gamma^L_M(\ell_1, 0)
\]

(10)
giving rise (amongst other features) to the Blair phase rule [19]. To perform the \( \ell \)- summation in this manner for heavy-ion reactions, ignoring the phase and magnitude dependence of radial integrals on \( \ell_1, \ell_f \), would throw away the subtlety and uniqueness of these reactions, and indeed would yield misleading results. For example, since \( Y_{L-2}^0(\pi/2, 0, 0) \) is quite comparable to \( Y_{L}^L(\pi/2, 0, 0) \) in magnitude, no \( |M| = L \) dominance can be a consequence of (10). Instead I use the asymptotic relations for \( Y_{L}^L \) and \( Y_{L}^M \) in

\[
\sum \ell_1 \ell_1^L \ell_1^M \ell_1^M \ell_1^M = 2 \pi \left[ \frac{4 \pi (2 L + 1)}{(2 L + 1)(2 L + 1)} \right]^{\frac{1}{2}} \sum \ell_1 \ell_1^L \ell_1^M \ell_1^M \ell_1^M
\]

retaining only the dependence on \( \Delta = \ell_f - \ell_1 \). Introducing the variables \( \ell = (\ell_1 + \ell_f)/2 \), \( \Delta \), converting summations to integrals, and simplifying the problem by setting \( \ell_1 = \ell_f = \ell \), \( \Gamma_1 = \Gamma_f = \ell \) there results

\[
\beta^L_M = \frac{\ell^{L}(\ell - 1)^M}{k_{L}^{L}(\ell_f)} \left[ \frac{2 \pi}{(2 L + 1)(2 L + 1)} \right]^{\frac{1}{2}} \frac{1}{\gamma} e^{i \Psi_0} \quad (11)
\]

where
and $1/\gamma^2 = 1/70^2 + 1/21^2$. Much of the simple physics of heavy-ion induced direct reactions can be discussed with this formula. A key element in its derivation was to recognize the importance of the variable $\Delta = \Delta^2 - \Delta^1$, even in a well matched situation where $\Delta^2 \approx \Delta^1$. The two terms in Eq. (11) when present together at an angle of observation $\theta_f$ will interfere producing narrow angular oscillations of period $\Delta \theta_f \approx \frac{2\pi}{2\theta_0 + 1} \approx \frac{2\pi}{kd}$. The first term in (11) may be associated with the dominant impact parameter $S(\theta)$ and a classical orbit at the energy in question. This term yields by itself a bell-shaped distribution with finite angular width $\sim 2(2)^2/\gamma$ because the sharp classical relations between $x$, $y$, and $s$ are smeared out. The second term is as we will see associated with a scattering through $\pi/2$ in an orbit on the opposite side of the nucleus. Since no negative values of $\gamma$ appeared in our $\Delta$-localized wave packet (8) one might refer to this second orbit as virtual. Finally, the function $I_N(L, \theta_f)$ is interpretable as a multipole radiation pattern, and it is its structure which both determines the $N$-population and helps in an understanding of the reaction mechanism.

A literal interpretation of (11) is then of two waves originating on opposite sides of the nucleus and interfering strongly if either the central scattering $\psi$ becomes small or the angular width $2(2)^2/\gamma$ becomes broad. Classical, physical optics analogies can be constructed, but some attention must be paid to the angles $\theta = \pi/2 \pm \theta_f/2$ about which the integrations for $I_N(L)$ in (12) are centered. Although the linear phase relation in (8) suggests the refraction through angles $\pm \psi_0$ is produced by straight optical paths, the classical orbits or paths in the present situation are actually curved in the external nuclear plus Coulomb field. Indeed the linear phase assumption is not carried through into the argument of $I_N(L)$. Also the parameters $\Delta', \psi, \Gamma$, in (11) are functions of energy and this lens aspect of the full DWBA calculation must eventually be evident.

