FISSION-PRODUCT AFTERHEAT — A REVIEW
OF EXPERIMENTS PERTINENT TO
THE THERMAL-NEUTRON FISSION OF ^{235}U

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ABSTRACT

We have reviewed experimental data relating to beta and gamma energy release rates following thermal-neutron fission of ^{235}U . We have proposed an afterheat function based on these experiments, for after-shutdown times from 1 sec to 5×10^5 sec, compared this function with the proposed ANS Standard ANS-5.1, and estimated the uncertainty in our proposed function. Our proposed function varies between 0.98 and 1.08 times the proposed standard function; we estimate the standard deviation to be between 10% and 15%.

Keywords: fission-products, afterheat, decay heat, thermal-neutrons, ^{235}U , experimental data, evaluation.

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1. INTRODUCTION

The American Nuclear Society, through one of its standards subcommittees (ANS-5), has proposed a standard function to describe fission-product decay heat following shutdown of a thermal reactor operated primarily with ^{235}U fuel.¹ The proposed standard is based on a 1961 evaluation of fission-product decay heat by K. Shure.² Shure's evaluation, in turn, rested primarily (for after-shutdown times greater than 1000 sec) on calculations by Perkins and King,³ in which concentrations and decay energies for individual fission-product nuclides were combined; and (for cooling times less than 1000 sec) on a prior evaluation by Stehn and Clancy⁴ of experimental afterheat determinations.

The proposed standard suggests that the uncertainty in the given function $F(t, \infty)$ is +20%, -40% for $t < 10^3$ sec; +10%, -20% for $10^3 < t < 10^7$ sec; and +25%, -50% for $t > 10^7$ sec. The significance of the stated uncertainty (e.g., whether one or two standard deviations of the mean) is not made clear.

In view of the substantial current interest in loss-of-coolant accident (LOCA) analysis in nuclear reactors and in view of the importance of fission-product decay heat to this analysis, we have undertaken a review of relevant experimental data, most of which has become available since Shure's evaluation.² The goal of this review was to try to establish a best estimate of the afterheat function, as deduced from the experimental data, to compare this estimate with the proposed ANS standard, and to estimate errors and confidence limits in the experimental afterheat function.

Our review has been focussed on thermal-neutron-induced fission of ^{235}U and on after-shutdown cooling times in the range from 1 to 10^5 sec. We were particularly concerned with the after-shutdown interval from 1 to 10^3 sec, because of its relevance to loss-of-coolant-accident analysis in light-water reactors and because it was in this range especially that the proposed standard was based on limited and uncertain evidence.

It should be noted (and is discussed in more detail in Sect. 3) that determination of afterheat at cooling times up to 10^3 sec following an extended period of reactor operation, if based on measured energy releases after short "bursts" of fission, may require information for times-after-fission much greater than 10^3 sec. Thus, our evaluation of experimental data does include longer cooling times to some extent.

In order to make the following discussion as clear as possible, it will be helpful to establish certain definitions.

$F(t, T) \equiv$ the fission-product decay heat at time t after shutdown of a reactor following operation at constant power for a period T . Conventional units are $(\text{MeV/sec})/(\text{fissions/sec}) = \text{MeV/fission}$ or, alternatively, a fraction of the operating power. The restriction to constant power is not essential, but is a convenient — and conventional — basis for comparison of information from different sources.

$f(t) \equiv$ energy release per unit time following an essentially-instantaneous burst of fissions. Units are MeV/sec/fission .

Subscripts β and γ may be used on either $f(t)$ or $F(t, T)$ to indicate the contribution of β particles or γ rays to the afterheat functions.

If we neglect production and removal of nuclides from the reactor by processes other than fission and radioactive decay, that is, if the concentrations of fission products resulting from fission that occurred at a time t' during the operating interval T are not significantly altered by subsequent exposure to the neutron flux,* then a simple relation exists between $f(t)$ and $F(t, T)$:

$$F(t, T) = \int_t^{t+T} f(t') dt' \quad (1a)$$

$$= \int_t^{\infty} f(t') dt' - \int_{t+T}^{\infty} f(t') dt' \quad (1b)$$

$$= F(t, \infty) - F(t + T, \infty) \quad (1c)$$

* The effects of neutron capture are discussed in Sect. 8.

We refer to $f(t)$ as the "differential" afterheat function, following a fission burst, and to $F(t, T)$ as the "integral" afterheat function following an operating period T . The proposed standard exhibits only the integral function for a very long operating period, $F(t, \infty)$, but notes that Eq. (1c) may be used to obtain the afterheat for finite operating periods, such as $T = 10^4$ hr.

It should be noted that the function $F(t, T)/T$, for $t \gg T$, approaches the function $f(t)$, or perhaps more accurately $f(t + \frac{T}{2})$. For $T \gg t$, $F(t, T)$ approaches $F(t, \infty)$, but the second term on the right-hand side of Eq. (1c) becomes arbitrarily small only for very large T , not for very small t .

Values of $f(t)$ or $F(t, T)$ for β 's and γ 's together or separately may be obtained from several different sources.

1) "Summation" calculations,^{3,5-16} (such as those of Perkins and King) in which concentrations of individual fission-product nuclides are calculated, both during and after reactor operation, based on their known or estimated yields, half-lives, and - in some calculations - neutron-capture cross sections; energy-release rates are then calculated from the known nuclide decay schemes or from estimated averaged decay energies.

2) Separate measurements of β or γ energy-release rates for combined fission products,¹⁷⁻³⁷ either following a fission burst, or during or after steady-state irradiation of fissile samples for various periods, T . Such experiments may involve measurement of the energy spectrum of the β - or γ -rays emitted by the fission products or only the total β or γ energy-release rate. (In some experiments, just the total number of β or γ rays is measured.)

3) Calorimeter measurements of combined β and γ power,³⁸⁻⁴⁰ following various periods of operation.

The first method, "summation" calculations, may be regarded as an experimental method, to the extent that the mathematical procedures for calculating fission-product concentration are made essentially exact and to the extent that the very large number of yields, half-lives, and β - and γ -decay energies are based on experimental results. However, these data are not available for many nuclides that can be expected (on the

basis of theoretical estimates) to be important, and thus some reliance must be placed on theoretical nuclear models or on nuclear-structure systematics to provide estimates of the "missing" quantities. This is especially true for many short-lived fission-product nuclides, and summation calculations characteristically fail for short cooling times (e.g., $t \lesssim 100$ sec). Thus, especially for short cooling times, the summation calculations are in a sense on trial, and validation must be sought in more direct experimental data.

(Here we sometimes encounter a different meaning for the terms "differential" and "integral" data than the ones we used above: the term "differential" may indicate data pertaining to individual fission-product nuclides or chains, and the term "integral" sometimes refers to the combined effect of all fission-products present at a given time, rather than to integration of the afterheat function, $f(t)$, over an operating interval.)

The third method — calorimeter measurements — has thus far been restricted to cooling times longer than about 10^3 sec³⁸⁻⁴⁰ and is therefore not directly helpful in verifying the afterheat function for the short cooling times that are most important in the LOCA analysis.

We therefore rely primarily on the second method, separate measurements of β and γ energy release rates.

In the following sections, we will present the afterheat function recommended in the proposed ANS standard (Sect. 2); summarize our best-estimate afterheat function derived from experimental data, and compare it with the draft standard (Sect. 3); exhibit the results of individual experiments and indicate how they were combined in our final weighted-average result (Sect. 4 and 5 for β 's and γ 's respectively); estimate errors and confidence limits for the composite experimental afterheat function (Sect. 6); review briefly the status of summation calculations and compare them with our composite experimental function (Sect. 7); comment on the effects of neutron capture Sect. 8. Details of our review of individual experiments will be found in appendices.

2. THE PROPOSED ANS STANDARD

As mentioned above, the proposed ANS standard (ANS-5.1)¹ is based directly on K. Shure's 1961 evaluation of fission-product afterheat for thermal-neutron fission of ²³⁵U (Ref. 2), following "infinite" operation at constant power.

Shure's recommended functions for combined β and γ afterheat and for γ 's alone are shown in Figs. 1 and 2, taken directly from Ref. 2; β power is obtained by subtraction.

These are the integral functions, $F(t, \infty)$. We could obtain corresponding values of the differential functions,

$$f(t) = \frac{\partial}{\partial t} F(t, \infty) , \quad (2)$$

but have chosen instead to compare differential functions from various sources through the quantities

$$\Delta_j F \equiv F(t_{j+1}, \infty) - F(t_j, \infty) \quad (3a)$$

$$= \int_{t_j}^{t_{j+1}} f(t') dt' , \quad (3b)$$

where the t_j are chosen as $1, 2, 5 \times 10^n$, with $n = 0, 1, 2, \dots$. Values of $F(t, \infty)$ and of $\Delta_j F$ are listed in Tables 1 and 3 in Sect. 3, along with our composite experimental values.

As noted previously, the afterheat following finite operating periods can be obtained from the data of Fig. 1 by subtraction, as indicated in Eq. (1c). The reduction in afterheat for various operating intervals and cooling times is shown in Fig. 3, as deduced from the data of Ref. 2.

3. EXPERIMENTAL AFTERHEAT FUNCTION

We have obtained composite or "best-estimate" experimental afterheat curves for β 's and γ 's (separately and in combination) as weighted-averages of experimental results enumerated in the next section. The

Fig. 1a. Proposed standard total absorbable energy released by fission products following "infinite" reactor operation at constant power; cooling time, 10^{-1} to 10^5 sec (from Shure²). (^{235}U fuel)

Fig. 1b. Total absorbable energy released by fission products following infinite reactor operation; cooling time, 10^5 to 10^9 sec (from Shure²).

Fig. 2a. Total gamma-ray energy released by fission products following infinite reactor operation; cooling time, 10^0 to 10^4 sec (from Shure²).

Fig. 2b. Total gamma-ray energy released by fission products following infinite reactor operation; cooling time, 10^4 to 10^8 sec (from Shure²).

Fig. 3. Relative afterheat versus irradiation time (approximate). (Based on data from ref. 2.)

results are given here and compared with the standard (i.e., with Shure's recommended values from Ref. 2). In Table 1 and in Fig. 4, we show the differential quantities ΔF (as defined in Sect. 2).

In Table 2 and in Fig. 5, we show the partially integrated function

$$\int_t^{t^*} f(t') dt' = F(t, \infty) - F(t^*, \infty), \quad (4)$$

where $t^* = 5 \times 10^5$ sec is the upper limit of the range over which we have seriously reviewed the experimental information. This function depends only on experimental data.

In Table 3 and in Fig. 6 we show the integral afterheat function, $F(t, \infty)$. Because of the limited time range covered by the experiments, the values of $F(t, \infty)$ listed as "experimental" cannot depend exclusively on experimental results, but include a contribution

$$F(t^*, \infty) = \int_{t^*}^{\infty} f(t') dt' \quad (5)$$

that must be supplied from some source other than the experiments. For this source, we have used Ref. 2 itself, partly because difficulties that cause the summation approach to break down at short cooling times (i.e., lack of data for short-lived fission products far from the line of stability) do not appear to cause trouble for cooling times longer than about 10^3 sec (see Sect. 7). However, in the case of the gamma afterheat, Table 1 and Fig. 4 suggest that $f(t)$ as deduced from experiments is perhaps 10% higher than corresponding values inferred from Ref. 2, for $t \sim 0.1 t^*$ to t^* . Lacking any more precise guidance, we have therefore used a value of $F_\gamma(t^*, \infty)$ that is 10% higher than Shure's (Ref. 2) in obtaining the values of $F(t, \infty)$ listed in Table 3.

