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MEASUREMENTS OF THE THERMAL CONDUCTIVITY
OF SOILS TO HIGH TEMPERATURES
Final Report

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Prepared by

D. R. Flynn

T. W. Watson

Institute for Applied Technology

National Bureau of Standards

Washington, D.C. 20234

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SC-CR 69-3059

MEASUREMENTS OF THE THERMAL CONDUCTIVITY OF SOILS TO HIGH TEMPERATURES

Final Report

Prepared by

D. R. Flynn and T. W. Watson
Institute for Applied Technology
National Bureau of Standards
Washington, D. C. 20234

for

Sandia Laboratories, Albuquerque

Contract No. ASB48-6179

April 1969

ABSTRACT

An apparatus is described which has been designed and built to enable measurement of the thermal conductivity of soils at hot face temperatures approaching 1700°C . The method utilizes radial heat flow through a hollow cylinder of soil contained between a central ceramic core and an outer water-cooled metal shell. Thermal conductivity values are reported for nine natural or artificial soils representative of subsoils found on most of the earth's surface. For each of these soils, measurements have been completed on two different specimens under particular but reproducible conditions of preparation.

Key Words: Conductance, conductivity, heat transfer, nuclear safety, soils, temperature, thermal conductance, thermal conductivity.

SUMMARY

The electrical-power systems in a number of present and planned space vehicles utilize radioactive isotopes as energy sources. This poses a potential safety hazard when the radioactive material undergoes random re-entry and impact on the Earth's surface. If the fuel capsule impacts on dry land, it may penetrate the soil to a depth approaching 1 meter. The soil surrounding such a buried capsule will thermally insulate it, causing a rise in the temperature of the capsule due to the heat generated by radioactive decay. In order to predict the temperature rise of the buried fuel capsule, and hence evaluate the safety hazard involved, it is necessary to know the thermal conductivity of the surrounding soil.

In order to carry out measurements of the effective thermal conductivity of soils at hot-side temperatures approaching 1700 °C, an existing apparatus was modified somewhat. The soil specimens were compacted into brass cans of 7.6 cm inside diameter and 43.6 cm inside length. An axial heater, constructed of platinum--40% rhodium wire in a 1.25 cm diameter alumina core, provided the heat which flowed radially in the specimen. The apparatus is described in detail. A rather complete mathematical analysis is given, including the effects of non-radial heat flows.

Nine soils were provided by Sandia Corporation. Characterization given for each of these soils includes composition, particle-size analysis, specific gravity, estimated melt range, thermo-gravimetric analysis to 500 °C, and differential thermal analysis to 1400 °C.

For each of these nine soils, effective thermal conductivity values were measured on two different specimens under particular but reproducible conditions of preparation. With the outer surface of the specimens held at temperatures near ambient, measurements were taken under steady-state conditions with the inner surface of the specimens at temperatures from 100 °C to 1400 °F or higher (up to 1650 °C). In all cases, thermal conductivity measurements on the two specimens of a given soil, compacted to about the same density, agreed within a few percent.

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NOMENCLATURE

Symbol	Units	Definition	Page
A_n	$W\ m^{-2}$	Fourier coefficient (eq. (26))	17
a	m	outer radius of ceramic core	6
b	m	inner radius of brass shell	6
C	$W\ m\ ^\circ C^{-1}$	longitudinal thermal conductance of ceramic core and heater wires	14
c	m	outer radius of brass shell	10
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β	$\Omega \text{ m}^{-1}$	electrical resistance per unit length of heater wire	14
λ	$\text{W m}^{-1} ^{\circ}\text{C}^{-1}$	thermal conductivity of specimen	4
λ_o	$\text{W m}^{-1} ^{\circ}\text{C}^{-1}$	$\lambda(T = T_o)$ or $\lambda(T = T_b)$	9,17
λ^*	$\text{W m}^{-1} ^{\circ}\text{C}^{-1}$	mean thermal conductivity of specimen (eq. (2))	4
λ_{app}^*	$\text{W m}^{-1} ^{\circ}\text{C}^{-1}$	apparent λ^* neglecting correction terms	12,18

1. Introduction

In conjunction with safety evaluation of space nuclear power systems, it is desired to be able to predict the maximum steady-state temperature which will be reached by a given nuclear power supply after reentry impact burial in earth. In order to do this it is necessary to have information regarding the thermal conductivity of soils under conditions in which heat flows from a source which may be as hot as 1700 °C to a sink at ambient temperatures.

Under Contract No. ASB48-6179 the National Bureau of Standards was asked to undertake an experimental study to measure the effective thermal conductivity, as a function of temperature, of nine soils (natural and artificial) selected and furnished by Sandia Corporation. The following conditions were imposed on these measurements:

1. The thermal conductivity shall be measured in a manner that simulates a heat source buried in soil.
2. Test temperatures shall be from room temperature to just below the molten range for each soil, with a maximum as near 1700 °C as possible wherever the melt range level permits.
3. Artificial gas pressure or inert gas shall not be used in these measurements.
4. No compaction load shall be applied to the soil samples during these measurements.
5. Duplicate runs shall be made on each type of soil using different samples.

2. Soil Samples

Samples of the following nine soils selected by Sandia Corporation were supplied to the National Bureau of Standards.

1. Calcareous Soil (natural weathered limestone)
2. Granitic Detrital Soil (weathered decomposed granite soil)
3. Dune Sand (windblown sand)
4. Magnesian Soil (magnesium aluminum silicate)
5. Podzol Soil (leached organic timberland soil)
6. Coastal Plains Clay (coastal flood plain soil)
7. Laterite Soil (tropical rain forest soil)
8. Estancia Playa (Dog Lake) Soil (highly saline playa soil)
9. Ottawa Sand (silica-artificial soil)

All of these soils were screened to a maximum particle diameter of 0.17 cm prior to shipment to NBS.

Sandia provided NBS with detailed information concerning each soil sample. These data include estimates of solid-state reaction temperatures and reaction magnitudes and of the melt range for each soil to be tested. In order to experimentally investigate the soil melt ranges, significant reaction temperatures, and rates and magnitudes of reactions, differential thermal analyses (DTA) of each soil sample were carried out by the Analytical Coordination Chemistry Section of the NBS Analytical Chemistry Division. The soil sample characterizations supplied by Sandia Corporation and the results of the NBS DTA determinations are given in Appendices A and B.

3. Method and Apparatus for Thermal Conductivity Measurements

3.1. Philosophy of Approach

The prime concern in deciding on the best experimental approach to this problem was to simulate as closely as possible the conditions which a buried heat source would experience. For a given radioactive capsule, the maximum temperature attained will depend on the composition and particle-size range of the surrounding soil, on the density of the soil, on the moisture content of the soil, on the degree to which the soil is disturbed during capsule burial, and on the thermal contact resistance between the capsule and the surrounding soil. A buried source may experience transient temperatures which are higher than the final steady-state temperature at the source after equilibrium is attained. This is so because the thermal conductivity may increase, e.g., due to sintering, with time at any given temperature. The best way to determine what would be the maximum transient temperature rise at the surface of a buried heat source would be by direct measurement of the temperature-time relationship in an experiment which closely simulates the conditions which a heat source buried in soil would encounter.

Investigation of all of the above-mentioned parameters would be prohibitively expensive and time-consuming if carried out for all types of soils of interest. Accordingly it was decided to concentrate on measuring the thermal conductivity of a number of soil

samples in an oven-dried condition. From this information it can presumably be decided which, if any, types of soil are most likely to cause difficulties insofar as the maximum temperature rise of a radioactive capsule is concerned. These soils could then be investigated more extensively to study the effects of particle size distribution, density, moisture content, and rate-of-heating. Within these constraints, the following considerations apply:

- a. The maximum steady-state temperature, T_m ($^{\circ}\text{C}$), at the surface of a heat source which is surrounded by a homogeneous soil or other material is given by

$$T_m = T_s + \frac{FQ}{\lambda^*}, \quad (1)$$

where T_s is the temperature ($^{\circ}\text{C}$) of the sink to which heat is lost. The quantity Q is the power output per unit area (W m^{-2}) from the source. The quantity F is a geometrical factor (m) which depends on the spatial configuration of the heat source and sink. The mean thermal conductivity, λ^* ($\text{W m}^{-1} ^{\circ}\text{C}^{-1}$), is defined by

$$\lambda^* = \lambda^*(T_s, T_m) = \frac{1}{T_m - T_s} \int_{T_s}^{T_m} \lambda(T) dT, \quad (2)$$

where $\lambda(T)$ is the temperature-dependent thermal conductivity of the soil. From this we see that it is not necessary to measure the actual thermal conductivity, $\lambda(T)$, as a function of temperature if the mean thermal conductivity, $\lambda^*(T_s, T_m)$, can be measured directly as a function of the hot side temperature. This can be done by measuring T_m as a function of Q for some configuration having a known geometrical factor, F , and then computing λ^* using eq. (1).

