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FUNDAMENTAL INVESTIGATION OF LIQUID-METAL LUBRICATED JOURNAL BEARINGS

Topical Report
Contract No. AT (II-I)-1228
Project Agreement No. 8
Modification No. 7

to

**U. S. ATOMIC ENERGY COMMISSION
Washington 25, D. C.**

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Department of Aerospace Propulsion Research

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to

U. S. ATOMIC ENERGY COMMISSION
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FOREWORD

This report was prepared by Southwest Research Institute under Contract No. AT(11-1)-1228, Project Agreement No. 8, Modification No. 7. The work was administered by the Engineering Development Branch, Division of Reactor Development, U.S. Atomic Energy Commission, Washington 25, D.C. with Mr. Nicholas Grossman as Project Engineer.

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ABSTRACT

A combined experimental and analytical program is reported, with the objective of providing basic design information for liquid-metal lubricated bearings, with emphasis on turbulence and film-inertia effects. The experiments make use of a large-scale bearing model, with 6-foot diameter and 0.5 in. film thickness. The working fluid is air, and Reynolds numbers comparable to those in liquid-metal lubricated bearings may be simulated. Data on velocity profiles, friction coefficients, and turbulence intensity are reported for concentric and eccentric operation, over a broad range of local Reynolds and Taylor numbers. Data are also reported for extremely distorted flows produced by placing a blockage in the film.

An analytical approach is reported, which exploits experimentally measured (or theoretically computed) profile parameters to predict pressures in bearing configurations. Data are given which show that inertial effects at the boundaries of the bearing film may at times be of primary importance in bearing calculations. Preliminary measurements of inertial effects at the entry to a tilted pad are reported.

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LIST OF SYMBOLS

a	Arbitrary constant
b	Local half-thickness of film
$\bar{b}$	Mean half-thickness of film
C_f	Friction coefficient
c	Radial bearing clearance
D	Bearing diameter
H	Dimensionless film thickness
h	Local film thickness
$\bar{h}$	Mean film thickness
h_c	Film thickness at point where only Couette flow prevails
L	Bearing length
n	Velocity profile exponent
P	Dimensionless pressure
$\bar{P}$	Mean pressure in bearing $\int P dX / L$
p	Pressure
R	Local Reynolds number, $b u_b / \nu$
$\bar{R}$	Mean Reynolds number, $\bar{b} u_{\bar{s}} / 2\nu$, $\bar{b} u_{bc} / \nu$
r	Bearing radius
U	See Eq. (69)
U^+	Dimensionless velocity
u	Velocity in direction of turning
u'	Turbulent velocity component in direction of turning
V	Velocity vector
W	See Eq. (54)

LIST OF SYMBOLS (Cont'd)

w	Velocity in axial direction
X	Dimensionless coordinate in direction of sliding
x	Coordinate in direction of turning
Y^+	Dimensionless wall distance
y	Coordinate normal to boundary
Z	Dimensionless axial coordinate
z	Axial coordinate
β	See Eq. (65)
δ^*	Displacement thickness [See Eq. (2)]
ϵ	Eccentricity ratio
ζ	See Eq. (39)
θ	Angular coordinate on bearing
μ	Absolute viscosity
ν	Kinematic viscosity
ξ	See Eq. (35)
ρ	Fluid density
τ	Shear stress
Φ	See Eq. (31)
ϕ	See Eq. (52)
ψ	See Eq. (55)
χ	See Eq. (46)
$( )_b$	Refers to midchannel
$( )_{bc}$	Refers to Couette flow at midchannel
$( )_{bh}$	Refers to midchannel velocity relative to moving wall

LIST OF SYMBOLS (Cont'd)

$( )_{bo}$	Refers to midchannel velocity relative to stationary wall
$( )_c$	Refers to Couette flow
$( )_h$	Refers to moving wall
$( )_o$	Refers to stationary wall
$( )_s$	Refers to surface velocity of journal

OBJECTIVE

The objective of this program is to investigate the fundamentals of liquid-metal lubricated hydrodynamic bearings, leading to the development of design criteria and performance estimates. This involves theoretical computations, and appropriate experimentation to provide basic data needed to guide these computations. Primary effort has been devoted to turbulent film phenomena and inertia effects.

INTRODUCTION

The need for this work arises from the fact that liquid-metal lubricated bearings of high reliability are required for advanced nuclear applications. Not only are such bearings expected to operate in regimes of speed and temperature which are unfamiliar, but the working fluids are known to be unforgiving of design errors, when compared with typical oils which have inherent lubricity. For these reasons, it has been judged advisable to undertake fundamental investigations where the goals would be: (1) to make best use of existing theories, (2) to define problem areas where existing theories cannot be extended, (3) to provide new approaches and data as needed, and (4) to develop design procedures and analytical predictions of performance.

Of particular interest are the problems resulting from turbulent transitions in the lubricant film, and the effects of fluid inertia. Both

are the consequence of the low kinematic viscosity of typical liquid metals, and the expected high operating speeds. These two effects are related both experimentally and analytically, and comprise the principal subjects of investigation in the work reported here.

Because it is felt that existing theoretical approaches have already been exploited to the limits of their validity, emphasis has been given to the experimental portions of the program. To accomplish this, an apparatus has been developed wherein the chief phenomena in lubricant flow can be simulated in a relatively large ($1/2$ in. thick) film, using air rather than liquid metal as the experimental fluid. This permits accurate measurement of local velocities throughout the film by hot-wire anemometer techniques; and from these measurements accurate estimates of shear stress at the wall can be obtained.

To set such data into perspective a specialized analytical program has been undertaken. Its original form was chosen to permit the experimental data to be inserted into the calculations as convenient dimensionless groups. However, with further study, these groups were found to be uniquely insensitive to the precise details of the flow, thus offering to provide accurate estimates of bearing behavior even when the flow parameters could only be estimated approximately. Because of this, the method appears to offer unique promise for calculation of

bearing film behavior. Some preliminary results in the exploitation of this approach are reported here.

Background of the Turbulence Problem

In typical hydrodynamic bearing operation, the fluid movement in the lubricant film is laminar. However, when fluids of low viscosity and high density are used in high speed bearings of relatively large clearance, the fluid movement departs from the simple laminar pattern and may become turbulent. This transition results in changes of pressure distribution, bearing power loss, eccentricity, and other factors. Through these effects turbulence undoubtedly influences vibratory stability (whirl) of the shaft-bearing system also.

In typical bearings the initial onset of this transition is not the random, disorderly movement characteristic of so-called "fully-developed" turbulence; it is, rather, an orderly formation of cellular vortices. If bearings are operated at Reynolds numbers considerably above this transition, a second change occurs, and the movement of the fluid becomes analogous to that found in turbulent pipe-flow.

In bearing flows, it is necessary to determine the effects of turbulence upon the interaction between "Couette flow" and "pressure flow." By "Couette flow" is meant the simple shear flow such as exists in the annulus between two concentric cylinders, when one is turned

relative to the other. By "pressure flow" is meant a pressure-induced fluid motion such as would take place between stationary walls. For the special case where the pressure and Couette flows cross at right angles, this might be thought of as an idealization of the practical problem of end-leakage through a thin annulus around a turning shaft. Other combinations of flow may represent the fluid movement in an actual bearing where the shaft is displaced from its central position, and pressure may vary both axially and circumferentially.

For laminar motion in bearings, linear superposition is possible with all combinations of such flows. For example, axial leakage is independent of the shaft speed for laminar flow, but it is clearly dependent upon this factor for turbulent flow. This dependence requires that, in turbulent-film bearings, the flows must be dealt with from the outset as combinations; or else account must be taken of some kind of "coupling" effect when superposition is attempted. In some cases the coupling effect would exceed the individual contribution from a flow component.

Before improved analyses of such flows are possible, experimental data are required of several aspects of the basic flows and their interactions. A breakdown follows on the more important of such topics for investigation:

- (1) Shear stress and velocity profile relationships in vortex flows, and full turbulent flow.

- (2) Effect of strong pressure flows when superposed on a Couette flow.
- (3) Interactions in an extremely skewed flow where a strong axial pressure flow interacts with a tangential Couette flow.
- (4) Effects of acceleration and deceleration on shear stress and velocity profile.
- (5) Circumferential transfer of turbulence so as to make solutions at a point in a film dependent upon the turbulent state elsewhere.
- (6) Effect of discontinuities in the flow, such as at the entry to a bearing pad or a step.

EXPERIMENTAL APPARATUS

Description of Test Apparatus

The experimental program was carried out using a large-scale bearing model wherein air, rather than liquid metal, served as the working fluid. This "test bearing" is approximately 6 ft in diameter and 2 ft long. The journal runs at speeds up to 120 rpm, generating, in the 0.54 in. film thickness, Reynolds numbers which are comparable to those expected in the much smaller bearings to be used with liquid metals.

A schematic diagram of the test apparatus is shown in Figure 1. The journal is supported on a shaft and bearings, and is mounted on a stand fixed to the floor. The bearing (or outer cylinder) is independently supported and, as shown in Figure 1, is mounted on a sled which can be moved back and forth in the horizontal plane to allow variation of journal eccentricity. The bearing can also be rotated around the journal on the ball bearing supports so that a pressure or velocity probe, fixed to the bearing, can be set at various attitude angles. A micrometer traversing-mechanism, mounted to the bearing, provides for radial as well as axial traversing of the air film with a velocity probe. This velocity probe is a hot-wire anemometer, of special design, with extra long needles to minimize the effect of the 1/8-in. diameter barrel of the probe on the flow.

Also shown in Figure 1 are removable sectors in the bearing, a feature which allows the simulation of a pad bearing. A photograph

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FIGURE 1. SCHEMATIC DIAGRAM OF TEST APPARATUS

of the journal is shown in Figure 2(a), and the bearing with its sled mount is shown in Figure 2(b).

Instrumentation

In the design of the test apparatus, a film thickness had to be chosen large enough so that good resolution would be obtained in the velocity measurements, yet small enough so that a realistic c/r ratio could be maintained. The 0.54-in. film thickness was chosen as one which would provide an optimum balance between these two requirements. To generate Reynolds numbers in this film equivalent to those expected in actual bearings lubricated with liquid metal, requires relatively low velocities, and therefore low pressures are encountered. In this experiment, the solution to the problem of measuring low velocities was solved rather simply by using a hot-wire anemometer, which also allows the measurement of turbulence quantities. The problem of low pressures was solved by developing a pressure measuring device, which is described later in this report.