Because of the non-experimental component, $F(t^*, \infty)$, in the values of

$$F(t, \infty) = \int_t^{t^*} f(t') dt' + F(t^*, \infty), \quad (6)$$

it is worth noting that the fractional contribution of this component is quite small for $t \ll t^*$. The relative magnitude of this component

Table 1. Differential afterheat

$$\Delta_j F = \int_{t_j}^{t_{j+1}} f(t') dt' \quad (\text{MeV/fission})$$

t_j	Betas			Gammas			Total		
	Experiment	Shure ^a	$\frac{\text{Experiment}}{\text{Shure}}$	Experiment	Shure ^a	$\frac{\text{Experiment}}{\text{Shure}}$	Experiment	Shure ^a	$\frac{\text{Experiment}}{\text{Shure}}$
1×10^0	0.614	0.44	1.40	0.327	0.25	1.31	0.941	0.69	1.36
2	0.819	0.47	1.74	0.539	0.49	1.20	1.408	0.96	1.47
5	0.626	0.40	1.57	0.499	0.43	1.16	1.125	0.83	1.36
1×10^1	0.581	0.50	1.16	0.524	0.50	1.05	1.105	1.00	1.11
2	0.660	0.70	0.94	0.737	0.71	1.04	1.397	1.41	0.99
5	0.428	0.39	1.10	0.575	0.57	1.01	1.004	0.96	1.05
1×10^2	0.367	0.53	0.69	0.532	0.52	1.02	0.899	1.05	0.86
2	0.416	0.53	0.79	0.593	0.63	0.94	1.009	1.16	0.87
5	0.292	0.34	0.86	0.401	0.37	1.08	0.693	0.71	0.98
1×10^3	0.284	0.29	0.98	0.396	0.30	1.32	0.680	0.59	1.15
2	0.314	0.30	1.05	0.482	0.42	1.15	0.796	0.72	1.11
5	0.164	0.22	0.75	0.300	0.25	1.20	0.464	0.47	0.99
1×10^4	0.145	0.16	0.91	0.232	0.22	1.05	0.377	0.38	0.99
2	0.178	0.189	0.94	0.207	0.191	1.08	0.385	0.38	1.01
5	0.104	0.101	1.03	0.123	0.112	1.10	0.227	0.213	1.07
1×10^5	0.0683	0.067	1.02	0.102	0.090	1.13	0.170	0.157	1.08
2	0.0549	0.056	0.98	0.105	0.094	1.12	0.160	0.150	1.07

^aReference 2.

Fig. 4. Differential afterheat: composite experimental results, relative to Shure.²

Table 2. Afterheat partially integrated to 10^5 sec

$$F(t, \infty) - F(t^*, \infty) = \int_t^{t^*} f(t') dt' \quad (\text{MeV/fission})$$

$$(t^* = 5 \times 10^5 \text{ sec})$$

t	Betas			Gammas			Total		
	Experiment	Shure ^a	<u>Experiment</u> <u>Shure</u>	Experiment	Shure ^a	<u>Experiment</u> <u>Shure</u>	Experiment	Shure ^a	<u>Experiment</u> <u>Shure</u>
1×10^0	6.115	5.683	1.076	6.725	6.147	1.094	12.84	11.83	1.085
2	5.501	5.243	1.049	6.396	5.897	1.085	11.90	11.14	1.068
5	4.682	4.773	0.981	5.809	5.407	1.074	10.49	10.18	1.030
1×10^1	4.056	4.373	0.928	5.310	4.977	1.067	9.366	9.35	1.002
2	3.475	3.873	0.897	4.786	4.477	1.069	8.261	8.35	0.989
5	2.815	3.173	0.887	4.049	3.767	1.075	6.864	6.94	0.989
1×10^2	2.387	2.783	0.858	3.473	3.197	1.086	5.860	5.98	0.980
2	2.020	2.253	0.897	2.941	2.677	1.099	4.961	4.93	1.006
5	1.604	1.723	0.931	2.348	2.047	1.147	3.952	3.77	1.048
1×10^3	1.312	1.383	0.949	1.947	1.677	1.161	3.259	3.06	1.065
2	1.028	1.093	0.941	1.551	1.377	1.126	2.579	2.47	1.044
5	0.714	0.793	0.900	1.069	0.957	1.117	1.783	1.75	1.019
1×10^4	0.550	0.573	0.960	0.769	0.707	1.088	1.319	1.28	1.030
2	0.405	0.413	0.981	0.537	0.487	1.103	0.942	0.900	1.047
5	0.227	0.224	1.013	0.330	0.296	1.115	0.557	0.520	1.071
1×10^5	0.123	0.123	1.00	0.207	0.184	1.125	0.330	0.307	1.075
2	0.055	0.056	0.98	0.105	0.094	1.117	0.160	0.150	1.067

^aReference 2.

Fig. 5. Afterheat partially integrated to 10^5 sec; composite experimental results relative to Shure.²

Table 3. Integral afterheat

$$F(t, \infty) = \int_t^{\infty} f(t') dt' \quad (\text{MeV/fission})$$

t	Betas			Gammas			Total		
	Experiment ^a	Shure	<u>Experiment</u> <u>Shure</u>	Experiment ^a	Shure	<u>Experiment</u> <u>Shure</u>	Experiment ^a	Shure	<u>Experiment</u> <u>Shure</u>
1 × 10 ⁰	6.442	6.01	1.072	7.080	6.47	1.094	13.52	12.48	1.084
2	5.828	5.57	1.046	6.753	6.22	1.086	12.58	11.79	1.067
5	5.009	5.10	0.982	6.164	5.73	1.076	11.17	10.83	1.032
1 × 10 ¹	4.383	4.70	0.933	5.665	5.30	1.069	10.05	10.00	1.005
2	3.802	4.20	0.905	5.141	4.80	1.071	8.94	9.00	0.994
5	3.142	3.50	0.898	4.404	4.09	1.077	7.55	7.59	0.994
1 × 10 ²	2.714	3.11	0.873	3.828	3.52	1.088	5.54	6.63	0.987
2	2.347	2.58	0.910	3.296	3.00	1.099	5.64	5.58	1.011
5	1.931	2.05	0.942	2.703	2.37	1.141	4.63	4.42	1.048
1 × 10 ³	1.639	1.71	0.958	2.302	2.00	1.151	3.941	3.71	1.062
2	1.355	1.42	0.954	1.906	1.70	1.121	3.261	3.12	1.045
5	1.041	1.12	0.929	1.424	1.28	1.113	2.465	2.40	1.027
1 × 10 ⁴	0.877	0.90	0.974	1.124	1.03	1.091	2.001	1.93	1.037
2	0.732	0.74	0.989	0.892	0.81	1.101	1.624	1.55	1.048
5	0.554	0.55	1.007	0.685	0.619	1.107	1.239	1.17	1.059
1 × 10 ⁵	0.450	0.450	1.00	0.562	0.507	1.108	1.012	0.957	1.057
2	0.382	0.383	0.997	0.460	0.417	1.103	0.842	0.800	1.053
5	0.327	0.327	1.0	0.355	0.323	1.1	0.682	0.650	1.049
1 × 10 ⁶	0.280	0.280	1.0	0.281	0.255	1.1	0.561	0.535	1.049

^aF(t*, ∞) = 1.0 × Shure for β's, 1.1 × Shure for γ's.

Fig. 6. Integral afterheat, $T = \infty$; composite experimental results relative to Shure.²

is shown in Fig. 7. In addition, in Fig. 8 we have plotted values of the ratio $F(t^*, \infty) \div F(t, \infty)$ for various values of t and t^* , so that the importance of the experimental data in various time ranges may be inferred.

The important features of the experimental functions, in comparison with the proposed standard, are the following.

1. The differential β function, as deduced from the experiments, appears to be significantly higher than the values inferred from Ref. 2 for cooling times from 1 to 10 or 20 sec after a fission burst. But it appears to be significantly lower than Ref. 2 for cooling times of 100 to 1000 sec. The peak in the ratio (Experiment/Ref. 2) in the vicinity of $t = 2000$ sec seems to be real, as will be discussed in Sect. 4.
2. The differential gamma function, as deduced from the experiments, appears to be systematically higher than the values inferred from Ref. 2 over most of the time range $1 < t < 5 \times 10^5$ sec.
3. The combined β and γ differential afterheat is higher than values deduced from Ref. 2 for short cooling times following a burst, lower for intermediate cooling times, and perhaps marginally higher for times in the range 5×10^4 to 5×10^5 sec.
4. The integral afterheat functions tend to be "smoother" than the differential functions; that is, the ratio (Experiment/Ref. 2) varies less widely for the integral than for the differential function. From Fig. 6, it appears that the integral gamma function, as deduced from the experiments, is higher than Ref. 2 for all cooling times (after a very long operating interval) in the range 10^0 to 10^6 sec.* On the other hand, the integral β function deduced from the experiments is lower than that inferred from Ref. 2 over most of this time range, while the combined afterheat function from the experiments is in substantial agreement with Ref. 2 over most of the range.

The curves of Fig. 6 suggest that the proposed standard (that is to say, Shure's 1961 evaluation) probably is low for the first few seconds and perhaps again in the vicinity of 1000 sec. (The peak that appeared at about 2000 sec in the differential functions for both β 's

* But note again the assumed value of $F_\gamma(t^*, \infty)$; see p. 9 and Fig. 7.

Fig. 7. $F(t^*, \infty) \div F(t, \infty)$ versus t for $t^* = 5 \times 10^5$ sec. (Combined betas and gammas.)

Fig. 8. $F(t^*, \infty) \div F(t, \infty)$ versus t^* for various values of t . (Combined betas and gammas.)

and γ 's is shifted slightly to shorter times in the integral functions; this is an expected result of the integration process.)

Anticipating the discussion of uncertainties that will be taken up in Sect. 6, we simply note here that we believe that there remains in the composite afterheat function, $F(t, \infty)$, an error (roughly in the sense of one standard deviation) of about 10% to 15%.

4. BETA ENERGY-RELEASE EXPERIMENTS

Our weighted-average composite experimental afterheat function for β 's includes results of four independent measurements of β energy-release rates, all of which have been reported since Shure's 1961 evaluation. Measurements were reported for times up to 5×10^5 sec after a fission burst, after initiating fission at a constant rate, or after shutdown following operation at a constant level for various periods of time. Typically, each experiment provides a redundant body of data for determination of $f(t)$ and of $F(t, \infty)$, the latter involving at least a small input of information from sources other than the experiment. The derivation of these functions from the experimental data is discussed in some detail in Appendix A. For the present, we simply identify the experiments, note the results, and compare them with each other and with the proposed standard.

M. E. Wyman and his associates at the University of Illinois^{21,22} measured the energy spectrum of fission-product β -rays as a function of time. McNair et al., at Aldermaston,²³ and MacMahon et al. at the Scottish Research Reactor Centre²⁴ measured the total β energy-release rate as a function of time following various periods of irradiation from 10 sec to 10^5 sec. Armbruster and Meister,¹⁸ at Munich, measured the number and energy of β particles for each fission-product chain, using a mass-spectrometer to separate the primary fission-fragments according to their atomic weights.

Results of these experiments are shown in Fig. 9 in terms of the function $t \times f(t)$ (MeV/fission), which varies much more slowly, as a function of time, than does $f(t)$ itself. This sort of plot therefore reveals small differences between experiments as well as systematic

Fig. 9. Differential beta energy release.

features of the functions more clearly than does the customary log-log plot of $f(t)$ versus t .

(If $w = \ln t$, then $t \times f(t) = f(w)$, the energy release per unit interval of $\ln t$. This quantity is convenient to use in much the same way as the neutron flux per unit lethargy so familiar to reactor physicists.)

We believe the "bumps" in the curves of Fig. 9 are real. Since the afterheat function represents the sum of many exponential terms, it isn't at all surprising that such irregularities should appear when the afterheat is plotted as a function of $\ln t$; and such irregularities should be especially noticeable when the function plotted is $t \times f(t)$ versus $\ln t$. (Of course $t^n \times f(t)$ versus $\ln t$ with $n > 1$ - e.g., $n = 1.2$ - would exaggerate the "bumps" even more; the choice $n = 1.2$ would be a reasonable one, since $f(t)$ is known to vary approximately as $t^{-1.2}$,⁴¹ at least for $10 < t < 10^6$ sec.)

In Fig. 10, the same data are plotted relative to the standard, i.e., relative to values inferred from Shure's evaluation.^{2*} While results of the four experiments disagree in absolute magnitude, it is clear that with regard to "shape" they agree pretty well among themselves, and disagree with the standard. From Fig. 10, it seems highly likely that the shape of the β afterheat function implied by the standard is not correct in detail, and will need adjustment.

