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METHODS FOR PREDICTING
AERODYNAMIC HEATING RATES TO
HIGH-VELOCITY BODIES
IN THE UPPER ATMOSPHERE

AEC Research and Development Report

RELEASED FOR ANNOUNCEMENT
IN NUCLEAR SCIENCE ABSTRACTS

ATOMICS INTERNATIONAL

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METHODS FOR PREDICTING
AERODYNAMIC HEATING RATES TO
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IN THE UPPER ATMOSPHERE

By
R. D. ELLIOTT

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ABSTRACT

The basic physical phenomena associated with aerodynamic heating are described. The methods currently used at Atomics International for predicting heating rates to bodies moving at high velocities through the earth's atmosphere are summarized. The methods described provide the principal input data for the analytical studies of SNAP reentry disintegration in the Aerospace Nuclear Safety Program.

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I. INTRODUCTION

This report describes the basic physical phenomena associated with aerodynamic heating and summarizes the methods currently used at Atomics International (AI) for predicting heating rates to bodies moving at high velocities through the atmosphere of the earth.

Accurate predictions of aerodynamic heating rates are important to the Aerospace Nuclear Safety Program because it is necessary to be able to predict reliably the modes of disintegration of SNAP systems and components upon reentry into the atmosphere. If it is required that some systems or components survive reentry, accurate predictions of the heating rates are equally important.

Early approaches to the problems of predicting aerodynamic heating rates were based on the premise that the flow phenomena could be classified into two categories: (1) free-molecule

flow and (2) equilibrium-continuum flow. Heating rates to reentering bodies were calculated for each of these two types of flow. Then the lesser of the two calculated rates at each instant during the trajectory was used in the disintegration analysis.

Further studies of the phenomena associated with aerodynamic heating have brought out the need for recognition of several intermediate flow regimes between free-molecule and equilibrium-continuum. Furthermore, it has been found necessary to consider in greater detail the effects of interactions between the flow and the surfaces of the reentering bodies.

Consideration of these refinements in future analyses will not invalidate previous conclusions concerning the safe reentry of SNAP systems, but it will enable AI to narrow the limits of uncertainty on the precise modes of disintegration.

II. THE REGIMES OF AERODYNAMIC FLOW AND HEATING PHENOMENA

A. THE ATMOSPHERE OF THE EARTH

The density of the atmosphere surrounding the earth has been determined by numerous rocket soundings up to altitudes of about 500,000 ft. Densities above this altitude have been deduced from observations of the decay characteristics of satellite orbits. These data, together with the results of other observations and theoretical studies, have been used in the preparation of standard atmosphere tables, such as the ARDC Model Atmosphere, 1959^{(1)*} and the U.S. Standard Atmosphere, 1962.⁽²⁾ These two standards differ only slightly up to altitudes of 300,000 ft, as shown in Figure 1. Above 300,000 ft, the two standard models

begin to diverge; and at 400,000 ft, the 1962 atmosphere has a tabulated density that is about 50% greater than that given for the 1959 atmosphere. AI is currently using the 1962 standard atmosphere in their studies of reentry heating.

The composition of the 1962 standard atmosphere is taken to be the same as that of sea level air up to an altitude of 90 km (295,000 ft). The mean molecular weight of the air is taken to be 28.964 up to that altitude. At higher altitudes, it decreases due to dissociation and diffusive separation, as shown in Figure 2. The standard variation of free air temperature with altitude is also shown in Figure 2.

*Superscript numerals refer to the References listed in Section VII of this report.

Figure 1. Atmospheric Density for 1959 and 1962 Standard Atmospheres

For thermal analysis of reentering bodies, it is assumed that the standard atmosphere is the same at all latitudes, and that it rotates with the earth as if it were a solid medium attached to the surface. These assumptions would not be used for studies on particle dispersion in the lower atmosphere, but they are quite adequate for studies of heating in the upper atmosphere.

If it is assumed that the atmosphere consists of uniform molecules, each having a molecular weight equal to the average value for air, and if these molecules are assumed to have a Maxwellian distribution of random velocities, then the mean speed of the molecules is given by the following equation:⁽²⁾

$$\bar{v}_{\infty} = \sqrt{\frac{8 R^* T_{\infty}}{\pi M}} \quad \dots (1)^*$$

* See Appendix A for definitions of symbols.

Figure 2. Air Temperature and Molecular Weight for 1962 Atmosphere

The average collision diameter, σ , for an "air" molecule in a field of other "air" molecules is taken to be 3.65×10^{-8} cm. This value is used in both the 1959 and the 1962 standard atmospheres. The mean free path of a molecule between successive collisions may be found as follows:⁽²⁾

$$\lambda_{\infty} = \left(n_{\infty} \sqrt{2} \pi \sigma^2 \right)^{-1} \quad \dots (2)$$

A convenient formula for calculating the mean free path as a function of the density of the ambient air may be found from the tabulated data of References 1 and 2 to be:

$$\lambda_{\infty} = \left(\frac{5.172 \times 10^{-10}}{\rho_{\infty}} \right) \left(\frac{M}{28.964} \right) \quad \dots (3)$$

Figure 3 shows values of λ_∞ as a function of altitude for the 1962 standard atmosphere.

B. THE PURE FREE-MOLECULE FLOW REGIME

As a satellite body orbits the earth, it collides with the molecules of gas in the atmosphere. With each collision, a small amount of momentum is transferred from the body to the gas. Thus, the satellite continually loses energy (kinetic plus potential), and the mean radius of its orbit decreases. Eventually, the perigee (minimum) altitude of its orbit decreases to such a low value that the satellite cannot complete another circuit of the earth, and it rapidly descends to the lower atmosphere. This critical altitude is nominally about 400,000 ft, although the exact value depends to some extent upon the ratio of the mass of the satellite to its effective drag area.

At high altitudes, where the mean free path of the molecules in the atmosphere is much greater than the size of the reentering body, aerodynamic forces and heating rates can be calculated by statistical analysis of the collisions between the body and the gas molecules. As the altitude of the body decreases, there is an increasing probability of collisions between molecules that have been in contact with the body and molecules that have not previously been disturbed by the body. The altitude range in which there is a negligible probability that gas molecules in the path of the body will be struck by molecules emitted from the body is called the pure free-molecule regime.

Roughly speaking, pure free-molecule flow occurs when the mean free path of the molecules is much larger than the radius of the body. Probstein⁽³⁾ has proposed a more rigid definition. He recommends use of the mean free path of the gas molecules which have been in contact with the body, brought into thermal equilibrium with the surface, and then emitted. In a coordinate system that travels with the body, the mean free path of these molecules in

Figure 3. Mean Free Path of Molecules in Ambient Air

a population of undisturbed molecules is given by the following equation:

$$\lambda_{ei} = \frac{\bar{v}_e}{n_\infty V_\infty \pi \sigma^2} \quad \dots (4)$$

The mean speed, $\bar{v}_e$, of the emitted molecules may be found by use of Equation 1, with the body surface temperature, T_w , used in place of the free air temperature.

Probstein then defined free-molecule flow as flow in which the ratio of λ_{ei} to the radius of the body, R_N , is greater than 10. Figure 4 shows the lower limit of the pure free-molecule flow regime as a function of body radius for a flight velocity of 26,000 ft/sec. (The variation with flight velocity is very small over the range of velocities from 15,000 to 50,000 ft/sec.) It may be noted that for bodies of moderate size, pure free-molecule flow occurs only at altitudes above 350,000 ft. At these high altitudes, the magnitude of the aerodynamic heating is so small that it is almost negligible in reentry disintegration analyses.

Figure 4. Flow Regimes for Hypervelocity Flight in the Upper Atmosphere

C. THE FIRST-COLLISION, FREE-MOLECULE FLOW REGIME

As the reentering body descends into denser air, there is an increased frequency of collisions between the emitted molecules and the incident molecules. If the surface of the body becomes hot enough to release an appreciable amount of vapor, these vapor molecules may also collide with the incident molecules. These collisions affect the flux of momentum and energy between the body and the air.

