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Abstract

In a previous report (ORNL CF 58-5-97) the temperature structure in the GCR-2 fuel elements and coolant stream was presented. The analysis depended on several assumptions, two of which were more controversial than the others. One was the stipulation that the "equilibrium" temperature profile (i.e. a temperature profile that did not change with axial location) had been achieved at the cross section being studied. The other simplification made was that the coolant could be thought of as consisting of two regions of constant thermal conductivity, one being the usual thermal conductivity and the other being a space average of conductivities found by using the Martinelli analogy which attempts to take into account the effects of eddy diffusivity.

It is the purpose of this report to lend validity to the first assumption and verify that the effect of the latter mentioned approximation is as was suggested in the aforementioned report.

To make the problem more tractable analytically, one dimensional rectangular geometry was chosen.

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Introduction

If a fluid flows between two infinite parallel plates it ultimately assumes a velocity distribution which does not change with further progress downstream. Similarly, if heat is introduced at the walls at a constant rate, the temperature structure will build up until a profile develops, the shape of which remains essentially unchanged with further travel downstream. While the magnitude of the temperature at corresponding points in this "equilibrium profile" will increase as the fluid flows downstream, the temperature at corresponding points in the cross section will increase at the same rate so that the shape of the temperature profile will remain the same.

Knowing the velocity field, the effective thermal conductivity of the fluid can be calculated as a function of position using the Martinelli analogy. This will lead to the temperature history in the fluid for a given heat flux at the boundaries.

In this report the buildup of the temperature field as determined numerically by use of the IBM 704 codes applicable for this purpose is reported. Also shown are the results of hand calculations comparing the temperature field at equilibrium as calculated using (1) a variable and continuous thermal conductivity as obtained from the Martinelli analogy and (2) a two region model in which the thermal conductivity is constant in each region.

Numerical values used were representative of those expected in the GCR-2 coolant channels. The established velocity distribution was assumed to prevail everywhere. The figure below shows the coordinate system employed.

Figure 1

Method of Analysis

If the heat conduction in the x direction compared to heat transport in that direction is neglected, the general heat conduction equation in rectangular geometry becomes:

$$\frac{\partial^2 T}{\partial y^2} + \frac{1}{K} \frac{\partial K}{\partial y} \frac{\partial T}{\partial y} = \frac{u \rho C_p}{K} \frac{\partial T}{\partial x} \quad (1)$$

With u and K as known functions of y , a numerical solution for T as a function of x and y is possible if boundary conditions are stipulated. The velocity is assumed to be the same as the "universal velocity" profile attributed to flow in straight tubes. It is broken up into three regions --- a laminar sub-layer, a buffer layer, and a turbulent core. The extent and velocities within the various regimes are given as follows:

$$\text{From } y = 0 \text{ to } y = \frac{5\mu}{\rho v \sqrt{f/2}}$$

$$u = \frac{f}{2} \frac{\rho v^2}{\mu} y \quad (2)$$

$$\text{From } y = \frac{5\mu}{\rho v \sqrt{f/2}} \text{ to } y = \frac{30\mu}{\rho v \sqrt{f/2}}$$

$$u = -3.05 v \sqrt{f/2} + 5.00 v \sqrt{f/2} \ln \frac{v \rho \sqrt{f/2}}{\mu} y \quad (3)$$

$$\text{From } y = \frac{30\mu}{\rho v \sqrt{f/2}} \text{ to } y = y_\delta$$

$$u = 5.5 v \sqrt{f/2} + 2.5 v \sqrt{f/2} \ln \frac{v \rho \sqrt{f/2}}{\mu} y \quad (4)$$

The thermal conductivity is obtained by assuming the eddy diffusivity for heat transfer is equal to the eddy diffusivity for momentum transfer and is expressed therefore by¹

$$K = k + C_p \frac{\frac{f}{2} \rho v^2}{du/dy} \left(1 - \frac{y}{y_\delta} \right) - C_p \mu \quad (5)$$

To facilitate machine calculations only the turbulent regime was investigated for the temperature buildup. The heat flux entering at each of the two bounding plates was fixed at 18.0 Btu/ft² sec and the inlet temperature into the system was considered constant across the cross section at 0°F. The results of the machine calculations are shown in Fig. 2 where the temperature is plotted against y for selected axial positions and again in Fig. 3 where the variation of temperature with x for various stream lines is shown. For the purposes of this calculation the turbulent core was considered to begin at y = .0005 feet and the fluid studied was helium.

¹ See for example McAdams, Heat Transmission, 3rd edition, McGraw-Hill, 1954, p 208

FIGURE 2

BUILDUP OF TEMPERATURE DISTRIBUTION IN A FLUID FLOWING BETWEEN TWO INFINITE PARALLEL PLANES WITH CONSTANT HEAT INPUT AT THE BOUNDARIES

HEAT FLUX - 18.0 BTU/FT² SEC. AT EACH PLANE

SPACING BETWEEN PLANES - .02 FEET

FLUID - HELIUM (properties evaluated at 725° and 300 psia)

VELOCITY PROFILE - "UNIVERSAL" VELOCITY DISTRIBUTION

THERMAL CONDUCTIVITY - VARIABLE AS CALCULATED USING

MARTINELLI ANALOGY

INLET TEMPERATURE ASSUMED TO BE 0°F.

The procedure used to compare the temperature structures obtained at equilibrium for three different gases and to compare the results one obtains using the "continuously varying thermal conductivity" model to the "two region" model was as follows:

The heat flux, geometry, and $\frac{\partial T}{\partial x}$ at equilibrium was considered the same in all instances.

