CROSS-SECTION SENSITIVITY OF BREEDING RATIO IN A FUSION-REACTOR BLANKET

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Abstract

For a particular fusion-reactor-blanket configuration, the changes in the tritium breeding ratio, i.e., in the number of tritium nuclei produced in the blanket per incident neutron, due to changes in nuclear cross-section data are calculated on the basis of linear perturbation theory. Results are presented for the changes in the breeding ratio due to changes in specific energy ranges of various partial cross sections of ${}^6\text{Li}$, ${}^7\text{Li}$, Nb, and C. The breeding ratio is found to be most sensitive to changes in the ${}^7\text{Li}(n,n')\alpha,t$ cross section, but the sensitivity to changes in this cross section is not large.

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I. INTRODUCTION

A power-producing fusion reactor will contain a plasma that is surrounded by a "blanket" that must have rather specific nuclear engineering properties. The design of such blankets has been studied by several investigators.* If the reactor is to operate on deuterium-tritium fusion, then tritium must be produced in the blanket at a rate that is larger than the rate of consumption of tritium in the plasma; i.e., the breeding ratio, defined to be the number of tritium nuclei produced in the blanket per incident neutron, must be greater than unity. This breeding ratio has been calculated for several configurations (see, for example, ref. 1), and usually a value greater than unity is obtained. However, the cross-section data available for many of the elements that are often used in blanket design are uncertain, and this introduces an unknown uncertainty in the calculated breeding ratios. In the present paper, the change in the breeding ratio in a particular blanket configuration due to changes in cross-section data is studied using linear perturbation theory. The power of the method employed here lies in the fact that the results of a forward and adjoint transport calculation are sufficient to obtain an estimate of the change in the breeding ratio due to a change in any partial cross section in any energy region and in any spatial region of the blanket. The limitation of the method lies in the fact that the range of validity of linear perturbation theory is dependent on the problem being considered and, in general, can be determined only by solving the perturbed problem.

*See ref. 1 and the references given therein.

In Section II linear perturbation theory as used here is described briefly, and the details of the calculations are given. In Section III the results are presented and discussed. In particular, in Section III the changes in the breeding ratios for assumed uncertainties in several partial cross sections are given and compared with calculated results obtained without the use of perturbation theory.

II. THEORY AND CALCULATIONAL DETAILS

The linear perturbation theory used here represents an application of the theory developed previously by several authors.²⁻⁷ For a more general discussion than that given below, the reader is referred to refs. 2-7, particularly ref. 5.

The blanket model considered here is the standard model that has been proposed for comparison of neutron-transport calculations⁸ and is shown schematically in Fig. 1. The tritium breeding ratio, R , is defined to be the number of tritium nuclei produced in the blanket per neutron incident on the blanket. This tritium breeding ratio for the perturbed and unperturbed systems may be expressed as

$$R_i = \frac{\int \Sigma_{ri}(\vec{r}, E) \phi_i(\vec{r}, E, \vec{\Omega}) d\vec{r} dE d\vec{\Omega}}{\int S(\vec{r}, E, \vec{\Omega}) d\vec{r} dE d\vec{\Omega}}, \quad (1)$$

where the subscript i takes values p for the perturbed system and u for the unperturbed system, the integrations are to be carried out over all phase space, and

$\phi_i(\vec{r}, E, \vec{\Omega})$ = the angular flux per unit energy range at position $\vec{r}$,
 $\Sigma_{ri}(\vec{r}, E)$ = the macroscopic tritium-production cross section at
 energy E and position $\vec{r}$,

Fig. 1. Schematic diagram of the geometry used in the calculations.

$S(\vec{r}, E, \vec{\Omega}) dE d\vec{\Omega}$ = the source neutrons per unit volume per unit time
emitted at $\vec{r}$ with energy in dE about E and direction
in $d\vec{\Omega}$ about $\vec{\Omega}$.

For convenience, it is assumed hereinafter that the source when integrated over all phase space is normalized to unity. For present purposes, the tritium production, Σ_{ri} , is the sum of the cross sections for the ${}^6\text{Li}(n, t)\alpha$ reaction and the ${}^7\text{Li}(n, n')\alpha, t$ reaction. If, now, δR is defined by the equation

$$\delta R = R_p - R_u, \quad (2)$$

then

$$\delta R = \int \left[\Sigma_{rp}(\vec{r}, E) \phi_p(\vec{r}, E, \vec{\Omega}) - \Sigma_{ru} \phi_u(\vec{r}, E, \vec{\Omega}) \right] d\vec{r} dE d\vec{\Omega} \quad (3)$$

where it has been assumed that the tritium-production cross section, as well as other cross sections, is being perturbed.

To further transform Eq. 3, it is necessary to introduce the time-independent neutron-transport equation and the time-independent adjoint equation. For both the perturbed and unperturbed systems, these equations may be written

$$H_i \phi_i = S(\vec{r}, E, \vec{\Omega}), \quad (4)$$

$$H_i^* \phi_i^* = \Sigma_{ri}(\vec{r}, E), \quad (5)$$

$$\begin{aligned} H_i \phi_i &= \vec{\Omega} \cdot \nabla \phi_i(\vec{r}, E, \vec{\Omega}) + \Sigma_{Ti}(\vec{r}, E) \phi_i(\vec{r}, E, \vec{\Omega}) \\ &- \int dE' d\vec{\Omega}' \Sigma_{Si}(\vec{r}, E' \rightarrow E, \vec{\Omega}' \rightarrow \vec{\Omega}) \phi_i(\vec{r}, E', \vec{\Omega}'), \end{aligned} \quad (6)$$

$$\begin{aligned} H_i^* \phi_i^* &= - \vec{\Omega} \cdot \nabla \phi_i^*(\vec{r}, E, \vec{\Omega}) + \Sigma_{Ti}(\vec{r}, E) \phi_i^*(\vec{r}, E, \vec{\Omega}) \\ &- \int dE' d\vec{\Omega}' \Sigma_{Si}(\vec{r}, E \rightarrow E', \vec{\Omega} \rightarrow \vec{\Omega}') \phi_i^*(\vec{r}, E', \vec{\Omega}'), \end{aligned} \quad (7)$$

where

$\Sigma_{Ti}(\vec{r}, E)$ = the total macroscopic cross section at position $\vec{r}$ and energy E ,

$\Sigma_{Si}(\vec{r}, E' \rightarrow E, \vec{\Omega}' \rightarrow \vec{\Omega}) dE d\vec{\Omega}$ = the differential scattering cross section which describes the probability per unit pathlength that a neutron with energy E' going in the direction $\vec{\Omega}'$ will undergo a collision at point $\vec{r}$ and that a neutron will emerge in the energy element dE about E and in the solid angle element $d\vec{\Omega}$ about $\vec{\Omega}$,

$\phi_i^*(\vec{r}, E, \vec{\Omega})$ = the adjoint flux at $\vec{r}$, E , and $\vec{\Omega}$.

It is to be noted that the same source S is used in both the perturbed and unperturbed transport equation. The perturbed and unperturbed adjoint equations may be combined to give

$$H_p^*[\phi_p^* - \phi_u^*] = [H_u^* - H_p^*] \phi_u^* + \Sigma_{rp}(\vec{r}, E) - \Sigma_{ru}(\vec{r}, E) , \quad (8)$$

and this equation, together with the definition of the adjoint operator, may be used in conjunction with Eq. 3 to give

$$\begin{aligned} \delta R = & \int \phi_p(\vec{r}, E, \vec{\Omega}) [H_u^* - H_p^*] \phi_u^* d\vec{r} dE d\vec{\Omega} \\ & + \int [\Sigma_{rp}(\vec{r}, E) - \Sigma_{ru}(\vec{r}, E)] \phi_p(\vec{r}, E, \vec{\Omega}) d\vec{r} dE d\vec{\Omega} . \end{aligned} \quad (9)$$

Equation 9 is exact and is valid for any specification of the perturbed cross sections. Here the perturbed differential cross sections will be specified to be proportional to the unperturbed differential cross sections in all

regions of phase space where the perturbed and unperturbed differential cross sections are assumed to be unequal; that is, it is assumed that

$$\Sigma_{sp}(\vec{r}, E \rightarrow E', \vec{\Omega} \rightarrow \vec{\Omega}') = C \Sigma_{su}(\vec{r}, E \rightarrow E', \vec{\Omega} \rightarrow \vec{\Omega}') , \quad (10)$$

where C is a constant which is independent of $\vec{r}$, E , E' , $\vec{\Omega}$, and $\vec{\Omega}'$ in all regions of phase space in which $C \neq 1$. It is to be understood that Eq. 10 may be applied to any partial differential cross section with $C = 1$ for all other partial differential cross sections.

