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Internal Report

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MASTER

COMPARISON OF LUMPED AND DISTRIBUTED INFLECTOR MAGNETS

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I. Introduction

The use of pulsed magnets as inflectors or ejectors for large accelerators was first proposed by G.K. O'Neill and V. Koreman^{1,2}.

These magnets have a field build-up time which is no longer than the time between particle bunches. The rise period is followed by a flattop of sufficient length to achieve the desired objective, for example the ejection of a beam. The flux required in the pulsed magnet is minimized by using an associated dc magnet system with high fields coupled by the betatron oscillations produced in the beam by the pulsed magnet.

Pulsed magnets designed for this application usually have an elaborate mechanical construction necessitated by the fast rise time requirement. This report considers the effect on electrical and magnetic characteristics of modifications to achieve a simpler construction.

II. The O'Neill -Koremen Magnet

O'Neill and Koreman proposed that the characteristics outlined above could be obtained by loading the magnet with distributed capacitance. The magnet behaves electrically as a transmission line with some characteristic impedance determined by the loading capacitors. The

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flattop is generated by discharging a pulse forming network into the magnet terminated with a suitable resistor. The circuit is similar to a line type radar modulator, see Fig. 1. When the switch is closed a fairly uniform magnetic field is established in the gap. The field propagates down the magnet with a velocity which is dependent on the per unit inductance and capacitance.

The behavior of the distributed magnet has been analyzed by several writers³ on the assumption that the magnet can be regarded as an infinitely distributed transmission line. In practice the magnet has a finite number of L - C sections, usually between ten and twenty. Thus insertion of the magnet in the discharge circuit shown in Fig. 1 causes degeneration of the inherent current rise time associated with the pulse-forming network. The field build-up time depends on the ultimate current rise time and the field propagation time down the magnet.

Consider the magnet shown in Fig. 1. If the dimensions are in centimeters and the ferrite permeability is high, then

$$L_2 = \frac{4\pi}{10^9} \frac{w}{h} l \quad \text{henries} \quad - - - (1)$$

The aperture and length are fixed by considerations arising out of the application. The aperture is set by the maximum betatron amplitude in the accelerator; this requirement is sometimes minimized by moving the magnet mechanically during the accelerating cycle. The magnet length is set by the required deflecting force. This means the designer must find some compromise solution by juggling such factors as the maximum pfn voltage, the lowest pfn impedance, the build-up time, etc. For any particular application we find L_2 is fixed. Thus the total distributed

capacity is given by:

$$C_2 = \frac{L_2}{R_o^2} \quad - - - (2)$$

where R_o is the characteristic impedance of the system.

The equivalent circuit of the discharge system is given in Fig. 2. " L_1 " is the equivalent series inductance of the switching device. The pfn is shown as a voltage generator E , in series with a resistor R_o . This equivalent circuit is adequate if the analysis is limited to time during the field build-up. " E " is the network charging voltage.

The field build-up time is seen to depend on three factors:

1. The series inductance L_1 .
2. Front edge degeneration because " n " is not infinite.
3. Propagation time of the magnet.

We will consider each effect separately.

2.1 " L_1 " introduces an inductive time constant into the expression for magnet current. The time constant is:

$$T_1 = \frac{L_1}{2R_o} \quad - - - (3)$$

and time from 10% to 90% points is

$$t_1 = 2.2 \frac{L_1}{2R_o} \quad - - - (4)$$

Substituting from Eq. (2):

$$t_1 = 1.1 L_1 \sqrt{\frac{C_2}{L_2}} \quad - - - (5)$$

2.2 The degeneration of rise time in a transmission line containing a finite number of L - C sections is a matter requiring involved theoretical considerations⁴. To a close approximation we find:

$$n = 1.1 \left(\frac{td}{t_2} \right)^{1.5} \quad - - - (6)$$

where n = number of sections

td = total delay time of line.

t₂ = rise time (10 - 90%).

$$\text{Now } td = \sqrt{L_2 C_2} \quad - - - (7)$$

thus re-arranging Eq. (6):

$$t_2 = \left(\frac{n}{1.1} \right)^{-2/3} \sqrt{L_2 C_2} \quad - - - (8)$$

2.3 The propagation time is given by Eq. (7), thus the time for 80% "fill" is:

$$t_3 = 0.8 \sqrt{L_2 C_2} \quad - - - (9)$$

2.4 The combined effect achieves an ultimate field build-up time given by:

$$t_r = \sqrt{t_1^2 + t_2^2} + t_3,$$

substituting:

$$t_r = \sqrt{(1.1)^2 L_1^2 \left(\frac{C_2}{L_2} \right) + \left(\frac{n}{1.1} \right)^{-4/3} (L_2 C_2)} + 0.8 \sqrt{L_2 C_2} \quad - - - (10)$$