(a)

(b)

FIGURE 2. PHOTOGRAPHS OF (a) ASSEMBLED LARGE-SCALE TURBULENCE APPARATUS, (b) STATIONARY CYLINDER (Bearing) SHOWING INSTRUMENTATION SLOT AND SLED MOUNT

VELOCITY PROFILES

Introduction

Not only are velocity profiles of direct interest in themselves, but when dealt with on a suitable nondimensionalized basis, they permit computation of wall shear stress. In the work reported, velocity profiles have been measured for the following conditions:

- (1) Variation of Reynolds number for fixed film thickness, and concentric journal placement.
- (2) Variation of film thickness for selected values of Reynolds number, accomplished by changes of journal eccentricity.
- (3) Creation of strong pressure flow, both tangential and axial, through blockage of the film.

The first item gives data which may be compared with those from the literature for other film thicknesses. In themselves they contribute to the understanding of Couette flow, and provide checks for analytical predictions.

The experiments indicated under the second item show the effects of changes of local film thickness, while the mean film thickness around the bearing is held constant. This provides information on the tangential transport of turbulence, and clarifies the relative importance of local and overall conditions in the film.

The experiments under the third item are best discussed in a separate section where the several aspects of blocked flows can be examined collectively.

Power Law Representation

A typical family of velocity profiles is shown in Figure 3. These are for pure Couette flow, where Reynolds number is varied through changes of turning speed. They are made at mid-span in the axial direction. For the very lowest Reynolds number they approach a straight diagonal line. As either turbulence or vortex flow appears at higher Reynolds numbers they show curvature, and ultimately seem to approach an asymptotic limiting profile shape.

When half of one of these profiles is plotted, it has much the same appearance as the velocity profile in a boundary layer or in the half-channel of pipe flow. To illustrate this, refer to Figure 4, where the data of Robertson and Reichardt are compared with a power law equation based upon independent data for pipe flow⁽¹⁾. The important observation here is that the profile shape in fully turbulent flow is not greatly sensitive to the details of the stress distribution. In the Couette flow, the shear stress is constant from the wall outward; in pipe flow (as in other parallel pressure flows) the shear stress varies linearly inward to zero at midchannel. Even with this extreme difference, velocity profiles for the two flows closely resemble one another.

The fact that these published profiles could so well be fitted by a power law suggests that this might provide a simple single-parameter method to characterize the velocity profile under any specified operating condition. The problem would be to determine the most appropriate exponent to use in an equation of the type:

FIGURE 3. EFFECT OF REYNOLDS NUMBER ON TYPICAL FULL-CHANNEL VELOCITY PROFILES FOR CONCENTRIC JOURNAL

FIGURE 4. HALF-CHANNEL TURBULENT VELOCITY PROFILES FOR PLANE COUETTE FLOW COMPARED WITH POWER LAW APPROXIMATION

$$u/u_b = (y/b)^n \quad (1)$$

It is clear that n would vary with Reynolds number, for example, since the profiles in Figure 3 obviously span a broad range of curvatures. For a typical example of such a fit at $\bar{R} = 2400$, refer to the linear and logarithmic plots in Figure 5. In the logarithmic plot, it is seen that a straight line does indeed conform to the data. In the linear plot, the fit may be assessed more easily in an undistorted frame of reference. From the slope of the line on the logarithmic plot, it is found that $n \doteq 1/8$, whereas the exponent in Figure 4 was $n = 1/7$.

Figure 6 shows corresponding profile plots for $\bar{R} = 740$. On the logarithmic plot, the data do not easily lend themselves to straight-line representation; consequently, three separate techniques were attempted to get a best fit. When the least-squares technique was applied to the entire data assembly, the uniformly-dashed broken line was found to represent the best fit. Clearly the slope of this line was affected disproportionately by the deviant lower points. Whether these represent true deviation, or are the result of small errors, magnified by the logarithmic representation, cannot be determined. Irrespectively, the least-squares approach gives inordinate weight to these points.

To remove the influence of the lower points, the solid line was obtained as best fit for the upper six points only. Thus it may be said to represent the best fit for the turbulent core. Referring to the corresponding linear plot, it is seen to provide a fair representation of the actual profile.

FIGURE 5. COMPARISON BETWEEN EXPERIMENTAL DATA AND POWER LAW CURVE FIT FOR $\bar{R} = 2400$
 (a) LINEAR PLOT, (b) LOGARITHMIC PLOT

FIGURE 6. COMPARISON BETWEEN EXPERIMENTAL DATA AND POWER LAW CURVE FIT FOR $\bar{R} = 740$
 (a) LINEAR PLOT, (b) LOGARITHMIC PLOT

As a third approach to curve fitting, a point near the knee of the profile was established precisely by repeated runs, and a power-law curve was drawn directly through this point. Again, the agreement with the overall distribution of data is fair, but not precise. The three profiles encompass a range of n from $1/6$ to $1/3$.

Displacement Thickness

For the reasons noted above, it was felt that the parameter n did not sufficiently well characterize the velocity profile for comparative plotting purposes. Thus recourse was made to the displacement thickness, which is perhaps a more fundamental parameter. At least, it has the advantage of being free from arbitrary judgements in the process of determination of its magnitude from profile data.

Displacement thickness in the half channel may be defined by the following equation:

$$\delta^* = \int_0^b (1 - u/u_b) dy \quad (2)$$

For empirical profiles the integration may be accomplished by the use of a planimeter with the profile-data plot. Figure 7 gives a number of such profiles for a wide range of experimental conditions. These profiles are for the half-channel on the journal side where y is measured from the bearing. For any typical profile on such a plot δ^*/b would represent the area to the upper left of the curve in ratio to the overall area of the square in which the normalized profile is set. It is measure of the velocity defect, or the discrepancy between the profile and a square one. For very high Reynolds number, the actual velocity profile may come close to the upper corner of the picture

FIGURE 7. EFFECT OF LOCAL CLEARANCE AND LOCAL REYNOLDS NUMBER ON NORMALIZED HALF-CHANNEL VELOCITY PROFILES FOR ECCENTRIC JOURNAL PLACEMENT
 (a) $h/\bar{h} = 1.67$, (b) $h/\bar{h} = 1.00$, (c) $h/\bar{h} = 0.46$, (d) $h/\bar{h} = 0.18$

and be nearly square, whereupon $\delta^*/b \rightarrow 0$. For laminar flow the profile becomes linear and $\delta^*/b \rightarrow 0.5$.

Each family of profiles in Figure 7 represents a given journal eccentricity. To designate each, the local film thickness h is expressed in ratio to $\bar{h}$, the mean clearance (difference between bearing radius and journal radius). Thus when $h/\bar{h} = 1.67$, this represents a portion of the clearance where the film thickness is much greater than the mean film thickness. In all cases, the profiles were measured at mid-span axially, and at the point of maximum or minimum eccentricity. The profiles were found to be very nearly those of Couette flow, since tangential pressure flow is very weak as a consequence of the small length/diameter ratio of the apparatus. In each case the profiles are seen to behave much as those presented above for changes of Reynolds number with fixed, uniform clearance.

Comparisons can best be made in terms of plots of displacement thickness variation as shown in Figures 8 and 9. The effects of local Reynolds number, R , upon the displacement thickness are shown in Figure 8, for different clearances. The curves appear to bend toward the laminar displacement thickness of 0.5 at different magnitudes of Reynolds number for each clearance. (Also plotted are data from the profiles of Robertson and Reichardt, for comparison purposes, these being for plane-Couette flow.)

The intersection between any given empirical curve with $\delta^*/b = 0.5$ should provide a measure of transition Reynolds number. Since Taylor

FIGURE 8. EFFECT OF LOCAL FILM THICKNESS ON THE RELATIONSHIP BETWEEN LOCAL REYNOLDS NUMBER R AND DISPLACEMENT THICKNESS, FOR ECCENTRIC JOURNAL PLACEMENT

FIGURE 9. EFFECT OF LOCAL FILM THICKNESS ON THE
RELATIONSHIP BETWEEN NOMINAL REYNOLDS NUMBER $\bar{R}$
AND DISPLACEMENT THICKNESS FOR ECCENTRIC
JOURNAL PLACEMENT

number determines transition in Couette flow, it might be hypothesized that local Taylor number controls the transition in the eccentric flows.

If there be a fixed Taylor number for transition, one would write:

$$R_{crit} = \text{Const.} \sqrt{r/c} \quad (3)$$

thus predicting that the Reynolds number of transition would be greatest when clearance is smallest. However, from Figure 8, it is apparent that the contrary trend prevails. Thus local Taylor number is eliminated as the correlating factor.

Forgetting local conditions, one might assume that the transition is simply a function of overall conditions around the circumference, and, therefore, a function of mean Reynolds number. Figure 9 is, therefore, based upon the Reynolds number R such that $u_s \bar{b}/2\nu \equiv \bar{R}$. This does indeed tend to draw the data together as compared to the previous case.

The data of Robertson and Reichardt were artificially transposed, by moving their transition to correspond with that for concentric journal placement. This, along with the experimental data, indicates that displacement thickness may be taken to be a function of Reynolds number in ratio to the transition Reynolds number. The data indicate further that mean Reynolds number $\bar{R}$ (rather than local R) tends to govern the transition. At least, $\bar{R}$ serves to draw the data together better than R does.