From Eq. 10 it follows that in the perturbed region of phase space

$$\Sigma_{Tp}(\vec{r}, E) = C \Sigma_{Tu}(\vec{r}, E) . \quad (11)$$

If δC is now defined by

$$\delta C = C - 1 , \quad (12)$$

Eqs. 10 and 11 may be used to obtain

$$\delta C = \frac{\Sigma_{sp}(\vec{r}, E \rightarrow E', \vec{\Omega} \rightarrow \vec{\Omega}') - \Sigma_{su}(\vec{r}, E \rightarrow E', \vec{\Omega} \rightarrow \vec{\Omega}')}{\Sigma_{su}(\vec{r}, E \rightarrow E', \vec{\Omega} \rightarrow \vec{\Omega}')} \quad (13)$$

and

$$\delta C = \frac{\Sigma_{Tp}(\vec{r}, E) - \Sigma_{Tu}(\vec{r}, E)}{\Sigma_{Tu}(\vec{r}, E)} , \quad (14)$$

so δC is the fractional change in the cross sections and is to be independent of $\vec{r}$, E , E' , $\vec{\Omega}$, and $\vec{\Omega}'$ in all regions of phase space where it is unequal to zero. In terms of δC , the quantity $[H_u^* - H_p^*] \Phi_u^*$, which occurs in Eq. 9, may be written

$$\begin{aligned}
[H_u^* - H_p^*] \phi_u^* &= \delta C [-\Sigma_{Tu}(\vec{r}, E) \phi_u^*(\vec{r}, E, \vec{\Omega}) \\
&+ \int dE' d\vec{\Omega}' \Sigma_{Su}(\vec{r}, E \rightarrow E', \vec{\Omega} \rightarrow \vec{\Omega}') \phi_u^*(\vec{r}, E', \vec{\Omega}')], \quad (15)
\end{aligned}$$

and the expression for δR becomes

$$\begin{aligned}
\delta R &= \left\{ \int \phi_p(\vec{r}, E, \vec{\Omega}) [H_u^* - H_p^*] \phi_u^* d\vec{r} dE d\vec{\Omega} \right. \\
&+ \left. \delta C \int \phi_p(\vec{r}, E, \vec{\Omega}) \Sigma_{Tu}(\vec{r}, E) d\vec{r} dE d\vec{\Omega} \right\}, \quad (16)
\end{aligned}$$

where Eq. 15 is to be used in Eq. 16 and the integrations in Eq. 16 are to be performed only over the perturbed regions of phase space in which $\delta C \neq 0$. It is to be understood that the second term on the right-hand side of Eq. 16 is present only if the tritium-production cross sections are perturbed. Equation 16 gives the change in δR for a given δC and is still exact for the assumed form of the perturbed cross sections. The right-hand side of Eq. 16, however, cannot be readily evaluated because ϕ_p depends on the choice of δC and can be determined only by solving the perturbed transport equation. The basic assumption of linear perturbation theory used here is that for small changes in cross sections ϕ_p in Eq. 16 may be approximated by ϕ_u , so

$$\begin{aligned}
\delta R &= \left[\int \phi_u(\vec{r}, E, \vec{\Omega}) [H_u^* - H_p^*] \phi_u^* d\vec{r} dE d\vec{\Omega} \right. \\
&+ \left. \delta C \int \phi_u(\vec{r}, E, \vec{\Omega}) \Sigma_{Tu}(\vec{r}, E) d\vec{r} dE d\vec{\Omega} \right]. \quad (17)
\end{aligned}$$

In this approximation, δR is proportional to δC (see Eq. 15) and may readily be evaluated for a complete range of values of δC . The results in Section III will be given in terms of $\frac{\delta R}{\delta C}$ and therefore will be independent of the magnitude of δC . It must be emphasized, however, that the range of validity

of the perturbation assumption is dependent on the problem being considered and, in general, can be determined only by solving the perturbed problem.

In considering Eq. 17, it must be remembered that δC is nonzero only in the perturbed region of phase space. Thus it is possible by the appropriate choice of the domain in which δC is taken to be nonzero to obtain the change in the breeding ratio R due to a change in a particular partial cross section in a given energy and spatial region. Furthermore, since δR is proportional to δC , it is possible to obtain the change in R due to different changes in cross sections in different energy and spatial regions. It is these properties of Eq. 17 that are exploited in obtaining the results presented in Section III.

All of the transport calculations considered here were carried out with the one-dimensional discrete ordinates code ANISN⁹ using P_3 S_{12} . The perturbation calculations, i.e., the evaluation of Eq. 17, were carried out with the code SWANLAKE.⁶ The cross-section data used in obtaining all of the results presented here were taken from ENDF/B-III, MAT. 1164 for Nb, MAT. 1115 for ${}^6\text{Li}$, MAT. 1116 for ${}^7\text{Li}$, and MAT. 1165 for C. The ${}^6\text{Li}$ and ${}^7\text{Li}$ total, elastic, and tritium production cross sections are shown as a function of energy in Appendix A.

In presenting the results, it will be instructive to distinguish between tritium production in ${}^6\text{Li}$ and ${}^7\text{Li}$. For this purpose, we define

$$R = R_{{}^6\text{Li}} + R_{{}^7\text{Li}}$$

$$R_{{}^6\text{Li}} = \int \Sigma_{{}^6\text{Li}u}(\vec{r}, E) \phi_u(\vec{r}, E, \vec{\Omega}) d\vec{r} dE d\vec{\Omega}$$

$$R_{{}^7\text{Li}} = \int \Sigma_{{}^7\text{Li}u}(\vec{r}, E) \phi_u(\vec{r}, E, \vec{\Omega}) d\vec{r} dE d\vec{\Omega} ,$$

where

$\Sigma_{6\text{Liu}}(\vec{r}, E)$, $\Sigma_{7\text{Liu}}(\vec{r}, E)$ = the unperturbed macroscopic tritium-
production cross sections at energy E
and position $\vec{r}$ in ${}^6\text{Li}$ and ${}^7\text{Li}$,
respectively,

and

$$\delta R = \delta R_{6\text{Li}} + R_{7\text{Li}}$$

$$\begin{aligned} \delta R_{6\text{Li}} = & \delta C \left\{ \int \phi_u(\vec{r}, E, \vec{\Omega}) [H_u^* - H_p^*] \phi_{6\text{Liu}}^* d\vec{r} dE d\vec{\Omega} \right. \\ & \left. + \int \phi_u(\vec{r}, E, \vec{\Omega}) \Sigma_{6\text{Liu}}(\vec{r}, E) d\vec{r} dE d\vec{\Omega} \right\} \end{aligned}$$

$$\begin{aligned} \delta R_{7\text{Li}} = & \delta C \left\{ \int \phi_u(\vec{r}, E, \vec{\Omega}) [H_u^* - H_p^*] \phi_{7\text{Liu}}^* d\vec{r} dE d\vec{\Omega} \right. \\ & \left. + \int \phi_u(\vec{r}, E, \vec{\Omega}) \Sigma_{7\text{Liu}}(\vec{r}, E) d\vec{r} dE d\vec{\Omega} \right\} , \end{aligned}$$

where

$\phi_{6\text{Liu}}^*$, $\phi_{7\text{Liu}}^*$ = the adjoint flux at $\vec{r}$, E , and $\vec{\Omega}$ when cross sections
 $\Sigma_{6\text{Liu}}$ and $\Sigma_{7\text{Liu}}$ are used in Eq. 5.