The approximate agreement of the experimental results as to the shape of the afterheat function, together with significant disagreement in magnitude, is one of the important aspects of the situation turned up by our review. It is shown perhaps even more clearly in Fig. 11, in which results of the individual experiments are compared with the weighted-average of all of them. For purposes of this comparison, it doesn't matter how the weighted average was obtained (though we'll come to that in a moment) since at each point in time the average is simply a unit of measure in terms of which the individual results are expressed.

Though significant and systematic differences in shape do occur (as discussed more fully in Appendix A), it is clear from Fig. 11 that

* The quantities plotted are actually ratios of integrals over time intervals 1-2, 2-5, 5-10 in each decade, plotted at the midpoint of each interval.

Fig. 10. Differential beta energy release relative to Shure.² (For identification of experiments, see Fig. 9.)

Fig. 11. Differential beta energy release, relative to weighted average. (For identification see Fig. 9.)

Experiments B1-B3 agree reasonably well in shape, while Experiment B4 differs from the others, at least for cooling times less than 100 sec.

For this reason, and for others discussed in Appendix A, a lower weight was assigned to Experiment B4 than to Experiments B1-B3. The intrinsic errors in B4 were judged to be probably twice as great as those of B1-B3 (e.g., $\pm 20\%$ as opposed to $\pm 10\%$), resulting in a relative weight of 0.308 for each of Experiments B1-B3 and of 0.076 for B4 over the time range within which data from all four experiments are available.

In developing an average outside the range of overlap, some advantage was taken of the regularity of the curves. That is, we assumed that the dominance of "normalization error" over "shape error" would persist if the range covered by each experiment were extended. In a sense, we extrapolated curves B1 and B3 to cover as wide a range as curve B2. However, virtual data should not be given as high a weight as real data. The actual procedure used in obtaining a weighted average was as follows:

Range I (1-5 sec): Only B2 and B4 are available; divide B2 by 0.95, representing extrapolation of a systematic "bias" evident for $t > 10$ sec; double the relative weight of B4 from 0.076 to 0.15.

Range II (5-10 sec): B2, B3, and B4 are available; extrapolate B1 from adjacent Range III by maintaining the same ratio of $B1/(B2 + B3)$ as the average over Range III; then proceed as in Range III.

Range III (10-5000 sec): All four experiments are available; use weights 0.308 for B1-B3, 0.076 for B4.

Range IV ($5 \times 10^3 - 1 \times 10^5$ sec): Only B2 and B3 are available; to allow for the "bias" evident in Range III, the average of B2 and B3 is divided by 0.925; B2 and B3 are given equal weight.

Range V ($1 \times 10^5 - 5 \times 10^5$ sec): Only B2 is available; we use it without bias, since it lies close to the weighted average in Range IV.

The values of weights and bias factors used in obtaining the weighted averages are also shown in Tables 4 to 6. Table 4 shows the bias factors used, as described above, and Table 5 shows the weights used in obtaining the average. However, in estimating variances, the virtual data implied by the biasing, or extrapolation procedure, were made explicit, in order to avoid underestimating the variability of the data. Thus the weights shown in Table 5 were distributed over the real and virtual data in the

Table 4. Bias factors used in obtaining weighted-average beta afterheat function^a

Range	Experiment			
	B1	B2	B3	B4
I	--	1.053	--	1.00
II	(1.00)	1.00	1.00	1.00
III	1.00	1.00	1.00	1.00
IV	--	1.081	1.081	--
V	--	1.00	--	--

^aParentheses indicate extrapolated data.

Table 5. Weighting factors used in obtaining weighted-average beta afterheat function^a

Range	Experiment			
	B1	B2	B3	B4
I	--	0.85	--	0.15
II	(0.308)	0.308	0.308	0.076
III	0.308	0.308	0.308	0.076
IV	--	0.50	0.50	--
V	--	1.00	--	--

^aParentheses indicate extrapolated data.

Table 6. Weighting factors used in estimating variability of beta afterheat data^a

Range	Experiment			
	B1	B2	B3	B4
I	(0.283)	0.283	(0.283)	0.151
II	(0.308)	0.308	0.308	0.076
III	0.308	0.308	0.308	0.076
IV	(0.333)	0.333	0.333	--
V	--	1.00	--	--

^a Parentheses indicate extrapolated data.

extrapolated ranges, as shown in Table 6, in which extrapolated data are indicated by parentheses. In calculating deviations from the mean, both real and extrapolated data are used without biasing, and of course the same value of the weighted average may be obtained from the unbiased real and extrapolated data, as from the biased data without extrapolation.

While the use of this biasing or extrapolation procedure may seem a bit arbitrary, it should be noted that the dominance of the overall systematic or "normalization" errors over random errors is a conspicuous feature of the data, and we believe that it is a better use of the available data to acknowledge this feature by use of the biasing procedure than to ignore it. It should also be noted that the effect of the biasing procedure is to raise the weighted average approximately 5% in Range I and approximately 8% in Range IV. No biasing was used in Range III, which, as may be seen from Fig. 8, is most important for cooling times of greatest interest in LOCA analysis for light-water reactors, i.e., 10 sec to a few hundred seconds.

The weighted-average beta afterheat differentials are listed in Table 7, along with values from the individual experiments.

Table 7. Differential Beta Afterheat Function

$$\Delta F = \int_{t_j}^{t_{j+1}} f(t') dt' \text{ (MeV/fission)}$$

t_j	Experiment ^{a,c}				Weighted ^b Average
	B1	B2	B3	B4	
1 x 10 ⁰	(0.742)	0.611	(0.574	0.447	0.614
2	(0.950)	0.782	(0.735)	0.794	0.819
5	(0.716)	0.592	0.560	0.673	0.626
1 x 10 ¹	0.679	0.540	0.512	0.630	0.581
2	0.762	0.609	0.589	0.743	0.660
5	0.483	0.398	0.387	0.498	0.428
1 x 10 ²	0.415	0.340	0.327	0.449	0.367
2	0.471	0.387	0.363	0.530	0.416
5	0.324	0.280	0.255	0.360	0.292
1 x 10 ³	0.313	0.273	0.256	0.327	0.284
2	0.370	0.293	0.260	0.390	0.314
5	(0.189)	0.165	0.139		0.164
1 x 10 ⁴	(0.167)	0.150	0.118		0.145
2	(0.205)	0.182	0.148		0.178
5	(0.119)	0.106	0.086		0.104
1 x 10 ⁵		0.0683			0.0683
2		0.0549			0.0549
5					
1 x 10 ⁶					

^aNumbers in parentheses represent extrapolated data (see text).

^bWeights and bias factors as given in Tables 4 and 5, or weights as given in Table 6.

^cB1: Ref. 22; B2: Ref. 23; B3: Ref. 24; B5: Ref. 18.

5. GAMMA ENERGY-RELEASE EXPERIMENTS

The situation with respect to gamma energy-release measurements is somewhat less satisfactory than for betas. In particular, there is less overlap among the limited number of experiments which appear to us to be most likely to be reliable, i.e., those in which the gamma-ray spectrum was measured.

We have selected five experiments for inclusion in our weighted-average afterheat function for gamma rays. These are summarized here and discussed in more detail in Appendix B.

Experiment G1: Fisher and Engel, at Los Alamos Scientific Laboratory,³² measured the spectrum and intensity of gamma rays following a short fission burst for times from 0.2 sec to 45 sec. Their ^{235}U foils were exposed to neutrons from the Godiva reactor, and the results therefore reflect fission-product yields appropriate to fast-neutron-induced fission, rather than thermal fission. While differences in fission-product yields between fast and thermal fission appear to be smaller than differences between different fissile nuclides,^{18,42} Fisher and Engel's results are used here (for thermal fission of ^{235}U) with lower weight than would otherwise be justified by the quality of the experiment (see Appendix B).

Experiment G2: Zobel and his colleagues at the Oak Ridge National Laboratory³¹ measured the spectrum and intensity of gamma rays for times from 1 to 1800 sec after thermal fission of ^{235}U .

Experiment G3: Bunney and Sam, at the U. S. Naval Radiological Defense Laboratory³⁷ measured the spectrum and intensity of gamma rays for times from 10^3 to 3×10^5 sec after thermal fission of ^{235}U .

Experiment G4: Warkentin, at the Phillips Petroleum Co.,³⁴ in Idaho, measured the response of a gamma-ray dosimeter for times from 1 to 10^4 sec after thermal fission of ^{235}U .

Experiment G5: Sakharov and Malofeev, in the Soviet Union,²⁵ observed the response of a Geiger counter to gamma rays from ^{235}U fission, for times after fission ranging from 1 to 1000 hr.

The results of these five experiments, expressed in terms of $t \times f(t)$ (MeV/fission), are shown in Fig. 12.

As explained in Appendix B, the following errors are attributed to these experiments for the purposes of calculating a weighted average gamma afterheat function and for estimating the variability of the data:

G1 (Fisher and Engle):	$\pm 15-20\%$
G2 (Zobel et al.):	$\pm 15\%$
G3 (Bunney and Sam):	$\pm 15\%$
G4 (Warkentin):	$\pm 20\%$
G5 (Sakharov):	$\pm 20\%$

Experiment G1 was given the same weight as G2 (within their range of overlap) in obtaining our recommended weighted average, designated as composite No. 3 (see below); but the effect of assigning lower weight (composite No. 4) was also determined.

It should be noted here that Experiment G4 does not have a reliable absolute normalization, owing to the way in which the actual observations are converted to energy release rates per fission (see Appendix B). While the data from G4 have been used without renormalization, we have, in a sense, normalized the data to the spectral measurements (G1-G3) by observing that the integral of $f(t')$ over the range covered by G4 ($1-10^4$ sec) agrees with the corresponding integral from G1-G3 to within 1% if bias factors are used on G1-G3 (see below). If no bias factors are used, the integral of $f(t)$ from $1-10^4$ sec for G4 exceeds that from G1-G3 by 4%.

With respect to the spectral measurements (G1-G3), despite the limited overlap of their time ranges, it appears that there is a pattern similar to that noted for the beta experiments; that is, they appear more nearly to agree in shape than in absolute magnitude. The ratio of G1 to G2 varies, over the range of overlap, from 1.16 to 1.30, with an average of 1.22. Over the very limited range of overlap of G2 and G3, the ratio G3/G2 is 1.23. Thus, in using these experiments (G1-G3) to obtain a composite differential afterheat function for gammas, we have applied bias factors analogous to those used for the beta experiments. However, since the case for doing so is much less clear for gammas than for betas, we have also obtained unbiased, as well as unweighted, averages to determine the sensitivity of the results to assumptions of this sort.

Fig. 12. Differential gamma energy release.

The bias factors and weights applied to the results of each experiment in each of several time ranges are listed in Tables 8 and 9. (As was the case with the beta experiments, the estimation of variances must not be based on biased data. In estimating variances for the biased composites, certain data were extrapolated and weights distributed as discussed in Sect. 6. The resulting distributed weights are shown in Table 10.) Four composites were obtained:

1. unweighted, G1-G3, with bias factors applied;
2. unweighted, G1-G3, without bias factors;
3. weighted, G1-G5, with bias factors;
4. weighted, G1-G5, without bias factors.

We are inclined to believe that composite No. 3 represents the best use of the available data and it is the one shown in Tables 1 through 3 and Figs. 4 through 6, in Sect. 3. However, the effect of using each of these composites to calculate the integral afterheat function,

$$\int_t^{t^*} f_\gamma(t') dt' ,$$

is discussed later in this section.

Results of the individual gamma experiments are listed in Table 11, along with their weighted average (composite No. 3). The results are also shown relative to the proposed standard, in Fig. 13. It seems likely, from this figure, that the gamma energy release rate proposed by Shure is somewhat too low. Shure's evaluation, for cooling times less than 10^3 sec, was based substantially on the ORNL data,²⁹ (with an allowance for low-energy gammas which has since been slightly reduced; see Appendix B), and it now appears that the ORNL data (Experiment G2) are on the low side of the range defined by all the available data.