FRICTION COEFFICIENT MEASUREMENTS

Empirical Wall-Law Equation

In the reported experiments the wall shear stress was very small (about 10^{-3} lb/ft² at the maximum), hence it could not be measured directly. Instead, a generally accepted alternative procedure was used, involving application of the "wall law" or "universal velocity profile" to a region in the vicinity of the solid boundary surface, to permit computation of the shear stress. Because this procedure appears to assume the shear-stress vs. profile relationship, and because of certain experimental considerations in its application, it is well to review the considerations which are involved. To begin with, in a field of uniform flow in the vicinity of a single wall, where any other boundaries are very far distant, it may be assumed that, for purposes of dimensional analysis, only the following variables describe the flow: y distance out from the wall, τ wall shear stress (which is also the shear stress throughout the flow), ρ fluid density, μ fluid viscosity and u local velocity. These variables fully specify both the fluid properties and the kinematic and geometric properties of the flow, whether it be turbulent or laminar. They may be grouped dimensionlessly in several ways; but, irrespectively, two dimensionless groups will result in each case. For example, the two dimensionless groups which characterize the above assembly of variables may be U^+ and Y^+ , where $U^+ = u\sqrt{\rho/\tau}$ and $Y^+ = \frac{y}{\nu}\sqrt{\frac{\tau}{\rho}}$. Otherwise, the two variables may be C_f and R , where C_f is defined as $2\tau/\rho u^2$ and R is defined as $\rho y u/\mu$. That these groups have a unique relationship to each other can be seen by noting that:

$U^+ = \sqrt{2/C_f}$ and $Y^+ = R\sqrt{C_f/2}$. Thus it is possible to write $Y^+ = f_1(U^+)$ or $C_f = f_2(R)$ and if the functional relationship is known for one case, it may be uniquely calculated for the other. Either U^+ vs. Y^+ , or C_f vs. R , may be plotted and either will shed special light on certain aspects of the flow. Of these two types of plot, the representation of U^+ vs. Y^+ is normally called the "universal velocity profile," or "wall law profile." The reason for this is that its appearance corresponds geometrically to the shape of the equivalent velocity profile in the vicinity of a wall. Numerous experimenters have attempted to determine this fundamental function or profile from measurements in actual flows. It has been fairly well established by such experiments that, for pipe flow, a single universal velocity profile function will apply for at least a third of the pipe radius out from the wall, and is a good first approximation right out to the center of the pipe. It has further been shown, though less exhaustively, that plane-Couette flow also follows the same universal velocity profile for a sizeable portion of the channel distance out from the wall. Boundary layer velocity profiles do the same.

Noting again that the "universal velocity profile" also corresponds uniquely to a C_f vs. R plot, it is not surprising that the correspondence between Couette flow and pressure flow velocity profiles is echoed by a similar correspondence between the C_f plots for the two types of flow.

This profile in the turbulent regime may be approximated to a fair degree by a simple power law and also by logarithmic expression. In the laminar regime very near to the wall the universal velocity profile

approaches $U^+ = Y^+$ and there is a transition region in the vicinity of $Y^+ = 10$ where neither of the simple expressions holds. Irrespective of this, if a sufficiently good empirical equation of the profile were available, the profile could be generated and could be used to produce both friction coefficient data and velocity profile data under circumstances similar to those which are involved in the definition of this universal velocity profile.

In order to apply the wall law to calculation of shear stress from known velocity, density, and viscosity data at the given distance, y , it is necessary to use some mathematical expression to represent known experimental data. To represent the data on the U^+ vs. Y^+ plot over any desired range with any degree of accuracy the expression

$$Y^+ = f(U^+) = U^+ + b(U^+)^2 + c(U^+)^3 + d(U^+)^4 \text{ ---} \quad (4)$$

could be used. Actually, the scatter of the data from different authors is such that the selection of the coefficients is arbitrary relative to the functionality in the most critical range around $U^+ = 10$. A more simple expression which gives a very adequate fit over the range of interest is

$$Y^+ = U^+ + \left(\frac{U^+}{a}\right)^n = U^+ + \left(\frac{U^+}{8.74}\right)^7 \quad (5)$$

It was found that the numerical constants used in the conventional power law gave a very adequate fit over the range of interest. Consequently this simple expression was used in a computer program to relate U^+ and Y^+ for measured values of velocity, wall distance, and fluid properties.

Figure 10 shows the above function on a semilog plot of U^+ vs. Y^+ , and Figure 11 shows a similar plot, expanded in the vicinity of $Y^+ = 10$, where

FIGURE 10. UNIVERSAL VELOCITY PROFILE SHOWING LOGARITHMIC APPROXIMATION AND PROPOSED APPROXIMATION AS COMPARED WITH NIKURADSE'S DATA

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FIGURE 11. EXPANDED PLOT SHOWING RELATIONSHIP BETWEEN MEASURED PROFILES AND UNIVERSAL VELOCITY PROFILE APPROXIMATION

a very critical comparison in the range of transition can be seen. This also shows the discrepancy between the different experimental data including those of Nikuradse⁽²⁾, Reichardt⁽³⁾, and Laufer⁽⁴⁾.

Experimental Results

A computer program was written to determine the value of wall τ necessary to force the experimental data to satisfy Equation (5). Since $U^+ = u\sqrt{\rho/\tau}$ and $Y^+ = y\sqrt{\tau/\rho}/\nu$, Equation (5) can be rewritten as

$$y\sqrt{\tau/\rho}/\nu = u\sqrt{\rho/\tau} + (u\sqrt{\rho/\tau}/8.74)^7 \quad (6)$$

Here ν and ρ are known constants, and y and u are obtained from the measured velocity profiles. Thus τ may be determined and converted to C_f where $C_f \equiv 2\tau/\rho u_b^2$. If the actual profile is of the same form as the universal velocity profile, and if the flow is simple Couette flow, the value of C_f computed for any pair of (u, y) coordinates will correspond to that for any other pair. If the values of computed τ vary out from the wall, this indicates that the velocity profile does not follow the universal velocity profile.

Figure 12(b) shows a number of profiles of computed C_f for concentric flow (where $C_f \equiv 2\tau/\rho u_b^2$), showing the computed value to remain nearly constant, but still dropping off toward midstream. This shows that the universal-velocity-profile shear-stress relationship underestimates the actual midstream shear stress; thus it indicates that an additional mode of momentum transfer must be at work, over and above the turbulent mode. This is undoubtedly Taylor vortex flow, still at work even when the flow is fully turbulent. By extrapolating the C_f profiles back to the wall, values are obtained which must be independent of this momentum transfer. The

FIGURE 12. COMPUTED NOMINAL FRICTION COEFFICIENT BASED ON ASSUMED WALL LAW PROFILE

NOTE: ACTUAL FRICTION COEFFICIENT IS GIVEN BY WALL INTERCEPT, $y/b = 0$ (y MEASURED FROM JOURNAL)

results of such extrapolations are shown in Figure 13, and it is seen that the C_f magnitudes are above those for plane-Couette flow as well as the thin-film values of Smith and Fuller. The data tend toward the earlier data of Taylor for even thicker films, though the transition is at higher Reynolds number than his. Critical Taylor number for transition for concentric operation is at $R \approx 80$, and corresponds well to the observed transition.

If the values of C_f determined by using the extrapolation technique are used to compute U^+ and Y^+ , the actual profiles may be compared with the universal velocity profile. This is shown in Figure 11 where in this case

$$U^+ = (u/u_b) \sqrt{2/C_f} \quad (7)$$

$$Y^+ = yR/U^+b \quad (8)$$

It is seen that the midstream values ($y = b$) are below the universal velocity profile. This again emphasizes that some factor other than simple turbulence contributes to the resistance to flow.

In order to determine the effect of film thickness variation on C_f , velocity profiles were obtained at three different journal eccentricities. The resulting values of C_f are shown in Figures 12(a), (c), and (d). When the extrapolated values of C_f are plotted against local Reynolds number, shown in Figure 13, it is seen that the values follow the same trend as those for the concentric journal.

Summary

As was previously mentioned, when the experimental values of C_f shown in Figure 13 are compared with the data for plane-Couette flow as

FIGURE 13. FRICTION COEFFICIENT AS A FUNCTION OF LOCAL REYNOLDS NUMBER IN LARGE-SCALE TURBULENCE APPARATUS ($\bar{b}/r = 0.00793$)

well as the thin-film values of Smith and Fuller, it is seen that the values reported here are generally higher, though they are in agreement with the measured velocity profiles. The data tend toward Taylor's earlier data for even thicker films. Though the transition appears as a smooth break-away from the laminar curve, compared to the relatively abrupt transitions of the Smith and Fuller, and Couette data, there is no contradiction here since the Reynolds numbers for the Smith and Fuller and Couette data are higher.

Figure 14 shows the theoretical transition of Di Prima⁽⁵⁾ as calculated for the $\bar{b}/r$ ratio of the experiment reported here. As will be seen in the figure, the predicted transition is more abrupt than the smooth transition which the experimental data show. The theoretical transition, however, is not unlike that of Taylor's experiment for the outer cylinder turning where more abrupt transitions than for inner cylinder turning were observed.

FIGURE 14. COMPARISON BETWEEN MEASURED FRICTION COEFFICIENT AND A PREDICTION FROM VORTEX FLOW THEORY, FOR CONCENTRIC JOURNAL PLACEMENT

TURBULENCE INTENSITY

The hot-wire anemometer lends itself to measurement of the instantaneous magnitude of the velocity component u parallel to the direction of flow. With an A. C. vacuum-tube voltmeter the root-mean-square amplitude of the fluctuating component of this velocity, u' , can be read, providing a measure of turbulence intensity. The magnitude of turbulence intensity can then be determined as a function of position in the film as well as Reynolds number, eccentricity and other variables. Such measurements may, in the long run, shed light on circumferential transport of turbulence as well as other factors.

Figure 15 shows comparisons between measured data for Couette flow (concentric journal), with representative values for pressure flow from the Literature. Data of Carper, Heilhecker and Logan⁽⁶⁾ are closely comparable with the reported work. These latter data are also supported by the independent measurements of Laufer⁽⁹⁾. Quite different results are reported by Reichardt⁽⁸⁾. Whereas this difference remains unexplained, it is felt that the independent corroboration of the other data supports their validity.

Figure 16 shows half-channel journal-side profiles of u' / u_s , for $\bar{R} = 1200$ and for four film thicknesses. Two aspects of these profiles are of interest; (1) the variation near the wall and (2) the variation in the bulk of the flow. Near the wall, at $(h-y)/b \doteq 0.1$, $u' / u_s \doteq 0.06$ for all clearances, thus the wall-turbulence is apparently responsive to local conditions. On the other hand midstream turbulence is high for the thin film and low for the thick film. This might tend to indicate that turbulence is being swept

FIGURE 15. VARIATION OF MEASURED TURBULENCE INTENSITY WITH DISTANCE FROM THE WALL, IN THE HALF-CHANNEL, FOR A NOMINAL REYNOLDS NUMBER OF $\bar{R} = 2400$.