Willis⁽⁴⁾ has calculated aerodynamic forces and heating rates for simple bodies in free-molecule flow, allowing for the finite probability of each incident molecule being struck once by an emitted molecule. The probability of more than one collision per incident molecule before it reaches the body was assumed to be negligible. This type of analysis is called first-collision, free-molecule theory.

Probstein⁽³⁾ holds that first-collision theory is valid down to values of λ_{ei}/R_N of about 1/3. The lower limit of the first-collision regime, so defined, is shown on Figure 4. Below a value of $\lambda_{ei}/R_N = 1/3$, multiple collisions become so frequent that the flow begins to take on the nature of a continuum.

D. THE TRANSITIONAL FLOW REGIME

This regime, as defined by Probstein, is bounded by the conditions that $\lambda_{ei}/R_N = 1/3$ and $\lambda_{\infty}/R_N = 1$. Mathematical analyses of aerodynamic forces and heating in this regime are very difficult to perform because, on one hand, the multiple collisions are too complex to be handled by free-molecule theory. On the other hand, if an analysis by continuum-flow theory is attempted, the usual simplifications of the Navier-Stokes equations of fluid dynamics to form the boundary-layer equations are not valid.

The solutions that have been found for flow in this regime have been for very special and limited problems, such as the solution of Liu and Lees⁽⁵⁾ for Couette flow. Couette flow is the flow that becomes established between two parallel plates of infinite extent, one plate moving with respect to the other in a direction contained within the plane of the plate. At low air densities, there were indications of a discontinuity or slip between the velocity of the wall and the velocity of the air adjacent to the wall.

Probstein⁽³⁾ states that, although solutions of the flow in the transitional regime are quite significant from a fundamental point of view, it is questionable whether such complete solutions will be needed in practical applications. The regime is fairly narrow in terms of altitude, and analytical solutions can be obtained on each side of the regime.

E. THE MERGED LAYER REGIMES

As the body descends still further, a region begins to form in the gas ahead of the body in which the thermodynamic state is distinctly

different from that of the free air. A large increase in the density of the gas occurs over a relatively small distance. This is the beginning of the formation of a bow shock wave. Between this incipient shock wave and the surface of the body, the gas may be treated as a continuum having a nonuniform distribution of entropy. Mixing processes in this region generate vorticity, and the terms for viscous shear forces must be included in the equations of flow.

As the density of the free atmosphere increases still further, the thickness of the shock region decreases and the region in which viscous effects are appreciable becomes separated from the shock. The flight regime in which the shock and the viscous layer are indistinguishable is called the merged layer regime. Probstein divides this regime into two parts. In the fully merged layer regime, the shock region is still thick; and in the incipient merged layer regime, the shock is thin enough that it may be analyzed separately from the viscous region.

The mean free path of molecules in the gas aft of the shock wave may be calculated by use of Equation 3, with the density aft of the shock, ρ_2 , used in place of the density of the ambient air. (The molecular weight of the partially dissociated gas after the shock is less than for free air, but the collision cross sections for the dissociated species are also lower. These two effects tend to cancel each other, and are neglected here.)

Ratios of the densities before and after normal shocks have been calculated and tabulated by Wittliff and Curtis⁽⁶⁾ for altitudes up to 300,000 ft and velocities up to 26,000 ft/sec. If we use the symbol, ϵ , to represent the ratio of ρ_∞ to ρ_2 , the mean free path aft of the shock is:

$$\lambda_2 = \lambda_\infty \cdot \epsilon \quad \dots (5)$$

Values of ϵ range from 0.05 to 0.10 for flight velocities ranging from 26,000 ft/sec down to 10,000 ft/sec over typical flight trajectories.

Probstein defines the lower limit of the fully merged layer regime by the condition that $\lambda_2/R_N = \epsilon/10$, and the lower limit of the incipient merged layer regime corresponds to $\lambda_2/R_N = \epsilon^{1.5}/10$.

F. THE VISCOUS LAYER REGIME

In the viscous layer regime, the shock wave becomes so thin that it may be treated mathematically as if it were a discontinuity in the flow. The region ahead of the body in which appreciable viscous effects occur is still thick. Analyses of flow in this regime have been made by Ho and Probstein⁽⁷⁾ and by Cheng.⁽⁸⁾

Probstein⁽³⁾ delimits this regime by the following set of inequalities:

$$\epsilon^{1.5} < 10 \frac{\lambda_s}{R_N} < \epsilon^{2.0} \quad \dots (6)$$

The boundaries of the viscous layer regime, so defined, are shown in Figure 4.

G. THE BOUNDARY LAYER REGIMES

The boundary layer is a region close to the surface of a body within which it may be assumed that all of the viscous effects are confined. The flow field outside of the boundary layer may be analyzed by use of the equations for inviscid flow.

Within the boundary layer, the general (Navier-Stokes) equations for fluid flow may be simplified by use of similarity considerations. That is, it is assumed that the profile of the curve of velocity vs distance from the surface remains invariant in shape as the flow proceeds around the body. Solutions of these simplified equations for incompressible and compressible flow are well known.

In hypervelocity flow, phenomena occur which complicate the analysis of the flow fields, both in the inviscid and in the boundary layer regions. A large fraction of the oxygen molecules and an appreciable fraction of the nitrogen molecules become dissociated aft of

the shock wave. Some of these dissociated species recombine on the surface of the body, releasing their heat of dissociation. Some of them recombine in the flow outside the boundary layer, and some of them remain dissociated until after the body has passed by.

The energy in the gas aft of the shock wave is bound up in various ways, as shown in Figure 5, which has been taken from a monograph by von Kármán.⁽⁹⁾ If the density of the flowing gas is very low and its velocity is high, release of the energy bound up in vibration and dissociation will not occur to a significant extent. In this situation, the flow is said to be frozen.

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Figure 5. Sketch of Diatomic Molecules Showing Various Degrees of Freedom

If, on the other hand, the density of the gas is high enough, its modes of energy storage at all points in the field will be nearly in accord with the equilibrium state for the local density and enthalpy. This type of flow is said to be in equilibrium.

Fay and Riddell⁽¹⁰⁾ have defined a recombination rate parameter, C_1 , as the ratio of the

time for a gas particle to diffuse through the boundary layer at the stagnation point to the mean time for dissociated species to recombine. They show that when $C_1 \leq 0.1$, dissociated species reach the surface of the body. The line corresponding to $C_1 = 0.1$ has been plotted in Figure 4. Values of C_1 were calculated by use of the equation given by Vaglio-Laurin and Bloom,⁽¹¹⁾ which is claimed to take into account more recent values of the rate parameters than those used by Fay and Riddell. Their equation is as follows:

$$C_1 = 0.2 \left(\frac{p_\infty / p_{S.L.}}{10^{-3}} \right)^2 \left(\frac{0.067}{\epsilon} \right)^{2.5} \left(\frac{9000}{T_s} \right)^{1.5} \left(\frac{20,000}{V_\infty} \right) \left(\frac{R_N}{1.0} \right) \quad \dots (7)$$

If the surface of the body is catalytic to recombination of dissociated species that diffuse through the boundary layer, the energy of dissociation will be released at the surface. In this case, there is no reduction in aerodynamic heating rates due to nonequilibrium dissociation, and equilibrium values can be used up to the lower limit of the viscous layer regime.

At very great flight velocities, above 30,000 ft/sec, a significant amount of thermal radiation is emitted from the region immediately after the shock wave. This heating is greatest at the lower altitudes and increases the heat flux to the surface. The analysis of the flow in this situation is complicated by the loss of heat which requires that the nonadiabatic effects be included in the equations.

Flow in the boundary layer may be characterized as either laminar or turbulent. In laminar flow, mixing of a macroscopic nature does not occur between adjacent lamina in the flow. In turbulent flow, the macroscopic mixing carries the energy of the high-velocity stream down closer to the surface of the body. If all other factors are alike, turbulent flow causes higher heating rates than laminar flow.