- b. If $\lambda^*(T_s, T_m)$, rather than $\lambda(T)$, is measured, the cold surface of the specimen can be kept at room temperature thus considerably simplifying the problem of containment.
- c. If $\lambda(T)$ is desired, it can be computed from measured values of $\lambda^*(T_s, T_m)$.
- d. Since the quantity of prime interest is the maximum temperature attained by the buried radio-active source, the values of λ^* which are appropriate are those obtained after monotonic heating of the hot surface of the specimen.
- e. The specimen size should be large enough that the heat flow path is much larger than the average particle size of the soil. For a maximum particle diameter of 0.17 cm, it would seem appropriate for the sample to be at least 2 to 3 cm thick from the hot surface to the cold surface. Similarly, the heat source in contact with the specimen should be large in comparison with the average particle size of the soil. This would preclude the use of "hot-wire" methods.
- f. Since the final steady-state temperature at the source surface is of principal concern, it would not be appropriate to attempt to derive thermal conductivity values from measured values of thermal diffusivity and specific heat. This is particularly so for materials such as soils which are subject to decomposition, phase changes, moisture migration, and other phenomena involving heat generation or absorption.

3.2. Method and Mathematical Analysis for

Radial Heat Flow

It was decided to measure the thermal conductivity of the soil samples using the method of radial heat flow in a right circular cylinder. A cross section of the apparatus is shown in figure 1. The specimen is contained within the annular space between the outer radius, a , of a ceramic core and the inner radius, b , of a brass shell. A measured quantity of heat per unit time generated electrically in the ceramic core flows radially through the specimen to the inner concentric water-cooled brass shell. The heat flow rate per unit area at the surface, $r=a$, of the ceramic core is designated as Q . The ceramic core has a concentric ring of equally spaced holes, parallel to the axis, at a radius, r' , each containing a heater wire. Temperatures are measured by an axial thermocouple in the ceramic core and by thermocouples attached to the outer surface of the inner brass shell at radius c .

Figure 1. Horizontal cross section of the apparatus for measuring the thermal conductivity of soils.

In figure 1, if the cylindrical surfaces $r = a$ and $r = b$ are each isothermal, the heat flow between these surfaces will be radial except near the ends of the apparatus. In general, the thermal conductivity of the specimen material will vary with temperature, and the heat flow rate per unit area through a cylindrical element of the sample is

$$-\lambda(T) \frac{dT}{dr} = \frac{Qa}{r}, \quad (3)$$

where temperature is denoted by the symbol T and the temperature-dependent thermal conductivity by $\lambda(T)$. If eq. (3) is rearranged and integrated, we obtain

$$\int_{T_b}^{T_a} \lambda(T) dT = Qa \ln(b/a), \quad (4)$$

where T_a and T_b are the temperatures at $r = a$ and $r = b$ respectively.

Let us define a mean thermal conductivity $\lambda^*(T_b, T_a)$ over the temperature range T_b to T_a :

$$\lambda^*(T_b, T_a) = \frac{1}{T_a - T_b} \int_{T_b}^{T_a} \lambda(T) dT. \quad (5)$$

Combining eqs. (4) and (5), we obtain

$$\lambda^*(T_b, T_a) = \frac{Qa \ln(b/a)}{T_a - T_b} \quad (6)$$

as the expression which can be used to compute λ^* from measurable quantities.

As an example of the use of (6) let us consider the case where the thermal conductivity of the sample can be assumed to vary linearly with temperature:

$$\lambda(T) = \lambda_0[1 + \alpha_0(T - T_0)] \quad , \quad (7)$$

where λ_0 is the thermal conductivity at an arbitrary reference temperature, T_0 , and α_0 is the corresponding temperature coefficient of thermal conductivity. Substitution of (7) into (5) yields, after integration,

$$\lambda^*(T_b, T_a) = \lambda_0 \left[1 + \alpha_0 \left(\frac{T_a + T_b}{2} - T_0 \right) \right] = \lambda \left(\frac{T_a + T_b}{2} \right) . \quad (8)$$

Thus, for a material for which the thermal conductivity varies linearly with temperature, eq. (6) yields a thermal conductivity corresponding to a temperature which is equal to the average of the temperatures at the hot and cold surfaces of the sample.

Equation (6) requires a knowledge of the temperatures T_a and T_b at the inner and outer surfaces of the sample. Experimental difficulties (e.g., finite size of temperature sensors and contamination of sensors by the sample) preclude direct measurement of T_a and T_b . Reference to figure 1 shows that there are four temperature drops which must be considered in deriving T_a and T_b from the measured temperatures:

1. The temperature drop between the thermocouple well in the center of the ceramic core and the surface, $r = a$, of the ceramic core.

2. The temperature drop due to the thermal contact resistance between the surface of the ceramic core and the inner surface of the sample, both surfaces being nominally at $r = a$.
3. The temperature drop due to the thermal contact resistance between the outer surface of the sample and the inner surface of the brass shell, both surfaces being nominally at $r = b$.
4. The temperature drop between the inner surface, $r = b$, of the brass shell, and the outer surface, $r = c$, where the temperature is measured.

If the circle of heater wires at $r = r'$ were a continuous cylindrical heat source, the entire region $r < r'$ inside the heater circle would be isothermal. For the case of a finite number of line heat sources at $r = r'$, the temperature measured at the axis is equal to the average temperature, T_h , at the radius of the heater circle [1,2]¹. Therefore the temperature drop between the thermocouple well in the center of the core and the surface of the core is given by

$$T_h - T_a' = \frac{Qa \ln(a/r')}{k_c}, \quad (9)$$

where k_c is the thermal conductivity of the ceramic core material and T_a' is the temperature at the outer surface, $r = a$, of the ceramic core (as opposed to T_a , which is the temperature at the inner surface, also $r = a$, of the sample).

¹Figures in brackets indicate the literature references at the end of this report.

If we designate R_a as the thermal contact resistance per unit area ($^{\circ}\text{C m}^2 \text{ W}^{-1}$) at the core-specimen interface ($r = a$), then the temperature drop across this interface is given by

$$T_a' - T_a = R_a Q \quad (10)$$

Similarly, the temperature drop at the specimen-shell interface ($r = b$) is given by

$$T_b - T_b' = \frac{a}{b} R_b Q, \quad (11)$$

where T_b' is the temperature at the inner surface, $r = b$, of the brass shell (as opposed to T_b , which is the temperature at the outer surface, also $r = b$, of the sample) and R_b is the thermal contact resistance per unit area at the specimen-shell interface.

The temperature drop across the brass shell is

$$T_b' - T_c = \frac{Qa \ln(c/b)}{k_s}, \quad (12)$$

where T_c is the temperature at the outer surface of the shell, $r = c$, and k_s is the thermal conductivity of the shell material.

In the apparatus configuration shown in figure 1, the temperatures measured correspond to T_h and T_c as defined above. Combining equations (6), and (9) through (12), we can eliminate all of the temperatures involved except T_h and T_c and thus obtain, instead of (6),

$$\lambda^*(T_b, T_a) = \frac{Qa \ln(b/a)}{T_h - T_c} \left[1 - \frac{Qa}{T_h - T_c} \left(\frac{\ln(a/r')}{k_c} + \frac{\ln(c/b)}{k_s} + \frac{R_a}{a} + \frac{R_b}{b} \right) \right]^{-1} \quad (13)$$

The magnitude of the correction terms in eq. (13) is more readily seen if we define

$$\lambda^*_{app} = \frac{Qa \ln(b/a)}{T_h - T_c} \quad (14)$$

as the apparent value of mean thermal conductivity obtained by neglecting the correction terms; combining (13) and (14) we obtain

$$\lambda^*(T_b, T_a) = \lambda^*_{app} \left[1 - \frac{\lambda^*_{app}}{\ln(b/a)} \left(\frac{\ln(a/r')}{k_c} + \frac{\ln(c/b)}{k_s} + \frac{R_a}{a} + \frac{R_b}{b} \right) \right]^{-1} \quad (15)$$

Equation (15) is the basic equation to be used for calculating mean thermal conductivity values if the heat is assumed to flow radially. The quantities a , b , Q , T_h , and T_c involved in the definition of λ^*_{app} (eq. (14)) are all directly measurable. The quantities k_c , k_s , R_a , and R_b (required for the correction terms in (15)) must either be known or the corresponding correction terms shown to be negligible.

3.3. Longitudinal Heat Flow Correction

The analysis given above is based on the assumption of purely radial heat flow in the alumina core and in the specimen. Owing to the finite length of the apparatus, consideration must be given to the effects of longitudinal heat flow in the core and in the specimen.

Peavy [2] has considered the effect of longitudinal heat flow in an apparatus such as that used in the present investigation. His analysis is exact for the case in which both the core and the specimen have constant thermal conductivity and the heat generation per unit length does not vary along the core. Flynn [1] has considered the effect of longitudinal heat flow along the core for the case in which the heat generation per unit length is a linear function of temperature. His analysis neglects

longitudinal heat flow in the specimen and assumes a constant thermal conductivity (i.e., independent of temperature) for the core.

In the present investigation the very large temperature differences within the apparatus make it imperative that the temperature dependence of the thermal conductivity of the core, the temperature dependence of the thermal conductivity of the specimen, and the temperature dependence of the heat generation per unit length all be taken into account. Peavy's analysis [2] would be very difficult to adapt to the case in which all these parameters have large temperature dependences. Flynn's analysis [1] can easily be modified to include the effect of the temperature dependence of the thermal conductivity of the core and with a little more effort be extended to include a more complicated (than linear) temperature dependence for the heat generation per unit length in the core. However, his analysis is inherently incapable of dealing with the effects of longitudinal heat flow in the specimen.