I would like to present a simple picture (Fig. 3) which connects the width $\Gamma$ and rate of change of phase $\psi$ of the quantum wave packet (8) with the angular argument of $I_N(L)$ and with the surface absorption. This picture is highlighted in the interesting limit $\gamma$ large whence

$$I_N(L, \theta_f) = \int_0^{\pi} d\theta \, y_N(L, \theta_f, \theta)$$

and

$$(-1)^N I_N(-\theta_f) = \int_0^{\pi} d\theta \, y_N(L, \theta_f, \theta).$$

If at the same time (consistently) $\Gamma$ is large the first term in (11) dominates and contributes only near $\theta_f = \psi$. This is the geometric optic limit. As shown in Fig. 3, an attempt at a diagrammatic representation, the $\theta_f = \psi$ term corresponds to a classical orbit for the well defined impact parameter $S(\theta_f)$. This orbit "strikes" the nucleus at the point $P_1$ and is bent through $\psi/2$ initially and $\psi/2$ in the exit channel. For those who like straight lines this ray can be thought of as reflected from a mirror along the tangent at $P_1$. Considering azimuthal symmetry the target nucleus reflects this incoming wave from the darkened annulus extending to $P_2$ (below). The resulting diverging, reflected, wave would produce no oscillations by itself. The broadened ($\Gamma, \gamma$ large) wave packet in $\Delta$-space would then truly have produced a well-defined orbit in configuration space. The significance of
is also clear if one notes the co-latitude and azimuth of $P_1$ are $\left(\frac{\pi}{2},\frac{\pi}{2}\right)$. Some ambiguity can be associated with an interpretation of the second term in $I^L_M$, i.e., the term proportional to $(-1)^M I^L_M(-\theta_f)$. A stationary phase treatment of elastic scattering [12] produces an interference phenomenon like the one considered here, associating the $-\psi$ term with an actual impact parameter $S_+$ which is a solution of $2\delta(S(\hat{\theta}))/d\hat{\theta} = -\psi$. Figure 4 shows the classical deflection curve $\Psi(S)$, the relation between $S_+, S_-$, and the dip in the $\Psi(S)$ curve near the "nuclear rainbow", all for a possible real potential.

I think it is useful to continue this view here, especially if one considers the continuity in description of the reaction from low to high energy. The impact parameter $S_+$ does not necessarily enter strongly into our wave packet. At $P_2$ for example the dominant wave at $S_+$ is not only reflected but also transmitted. This transmitted wave which then probably moves about the nuclear sphere as a surface wave (Fig. 3) can arrive at $P_3$ and for the particular observation angle $\theta_f = \psi$ reappear externally along the lower orbit ($-\psi$) in Fig. 3. Since $P_3$ has the angular position $(\pi/2, \pi/2, x)$ this also explains

$$(-1)^M I^L_M(-\theta_f) \approx y^L_M(\frac{\pi}{2} - \psi, \pi)$$

In addition the damping of oscillations at $\theta_f = \psi$ is explained by the distance travelled by the surface wave between $P_2$ and $P_3$, a distance proportional to $\psi$. The damping of oscillations at any other angle and corresponding arguments of $I^L_M$ are also described by Fig. 3 as at least a mnemonic device. For example, transmitted surface waves at both $P_1$ and $P_2$ can interfere for $\theta_f = 0$ after each travels through distances $\approx \psi/2$. Oscillations for our Gaussian wave packet are then maximal for forward directions.

Of course to produce appreciable oscillations we want $\Gamma$ small and as we find realistically $\Gamma$ is also small. Our sharp orbit picture is then a little fuzzed; the relations between $\psi, \zeta, S$ are spread out. In particular, contributions to the integral for $I^L_M$ come from an angular region width $2\pi/\psi$ about the central angle. Had one used other than Gaussian windows in $x$-space the broader diffraction minima associated with the reflected wave at $P_1$ might appear. Nevertheless the previous picture remains useful.

It is clear from the above that absorption and the smallness of $\psi$ both play a big role in the amplitude of the fine angular oscillations. It appears as we raise the energy of a given reaction that it is a dramatic decrease in $\psi$ which heralds the onset of oscillations [6]. Some analyses of one nucleon transfer data indicate an increase in surface absorption with energy, not surprising perhaps. The likelihood of seeing the effects of the nuclear rainbow is then decreased. Perhaps two-nucleon transfer will sense inner regions.