During reentry of satellite bodies, the flow is usually laminar during the period of interest for heating analysis. Turbulent flow occurs only at the lowest altitudes, after the body has slowed down to such a velocity that heating is unimportant. For long bodies, such as reentry vehicles, turbulent flow may occur over the aft portions while the heating rates are still high. Furthermore, irregularities in the surfaces of

the bodies may cause local turbulence to occur at high altitudes. Therefore, the turbulent flow regime cannot be overlooked in reentry analysis.

The following sections discuss specific methods for calculating aerodynamic heating rates to bodies in the regimes described in this section.

III. AERODYNAMIC HEATING IN FREE-MOLECULE FLOW

The efficiency of converting the kinetic energy of a reentering body into heating of that body is greatest in the pure free-molecule flow regime. Heating rates in the other flight regimes may be expressed as ratios with respect to the free-molecule rate at any given flight velocity and local atmospheric density. Therefore, an understanding of the equations of free-molecule heating is fundamental to further discussion.

Consider a flat plate of area, A , moving through the atmosphere at a velocity, V_∞ . Let the plate be aligned so that the normal to its surface makes an angle, θ , with respect to the direction of motion, as shown in the following sketch:

This flat plate may be considered to be an element of the surface of a larger body.

Let the local atmosphere be of density, ρ_∞ , and assume that this atmosphere consists of n_∞ particles per unit volume, each particle having a mass, m . Let these particles be in random motion with a Maxwellian distribution of velocities corresponding to the temperature, T_∞ . Then the mean speed, $\bar{v}_\infty$, of these particles can be found by use of Equation 1.

In this discussion, a frame of reference is used that is fixed in the atmosphere, with the body moving through it. The number of particles encountered by the element of area, A , per unit time is $n_\infty V_\infty A \cos \theta$. As each particle comes within the field of influence of the atoms forming the surface of the plate, it is deflected. After one or more deflections, it is emitted from the surface, assuming that it does not combine chemically or physically with the plate material. Experiments with many types of surfaces have shown that the emission is diffuse rather than specular for most ordinary surfaces.⁽¹²⁾ That is to say, the directions taken by the many particles in a stream have a cosine distribution about the normal to the surface.

The extent to which a stream of particles tends to approach thermal equilibrium with the temperature of the surface is measured by the thermal accommodation coefficient, a , which is defined as follows:

$$a = \frac{H_{in} - H_{em}}{H_{in} - H_w} \quad \dots(8)$$

Values of the accommodation coefficient have been determined experimentally for many materials in streams of several different gases. Typical values determined⁽¹³⁾ for air in contact with metal surfaces range from 0.87 to 0.97.

After the particles have been emitted from the plate, they will have a mean speed, $\bar{v}_e$, corresponding to the enthalpy level, H_{em} . The experiments upon which Equation 8 is based were made with fixed plates and moving streams of particles. In the case of a moving plate in a stationary atmosphere, we must consider the energy added to the particles which corresponds to the flight velocity, V_∞ . If we assume that $\bar{v}_e$ is of negligible magnitude in comparison with V_∞ , as will generally be true for satellite re-entry, then the drag force on the plate is equal to the rate of change of momentum of the particles which are accelerated from rest and released at a velocity, V_∞ :

$$D = \rho_\infty V_\infty^2 A \cos \theta \quad \dots (9)$$

The rate of energy dissipation by the body is the rate at which it does work against the drag force:

$$Q_{total} = D V_\infty = \rho_\infty V_\infty^3 A \cos \theta \quad \dots (10)$$

The rate at which energy is imparted to the emitted particles is:

$$Q_{stream} = \rho_\infty V_\infty A \cos \theta \left(\frac{V_\infty^2}{2} + H_{em} - H_\infty \right) \quad \dots (11)$$

If Equation 11 is subtracted from Equation 10, it is seen that the rate at which energy is imparted to the plate is:

$$Q_{plate} = \rho_\infty V_\infty A \cos \theta \left(\frac{V_\infty^2}{2} + H_\infty - H_{em} \right) \quad \dots (12)$$

Defining the stagnation enthalpy of the stream with respect to the plate as:

$$H_s = \frac{1}{2} V_\infty^2 + H_\infty \quad \dots (13)$$

and using the accommodation coefficient defined by Equation 8, we have:

$$Q_{plate} = a \rho_\infty V_\infty A \cos \theta (H_s - H_w) \quad \dots (14)$$

It is interesting to note that in the particular case of an accommodation coefficient of 1.0 and a plate surface temperature equal to the kinetic temperature of the surrounding air, the heating rate to the plate is exactly half of the rate of energy dissipation. It will be shown later that, for continuum flow, the fraction of energy converted is much less than one half. At low altitudes, the fraction converted may be 1% or less.

The projected area of the plate in the direction of its motion is $A \cdot \cos \theta$. If the plate is an element of a larger body, then the free-molecule heating rate per unit projected area is given by the formula:

$$q_{fm} = a \rho_\infty V_\infty (H_s - H_w) \quad \dots (15)$$

It is implicit in this equation that the body be flat or convex to the stream, since concave surfaces would cause multiple scatterings of the air molecules. The equation for heating in this case would be considerably more complex.

It is convenient to define a heating rate coefficient, C_H , in the following way:

$$C_H \equiv \frac{q}{\rho_\infty V_\infty (H_s - H_w)} \quad \dots (16)$$

This coefficient is known as the Stanton number. Comparison of Equations 15 and 16 shows that the Stanton number for pure free-molecule flow as described above is equal to the accommodation coefficient.

IV. STAGNATION POINT HEATING IN CONTINUUM FLOW

A. PRELIMINARY DISCUSSION

Passing over temporarily the transitional flow regimes, this section will proceed to discuss methods for predicting aerodynamic heating rates in the regime where a thin shock wave forms ahead of the body and viscous effects are confined to a thin layer near the surface of the body.

Consider the flow over an axisymmetric body that is tipped by a portion of a sphere of radius R_N . A cross section of a portion of the flow pattern is shown in Figure 6. There is one point on the front of the body, marked s in the figure, away from which the flow along the surface departs radially in all directions. This point is called the stagnation point. Since, for a solid body, there can be no normal component of velocity through the surface, the velocity of the flow at the stagnation point is zero. (In this discussion, and throughout the rest of this report, a frame of reference will be used that is fixed in the body; and the air will be considered to be moving over the body.)

The location of the stagnation point on a body may shift about as the angle of attack of the body changes. In two-dimensional flow, such as flow across a cylinder of infinite length, the stagnation point becomes a stagnation line.

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Figure 6. Hypersonic Flow at the Nose of a Blunt Body

The stagnation streamline is a line in the flow field which starts in the undisturbed flow ahead of the body and terminates at the stagnation point. Consider a tubular surface in the flow field, enclosing all of the flow that passes within an arbitrarily small distance from the stagnation point. Now let that arbitrary distance approach zero and the tubular surface will, in the limit, shrink to a line. This is the stagnation streamline.

Typical variations in the thermodynamic parameters in the flow along the stagnation streamline aft of the shock are shown in Figure 7. As the gas crosses the shock wave, there is a sharp increase in its pressure, density, and temperature and a sharp decrease in its velocity. As the flow continues behind the

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Figure 7. Typical Variations in Thermodynamic Parameters Along the Stagnation Streamline

shock, the diatomic species of nitrogen and oxygen tend to dissociate into monatomic species, as shown in Figure 8. If the time of transit between the shock and the boundary layer on the body is long enough, the species tend to approach their equilibrium values.

The flow along the other streamlines changes in a similar manner, and a region of subsonic velocity exists within an angle of about 60 degrees around the stagnation point. Outside of this region, the flow accelerates to supersonic velocities, the rate of change of velocity depending upon the shape of the body.

In continuum flow, the velocity of the gas at the interface between the gas and the body is zero with respect to the surface. At a comparatively small distance away from the surface, the velocity of the gas in a direction parallel to the surface is essentially equal to the value, u_e , which may be found by solution of the equations

Figure 8. Typical Species Concentrations Aft of a Normal Shock

of the inviscid flow field. The region within which the velocity varies from zero to 99% of u_e is called the boundary layer.