Since the apparatus design permits direct experimental measurement of the temperature distribution along the axis of the core, we have more information than is available in boundary value problems in which only the conditions on the outside of the system are prescribed. This additional information enables us to compute the effects of longitudinal heat flow in a rather straightforward manner with only minor approximations being required. If necessary even these approximations could be circumvented by a simple process of iteration.

Let us first consider the situation which exists in the alumina core. The thermal conductivity of high-density, high-purity alumina decreases by a factor of more than six between room temperature and 1600 °C. The electrical resistivity of the platinum-rhodium alloy used for the heater winding increases by a factor of three between room temperature and 1600 °C. Both the thermal conductivity of the core and the electrical resistivity of the heater (and hence the heat generation per unit length) are explicit functions of temperature. Since we experimentally measure the temperature distribution along the axis of the core and since the temperature at the axis of the core is essentially the same as the temperature of the heater winding, we can determine the thermal conductivity of the core (inside the circle of heater wires) and the heat generation per unit length as functions of position. For a given test we can treat both of these quantities as if they were explicit functions of position.

Consider an element of the core of length dz . The heat generated in this element is $mI^2\beta dz$, where m is the number of heater wires, I is the electric current flow through the heater winding, and β is the electrical resistance per unit length of heater wire. The radial heat flow from the convex surface of this element into the specimen is $2\pi aQdz$, where now $Q = Q(z)$ varies with longitudinal position. The heat flowing into this element in the positive z -direction is $-C(d\bar{T}/dz)$, where C is the ("position-dependent") thermal conductance of the core,

including heater wires, and $\bar{T}$ is the average temperature of the core at longitudinal position z . The heat leaving the element in the positive z -direction is

$$-C(d\bar{T}/dz) + (d/dz)[-C(d\bar{T}/dz)]dz.$$

Under steady-state (i.e. temperature independent of time) conditions, conservation of energy enables us to write

$$\frac{d}{dz} \left(C \frac{d\bar{T}}{dz} \right) - 2\pi a Q(z) = -mI^2 \beta(z) \quad (16)$$

as the differential equation relating $\bar{T}(z)$, $Q(z)$, and $\beta(z)$. If $\bar{T}$ and β are known or measured, the heat flux into the specimen at any longitudinal position is

$$Q(z) = \frac{1}{2\pi a} \left[mI^2 \beta + \frac{d}{dz} \left(C \frac{d\bar{T}}{dz} \right) \right] \quad (17)$$

Mathematically, the average core temperature at any longitudinal position is given by

$$\bar{T}(z) = \frac{2}{a^2} \int_0^a r \bar{T}(r, z) dr \quad (18)$$

With this definition of $\bar{T}$, eq. (17) is completely rigorous.

Except very near the ends of the apparatus, most of the heat flow in the core is radial rather than longitudinal (see figure 6 and accompanying text). Hence, to a good approximation, the temperature at any position in the core is given by

$$T(r, z) = \begin{cases} T(0, z) & , 0 < r < r' \\ T(0, z) - \frac{mI^2 \beta}{2\pi} \int_{r'}^r \frac{1}{k_c} \frac{dr}{r} & , r' < r < a \end{cases} \quad (19)$$

where $T(0,z) = T_h(z)$ is the experimentally determined temperature distribution along the axis of the core. Substituting this expression into (18), the average temperature of the core at any cross section is given very closely by

$$\bar{T}(z) = T(0,z) - \frac{mI^2\beta}{\pi a^2} \int_{r'}^a r dr \int_{r'}^r \frac{1}{k_c} \frac{dr}{r} \quad (20)$$

For the relatively small radial temperature differences between $r = r'$ and $r = a$ we can justifiably neglect the temperature dependence of k_c and thereby reduce (20) to

$$\begin{aligned} \bar{T}(z) &= T(0,z) - \frac{mI^2\beta}{\pi a^2 k_c} \int_{r'}^a r \ln \frac{r}{r'} dr \\ &= T(0,z) - \frac{mI^2\beta}{4\pi k_c} \left[2\ln \frac{a}{r'} - 1 + \frac{r'^2}{a^2} \right] \end{aligned} \quad (21)$$

This equation can be substituted into (17) to enable calculation of $Q(z)$, the radial heat flux into the specimen at all longitudinal positions.

By writing a differential equation (eq. (16)) in terms of $\bar{T}$, the average core temperature, we were able to determine the heat flux, $Q(z)$, into the inner surface, $r = a$, of the specimen. We now show the analysis needed to permit computation of the thermal conductivity of the specimen. Consider an apparatus, having the horizontal cross section shown in figure 1, which is of finite length, extending from $z = 0$ to $z = \ell$, with the flat ends water cooled. We neglect contact resistances, and write the boundary conditions for the specimen as

$$\begin{array}{lll} a < r \leq b & z = 0 \text{ and } z = \ell & T = T_b, \\ r = b & 0 \leq z \leq \ell & T = T_b, \\ r = a & 0 < z < \ell & -\lambda(T) \frac{\partial T}{\partial r} = Q(z). \end{array} \quad (22)$$

In order to simplify the boundary condition at the surface of the core, $r = a$, we introduce a new variable defined by

$$y = \frac{1}{\lambda_0} \int_{T_b}^T \lambda(T) dT = \frac{\lambda^*(T_b, T)}{\lambda_0} (T - T_b) \quad (23)$$

where $\lambda_0 = \lambda(T = T_b)$. In terms of y , the boundary conditions, (22), can be written as

$$\begin{aligned} a < r \leq b & \quad z = 0 \text{ and } z = \ell & \quad y = 0 & , \\ r = b & \quad 0 \leq z \leq \ell & \quad y = 0 & , \\ r = a & \quad 0 < z < \ell & \quad -\lambda_0 \frac{\partial y}{\partial r} = Q(z) & . \end{aligned} \quad (24)$$

With these boundary conditions, the solution of the Laplacian equation for the potential, y , is given by Carslaw and Jaeger [3, p. 221] as

$$y(r, z) = \frac{-1}{\lambda_0} \sum_{n=1}^{\infty} \frac{\ell A_n F_0(n\pi r/\ell; n\pi b/\ell)}{n\pi F_1(n\pi a/\ell; n\pi b/\ell)} \sin \frac{n\pi z}{\ell} , \quad (25)$$

where

$$A_n = \frac{2}{\ell} \int_0^{\ell} Q(z') \sin \frac{n\pi z'}{\ell} dz' , \quad (26)$$

$$F_0(x; y) \equiv I_0(x)K_0(y) - I_0(y)K_0(x) , \quad (27)$$

$$\text{and } F_1(x; y) \equiv I_1(x)K_0(y) + I_0(y)K_1(x) , \quad (28)$$

$I_n(x)$ and $K_n(x)$ being modified Bessel functions of the first and second kind, respectively, and of order n .

Evaluation of (25) at $r = a$ and comparison with (23) enables us to write

$$\lambda^*(v) = \frac{-1}{T_a(z) - T_b} \sum_{n=1}^{\infty} \frac{\ell A_n F_0(n\pi a/\ell; n\pi b/\ell)}{n\pi F_1(n\pi a/\ell; n\pi b/\ell)} \sin \frac{n\pi z}{\ell} . \quad (29)$$

Since we know that the heat flow will be essentially radial near the center of the apparatus, it is useful to rewrite (29) in the form

$$\lambda^*(T) = \frac{Q(z) \cdot a \ln(b/a)}{T_a(z) - T_b} [1 - U(z)] , \quad (30)$$

where $U(z)$, a correction term for the effects of longitudinal heat flow in the specimen, is given by

$$U(z) = \frac{1}{Q(z)} \sum_{n=1}^{\infty} A_n \left\{ 1 - \frac{1}{\ln(b/a)} \cdot \frac{\ell}{\pi a} \frac{F_0(n\pi a/\ell; n\pi b/\ell)}{F_1(n\pi a/\ell; n\pi b/\ell)} \right\} \sin \frac{n\pi z}{\ell} . \quad (31)$$

In deriving eq. (30), we neglected the influence of thermal contact resistances. Since $U(z)$ is a small correction, we assume that it is still valid even if thermal contact resistances are taken into account.

Comparing eqs. (14), (15), and (30), we write

$$\lambda^*(T_b, T_a) = \frac{\lambda_{app}^* [1 - U(z)]}{\left[1 - \frac{\lambda_{app}^*}{\ln(b/a)} \left(\frac{\ln(a/r')}{k_c} + \frac{\ln(c/b)}{k_s} + \frac{R_a}{a} + \frac{R_b}{b} \right) \right]} , \quad (32)$$

where λ_{app}^* now has the more general definition

$$\lambda_{app}^* = \frac{Q(z) \ln(b/a)}{T_h(z) - T_c} , \quad (33)$$

where $Q(z)$ is given by eq. (17). Equation (32) is the equation used in the present investigation to compute λ^* .

3.4. Apparatus

A vertical cross section of the apparatus designed and built for the thermal conductivity measurements is shown in figure 2. The specimen is contained in the annular space between the central ceramic core, which is supported at both ends, and the inner brass shell. Cooling water flows in the 3-mm annulus between this inner brass shell and a coaxial outer brass shell.