FIGURE 16. VARIATION OF MEASURED TURBULENCE INTENSITY FOR A NOMINAL REYNOLDS NUMBER OF $\bar{R} = 1200$.

into the thin-film region from some more highly turbulent zone.

On the other hand, reference to Figure 17 shows that even for concentric operation ($h/\bar{h} = 1.0$), midstream turbulence level is above the wall level for low $\bar{R}$.

To make possible better comparisons among the changes of overall turbulence level, the integrated value of turbulence level is plotted relative to $\bar{R}$ in Figure 18, and R in Figure 19. The data are seen to agree fairly well on the basis of local Reynolds number, but to spread broadly on the basis of $\bar{R}$. Had $(\overline{u'}/u_s)^2$ been plotted instead of $(\overline{u'}/u_s)^2 h/\bar{h}$, the data on the $\bar{R}$ plot would have been the more closely grouped, but not much better than that shown in Figure 19.

The group $(\overline{u'}/u_s)^2 h/\bar{h}$ plotted is a measure of turbulence flux through the specified location, and one must conclude that, except near transition, this is largely determined by local Reynolds number. Whether the transition, as indicated by this quantity, occurs at a given $\bar{R}$ cannot be verified by the points plotted, since not enough extend into the lower range of $(\overline{u'}/u_s)^2 h/\bar{h}$. Without serious distortion the curves shown could be replaced by a family tending to group together at $\bar{R} = 80$ to 100 for extremely low turbulence values. Thus the possibility is not eliminated that $\bar{R}$ does indeed control transitions as has been suggested on the basis of the δ^*/b data.

FIGURE 17. VARIATION OF MEASURED TURBULENCE INTENSITY FOR A NOMINAL REYNOLDS NUMBER OF $\bar{R} = 250$.

FIGURE 18. EFFECT OF LOCAL FILM THICKNESS ON TURBULENCE INTENSITY AS A FUNCTION OF NOMINAL REYNOLDS NUMBER $\bar{R}$

FIGURE 19. EFFECT OF LOCAL FILM THICKNESS ON TURBULENCE INTENSITY AS A FUNCTION OF LOCAL REYNOLDS NUMBER R

EXPERIMENTS WITH DISTORTED FLOWS

Skewed Flows

Very little has been published on extremely skewed flows, like that where a strong axial pressure flow interacts with a strong tangential Couette flow. In an attempt to produce such flows experimentally, it was found that, even at large journal eccentricities in the full bearing, insufficient leakage flow would arise to permit a critical set of measurements on skewed flows. As a second approach to producing such a flow, the journal was set concentric and a barrier placed in the film in the form of a 0.5×0.5 -in. board which blocked the flow in the tangential direction and forced the fluid to divert and pass out the ends of the bearing clearance.

Figure 20 shows a map of the local velocity vectors at the half-channel thickness as the flow approaches the blockage. Figure 20(a) is for $\bar{R} = 550$, and Figure 20(b) is for $\bar{R} = 2400$. Here z is measured from the mid-span of the bearing outward toward the edge, and x is measured from the blockage. Both measurements are in inches. Note that the velocity at the mid-span ($z = 0$) simply decelerates. As z increases, the velocity turns into the axial direction and increases in magnitude. The differences between the flow patterns for the two Reynolds numbers is apparently due to inertial effects, since for the higher Reynolds number a measurable tangential component of velocity still persists even at $x = 1$.

FIGURE 20. HALF-CHANNEL VELOCITY VECTORS IN
BLOCKED FLOW FOR (a) $\bar{R} = 550$ AND (b) $\bar{R} = 2400$

Figure 21 shows typical measurements of the velocity direction across the film for several given (x, z) points, for $\bar{R} = 2400$. Here θ is the angle that the velocity vector, V , makes with the z axis, therefore $\theta = 90^\circ$ at the journal surface ($y/h = 1$).

Another way of presenting these profiles is shown in Figure 22, where the x and z components of the velocity vectors have been computed from the data at one of the (x, z) points shown in Figure 21.

At the journal surface the normalized circumferential component of velocity, u/u_s , is unity (corresponding to the journal surface speed), and the normalized axial component is zero. Near the bearing surface both components of velocity tend to zero as they should.

Pressures were measured for the flow situation shown in Figure 20(b), and are shown in Table I, where $P \equiv p/\rho(u_s/2)^2$.

Analysis of these data on skewed flow involves the complete solution for bearings with end leakage and will be attempted in the future. At present these data are reported for their direct interest value.

Decelerated Flows

Another important topic investigated in the above setup was the effect of acceleration or deceleration on shear-stress and velocity profiles. A strongly decelerated flow occurred at midspan when the blockage was placed across the channel. In effect, this blockage also produced a pressure flow opposing the

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FIGURE 21. ANGLE OF VELOCITY VECTOR VS DIMENSIONLESS WALL DISTANCE FOR SKEWED FLOW AND $\overline{R} = 2400$, (AT TWO LOCATIONS)

FIGURE 22. TANGENTIAL AND AXIAL COMPONENTS OF VELOCITY PROFILE FOR SKEWED FLOW

TABLE I . PRESSURE DISTRIBUTION IN
SKEWED FLOW ($\bar{R} = 2400$)

<u>x, in.</u>	<u>z, in.</u>	<u>P</u>
1	0	0.410
	2	0.382
	4	0.356
	6	0.289
2	0	0.364
	2	0.356
3	0	0.340
	4	0.297
	6	0.232
4	0	0.314
	2	0.314
5	0	0.297
	4	0.267
	6	0.210

Couette flow. Figure 23 shows full-channel velocity profiles measured at the axial mid-span of the bearing, for two values of Reynolds number. For different distances from the blockage (indicated by $x = 2, 3$, and 4 inches, measured from the blockage), the profiles go very nearly normal to the stationary wall and show sharp gradients at the moving wall. These full-channel profiles are quite different from those shown in Figure 3 for unaccelerated Couette flow but will be shown to correspond closely to those in half-channel plots. Referring again to Figure 23, note that both sets of profiles show the same general configuration though the Reynolds numbers differ by a factor of 10.

Figure 24 shows the journal-side half-channel profiles taken from the data shown in Figure 23. These resemble closely the half-channel profiles for simple Couette flow, as shown in Figure 7(b). Table II shows how the displacement thickness parameter δ^*/b , previously defined, varied with Reynolds number for the two types of flow. In the table $(u_s - u_b)/u_s$ is a measure of the centerline velocity relative to the moving surface. The magnitude 0.5 represents half of u_s as would prevail in Couette flow; a value of unity represents centerline velocity corresponding to the relative velocity of the opposite wall. Considering how greatly the flow has been distorted here, it is interesting that δ^*/b is nearly the same for the two cases shown in Figure 24. For comparison purposes, the values of δ^*/b for simple Couette flow $[(u_s - u_b)/u_s = 0.5]$ for two corresponding Reynolds numbers are given. Data from Robertson are included for $(u_s - u_b)/u_s = 0.5$ and 0.3. Again these values are in general agreement.

(a)

(b)

FIGURE 23. NORMALIZED FULL-CHANNEL VELOCITY PROFILES IN FIELD WITH LARGE PRESSURE GRADIENT FOR CONCENTRIC OPERATION
(a) $\bar{R} = 2400$ AND (b) $\bar{R} = 250$

NOTE: x REPRESENTS DISTANCE FROM BLOCKAGE IN FILM

(a)

(b)

FIGURE 24. NORMALIZED HALF-CHANNEL VELOCITY PROFILES CORRESPONDING TO FIG. 23 FOR (a) $\bar{R} = 2400$ AND (b) $\bar{R} = 250$

TABLE II. PARAMETERS OF HALF-CHANNEL PROFILES
WITH STRONG PRESSURE FLOW

$\frac{(u_s - u_b)}{u_s}$	$\overline{R}$	δ^*/b
0.5	250	0.25
1.0	250	0.19
1.0	250	0.18
1.0	250	0.18
0.5	2400	0.15
1.0	2400	0.15
1.0	2400	0.12
1.0	2400	0.11
0.5(a)	10,000	0.125
0.3(a)	10,000	0.124

(a) After Robertson, Ref. (9).

Figure 25 has been included for comparison with Figures 15, 16, 17, and shows data on the turbulence level for the journal-side half-channel, corresponding to the velocity profiles in Figure 24. The turbulence level appears to be higher than in Couette flow. This may be attributable to a storage effect due to the sudden deceleration, or may be due to increased stability of the wall layers in the adverse pressure-gradient field.

(a)

(b)

FIGURE 25. VARIATION OF MEASURED TURBULENCE INTENSITY IN A REGION OF STRONG PRESSURE GRADIENT FOR
(a) $\bar{R} = 250$ AND (b) $\bar{R} = 2400$

PRESSURE MEASUREMENTS

Development of Pressure Measuring Device

As was mentioned previously, static pressures of extremely low order of magnitude were generated in the large scale bearing apparatus, typically 10^{-5} psi and lower. Although there are instruments available which will resolve pressures of approximately this order, difficulties associated with their use suggested the development of a simpler and more rugged device for use in measuring the static pressure distribution around the bearing.

The device which was developed employs a halogen detector with its sensing element placed in a tube, connected to a static pressure tap on the bearing. A halogen sample is injected into the tube, a known distance upstream of the sensing element. This is forced along the tube toward ambient by the pressure to be measured. The time required for the halogen to reach the sensing element is recorded. Using the Poiseuille flow equation, the unknown pressure can then be computed.

As schematic of the complete system is shown in Figure 26. As shown in the figure, halogen (Freon 12 in this case) is admitted into the sample chamber by a regulator. The pressure difference ($p_1 - p_2$) then causes a flow, in the direction shown, down the tube and across the sensing element of the halogen detector. The pressure p_2 can be either atmospheric, for determining gage pressure p_1 , or it may also be a pressure in the bearing, for measurement of the gradient between two points.