There remains now the treatment of $H$-population for realistic $\Gamma, \gamma$. In practice potential models which fit elastic scattering and describe the oscillations in one particle transfer yield $\Gamma \approx 6$ and $\gamma \approx 2-4$. The $I^L_M(\pm \theta_f)$ are then obtained by averaging $I^L_M(\hat{\theta})e^{iL\phi}$ over an angular region $\Delta \theta \sim 2\pi/\psi$ about $\theta = \pi/2 \pm \theta_f/2$. Angular patterns for $Y^L_+\gamma$ and $Y^L_-\gamma$ are shown in Fig. 5.

Clearly for reasonably high energy where only small $\theta_f$ are relevant, and for well-matched transfer ($|\Delta_0| < L$), the averaging strongly favours $N = \pm L$ over $N = \mp (L-2)$. The nodes in $Y^L_\pm L, Y^L_{L-2}$, etc. greatly reduce cross-section in the $|H| \neq L$-states. The odd-states $N = L-1$, etc. are also greatly reduced, possessing a node at $\theta = \pi/2$, but are non-zero for non-vanishing $\theta_f$ or $\Delta_0$. Figures 6 and 7 show explicit calculations of $H$-state population as functions of $\gamma$ and $\Delta_0$. The reduction in $|H| \neq L$ states is quite drastic for the realistic range of $\gamma$, $2 < \gamma < 4$. For poorly matched transfer signalled here by $|\Delta_0| \geq L$, $N$-states are more haphazardly populated. Near barrier energies the large $\gamma$-result
\[ I_M^L(\theta_f) \sim Y_M^L(\frac{\pi + \theta_f}{2}, 0) \]

is almost applicable. The result is again a more even magnetic state population. In general the population is a strong function of angle of observation.

When \( \gamma \) is not too small (\( \geq 2 \)) an analytic evaluation of \( I_M^L(\theta_f) \) may be obtained from (12) [13]. The situation for low values of transferred angular momentum (\( L \leq 4 \)) are easily handled. A general result of some surprise is obtained for \( \Delta_0 = 0 \) (and seen in DWBA calculations, e.g., \( ^{40}\text{Ca}(^{13}\text{C},^{14}\text{N}) \) at 40 MeV):
\[
(-1)^M I_M^L(-\theta_f) = (-1)^L I_M^L(\theta_f)
\]

One normally expects the oscillations for \((L+M)\) even and \((L+M)\) odd to be just out of phase, i.e.,
\[
(-1)^M I_M^L(-\theta_f) = (-1)^{L+M} I_M^L(\theta_f)
\]

Finally for the forward angular region \( \theta \leq M/L \) I refer you to the paper with Bond and Chasman [13]. The result here is an amplitude
\[
\beta^{LM} \sim J_M((L_f \frac{0+1}{2}) \theta_f) I_M^L(\theta_f) e^{-\gamma^2/8}
\]

The most forward peak position \( \theta_M \) is determined by the first maximum in \( J_M \) and satisfies [16]
\[
(M(M+2))^\frac{1}{2} \leq (L_f \frac{0+1}{2}) \theta_M < (2M(M+1))^\frac{1}{2}
\]

When the \( |M| = L \) rule holds there will then be a \( L \)-dependence, frequently observed in data.

Some other observations of interest can be made. Bond and I find that imagining the \( \delta_1 \)-sum in (3) to be done and directly parametrizing the \( L_f \)-sum can be tricky. With our model (8) the \( \delta_1 \) sum can be performed and for \( L \geq 0 \) and given \( M \) the \( L_f \)-distribution is necessarily multi-peaked, with a specific separation between peaks and \( L_f^0 \) an apparent function of \( M \). This result has direct consequences on possible asymmetry about \( \theta = \gamma \) of the classical bell-shaped part of the differential cross-section; the specific form of \( I_M^L(\theta_f) \) can destroy the otherwise evident symmetry.