The central core is an extruded alumina rod, 46 cm long and 1.25 cm in diameter. A horizontal cross-section of this core is shown in figure 3. Sixteen equally-spaced holes, 0.09 cm in diameter, extend the entire length of the rod. The centers of these holes form a circle of 0.44 cm radius. The core heater, which provides the heat flowing radially through the specimen, consists of lengths of platinum-40 percent rhodium (0.06 cm diameter) wire threaded back and forth through the sixteen holes. The core is held from beneath by a ceramic insulating support designed to permit free expansion of the heater wire. The upper end of the core passes through a hole in the removable flange at the top of the apparatus; this hole is a sufficiently loose fit to permit free expansion of the core. Current leads and voltage taps are attached to the heater winding at the upper end of the core.

Figure 2. Vertical cross section of the apparatus for measuring the thermal conductivity of soils.

Figure 3. Horizontal cross section of the ceramic core.

In the center of the alumina core there is an axial hole 0.25 cm in diameter. This hole accommodates a thermocouple which can be moved vertically by exterior manipulation. This thermocouple is fabricated from 0.04 cm diameter platinum and platinum - 10 percent rhodium wire and is contained in double-bore alumina tubing which is a snug slip fit in the thermocouple well. The temperature measured at the midlength of the apparatus is designated T_h . This thermocouple can also be utilized to obtain the longitudinal temperature distribution along the core - this information is needed to make corrections for longitudinal heat losses.

With the exception of the ceramic core, all surfaces in contact with the specimen are water-cooled to maintain them at room temperature. As shown in figure 2, the cooling water enters the center of the bottom of the apparatus, passes upward in the annulus between the inner and outer brass shells, and exits at the top of the apparatus. A circulating system, shown in figure 4, is used to maintain a constant temperature in the apparatus. The water leaving the apparatus is pumped through a heat exchanger which cools the water to a temperature lower than that desired at the inlet to the apparatus. From the heat exchanger, the water enters a small commercial hot water heater where it is reheated to the desired temperature for circulation to the apparatus. A resistance thermometer at the exit from the tank is used to control the power to the heater in the tank and thus maintain the desired water temperature.

Figure 4. The circulating system used to cool the apparatus.

Two copper versus constantan thermocouples (0.025 cm diameter) are attached to the outer surface of the inner brass shell at the midplane of the apparatus. The junctions of these thermocouples are thermally insulated from the cooling water by plastic electrical tape. The average of these two temperatures is designated T_c . Additional copper-constantan thermocouples are used to monitor temperatures at different locations in the circulating system.

Individual cold junctions in an ice bath are used with the platinum versus platinum-10 percent rhodium thermocouple used to measure T_h . The constantan leads of the copper versus constantan thermocouples are brought to an isothermal zone box at room temperature. A thermocouple with one junction in the zone box and one in an ice bath is placed in series with a double-pole selector switch, so that each measuring thermocouple is automatically referenced against the ice bath[4].

The ceramic core heater, which provides the heat flowing radially through a specimen, is fed a-c power from a saturable core reactor which is controlled by a current-adjusting-type proportional controller incorporating automatic rate and reset action. This controller has been modified to operate in either of two modes. It can adjust the power to the core heater so as to maintain the temperature, T_h , at the center of the core at a constant value. Alternatively, it can control the saturable core reactor so as to provide constant current to the core heater. The power dissipated in the core heater is determined by measuring the current through the heater and the voltage drop across the heater utilizing electrodynamic voltmeters and ammeters having an accuracy of one-quarter percent of full scale.

4. Testing Procedure and Calculation Procedure

4.1. Specimen Preparation

Sandia Corporation provided us with desired dry bulk density values for each of the soil samples supplied. These desired densities, which are given in Appendix A, were based on the maximum densities attained in a series of tests using ASTM Method of Test D698. This test method provides a specified compaction procedure based on dropping a weight a fixed distance a given number of times. This compaction procedure had been carried out on each soil for a number of samples having different moisture contents; the moisture contents which resulted in the highest dry bulk densities were reported to us.

Adaptation of ASTM D698 to the geometry of our thermal conductivity apparatus required considerable effort. We were successful in developing a compaction procedure, based on ASTM D698, which enabled rapid, reliable compaction of the soil sampled with reproducible densities. The bulk densities obtained with our compaction rig did not always agree with those densities obtained with ASTM D698 for similar samples at nominally the same moisture content and compaction energy. We were always able to come reasonably close to the desired bulk densities with only a few trials, however.

Each thermal conductivity sample was compacted into a new brass can (the inner brass shell shown in figure 2). This can had a centered hole in the bottom which held the lower end of the ceramic core (heater) coaxially within the can. During the compaction operation, a combination weight guide and core support, as shown in figure 5, was affixed to the upper end of the brass can which was to contain the specimen. The core support held the upper end of the ceramic core (heater) coaxially within the can during compaction; the core support was spring-loaded to hold the ceramic core down during compaction. A brass weight was used to compact the soil in the specimen container. This weight had an outside diameter slightly smaller than the inside diameter of the specimen container and was provided with a keyhole-shaped slot to clear the core support and web shown in figure 5. An attachment extending above the top of the weight prevented the weight from rotating even when the weight was totally within the specimen can. The weight guide could be rotated when desired so that the keyhole-shaped slot in the weight would not always impinge on the same position of the specimen.

Specimen preparation consisted of the following steps:

1. A small amount of the soil was oven-dried to constant weight at 110 °C in order to determine the existing moisture content.

Figure 5. Rig used for compacting the soil samples.

2. Meanwhile the brass can (which would later receive the specimen) and the ceramic core (heater) were weighed.
3. Sufficient additional moisture was added to a soil sample to attain the moisture content desired.
4. The compaction rig shown in figure 5 was affixed to the top of the specimen container with the heater centered in place.
5. A mass of moisture-conditioned soil equal to one-tenth of the total mass needed to achieve the desired density was placed in the specimen container.
6. The weight was dropped a fixed distance a given number of times to achieve the desired compaction energy.
7. Another layer of soil was placed in the specimen container and this layer was compacted as above. This procedure was repeated until the specimen container was filled.
8. The filled specimen container was oven-dried to constant weight at 110 °C.
9. The specimen and container were weighed and the specimen dry bulk density was computed.
10. If the dry bulk density obtained was sufficiently close to the desired value, testing proceeded. Otherwise, the specimen was removed and repacked using a different compaction energy.

A few of the soil samples did not lend themselves to the above compaction procedure. The specimens of Ottawa sand, dune sand, and coastal plains clay were packed to the desired density by filling the specimen container slowly with dry soil while rapping it with a small hammer.

4.2. Testing Procedure

A specimen container, with the ceramic core heater and the compacted specimen installed, was placed in the outer brass shell as shown in figure 2. Rubber O-rings, not shown in figure 2, provided water tight seals at the top of the apparatus. The current leads and voltage taps were attached to the heater leads extending from the top of the ceramic core. A thermocouple was installed in the axial hole in the ceramic core so that the hot junction was at the mid-height of the apparatus. The upper end of this thermocouple was attached to a height gage which permitted moving the thermocouple vertically by known distances.

The circulating system shown in figure 4 was activated to maintain the outside of the specimen container at the desired temperature. This involved adjusting the amount of chilled water passing through the heat exchanger as well as the setting of the temperature controller connected to the resistance bulb in the water tank. During tests at the highest specimen hot-side temperatures, the chilled water system could not maintain the desired temperature at the outside of the specimen container. Since all of the data were later adjusted to correspond to a 25 °C cold-side temperature, this caused no significant errors.

The specimen heater was energized with the proportional controller operating in a constant-temperature mode so as to bring and maintain the temperature at the core thermocouple to the desired value. This procedure effectively prevented significant temperature overshoot, allowing monotonic heating of the hot face of the sample but yet permitting rapid equilibrium to be attained. When the temperature indicated by the core thermocouple was steady and when the electrical power required to maintain this temperature had also leveled out to a steady value, the proportional controller was switched to a constant-current mode so as to maintain the current to the heater at the value it had before switching over.

After any small disturbances due to mode-changing had time to die out, several successive readings were taken of the current through the heater, the voltage drop across the heater, the emf of the Pt - 10% Rh: Pt thermocouple (at the mid-height of the apparatus) in the axial hole in the ceramic core, and the emfs of the two copper:constantan thermocouples on the outside of the specimen container. After satisfactory data were obtained with the core thermocouple at the mid-height of the apparatus, the thermocouple junction was moved to a position approximately level with the bottom of the specimen. The emf of the core thermocouple was read at this position and then at approximately 2.5 cm intervals

along the entire length of the specimen, time being allowed at each position for the thermocouple junction to reach thermal equilibrium with the ceramic core. The core thermocouple junction was then returned to the mid-height of the apparatus and readings were again taken of the heater current, heater voltage, and thermocouple emfs.

Following completion of one set of data, the heater controller was switched to constant temperature mode and the heater and both controllers were adjusted to bring the specimen to the next condition desired. Data were normally taken at 200 °C intervals to about 1100 °C (hot-side) and then at 100 °C intervals until either the heater burned out or the core thermocouple ceased to function properly.

For the first tests conducted, the heater was fabricated by connecting all sixteen heater wires in series (Type 1 heater). With this wiring arrangement, the current leads to and from the heater were in adjacent holes in the ceramic core. This resulted in large voltage gradients within the alumina core with consequent dielectric breakdown of the alumina at core temperatures around 1400 °C. The heater wiring was subsequently changed to a series-parallel arrangement in which the heater was wired as two sets of eight series-connected wires with these two sets connected in parallel (Type 2 heater). This greatly reduced the voltage gradients in the ceramic core and we had no further trouble with dielectric breakdown.