FIGURE 26. SCHEMATIC DIAGRAM OF PRESSURE MEASURING DEVICE

The procedure for operating the device is best understood by referring to Figures 26 and 27, where Figure 27 is a schematic of the operation of valves A, B, C, and D. First the 3-way valve A is turned so that it blocks the flow path from the bearing, valves B and C are shut, and valve D is opened so as to charge the sample chamber with Freon as shown in Figure 27(a). Next, valve D is shut, and C is opened so that the sample chamber exhausts the Freon to ambient pressure as shown in Figure 27(b). Now valve C is shut and A and B are turned simultaneously as shown in Figure 27(c) so that the freon sample begins to move in the tube towards the halogen detector. At this point a stop-watch is started and is stopped when a response is obtained from the halogen detector.

A typical response of the halogen detector is shown in Figure 28, which is a recorder chart of detector voltage versus time. The time required for the detector voltage to increase from the background level to the peak is approximately 3 seconds while the total time for the halogen to reach the detector is of the order of at least 30 seconds. Thus, the error in measuring the total time is less than 10 percent. Typical resolution of the instrument is 10^{-6} psi.

Calibration Equation

The equation for relating the time-to-response of the halogen detector to the static pressure in the bearing is derived as follows: The Poiseuille equation for laminar flow in a tube may be written

$$\Delta p = 8\mu \frac{\ell}{R^2} \bar{u} \quad (9)$$

FIGURE 27. ILLUSTRATION OF VALVE SETTINGS
FOR VALVE OPERATION SEQUENCE

FIGURE 28. TYPICAL RESPONSE OF HALOGEN DETECTOR
DURING PRESSURE MEASUREMENT PROCESS

where

Δp = pressure drop across the tube

μ = viscosity of fluid

ℓ = length of tube

R = radius of tube

$\bar{u}$ = average flow velocity in the tube

In this case the total length of the tube is made up of various fittings and tubes with different lengths and diameters, and a variable-length tube to allow measurements over different pressure ranges. Then

$$\ell = \ell_1 + \ell_2 + \dots + \ell_v + \dots + \ell_n \quad (10)$$

where ℓ_v is the length of the variable-length tube. The total pressure drop is the sum of the drops across each tube or

$$\Delta p_T = \Delta p_1 + \dots + \Delta p_v + \dots + \Delta p_n \quad (11)$$

substituting Equation (9) into Equation (11) for each tube and using tube diameter instead of radius

$$\Delta p_T = 8\mu \left[\frac{\ell_1 u_1}{\left(\frac{D_1}{2}\right)^2} + \frac{\ell_2 u_2}{\left(\frac{D_2}{2}\right)^2} + \dots + \frac{\ell_v u_v}{\left(\frac{D_v}{2}\right)^2} + \dots + \frac{\ell_n u_n}{\left(\frac{D_n}{2}\right)^2} \right] \quad (12)$$

From continuity

$$u_i = \left(\frac{D_v}{D_i}\right)^2 u_v \quad (13)$$

where

u_i = velocity in any particular tube

u_v = velocity in the variable-length tube

Then substituting Equation(13) into Equation (12)

$$\Delta p_T = 32\mu \left[\ell_1 \left(\frac{D_v}{D_1} \right)^2 \frac{u_v}{(D_1)^2} + \ell_2 \left(\frac{D_v}{D_2} \right)^2 \frac{u_v}{(D_2)^2} + \dots + \frac{\ell_v u_v}{(D_v)^2} + \dots + \ell_n \left(\frac{D_v}{D_n} \right)^2 \frac{u_v}{(D_n)^2} \right] \quad (14)$$

or

$$\Delta p_T = \frac{32\mu u_v}{(D_v)^2} \left[\ell_1 \left(\frac{D_v}{D_1} \right)^4 + \ell_2 \left(\frac{D_v}{D_2} \right)^4 + \dots + \ell_v + \dots + \ell_n \left(\frac{D_v}{D_n} \right)^4 \right] \quad (15)$$

The total time for the halogen sample to reach the detector is the sum of the times for each tube between the point of injection and the sensing element or

$$t = \frac{\ell_v}{u_v} + \dots + \frac{\ell_n}{u_n} \quad (16)$$

Again from continuity

$$u_i = \left(\frac{D_v}{D_i} \right)^2 u_v \quad (17)$$

Then

$$t = \frac{\ell_v}{u_v} + \dots + \frac{\ell_n \left(\frac{D_n}{D_v} \right)^2}{u_v} \quad (18)$$

or

$$t = \frac{1}{u_v} \left[\ell_v + \dots + \ell_n \left(\frac{D_n}{D_v} \right)^2 \right] \quad (19)$$

Solving for u_v

$$u_v = \frac{1}{t} \left[\ell_v + \dots + \ell_n \left(\frac{D_n}{D_v} \right)^2 \right] \quad (20)$$

Substituting Equation (20) into (15)

$$\Delta p_T = \frac{32\mu\ell_v^2}{(D_v)^2 t} \left[\sum_{i=v}^n \frac{\ell_i}{\ell_v} \left(\frac{D_i}{D_v} \right)^2 \right] \left[\sum_{i=1}^n \frac{\ell_i}{\ell_v} \left(\frac{D_v}{D_i} \right)^4 \right] \quad (21)$$

In Equation (21) all lengths and diameters of tubes are known, the time t is measured, and the pressure Δp_T can be calculated. In this report Δp_T represents the gage static pressure in all cases since one end of the tube was attached to a static pressure tap on the bearing, and the other end exhausted to the atmosphere.

Experimental Results with 360° Bearing

To check the validity of Equation (21) two different tubes were used as the variable-length tube in the device to measure the pressure at the same point on the bearing. In one case, the two pressures agreed within 10 percent and in another case within 3 percent. The error in reading the exact time-to-response of the halogen detector can account for the slight disagreement.

Pressures in the large scale turbulence apparatus range upward to the magnitude of $P = 4p/\rho u_g^2 = 3 \times 10^{-2}$.

Since the "velocity pressure" is $\rho u_g^2/4$, even a one-percent change in velocity would create a pressure increment such that $\Delta P = 10^{-2}$.

Thus even a very small disturbance can create pressure changes that mask the true pressure. As a consequence of this delicacy of measurement, a great many runs were required to establish confident data. The end result showed a gratifying consistency, as indicated in Figures 29 and 30. In Figure 29, the time-readings of the pressure measurement instrument are shown, thus indicating a considerable variation in actual pressure from run to run. In Figure 30, the dimensionless pressure P is plotted for the three runs and is seen to correspond well within expected experimental scatter despite the change in Reynolds number from run to run. The direction of turning of the journal with respect to the point of minimum film thickness, 0° , is shown on the figures. These values sustain the arguments that both circumferential and axial flow are significant even in this short bearing experimental setup ($L/D = 1/3$).

Experimental Results with Pad Bearings

As was mentioned in the description of the test apparatus, pad bearings can be studied by removing panels from the 360° bearing. Some pressure data were obtained near the leading edge of a pad, with the journal running concentric, and with an air film thickness of 0.54 in. These data are shown in Figure 31. Two entry conditions were used, one with a step at the entry, the other without the step. As shown in the figure, the pressure is negative at entry when the step is not used, and

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FIGURE 29. PRESSURE INSTRUMENT READINGS VS ANGULAR POSITION FOR JOURNAL ECCENTRICITY OF 0.5

FIGURE 30. DIMENSIONLESS PRESSURE DISTRIBUTION
CORRESPONDING TO FIG. 29

FIGURE 31. PRESSURE DISTRIBUTION AT ENTRY TO A PAD-TYPE BEARING

as flow proceeds away from entry the pressure begins to rise, still being negative, however, even at 90° behind the entry. When the step is fixed to the pad, the pressure is relatively high under the step, but as soon as the flow enters under the pad the pressure again goes negative and is still negative at 90° behind the entry. Some "ram" effect is noted since the pressures, though negative, are higher than those for the pad with no step. A flow model is suggested in Figure 32. As shown in the figure, evidently separation occurs just prior to entry of the flow under the pad causing a "dead water" region of low pressure. The profile then begins to take the shape shown in the figure, the velocity varying across the film from that of the journal surface, u_s , to zero at the leading edge of the pad. At some distance downstream from this point the profile has changed to the shape shown where u_b is the mid-film velocity $\doteq u_s/2$. The momentum equation for this flow shows that there must be a pressure rise in the direction of flow which agrees with that actually observed.

FIGURE 32. ILLUSTRATION OF FLOW AT ENTRANCE TO A PAD-TYPE BEARING

ANALYSIS OF BEARING FILMS WITHOUT END LEAKAGE

Introduction

Because of the emphasis upon profile parameters and friction coefficient in the experimental program, it has been found desirable to carry through an analytical treatment of bearing films, in terms of these quantities. As a consequence it has been found that the choice of dimensionless form of the variables takes maximal advantage of the inherent simplicities of turbulent shear flows. As will be shown, the "film equation" which results is a straightforward differential equation with four coefficients which may be held constant to "linearize" the problem, or which may be allowed to acquire experimentally or analytically determined values to correct the equation to apply to actual flows. Three of the coefficients are found to be near unity in magnitude and are relatively insensitive to the details of the flow. Thus, a "grouping" of variables has been achieved wherein the chief nonlinearities of the problem tend to cancel one another.

To be specific, an equation for circumferential flow (in an annulus of varying thickness) has been derived from the Navier-Stokes equation in the X direction and the continuity equation $\partial u / \partial x + \partial v / \partial y = 0$ ($u_o = 0$ at h_o) such that:

$$\xi (1 + U) \frac{\partial U}{\partial X} + 2C_f \xi \frac{U}{H} = -\frac{dP}{dX} \quad (22)$$

where

$$\frac{\Delta u_b}{u_{bc}} \equiv U, \quad x/h_c \equiv X, \quad h/h_c \equiv H, \quad p/\rho u_{bc}^2 \equiv P, \quad C_{fc} \equiv C_f \quad (23)$$

and

$$\xi \equiv \frac{C_{fo} (u_{bc} + \Delta u_b)^2 - C_{fh} (u_{bc} - \Delta u_b)^2}{4C_{fc}(u_{bc})\Delta u_b} \quad (24)$$

$$\zeta = \frac{\int_0^h u^2 dy}{h u_b^2} \quad (25)$$

The last three quantities are shown to be weak variables and, of the order of unity.