There are steep gradients of velocity and temperature within the boundary layer, as shown in Figure 9. In hypervelocity flow, there are also gradients in the concentrations of dissociated species. These species carry an enthalpy of dissociation which may be released if two atoms of gas collide with a molecule of the surface and recombine.

B. THE METHOD OF FAY AND RIDDELL

In calculating the aerodynamic heating rates near the stagnation points of spherical bodies, the methods developed by Fay and Riddell⁽¹⁰⁾ are followed. Basically, the heat transfer rate from a boundary layer to a surface is the sum of a conductive term and the sum of a series of terms for the diffusion of each gas species, as follows:

$$q = \left(\frac{k}{c_p} \frac{\partial H}{\partial y} \right)_{y=0} + \sum_i \rho (H_i - H_i^0) \left[D_i \left(\frac{\partial C_i}{\partial y} \right) + D_i^T \frac{C_i}{T} \frac{\partial T}{\partial y} \right]_{y=0} \quad \dots (17)$$

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Figure 9. Schematic Drawing of Boundary Layer Flow

Fay and Riddell neglect the term for the thermal diffusivity of species as being unimportant for equilibrium boundary layers at temperatures less than 18,000°R. Velocities near 26,000 ft/sec are just below this limit.

If the velocity and temperature profiles across the boundary layer at various distances from the stagnation point are assumed to be similar to each other, transformations of the flow equations can be made which facilitate their solution. Fay and Riddell applied these transformations and derived the following equation for the heating rate at the stagnation point of a spherical body in laminar flow:

$$q = \frac{Nu}{\sqrt{Re}} \sqrt{\rho_w \mu_w} \left(\frac{du_e}{ds} \right)_{s=0}^{0.5} \frac{H_s - H_w}{Pr} \quad \dots (18)$$

In Equation 18, the local Nusselt number at a distance, s , from the stagnation point is defined as follows:

$$Nu \equiv \frac{q s c_{p,w}}{k_w (H_s - H_w)} \quad \dots (19)$$

The local Reynolds number is defined as:

$$Re \equiv \frac{\rho_w \mu_e s}{\mu_w} \quad \dots (20)$$

The value of the parameter, $Nu/\sqrt{Re}$, as $s \rightarrow 0$, was calculated by Fay and Riddell by numerical solutions of the boundary layer equations. They found:

$$\frac{Nu}{\sqrt{Re}} = 0.67 \left(\frac{\rho_s \mu_s}{\rho_w \mu_w} \right)^{0.4} \left[1 + (Le^b - 1) \frac{H_d}{H_s} \right] \quad \dots (21)$$

H_d is the enthalpy of dissociation, as explained in Section V.B.

The value of the exponent, b , was found to be 0.52 for equilibrium boundary layers and 0.63

for frozen boundary layers. In a frozen boundary layer, the relative concentrations of species reaching the wall is the same as they are in the inviscid flow outside the boundary layer. Fay and Riddell recommend a value of 1.4 for the Lewis number, Le , and 0.71 for the Prandtl number, Pr .

Equations 16, 18, and 21 may be combined to obtain the following expression for the Stanton number:

$$C_H = \frac{0.67}{Pr} \left(\frac{\rho_s \mu_s}{\rho_w \mu_w} \right)^{0.4} \frac{(\rho_w \mu_w)^{0.5}}{\rho_\infty V_\infty} \cdot \sqrt{\left(\frac{du_e}{ds} \right)_{s=0}} \left[1 + (Le^b - 1) \frac{H_d}{H_s} \right] \quad \dots (22)$$

It is useful to define a Reynolds number based on the nose radius of the body and on the conditions aft of the shock:

$$Re_s \equiv \frac{\rho_2 V_2 R_N}{\mu_2} \quad \dots (23)$$

Since the continuity condition through a shock wave requires that $\rho_1 V_1 = \rho_2 V_2$, Equation 23 is equivalent to:

$$Re_s = \frac{\rho_1 V_1 R_N}{\mu_2} \quad \dots (24)$$

Combining Equations 22 and 24 yields the following:

$$C_H = \frac{0.67}{Pr} \left(\frac{\rho_w \mu_w}{\rho_s \mu_s} \right)^{0.1} \sqrt{\frac{1}{\epsilon}} \cdot \sqrt{\left(\frac{du_e^*}{ds^*} \right)_{s=0}^*} \left[1 + (Le^b - 1) \frac{H_d}{H_s} \right] \cdot \frac{1}{\sqrt{Re_s}} \quad \dots (25)$$

In Equation 25, the velocity gradient at the stagnation point has been normalized by use of the terms $u_e^* = u_e/V_\infty$ and $s^* = s/R_N$. The detailed procedures for evaluating the several terms in the equation are given in Section V. B.

It is found upon evaluation of Equation 25 that, over a wide range of flight velocities and altitudes, the product, $C_H \sqrt{Re_s}$ does not vary greatly. Furthermore, it has been found that the viscosity, μ_s , evaluated at the stagnation temperature is nearly proportional to the flight velocity, V_∞ . Thus, it is found that C_H varies nearly inversely as the square root of the product $\rho_\infty R_N$.

It may be further noted that, according to Equation 3, the mean free path, λ_∞ , is inversely proportional to ρ_∞ at altitudes below 300,000 ft where the molecular weight of the air does not vary with altitude. Therefore, C_H is found to vary directly as the square root of the Knudsen number, Kn. Some representative values are shown in Figure 10.

The theoretical solutions of Fay and Riddell have been verified by many experiments, such

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Figure 10. Stanton Number Variation for Stagnation Point Heating

*Chapman⁽²¹⁾ has presented Equation 26 in another form:

$$q = \frac{17600}{\sqrt{R_N}} \left(\frac{\rho_\infty}{\rho_{SL}} \right)^{0.5} \left(\frac{V_\infty}{26,000} \right)^{3.15} \quad \dots (27)$$

In this equation, the number 26,000 is a constant that was chosen because it is a round number close to the value, in ft/sec, of the velocity of a near-earth satellite. Chapman did not include the correction for surface temperature.

as those of Rose and Stark.⁽¹⁴⁾ Although the theory was derived for use in the equilibrium boundary layer regime, it gives correct heating rates for nonequilibrium boundary layers if the surface of the body is highly catalytic to atom recombination.

Experiments have been made to determine the heating rates to blunt bodies in low-density flows, where viscous effects are important in the region aft of the shock wave. Data taken by Ferri and Zakkay,⁽¹⁵⁾ Hickman and Giedt,⁽¹⁶⁾ Wittliff and Wilson,^(17,18) and Potter and Miller⁽¹⁹⁾ in the range of Reynolds numbers, Re_s , between 100 and 1000 have shown that the heating rates are as much as 20% higher than those predicted using the methods of Fay and Riddell.

At Reynolds numbers below about 100, the experimentally determined heating rates fall below the Fay and Riddell values. A method for computing heating rates at low Reynolds numbers has been worked out by Cheng,⁽⁸⁾ and a comparison of his results with the experimental data is shown in Figure 11.

C. THE CORRELATION EQUATION OF DETTRA AND HIDALGO

Detra and Hidalgo⁽²⁰⁾ have formulated an empirical equation that closely matches the results of Fay and Riddell for heating of catalytic surfaces. The agreement is within 10% over the range of flight velocities from 6000 to 26,000 ft/sec, and for altitudes from sea level to 230,000 ft. The equation is as follows:

$$q = \frac{865}{\sqrt{R_N}} \left(\frac{\rho_\infty}{\rho_{SL}} \right)^{0.5} \left(\frac{V_\infty}{10^4} \right)^{3.15} \frac{H_s - H_w}{H_s - H_{540}} \quad (26)^*$$

This equation gives heating rates in units of $\text{Btu/ft}^2\text{-sec}$. The nose radius, R_N , must be given in ft, and the velocity, V_∞ , in ft/sec. The density of the free air, ρ_∞ , appears as a ratio with respect to the standard, sea-level value, and thus any consistent units may be used.