Another problem that caused some concern was that of a-c pickup at high temperatures by the core thermocouple. The physical arrangement of the apparatus made it difficult to shield this thermocouple. Probably one of the best procedures would have been to use commercially-available metal-sheathed thermocouples fabricated by swaging techniques. The temperatures involved would have required the use of thermocouples with noble-metal sheaths, thus greatly increasing the cost of the thermocouples. Since it was desired to use a new thermocouple for each specimen (the high temperatures could cause thermocouples to lose their calibration), the total cost would have been quite high. It was found possible to use unshielded thermocouples by reading the output using a electronic integrating digital voltmeter having 140 dB common-mode rejection.

4.3. Calculation Procedure

A computer program was written to enable calculation of thermal conductivity values from the data which were obtained. The input to the computer consisted of the voltmeter and ammeter readings and the thermocouple positions and emfs for each test. The calculation procedure was as follows:

1. The ammeter and voltmeter readings were corrected slightly, if necessary, by sub-programs which contained the meter calibrations.

2. Temperatures were computed from the thermocouple emfs using sub-programs which contained the thermocouple calibrations.
3. The power output per unit length in the absence of longitudinal heat losses was computed for each position along the ceramic core using the leading term in eq. (17). The resistance per unit length, β , of the heater wire was computed using an equation representing NBS-obtained data on the temperature-dependence of Pt - 40% Rh alloy. It was assumed that the heater wire was at the temperature, T_h , measured at the center of the core.
4. The average core temperature at each longitudinal position was computed using eq. (21).
5. A fourth-order polynomial was fitted (by least-squares) to the (usually 19) values of $\bar{T}(z)$. From this equation and an equation which was used to compute C, the longitudinal thermal conductance of the core, the second term in eq. (17) was evaluated.
6. From these values of $Q(z)$, the mean thermal conductivity, $\lambda^*(T_b, T_a)$, was computed at each position using eqs. (32) and (33). The computer program contained a value for k_s , the thermal conductivity of the brass shell, and an equation for computing k_c , the thermal conductivity of the ceramic core, as

a function of temperature[5]. Since no knowledge was available regarding the thermal contact resistances, R_a and R_b were set equal to zero; thus the λ^* values obtained are effective mean thermal conductivity values. A separate subprogram was used to compute $U(z)$, the correction for longitudinal heat flow in the specimen.

7. An equation of the form $\lambda^*(25, T_a) = a + bT_a + cT_a^2 + dT_a^n$ ($n \geq 3$) was fitted (least-squares) to the computed $\lambda^*(T_b, T_a)$ values corresponding to the mid-height of the apparatus, the λ^* values being adjusted slightly so that they all corresponded to a cold side temperature of $T_b = 25^\circ\text{C}$. The value of n selected was that which yielded the lowest standard deviation of the residuals.

8. The effective thermal conductivity was computed from this equation using the relation

$$\lambda(T_a) = \lambda^*(T_b, T_a) + (T_a - T_b) \frac{\partial \lambda^*(T_b, T_a)}{\partial T_a} \quad (34)$$

which is obtained by differentiation of eq. (5).

9. The computed λ^* values and the fitted curves for λ^* and λ were stored on a magnetic tape which later was used to generate the plots (figures 6 through 16) in the next section.

5. Results

In order to illustrate the type of data which were obtained and the relative magnitudes of the various corrections involved, we first present a set of data and a typical calculation for one of the Podzol soil specimens for a hot side temperature near 1100 °C. Podzol was selected for illustrative purposes because its thermal conductivity lies near the median for all nine soil samples tested.

The upper curve in figure 6 illustrates the measured values of T_h , the temperature along the axis of the ceramic core. The dashed curve, labelled T_a' , is the computed longitudinal temperature distribution at the surface, $r = a$, of the core. The electrical resistance per unit length of the heater wire, corresponding to the measured values of T_h , was used to compute the power generated per unit length--this quantity, the leading term in eq. (17), is shown by the curve labelled Q_g in figure 6. The average core temperature at any longitudinal position, computed by eq. (21), was used in eq. (17) to correct Q_g for the effects of longitudinal heat flow in the core and thus obtain the heat flux into the specimen at any longitudinal position--this quantity is shown by the curve labelled Q in figure 6. Using this curve for $Q(z)$, $\lambda^*(T_b, T_a)$ values were computed at a number of longitudinal positions using eqs. (32) and (33). Only the thermal conductivity values corresponding to the midplane position of the apparatus were used for the curves shown in this report.

Figure 6. Temperature and heat flux distribution for a specimen of Podzol soil at hot side temperature of about 1100 °C. The four curves are identified in the text.

For the test discussed above and in figure 6, the different parameters defined in sections 3.2 and 3.3 had the following values:

T_h	1131.1 °C	Q_g	6.04 W cm ⁻²
$\bar{T}$	1124.7 °C	Q	6.08 W cm ⁻²
$T_a = T_a'$	1108.3 °C		
$T_b = T_b'$	36.8 °C	U	-.007
T_c	36.5 °C		

$$\lambda^*(36.8, 1108.3) = .646 \text{ W m}^{-1} \text{ °C}^{-1}$$

Prior to undertaking measurements of the thermal conductivity of any of the supplied soil samples, we measured the thermal conductivity of a sample of diatomaceous earth. We had previously [1] measured the thermal conductivity of this same lot of diatomaceous earth in an apparatus in which both the hot and the cold sides of the specimen were heated so that λ could be measured directly rather than being computed from λ^* . Further, in this earlier apparatus the ends were heated so that there was no need to make corrections for longitudinal heat flow in the core or in the specimen. Measurement of this lot of diatomaceous earth in the present apparatus provided a check on the corrections for longitudinal heat flow (which were rather large because of the low thermal conductivity of diatomaceous earth) and also provided a check on the procedure used to compute λ from the measured λ^* values.

Figure 7. Thermal conductivity of diatomaceous earth at a density of $0.15 \times 10^3 \text{ kg m}^{-3}$. The solid curve represents an equation of the form $\lambda^*(25, T_a) = a + bT_a + cT_a^2 + dT_a^3$. The open circles represent the smoothed values obtained earlier by Flynn[1].

The solid triangles shown in figure 7 represent the $\lambda^*(25, T_a)$ data points obtained in the present apparatus plotted versus T_a , the hot side temperature. The solid curve is the cubic of least-squares fit to these data. The dashed curve represents the $\lambda(T_a)$ -values computed using eq. (34). The open circles represent the smoothed thermal conductivity values obtained earlier by Flynn[1] on diatomaceous earth from the same lot as was used in the present measurements. The agreement is seen to be good.

Figures 8 through 17 show the measured λ^* -values, the least-squares λ^* -curves, and the derived λ -curves for each of the nine soil samples supplied by Sandia. The data correspond to monotonic heating to the maximum temperatures shown. Measurements have been completed on two samples, designated by solid triangles (Test A) and open squares (Test B), at nominally the same density, for eight of the soils. At the request of Sandia, the second run on Ottawa sand was made with a lower density than was used for the first specimen. Each of the eighteen tests which were carried out are briefly discussed below.

CALCAREOUS (Figure 8)

Test A. This was the second soil test conducted. The Type 1 heater burned out at a fairly low temperature. Difficulties were encountered due to a-c pickup in the axial thermocouple; however, cross checks indicated that the thermocouple emf values obtained were not seriously in error.

Figure 8. Thermal conductivity of calcareous soil at densities of 2.01×10^3 and $1.99 \times 10^3 \text{ kg m}^{-3}$. The solid curve represents an equation of the form $\lambda^*(25, T_a) = a + bT_a + cT_a^2 + dT_a^{13}$.

Test B. Neither the Type 2 heater nor the thermocouple burned out. The specimen lost 4.0% of its weight during testing.

GRANITIC DETRITAL (Figure 9)

Test A. The Type 1 heater burned out while we were attempting to achieve a 1500 °C hot-side temperature (which is well above the melting range for this soil). The specimen lost 5.6% of its weight during testing.

Test B. Both the Type 2 heater and the thermocouple failed while we were attempting to achieve a hot-side temperature near 1600 °C (again well above the melting range). The apparent weight loss during testing was only 0.9 percent.

DUNE SAND (Figure 10)

Both of the specimens were packed dry by vibration rather than by compaction of wet soil.

Test A. The heater for this test was a somewhat modified version of a Type 1 heater; it burned out while we were attempting to achieve a hot-side temperature above 1550 °C.

Test B. Neither the Type 2 heater nor the thermocouple failed. The specimen lost 1.1% of its weight.

MAGNESIAN (Figure 11)

Test A. This was the first soil test conducted. Considerable difficulties were encountered with a-c pickup by the thermocouple - the validity of the data for the highest temperature test (the triangle at 1138 °C in figure 11) is questionable. The Type 1 heater was prematurely burned out by an inadvertent external short circuit. The specimen lost 0.3% of its weight.

Figure 9. Thermal conductivity of two specimens of granitic detrital soil, each at density $1.92 \times 10^3 \text{ kg m}^{-3}$. The solid curve represents an equation of the form $\lambda^*(25, T_a) = a + bT_a + cT_a^2 + dT_a^{15}$.