Description of Film Equations

An expression of Newton's second law for such a film* may be written as follows (under the assumption of steady flow with the local velocity u in the x direction, and where pressure is assumed to be independent of y , the coordinate measured across the film thickness, which ranges from 0 to h):

$$\rho \int_0^h \left(u \frac{\partial u}{\partial x} + v \frac{\partial u}{\partial y} \right) dy + \frac{\partial p}{\partial x} h = \tau_h - \tau_o \quad (26)$$

It is now possible to introduce the definition

$$C_f = \frac{2|\tau|}{\rho u_b^2} \quad (27)$$

such that C_f is defined relative to the wall on which the shear stress is to be measured, and u_b is defined as midchannel velocity relative to that wall.

Substituting into Equation (26)

$$\frac{\partial}{\partial x} \int_0^h \rho u^2 dy + \frac{\partial p}{\partial x} h = \left[\frac{C_{fh} \rho (u_{bh})^2}{2} \right] - \left[\frac{C_{fo} \rho (u_{bo})^2}{2} \right] \quad (28)$$

Refer to Figure 33 in which the definition of the relative velocities u_{bh} and u_{bo} are defined.

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* For turbulent or laminar film in steady flow.

FIGURE 33. NOMENCLATURE FOR
ANALYSIS OF BEARINGS
WITHOUT END LEAKAGE

At this point it is desirable to refine further the definition of u_b in order to examine the case of a pressure flow, acting codirectional with a Couette flow. As illustrated in Figure 33(b), a Couette flow with centerline velocity u_{bc} is incremented to the right by a pressure flow, where the incrementation of the centerline velocity is of the magnitude Δu_b .

In view of the definitions of u_{bo} and u_{bh} , these become

$$|u_{bo}| = |u_{bc} + \Delta u_b|$$

$$|u_{bh}| = |u_{bc} - \Delta u_b|$$

Here it is recognized that the Couette flow bears such symmetry relative to each wall that the midchannel velocity is, in either case, designated by the same quantity u_{bc} , relative to either wall, for the Couette flow acting alone.

In terms of this, Equation (28) becomes

$$\frac{\partial}{\partial x} \int_0^h \underbrace{\rho u^2 dy}_I + \underbrace{\frac{\partial p}{\partial x} h}_{II} = \underbrace{\frac{C_{fh} \rho (u_{bc} - \Delta u_b)^2}{2} - \frac{C_{fo} \rho (u_{bc} + \Delta u_b)^2}{2}}_{III} \quad (29)$$

The equation has been divided into three parts for purposes of discussion, part I being the "momentum flux" term, part II being the "pressure gradient" and part III being the "wall shear" term.

Wall Shear Term

If C_{fc} is the wall shear stress for the Couette flow before its incrementation by Δu , it is possible to write for other conditions the relation

$$\Phi = \frac{C_f}{C_{fc}} \quad (30)$$

Hence ΦC_{fc} will permit the computation of actual friction coefficient C_f when Φ is known. To obtain some idea of the type of variation expected of Φ we may write the approximate expression:

$$\Phi = \frac{C_f}{C_{fc}} = \left(\frac{R_c}{R} \right)^m \quad (31)$$

In this case we have assumed that the C_f vs R relationship is continuous and can be approximated by suitable selection of m . This would not necessarily be the same m that would best represent the Reynolds number variation for the Couette flow alone, though evidence has been presented (see section on friction coefficient) which suggests that m will not differ greatly from that magnitude. In terms of this, Term III of Equation (29) becomes

$$(III) = \rho C_{fc} \left(\frac{u_{bc}}{u_{bc} - \Delta u_b} \right)^m \frac{(u_{bc} - \Delta u_b)^2}{2} - \rho C_{fc} \left(\frac{u_{bc}}{u_{bc} + \Delta u_b} \right)^m \frac{(u_{bc} + \Delta u_b)^2}{2} \quad (32)$$

Simplifying Equation (32)

$$(III) = \rho \frac{C_{fc} u_{bc}^2}{2} \left[\left(1 - \frac{\Delta u_b}{u_{bc}} \right)^{2-m} - \left(1 + \frac{\Delta u_b}{u_{bc}} \right)^{2-m} \right] \quad (33)$$

If $m = 0$ (in which case $C_f = C_{fc} = \text{Const.}$), Equation (33) would reduce to

$$(III) = - \rho \frac{C_{fc} u_{bc}^2}{2} \left[\frac{4\Delta u_b}{u_{bc}} \right] \quad (34)$$

This would be expected to prevail where wall roughness causes C_f to depart from smooth-wall behavior and level out at a nearly constant value at high Reynolds number.

Using the results of Equation (33) and Equation (34), a useful comparison is the consequence when the following quotient is determined:

$$\xi = \frac{\left[\left(1 + \frac{\Delta u_b}{u_{bc}} \right)^{2-m} - \left(1 - \frac{\Delta u_b}{u_{bc}} \right)^{2-m} \right]}{4(\Delta u_b/u_{bc})} \quad (35)$$

The quantity ξ would compare the effect of C_f varying, with that of constant C_f as to their influence on Term III. Table III shows comparisons of the variation of ξ with $\Delta u_b/u_{bc}$ for $m = 1/2$ and $m = 1/4$, which would be expected to represent a realistic range of magnitudes.

TABLE III. EFFECT OF PRESSURE FLOW ON ξ

	ξ	
	$m = 1/2$	$m = 1/4$
$\Delta u_b/u_{bc} \rightarrow 0$	0.75	0.875
$\Delta u_b/u_{bc} = \pm 1/2$	0.741	0.866
$\Delta u_b/u_{bc} = \pm 1$	0.70	0.84

Note that for $\Delta u_b/u_{bc} \rightarrow 0$, $\xi \rightarrow 1 - \frac{m}{2}$. Also note that $\Delta u_b/u_{bc} = 1$, represents the limit of applicability of Equation (29), as written, since exceeding of this magnitude would require a change of the sign in Term III of Equation (29), since the shear stress at one wall would reverse direction.

In terms of ξ , and Equation (33), Term III becomes

$$(III) = -\rho \frac{C_{fc} u_{bc}^2}{2} \xi \left[\frac{4\Delta u_b}{u_{bc}} \right] \quad (36)$$

In recognizing that $\rho C_{fc} u_{bc}^2/2$ is the wall shear stress τ_c of the original Couette flow acting alone

$$(III) = -\tau_c \xi \left[\frac{4\Delta u_b}{u_{bc}} \right] \quad (37)$$

This is a useful form of the term since ξ is not widely variable, as is seen in Table III. As a first approximation, it could well be $\xi = 1 - \frac{m}{2}$. If more

accuracy is desired any reasonable choice of m will give a fair approximation since, for the wide range indicated, the ratio of ξ for the two values of m is about 0.85; or, only a 15% error in Term III would have resulted if one value of m had been used for computation when the other should apply. Somewhere between these extremes even better agreement should result.

Should we, in measurement, be able to determine m within 20% of its best value, only a very small error in Term III would result. This suggests the feasibility of obtaining realistic predictions of the "wall shear" term by reasonably accurate experimentation.

Momentum Flux

In Equation (29) the momentum flux term, Term I, may be written as

$$(I) = \frac{\partial}{\partial x} \int_0^h \rho u^2 dy = \frac{\partial}{\partial x} (\zeta \rho u_b^2 h) \quad (38)$$

A new dimensionless quantity ζ is introduced to account for the variation of u . It is of interest to determine the general magnitude of ζ and its sensitivity to the shape of the velocity profile. We note that

$$\zeta = \frac{\int_0^h u^2 dy}{h u_b^2} \quad (39)$$

Though some additional analysis is possible, we shall proceed directly to an order-of-magnitude investigation of factors influencing the term by the assumption that both u_c and Δu vary out from a wall into either half channel

according to an equation of the form $u/u_b = (y/b)^n$. In addition to approximating the flow in each half channel by a power-law profile, it assumes that the law is identical for each*. In terms of this, Equation (38) becomes

$$\zeta h u_b^2 = \int_0^b \left\{ (u_{bc} + \Delta u_b) (y_o/b)^n \right\}^2 dy_o + \int_0^b \left\{ 2u_{bc} - (u_{bc} - \Delta u_b) (y_h/b)^n \right\}^2 dy_h \quad (40)$$

To obtain some information of the variation of ζ , it is desirable to insert some numerical value of n , which for the present has been chosen as $1/7$. In addition to ζ , two additional parameters have been included in the calculations, and these have been defined as follows

$$A \equiv \frac{\int_0^h u^2 dy}{u_{bc}^2 h} \quad (41)$$

$$B \equiv \frac{(u_{bc} + \Delta u_b)^2}{u_{bc}^2}$$

and

$$A/B = \zeta \quad (42)$$

Table IV compares A , B and ζ for different values of $\Delta u_b/u_{bc}$.

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*Also it is assumed that $\partial n / \partial x \rightarrow 0$.

TABLE IV. EFFECT OF PRESSURE FLOW ON ζ

$\Delta u_b / u_{bc}$	<u>A</u>	<u>B</u>	<u>ζ</u>
0	1.028	1	1.028
+1	3.555	4	0.888
-1	0.056	0	∞

Note that though $\zeta \rightarrow \infty$ for $\Delta u_b / u_{bc} \rightarrow -1$, this is not to be misinterpreted as making ζ a useless parameter. Comparison of A and B shows that the momentum flux prediction using the approximate profile, follows the simple prediction reasonably well. Thus, if ζ were assumed to be unity and multiplied by $B = 0$, it would predict $A = 0$ rather than the very small value of 0.056, computed by the more accurate estimate of the profile.

Continuity

The continuity equation for an incompressible lubricant film may be written as follows:

$$(h_c - h) u_{bc} = \int_0^h \Delta u \, dy \quad (43)$$

Here there is no normal motion between surfaces, and h_c is film thickness where only Couette flow would prevail. It may, in fact, be a hypothetical thickness which does not exist in a bearing, but nevertheless serves as a parameter to characterize the flow.