If the cases for n = 11 and 22, and L₁ = 0 or L₁ = 1/2 L₂ are considered the following table for t_r can be made.

	n = 11	n = 22
$L_1 = 0$	$1.02 \sqrt{L_2 C_2}$	$0.94 \sqrt{L_2 C_2}$
$L_1 = \frac{L_2}{2}$	$1.38 \sqrt{L_2 C_2}$	$1.34 \sqrt{L_2 C_2}$

TABLE I. Values of t_r

The table shows that it is important to keep L_1 small, but little advantage is gained by making "n" large.

III. The Lumped Magnet

The picture frame magnet shown in Fig. 3 has the same magnetic characteristics as the O'Neill magnet. Thus the inductance is the same for any given application. The magnet is connected to the same pfn, though a capacitor is used in shunt with the load to decrease rise time. The circuit has a resemblance to a shunt compensated video amplifier.

Using the notation of Fig. 4 it can be shown the magnet current is given by:

$$I = E \left(\frac{1}{2R_o} + \frac{1}{R_o} \left(\frac{R_o}{L} - \frac{1}{\frac{1}{2} (C_1 R_o + \frac{L}{R_o})} \right) \right) e^{-\frac{t}{T_2}} - \frac{1}{2R_o} e^{-\frac{t}{T_2}} \quad (11)$$

where $L = L_1 + L_2$

$$\text{and } \frac{1}{T_2} = \frac{1}{2} \left(\frac{1}{R_o C_1} + \frac{R_o}{L} \right) \quad (12)$$

If C_1 is chosen for critical damping the value must satisfy:

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$$\left(\frac{1}{R_o C_1} + \frac{R_o}{L}\right)^2 = \frac{8}{LC_1} \quad - - - (13)$$

this can be solved to yield:

$$C_1 \approx \frac{1}{6} \frac{L}{R_o^2} \quad - - - (14)$$

Combining Eq. (12) and (13) yields

$$T_2 = \frac{1}{\sqrt{2}} \sqrt{LC_1} \quad - - - (15)$$

and the rise time is thus

$$t_4 = \frac{2.2}{\sqrt{2}} \sqrt{LC_1} = 1.56 \sqrt{LC_1} \quad - - - (16)$$

The capacitance per unit length of the lumped magnet is less than the distributed magnet by several orders of magnitude. Thus the propagation velocity approaches the speed of light. For most applications the propagation time is small and can be ignored. Hence the field build-up time is given by Eq. (16), yielding the following values of t_4 and C_1 for the range of L_1 considered in Par. 2.

	$L_1 = 0$	$L_1 = \frac{L_2}{2}$
t_4	$1.56 \sqrt{L_2 C_1} = 0.64 \sqrt{L_2 C_2}$	$1.9 \sqrt{L_2 C_1} = 0.95 \sqrt{L_2 C_2}$
C_1	$\frac{1}{6} C_2$	$\frac{1}{4} C_2$

TABLE II. Values of t_4 and C_1

IV. Conclusion

A comparison of Tables I and II will show that the field build-up time with either kind of magnet is comparable in each case. The figures are calculated for 80% of field rise. In the case of the distributed magnet the time is linearly related to this factor of 0.8. For the lumped magnet the build-up is, of course, exponential and complete fill requires many time constants. In practice the exponential character can be improved by mismatching the first section of the pulse-forming network. On the credit side of the lumped magnet system is the greatly simplified construction. The shunt capacitor, " C_1 ", can be a commercial item mounted at the load. Its value is only a fraction of that capacitance required for the distributed magnet.

It is proposed to build the fast kicker magnet of the AGS beam ejector system using a lumped magnet. A further report will describe the design and results obtained with this magnet.

Acknowledgment

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Fig. 1a Construction of Distributed Magnet.

Fig. 1b Discharge Circuit.

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Fig. 2 Equivalent Circuit of Discharge System.

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Fig. 3 "Picture Frame" Magnet.

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Fig. 4 Equivalent Circuit of Lumped Magnet Discharge System.