The term involving the stagnation enthalpy, H_s , corrects for the effect of the surface temperature. H_w is the enthalpy of air at the surface temperature, T_w , and H_{540} is the enthalpy of

air at a temperature of 540°R . When the surface temperature is 540°R , the value of the enthalpy ratio in Equation 26 is 1.00. Since H_s is usually much larger than H_w , the correction for surface temperature is frequently ignored. It can, however, be important in re-entry disintegration analysis where surface temperatures become high and velocities low. The possible magnitude of this effect is shown in Figure 12.

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Figure 12. Correction Factor for Effect of Body Surface Temperature

Equation 26 may be revised to give an expression for the Stanton number, C_H , in terms of the Knudsen number, based on the mean free path in the free air:

$$C_H = 1.216 \left(\frac{V_\infty}{10^4} \right)^{0.15} \sqrt{Kn} \quad \dots (28)$$

In the derivation of this equation, it was assumed that the temperature of the free air is 540°R. The effect of using actual air temperatures would be quite small. Values found by use of this equation for velocities of 6000 and 26,000 ft/sec are shown in Figure 10.

D. RECOMMENDED METHODS FOR USE IN SPECIAL CASES

1. Noncatalytic Surfaces

If the surface of a body is not highly catalytic to atom recombination, and if the body is in the nonequilibrium boundary layer flight regime, the methods of Fay and Riddell should be used for calculating the heating rates. If the surface is completely noncatalytic, the diffusion of dissociated species through the boundary layer will not contribute to the heating. The effect of this

can be simulated by using a value of zero for the Lewis number in Equation 21 or 25.

It was calculated by Fay and Riddell⁽¹⁰⁾ that the reduction in heating to a noncatalytic surface may be as much as 50% at the higher altitudes. Fortunately, most metallic surfaces are highly catalytic.

2. Bodies with Nonspherical Noses

The heating rate at the stagnation point of a blunt body of arbitrary shape may be calculated by use of Equation 25 if the value of the velocity gradient, du_e/ds , is available. This gradient may be calculated from analytical or experimental determinations of the pressure distribution around the stagnation point. Boison and Curtiss⁽²²⁾ have measured pressure distributions for a variety of nose shapes.

Equation 26 applies only to bodies having spherical surfaces at the stagnation points. If the body is curved in only one direction, as is a cylinder in cross flow, the Stanton number, as found by use of Equation 25, should be divided by $\sqrt{2}$. If Equation 26 is used, an effective radius equal to twice the radius of the cylinder should be used in place of R_N .

The heating rate at the stagnation point of a flat-ended cylinder, moving lengthwise through the atmosphere, is half of the rate given by either Equation 25 or 26.

The methods of Fay and Riddell do not apply for sharp-nosed bodies that do not have a bow shock normal to the stream. This limitation is not very restrictive, since most practical reentry bodies do not have sharp leading edges. Those that do usually melt back to rounded shapes early during reentry.

3. Heating Rates in the Transition Regimes

As previously mentioned, Cheng⁽⁸⁾ has computed heating rates in the low-density regimes and correlated his results with experimental data. He solved the equations of flow in the bow-shock region by use of finite-difference methods. His results for a velocity of 26,000 ft/sec are shown in Figure 10. It should be noted that Cheng's values cross and fall below the extension of the line plotted from the Detra and Hidalgo equation at a value of C_H of about 0.5.

Since it would not be practical to solve a set of finite-difference equations for each reentry condition, an approximate method of fairing between the continuum line and the free-molecule line is needed. If it is assumed that the shape of the transition curve is the same at other flight velocities, but that the curve is shifted in the horizontal direction, then a general correction chart can be drawn as shown in Figure 13.

The procedure for predicting heating rates in the transition regimes is then as follows:

- 1) At each flight velocity and altitude of interest, calculate the heating rate at the stagnation point of the body by use of the Detra and Hidalgo equation, or by the

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Figure 13. Correction for Stanton Number in Transition Regime

methods of Fay and Riddell. Make the necessary corrections for the body nose radius and shape. The correction for body surface temperature may be omitted at this step.

- 2) At each of the same flight conditions used in the previous step, calculate the heating rate for free-molecule flow with an accommodation coefficient of 1.0. If the surface temperature correction was omitted previously, omit it here.

- 3) Divide the result of Step 1 by the result of Step 2. If the ratio is 0.5 or less, use it as the Stanton number. Optionally, the heating rate from Step 1 can be used directly, with the correction for surface temperature made now if it was not made previously.

- 4) If the ratio found in Step 3 is greater than 0.5, use the graph of Figure 13 or else find the Stanton number by use of the following formula:

$$C_H = 1 - \frac{0.266}{C_{HP}} + \frac{0.008}{C_{HP}^2} \quad \dots (29)$$

In this equation, C_{HP} is the pseudo Stanton number found in Step 3.

V. HEATING AT LOCATIONS AWAY FROM THE STAGNATION POINT

A. PRELIMINARY DISCUSSION

The analytical prediction of aerodynamic heating rates at locations other than at stagnation points is possible for some bodies of relatively simple shape. Heating rate distributions on the forward-facing surfaces of spheres, cylinders, cones, and wedges have been determined experimentally many times, and the analytical methods have been refined so that they give results that agree closely with experimental data.

The analytical methods become more difficult and less reliable for bodies that are not smooth and symmetrical, or for symmetrical bodies at angles of attack other than zero. Most of the analytical methods require that an assumption be made about the similarity of the boundary-layer profiles at various stations along the body. If the flow in the boundary layer separates from the body or if it is retarded excessively, the similarity assumption does not hold. The situation in a typical separated flow is shown in Figure 14, taken from a paper by Bogdonoff and Vas.⁽²³⁾

The intersection of a shock wave from one part of a body with the surface of another part of the body causes a disturbance of the boundary

layer and possible large increases in the local heating rates. The location and interaction of shock waves can be determined from schlieren photographs of the flow about a model of the body in a wind tunnel. In many cases, it is possible to calculate the velocities in the flow field and the locations of the shocks by use of the method of characteristics. This method uses finite-difference techniques for solution of the parabolic equations of supersonic flow.

A procedure for calculating aerodynamic heating rates to blunt-nosed, axisymmetric bodies in hypersonic flow was developed for AI by Hidalgo. The procedure is based mainly on the work of Fay and Riddell⁽¹⁰⁾ for the stagnation region; Kemp, Rose, and Detra⁽²⁴⁾ for the laminar-flow region; and Rose, Probst, and Adams⁽²⁵⁾ for the turbulent-flow region. The procedure has been programmed for calculation with a high-speed digital computer (IBM 7094; FORTRAN-II). The program has been named HYHEAT because it is used for calculating heating rates in hypervelocity flow. An abridged description of the procedures used is given in the next section of this report.

B. DESCRIPTION OF THE HYHEAT COMPUTER PROGRAM

1. General Discussion

The HYHEAT program is applicable, in its present form, to the calculation of aerodynamic heating rates to blunt-nosed bodies that have either axial or plane symmetry with respect to the direction of flight. Typical body configurations that may be analyzed are as follows:

- 1) Spheres
- 2) Circular cylinders in cross flow
- 3) Circular cylinders with hemispherical tips, in axial flow
- 4) Cones with hemispherical tips, in axial flow

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Figure 14. Typical Examples for Boundary Layer Separation

5) Wedges with cylindrical leading edges, in cross flow

6) Flat plates with cylindrical leading edges

7) Composite shapes made up of hemispheres, cylinders, and cones

8) Combinations of semicylinders, flat plates, and wedges.

With some special preparation of input data, it is also possible to use the HYHEAT program for calculating heating rates to bodies having ellipsoidal noses or arbitrary variations in surface slope, within certain limits. Regions of separated flow cannot be calculated. Arbitrary variations of the surface temperature may be specified.