A pattern somewhat similar to that of the beta experiments (Figs. 7, 8, 4) is also noticeable for the gamma experiments (Figs. 12, 13, 4). There does seem to be a "bump" between 1000 and 2000 sec which is not fully revealed by the standard. The experiments suggest that the standard is low for the first 20 sec or so, in reasonable agreement with the experiments from 20 to 500 sec, and low again for times from 500 sec to the upper limit of the range covered by these experiments. (Recall that

Table 8. Bias factors used in obtaining weighted-average gamma afterheat function

Range ^a	Time interval (sec)	Experiment				
		G1	G2	G3	G4	G5
I	1-50	0.907	1.115	--	1.0	--
II	50-1000	--	1.115	--	1.0	--
III	1000-2000	--	1.115	0.907	1.0	--
IV	2000-5000	--	--	0.907	1.0	--
V	5000-10000	--	--	0.907	1.0	1.0
VI	10000-500000	--	--	0.907	--	1.0

^aThese are different from the ranges identified for the beta experiments.

Table 9. Weighting factors used in obtaining weighted-average gamma afterheat function

Range	Time interval (sec)	Experiment				
		G1	G2	G3	G4	G5
Composite #3						
I	1-50	0.39 ^a	0.39	--	0.22	--
II	50-1000	--	0.64	--	0.36	--
III	1000-2000	--	0.39	0.39	0.22	--
IV	2000-5000	--	--	0.64	0.36	--
V	5000-10000	--	--	0.47	0.265	0.265
VI	10000-500000	--	--	0.64	--	0.36
Composite #4						
I		0.265 ^b	0.47	--	0.265	--
II-VI		Same as for Composite #3				

^aConsistent with derivation of bias factors.

^bConsistent with reduced weight assignment since G1 was for fast fission.

Table 10. Weighting factors used in estimating variances for composite #3 (gamma afterheat)^a

Range	Time interval (sec)	Experiment				
		G1	G2	G3	G4	G5
I	1-50	0.39	0.39	--	0.22	--
II	50-1000	--	0.32	(0.32)	0.36	--
III	1000-2000	--	0.39	0.39	0.22	--
IV	2000-5000	--	(0.32)	0.32	0.36	
V	5000-10000	--	(0.235)	0.235	0.265	0.265
VI	10000-500000	--	(0.32)	0.32	--	0.36

^aExtrapolated unbiased data (used in place of biased "real" data).

Table 11. Gamma afterheat differentials

$$\Delta F_{\gamma} = \int_{t_j}^{t_{j+1}} f_{\gamma}(t') dt' \text{ (MeV/fission)}^a$$

t_j	G1	G2	G3	G4	G5	Weighted average ^b
1 x 10 ⁰	0.326	0.281		0.412		0.327
2	0.610	0.520		0.672		0.589
5	0.535	0.422		0.571		0.499
1 x 10 ¹	0.573	0.442		0.583		0.524
2	0.793	0.672		0.751		0.737
5		0.538	(0.661)	0.533		0.576
1 x 10 ²		0.496	(0.610)	0.495		0.532
2		0.533	(0.656)	0.591		0.593
5		0.363	(0.446)	0.395		0.401
1 x 10 ³		0.367	0.451	0.352		0.396
2		(0.465)	0.572	0.416		0.482
5		(0.289)	0.355	0.277	0.284	0.300
1 x 10 ⁴		(0.215)	0.265		0.217	0.232
2		(0.181)	0.222		0.218	0.207
5		(0.107)	0.132		0.128	0.123
1 x 10 ⁵		(0.089)	0.110		0.107	0.102
2		(0.086)	(0.106)		0.122	0.105
5					0.088	

^aParentheses indicate extrapolated data.

^bComposite #3.

Fig. 13. Differential gamma energy release, relative to Shure. (For identification of experiments, see Fig. 12.)

these are values of $f(t)$, as a function of time following a fission burst.)

For the gamma experiments, as for the beta experiments, the absolute magnitude of the energy release rate seems to present more of a problem than the shape, or relative time dependence. This may be seen from Fig. 12 and from Fig. 14, in which results of the experiments are plotted relative to their weighted average (composite No. 3). The spectral measurements (G1-G3), at least, appear to be uniformly high or low (see discussion on page 29). Experiment G4, however, differs from the spectral data with respect to shape, being appreciably higher than G1 or G2 for short times after fission, and appreciably lower than G3 from 10^3 to 10^4 sec, their range of overlap. Thus G4, even with reduced weight, has the effect of raising the average for short times after fission (i.e., 1 to 50 sec) and lowering the average for longer times (50 sec to 10^4 sec).

The effect of using the available data in different ways is shown in Fig. 15, in which four different integral gamma afterheat functions deduced from the experiments are shown relative to Shure's 1961 evaluation. For the first hundred seconds or so after shutdown, the differences are quite small (<5%), while for cooling times around 1000 sec, a variation of about 15% is noted. A comparison of curves 1 and 3, or of curves 2 and 4 (in Fig. 15) shows that including or excluding Experiments G4 and G5 has a rather small effect on the composite function. But a comparison of curves 1 and 2, or of curves 3 and 4, shows that inclusion of the bias factors described above (and listed in Table 8) appreciably alters the composite afterheat function for the longer cooling times. Lowering the data of Experiment G3 (by the factor 0.907) produces a relatively larger effect for cooling times greater than 10^3 than raising G2 (by a factor of 1.115) produces for times less than 10^3 sec. For cooling times of greatest interest in LOCA analysis for light-water reactors, the uncertainty associated with the treatment of available data is rather small.

Fig. 14. Differential gamma energy release relative to weighted average. (For identification of experiments see Fig. 12.)

Fig. 15. Integral gamma afterheat function, relative to Shure, for various weighting and biasing procedures. (See Tables 8 through 10.)

6. ERROR ESTIMATES

Standard errors in the composite experimental afterheat functions described in Sect. 3-5 have been estimated by normal statistical procedures, but it is doubtful if very meaningful quantitative statements can be made concerning the very small probability that the nominal functions are substantially in error.

An estimate of the variance of the differential functions, $f(t)$, for each time interval (1-2, 2-5, 5-10 sec, etc.), may be based on the spread of the data in each interval and on the presumed uncertainties in the individual experiments, as cited in Sects. 4 and 5.

With the assigned errors, σ_n , for each experiment, we apply the chi-squared test,

$$\left(\frac{\chi^2}{\nu}\right) = \frac{1}{N-1} \sum_{n=1}^N \left(\frac{x_n - \bar{x}}{\sigma_n} \right)^2, \quad (7)$$

where

$$\bar{x} = \frac{\sum_{n=1}^N x_n / \sigma_n^2}{\sum_{n=1}^N 1 / \sigma_n^2}, \quad (8)$$

N is the number of independent experiments and ν is the number of degrees of freedom (see p. 43).

The value of (χ^2/ν) forms the basis for a decision rule to select the larger of two estimates of the variance, i.e., one based on the assumed errors, σ_n , of the individual experiments and the other on the spread of the experimental results. If $(\chi^2/\nu) > 1$, the variance is given by

$$\begin{aligned} V(\bar{x}) &= \frac{(\chi^2/\nu)}{\sum_{n=1}^N (1/\sigma_n^2)} \\ &= \frac{1}{N-1} \frac{\sum_{n=1}^N \left(\frac{x_n - \bar{x}}{\sigma_n} \right)^2}{\sum_{n=1}^N \left(\frac{1}{\sigma_n} \right)^2}. \end{aligned} \quad (9)$$

This estimate of the variance is not changed if all σ_n are multiplied by a constant factor; that is, it depends only on the relative weights assigned to the various experiments and on the scatter among the experiments.

If $(\chi^2/\nu) \leq 1$, the variance is given by

$$V(\bar{x}) = \left[\sum_{n=1}^N (1/\sigma_n^2) \right]^{-1} . \quad (10)$$

$(\chi^2/\nu) > 1$ indicates that some or all of the error assignments, σ_n , may have been too small, while $(\chi^2/\nu) < 1$ suggests the converse. Either the assumed uncertainties in the experiments or the data spread may determine the estimated variance in the mean, and the larger estimate is chosen.

The only unconventional part of the treatment of errors was the inclusion of extrapolated, or "virtual," data, along with real data, as indicated in Tables 6 and 10. In redistributing weights to include extrapolated data (compare Tables 5 and 6, Tables 9 and 10), we alter the assumed individual errors, σ_n , for the experiments subject to extrapolation, bias, or redistributed weight (B1-B3, G1-G3) so as to preserve both the absolute weights and the assumed errors in the remaining experiments. The weights are proportional to

$$w_n = \frac{1/\sigma_n^2}{\sum_{n=1}^N 1/\sigma_n^2} . \quad (11)$$

For the differential beta function, $f_\beta(t)$, (χ^2/ν) was greater than unity for nearly all time intervals. The spread in the data (see Fig. 11) indicates that the initial error assignments may have been too small, by an average factor of about 1.12 for $t < 2000$ sec, and by a factor of about 1.7 for $t > 2000$ sec, reflecting the increasing divergence of the experimental results for the longer times.

For the differential gamma function, $f_\gamma(t)$, (χ^2/ν) was approximately equal to unity for $t < 20$ sec, and appreciably less than unity for $t > 20$ sec, which might be thought to indicate that for the longer times the initial error assignments were too large (again by a factor of about 1.7). This is misleading, however, because of the inclusion of interpolated and extrapolated data which preserve the same spread as shown by the real data in the regions of overlap, but which should have

attributed to them the larger assumed errors that are reflected in the low values of χ^2/ν .

(This comment doesn't apply to the beta experiments because — except for $t < 5$ sec — the regions in which extrapolated data were used included only experiments that were initially assumed to be of equal weight. Thus, the assumed uncertainties were not actually changed in these regions, since it turns out that neither the weighted average nor the estimated standard deviation would have been altered thereby.)

The resulting error estimates for the differential beta and gamma functions are shown in Table 12. The variations in the error estimates from one time range to another are associated with the number of experiments represented in each range, with differences in the spread of the available data, and with the presence or absence of extrapolated data, as may be understood by reference to Figs. 11 and 14, and Tables 6 and 10.

Table 12. Estimated errors in differential beta and gamma energy-release rates

(One standard deviation of the mean)^a

Betas		Gammas	
Time range	Standard deviation (%)	Time range	Standard deviation (%)
1 - 10 sec	8	1 - 50 sec	9
10 - 2×10^3	6.5	50 - 10^3	12
2×10^3 - 10^4	9	$(1-2) \times 10^3$	9
10^4 - 5×10^5	10	2×10^3 - 5×10^5	12

^a $\sigma(\bar{x}) = [V(\bar{x})]^{1/2}$, $V(\bar{x})$ = variance of the mean.

While the error estimates just discussed relate to the differential functions, $f(t)$, we are primarily concerned with the integral function, $F(t, \infty)$. As noted previously, this contains a component $F(t^*, \infty)$ for contributions beyond the upper limit, t^* , of the time range covered by the experiments discussed here.

In estimating errors for $F(t, \infty)$, we proceed in two steps. First, we estimate the error in

$$F(t, \infty) - F(t^*, \infty) = \int_t^{t^*} f(t') dt'$$

(which is based on the experiments), by assuming that the errors in the various time intervals are fully correlated. While this assumption is not strictly correct, it is consistent with the observation that uncertainties in absolute magnitude appear to be much larger than uncertainties in relative time dependence. Thus, we add positive deviations from the nominal $f(t)$ over the whole range t to t^* to obtain an estimate of the standard deviation in $F(t, \infty) - F(t^*, \infty)$, using the standard deviations of $f(t)$ listed in Table 12.

Second, we assume an uncertainty in the value of $F(t^*, \infty)$ taken from Shure's 1961 evaluation; we have assumed $\pm 15\%$. We then calculate the error in

$$F(t, \infty) = [F(t, \infty) - F(t^*, \infty)]_{\text{expt}} + F(t^*, \infty)_{\text{Shure}}$$

by combining the errors in the two terms as the square root of the sum of the squares. That is, we assert that these errors are not correlated.