Figure 10. Thermal conductivity of two specimens of dune sand, each at a density of $1.57 \times 10^3 \text{ kg m}^{-3}$. The solid curve represents an equation of the form $\lambda^*(25, T_a) = a + bT_a + cT_a^2 + dT_a^6$.

Figure 11. Thermal conductivity of magnesian soil at densities of 1.76×10^3 and $1.82 \times 10^3 \text{ kg m}^{-3}$. The solid curve represents an equation of the form $\lambda^*(25, T_a) = a + bT_a + cT_a^2 + dT_a^9$.

Test B. This test was terminated when the axial thermocouple became stuck at the midplane position in the core. The data corresponding to this condition were not used. The highest hot-side temperature attained was near the upper end of the estimated melting range. The specimen lost 0.6% of its weight.

PODZOL (Figure 12)

Test A. The Type 1 heater burned out. The specimen lost approximately 0.9% of its weight.

Test B. Neither the Type 2 heater nor the thermocouple failed. The specimen lost 1.4% of its weight.

COASTAL PLAINS (Figure 13)

The large shrinkage exhibited by this soil made it impossible to prepare specimens by compaction of moist soil. Both specimens were prepared by vibration of dry soil.

Test A. The Type 2 heater did not burn out. The thermocouple failed and had to be replaced for the highest temperature test. The specimen lost 0.7% of its weight.

Test B. Neither the Type 2 heater nor the thermocouple failed. The specimen weight did not change measurably during testing.

LATERITE (Figure 14)

The first specimen of this soil chemically attacked the alumina heater core causing failure of the Type 2 heater at a rather low temperature. There was considerable a-c pickup by the core thermocouple. All-in-all, this test was very unsatisfactory and hence was not used. Two additional specimens were prepared.

Figure 12. Thermal conductivity of podzol soil at densities of 1.76×10^3 and 1.74×10^3 kg m⁻³. The solid curve represents an equation of the form $\lambda^*(25, T_a) = a + bT_a + cT_a^2 + dT_a^3$.

Figure 13. Thermal conductivity of coastal plains clay at densities of 1.32×10^3 and 1.35×10^3 kg m⁻³. The solid curve represents an equation of the form $\lambda^*(25, T_a) = a + bT_a + cT_a^2 + dT_a^{11}$.

Figure 14. Thermal conductivity of two specimens of laterite soil, each at a density of $1.49 \times 10^3 \text{ kg m}^{-3}$. The solid curve represents an equation of the form $\lambda^*(25, T_a) = a + bT_a + cT_a^2 + dT_a^6$.

Test A. The Type 2 heater did not fail but it was necessary to replace the axial thermocouple twice. The specimen lost 7.0% of its weight during testing.

Test B. No particular difficulties were encountered in achieving a hot-side temperature of about 1648 °C. After the specimen had cooled back to room temperature, the heater core was found to be loose in the specimen, presumably due to specimen shrinkage. The specimen lost 7.9% of its weight during testing.

ESTANCIA PLAYA (Figure 15)

Difficulties were encountered in preparing these two specimens. Drying at 110 °C apparently did not remove all of the water which was added when the samples were prepared for compaction. In order to remove this added water, the specimens were baked at 140 °C. When a moist, freshly-compacted specimen was placed in an oven at 140 °C, the specimen shrunk too much to be usable. However, when the moist specimens were dried to constant weight at 110 °C and then to a new constant weight at 140 °C, satisfactory specimens were obtained.

When the hot-side temperature of both specimens reached about 1200 °C, a disagreeable odor emanated from the specimens. This presumably was sulfur dioxide resulting from the decomposition of the gypsum in this soil.

Figure 15. Thermal conductivity of estancia playa soil at densities of 1.54×10^3 and 1.52×10^3 kg m⁻³. The solid curve represents an equation of the form $\lambda^*(25, T_a) = a + bT_a + cT_a^2 + dT_a^{13}$.

Test A. The data at hot-side temperatures of approximately 300 and 500 °C (triangles in figure 15) are probably erroneously high due to an inadvertent error in the data-taking procedure. The Type 2 heater burned out in trying to attain a hot-side temperature near 1650 °C (well above the estimated melting range of this soil). The specimen lost 12.7% of its weight during testing.

Test B. During the higher temperature tests with this specimen, considerable difficulties were encountered due to fuses blowing in the specimen heater circuit. Thus the specimen was cooled down several times before the data were all obtained. The Type 2 heater burned out in trying to reach a hot-side temperature above 1600 °C. The specimen lost 10.1% of its weight during testing.

OTTAWA SAND (Figures 16 and 17)

As stated previously, these specimens were intentionally run at quite different bulk densities.

Test A. This test (figure 16) was the first test using a Type 2 heater. The heater performed satisfactorily but both the original axial thermocouple and a replacement thermocouple failed.

Test B. Several data points were taken at higher temperatures than are shown in figure 17. However, the specimen had cooled down between the time the plotted data were taken and the time these higher-temperature data were taken. This cooling of the specimen caused settling with an irreversible increase in bulk density and a consequent increase in thermal conductivity.

Figure 16. Thermal conductivity of Ottawa sand at a density of $1.76 \times 10^3 \text{ kg m}^{-3}$. The solid curve represents an equation of the form $\lambda^*(25, T_a) = a + bT_a + cT_a^2 + dT_a^3$.

Figure 17. Thermal conductivity of Ottawa sand at a density of $1.57 \times 10^3 \text{ kg m}^{-3}$. The solid curve represents an equation of the form $\lambda^*(25, T_a) = a + bT_a + cT_a^2 + dT_a^3$.

The effective mean thermal conductivity values, $\lambda^*(25, T_a)$, of the nine samples are given in table 1 and are plotted in figure 18. The derived thermal conductivity values, $\lambda(T_a)$, of the nine types of soil are given in table 2 and are plotted in figures 19 and 20. Note that the thermal conductivity values tend to converge to a common region at high temperatures. The key to the curves in figures 18, 19, and 20 is given below.

Symbol	Specimen	Average Density kg m ⁻³
DE	diatomaceous earth	150
C	calcareous	2000
GD	granitic detrital	1920
DS	dune sand	1570
M	magnesian	1790
P	podzol	1750
CP	coastal plains	1335
L	laterite	1490
EP	estancia playa	1530
OS ₁	Ottawa sand	1760
OS ₂	Ottawa sand	1570

Because of the many factors involved, it is difficult to estimate how well the thermal conductivity values obtained in the present investigation correspond to in situ thermal conductivity values on similar soils. Excluding possible errors due to contact effects at the interface between the heater core and the specimen, we estimate that the $\lambda^*(25, T_a)$ values

Table 1. Effective mean thermal conductivity, $\lambda^*(25, T_a)$,
(expressed in $W m^{-1} ^\circ C^{-1}$) of the nine types of soil

Type of Soil	Hot-Side Temperature, $^\circ C$											
	100	200	400	600	800	1000	1200	1300	1400	1500	1600	1650
Calcareous	.78	.74	.68	.63	.58	.54	.54	.62	.82	1.32	2.4	3.4
Granitic Detrital	.88	.83	.77	.74	.75	.82	1.05	1.44	2.5	5.2*	----	----
Dune Sand	.30	.34	.40	.44	.51	.70	1.25	1.77	2.5	3.7	5.2*	----
Magnesian	.66	.65	.64	.65	.67	.73	.93	1.17	1.6	----	----	----
Podzol	.52	.53	.51	.52	.64	.98	1.63	2.1	2.7	3.4	4.2*	----
Coastal Plains	.28	.30	.34	.40	.47	.56	.75	.96	1.38	2.2	3.7	4.8
Laterite	.32	.27	.19	.13	.13	.30	.85	1.38	2.2	3.3	4.9	5.9
Estancia Playa	.38	.40	.42	.41	.35	.28	.38	.76	1.8	4.4	----	----
Ottawa Sand ^{a/}	.43	.51	.68	.85	1.04	1.24	1.45	1.56	1.68	1.80	----	----
Ottawa Sand ^{b/}	.30	.34	.45	.57	.75	.98	1.29	1.48	1.70	----	----	----

*Extrapolated value.

^{a/}1760 $kg m^{-3}$ density.

^{b/}1570 $kg m^{-3}$ density.

Figure 18. Effective mean thermal conductivity, $\lambda^*(25, T_a)$, of all of the specimens measured plotted versus the hot-side temperatures.

Table 2. The derived thermal conductivity, $\lambda(T_a)$,
(expressed in $W\ m^{-1}\ ^\circ C^{-1}$) of the nine types of soil

Type of Soil	Hot-Side Temperature, $^\circ C$											
	100	200	400	600	800	1000	1200	1300	1400	1500	1600	1650
Calcareous	.79	.77	.74	.71	.68	.66	.64	.63	.64	.66	.74	.80
Granitic Detrital	.90	.88	.83	.80	.79	.79	.81	.84	.91	1.10*	----	----
Dune Sand	.29	.31	.34	.37	.39	.43	.52	.60	.71	.87	1.09*	----
Magnesian	.66	.66	.65	.65	.65	.66	.69	.71	.76	----	----	----
Podzol	.51	.52	.52	.52	.53	.58	.70	.79	.91	1.05	1.23*	----
Coastal Plains	.27	.28	.30	.33	.35	.39	.43	.46	.51	.59	.74	.84
Laterite	.34	.31	.27	.23	.20	.20	.26	.32	.42	.58	.80	.94
Estancia Playa	.36	.38	.40	.40	.40	.38	.37	.38	.44	.61	----	----
Ottawa Sand ^{a/}	.41	.44	.52	.61	.69	.79	.88	.93	.98	1.03	----	----
Ottawa Sand ^{b/}	.28	.30	.35	.41	.47	.55	.65	.71	.77	----	----	----

*Extrapolated value.