Let me add some general comments not necessarily connected with the above but which arise from detailed analysis of data. First it appears the transfer processes occur well out into the nuclear surface where the nuclear real potential is changing rapidly. This region which is near the distance of closest approach for the peak partial waves \( \delta_1^0 \), \( \delta_f^0 \) is also important for elastic scattering and inelastic excitation. These latter processes are not so well localized as transfer, however, and consequently do not exhibit the Fraunhoffer-type diffraction we have been discussing as strongly or at as low energies as in transfer. The real optical potential is of common importance for all quasi-elastic processes and is probably well determined in the exterior regions by elastic data. The transfer is clearly more sensitive to surface absorption, which we must conclude is weak. It should be emphasized that because of a generally steeper drop in its form factor two nuclear transfer takes place a bit further into the potential than single nucleon transfer. For these reactions then the slightly more interesting region of the nuclear potential, near a possible nuclear rainbow or near resonances, may be seen. On the other hand, a
very low energy oscillation threshold in the 40 MeV one-proton transfer reaction
\( {^{40}\text{Ca}(^{13}\text{C},^{14}\text{N})^{39}\text{K}} \) (gs) [21] seems to demand a sharply rising absorption at overlapping ion-target separations, clearly indicating a sensitivity to these inner regions. The DWBA analysis of two particle transfers performed recently [9,10, 20] required similar imaginary potentials, more transparent in the surface than one imagined as well as more steeply rising in the interior. Such potentials may in fact conform better to reality: with a volume, small diffusivity, Woods-Saxon coming from the more violent compound processes and a surface longer-tailed piece arising from the direct reactions themselves.

Of potentially greater interest are some oscillating angular distributions [22] indicating disagreements with the predictions of specific DWBA calculations. One such reaction \( {^{40}\text{Ca}(^{13}\text{C},^{14}\text{N})^{39}\text{K}} \) at 68 MeV, to be discussed in a contribution to this conference, shows oscillations 180° out of phase with predictions. The problem, which is not cured by including recoil (non-normal parity transfer), seems to be one of incorrect calculation of the N-population. The solution may come from varying the inner regions of the real potential, for example to produce a true resonance; or it may lie in the introduction of multi-step reactions. Either explanation at this time is somewhat of a fond hope.

I will conclude then by saying that the degree of success of an essentially semi-classical analysis is to me surprising but heartening. The quantum phenomena of interference and diffraction surely fit in well under a semi-classical umbrella. The models we use here rely on a one body treatment in terms of distance between centers of mass of rather large nuclei. The many-body aspects of the problem must eventually play a role.
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FIGURE CAPTIONS

Fig. 1 The variation of phase and amplitude with $\phi_f$ for the reaction $^{48}\text{Ca}(^{12}\text{N},^{13}\text{C})^{49}\text{Sc}$ gs at 50 MeV. There is a dramatic change in phase at the low end of the $\phi$-window. Direct elimination of partial waves indicates little effect from this.

Fig. 2 Another extreme in phase variation. The reaction $^{40}\text{Ca}(^{13}\text{C},^{14}\text{N})^{39}\text{K}$ gs at 40 MeV with the deflection function $\psi(S) = 2dS/\delta$ plotted to emphasize, here, an appreciable minimum in $\psi(S)$ in the middle of the $\phi$-window. The surface absorption is here very weak. It should be pointed out that the phases discussed here are from the transfer amplitude and are not the elastic phases which more properly describe "orbits".

Fig. 3 Schematic representation of Eq. (11) as model for heavy-ion induced transfer.

Fig. 4 Classical deflection curve, scattering angle versus impact parameter for a possible real potential.

Fig. 5 Plots of spherical harmonics to show the strong effects of averaging near $\theta = \pi/2 \pm \phi_f/2$.

Fig. 6 M-populations as function of $\gamma$ for forward angles and $\Delta_0 = 0$, i.e., well matched and reasonably high energy $\sim$ twice barrier energies. Also for $\Delta_0 \sim L$, bad matching.

Fig. 7 M-populations for well matched non-forward angles. Cross-sections are appreciable at these angles for lower energies.
Figure 2.
Figure 4.
Figure 5.
Figure 7.