The resulting error estimates are shown in Fig. 16. (The contribution of $F(t^*, \infty)$ to $F(t, \infty)$ for betas and gammas is approximately as shown in Fig. 7 for the combined afterheat.) We note that the errors in $F(t, \infty)$ for $t < 1000$ sec are 7 to 8% for betas and 10 to 11% for gammas.

In estimating the error in the total afterheat function, $F_{\beta}(t, \infty) + F_{\gamma}(t, \infty)$, we assume that the errors in these two functions are uncorrelated, and combine the errors quadratically; i.e.,

$$\sigma(F_{\beta} + F_{\gamma}) = [\sigma^2(F_{\beta}) + \sigma^2(F_{\gamma})]^{1/2}.$$

The resulting error in $F(t, \infty)$ is between 6.5 and 7% over most of the range from 1 to 10^5 sec (see Fig. 16), rising to 10.6% at 5×10^5 sec as a result of our assumption of a 15% uncertainty in $F_{\beta}(t^*, \infty)$ and $F_{\gamma}(t^*, \infty)$. The assumption that the systematic uncertainties from unascertained sources are uncorrelated is a central one and difficult to prove; otherwise the combined uncertainty would be nearer 9%.

Fig. 16. Estimated standard deviations in integral afterheat functions.

In order to calculate the probability that the true value of the afterheat may exceed our nominal value by any given amount, we make use of Student's t-distribution, applicable where variance is determined by scatter. That is, the probability is P that the true value of x exceeds $x^* = \bar{x} + t_{\nu, P} \sigma(\bar{x})$, where $\bar{x}$ is the mean value from the few observations, $\sigma(\bar{x})$ the standard deviation of the mean, ν the number of degrees of freedom associated with the variance of $\bar{x}$, and $t_{\nu, P}$ is the variable presented in tables of the t-distribution that allow interpolation of the desired value of P. The number of degrees of freedom for the sum of two quantities with unequal variances (as applied to our present case) is given by

$$\nu = \left[\frac{c^2}{\nu_\beta} + \frac{(1-c)^2}{\nu_\gamma} \right]^{-1}, \quad (12)$$

where

$$\begin{aligned} c &= V(F_\beta) / [V(F_\beta) + V(F_\gamma)] \\ &= V(F_\beta) / V(F_\beta + F_\gamma) \end{aligned}$$

and the V's are the variances of the integral afterheat functions.

In estimating ν_β and ν_γ , we note that while none of the measurements of $f_\beta(t)$ or $f_\gamma(t)$ covers the whole range of cooling times between 1 and 5×10^5 sec, we have assumed the errors in the averages f_β (and f_γ) to be fully correlated over the whole time range. That is, we have assumed that each experiment (with the exceptions noted in Sections 4 and 5) tends to be uniformly high or low throughout its time range (see Figs. 11 and 14), and have made use of this assumption in applying the bias factors discussed in the previous sections. This assumption effectively reduces the number of degrees of freedom for the integral functions to one less than the number of experiments from which f_β and f_γ have each been obtained. In addition, for f_γ , some of the experiments cover a very limited range (e.g., G1, G5) or have in effect been normalized to the other measurements (G4). We conclude that we have three degrees of freedom for F_β and two for F_γ . Applying Eq. (12), we find about three degrees of freedom for $F_\beta + F_\gamma$ over the range 10^{-10} - 10^3 sec, rising to about four at 1 sec and 10^4 sec.

We have therefore estimated the probability of large departures from our nominal afterheat curve using a standard deviation of 7% and

3 degrees of freedom. The results are shown in Figs. 17 and 18 for mean values of 1.00 and 1.05, in units of the proposed standard (see Fig. 6).

The assertion that the standard error in the afterheat function $F(t, \infty)$ is as small as $\pm 7\%$ seems a bit surprising, when the results of individual experiments are as discrepant as they are shown to be in Figs. 11 and 14. In Fig. 11, for example, it may be seen that over much of the time range covered all the measurements lie outside or barely within the error band (± 7 to 8%) deduced for $\bar{f}_\beta(t)$. The pattern for gammas is somewhat similar though less clear cut. It is our feeling, on the basis of many years of experience with nuclear physics measurements, including those of the type being evaluated, that the uncertainty in $F(t, \infty)$ would more conservatively be placed at $\pm 15\%$. It would not be proper, however, to double the error arbitrarily and then calculate the probability of large deviations using a distribution with three degrees of freedom. The intuitive enlargement of the error estimate has primarily to do with a judgment regarding the probability of relatively small (10 to 20%) deviations from our nominal function, and if probabilities are to be estimated at all for the larger error, it would seem more appropriate to base the estimate on a normal distribution (i.e., infinitely many degrees of freedom). Results of such estimates are also shown in Figs. 17 and 18. It may be seen that the probability of small and intermediate deviations is higher with the enlarged error and a normal distribution but that for larger deviations, with smaller probabilities, the result from the t-distribution becomes relatively larger. For the 1% probability level, the corresponding deviations are nearly the same.

An alternative approach for dealing with the possibility of large departures from our nominal afterheat function is to establish an upper bound, or envelope, that includes all of the data thought to be sufficiently reliable for inclusion in this evaluation. These "upper bounds," though not to be understood as absolute upper limits, represent a very conservative interpretation of the available experimental data.

Fig. 17. Probability that the true value of F exceeds F^* for the case $\bar{F} = 1.00$.

Fig. 18. Probability that the true value of F exceeds F^* for the case $\bar{F} = 1.05$.

Upper bounds for the beta and gamma differential data are shown in Figs. 19 and 20, which are the same as Figs. 11 and 14, with the addition of the upper bounds. The integral total afterheat function, $F^+(t, \infty)$, as derived from these upper bounds for $f_\beta(t)$ and $f_\gamma(t)$, is shown in Fig. 21 relative to the proposed standard. The contribution $F(t^*, \infty)$, which must be added to the integral of $f_\beta(t) + f_\gamma(t)$ from t to t^* , in order to obtain $F^+(t, \infty)$, was taken as 1.25 times Shure's value.² As before, the fractional contribution of this term to $F^+(t, \infty)$ is given approximately by Fig. 7.

Over most of the range $1-10^5$ sec after shutdown, the upper bound for the afterheat lies between 1.20 and 1.25 times the standard, and is less than 1.30 times the standard over the entire range.

7. SUMMATION CALCULATIONS

Calculations of fission-product afterheat by adding up the contributions of a very large number of individual fission products are often referred to as "summation calculations." Such calculations have been done many times in many different laboratories, and it is not our intention to review these in detail or to provide an extensive bibliography. We simply observe that there is a general recognition^{15,16,43} of the fact that such calculations tend to be unreliable, particularly for short cooling times, because of inadequate information concerning yields, half-lives, and decay energies for short-lived nuclides far from the line of nuclear stability.

K. Shure has recently published results of a careful review of data and methods used for such calculations at the Bettis Atomic Power Laboratory, and his report⁴³ provides a good indication of the present status of this approach. (Methods and data for calculating fission-product concentrations, activities, decay heat, etc., are also being reviewed currently by an AEC Task Force within the Cross Section Evaluation Working Group.)

Some of Shure's key conclusions (of the 1972 study⁴³) are indicated in Fig. 22, in which several different determinations of fission-product decay heat are compared. The horizontal line labelled curve 1

Fig. 19. "Upper bound" for differential beta energy release.

Fig. 20. "Upper bound" for differential gamma energy release.

Fig. 21. "Upper bound" for integral afterheat, relative to Shure (1961).²

Fig. 22. Relative integral afterheat, $F(t, T)$, from various sources (relative to [1]).
(See text for identification.)

represents a prediction based on Shure's 1961 evaluation² either for "infinite" operation at constant power or for England's 10000-hr "fiducial case."¹² Curve 2 represents our present best estimate, based on experiments, for "infinite" operation at constant power, relative to Shure 1961. Curve 3 represents Shure's 1972 calculation, for the same operating history, using the FSTAB code,⁴³ as shown in the last column of Table IV of Ref. 43. This calculation, without any adjustment for neglected contributions from short-lived nuclides, shows the short-fall that is characteristic of unadjusted summation calculations at short cooling times. This is perhaps best indicated by a comparison of curves 2 and 3. (Other calculations, using different numbers of nuclides, that is, chain lengths, and possibly different data libraries, would not necessarily fall short by the same amount.) The agreement among curves 1, 2, and 3 is quite good for times longer than 100 sec.

Curves 4 and 5 represent calculations using the CINDER code¹¹ and FSTAB code,⁴³ respectively, relative to a prediction based on Shure 1961, for England's 10000-hr fiducial case. In this case, (in contrast with curve 3), the FSTAB calculation was adjusted, for $t < 10^3$ sec, to force agreement with Shure 1961, but the CINDER calculation was not adjusted.

For $t > 100$ sec, data from various recent sources seem to be in quite good agreement (i.e., with differences less than about 5%). The peaks around 10^3 sec uniformly show the need for a small adjustment in the standard (based on Shure 1961). The fact that curve 2 approaches a relative value of 1.05 for times in the range 10^5 to 10^6 sec is a direct consequence of the assumption, discussed in Sect. 5, concerning the most probable value of gamma decay heat for $t > 10^5$ sec, and may be a small indication that our chosen value for $F_\gamma(t^*, \infty)$ is slightly high. However, we do not think that discrepancies of this magnitude can presently be resolved. Thus, Fig. 22 supplements Fig. 5 and further justifies the use of Ref. 2 to obtain $F(t^*, \infty)$.

8. NEUTRON CAPTURE EFFECTS

Most of the foregoing discussion has been predicated on the assumption that coupling of fission-product chains by neutron capture may be neglected. This assumption has been tested, and appears to be justified for flux levels normally encountered in power reactors.

Shure, in Ref. 43, reports the results of CINDER calculations in which neutron flux levels were varied over a range of 10^4 (from 0.001 to 10 times the flux level in England's fiducial case¹²) while maintaining the same fission-rate history. Substantial differences in decay heat were noted at long cooling times, due mainly to buildup of ^{134}Cs by neutron capture in stable ^{133}Cs ; but for short cooling times, the differences are quite negligible, as indicated in Table 13 (data taken from Ref. 43).

We have made an analogous study of neutron-capture effects, using the ORIGEN code,⁴⁴ by "switching off" the capture cross sections of all fission products while maintaining all other conditions the same, for a representative PWR with fuel irradiated at an average specific power of 15 MW(th)/t to an exposure of 32000 MWd/t. In these calculations, the decay heat with neutron capture in fission products was less than 2% higher than the decay heat without neutron capture for cooling times less than 10^3 sec; 4% higher at 10^4 sec and 7% higher at 10^5 sec. For a much shorter fuel exposure (10000 hr, 6300 MWd/t) the difference was less than 2% for all cooling times less than 10^5 sec.

We conclude, as did Shure,⁴³ that neutron coupling effects are very minor for normal fuel specific powers and for shutdown times of interest in LOCA analysis.

Table 13. Effect of neutron absorption on ^{235}U fission-product decay energy for 10000-hour fiducial case^a

t (sec)	(ϕ/ϕ_0)		
	10^{-3}	5	10
0	1.00	1.01	1.01
10^3	1.00	1.01	1.02
10^4	1.00	1.01	1.03
10^5	0.98	1.02	1.05
10^6	0.98	1.02	1.05
10^7	0.98	1.05	1.10
10^8	0.92	1.30	1.60

^aData taken from Ref. 43.

APPENDIX A

Beta-Ray Energy Release Experiments

1. Experiment B1, M. E. Wyman et al. (Refs. 21,22,45)

Summary

Wyman has been studying beta rays from fission for approximately 15 years, first at LASL and later at the University of Illinois. Reference 22 apparently represents the latest results and supersedes Ref. 21. Data for shorter decay periods (i.e., < 5 sec) given in Ref. 21 are specifically discarded in Ref. 22.

A plastic scintillator was used for measurements of electron energy. A surface barrier detector was used to determine the fission rate. Spectra were obtained for a reactor burst (15 msec width) and for various times during and after an 8-hour irradiation. Typical are the pulse data, shown in Table A-1.