III. RESULTS AND DISCUSSION

Calculations have been carried out to determine the change in the tritium breeding ratio due to a variety of changes in the ${}^6\text{Li}$, ${}^7\text{Li}$, Nb, and C cross sections. In all cases, the contributions to the change in the breeding ratio due to changes in tritium production in ${}^6\text{Li}$ and ${}^7\text{Li}$ are presented separately. The unperturbed cross sections give breeding ratios of

$$R_{{}_6\text{Li}} = 0.941 \frac{\text{tritium nuclei}}{\text{incident neutron}}$$

$$R_{{}_7\text{Li}} = 0.527 \frac{\text{tritium nuclei}}{\text{incident neutron}}$$

$$R = 1.468 \frac{\text{tritium nuclei}}{\text{incident neutron}}$$

In Fig. 2 the changes in the breeding ratios (i.e., $\delta R_{{}_6\text{Li}}$ and $\delta R_{{}_7\text{Li}}$), caused by an increase in the ${}^6\text{Li}(n,t)\alpha$ cross section in specific energy ranges, are shown as a function of energy. The ordinate in the figure gives the quantity $\frac{1}{\Delta u} \frac{1}{R} \frac{\delta R_j}{\delta C}$, where Δu is the change in lethargy defined by

$$\Delta u = \ln \left[\frac{E_{i+1}}{E_i} \right],$$

where E_i and E_{i+1} are the energy limits of the histogram intervals given in the figure, and the other quantities have been defined in Section II. Each histogram value in the figure corresponds to a different choice of the energy region over which δC has been taken to be nonzero; that is, in determining the histogram values in a given energy region the perturbed and unperturbed cross sections are taken to differ by a constant in the energy interval and to be equal outside of the energy interval. The histogram values given in the figure are independent of the magnitude of δC , since it was shown in Section II that δR_j is proportional to δC in the approximation used here. The

Fig. 2. Breeding-ratio sensitivity per unit lethargy vs energy for the ${}^6\text{Li}(n,t)\alpha$ cross section.

histogram values are divided by Δu in order to make the values approximately independent of the width of the histogram interval. Division by $\Delta E (= E_{i+1} - E_i)$ would have been equally satisfactory, but Δu is used here since a logarithmic scale is used on the abscissa. For plotting purposes, in Fig. 2 and throughout the paper the lethargy interval, Δu , for the thermal group is defined to be $\ln 4.14$.

When multiplied by Δu , each histogram value in Fig. 2 may be thought of as being the percent change in the breeding ratio R_j for a 1% increase in the ${}^6\text{Li}(n,t)\alpha$ cross section in a given energy interval. In Fig. 2 and throughout the paper, negative values of the quantity $\frac{1}{\Delta u} \frac{1}{R} \frac{\delta R_j}{\delta C}$ are indicated by solid lines and positive values of this quantity are indicated by dashed lines. The quantity $\frac{1}{\Delta u} \frac{1}{R} \frac{\delta R}{\delta C}$, of course, can be obtained by adding the two histograms. If the histograms in Fig. 2 are integrated over lethargy, the resulting values are the percent changes in the breeding ratios R_j for a 1% increase in the ${}^6\text{Li}(n,t)\alpha$ cross section at all energies. Since each histogram value is independent, it is also possible to multiply each histogram value by a different value of δC before carrying out the integration over lethargy and in this manner obtain the percent change in R_j when the percent change in the cross section is not the same at all energies. Histograms such as those shown in Fig. 2 will hereinafter be referred to as breeding-ratio-sensitivity profiles for a particular cross section.

In Fig. 2 the ${}^6\text{Li}$ breeding ratio shows a positive sensitivity to increases in the ${}^6\text{Li}(n,t)\alpha$ cross section at all energies. This is to be expected since by increasing the breeding cross section the probability that neutrons contribute to the breeding is increased and the probability that neutrons are lost from the system without contributing to the breeding is

decreased. In Fig. 2 and in most of the figures, there is usually an increase in sensitivity in the highest energy groups because of the high-energy source used. The depression in the sensitivity curve at approximately 2.8×10^{-1} MeV is due to the fact that the ${}^6\text{Li}$ and ${}^7\text{Li}$ total cross sections have a resonance in this energy region.*¹⁰ The ${}^6\text{Li}$ breeding ratio is most sensitive to changes in the ${}^6\text{Li}(n,t)\alpha$ cross section in the vicinity of the resonance in this cross section at approximately 2.3×10^{-1} MeV.

In Fig. 2 the ${}^7\text{Li}$ breeding ratio shows a negative sensitivity to increases in the ${}^6\text{Li}(n,t)\alpha$ cross section for energies ≥ 3 MeV. This is due to the fact that an increase in the ${}^6\text{Li}(n,t)\alpha$ cross section at these higher energies removes neutrons from the system, and this reduces the possibility of a ${}^7\text{Li}(n,n')\alpha,t$ reaction. The ${}^7\text{Li}(n,n')\alpha,t$ reaction has a threshold of approximately 3 MeV, and thus the sensitivity of the ${}^7\text{Li}$ breeding ratio to increases in the ${}^6\text{Li}(n,t)\alpha$ cross section is zero below this threshold; i.e., changes in the ${}^6\text{Li}(n,t)\alpha$ cross section below this threshold do not change the neutron flux above this threshold and thus do not affect the ${}^7\text{Li}$ breeding ratio.

The sensitivity of the breeding ratios to increases in the ${}^7\text{Li}(n,n')\alpha,t$ cross section in specific energy ranges is shown in Fig. 3 as a function of energy. The ${}^6\text{Li}$ breeding ratio shows a negative sensitivity to increases in the ${}^7\text{Li}(n,n')\alpha,t$ cross section at energies ≥ 10 MeV and a positive sensitivity to increases in the ${}^7\text{Li}(n,n')\alpha,t$ cross section at energies ≤ 10 MeV. The negative sensitivity at the higher energies is probably due to the fact that an increase in the ${}^7\text{Li}(n,n')\alpha,t$ reaction decreases the number of $(n,2n)$

*The ${}^6\text{Li}$ and ${}^7\text{Li}$ total, elastic, and tritium production cross sections are shown as a function of energy in Appendix A.

Fig. 3. Breeding-ratio sensitivity per unit lethargy vs energy for the ${}^7\text{Li}(n,n')\alpha,t$ cross section.

reactions and thereby decreases the number of neutrons available for the ${}^6\text{Li}(n,t)\alpha$ reaction. At the lower energies, i.e., at energies where the $(n,2n)$ cross sections have become small or zero, the sensitivity is positive because the neutrons from the ${}^7\text{Li}(n,n')\alpha,t$ reaction are emitted into a lower energy range where the ${}^6\text{Li}$ breeding cross section is higher. This latter effect is also present at the higher energies where there are appreciable $(n,2n)$ cross sections, but at these energies the sensitivity is presumably dominated by the decrease in the number of $(n,2n)$ reactions.

The ${}^7\text{Li}$ breeding ratio in Fig. 3 shows a positive sensitivity to increases in the ${}^7\text{Li}$ breeding cross section as it must, since increasing the breeding cross section necessarily increases the breeding ratio. The ${}^7\text{Li}$ breeding ratio is considerably more sensitive to the ${}^7\text{Li}(n,n')\alpha,t$ cross section than is the ${}^6\text{Li}$ breeding ratio.

The sensitivity of the breeding ratios to increases in the ${}^6\text{Li}$ total collision cross section in specific energy ranges is shown in Fig. 4 as a function of energy.* The ${}^6\text{Li}$ breeding-ratio sensitivity profile is positive at almost all energies but becomes negative in the vicinity of 4 MeV. The very rapid change of the ${}^6\text{Li}$ breeding ratio in the vicinity of 4 MeV is probably due to the fact that the ${}^6\text{Li}$ and ${}^7\text{Li}$ total collision cross sections have resonances in this general energy region. It is to be noted that by comparing Figs. 4 and 2 the sensitivity of the ${}^6\text{Li}$ breeding ratio to increases in the ${}^6\text{Li}$ total cross section is very nearly the same as the sensitivity to increases in the ${}^6\text{Li}(n,t)\alpha$ cross section at energies ≤ 2 MeV. The ${}^7\text{Li}$ breeding ratio in Fig. 4 shows a rather appreciable negative sensitivity to increases in the ${}^6\text{Li}$ total collision cross section at the higher

*Breeding-ratio sensitivity profiles for the partial cross sections of ${}^6\text{Li}$ and the other elements in the blanket, i.e., ${}^7\text{Li}$, Nb, and C, are given in Appendix B.