Heating rates are computed for both laminar and turbulent boundary layers. The Reynolds number based on the momentum thickness of the boundary layer is given as output, so that the user can judge where transition may occur. The solutions of the boundary-layer equations are based on the concept of local similarity. This excludes the use of the program in regions of highly retarded or separated flow. It is also required that the chemistry of the flow in the boundary layer be in equilibrium.

The input to the program consists of a description of the body in terms of radii, axial stations, and cone angles. The type of flow is specified by use of a discriminant, j , where $j=0$ for two-dimensional flow or $j=1$ for axisymmetric flow. Since the program integrates certain parameters stepwise along the surface of the body, the number of steps to be used for each component must be specified. Tables of surface temperatures at various locations are required. The flight velocity and altitude must be specified.

2. Calculation of Local Parameters in the Inviscid Flow Field

The first operation in the program is to divide the surface of the body into small increments

along the direction of the flow. Given the overall dimensions and shape of the body, the program finds the coordinates and local slope of each increment.

The state of the air aft of the bow shock is found next. The flight velocity and altitude are supplied as input data. This fixes the conditions ahead of the shock. The conditions aft of the shock are found by an iterative simultaneous solution of the following conservation equations:

Continuity:

$$\rho_1 V_1 = \rho_2 V_2 \quad , \quad \dots (30)$$

Momentum:

$$p_1 + \rho_1 V_1^2 = p_2 + \rho_2 V_2^2 \quad , \quad \dots (31)$$

and Energy:

$$H_1 + \frac{1}{2} V_1^2 = H_2 + \frac{1}{2} V_2^2 = H_g \quad . \quad \dots (32)$$

In addition, an equation of state is needed, both for the free air and for the partially dissociated air aft of the shock. Ahead of the shock, the ideal gas relationship is used:

$$p_1 = \rho_1 R T_1 \quad . \quad \dots (33)$$

Aft of the shock, a correlation equation derived by Cohen⁽²⁶⁾ for equilibrium air properties is used:

$$\frac{p_2}{p_o} = \frac{2.94 \times 10^{-2} \left(\frac{p_2}{p_o} \right)^{0.965}}{1.0477 \left(\frac{H_2}{250 R T_o} \right)^{0.6123} - 0.0477} \quad . \quad \dots (34)$$

The stagnation pressure is found by use of the incompressible Bernoulli equation, as recommended by Wittliff and Curtis:⁽⁶⁾

$$P_s = P_2 + \frac{1}{2} \rho_2 V_2^2 \quad \dots (35)$$

Pressures at other locations along the surface of the body are found by use of different methods, depending on the geometry of the body. Modified Newtonian theory is used for locations on the nose within about 60 degrees of the stagnation point. This theory is based on the observation that the shape of the shock wave about a body in hypersonic flow closely parallels the shape of the body. A momentum balance across the shock, similar to that of Equation 31 leads to the following equation:

$$P_e = P_s - (P_s - P_1) \sin^2 \theta \quad \dots (36)$$

In this equation, θ is the angle that a normal to the surface of the body makes with respect to the direction of flight. At the stagnation point, $\theta = 0$.

The local density, ρ_e , enthalpy, H_e , and velocity, u_e , can all be related to the stagnation enthalpy, H_s , and the stagnation pressure, P_s , by the use of Equation 34 and standard thermodynamic relationships for isentropic flow.

The local viscosity, μ_e , is found by use of another equation derived by Cohen⁽²⁶⁾ for equilibrium air:

$$\frac{\rho_e \mu_e}{\rho_o \mu_o} = \frac{0.225 \left(\frac{P_e}{P_o} \right)^{0.992}}{1.0213 \left(\frac{H_e}{250 R T_o} \right)^{0.3329} - 0.0213} \quad \dots (37)$$

The local temperature, T_e , is found by use of a set of stored tables, based on the data of Hansen.⁽²⁷⁾ This temperature is used to calculate the amount of energy which is tied up in dissociation, as follows:

$$H_d = H_s - \frac{1}{2} u_e^2 - \frac{9}{2} R T_e \quad \dots (38)$$

In this equation, the $9/2$ factor is based on the assumption that the gas molecules are fully excited in the vibrational modes as well as in rotation.

The location on the body at which u_e becomes equal to the local velocity of sound is called the sonic point. On a spherical nose, this usually is found to be at an arc of about 60 degrees from the stagnation point. It has been found that an effective value of $\gamma_e = 1.18$ can be used for finding the sonic velocity and the critical pressure ratio.

Beyond the sonic point, the local pressures are calculated by means of the Prandtl-Meyer expansion method.⁽²⁸⁾ This method relates the changes in velocity of a supersonic flow to the changes in its direction. Thus, starting at the sonic point, and knowing the slope of the wall as a function of distance along the surface, we can find the velocity at other locations. It is implicit in this method that the expansion be isentropic; thus, the calculation cannot be continued through any secondary shock waves. An effective value of $\gamma_e = 1.18$ is used in the Prandtl-Meyer equations.

With the local velocities known, the local enthalpies, pressures, densities, viscosities, and temperatures can all be found.

If the Prandtl-Meyer methods are used along a surface of constant slope, the pressure is found to be constant. This is in agreement with experimental data for conical flow, but is not accurate for flow along flat plates or cylinders that are aligned with the direction of flight. For these cases, the blast-wave analogy⁽²⁹⁾ is used. In this analogy, the body is treated as a line source creating an expansion wave moving radially outward. The formula for the pressure as a function of distance along the surface is:

$$P_e = P_\infty + (P_{e,a} - P_\infty) \frac{s_a}{s} \quad \dots (39)$$

In this equation, the subscript, a, refers to the known conditions at the start of the cylindrical or flat section.

If a cylinder or cone is followed by a cone of a much greater angle, a secondary shock wave will form. The pressure on the portion aft of this shock may be found by use of the Newtonian theory. If the shock is very strong, separation of the boundary layer may occur. The computer program, in its present form, cannot handle such cases. It is possible, however, for the user to put in a complete set of body coordinates and local pressures, which he may have measured or calculated by hand.

The surface temperature, T_w , as a function of distance, s , along the surface is supplied as input to the program. The local density at the surface is found by use of the ideal gas equation of state, Equation 33, with the surface pressure taken as equal to p_e and the temperature as T_w . The viscosity of air at the temperature, T_w , is found by use of the Sutherland equation:

$$\frac{\mu_w}{\mu_o} = \left(\frac{T_w}{T_o} \right)^{1.5} \left(\frac{T_o + 203.4}{T_w + 203.4} \right) \quad \dots (40)$$

The enthalpy of the gas at the wall is found by use of ideal gas properties as follows:

$$H_w = \frac{7}{2} R T_w \quad (41)$$

where $7/2 R$ is the specific heat capacity, c_p , of cold air.*

The velocity gradient at the stagnation point is found by use of the following equation:⁽²⁴⁾

$$\left(\frac{du_e}{ds} \right)_s = \frac{1}{R_N} \sqrt{\frac{2(p_s - p_1)}{\rho_s}} \quad \dots (42)$$

This equation applies for spherical surfaces with pressure distributions that follow the modified Newtonian theory, as given by Equation 36. If desired, a modifying factor may be put in by the user of the program.

As explained in Reference 24, the following integral is evaluated at each location, starting at the stagnation point:

$$\xi = \frac{\int_0^s (\rho_w \mu_w) u_e r^{2j} ds}{(\rho_w \mu_w) u_e r^{2j}_s} \quad \dots (43)$$

The velocity gradient at each station along the body is found by numerical differentiation. This velocity gradient is then used to evaluate the following parameter at each station:

$$\beta = \frac{2 s \xi}{u_e} \left(\frac{du_e}{ds} \right) \quad \dots (44)$$

At the stagnation point, it may be shown that when Equation 42 applies, $\beta_s = (1+j)^{-1}$ and $\xi_s = (2+2j)^{-1}$.