Integrations of the spectra over energy are given for five decay periods for the pulse data. Integration of these data over time gave a total energy yield per fission of 6.9 ± 0.5 MeV as reported in Ref. 22. A different fit to these same data, given in Ref. 45, increases the total energy yield to $\sim 7.5 \pm 0.5$ MeV per fission according to Wyman, but does not change the results for the five decay periods.

Error estimates are given in detail; $\pm 10\%$ is taken as an upper limit for the uncertainties for the tabulated energy yields.

Detailed Discussion

As noted above, the experimental results were obtained as energy release rates: (1) after reactor startup (during operation), (2) after shutdown (following operation for either 3 hours or 8 hours), and (3) following a short reactor pulse (full width at half-maximum amplitude = 15 msec). It was therefore necessary to reduce these data to a common form, for example the function $f(t)$, the energy-release rate following a burst. Since the observed startup function may be written

Table A-1. Data from pulsed-reactor operation (Expt. B1)

t (sec)	$\bar{t}$ (sec)	E_{β}^a (MeV·f ⁻¹ ·s ⁻¹)	$\bar{t}E_{\beta}$ (MeV·f ⁻¹)	$\pm\Delta(\bar{t}E_{\beta})^b$ (MeV·f ⁻¹)	A_{β}^c (β·f ⁻¹ ·s ⁻¹)	$\bar{t}A_{\beta}$ (β·f ⁻¹)	$\pm\Delta(\bar{t}A_{\beta})^d$ (β·f ⁻¹)
10-16	13	0.777×10^{-1}	1.01	0.10	0.421×10^{-1}	0.55	0.055
60-72	66	0.106×10^{-1}	0.70	0.070	0.815×10^{-2}	0.54	0.054
180-228	204	0.252×10^{-2}	0.51	0.051	0.228×10^{-2}	0.47	0.049
900-1020	960	0.466×10^{-3}	0.45	0.045	0.511×10^{-3}	0.49	0.049
3600-3900	3750	0.980×10^{-4}	0.37	0.037	0.135×10^{-3}	0.51	0.051

^a E_{β} = beta energy release rate (pulsed reactor data).

^b ΔE_{β} = standard error in E_{β} , taken to be ±10%.

^c A_{β} = beta particle emission rate (pulsed reactor data).

^d ΔA_{β} = standard error in A_{β} , taken to be ±10%.

$$S(t) = \int_0^t f(t') dt' , \quad (A.1)$$

and the after-shutdown function may be written

$$F(t, T) = \int_t^{t+T} f(t') dt' , \quad (A.2)$$

the function $f(t)$ is derived from $S(t)$ and $F(t, T)$ by differentiation. This is not a completely satisfactory procedure, for several reasons:

(a) There are only a few observations for each class of measurement (8 points for $S(t)$, 7 points for $F(t, T)$, 5 points for the pulse data).

(b) The measurements are integrals of the respective functions over intervals of time, and for the shorter cooling times the intervals (e.g., 0-12 sec for $S(t)$, $F(t, T)$) are relatively too wide to permit a meaningful differentiation.

(c) For times t greater than 500-1000 sec, the slope of $S(t)$ is relatively too small to provide good values of $f(t)$.

(d) For times t greater than 1000 sec, the values of $F(t, T)$ are a composite of measurements following an 8-hour irradiation period ($E_\beta < 1$ MeV) and measurements following a 3-hour irradiation period ($E_\beta > 1$ MeV), with some adjustment in normalization being required to match the energy spectra at 1 MeV.

For all these reasons a selection was made from the data in arriving at an average value of $f(t)$ from the three different sets of data; that is, we used

- the startup data for $15 \leq t \leq 240$ sec;
- the shutdown data for $15 \leq t \leq 1020$ sec;
- the pulse data for $10 \leq t \leq 3900$ sec.

The authors themselves have derived an expression for the beta particle emission rate which they believe best fits all of their data*:

$$n(E, t) = e^{A(t + \tau)^B} , \quad (A.3)$$

*Reference 45 and private communication from M. E. Wyman, January 1973.

where

$$A = - (0.833 + 0.055E + 0.075E^2),$$

$$B = - (1.0 + 0.109E),$$

$$\tau = 2.5 \text{ sec.}$$

From this, we may obtain

$$f(t) = \int_0^{10 \text{ MeV}} n(E,t) E dE. \quad (\text{A.4})$$

In Fig. A-1, we show the data for $S(t)$ and $F(t,T)$, as given in Ref. 22, and in Fig. A-2 we show the agreement between the $F(t,T)$ data and a curve calculated (by us) from Eqs. (A.3, A.4, and A.2).

In Fig. A-3 are shown values of $t \times f(t)$ deduced from the three sets of data in experiment B1, along with curves representing our own empirical fit to the data (which we used in obtaining the results of this report), the authors' fit, as reproduced by us through Eqs. (A.3) and (A.4), a fit to the pulse data alone making use in part of the shapes of results from experiments B2 and B3, and finally, for comparison, our composite fits to the data of experiments B2 and B3 (McNair et al., and MacMahon et al., respectively; see following sections).

It is evident from Fig. A-3 that the differences in the differential β afterheat function, $f(t)$, for the three sets of data from experiment B1 (as reduced by us to a common basis) are smaller than the differences between all of them as an ensemble and the results of experiments B2 and B3. An error analysis on the three (B1) data sets indicates that the standard deviation of each set with respect to the mean of all three sets is less than 6%. Taken together with the authors' estimate of $\pm 8\%$ for errors associated with factors that should be common to all three data sets, this suggests an overall uncertainty of $\pm 10\%$. Most of this is correlated error, applying in the same way to all the data of the experiment.

Fig. A-1. Startup and shutdown afterheat functions, $S(t)$ and $F(t,T)$, from experiment B1 (ref. 22).

Fig. A-2. Comparison between shutdown data, $F(t,T)$ and analytical fit ($T = 2.88 \times 10^4$ sec), (refs. 22 and 45).

Fig. A-3. Differential beta afterheat function, experiment B1 (ref. 22).

2. Experiment B2, McNair et al. (Ref. 23)

Summary

Exposures of ^{235}U to thermal neutrons were made for periods ranging from 10 to 10^5 sec, with similar decay periods. A plastic scintillator and photomultiplier were used as a current source for ^{235}U exposures in a reactor thermal column, and as a counter with weighting proportional to pulse height for ^{235}U exposures in a weak flux due to a neutron source in water.

A detailed discussion of sources of error is given. The effect of nonlinear scintillator response versus electron energy is not adequately described and the energy spectral variations shown by Tsoulfanidis et al.²³ are large. The error assignment for other effects such as the energy release by a standard beta-emitter may be optimistic. The lower energy limit of the detector is not estimated. The absolute calibration error is estimated as $\sim 4-5\%$, and the error in the relative magnitudes of the decay curves $4-12\%$ ($4-5\%$ for all but the shortest cooling times).

Results of this experiment appear to be consistent with a total beta energy release rate of 6.9 ± 0.5 MeV/fission; (see MacMahon²⁴).

Detailed Discussion

The full matrix of data for $F(t, T)$, for 5 values of T from 10 to 10^5 sec and up to 18 values of t from 1 to $500,000$ sec (each data point representing the average of several determinations) presents a complex, redundant data set from which we must unfold either or both of the functions $f(t)$ and $F(t, \infty)$, to permit direct comparison with data from other sources. The data for short cooling times following long irradiations approach the integral function $F(t, \infty)$, but as noted in Sect. 1, a correction is still required, even for small t , to refer the results to an "infinite" irradiation period. The differential function may be deduced from $F(t, T)$, for $t \gg T$ ($t/T \geq 2$), by the expression

$$f(t) = \frac{1}{T} F\left(t - \frac{T}{2}, T\right). \quad (\text{A.5})$$

For shorter cooling times, the expression

$$f(t) = - \frac{\partial}{\partial t} F(t, T) + f(t + T) \quad (\text{A.6})$$

is preferable. The second term on the right must be obtained from other data [e.g., from the data for longer cooling times, via Eq. (A.5)], and will represent a small fractional contribution if $(t/T \ll 1$ (e.g., $\lesssim 5\%$ for $t/T < 0.1$, $\lesssim 15\%$ for $t/T < 0.2$, $\lesssim 30\%$ for $t/T < 0.5$). Thus a combination of the two methods is used to deduce $f(t)$, with Eq. (A.5) dominating for $t/T \geq 2$, Eq. (A.6) dominating for $t/T \leq 0.1$ and a combination of the two for $t/T \sim 1$. Given the range of irradiation periods, $10 \leq T \leq 10^5$ sec, the entire range of cooling times $1 \leq t \leq 5 \times 10^5$ sec is adequately covered, though the reliability of the data diminishes for $t \gtrsim 10^5$ sec.

The differentiation was performed graphically by using a drafting machine to measure the slopes of the functions $F(t, T)$, plotted on log-log paper. The plotting was done twice and the slopes measured three times on each plot. The reproducibility of this procedure was very good, and the error in $f(t)$ introduced in this way is far less than that attributed to other causes. An alternate approach using a least squares fitting procedure on a digital computer was in excellent agreement with the approach described here over most of the range of cooling times, but introduced some difficulties at both ends of the range because of an extrapolation procedure incorporated in the code.

Figure A-1 shows the functions $F(t, T)$ for $1 \leq t \leq 20$ sec, taken directly from the tables given in Ref. 23 (B2) as well as the corresponding data from Ref. 24 (B3). It may be seen that very little smoothing of the curves is necessary, to allow for random errors in the tabulated data. Furthermore, the two experiments agree very well with each other, except for a systematic difference in normalization. The results from B2 are systematically 7% higher than those from B3 for $T = 10^3$ and 10^4 sec, and 4% higher for $T = 10^2$ sec. (For the smallest values of t , there is an odd deviation of the data from B3, and we have neglected the data from this experiment for $t < 5$ sec.) For the McNair

Fig. A-4. Beta afterheat function, $F(t, T)$ for short cooling times, experiments B2 and B3 (refs. 23 and 24).

data (B2), the standard deviation in the mean of four determinations of $f(t)$ (using Eq. A.6) for $T = 10^2, 10^3, 10^4$ and 10^5 sec and for $1 \leq t \leq 20$ sec was less than 3% over the entire range, indicating a high degree of internal consistency throughout the data matrix, as well as a high degree of reproducibility in the graphical differentiation.

In Fig. A-5, we show the function $t^{1.2} \times f(t)$ with data from each of the different irradiation periods as derived by use of Eq. (A.5) or Eq. (A.6). Recall that Eq. (A.5) (open points) should be more reliable for the longer cooling times and Eq. (A.6) (solid points) for the shorter times. The deviations of $f(t)$ from a strict $t^{-1.2}$ time-dependence are undoubtedly real and — as discussed in Sect. 4 — are not unexpected.

The final smoothed curve for $t \times f(t)$ for experiment B1 is shown in Fig. A-3 and in Fig. 9 (Sect. 4).

The integrals

$$\int_{t_j}^{t_{j+1}} f(t) dt = \Delta F_j \quad (t_j = 1, 2, 5 \times 10^n)$$

were obtained by reading values at closely spaced points from the final smoothed curve of $t \times f(t)$, noting that

$$\int f(t) dt = \int t \times f(t) d(\ln t) .$$

These are the integrals recorded in Table 7.

Our primary concern from the outset, of course, was to deduce the integral afterheat function $F(t, \infty)$; we derived the differential function, $f(t)$, in order to facilitate comparisons with data obtained from short irradiations and from fission bursts. We therefore obtain $F(t, \infty)$ from the expression

$$F(t, \infty) = F(t, T) + F(t + T, \infty), \quad (\text{A.7})$$

as well as by adding the differentials listed in Table 7. The second term on the right in Eq. (A.7) must be supplied at least in part from data independent of the experiment being evaluated. Values of $F(t, \infty)$

 Fig. A-5. Beta afterheat function, $f(t)$, experiment B2 (ref. 23).

obtained in these two ways - by integration of $f(t)$ or by use of Eq. (A.7) - always agreed within 2% and usually within 1%. In either case, the result is "contaminated" by data from outside sources approximately to the extent shown in Fig. 7. (Note that Fig. 7 is for combined beta and gamma energy release rates; the separate components don't behave in exactly the same way, but Fig. 7 is a rough indication for either betas or gammas.)