Fig. 4. Breeding-ratio sensitivity per unit lethargy vs energy for the ^6Li total collision cross section.

energies. By comparing the ${}^7\text{Li}$ breeding-ratio histograms in Figs. 2 and 4, it can be seen that the ${}^7\text{Li}$ breeding ratio is considerably more sensitive to the ${}^6\text{Li}$ total cross section than to the ${}^6\text{Li}(n,t)\alpha$ cross section.

The sensitivity of the breeding ratios to increases in the ${}^7\text{Li}$ total collision cross section in specific energy ranges is shown in Fig. 5 as a function of energy. The ${}^6\text{Li}$ breeding ratio has both positive and negative sensitivity to the ${}^7\text{Li}$ total cross section depending on the energy range. The rapid change in the ${}^6\text{Li}$ sensitivity in the vicinity of 10 MeV is probably due in part to the ${}^7\text{Li}(n,n')\alpha,t$ cross section, as discussed in regard to Fig. 3, and in part to the ${}^7\text{Li}(n,2n')$ cross section.* The peak in the ${}^6\text{Li}$ breeding ratio at approximately 2×10^{-1} MeV is similar to, but opposite in sign from, that which occurs in this energy region in the ${}^6\text{Li}$ breeding ratio shown in Fig. 2 and, again, is probably due to the resonance in the ${}^6\text{Li}(n,t)\alpha$ cross section at 2.3×10^{-1} MeV. The very pronounced structure in the ${}^6\text{Li}$ breeding-ratio histogram in the vicinity of 10^{-4} to 10^{-3} MeV is probably due to resonances in the niobium cross section. The sensitivity of the ${}^7\text{Li}$ breeding ratio to increases in the ${}^7\text{Li}$ total collision cross section is positive and is very similar to its sensitivity to increases in the ${}^7\text{Li}(n,n')\alpha,t$ cross section (see Fig. 3).

The sensitivity of the breeding ratios to the niobium total collision cross section in specific energy regions is shown in Fig. 6 as a function of energy. The positive sensitivity of the ${}^6\text{Li}$ breeding ratio at the higher energies (≥ 9 MeV) to increases in the niobium total cross section is due primarily to increases in the niobium $(n,2n)$ cross section.* The pronounced lack of sensitivity of the ${}^6\text{Li}$ breeding ratio in the vicinity of 4 MeV is reminiscent of that shown in Fig. 4 in this energy region and probably arises

*See Appendix B.

Fig. 5. Breeding-ratio sensitivity per unit lethargy vs energy for the ^7Li total collision cross section.

Fig. 6. Breeding-ratio sensitivity per unit lethargy vs energy for the Nb total collision cross section.

from the fact that the ${}^6\text{Li}$ and ${}^7\text{Li}$ total cross sections have resonances in this energy region. The local minimum in the ${}^6\text{Li}$ sensitivity profile in the vicinity of 3×10^{-1} MeV is probably associated with the resonance in the ${}^6\text{Li}(n,t)\alpha$ cross section at approximately 2.3×10^{-1} MeV. Below approximately 1 MeV the sensitivity of the ${}^6\text{Li}$ breeding ratio to an increase in the niobium total cross section is due primarily to the increase in the niobium absorption cross section. At energies of $\leq 10^{-4}$ MeV where the ${}^6\text{Li}$ sensitivity exhibits changes in sign, there are significant contributions to the profile from increases in both the niobium absorption cross section and the niobium elastic-scattering cross section. The regions of high negative sensitivity correspond to peaks in the absorption cross section while the regions of low positive sensitivity correspond to minima in the absorption cross section where elastic scattering dominates. The sensitivity of the ${}^6\text{Li}$ breeding ratio to an increase in the niobium absorption cross section must, of course, be negative since an increase in the niobium absorption cross section decreases the number of neutrons that are available to produce a breeding reaction in the ${}^6\text{Li}$.

The sensitivity of the ${}^7\text{Li}$ breeding ratio to an increase in the total niobium cross section is negative at all energies. It is quite appreciable at the highest energies but decreases rapidly as energy decreases. At the highest energies, there is a considerable contribution to the sensitivity profile from increases in the niobium $(n,2n)$ cross section and in the niobium inelastic cross section, but below approximately 9 MeV the major contribution comes only from the increase in the niobium inelastic cross section.*

*See Appendix B.

In Fig. 7 the sensitivity of the breeding ratios to increases in the carbon total collision cross section is shown as a function of energy. The ${}^6\text{Li}$ breeding-ratio sensitivity is positive at all energies except those in the thermal group, and the ${}^7\text{Li}$ breeding-ratio sensitivity is negative at all energies. The carbon region in the blanket serves primarily as a neutron reflector and moderator. It is to be expected that the ${}^6\text{Li}$ breeding ratio will have a positive sensitivity to the carbon elastic and inelastic cross sections, since by increasing these cross sections the number of reflected neutrons is increased and the chance of their producing a ${}^6\text{Li}(n,t)\alpha$ reaction is increased. On the other hand, the ${}^6\text{Li}$ breeding ratio will have a negative sensitivity to increases in the carbon absorption cross section, since when neutrons are absorbed in the carbon the probability of their producing a ${}^6\text{Li}(n,t)\alpha$ reaction is reduced. Since the ${}^6\text{Li}$ breeding ratio is positive at all energies above the thermal group, it follows that elastic and inelastic scattering in the carbon has a more important effect than absorption in the carbon on the ${}^6\text{Li}$ breeding ratio. This is borne out when the sensitivity of the breeding ratio to carbon partial cross sections is considered.* The ${}^7\text{Li}$ breeding-ratio sensitivity shown in Fig. 7 contains significant contributions from the carbon elastic, inelastic, and absorption cross sections.

In obtaining the results presented in Figs. 2-6, the cross-section perturbation was defined to be the same in all spatial regions of the blanket; i.e., δC was specified to be nonzero in a given energy region in all spatial regions of the blanket. By an appropriate choice of the spatial regions in which δC is specified to be zero, it is also possible to obtain information on the sensitivity of the breeding ratios to cross-section perturbations in specific spatial intervals.

*See Appendix B.

Fig. 7. Breeding-ratio sensitivity per unit lethargy vs energy for the C total collision cross section.

In Tables 1-4 the sensitivity of the breeding ratios to increases in the various partial cross sections and the total collision cross sections of ${}^6\text{Li}$, ${}^7\text{Li}$, Nb, and C, respectively, in particular spatial zones (see Fig. 1) is given. In obtaining the results presented in the tables for a particular cross section, δC has been specified to be nonzero at all energies in the specified spatial interval. A given entry in the tables may be thought of as the percent change in the breeding ratio being considered for a 1% increase at all energies in a particular partial cross section in a particular spatial zone. The last column in each of the tables is the sum of the individual contribution from each zone, and the results presented at the bottom of each table for the total collision cross section indicate the sum of the results for the partial cross sections.

In Tables 1 and 2 the breeding-ratio-sensitivity values are smaller in zones 4 and 8 than in zone 6, as would be expected, since zone 6 is considerably larger than the other two zones. In Table 1 the ${}^6\text{Li}$ breeding ratio is much more sensitive to the ${}^6\text{Li}(n,t)\alpha$ cross section, i.e., to the ${}^6\text{Li}$ breeding cross section, than to any of the other partial cross sections. The ${}^7\text{Li}$ breeding ratio is most sensitive to the ${}^6\text{Li}$ inelastic cross section, but this sensitivity is not large. It is interesting to note that the sensitivity of the total breeding ratio, i.e., the sum of the ${}^6\text{Li}$ and ${}^7\text{Li}$ breeding ratios, over the entire blanket to the ${}^6\text{Li}$ total collision cross section is very nearly the same as the sensitivity of the ${}^6\text{Li}$ breeding ratio in zone 6 to the ${}^6\text{Li}(n,t)\alpha$ cross section.