To determine how the intergral $\int_0^h \Delta u \, dy$ is influenced by velocity profile form, let us use, as previously, a profile in the half channel (for

velocity relative to the nether wall) $\Delta u / \Delta u_b = (y/b)^n$. Equation (43)

becomes

$$(h_c - h)u_{bc} = 2 \int_0^b \Delta u_b (y/b)^n dy$$

or integrating

$$(h_c - h)u_{bc} = \frac{2\Delta u_b b}{n+1} = \frac{\Delta u_b h}{n+1} \quad (44)$$

Had the velocity Δu_b prevailed across the channel as a "constant" increment to the flow the result would have been

$$(h_c - h)u_{bc} = \Delta u_b h \quad (45)$$

The ratio of the right-hand term in Equation (44) to the equivalent term in Equation (45) may be designated by χ and would be, for $n = 1/7$,

$$\chi = \frac{1}{n+1} = 0.875 \quad (46)$$

Again the critical quantity is close to unity. Hence the correction due to the choice of n tends not to be a major factor. Irrespective of the particular value of n , the continuity equation written in the following form must be correct:

$$(h_c - h)u_{bc} = \chi \Delta u_b h \quad (47)$$

Extreme Conditions

It has been mentioned that the preceding analysis cannot be applied without alteration outside the range $-1 < \Delta u_b / u_{bc} < +1$. Furthermore, the approximate power law expressions for C_f and velocity profile must break down where $\frac{u_{bc} - \Delta u_b}{u_{bc}} \rightarrow 0$. On the other hand, shear stress tends also to go to zero in this case, which makes the precise magnitude relatively unimportant.

To show how insensitive a typical quantity might be to gross departures of velocity profile from the chosen form, let us examine the somewhat more realistic power law profile approximation shown in Figure 34, with respect to the continuity equation.

$$h_c u_{bc} = \int_0^h u \, dy = \int_0^h 2u_{bc} \left(\frac{y}{h}\right)^n \, dy = \frac{2u_{bc}h}{(n+1)} \quad (48)$$

Taking difference between $u_{bc} h_c$ from Equation (48), and $u_{bc} h$

$$u_{bc} (h_c - h) = \frac{(1-n)u_{bc}h}{(n+1)} \quad (49)$$

When Equation (49) is compared with Equation (47), with the condition $\Delta u_b / u_{bc} \rightarrow 1$, we note that

$$\chi = \frac{1-n}{(n+1)} \quad (50)$$

If, for instance, $n = 1/7$, $\chi = 0.75$, which is not greatly different from the value of 0.875 given by the previous computation in Equation (46).

Summary

Though additional comparisons can be made, it is futile to do so without further knowledge of actual velocity profiles. There is nevertheless reassurance in the observation that ζ , ξ and χ are all near unity and are all relatively insensitive to the details of the velocity profile or the C_f variation function.

Again it should be reiterated that the defining-equations for ζ , ξ and χ are not dependent on special assumptions as to the particular velocity profile or C_f variations. They remain critical parameters in the film

FIGURE 34. ILLUSTRATION OF APPROXIMATE VELOCITY PROFILE WHEN $(\Delta u_b)/u_{bc} \rightarrow 1$.

equation for determining the influence of turbulence quantities. They may be used to compare various turbulence "theories" as to differences in their predictions. They are also the quantities to be evaluated by our experimental measurements.

SHORT-BEARING EQUATIONS

The experimental apparatus represents the "short-bearing" regime of film behavior since the L/D ratio is approximately 0.3. Thus, it is pertinent, in the paragraphs which follow, to set up equations especially for turbulent film behavior applicable exactly to the limiting case of $L/D \rightarrow 0$, and presumably applicable with diminishing accuracy as L/D becomes larger.

Pressure Gradient

The short-bearing limiting case may be characterized by a tangential Couette flow and an axial pressure flow. The combined flow may be characterized by a friction coefficient acting on each wall of the type

$$\tau_{\text{wall}} = C_f \rho V^2 / 2 \quad (51)$$

where

$$V \equiv \sqrt{w_b^2 + u_b^2}$$

Note that the symmetry of the flows (at least in the limiting case of $b/r \ll 1$, is such that V would bear the same magnitude relative to either surface.

The component of wall shear stress opposing the pressure flow would be

(on the assumption that τ_{wall} is codirectional with V) $\tau_{\text{wall}} \frac{w_b}{V}$. Allowing for the fact that such codirectionality may not prevail, and that C_f may depart from C_{fc} (which would prevail for the Couette flow), we may introduce a factor ϕ to be evaluated experimentally, and write

$$h \frac{\partial p}{\partial z} = \frac{2\phi C_{fc} \rho V^2}{2} \frac{w_b}{V} \quad (52)$$

Recognizing that w and V are orthogonal

$$\phi C_{fc} \rho u_b^2 \left(\frac{w_b}{u_b} \right) \sqrt{1 + \left(\frac{w_b}{u_b} \right)^2} = \frac{h}{\bar{h}} \left(\frac{\partial p}{\partial z / \bar{h}} \right) \quad (53)$$

One approach to nondimensionalization of this equation is provided by the following definitions: $P \equiv p / \rho u_{bc}^2$; $H \equiv h / \bar{h}$; $X \equiv x / \bar{h}$; $Z \equiv z / \bar{h}$; $W \equiv w_b / u_b$; $u_b \equiv u_{bc}$ and

$$C_{fc} \phi W \sqrt{1 + W^2} = H \frac{\partial P}{\partial Z} \quad (54)$$

Continuity Considerations

If pressure flow in the tangential direction is assumed to be zero, the continuity equation for an incompressible flow in a lubricant film is

$$-\psi z u_{bc} \frac{dh}{dx} = w_b h \quad (55)$$

where it is assumed that $\partial h / \partial z = 0$, $w = 0$ at $z = 0$, $\frac{\partial \psi}{\partial z} = 0$, and $u_b \equiv u_{bc}$. The profile factor ψ is found to be $n + 1$ for power-law profiles.

In the above dimensionless terminology, this becomes

$$\psi \frac{Z}{H} \frac{dH}{dX} = -W \quad (56)$$

Substituting back into Equation (54)

$$-C_{fc} \psi \phi \frac{Z}{H} \frac{dH}{dX} \sqrt{1 + \left(\frac{Z}{H} \frac{dH}{dX} \right)^2} = H \frac{\partial P}{\partial Z} \quad (57)$$

Integration of Film Equation

Let

$$a \equiv \frac{1}{H^2} \frac{dH}{dX} \text{ and } b \equiv \left(\frac{dH}{dX} \right)^2 \frac{1}{H^2} \quad (58)$$

In terms of this, Equation (57) becomes

$$-C_{fc} \psi a Z \sqrt{1 + b Z^2} = \frac{\partial P}{\partial Z} \quad (59)$$

Trial Approximation

Integrating Equation (59) with respect to Z (assuming ϕ constant)

$$-C_{fc} \psi \frac{1}{3} \phi \left(\frac{a}{b} \right) \left(1 + bZ^2 \right)^{3/2} = P + f(X) \quad (60)$$

For a bearing of uniform width, $P \equiv 0$, $Z = Z_m (\equiv Z_{\max})$. Thus

$$-f(X) = \psi \phi \frac{C_{fc}}{3} \left(\frac{a}{b} \right) \left(1 + bZ_m^2 \right)^{3/2} \quad (61)$$

Continuing Equation (60) and Equation (61), and substituting for a and b

$$\begin{aligned} -P = \psi C_{fc} \frac{1}{3} \phi \left(\frac{1}{dH/dX} \right) & \left\{ \left[1 + \left(\frac{dH}{dX} \right)^2 \frac{Z^2}{H^2} \right]^{3/2} - \right. \\ & \left. \left[1 + \left(\frac{dH}{dX} \right)^2 \frac{Z_m^2}{H^2} \right]^{3/2} \right\} \end{aligned} \quad (62)$$

If $H = 1 - \epsilon \cos \beta X$, then $dH/dX = \beta \epsilon \sin \beta X$, and near the point where the measured pressure maximum was indicated $\sin \beta X = 1$. This maximum value of dH/dX would also locate the maximum of P in Equation (62).

Thus, at $Z = 0$, the additional condition for locating the maximum pressure,

$$P_{\max} = -\frac{\psi \phi}{3(\beta \epsilon)} C_{fc} \left\{ 1 - \left[1 + (\beta \epsilon)^2 (Z_m)^2 \right]^{3/2} \right\} \quad (63)$$

To evaluate this further note that $X = x/\bar{h}$, and

$$\theta = \frac{x}{r} = \frac{X\bar{h}}{r} = X\beta$$

or

$$\beta = \bar{h}/r$$

In the present bearing-model experiments $\epsilon = 0.5$ and $r \doteq 3$ ft, and

$$z_{\max} = 5/6 \text{ ft}, \quad \beta = 0.015 \text{ and } (\beta \epsilon)^2 (Z_m)^2 \ll 1.$$

$$P_{\max} = \phi (\beta \epsilon) (Z_{\max})^2 \frac{\psi C_{fc}}{2} \quad (66)$$

Thus

$$P_{\max} = 1.28 \phi \psi C_{fc} \quad (67)$$

Remembering that $P_{\max} \equiv p_{\max} / \rho u_{bc}^2$ then, for the measured $P_{\max}$

$$\psi \phi C_{fc} = \frac{P_{\max}}{1.28 \rho u_{bc}^2} \quad (68)$$

since $p_{\max} / \rho u_b^2 \doteq 3 \times 10^{-2}$, $\psi = n + 1$, $n = \frac{1}{7}$ (and assuming $\phi = 1$),

$C_{fc} = 2.61 \times 10^{-2}$. This figure for C_{fc} is not in line with what might have been expected. The difference might be attributable to neglect of tangential pressure flow, but is more likely due to neglect of axial momentum terms.