3. Heating Rates With Laminar Boundary Layers

The heating rates for the regions of laminar boundary-layer flow may then be calculated. At the stagnation point, the equation used by the program is:

$$q_s = \frac{0.76}{(Pr)^{0.6}} \sqrt{\frac{1+j}{2}} \left(\frac{p_e \mu_e}{\rho_w \mu_w} \right)^{0.4} (\rho_w \mu_w)^{0.5} \cdot \sqrt{\left(\frac{du_e}{ds} \right)_s} \left[1 + (Le^{0.52} - 1) \frac{H_d}{H_s} \right] \cdot (H_s - H_{w,s}) \quad \dots (45)$$

*For very hot walls, the real gas properties should be used.

This equation follows from Equations 18 through 21, with a modification suggested by Fay and Riddell that the 0.6 power of the Prandtl number be used and that the constant be revised to match at $Pr = 0.71$.

At stations other than the stagnation point, the following equation is used:

$$\frac{q}{q_s} = \sqrt{\frac{\rho_w \mu_w}{\rho_s \mu_s}} \sqrt{\frac{u_e}{s} \left(\frac{du_e}{ds} \right)}^{-0.5} \cdot \sqrt{\frac{\xi_s}{\xi}} \frac{1+0.096}{1+0.096} \frac{\sqrt{\beta}}{\sqrt{\beta_s}} \frac{H_s - H_w}{H_s - H_{w,s}} \dots (46)$$

4. Heating Rates With Turbulent Boundary Layers

The method of Rose, Probst, and Adams⁽²⁵⁾ is used for calculating the heating rates in turbulent flow, with an additional correction for the effect of the pressure gradient along the surface, as explained in Reference 30.

The integral, G , is calculated at each station along the surface by use of the equation:

$$G = \frac{\int_0^s \left(r^j \rho_e u_e^2 \right)^{\frac{5}{4}} \left(\frac{\mu_e}{\rho_e u_e} \right)^{\frac{1}{4}} ds}{\left(r^j \rho_e u_e^2 \right)^{\frac{5}{4}} \left(\frac{\mu_e}{\rho_e u_e} \right)^{\frac{1}{4}} s} \dots (47)$$

(G as stated here is the reciprocal of the parameter described in Reference 30.)

The Reynolds number based on surface length is found at each station along the surface, as follows:

$$Re_x = \frac{\rho_e u_e s}{\mu_e} \dots (48)$$

Then the Nusselt number for turbulent flow is found as follows:⁽³⁰⁾

$$Nu_T = 0.029 (Pr)^{1/3} (Re_x)^{0.8} \left(1 + 0.4 \frac{H_d}{H_s} \right) \cdot (1.037 G^{-0.2}) \dots (49)$$

The term involving H_d is similar to the term involving the Lewis number in Equation 21. Values of H_d are found by use of Equation 38. Unless otherwise specified, the Prandtl number is taken to be 0.71.

The heating rate at each station is then calculated by use of the following formula:

$$q_T = \frac{Nu_T \mu_e}{Pr \cdot s} (H_s - H_w) \dots (50)$$

5. Calculation of the Thickness of the Boundary Layer

It is recommended that the flow in the boundary layer be considered to be turbulent when the Reynolds number based on momentum thickness exceeds 250. Surface roughness could cause earlier transition, so the program presents both laminar and turbulent values for selection by the user.

The momentum thickness of the boundary layer is found by the following formula derived from expressions given in Reference 30:

$$\theta_m = s \sqrt{2 \xi} (Re_x)^{-0.5} \left(\frac{\rho_e \mu_e}{\rho_w \mu_w} \right)^{-0.114} \cdot \left[0.491 (1 - 0.09 \beta^{0.4}) \right] \dots (51)$$

The Reynolds number based on this thickness can be found as follows:

$$Re_\theta = \frac{\rho_e u_e \theta_m}{\mu_e} \dots (52)$$

C. EXPERIMENTAL DETERMINATION OF HEATING RATES

The determination of aerodynamic heating rates on bodies that are not of simple, regular geometry requires experimental investigations. Even for simple bodies at angles of attack other than zero, it is not generally possible to predict heating rates by analytical means. Therefore, any serious investigation of heating of a reentry body requires the use of wind tunnel tests and, as a final proof, flight tests.

An experimental investigation of the relative distribution of heating rates on the SNAP 10A reactor was conducted at AI in 1964. These tests were conducted in a "hotshot" type of hypersonic wind tunnel, as described in Reference 31.

The geometry of the SNAP 10A reactor is quite complex, as shown in Figure 15. The coolant pipes and the pump in front of the reactor create shock waves that affect the heating of the head and sides of the vessel. In order to make possible the predictions of the mode of failure of the SNAP 10A during reentry, as given in Reference 32, it was necessary to have at least an approximate survey of the heating rates.

The technique used in the tunnel tests was to paint the entire model with a proprietary type of black paint. The model was then mounted in the tunnel, and a measured amount of air was heated in a closed chamber by discharge of electrical energy stored in a bank of capacitors. The increase of pressure in the chamber ruptured a diaphragm which allowed the gas to flow through a divergent nozzle and over the model.

Although the duration of the flow was less than 10 msec, its specific enthalpy was high enough to simulate flight at a velocity of 25,500 ft/sec. Stagnation pressures equivalent to those which would occur at altitudes of 250,000 and 275,000 ft were used. During the brief period of the test, a shock wave formed

over the model, and the surface of the paint was heated. The paint responded to this heating in such a way that it became partially clear for a depth that was approximately proportional to the total heat absorbed during the test. The depth of the clear layer was determined after the test by visual inspection of the apparent colors seen when light was reflected from the surface.

During each test, a calibration sphere of known radius was mounted in the tunnel near the model. The colors seen on this sphere could be matched with those seen on the model. Since the relative distribution of heating rates on a sphere can be calculated, the relative heating rates on the model could be identified. Figures 16 through 18 show typical heating rate distributions found during these tests.

There were some questions raised concerning the accuracy of the simulation of flight conditions in the tunnel. It was suspected that the chemical species in the flow froze in a partially dissociated state not far downstream from the throat of the nozzle. Thus, the model was being tested in a stream that was initially dissociated. Furthermore, the paint responded to the total heat pulse, including the blowdown period. Thus, the precise flight condition being simulated could not be stated.

In order to obtain a better understanding of the actual flow conditions in the tunnel, a series of calculations was made for the non-equilibrium expansion processes in the nozzle. The results for a typical test run are shown in Figures 19 through 22. It was found for this test that the air was 42% dissociated at all positions in the nozzle greater than about 2 in. aft of the throat. The equilibrium dissociation at the surface of the calibration sphere (and at the model) was 72%.

Heating rates were calculated for the calibration sphere by means of the Detra and Hidalgo equation, the Fay and Riddell method for a catalytic wall, and the Fay and Riddell

Figure 15. Full-Scale Model of SNAP 10A Reactor as Tested in Wind Tunnel

7611-0375

Figure 16. Local Heating Coefficients on Coolant Pipe

6-1-65

7611-0873

NAA-SR-11704

Figure 17. Relative Local Heating Rates on Top Head of Reactor Vessel

Figure 19. Nonequilibrium Gas Species During Expansion in Wind Tunnel

Figure 20. Nonequilibrium Gas Species Aft of Shock in Wind Tunnel Test

Figure 21. Velocity and Temperature Along Axis of Tunnel Nozzle

Figure 22. Calculated Stagnation Point Heating Rate Variation With Time During Wind Tunnel Test

method for a noncatalytic wall ($Le = 0$). The results in terms of the altitude and velocity simulated are given in Table 1.

TABLE 1
CALCULATED EQUIVALENT ALTITUDES
FOR CALIBRATION SPHERE USED IN WIND
TUNNEL TESTS

Run No.	Velocity (ft/sec)	Altitude (kilofeet)		
		Detra and Hidalgo	Fay and Riddell-Catalytic	Fay and Riddell Noncatalytic
1	25,000	262	267	266
2	25,000	283	286	286
3	28,200	253	254	253

The calculated rate of heating of the calibration sphere during the blowdown period of Run No. 1 is shown in Figure 22. It is evident that a considerable variation in the heating conditions does exist during the period of a test.