In Table 2 the ${}^7\text{Li}$ breeding ratio is much more sensitive to the ${}^7\text{Li}(n,n')\alpha,t$ cross section, i.e., to the ${}^7\text{Li}$ breeding cross section, than to the other ${}^7\text{Li}$ partial cross sections. The ${}^6\text{Li}$ breeding ratio is most sensitive to the ${}^7\text{Li}$ elastic scattering and $(n,2n)$ cross sections, but this

Table 1

Breeding-Ratio Sensitivity to Increases in Various ${}^6\text{Li}$
Cross Sections in Various Spatial Zones (see Fig. 1)

${}^6\text{Li}$ Cross-Section Type	Breeding Material ^a j	$\frac{1}{R} \frac{\delta R_j}{\delta C}$			
		Zone 4	Zone 6	Zone 8	All Zones
$\Sigma_{(n,t)\alpha}$	${}^6\text{Li}$	9.24×10^{-3}	1.25×10^{-1}	1.21×10^{-2}	1.46×10^{-1}
	${}^7\text{Li}$	-1.79×10^{-4}	-1.13×10^{-3}	-1.24×10^{-6}	-1.31×10^{-3}
	${}^6\text{Li}$ and ${}^7\text{Li}$	9.06×10^{-3}	1.24×10^{-1}	1.21×10^{-2}	1.45×10^{-1}
$\Sigma_{(n,\gamma)}$	${}^6\text{Li}$	-7.81×10^{-7}	-1.58×10^{-5}	-9.15×10^{-7}	-1.75×10^{-5}
	${}^7\text{Li}$	-6.17×10^{-8}	-3.45×10^{-7}	-2.51×10^{-10}	-4.07×10^{-7}
	${}^6\text{Li}$ and ${}^7\text{Li}$	-8.43×10^{-7}	-1.61×10^{-5}	-9.15×10^{-7}	-1.79×10^{-5}
$\Sigma_{(n,p)}$	${}^6\text{Li}$	-1.13×10^{-4}	-7.33×10^{-4}	-5.85×10^{-7}	-8.47×10^{-4}
	${}^7\text{Li}$	-5.05×10^{-5}	-3.24×10^{-4}	-3.77×10^{-7}	-3.75×10^{-4}
	${}^6\text{Li}$ and ${}^7\text{Li}$	-1.64×10^{-4}	-1.06×10^{-4}	-9.62×10^{-7}	-1.22×10^{-3}
Σ_{ELASTIC}	${}^6\text{Li}$	-7.56×10^{-6}	-5.04×10^{-3}	1.12×10^{-4}	-4.94×10^{-3}
	${}^7\text{Li}$	-3.22×10^{-4}	-1.95×10^{-3}	-2.55×10^{-6}	-2.28×10^{-3}
	${}^6\text{Li}$ and ${}^7\text{Li}$	-3.30×10^{-4}	-6.99×10^{-3}	1.09×10^{-4}	-7.22×10^{-3}
$\Sigma_{(n,n')\gamma}$	${}^6\text{Li}$	-5.55×10^{-4}	-6.72×10^{-4}	6.45×10^{-6}	-1.22×10^{-3}
	${}^7\text{Li}$	-1.74×10^{-3}	-1.09×10^{-2}	-9.30×10^{-6}	-1.26×10^{-2}
	${}^6\text{Li}$ and ${}^7\text{Li}$	-2.30×10^{-3}	-1.16×10^{-2}	-2.85×10^{-6}	-1.38×10^{-2}
$\Sigma_{(n,2n')\alpha,p}$	${}^6\text{Li}$	4.50×10^{-4}	2.46×10^{-3}	1.45×10^{-6}	2.91×10^{-3}
	${}^7\text{Li}$	-3.84×10^{-4}	-1.83×10^{-3}	-6.84×10^{-7}	-2.21×10^{-3}
	${}^6\text{Li}$ and ${}^7\text{Li}$	0.66×10^{-4}	0.63×10^{-3}	0.76×10^{-6}	0.70×10^{-3}
Σ_{TOTAL} COLLISION	${}^6\text{Li}$	9.01×10^{-3}	1.21×10^{-1}	1.20×10^{-2}	1.42×10^{-1}
	${}^7\text{Li}$	-2.63×10^{-3}	-1.61×10^{-2}	-1.42×10^{-5}	-1.87×10^{-2}
	${}^6\text{Li}$ and ${}^7\text{Li}$	6.38×10^{-3}	1.05×10^{-1}	1.20×10^{-2}	1.23×10^{-1}

- a. When the breeding material is specified to be both ${}^6\text{Li}$ and ${}^7\text{Li}$, the values given in the table are for the quantity

$$\frac{1}{R} \left[\frac{\delta R_{{}^6\text{Li}} + \delta R_{{}^7\text{Li}}}{\delta C} \right]$$

Table 2
Breeding-Ratio Sensitivity to Increases in Various ${}^7\text{Li}$
Cross Sections in Various Spatial Zones (see Fig. 1)

${}^7\text{Li}$ Cross-Section Type	Breeding Material ^a j	$\frac{1}{R} \frac{\delta R_j}{\delta C}$			
		Zone 4	Zone 6	Zone 8	All Zones
$\Sigma(n, \gamma)$	${}^6\text{Li}$	-1.12×10^{-3}	-4.30×10^{-3}	-9.71×10^{-7}	-5.42×10^{-3}
	${}^7\text{Li}$	-6.72×10^{-4}	-2.93×10^{-3}	-8.48×10^{-7}	-3.60×10^{-3}
	${}^6\text{Li}$ and ${}^7\text{Li}$	-1.79×10^{-3}	-7.23×10^{-3}	-1.82×10^{-6}	-9.02×10^{-3}
$\Sigma(n, d)$	${}^6\text{Li}$	-9.83×10^{-6}	-2.63×10^{-4}	-7.20×10^{-5}	-3.45×10^{-4}
	${}^7\text{Li}$	-7.70×10^{-7}	-4.30×10^{-6}	-3.14×10^{-9}	-5.08×10^{-6}
	${}^6\text{Li}$ and ${}^7\text{Li}$	-1.06×10^{-5}	-2.67×10^{-4}	-7.20×10^{-5}	-3.50×10^{-4}
Σ_{ELASTIC}	${}^6\text{Li}$	-7.25×10^{-3}	-3.59×10^{-2}	1.88×10^{-3}	-4.13×10^{-2}
	${}^7\text{Li}$	-5.22×10^{-3}	-2.86×10^{-2}	-4.75×10^{-5}	-3.38×10^{-2}
	${}^6\text{Li}$ and ${}^7\text{Li}$	-1.25×10^{-2}	-6.45×10^{-2}	1.83×10^{-3}	-7.51×10^{-2}
$\Sigma(n, n')\gamma$	${}^6\text{Li}$	-1.10×10^{-3}	-1.31×10^{-3}	4.48×10^{-5}	-2.36×10^{-3}
	${}^7\text{Li}$	-1.77×10^{-3}	-1.28×10^{-2}	-1.79×10^{-5}	-1.46×10^{-2}
	${}^6\text{Li}$ and ${}^7\text{Li}$	-2.87×10^{-3}	-1.41×10^{-2}	2.69×10^{-5}	-1.70×10^{-2}
$\Sigma(n, n')\alpha, t$	${}^6\text{Li}$	-5.47×10^{-3}	-7.37×10^{-3}	4.12×10^{-5}	-1.28×10^{-2}
	${}^7\text{Li}$	3.46×10^{-2}	2.01×10^{-1}	6.33×10^{-4}	2.36×10^{-1}
	${}^6\text{Li}$ and ${}^7\text{Li}$	2.91×10^{-2}	1.94×10^{-1}	6.74×10^{-4}	2.23×10^{-1}
$\Sigma(n, 2n')$	${}^6\text{Li}$	4.24×10^{-3}	2.13×10^{-2}	1.08×10^{-5}	2.56×10^{-2}
	${}^7\text{Li}$	-3.71×10^{-3}	-1.64×10^{-2}	-4.98×10^{-6}	-2.01×10^{-2}
	${}^6\text{Li}$ and ${}^7\text{Li}$	0.53×10^{-3}	0.49×10^{-2}	0.58×10^{-5}	0.55×10^{-2}
Σ_{TOTAL} COLLISION	${}^6\text{Li}$	-1.07×10^{-2}	-2.79×10^{-2}	1.90×10^{-3}	-3.67×10^{-2}
	${}^7\text{Li}$	2.32×10^{-2}	1.40×10^{-1}	5.62×10^{-4}	1.64×10^{-1}
	${}^6\text{Li}$ and ${}^7\text{Li}$	1.25×10^{-2}	1.12×10^{-1}	2.48×10^{-3}	1.27×10^{-1}

a. When the breeding material is specified to be both ${}^6\text{Li}$ and ${}^7\text{Li}$, the values given in the table are for the quantity

$$\frac{1}{R} \left[\frac{\delta R_{{}^6\text{Li}} + \delta R_{{}^7\text{Li}}}{\delta C} \right]$$