It should also be noted that in obtaining $F_{\beta}(t, \infty)$ as a composite of all the experiments we first obtained a weighted average $f(t)$ and then integrated, rather than attempting to deduce $F_{\beta}(t, \infty)$ from each experiment and then averaging. This procedure was followed because the various experiments cover different ranges of cooling times, none of them by itself containing enough information to define $F(t, \infty)$ completely.

3. Experiment B3, MacMahon et al. (Ref. 24)

Summary

Exposures of ^{235}U to thermal neutrons were made for periods from 10 sec to 10^4 sec, and the beta energy release rates were observed for decay times from 5 sec to 10^5 sec. In this experiment, the necessary separation of fission product beta particles and gamma rays was accomplished by means of a magnetic field. A plastic scintillator and photomultiplier were used as a current source, calibrated by a standard $^{90}\text{Sr}/^{90}\text{Y}$ source. Linearity of the detector was checked with a series of sources having beta end-point energies ranging from 1.7 to 5.2 MeV. The number of fissions in the uranium foils was determined by comparison with a thin ^{235}U fission chamber, or by radiochemical determination of ^{99}Mo or by both methods.

Overall uncertainties in the final results of the experiment were estimated by the authors to be about $\pm 5\%$ for cooling times between 10 and 10^4 sec, about 6.5% for $t < 5$ sec and about 9% for $t > 10^4$ sec.

The authors estimate that the total beta-energy release per fission is 6.3 ± 0.5 MeV.

Detailed Discussion

The data from this experiment are presented in the same form as those from B2, and were analyzed in the same way. Results for $t < 20$ sec were shown in Fig. A-4.

Values of $f(t)$ derived from data for the different irradiation periods are shown in Fig. A-6 (plotted as $t^{1.2} \times f(t)$ to emphasize the structure in the functional dependence on decay time). The internal agreement among the various determinations of $f(t)$ from this experiment is apparent, as is the significant discrepancy between the results of this experiment and those of B2, especially for the longer decay times.

An exception to the internal consistency of the data is found for $T = 10^4$ sec, $2.5 \times 10^4 \leq t \leq 1.05 \times 10^5$ sec (not plotted in Fig. A-6). Values of $f(t)$ deduced from these data, using Eq. (A.5), are in significant disagreement with the average of data for $t = 10^2$ and 10^3 sec, being lower by about 11% at $t \sim 2.5 \times 10^4$ sec, by about 15% at $t \sim 5 \times 10^4$ sec, and by about 25% at $t \sim 1 \times 10^5$ sec. These particular data (for $T = 10^4$ sec) were ignored in determining an average $f(t)$ from this experiment. Their inclusion would have lowered the weighted average $f_{\beta}(t)$ from all the experiments by about 2.5% at $t = 5 \times 10^4$ and by about 4% at $t = 10^5$ sec. The effect on $F(t, \infty)$ for smaller values of t (e.g., $t < 10^3$ sec) would be entirely negligible.

4. Experiment B4, Armbruster and Meister (Ref. 18)

A plastic scintillator was used to measure the energy spectra due to beta rays from fission products separated by mass (with low mass resolution) in a magnetic field. What was measured was $\int_0^t A_{\beta}(A, t') dt'$ and $\int_0^t E_{\beta}(A, t') dt'$ for $0.1 < t < 3 \times 10^3$ sec, where A_{β} is the number of beta rays per second per fission at time t after fission and A is the mass number. In order to obtain $A_{\beta}(A, t)$ and $E_{\beta}(A, t)$ ($\equiv f(t)$ in our notation), the above data were differentiated (graphically) by the authors, thus introducing a potentially large uncertainty. A summation over A is required to obtain $A_{\beta}(t)$ and $E_{\beta}(t)$ and this required some allowance for unmeasured regions of the fission-product yield curve. (Because of the characteristics of the mass separator, measurements

Fig. A-6. Beta afterheat function, $f(t)$, experiment B3 (ref. 24).

were made only in the regions $88 \leq A \leq 103$ and $130 \leq A \leq 145$, with a resolution of ± 5 mass units). Determining $A_\beta(t)$ and $E_\beta(t)$ was not the primary purpose of the measurement as is indicated by the title of the published paper. Clearly the integral quantities would be determined more accurately. If results are taken from the published mini curves, however, this potential advantage may largely disappear.

A "transmission" coincidence counter was used before the scintillator to discriminate against gamma rays. To determine the background due to gamma rays in coincidence with the beta rays, which would not be thus removed, an absorber was introduced and measurements made with and without the absorber. The lower energy limit for the scintillator was $E_\beta = 0.14$ MeV. An estimate of the contribution ($\sim 1\%$) from lower energies was included.

The rate of arrival of separated fission products was counted absolutely with a CsI(Tl) scintillator. Integration gives the corresponding number of fissions and a coincidence measurement with the transmission counter yields the number of fission products traveling in that direction.

Uncertainties in the integrals over time (the measured quantities) are given as $\pm \sim 5\%$. No uncertainties are given for $A_\beta(t)$ and $E_\beta(t)$. Reading the log-log plots with one decade = 10 mm might introduce errors as large as 10%.*

As is well known, integration over time of $A_\beta(t)$ and $E_\beta(t)$ from this paper gives results which are higher than generally accepted. Story, according to Unik and Gindler,⁴⁶ suggests that $\int_0^{3000} E_\beta(t) dt$ be reduced by 13%. Applying this factor to $E_\beta(t)$ and $A_\beta(t)$ may give a better estimate of the "best" value of Armbruster than making no correction to the published data. Overall, without further input, it is hard to see how the error could be considered to be less than $\sim 20\%$. Furthermore, not reducing Armbruster and Meister's values by 13%, as suggested by Story,⁴⁶ would have raised the weighted-average $f_\beta(t)$ by an additional 2% or less, and $F(t, \infty)$ by an additional 0.6% at most (much less over most of the range of cooling times covered by the experiment).

* It should be noted that a second reading of values for $f(t)$ from Fig. 9 of Ref. 18, after completion of all the work described in this report gave results averaging 8% higher than our previous reading. Use of the newer readings would have raised the weighted-average $f_\beta(t)$ by less than 1% and the overall afterheat function, $F(t, \infty)$, by less than 0.3%.

5. Other Experiments

Not included in the evaluation of $f_{\beta}(t)$ were the following experiments:

a. Von Alzmann (Ref. 17)

This was a measurement similar to that of Armbruster and Meister,¹⁸ except that only the beta counting rate was determined. Values were obtained from a log-log plot with one decade ≈ 20 mm.

b. Specht and Seyfarth (Refs. 19, 20)

The number of beta rays, not the energy spectrum, was measured from 10^{-2} sec to 10 hr after irradiation in a filtered thermal neutron beam. A 2-wire 4π gas-flow counter measured both the beta activity (with claimed discrimination against gamma rays) and, with different voltage and absorbers, the fission rate.

Quoted errors of 4% in the absolute calibration and 2-5% in decay rate combine to give a claimed overall error of $\sim 6\%$. No tabulated results are available. Values were taken from a log-log plot with one decade ≈ 14 mm.

Since these experiments measured only the number of beta rays emitted as a function of time after fission, not their energy, they can't be used directly to determine $f_{\beta}(t)$. It seems best, therefore, just to compare the measured activities. In Fig. A-7, we compare beta particle emission rates from Refs. 17, 19, 20 with those from Refs. 18 and 22, which were used in obtaining $f_{\beta}(t)$. Perhaps all we can conclude from these data is that they present a spread of activity measurements comparable to the spread in the energy measurements that were used in obtaining $f_{\beta}(t)$. To the extent that two of the experiments^{18,22} provided both activity and energy data, there appear to be no major inconsistencies between the two types of data. That is, the ensemble of activity data shown in Fig. A-7 does not suggest that any change in $f_{\beta}(t)$ might be appropriate.

(Values of $\bar{E}_{\beta}(t) \equiv f_{\beta}(t) \div A_{\beta}(t)$, where available,^{18,22} are not in sufficiently good agreement to warrant using them to infer $f_{\beta}(t)$ from experiments in which only $A_{\beta}(t)$ was measured (see Fig. A-8).

Fig. A-7. Beta particle emission rate following fission.

Fig. A-8. Average energy of beta rays as a function of time after fission.

6. Summary: Beta-Ray Energy Release from Fission Products

It is perhaps dangerous to lower the Armbruster results¹⁸ by $\sim 15\%$ without really knowing why they might be too high. Treating these modified data with reduced weight (like 20% error), the four inputs for $f_{\beta}(t)$ are nearly consistent within the errors considered (20% for Armbruster¹⁸ and 10% for the others^{22,23,24} (see Fig. 11). Increasing the error to $\pm 15\%$ for the combined result for $f_{\beta}(t)$ seems adequate to attain consistency.

From the beta-ray counting rate data, Fig. A-7, the Specht and Alzmann results tend to confirm the Tsoulfanidis points at the 20% level of uncertainty.

APPENDIX B

Gamma-Ray Energy Released Experiments

1. Experiment G1, Fisher and Engle (Ref. 32)

Gamma-ray spectra were measured at five time intervals after fission. The spectra do not change greatly. Results integrated over energy, as taken directly from the reference, are given in Table B-1. The time, t , is the decay time to the measurement after a short irradiation at Godiva II (burst width <0.01 sec with a tail <0.1 sec).

The error in E_γ ,* the gamma-ray energy release at decay time, $\bar{t}$, is estimated by the authors to be $\pm 12\%$. This seems like a reasonable and meaningful estimate. It is backed by a table of significant energy-dependent contributions to the overall error.

An additional and independent uncertainty applies if these data are used for thermal-neutron fission of ^{235}U . Bunney and Sam show a difference of $+13\%$ for photon emission rates between fission products from fast and thermal-neutron induced fission of ^{235}U at 15 sec after fission. For longer times, differences $+33$ to -15% were observed. The average magnitude of the difference between fast and thermal-neutron induced fission observed by Bunney and Sam is $+18.5\%$. Making the crude assumption that this difference may be considered as an additional uncertainty for the time range of Fisher and Engle data gives an overall error of $\pm 22\%$.

2. Experiment G2, Zobel et al. (Refs. 29-31)

Results included in progress reports prior to Ref. 29 should be ignored. They were described as tentative and subject to change, and they were changed.

No table of results is given in the published papers. The analysis of the experiment was never finished (due to the demise of the ANP Project). Reference 29 contains results in graphical form as ORNL-LR-DWG-20558A. References 30 and 31 contain almost but not quite the same graph. No differences in values of E_γ are evident among the various

* $E_\gamma = f_\gamma(t)$, in the notation of the present report.

Table B-1. Photon and energy emission rates³²

t (sec)	$\bar{t}$ (sec)	E_{γ} (MeV·f ⁻¹ ·s ⁻¹)	$\bar{t}E_{\gamma}$ (MeV·f ⁻¹)	$\pm\Delta(\bar{t}E_{\gamma})$ (MeV·f ⁻¹)	A_{γ} (γ·f ⁻¹ ·s ⁻¹)	$\bar{t}A_{\gamma}$ (γ·f ⁻¹)	$\Delta(\bar{t}A_{\gamma})$ (γ·f ⁻¹)
0.2-0.5	0.35	0.564	0.197	0.023	0.613	0.214	0.026
1.0-2.0	1.5	0.311	0.47	0.056	0.324	0.485	0.058
4.0-5.5	4.75	0.153	0.73	0.087	0.163	0.802	0.096
10-13	11.5	0.0706	0.81	0.098	0.0775	0.89	0.11
35-45	40.0	0.0221	0.88	0.106	0.0225	0.90	0.11

graphs. The tabulated results shown in Table B-2 were taken from ORNL-LR-DWG-20558A with an error in reading from the graph judged to be no more than $\pm 3\%$.