There were not enough tests run in the hot-shot tunnel to enable definite conclusions to

be made regarding the variations in heating rate patterns with simulated flight conditions. The relative heating rates at various locations on the models were reduced to ratios with respect to the heating rate at the stagnation point of a hypothetical sphere of 1 ft radius flying at the same speed and altitude. These ratios, called F_q , were then used as constants over the entire range of flight conditions in the continuum flow regime.

While this type of wind tunnel test is useful for giving an overall picture of the heating patterns, it appears that measurements with calibrated heat meters in a regular hypersonic wind tunnel would be of more quantitative value.

The actual flight environment is, of course, best reproduced in a flight test. Such a test was made on a full-scale mockup of the SNAP 10A reactor,⁽³³⁾ and the information obtained was very useful in verifying and supplementing the theoretical predictions. Flight testing is, however, more expensive than tunnel testing; and interpretation of the data is more difficult because the flight velocities, altitudes, and angles of attack vary with time.

VI. CONCLUDING REMARKS

This report presents a summary of the methods currently being used at Atomics International for predicting aerodynamic heating rates in the various flight regimes encountered by reentering SNAP systems. The discussion has been confined to descriptions of fundamental methods for finding heating rates to selected locations on bodies of various shapes, given the flight velocity and altitude.

The methods for hypervelocity, free-molecule flow are shown to be fundamentally simple, at least for bodies that are not concave.

The methods of Fay and Riddell are shown to be applicable for the continuum flow regime; and for bodies having surfaces that are catalytic to atom recombination, they are applicable up to Knudsen numbers of about 0.1.

A method, based on a wealth of experimental data, is given for predicting stagnation-point heating rates in the transition regimes between continuum and free-molecule flow.

Methods are presented for calculating heating rates to various locations on bodies that

are of simple shape and with their principal axes aligned with their direction of flight. These methods apply to both laminar and turbulent flow around the bodies.

A brief description is given of an experimental method that has been used for estimating relative heating rate distributions on complex bodies at various angles of attack. Some limitations of this method are discussed.

The methods described in this report are being used to provide input data for the analyses of SNAP reentry disintegration in the Aerospace Nuclear Safety Program. Wherever possible, theoretical predictions and experimental data are being compared, and the theoretical methods are being refined accordingly. It appears that the primary need for better methods and experiments is in connection with heating of bodies of complex shapes. Flow separation and shock wave interactions, such as occur on typical reactor systems, cannot be adequately handled by means of the present theoretical methods.

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APPENDIX A NOMENCLATURE

Roman Letters

<u>Symbol</u>	<u>Description</u>	<u>Dimension</u>
A	Area	ft ²
a	Accommodation coefficient	*
b	Exponent of the Lewis number	*
C ₁	Recombination rate parameter	*
C _H	Stanton number; heating coefficient	*
C _{HP}	Pseudo Stanton number	*
C _i	Concentration of species, i, in gas	*
c _p	Specific heat capacity	ft-lb/slug-°R
D	Drag force on body	lb
D _i	Diffusivity of species, i, due to concentration gradient	ft ² /sec
D _i ^T	Diffusivity of species, i, due to temperature gradient	ft ² /sec
F _q	Heating rate relative to q _{ref}	*
G	Pressure-gradient parameter	*
H _d	Enthalpy of dissociation	ft-lb/slug
H _i	Enthalpy of species, i, in gas	ft-lb/slug
H _i ^o	Enthalpy of formation of species, i, at 0°R	ft-lb/slug
H	Enthalpy of gas mixture	ft-lb/slug
j	Discriminant for type of flow; j = 0 for two-dimensional j = 1 for axisymmetric	*
K _n	Knudsen number; λ/R _N	*
k	Thermal conductivity of gas	ft-lb/ft-sec-°R
Le _i	Lewis number; D _i /α	*
M	Molecular weight of a gas	slug/slug-mole
m	Mass of a particle	slug
Nu	Nusselt number	*
N	Number of degrees of freedom for energy storage in a gas	*
n	Numerical density of particles	1/ft ³

*Denote dimensionless parameters

Pr	Prandtl number; ν/α	*
p	Pressure	lb/ft ²
Q	Heating rate over an area, A	ft-lb/sec
q	Heating rate per unit area	ft-lb/ft ² -sec
q _{fm}	Heating rate in free-molecule flow	ft-lb/ft ² -sec
q _{FR}	Heating rate at stagnation point, by method of Fay and Riddell	ft-lb/ft ² -sec
q _{DH}	Heating rate at stagnation point, by Detra and Hidalgo equation	Btu/ft ² -sec
q _{ref}	Heating rate in continuum flow at stagnation point of a sphere of 1.0 ft radius and with $T_w = 540^\circ\text{R}$	ft-lb/ft ² -sec
R*	Universal gas constant	ft-lb/slug-mole-°R
R	Gas constant for undissociated air	ft-lb/slug-°R
Re	Reynolds number	*
R _N	Radius of the nose of the body	ft
r	Radius at local station on an axisymmetric body	ft
s	Distance along surface, starting at stagnation point	ft
T	Temperature	°R
t	Time	sec
u	Component of velocity parallel to the surface	ft/sec
V	Magnitude of total velocity vector	ft/sec
$\bar{v}$	Mean speed of particles in a gas	ft/sec
y	Distance normal to surface of body	ft

*Denote dimensionless parameters

Greek Letters

<u>Symbol</u>	<u>Description</u>	<u>Dimension</u>
α	Thermal diffusivity	ft ² /sec
β	Parameter involving the velocity gradient along the body	*
γ	Ratio of specific heats or (N+2)/N	*
ϵ	Density ratio across shock, ρ_1/ρ_2	*
θ	Angle between direction of flight velocity and normal to local surface of body	degree
θ_m	Momentum thickness of boundary layer	ft
λ	Mean free path of molecules in gas	ft
μ	Absolute viscosity of gas	lb-sec/ft ² or slug/ft-sec
ν	Kinematic viscosity of gas	ft ² /sec
ξ	Parameter involving the integral of the dissipation, $\rho_w \mu_w$	*
ρ	density of a gas	slug/ft ³
σ	Collision diameter for molecular scattering	ft

Subscripts

0	Reference value at standard conditions
1	Conditions ahead of the bow shock
2	Conditions immediately aft of the bow shock
∞	Conditions in the free stream (Same as 1)
a	Conditions at start of straight section (Equation 39)
d	Related to dissociation, as in H_d
e	Conditions at outer "edge" of boundary layer
em	Emitted, as in H_{em}
e, i	Emitted particles in a field of incident particles
in	Incident, as in H_{in}
i	Species identification, as in C_i
s	Conditions at the stagnation point
SL	Conditions in standard, sea-level atmosphere
w	Conditions at the "wall" or surface of the body

*Denote dimensionless parameters

APPENDIX B
STANDARD AND REFERENCE VALUES USED IN THIS REPORT

1. Reference Values for Correlation of Thermodynamic Properties of Air:

$$\begin{aligned}p_o &= 2117.0 \text{ lb/ft}^2 \\T_o &= 491.69 \text{ }^\circ\text{R} \\ \rho_o &= 0.002498 \text{ slug/ft}^3 \\R_o &= 1724.0 \text{ ft-lb/slug-}^\circ\text{R} \\M_o &= 28.9644 \text{ slug/slug-mole} \\g_o &= 32.1741 \text{ ft/sec}^2 \\\mu_o &= 3.584 \times 10^{-7} \text{ lb-sec/ft}^2 \text{ or slug/ft-sec} \\R^* &= 49,719 \text{ ft-lb/slug-mole-}^\circ\text{R}\end{aligned}$$

2. Standard Values at Sea-Level Used for Atmospheric Properties:

$$\begin{aligned}P_{SL} &= 2116.22 \text{ lb/ft}^2 \\T_{SL} &= 518.670 \text{ }^\circ\text{R} \\\rho_{SL} &= 0.002377 \text{ slug/ft}^3 \\R_{SL} &= 1716.55 \text{ ft-lb/slug-}^\circ\text{R}\end{aligned}$$