Table 3

Breeding-Ratio Sensitivity to Increases in Various Nb Cross Sections in Various Spatial Zones (see Fig. 1)

Nb Cross-Section Type	Breeding Material ^a j	$\frac{1}{R} \frac{\delta R_j}{\delta C}$					
		Zone 3	Zone 4	Zone 5	Zone 6	Zone 8	All Zones
$\Sigma(n, \gamma)$	⁶ Li	-1.02×10^{-2}	-3.79×10^{-3}	-1.06×10^{-2}	-8.67×10^{-2}	-7.62×10^{-4}	-1.12×10^{-1}
	⁷ Li	-1.04×10^{-5}	-3.50×10^{-6}	-8.87×10^{-6}	-2.34×10^{-5}	-3.05×10^{-8}	-4.62×10^{-5}
	⁶ Li and ⁷ Li	-1.02×10^{-2}	-3.79×10^{-3}	-1.06×10^{-2}	-8.67×10^{-2}	-7.62×10^{-4}	-1.12×10^{-1}
$\Sigma(n, p)$	⁶ Li	-1.01×10^{-3}	-2.99×10^{-4}	-6.94×10^{-4}	-1.35×10^{-3}	-4.84×10^{-7}	-3.36×10^{-3}
	⁷ Li	-5.51×10^{-4}	-1.77×10^{-4}	-4.33×10^{-4}	-9.02×10^{-4}	-4.19×10^{-7}	-2.06×10^{-3}
	⁶ Li and ⁷ Li	-1.56×10^{-3}	-4.75×10^{-4}	-1.13×10^{-3}	-2.25×10^{-3}	-9.03×10^{-7}	-5.42×10^{-3}
$\Sigma(n, \alpha)$	⁶ Li	-3.01×10^{-4}	-8.77×10^{-5}	-2.02×10^{-4}	-3.75×10^{-4}	-1.16×10^{-7}	-9.66×10^{-4}
	⁷ Li	-1.65×10^{-4}	-5.22×10^{-5}	-1.27×10^{-4}	-2.52×10^{-4}	-1.01×10^{-7}	-5.96×10^{-4}
	⁶ Li and ⁷ Li	-4.66×10^{-4}	-1.40×10^{-4}	-3.29×10^{-4}	-6.27×10^{-4}	-2.17×10^{-7}	-2.10×10^{-3}
$\Sigma_{ELASTIC}$	⁶ Li	-2.37×10^{-4}	-8.52×10^{-5}	-2.83×10^{-4}	-2.90×10^{-3}	7.76×10^{-4}	-2.73×10^{-3}
	⁷ Li	-4.27×10^{-4}	-1.50×10^{-4}	-4.06×10^{-4}	1.99×10^{-4}	1.03×10^{-6}	-7.82×10^{-4}
	⁶ Li and ⁷ Li	-6.64×10^{-4}	-2.35×10^{-4}	-6.89×10^{-4}	-2.70×10^{-3}	7.75×10^{-4}	-3.51×10^{-3}
$\Sigma(n, n')\gamma$	⁶ Li	-5.31×10^{-3}	-1.43×10^{-3}	-3.10×10^{-3}	-1.38×10^{-3}	3.19×10^{-5}	-1.12×10^{-2}
	⁷ Li	-1.83×10^{-2}	-6.17×10^{-3}	-1.57×10^{-2}	-4.09×10^{-2}	-4.02×10^{-5}	-8.11×10^{-2}
	⁶ Li and ⁷ Li	-2.36×10^{-2}	-7.60×10^{-3}	-1.88×10^{-2}	-4.23×10^{-2}	-0.83×10^{-5}	-9.23×10^{-2}
$\Sigma(n, 2n')$	⁶ Li	2.29×10^{-2}	6.99×10^{-3}	1.67×10^{-2}	3.61×10^{-2}	1.90×10^{-5}	8.27×10^{-2}
	⁷ Li	-1.93×10^{-2}	-6.07×10^{-3}	-1.47×10^{-2}	-2.76×10^{-2}	-8.83×10^{-6}	-6.75×10^{-2}
	⁶ Li and ⁷ Li	0.36×10^{-2}	0.92×10^{-3}	0.20×10^{-2}	0.85×10^{-2}	1.02×10^{-5}	1.52×10^{-2}
Σ_{TOTAL} COLLISION	⁶ Li	5.82×10^{-3}	1.32×10^{-3}	1.77×10^{-3}	-5.67×10^{-2}	6.36×10^{-5}	-4.77×10^{-2}
	⁷ Li	-3.88×10^{-2}	-1.26×10^{-2}	-3.13×10^{-2}	-6.96×10^{-2}	-4.88×10^{-5}	-1.52×10^{-1}
	⁶ Li and ⁷ Li	-3.30×10^{-2}	-1.13×10^{-2}	-2.95×10^{-2}	-1.26×10^{-1}	-1.48×10^{-5}	-2.00×10^{-1}

a. When the breeding material is specified to be both ⁶Li and ⁷Li, the values given in the table are for the quantity

$$\frac{1}{R} \left[\frac{\delta R_{6Li} + \delta R_{7Li}}{\delta C} \right]$$

Table 4
Breeding-Ratio Sensitivity to Increases in Various C Cross
Sections in the Spatial Zone Containing C (see Fig. 1)

C Cross-Section Type	Breeding Material ^a j	$\frac{1}{R} \frac{\delta R_j}{\delta C}$
		Zone 7
$\Sigma_{\text{ABSORPTION}}$	${}^6\text{Li}$	-4.35×10^{-3}
	${}^7\text{Li}$	-6.12×10^{-4}
	${}^6\text{Li and } {}^7\text{Li}$	-4.96×10^{-3}
Σ_{ELASTIC}	${}^6\text{Li}$	6.48×10^{-2}
	${}^7\text{Li}$	9.18×10^{-5}
	${}^6\text{Li and } {}^7\text{Li}$	6.49×10^{-2}
$\Sigma_{(n,n')\gamma}$	${}^6\text{Li}$	4.40×10^{-3}
	${}^7\text{Li}$	-1.32×10^{-3}
	${}^6\text{Li and } {}^7\text{Li}$	3.08×10^{-3}
$\Sigma_{\text{TOTAL COLLISION}}$	${}^6\text{Li}$	6.49×10^{-2}
	${}^7\text{Li}$	-1.84×10^{-3}
	${}^6\text{Li and } {}^7\text{Li}$	6.31×10^{-2}

a. When the breeding material is specified to be both ${}^6\text{Li}$ and ${}^7\text{Li}$, the values given in the table are for the quantity

$$\frac{1}{R} \left[\frac{\delta R_{{}^6\text{Li}} + \delta R_{{}^7\text{Li}}}{\delta C} \right] .$$

sensitivity is not large. The sensitivity of the total breeding ratio over the entire blanket to the ${}^7\text{Li}$ total collision cross section is comparable to, but less than, the sensitivity of the ${}^7\text{Li}$ breeding ratio in zone 6 to the ${}^7\text{Li}(n,n')\alpha,t$ cross section. By comparing Tables 1 and 2, it can be seen that the sensitivity of the total breeding ratio over the entire blanket to the total ${}^6\text{Li}$ and ${}^7\text{Li}$ collision cross sections is very nearly the same.

The ${}^6\text{Li}$ and the ${}^7\text{Li}$ breeding ratios show the most sensitivity to niobium cross-section changes in zone 6, the largest zone, but the sensitivity to niobium cross-section changes in zones 3, 4, and 5 is not negligible compared to that in zone 6. The sensitivity of the breeding ratios to niobium cross-section changes in zone 8 is very small. The ${}^6\text{Li}$ breeding ratio is most sensitive to changes in the $\text{Nb}(n,\gamma)$ cross section, while the ${}^7\text{Li}$ breeding ratio is most sensitive to the niobium inelastic and $(n,2n)$ cross sections. The sensitivity of the total breeding ratio over the entire blanket to the niobium total cross section is comparable to, but opposite in sign from, that found for the ${}^6\text{Li}$ and ${}^7\text{Li}$ total cross sections.