^{a/}1760 $kg\ m^{-3}$ density.

^{b/}1570 $kg\ m^{-3}$ density.

Figure 19. The derived thermal conductivity, $\lambda(T_a)$, of all of the specimens measured plotted versus temperature. The curves are truncated near $\lambda = 1.4 \text{ W m}^{-1} \text{ } ^\circ\text{C}^{-1}$.

Figure 20. The derived thermal conductivity, $\lambda(T_a)$, of all the specimens measured plotted versus temperature. The vertical scale has been changed to permit showing the high-temperature behavior.

obtained for any given specimen are uncertain by not more than about 5 percent in the temperature range 100 to 1200 °C with the uncertainty increasing to perhaps 10 percent at the highest temperatures achieved. If contact effects for a buried radioactive capsule were similar to the contact effects which prevailed in our measurements, these λ^* -values could be used, with no serious error due to contact effects, to predict the capsule temperature when steady-state is reached. At temperatures up to, say, 1000 °C, the derived $\lambda(T_a)$ values should not be seriously inferior in accuracy to the measured $\lambda^*(25, T_a)$ values; at higher temperatures, however, where sintering, melting, and increased thermal radiation cause very rapid increases in thermal conductivity, the $\lambda(T_a)$ values are definitely rather uncertain. However, as discussed in section 3.1, λ^* , rather than λ , is the controlling parameter affecting the steady-state temperature rise of a buried heat source. As also discussed in section 3.1, the maximum temperature rise of a buried capsule also would depend on such factors as density, particle size, moisture content, and rate of temperature rise. These factors could be studied by carrying out both steady-state and transient measurements on a few selected soil samples.

6. Appendix A: Soil Sample Characterization

The University of New Mexico carried out X-ray diffraction analyses of the soil samples and then made estimates of the composition and melting range of each soil[6]. These results are presented below:

CALCAREOUS Common to arid and semi-arid regions of world. At high temperatures the calcite decomposes, with attendant gassing, to calcium oxide which has a very high melting range.

Soil Constituents	Estimated Amount wt %
Calcite	75
Montmorillonite	.8
Quartz	8
Dolomite	3
Muscovite-illite	2
Goethite	1
Oligoclase	1
Trace Minerals	2
	<hr/> 100

Estimated Soil-Melt Range: 1950 to 2050 °C

GRANITIC DETRITAL

These soils with a high-alkali feldspar content have a low-melting range. They are typical of many detrital soils, which comprise about a quarter of total land area of world.

Soil Constituents	Estimated Amount wt%
Quartz	45
Albite	20
Microcline	10
Calcite	6
Montmorillonite	7
Illite-mica	3
Chlorite	3
Mixed-layer clays	4
Trace Minerals	2
	<hr/> 100

Estimated Soil-Melt Range: 1200 to 1320 °C

DUNE SAND

Typical of barchan sand dunes of world, representative of about 7% of total land area of world. Consists largely of siliceous sand, SiO_2 , comprising nearly 60% of oxides of earth's crust.

Soil Constituents	Estimated Amount wt%
Quartz	60
Oligoclase	25
Microcline	10
Clinopyroxene	5
	<hr/> 100

Estimated Soil-Melt Range: 1300 to 1450 °C

MAGNESIAN

Representative of soils derived from gabbros, basalts, and dolomites. MgO is one of the 12 most plentiful oxides of the earth's crust.

Soil Constituents	Estimated Amount wt%
Talc	35
Penninite	30
Oligoclase	25
Calcite	5
Trace minerals	5
	<hr/> 100

Estimated Soil-Melt Range: 1350 to 1435 °C

PODZOL

Leached organic soils of woodland regions of temperate zones of world, which comprise about a quarter of total land area of world.

Soil Constituents	Estimated Amount wt %
Quartz	60
Microcline	15
Albite	10
Chlorite	5
Hornblende	5
Trace Minerals	5
	<hr/> 100

Estimated Soil-Melt Range: 1350 to 1450 °C

COASTAL PLAINS CLAY Typical of well-weathered soils, representative of about 15% of total land area of world. Soil may seal and build gas pressure when saturated.

Soil Constituents	Estimated Amount wt%
Quartz	45
Kaolinite	25
Montmorillonite	25
Trace minerals	5
	<hr/> 100

Estimated Soil-Melt Range: 1600 to 1620 °C

LATERITE Representative of tropical humid regions of world, comprising more than 10% of total land area of world. FeO and Fe₂O₃ are two of most plentiful oxides of the earth's crust.

Soil Constituents	Estimated Amount wt%
Goethite	55
Gibbsite	30
Magnetite	5
Maghemite	5
Ilmenite	5
	<hr/> 100

Estimated Soil-Melt Range: 1900 to 1950 °C

ESTANCIA PLAYA Typical of very saline bolson playas of the world; most extreme chemical corrosive action upon heating.

Soil Constituents	Estimated Amount wt%
Gypsum	31.5
Calcite	13.0
Dolomite	24.5
Quartz	7.5
Feldspar	5.0
Halite	7.5
Clays	5.0
Trace minerals	6.0
	<hr/> 100.0

Estimated Soil-Melt Range: 1350 to 1400 °C

OTTAWA SAND Commercially available material that might possibly be used as a standard in comparing results from different laboratories.

Essentially pure quartz

Estimated Melting Point: 1720 °C

A private testing laboratory carried out measurements of the particle size distribution of the nine soil samples supplied by NBS. The results of these measurements are given in Table 3. The results of some additional characterization measurements are given in Table 4.

Table 3. Particle size analysis of the nine soils investigated. The table entries indicate, in weight percent, the cumulative amount finer than the size shown at the head of each column. The distribution of particle sizes smaller than 53 μ m was determined by a sedimentation process. See ASTM D-422.

Type of Soil	Maximum Particle Size, μ m											
	4760 (4) ^{a/}	2000 (10)	1680 (12)	595 (30)	250 (60)	149 (100)	74 (200)	53 (270)	20	10	5	2
Calcareous	-----	-----	100.0	61.5	43.7	36.6	30.4	28.6	---	---	---	---
Granitic Detrital	-----	100.0	-----	98.0	94.7	79.7	35.2	32.7	---	---	---	---
Dune Sand	100.0	95.4	-----	90.7	85.2	31.5	6.0	4.2	---	---	---	---
Magnesian	-----	100.0	-----	94.0	72.3	56.7	41.7	37.0	---	---	---	---
Podzol	100.0	99.8	-----	86.4	57.9	39.6	29.5	27.7	---	---	---	---
Coastal Plains	-----	-----	100.0	99.9	98.2	91.7	69.2	65.9	64.	62.	61.	60.
Laterite	-----	-----	100.0	82.4	73.5	68.4	60.6	57.4	45.	36.	30.	25.
Estancia Playa	-----	-----	100.0	99.5	98.0	95.5	89.3	86.7	82.	74.	54.	7.
Ottawa Sand ^{b/}	-----	100.0	-----	0.6	0	0	0	0	---	---	---	---

^{a/} The numbers in parentheses indicate the sieve sizes used (ASTM E-11).

^{b/} Natural silica sand from Ottawa, Ill., graded to pass a No. 20 (841 μ m) sieve and retained on a No. 30 (595 μ m) sieve. See ASTM C-190.

Table 4. Maximum bulk density, Atterberg limits, and specific gravity of the nine soils investigated

Type of Soil	Compaction Test ^{a/}		Atterberg Limit Tests ^{b/}			Specific Gravity ^{f/}
	Maximum Density	Moisture Content	Liquid Limit ^{c/}	Plastic Limit ^{d/}	Shrinkage Limit ^{e/}	
	kg m ⁻³	wt. %	wt. %	wt. %	wt. %	
Calcareous	2050	11	19	13	--	2.71
Granitic Detrital	1890	14	23	11	--	2.69
Dune Sand	1570	19	--	--	--	2.68
Magnesian	1770	17	30	18	--	2.75
Podzol	1790	17	25	17	--	2.67
Coastal Plains	1350	24	79	21	16	2.72
Laterite	1570	29	35	25	1	3.20
Estancia Playa	1570	25	53	22	18	2.91
Ottawa Sand	1760 ^{g/}	--	--	--	--	2.65

^{a/}ASTM D-698

^{b/}Measurements made on soil finer than 40 mesh (420 μ m)

^{c/}ASTM D-423

^{d/}ASTM D-424

^{e/}ASTM D-427

^{f/}ASTM D-854

^{g/}Compaction test not feasible; maximum density value obtained through vibration and saturation in container of same volume as used in compaction tests.