INERTIAL EFFECT ON A PAD WITHOUT END LEAKAGE AND WITH TURBULENT FLOW

Rewriting Equation (29), for flow in a bearing without end leakage in terms of the above defined quantities [Eqs. (23), (24) and (25)] and neglecting $\partial \zeta / \partial x$

$$-\frac{dp}{dx} = \rho \zeta (u_{bc} + \Delta u_b) \frac{\partial \Delta u_b}{\partial x} + 2 C_{fc} \rho \xi \frac{(u_{bc}) \Delta u_b}{h} \quad (69)$$

if

$$\frac{\Delta u_b}{u_{bc}} \equiv U, \quad \frac{x}{h_c} \equiv X, \quad \frac{h}{h_c} \equiv H, \quad \frac{p}{\rho u_{bc}^2} \equiv P, \quad C_{fc} \equiv C_f$$

then

$$-\frac{dP}{dX} = \zeta (1 + U) \frac{\partial U}{\partial X} + 2 C_f \xi \frac{U}{H} \quad (70)$$

In the absence of normal motion of surfaces, the continuity equation becomes

$$\left(\frac{1}{H} - 1 \right) = U \chi$$

Substituting Equation (71) into Equation (70), we have

$$- \frac{dP}{dX} = \frac{\xi}{H\chi^2} \frac{d\left(\frac{1-H}{H}\right)}{dX} + \frac{\xi(\chi-1)}{\chi^2} \frac{d\left(\frac{1-H}{H}\right)}{dX} + 2C_f \frac{\xi}{\chi} \left(\frac{1-H}{H^2}\right) \quad (72)$$

Letting $C_f' = C_f \frac{\xi}{\chi}$, this simplifies to

$$- \frac{dP}{dX} = \frac{\xi}{\chi^2} \frac{dH/dX}{H^3} - \frac{\xi(\chi-1)}{H^2\chi^2} \frac{dH}{dX} + 2C_f' \left(\frac{1-H}{H^2}\right) \quad (73)$$

For an inclined flat slider on a flat surface

$$H = 1 - aX$$

$$\frac{dH}{dX} = -a$$

Substituting this into Equation (72)

$$dP = \frac{\xi}{\chi^2} \frac{dH}{H^3} + \frac{\xi(\chi-1)}{\chi^2} \frac{dH}{H^2} + \frac{2C_f'}{a} \frac{(1-H)}{H^2} dH \quad (74)$$

If it is assumed, as outlined in the previous section, that ξ/χ^2 and C_f' may be dealt with approximately as constants

$$-P = \frac{\left(\xi/2\chi^2 H + \frac{2C_f'}{a}\right)}{H} + \frac{2C_f'}{a} \ln H + \frac{\xi(\chi-1)}{\chi^2} \frac{1}{H} \quad (75)$$

Since ξ , ξ and χ are all near unity, the term $(\xi/\chi^2)(\chi-1)(1/H)$ in Equation (75) may be neglected because $(\chi-1)$ is very small. Thus, Equation (75) may be written as

$$-P = \left(\frac{\xi}{2\chi^2}\right) \left(\frac{1}{H^2}\right) + \frac{2}{a} \frac{\xi}{\chi} C_f \left(\frac{1}{H} + \ln H\right) \quad (76)$$

Under the same simplifying assumption, P may be integrated a second time; however, Equation (76) was considered to be sufficiently simple for numerical integration. Hence, a program was set up for use with a computer, for the combination of variables shown in Table V.

TABLE V. RANGE OF VARIABLES USED IN
TILTED PAD COMPUTATIONS

ζ/χ	C_f	a	X_o		
0	0.005	0.001	-200	-400	-800
0	0.010	0.001	-200	-400	-800
0	0.015	0.001	-200	-400	-800
1	0.005	0.001	-200	-400	-800
1	0.010	0.001	-200	-400	-800
1	0.015	0.001	-200	-400	-800

The choice of $\zeta/\chi = 0$ simply eliminates the inertial term from Equation (72). The quantity C_f represents a plausible magnitude from the previously shown charts. The quantity X_o measures the position of the pad's leading edge from an arbitrary point, where H is assigned a value of unity. This arbitrary point corresponds approximately to the mean film thickness.

Pressure Profiles

Figure 35 shows a set of pressure profiles, showing the effect of pad length L (expressed as a multiple of mean film thickness), and ζ/χ (the inertial factor). Because L was not one of the preset parameters in the program, exactly comparable values are not available for all of the magnitudes of ζ/χ . On the other hand, they are sufficiently close to permit useful comparisons. The principal feature to note in these plots is that the higher the inertial factor, ζ/χ is, the lower the profile is. Figures 36 and 37 show similar relations for different friction coefficients C_f .

FIGURE 35. PRESSURE DISTRIBUTION UNDER TILTED PADS SHOWING EFFECT OF INERTIA FOR $C_f = 0.005$

FIGURE 36. PRESSURE DISTRIBUTION UNDER TILTED PADS SHOWING EFFECT OF INERTIA FOR $C_f = 0.01$

FIGURE 37. PRESSURE DISTRIBUTION UNDER TILTED PADS SHOWING EFFECT OF INERTIA FOR $C_f = 0.015$

Location of Center of Pressure

In Table VI, the symbol C_p/L represents the distance back from the leading edge to the center of pressure, expressed as a fraction of pad length. The center of pressure is behind the pad center, as is true in laminar flow.

TABLE VI. EFFECT OF BEARING PARAMETERS ON
COMPUTED CENTER-OF-PRESSURE LOCATION

<u>a</u>	<u>C_f</u>	<u>ξ / λ</u>	<u>L</u>	<u>C_p/L</u>
.001	.005	0	358	.523
	.005	0	661	.543
	.005	0	1188	.571
	.005	1	204	.510
	.005	1	530	.533
	.005	1	1082	.562
	.01	0	358	.523
	.01	0	661	.542
	.01	0	1188	.570
	.01	1	279	.520
	.01	1	594	.537
	.01	1	1134	.565
	.015	0	358	.524
	.015	0	661	.541
	.015	0	1188	.571
	.015	1	305	.521
	.015	1	616	.571
	.015	1	1151	.567

Mean Dimensionless Pressure

Figures 38, 39 and 40 show the variation of mean dimensionless pressure with bearing lengths for three values of friction coefficient C_f . Here it is seen that inclusion of inertia only results in a correction of the order of ten percent, for all lengths and slopes. The greater slopes and lengths lead to the largest pressures, as would be expected.

FIGURE 38. MEAN PRESSURE AS A FUNCTION OF LENGTH TO CLEARANCE RATIO, SHOWING EFFECT OF INERTIA
FOR $C_f = 0.005$

FIGURE 39. MEAN PRESSURE AS A FUNCTION OF LENGTH TO CLEARANCE RATIO, SHOWING EFFECT OF INERTIA FOR $C_f = 0.01$

FIGURE 40. MEAN PRESSURE AS A FUNCTION OF LENGTH TO CLEARANCE RATIO, SHOWING EFFECT OF INERTIA
FOR $C_f = 0.015$

CONCLUSIONS

On the basis of the reported results we conclude that the large scale apparatus is, indeed, capable of providing unique data for use in turbulent film bearing design. Velocity profiles measured therein show regular relationships to Reynolds number; and friction coefficients calculated from these fall into an orderly relationship with prior measurements by others.

On the basis of the velocity profile data it has become apparent that, for concentric operation, local Reynolds number does not determine velocity profile shape so much as does the mean Reynolds number in the film.

Friction coefficients in eccentric operation have been shown, at different localities, to correspond well with those of simple Couette flow. This was not unexpected on the basis of earlier, published comparisons which indicated that both friction coefficient and velocity profile shape are largely independent of the details of the flow.

Measurements of turbulence intensity (which are given here on a preliminary basis, subject to fuller exploitation later) offer to provide more precise information on transition and the circumferential transport of turbulence.

Experiments with distorted flows, produced by a blockage in the film, have provided maps for subsequent use to test theoretical analyses. They have shown, for present exploitation, that half-channel profiles are not greatly different in such flow from the corresponding profiles

in Couette flow. Thus they confirm what has previously been only a useful assumption.

The analyses which have been reported here are shown to simplify the turbulent film problem greatly, dealing as they do with quantities of the flow integrated across the film, rather than details within the film. Checks with profile- and friction-coefficient representations, typical of experimental values, show the chief film parameters in the derivation are insensitive to the details of the flow, and vary within prescribable limits. As a consequence, the derived pressure and velocity distributions may be considered accurate so long as they satisfy the full boundary conditions. This, at first, does not appear to be so in the case of the short bearing, where pressure data are compared with computed results. On the other hand, it should be recognized that large momentum flux is associated with the flow. It is concluded that, rather than arouse questions as to the fundamental analysis, this points up the necessity for further work on momentum effects at film boundaries and interiors. From the initial measurements of such effects at the entry to a pad, it has been demonstrated that they can be quite significant. It is expected that such effects will be clarified in the near future and incorporated into subsequent bearing analyses. With this inclusion, and extension of the derivations to full two-dimensional flow (tangential and axial), it is expected that a family of bearing performance plots can be calculated in the near future.

REFERENCES

1. Burton, R. A., "On the Close Relationship Between Turbulent Plane-Couette and Pressure Flows," J. of the Aerospace Sciences, Vol. 29, No. 8.
2. Nikuradse, J., "Gesetzmässigkeit der turbulenten Strömung in glatten Rohren," VDI Forschungsheft, Vol. 356, 1932.
3. Reichardt, H., "Vollständige Darstellung der Turbulenten Geschwindigkeitsverteilung in glatten Leitungen," Z. Angew. Math. and Mech., Vol. 31, No. 7, July 1951, pp. 208-214.
4. Laufer, J., "The Structure of Turbulence in Fully Developed Pipe Flow," NACA TN 2954, 1956.
5. DiPrima, R. C., "Viscous Flow Between Rotating Concentric Cylinders with a Circumferential Pressure Gradient at Speeds Above Critical," ASLE Trans., Vol. 7, No. 4, October, 1964, pp. 333-341.
6. Carper, H. J., Heilhecker, J. K., and Logan, E., "The Effect of a Change in Wall Roughness on Turbulence Intensity and Shear Stress Distribution in a Two-Dimensional Channel Flow," 8th Midwestern Mechanics Conference, Case Institute of Technology, 1963.
7. Laufer, J., "Investigation of Turbulent Flow in a Two-Dimensional Channel," NACA Report No. 1953, 1951.
8. Reichardt, H., "Messungen turbulenter Schwankungen," Naturwissenschaften, Vol. 404, 1938.
9. Robertson, J. M., "On Turbulent Plane-Couette Flow," 6th Midwestern Conference on Fluid Mechanics, University of Texas, 1959.