In Table 4 the ${}^6\text{Li}$ breeding ratio is shown to be considerably more sensitive than the ${}^7\text{Li}$ breeding ratio to the carbon total collision cross section. This is to be expected since the carbon region in the blanket serves primarily as a neutron reflector and, in general, the reflected neutron will be too low in energy to produce a breeding reaction in the ${}^7\text{Li}$. The ${}^6\text{Li}$ breeding ratio is most sensitive to the carbon elastic cross section and the ${}^7\text{Li}$ breeding ratio is most sensitive to the carbon inelastic cross section.

The results presented in Figs. 2-7 and in Tables 1-4 are independent of δC in the linear-perturbation approximation. The change in the breeding ratio due to specified changes in the cross sections can be obtained by

multiplying the values in the figures and tables by the appropriate values of δC and carrying out any required integrations. The changes in the breeding ratios for assumed cross-section uncertainties in several partial cross sections are given in Table 5. If the cross-section error specifications are large, the corresponding changes in the breeding ratios will be large and may be invalid because of the linear-perturbation assumption. To test the validity of the linear-perturbation assumption, in general it is necessary to solve the transport equation with the perturbed cross sections. Several XSDRN¹¹ calculations with perturbed cross sections in the geometry shown in Fig. 1 have been carried out by Tobias and Steiner,¹² and it is thus possible to test the perturbation results obtained here by comparing with the results presented in ref. 12.

Comparisons of the percent change in the ^6Li , ^7Li , and total breeding ratios obtained by perturbation theory and by direct calculation are shown in Table 5 for several different error assumptions. In all of the cases considered, the perturbation results are in reasonable agreement with the results obtained by direct calculation, and thus for the cases considered the perturbation is quite valid. These results, however, are very dependent on the cases considered. If larger and larger errors are assumed, the perturbation theory will become more and more incorrect.

In general, the sensitivity of the breeding ratios to changes in the various cross sections has been found to be relatively small for the system considered here. The magnitude of the change in the breeding ratio is dependent on the errors which exist in the various cross sections and can be evaluated only when these errors are specified. In this regard, it should be noted that to a considerable extent the lack of sensitivity of the breeding ratio to cross-section changes is due to cancellation, i.e., positive changes in R due to increases in one cross section being cancelled by

Table 5

Comparison of Results Obtained from Perturbation Calculations and Discrete Ordinate Calculations with Perturbed Cross Sections

Element Perturbed	Cross-Section Type Perturbed	δC .100%	Perturbation Calculation			Discrete Ordinates Calculation ^a		
			$\frac{\delta R_{6Li}}{R}$	$\frac{\delta R_{7Li}}{R}$	$\frac{\delta R}{R}$	$\frac{\delta R_{6Li}}{R}$	$\frac{\delta R_{7Li}}{R}$	$\frac{\delta R}{R}$
			.100%	.100%	.100%	.100%	.100%	.100%
Nb	$\Sigma_{(n,n')}$	25 at all energies	-0.28	-2.03	-2.31	-0.23	-1.92	-2.15
Nb	$\Sigma_{(n,n')}$	-25 at all energies	0.28	2.03	2.31	0.24	2.13	2.37
Nb	$\Sigma_{(n,2n')}$	25 at all energies	2.07	-1.69	0.38	2.09	-1.63	0.46
Nb	$\Sigma_{(n,2n')}$	-25 at all energies	-2.07	1.69	-0.38	-2.23	1.74	-0.49
⁷ Li	$\Sigma_{(n,n')\alpha,t}$	20 at all energies	-0.26	4.72	4.46	-0.21	4.31	4.10
⁷ Li	$\Sigma_{(n,n')\alpha,t}$	-20 at all energies	0.26	-4.72	-4.46	0.21	-4.94	-4.73
⁶ Li	$\Sigma_{(n,t)\alpha}$	$\left\{ \begin{array}{l} 3 \text{ for } E \leq 0.01 \text{ MeV} \\ 5 \text{ for } 0.01 < E \leq 2 \text{ MeV} \\ 10 \text{ for } 2 < E \leq 14.9 \text{ MeV} \end{array} \right\}$	0.49	0.00	0.49	0.49	-0.01	0.48
⁶ Li	$\Sigma_{(n,t)\alpha}$	$\left\{ \begin{array}{l} -3 \text{ for } E \leq 0.01 \text{ MeV} \\ -5 \text{ for } 0.01 \leq E \leq 2 \text{ MeV} \\ -10 \text{ for } 2 \leq E \leq 14.9 \text{ MeV} \end{array} \right\}$	-0.49	0.00	-0.49	-0.51	0.01	-0.50

- a. M. L. Tobias and D. Steiner, "Cross-Section Sensitivity of Tritium Breeding in a Fusion Reactor Blanket: Effects of Uncertainties in Cross Sections of ⁶Li, ⁷Li, and ⁹³Nb," ORNL-TM-4200, Oak Ridge National Laboratory (1973).

negative changes in R due to increases in some other cross section, or to increases in the same cross section in a different energy range. Therefore, the magnitude of δR depends not only on the magnitude of the cross-section error but also on the sign of the error; that is, the maximum change in R , in general, will not be obtained by the simple assumption that all cross-section errors are either positive or negative.

The results presented here are dependent on the blanket design considered. Furthermore, the breeding ratio considered here is only one of many nuclear-engineering properties of fusion-reactor blankets. Since the sensitivity of the breeding ratio to cross-section uncertainties for the system considered is not large, it is possible that nuclear properties other than the breeding ratio will determine the cross-section accuracy required for the design of fusion-reactor blankets.

APPENDIX A

In this appendix the ${}^6\text{Li}$ and ${}^7\text{Li}$ total, elastic, and tritium production cross sections, which are significant in interpreting the results presented in Section III, are given. The ${}^6\text{Li}$ and ${}^7\text{Li}$ cross sections are shown as a function of neutron energy in Figs. A.1 and A.2, respectively.

Fig. A.1. ${}^6\text{Li}$ total, elastic, and tritium breeding cross sections as a function of neutron energy.

Fig. A.2. ${}^7\text{Li}$ total, elastic, and tritium breeding cross sections as a function of neutron energy.

APPENDIX B

In this appendix ${}^6\text{Li}$ and ${}^7\text{Li}$ breeding-ratio-sensitivity profiles are presented for increases in various partial cross sections of ${}^6\text{Li}$, ${}^7\text{Li}$, Nb, and C. A list of the figures included in this appendix is given in Table B.1.

Sensitivity profiles are not given for the changes in the breeding ratios due to changes in the ${}^6\text{Li}(n,\gamma)$ cross section and the ${}^7\text{Li}(n,d)$ cross section because the sensitivity was found to be very small at all energies. In Fig. B.29 the change in the ${}^6\text{Li}$ breeding ratio due to changes in the C absorption cross sections at specific energies is shown. In this figure, the sensitivity at the higher energies (> 5 MeV) is due entirely to the $\text{C}(n,\alpha)$ cross section and at the lower energies ($< 5 \times 10^{-4}$ MeV) it is due entirely to the $\text{C}(n,\gamma)$ cross section.