The heat absorbed per unit area from y_1 to y_2 is $\int_{y_1}^{y_2} \rho c_p u \frac{\partial T}{\partial x} dy$

and from the expression for u in the three regions the heat absorbed per unit area between y_1 and y_2 is found to be

for the laminar sub-layer

$$q_{1-2} = \frac{\rho^2 c_p f v^2}{4 \mu} \frac{\partial T}{\partial x} \left[y^2 \right]_{y_1}^{y_2}$$

for the buffer layer

$$q_{1-2} = \rho c_p \sqrt{f/2} v \frac{\partial T}{\partial x} \left[5.00 y \ln \frac{v \rho \sqrt{f/2}}{\mu} y - 8.05 y \right]_{y_1}^{y_2}$$

for the turbulent core

$$q_{1-2} = \rho c_p \sqrt{f/2} v \frac{\partial T}{\partial x} \left[2.5 y \ln \frac{v \rho \sqrt{f/2}}{\mu} y + 3.0 y \right]_{y_1}^{y_2}$$

The above relations can be used to find the heat flux at any position in y and this in turn can be divided by the known local thermal conductivity as expressed by Eq. (5) yielding $\frac{\partial T}{\partial y}$ as a function of y . A numerical integration of the plot of $\frac{\partial T}{\partial y}$ against y thus yields the temperature difference between any two points and hence the entire equilibrium profile.

This is the procedure followed in obtaining the curves of Figs. 4, 5, and 6 labelled "Martinelli Analogy".

The curves labeled "Two Regions of Constant Thermal Conductivity" were obtained in similar fashion. The heat flowing at any position in y was computed as before but the thermal conductivities by which the flux was divided were either the static thermal conductivity or a space averaged conductivity obtained by finding the average thermal conductivity as predicted from the Martinelli analogy. The average conductivity between y_1 and y_2 is given generally by

$$\frac{1}{y_2 - y_1} \int_{y_1}^{y_2} K \, dy$$

and substituting the expression for K in the turbulent regime this yields (if $y \equiv \frac{y_\delta}{2}$)

$$\bar{K} = \frac{1}{y_2 - y_1} \left[k y - C_p \mu y + 0.4 C_p \sqrt{f/2} \rho v \left(\frac{y^2}{2} - \frac{y^3}{3y_\delta} \right) \right]_{y_1}^{y_2}$$

For y greater than $\frac{y_\delta}{2}$, the thermal conductivity was considered equal to the value at $\frac{y_\delta}{2}$.

The extent of the two regions was determined by plotting the velocity as a function of y on log-log paper. Extrapolating the curves obtained from Eq. 2 and 4 for the velocity in the laminar sub-layer and turbulent core respectively, gave a point of intersection which was assumed to be the position where the laminar sub-layer ended and the turbulent core

began. The buffer layer was thus obliterated or "absorbed" into the other two flow regimes. The results of studies using helium, carbon dioxide and water vapor as the coolant are shown in Figs. 4, 5, and 6 respectively. All curves on these figures have been normalized so that the mixed mean temperature is zero. The numerical values used in these calculations are summarized in the table below.

	Helium	Carbon Dioxide	Water Vapor
Distance between plates, ft	.02	.02	.02
Density, lb/ft ³	.10	1.04	.4487
Specific heat, Btu/lb °F	1.25	.279	.525
Average velocity, ft/sec	210	90.44	111.4
Static thermal conductivity, Btu/sec ft °F	$.408 \times 10^{-4}$	7.58×10^{-6}	7.75×10^{-6}
Dynamic viscosity, lb/ft sec	2.42×10^{-5}	1.96×10^{-5}	1.61×10^{-5}

FIGURE 4

COMPARISON OF EQUILIBRIUM TEMPERATURE PROFILES IN
HELIUM DURING HEATING FROM A FLAT PLATE

Martinielli Analogy —

Two Regions of Constant Thermal Conductivity ---

FIGURE 5

COMPARISON OF EQUILIBRIUM TEMPERATURE PROFILES IN CARBON
DIOXIDE DURING HEATING FROM A FLAT PLATE

FIGURE 6

**COMPARISON OF EQUILIBRIUM TEMPERATURE PROFILES IN WATER
VAPOR DURING HEATING FROM A FLAT PLATE**

Discussion and Conclusion

Reference to Figs. 2 and 3 shows that the equilibrium temperature profile is reached at a distance of about half a foot from the inlet section for the conditions of the problem investigated. This corresponds to an L/D of $12\frac{1}{2}$, much less than the L/D of the section studied in the GCR-2 coolant channel. It is therefore concluded that the assumption of the existence of an equilibrium temperature structure in the aforementioned study was justified.

Reference to Figs. 4, 5, and 6 shows that the profiles obtained by the two methods compare to each other roughly as was suggested in Fig. III of ORNL CF 58-5-97.

It is believed that statements concerning the shortcomings of the present septafoil fuel element design are well founded in theory and that experimental evidence to this effect is bearing out the theory.

Nomenclature

C_p	Specific heat
f	Friction factor
k	Static thermal conductivity
K	Total thermal conductivity
T	Temperature above inlet conditions
u	Velocity at a given stream line
V	Average velocity
$\left. \begin{matrix} x \\ y \end{matrix} \right\}$	Coordinate directions
y_s	Half distance between plates
μ	Dynamic viscosity
ρ	Density

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