The lower energy limit was 0.28 MeV. An estimate for the energy release below 0.28 MeV can be based on the longest decay spectrum of Fisher and Engle and the shortest decay spectrum of Bunney and Sam. Rough evaluations give 4.4% ($9.8 \times 10^{-4} \div 2.21 \times 10^{-2} = 4.4\%$ at 40 sec) and 4.0% (Σ bins 1-17 + $0.75 \times \text{bin 18} = 2.7 \times 10^6 \div 7.0 \times 10^{-4} = 4.0\%$ at 905 sec) respectively for the additional energy below 0.28 MeV. Therefore, a correction factor of 1.04 seems appropriate for this effect for all time intervals. This factor of 1.04 has been applied in the tabulated results for energy release shown below. (Table B-2).

A defensible error analysis is not available for the Oak Ridge data. Based purely upon distant memory and experience with similar measurements, Zobel estimates the errors overall to be $\pm 20\%$. Maienschein and Peele estimate $\pm 15\%$. The tabulated results shown here include an error of $\pm 16\%$.

For the number of photons/fission, the extrapolated contribution below 0.28 MeV is logically much larger ($\sim 34\%$ from the Fisher and Engle data and 22% from Bunney and Sam). The variable correction factor, k, assumed is indicated in the table and the uncertainties are increased to $\pm 21\%$, including $\pm 15\%$ for Δk .

3. Experiment G3, Bunney and Sam (Ref. 37)

Gamma-ray energy spectra are given for nine times after fission. Integration over the 100 energy bins by us gave the results listed below (t is the time from the center of the irradiation time to the count interval).

Unfortunately, no error estimates are given by the authors. They report good reproducibility (6-12%) which is plausible, and the error in determining the fission rate, from ^{99}Mo analysis, was estimated to be $\pm 10\%$. On August 25, 1972, Sam, at NRDL, in a telephone conversation with F. C. Maienschein, estimated the overall error to be $\pm 15\%$. Considering the problems of determining the response function of the gamma-ray spectrometer used [5 in. NaI(Tl)], this seems to be a reasonable error estimate.

Table B-2. Photon and energy emission rates²⁹

$\bar{t}$ (sec)	E_γ (MeV·f ⁻¹ ·s ⁻¹)	$1.04 \bar{t} E_\gamma$ (MeV·f ⁻¹)	$\pm(1.04 \Delta \bar{t} E_\gamma)$ (MeV·f ⁻¹)	A_γ (γ·f ⁻¹ ·s ⁻¹)	$k \bar{t} A_\gamma$ (γ·f ⁻¹)	$\Delta \bar{t} A_\gamma$ (γ·f ⁻¹)
1.2	0.29	0.36	0.057			
2.0	0.22	0.46	0.073	0.24	0.64 (k=1.34)	0.14
3	0.18	0.56	0.081			
4	0.15	0.62	0.099			
5	0.12	0.62	0.099	0.117	0.78 (k=1.34)	0.16
6	9.5×10^{-2}	0.59	0.094			
8	7.3	0.61	0.096			
1.0×10^1	6.1	0.63	0.101	0.071	0.95	0.20
2.0	3.2	0.66	0.106	0.0336	0.90 (k=1.34)	0.19
3.0	2.35	0.73	0.116			
4.0	1.8	0.75	0.119			
5.0	1.5	0.78	0.124	1.43×10^{-2}	0.96 (k=1.34)	0.20
6.0	1.25	0.78	0.124			
8.0	8.8×10^{-3}	0.73	0.116			
1.0×10^2	7.2	0.75	0.119	7.2×10^{-3}	0.96 (k=1.34)	0.20
1.5	4.6	0.72	0.114			
2.0	3.1	0.65	0.102	3.14×10^{-3}	0.83 (k=1.32)	0.17
3.0	1.92	0.60	0.095			
4.0×10^2	1.40	0.58	0.093			
5.0	1.01	0.53	0.084	1.16×10^{-3}	0.74 (k=1.28)	0.15
6.0	8.4×10^{-4}	0.53	0.084			
8.0	6.4	0.53	0.085			
1.0×10^3	5.0	0.52	0.084	6.2×10^{-4}	0.75 (k=1.22)	0.16
1.5	3.2	0.50	0.079			
1.8	2.8	0.52	0.084			

Table B-3. Photon and energy emission rates³⁷

t (sec)	$\bar{t}$ (sec)	E_{γ} (MeV·f ⁻¹ ·s ⁻¹)	$\bar{t}E_{\gamma}$ (MeV·f ⁻¹)	$\pm\Delta(\bar{t}E_{\gamma})$ (MeV·f ⁻¹)	A_{γ} (γ·f ⁻¹ ·s ⁻¹)	$\bar{t}A_{\gamma}$ (γ·f ⁻¹)	$\pm\Delta(\bar{t}A_{\gamma})$ (γ·f ⁻¹)
845 - 965	905	7.0×10^{-4}	0.63	0.095	8.0×10^{-4}	0.72	0.11
1,685 - 1,925	1,805	3.6×10^{-4}	0.65	0.097	4.0×10^{-4}	0.77	0.12
3,422 - 3,782	3,602	1.71×10^{-4}	0.62	0.092	1.99×10^{-4}	0.72	0.11
6,844 - 7,564	7,204	7.1×10^{-5}	0.51	0.77	7.9×10^{-5}	0.57	0.085
17,110 - 18,910	18,010	1.8×10^{-5}	0.32	0.049	1.93×10^{-5}	0.35	0.052
35,120 - 36,920	36,020	6.2×10^{-6}	0.22	0.033	7.9×10^{-6}	0.28	0.043
85,240 - 87,640	86,440	2.12×10^{-6}	0.183	0.027	3.6×10^{-6}	0.31	0.047
170,140 - 175,540	172,840	8.5×10^{-7}	0.147	0.022	1.62×10^{-6}	0.28	0.043
255,640 - 262,840	259,240	5.0×10^{-7}	0.125	0.019	9.4×10^{-7}	0.24	0.037

The relative error among points at various times could (and would) be less than 15% since uncertain factors are common to many or all points, but this is of no significance in assessing the absolute error in the value for any decay time. The lower energy limit was 0.07 MeV.

4. Experiment G4, Warkentin (Ref. 34)

Gamma-ray dosimeters were used to measure the time response after the excursion of a compact reactor (SNAPTRAN). The accuracy of the data in a relative sense would be limited by the dynamic range of the dosimeters and by their variations of response with gamma-ray energy.

An attempt to provide an absolute normalization for the data was based on calculated gamma-ray leakage from the reactor core, assuming a uniformly distributed source, and seems too complex to inspire confidence. Nonetheless, though the curve for $f_{\gamma}(t)$ from this experiment is different in shape from the others, the integral over the time range covered (1 to 10^4 sec) is in good agreement with that obtained from a composite of the spectral measurements (G1-G3). We use the data with a relatively low weight, commensurate with an assumed uncertainty of $\pm 20\%$. No error estimates or tabulated results are given by the author. Values for the graph included here (Fig. 12) were read from a log-log plot with one decade = 39 mm.

5. Experiment G5, Sakharov and Malofeev (Ref. 25)

These measurements extend from 1 to 1000 hr. A Geiger counter of claimed constant efficiency versus gamma-ray energy was used to compare fission-product activity for ^{235}U to the activity of a ^{24}Na source irradiated in the same neutron flux as the ^{235}U . The paper on the Geiger counter⁴⁷ indicated variations of $\pm 15\%$ in efficiency. The efficiency was determined using sources calibrated using an ionization chamber. Apparently the energy dependence of the ion chamber was ignored and it was calibrated with a ^{60}Co source. In any event, no check of efficiency of the counter was made above 2.75 MeV. The angular response of the counter was nonisotropic, varying from 1.0 to 0.6, depending upon energy. Overall, the claimed accuracy of $\pm 20\%$ seems optimistic.

No tabulated results are included. Values included here (Fig. 12) were read from a log-log plot with one decade = 18 mm.

6. Other Experiments, not Included in Evaluation of $f_{\gamma}(t)$

a. Zigman and Mackin (Refs. 27, 38, 48)

Reference 48 reports results of calculating gamma-ray spectra from ^{235}U fission products and corresponding gamma-ray dose levels. The calculations were based on data from Refs. 29 and 27, which report counting measurements made with a NaI(Tl) well-type scintillation spectrometer. However, in Ref. 27, only decay rates were determined, and no attempt was made to determine gamma energy-release rates. No information is provided on the variation of detection efficiency with gamma energy.

In Ref. 27, an attempt is made to extrapolate the Oak Ridge data²⁹ to lower energies. This extrapolation can now be made much better on the basis of the Fisher and Engle³² and Bunney and Sam³⁷ spectra. Thus the Zigman and Mackin data add nothing to the spectral data.

b. Petrov (Ref. 26)

The data obtained with an ionization chamber or Geiger counter cover decay times from 0.6 sec to 11 hr. The attempts to provide an absolute calibration for these data are scarcely to be trusted. They are:

(1) integration of the spatial distribution of dose in water measured with an air-equivalent ionization chamber; (2) assuming arbitrarily that the average gamma-ray energy at all times was 1.5 MeV; and (3) comparing ^{235}U activity with that of ^{24}Na using a thin-aluminum-wall Geiger counter in the manner of Sakharov.

Furthermore, the failure to detect any difference between the energy release in ^{235}U and ^{239}Pu in the face of the clear differences seen by Fisher and Engle (for fission by fast neutrons) casts further doubt. To be fair, the differences seen by Fisher and Engle are only ~10% which is the error claimed by Petrov. His absolute error is likely far greater.

The gamma-ray spectrum was apparently assumed to be constant with time at least up to 5 min. The carefully constructed Geiger counter of Sakharov was evidently not used and the time constant in the amplifier

was reported to be 10^{-2} sec. Dead-time losses could be large at shorter decay times.

No tabulated results are included. Values for the graph included here (Fig. B-1) were read from a log-log plot with one decade = ~ 9 mm.

It may be seen from Fig. B-1 that Petrov's results are in marked disagreement with all the other available measurements, and presumably should be disregarded.

c. Iyer and Ganguly (Ref. 33)

Relative data were obtained with a NaI(Tl) detector for times from 10^{-1} to 10^3 sec. These data are normalized to Zigman and Mackin. No tabulated results are included.

d. Oi (Ref. 35)

Relative spectra only obtained with a Li-Ge gamma-ray spectrometer.

e. Berick et al. (Ref. 36)

A plastic scintillator was used to measure the number of photons per fission for the time range from 0.01 to 4.0 sec after fission and for gamma energies above 380 keV. The data were normalized to those of Fisher and Engle.³² A correction of up to 18% was made to account for changes of efficiency as the spectrum varies with decay time.

No energy release data are included and the photon/fission decay rate would have the same time dependence as the energy release only insofar as the gamma-ray spectrum is unchanging with time.

7. Summary - Gamma-Ray Energy Release from Fission Products

The known sources of primary information concerning the total energy release in the form of gamma rays within about 1 to 10^5 sec after thermal-neutron fission of ^{235}U have been examined.

The energy releases for the three spectral measurements, which partially overlap in their time ranges are consistent within the errors of $\pm 15\%$, as may be seen in Fig. 14.

The Sakharov Geiger-counter data, which must be truly independent, are also consistent within errors of $\pm 15\text{-}20\%$. This agreement may be fortuitous. The Petrov ion-chamber and Geiger-counter data are grossly

Fig. B-1. Comparison of differential gamma afterheat functions.

discrepant and can only be disregarded in the face of consistency of the other sets of data.

Warkentin's ion-chamber results may be considered as relative only and perhaps poorly normalized. In the face of no error estimates, the deviations from the spectral data at the extremes of the time range of Warkentin of -15% and $+25\%$ are probably not too worrisome.

Berick's relative measurements for short times add little to the time range of interest for reactor afterheat.

Therefore, overall one is left with the time dependence for the three sets of spectral data standing in reasonably satisfactory consistency. Little or nothing is added or subtracted by considering the other various measurements. The effective error remains in the range of $\pm 15\%$. Any further reduction in uncertainty in the afterheat function for gammas, as well as for betas, will require additional experiments.

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