Table B.1
List of Figures

Figure No.	Breeding Material	Element Perturbed	Cross-Section-Type Perturbed
B.1	${}^6\text{Li}$	${}^6\text{Li}$	$\Sigma_{(n,p)}$
B.2	${}^7\text{Li}$	${}^6\text{Li}$	$\Sigma_{(n,p)}$
B.3	${}^6\text{Li}$	${}^6\text{Li}$	Σ_{ELASTIC}
B.4	${}^7\text{Li}$	${}^6\text{Li}$	Σ_{ELASTIC}
B.5	${}^6\text{Li}$	${}^6\text{Li}$	$\Sigma_{(n,n')\gamma}$
B.6	${}^7\text{Li}$	${}^6\text{Li}$	$\Sigma_{(n,n')\gamma}$
B.7	${}^6\text{Li}$	${}^6\text{Li}$	$\Sigma_{(n,2n')\alpha,p}$
B.8	${}^7\text{Li}$	${}^6\text{Li}$	$\Sigma_{(n,2n')\alpha,p}$
B.9	${}^6\text{Li}$	${}^7\text{Li}$	$\Sigma_{(n,\gamma)}$
B.10	${}^7\text{Li}$	${}^7\text{Li}$	$\Sigma_{(n,\gamma)}$
B.11	${}^6\text{Li}$	${}^7\text{Li}$	Σ_{ELASTIC}
B.12	${}^7\text{Li}$	${}^7\text{Li}$	Σ_{ELASTIC}
B.13	${}^6\text{Li}$	${}^7\text{Li}$	$\Sigma_{(n,n')\gamma}$
B.14	${}^7\text{Li}$	${}^7\text{Li}$	$\Sigma_{(n,n')\gamma}$
B.15	${}^6\text{Li}$	${}^7\text{Li}$	$\Sigma_{(n,2n')}$
B.16	${}^7\text{Li}$	${}^7\text{Li}$	$\Sigma_{(n,2n')}$
B.17	${}^6\text{Li}$	Nb	$\Sigma_{(n,\gamma)}$
B.18	${}^7\text{Li}$	Nb	$\Sigma_{(n,\gamma)}$
B.19	${}^6\text{Li}$	Nb	$\Sigma_{(n,p)}$
B.20	${}^7\text{Li}$	Nb	$\Sigma_{(n,p)}$
B.21	${}^6\text{Li}$	Nb	$\Sigma_{(n,\alpha)}$
B.22	${}^7\text{Li}$	Nb	$\Sigma_{(n,\alpha)}$
B.23	${}^6\text{Li}$	Nb	Σ_{ELASTIC}
B.24	${}^7\text{Li}$	Nb	Σ_{ELASTIC}
B.25	${}^6\text{Li}$	Nb	$\Sigma_{(n,n')\gamma}$
B.26	${}^7\text{Li}$	Nb	$\Sigma_{(n,n')\gamma}$
B.27	${}^6\text{Li}$	Nb	$\Sigma_{(n,2n')}$
B.28	${}^7\text{Li}$	Nb	$\Sigma_{(n,2n')}$
B.29	${}^6\text{Li}$	C	$\Sigma_{\text{ABSORPTION}}$
B.30	${}^7\text{Li}$	C	$\Sigma_{\text{ABSORPTION}}$
B.31	${}^6\text{Li}$	C	Σ_{ELASTIC}
B.32	${}^7\text{Li}$	C	Σ_{ELASTIC}
B.33	${}^6\text{Li}$	C	$\Sigma_{(n,n')\gamma}$
B.34	${}^7\text{Li}$	C	$\Sigma_{(n,n')\gamma}$

Fig. B.1. ${}^6\text{Li}$ breeding-ratio sensitivity per unit lethargy vs energy for the ${}^6\text{Li}(n,p)$ cross section.

Fig. B.2. ${}^7\text{Li}$ breeding-ratio sensitivity per unit lethargy vs energy for the ${}^6\text{Li}(n,p)$ cross section.

Fig. B.3. ⁶Li breeding-ratio sensitivity per unit lethargy vs energy for the ⁶Li elastic cross section.

Fig. B.4. ${}^7\text{Li}$ breeding-ratio sensitivity per unit lethargy vs energy for the ${}^6\text{Li}$ elastic cross section.

Fig. B.5. ${}^6\text{Li}$ breeding-ratio sensitivity per unit lethargy vs energy for the ${}^6\text{Li}(n,n')\gamma$ cross section.

Fig. B.6. ${}^7\text{Li}$ breeding-ratio sensitivity per unit lethargy vs energy for the ${}^6\text{Li}(n,n')\gamma$ cross section.

Fig. B.7. ${}^6\text{Li}$ breeding-ratio sensitivity per unit lethargy vs energy for the ${}^6\text{Li}(n,2n')\alpha,p$ cross section.

Fig. B.8. ${}^7\text{Li}$ breeding-ratio sensitivity per unit lethargy vs energy for the ${}^6\text{Li}(n,2n')\alpha,p$ cross section.

Fig. B.9. 6Li breeding-ratio sensitivity per unit lethargy vs energy for the $7\text{Li}(n,\gamma)$ cross section.

Fig. 3.10. ^7Li breeding-ratio sensitivity per unit lethargy vs energy for the $^7\text{Li}(n,\gamma)$ cross section.

Fig. B.11. ${}^6\text{Li}$ breeding-ratio sensitivity per unit lethargy vs energy for the ${}^7\text{Li}$ elastic cross section.

Fig. B.12. ${}^7\text{Li}$ breeding-ratio sensitivity per unit lethargy vs energy for the ${}^7\text{Li}$ elastic cross section.

Fig. B.13. ${}^6\text{Li}$ breeding-ratio sensitivity per unit lethargy vs energy for the ${}^7\text{Li}(n,n')\gamma$ cross section.

Fig. B.14. ${}^7\text{Li}$ breeding-ratio sensitivity per unit lethargy vs energy for the ${}^7\text{Li}(n,n')\gamma$ cross section.

Fig. B.15. ${}^6\text{Li}$ breeding-ratio sensitivity per unit lethargy vs energy for the ${}^7\text{Li}(n,2n')$ cross section.

Fig. B.16. ${}^7\text{Li}$ breeding-ratio sensitivity per unit lethargy vs energy for the ${}^7\text{Li}(n,2n')$ cross section.

Fig. B.17. ${}^6\text{Li}$ breeding-ratio sensitivity per unit lethargy vs energy for the $\text{Nb}(n,\gamma)$ cross section.

Fig. B.18. ${}^7\text{Li}$ breeding-ratio sensitivity per unit lethargy vs energy for the $\text{Nb}(n,\gamma)$ cross section.

Fig. B.19. ${}^6\text{Li}$ breeding-ratio sensitivity per unit lethargy vs energy for the Nb(n,p) cross section.

Fig. B.20. ${}^7\text{Li}$ breeding-ratio sensitivity per unit lethargy vs energy for the $\text{Nb}(n,p)$ cross section.

Fig. B.21. ${}^6\text{Li}$ breeding-ratio sensitivity per unit lethargy vs energy for the $\text{Nb}(n,\alpha)$ cross section.

Fig. B.22. ${}^7\text{Li}$ breeding-ratio sensitivity per unit lethargy vs energy for the $\text{Nb}(n,\alpha)$ cross section.

Fig. B.23. ${}^6\text{Li}$ breeding-ratio sensitivity per unit lethargy vs energy for the Nb elastic cross section.

Fig. B.24. ${}^7\text{Li}$ breeding-ratio sensitivity per unit lethargy vs energy for the Nb elastic cross section.

Fig. B.25. ${}^6\text{Li}$ breeding-ratio sensitivity per unit lethargy vs energy for the $\text{Nb}(n,n')\gamma$ cross section.

Fig. 3.26. ^7Li breeding-ratio sensitivity per unit lethargy vs energy for the $\text{Nb}(n,n')\gamma$ cross section.

Fig. B.27. ⁶Li breeding-ratio sensitivity per unit lethargy vs energy for the Nb(n,2n') cross section.

Fig. B.28. ^7Li breeding-ratio sensitivity per unit lethargy vs energy for the $\text{Nb}(n,2n')$ cross section.

Fig. B.29. ${}^6\text{Li}$ breeding-ratio sensitivity per unit lethargy vs energy for the C absorption cross section.

Fig. B.30. ${}^7\text{Li}$ breeding-ratio sensitivity per unit lethargy vs energy for the C absorption cross section.

Fig. B.31. ${}^6\text{Li}$ breeding-ratio sensitivity per unit lethargy vs energy for the C elastic cross section.

Fig. B.32. ${}^7\text{Li}$ breeding-ratio sensitivity per unit lethargy vs energy for the C elastic cross section.

Fig. B.33. ${}^6\text{Li}$ breeding-ratio sensitivity per unit lethargy vs energy for the $\text{C}(n,n')\gamma$ cross section.

Fig. B.34. ^7Li breeding-ratio sensitivity per unit lethargy vs energy for the $\text{C}(n,n')\gamma$ cross section.

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