7. Appendix B: Results of Differential Thermal Analyses of Soil Samples

The soil samples were ground to pass through a 100-mesh sieve. After drying each sample at 140 °C for two hours, the analyses were performed using an R. L. Stone #230 differential thermal analysis (DTA) instrument. Each sample was placed in a platinum cup and tapped lightly to fill possible air spaces. The aluminum oxide powder used as a reference material was placed in a similar cup. The sample and reference cups were installed within a platinum enclosure in the furnace chamber. The temperature difference between the sample and reference cups was monitored by a differential thermocouple with a junction in each cup. The enclosure temperature was monitored by a thermocouple having its hot junction located between the sample and reference cups.

For each soil sample, DTA curves were obtained at heating rates of 10, 20, and 30 °C/minute with helium flowing through the sample enclosure; on certain samples, cooling curves at one or more rates were also obtained. Heating curves at 30 °C/minute with dry air flowing through the sample enclosure were also obtained for each sample. No significant differences were apparent between the DTA curves in helium and in air.

The following table summarizes the DTA results obtained on heating. The characteristic temperatures A, B, and D were determined as shown in figure 21.

In addition to carrying out differential thermal analyses (DTA) on each sample to about 1400 °C, thermogravimetric analyses (TGA) were made on each sample to 500 °C or higher in order to determine the amount of free or bound water coming out of the sample. The TGA measurements were made on undried samples in air at a heating rate of 15 °C/minute.

Representative results of the DTA and TGA tests are shown in figures 21 through 38.

Figure 21. Scheme utilized to determine the temperatures A, B, and D which characterize a particular endotherm or exotherm. The ordinate represents the temperature difference between the sample and reference cups; the abscissa represents the temperature of the sample enclosure.

Table 5. Differential Thermal Analysis Results from Heating Curves

Material	Run	Atmosphere	Heating Rate	Endotherm			Exotherm
				A	B	D	
			°C/min.	°C	°C	°C	°C
Calcareous	1	Helium	20	639	729	826	1268
	2	Helium	10	---	---	(372)	----
	3	Helium	10	704	715	782	----
	4	Helium	30	656	683	784	1265
	5	Helium	30	---	---	886	----
	6	Air	30	683	774	891	1262
Granitic Detrital	1	Helium	20	563	563	564	892
				683	683	738	---
				---	---	1156	---
	2	Helium	10	560	560	563	---
				---	---	720	---
	3	Helium	30	561	561	568	904
				725	725	782	---
				1119	1119	1160	---
	4	Air	30	562	562	570	---
				724	724	784	900
				1107	1107	1154	---
Dune Sand	1	Helium	20	---	---	224	---
				560	560	565	---
				---	---	694	---
	2	Helium	10	---	---	300	---
				561	561	567	---
	3	Helium	30	558	558	567	---
				639	660	766	---
	4	Air	30	567	567	571	---
Magnesian				635	675	767	---
	1	Helium	20	---	---	293	---
				---	---	563	---
				---	---	723	---
				---	---	791	---
	2	Helium	10	---	---	325	---
				---	---	581	---
				---	---	701	---
				---	---	716	---
	3	Helium	30	---	---	228	---
				397	526	589	---
				671	703	766	---
				---	---	1214	---
	4	Air	30	---	---	154	---
				---	---	241	---
				---	---	593	---
				671	698	770	---
				---	---	1208	---

Table 5. (cont)

Podzol	1	Helium	10	558	558	559	---
	2	Helium	20	570	570	576	---
	3	Helium	30	---	---	565	---
	4	Air	30	1009	1009	1009	---
Coastal Plains				563	563	574	---
	1	Helium	20	467	467	515	915
				---	---	571	---
	2	Helium	10	(270)	(331)	508	---
				---	---	567	---
	3	Helium	10	---	---	514	---
				---	---	561	---
				---	---	771	---
Laterite	4	Helium	30	(353)	(438)	537	914
	5	Air	30	---	438	539	914
	1	Helium	20	269	286	326	---
	2	Helium	10	263	287	323	---
	3	Helium	20	---	---	328	---
	4	Helium	30	276	287	334	---
	5	Air	30	300	313	348	---
Estancia Playa	1	Helium	10	---	---	111	---
				---	---	197	---
	2	Helium	20	---	---	121	---
				---	---	567	---
				---	---	672	---
				---	---	719	---
				---	---	1222	---
				---	---	1345	---
	3	Helium	20	---	---	154	---
				---	---	570	---
				---	---	676	---
				---	---	715	---
	4	Helium	30	---	---	129	---
				---	---	589	---
				---	---	648	---
				---	---	748	---
				---	---	1231	---
				---	---	1337	---
	5	Helium	10	---	---	111	---
				---	---	340	---
				---	---	556	---
				---	---	702	---
				---	---	887	---
				---	---	950	---
	6	Air	30	---	---	607	---
				---	---	658	---
				---	---	697	---
				---	---	766	---

Table 5. (cont)

Ottawa Sand	1	Helium	10	---	---	182	---
				559	559	559	---
	2	Helium	20	---	---	226	---
				562	562	562	---
	3	Helium	20	---	---	221	---
				567	567	567	---
	4	Helium	30	439	558	564	---
	5	Air	30	366	561	569	---

Figure 22. Thermogravimetric analysis of calcareous soil

CALCAREOUS SOIL

Figure 23. Differential thermal analyses of calcareous soil

Figure 24. Thermogravimetric analysis of granitic detrital soil

GRANITIC DETRITAL SOIL

Figure 25. Differential thermal analyses of granitic detrital soil

Figure 26. Thermogravimetric analysis of dune sand

DUNE SAND

Figure 27. Differential thermal analyses of dune sand

Figure 28. Thermogravimetric analysis of magnesian soil

MAGNESIAN SOIL

Figure 29. Differential thermal analyses of magnesian soil.

Figure 30. Thermogravimetric analysis of podzol soil

PODZOL SOIL

Figure 31. Differential thermal analyses of podzol soil.

Figure 32. Thermogravimetric analysis of coastal plains clay.

COASTAL PLAINS CLAY

Figure 33. Differential thermal analyses of coastal plains clay

Figure 34. Thermogravimetric analysis of laterite soil

LATERITE SOIL

Figure 35. Differential thermal analyses of laterite soil

Figure 36. Thermogravimetric analysis of estancia playa soil

ESTANCIA PLAYA SOIL

Figure 37. Differential thermal analyses of estancia playa soil

Figure 38. Thermogravimetric analysis of Ottawa sand

OTTAWA SAND

Figure 39. Differential thermal analyses of Ottawa sand

8. Appendix C. Sandia Post-Mortem of Soil Test Samples

All thermally tested soil samples were shipped by the NBS to Sandia in the brass cylinders in which they were tested, except for one coastal plains specimen which was opened at NBS per Sandia request and thus was not shippable. Each cylinder was sectioned at its midpoint; the sectioned surfaces then were photographed to permit visual comparison of thermal effects on the soils.

Definitive photos of the Ottawa sand specimens were not possible because test temperatures were too low to even sinter this high melt-range material; hence, these photographs were omitted.

The lone coastal plains specimen was photographed differently to show in better detail the sintered and melted portions of soil which surrounded the heater core. These views show typical gas-produced voids in the glass adjacent to the heater.

Examination of all the test specimens verified the test achievement of uniform heating at the hot side and uniform cooling at the outside diameter, with expected tapering at both ends. Also, the appearance of these laboratory test specimens is very similar to specimens retrieved from field burial tests conducted by Sandia.

Figure 40. Calcareous Soil - Maximum Test Temperature 1625°C , Estimated Melt Range $1950\text{-}2050^{\circ}\text{C}$

Figure 41. Granitic Detrital Soil - Maximum Test Temperature 1500°C, Estimated Melt Range 1200-1320°C

Figure 42. Dune Sand - Maximum Test Temperature 1600°C, Estimated Melt Range 1300-1450°C

Figure 43. Magnesian Soil - Maximum Test Temperature 1400°C , Estimated Melt Range $1350-1435^{\circ}\text{C}$

Figure 44. Podzol Soil - Maximum Test Temperature 1600°C , Estimated Melt Range $1350\text{-}1450^{\circ}\text{C}$

Figure 45. Coastal Plains Soil, Maximum Test Temperature 1650°C

Figure 46. Estimated Melt Range 1600-1620°C

Figure 47. Laterite Soil, Maximum Test Temperature 1650°C

Figure 48. Estancia Playa Soil - Maximum Test Temperature 1575°C , Estimated Melt Range $1350\text{-}1400^{\circ}\text{C}$

9. References

- [1] D. R. Flynn, A radial flow apparatus for determining the thermal conductivity of loose-fill insulations to high temperatures, J. Research NBS 67C, 129-137 (1963).
- [2] B. A. Peavy, Steady state heat conduction in cylinders with multiple continuous line heat source, J. Research NBS 67C, 119-127 (1963).
- [3] H. S. Carslaw and J. C. Jaeger, Conduction of Heat in Solids, 2nd Edition (Oxford University Press, 1959).
- [4] W. F. Roeser, Thermoelectric thermometry, J. Appl. Phys. 11 404-405 (1940); also in Temperature, Its Measurement and Control in Science and Industry, p. 202 (Reinhold Publishing Corp., New York, New York, 1941).
- [5] This equation was fitted to the recommended values for the thermal conductivity of alumina as given by R. W. Powell, C. Y. Ho, and P. E. Liley, Thermal Conductivity of Selected Materials, NSRDS-NBS 8 (November, 1966).
- [6] J. L. Post, The Fusion of Soils, SC-CR-67-2688 (July, 1967).

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