NUMERICAL SIMULATIONS OF PLASMAS
WITH SMOOTHING IN PHASE SPACE
AND FILTERING IN TIME

Progress Report

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Evanston, Illinois

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MASTER

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ABSTRACT

Research under the present contract AT(11-1)-2200 was initiated on February 1, 1972. The program is supported jointly by the U.S. Atomic Energy Commission and the Office of Naval Research (Physics Branch), each agency contributing \$15,000 yearly during the present period.

As stated in the original proposal, the research is directed toward the development of numerical simulation methods for collisionless plasmas using

A. Periodic Smoothing in Phase Space

B. Filtering in Time

and the application of these methods to physical problems of current interest to Controlled Thermonuclear Research.

During the past eight months, research on the project has been concentrated on Part A. A computer program has been developed and tested to simulate plasma dynamics with self electric and magnetic interactions and a numerical study of trapping effects on the evolution of whistler wavetrains, using this computer program, has been initiated.

RESEARCH ACCOMPLISHED SINCE THE START OF PROJECT

During the past eight months, research has been concentrated on Part A of the original proposal (Particle Simulations with Periodic Smoothing in Phase Space).

1. A computer program has been developed to simulate plasma dynamics with self electric and magnetic interactions neglecting radiation effects (see Appendix A). A novel feature of this program is a smoothing operation which is carried out periodically in phase space to reduce the noise due to individual particle interactions and allow the study of low amplitude phenomena. An analysis of the effects of discrete space and velocity grids in the smoothing operation is currently being made. This analysis should be completed by the end of the present contract period and will be published in the open literature.
2. A study of the effect of trapped electrons on the nonlinear evolution of whistler-mode wavetrains propagating along the lines of force of a constant magnetic field has been initiated. When resonant electrons are trapped in the potential of a whistler wave during their time of transit through the wavetrain, they emerge with phase correlated velocities. The phase correlated electrons are subject to an instability and are expected to generate new whistler waves thus lengthening the original wavetrain.*

This study has been motivated by the dramatic increase in length of whistler wavetrains propagating in the magnetosphere when the above trapping condition is satisfied. For a stationary observer the signal duration is increased and is generally accompanied by a significant change in frequency toward the trailing end.

* R. N. Sudan and E. Ott, J. Geophys. Res. 76, 4463 (1971)

In laboratory plasmas, electron trapping in whistler waves are significant to plasma confinement in mirror devices since they cause a collisionless diffusion of particles into the loss cone. Whistler wavetrains also play an important role in the formation of collisionless shocks and it is expected that the above computer program will contribute to the study of these effects.

An earlier study of trapped effects in electrostatic wavetrains has been carried out in cooperation with Professor R. N. Sudan of Cornell University.* In the electrostatic case numerical simulations showed a considerable lengthening of the wavetrain when the trapping condition is satisfied and preliminary numerical results recently obtained from the above computer program indicate a similar behavior in the whistler case (see Appendix B).

*"Effect of Trapped Particles on the Nonlinear Evolution of a Wave Packet," J. Denavit and R. N. Sudan, Phys. Rev. Lett. 28, 404 (1972)

PERSONNEL AND TIME ALLOTMENT

During the present period, research on the project is carried out by the following personnel:

1. Principal Investigator: Professor J. Denavit

Full time from 8 August 1972 to 31 August 1972.

10% time from 1 September 1972 to 31 January 1973.

2. Research Associate: Dr. B. L. Bogema

Half-time from 1 September 1972 to 31 January 1973.

3. Student assistant: Mr. W. L. Gibbons

Half-time from 16 June 1972 to 31 August 1972.

A summary of Dr. Bogema's professional background is attached in Appendix C. Dr. Bogema will work half time on the present project and half time on the related ONR project. Mr. Gibbons is an undergraduate student in computer sciences at Northwestern. He is an honor student expected to receive his B.S. degree in December 1973.

APPENDIX A

NUMERICAL SIMULATION METHOD

The computer program is designed to simulate the dynamics of an electron plasma with a uniform neutralizing background. The computations are one-dimensional with spatial variations in one direction only (the x direction) and the distribution function depends on x , the three velocity components v_x , v_y , v_z and time. The self electric and magnetic fields are computed at each time step neglecting retardation effects (no radiation). External fields may be applied and by specifying an external magnetic field in the x direction, whistler waves and wavetrains may be studied numerically.

The algorithm is based on the Darwin Lagrangian from which finite difference equations are derived by the variational method of H. R. Lewis.* Two versions of the program have been written using either linear or quadratic base functions in the formulation. The canonical momenta are used to advance the transverse velocities.**

Weighted particles representing values of the distribution function are advanced in phase space along electron trajectories for a number of time steps (usually 10 to 20 steps). The distribution function is then reconstructed by a local smoothing operation as in numerical solutions of the Vlasov equation and the dynamical computations are restarted, now advancing weighted particles representing the updated distribution function.***

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H. R. Lewis, "Methods of Computational Physics," vol. 9, p. 307. Academic Press, New York, 1970.

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I. Haber, C. E. Wagner, J. P. Boris and J. M. Dawson, Proceedings of the Fourth Conference on Numerical Simulation of Plasmas, Naval Research Laboratory, Washington, D. C. November 2-3, 1970, p. 126.

J. Denavit, J. Computational Physics, 9, 75 (1972).

To avoid the use of a four-dimensional grid the distribution function is reconstructed over a grid covering the (x, v_x) plane only and values of the new particle weights as well as v_y and v_z are represented as multiple valued functions of x and v_x . This partial smoothing method retains the desirable effects noted in earlier one-dimensional studies,

- a) To set up new particles as small regions of phase space are extended into filaments in the (x, v_x) plane
- b) To reduce the effect of individual particle interactions and beaming instabilities.

APPENDIX B

EFFECT OF TRAPPED ELECTRONS ON THE EVOLUTION OF WHISTLER WAVETRAINS

Consider a whistler wave of wavelength $\lambda = 2\pi/k$ propagating along the lines of force of a uniform magnetic field B_0 (in the x direction). With respect to the wave coordinate system moving with the phase velocity ω/k with respect to the laboratory system, the electric field vanishes and the wave magnetic field $B_{\perp}$ has a helical space dependence as shown in Fig. 1(a). Assuming an electron population consisting of cold electrons with density n_0 and energetic electrons with density $n_E \ll n_0$, the phase and group velocities may be approximated from the cold plasma dispersion relation as shown in Fig. 1(b). Energetic electrons with velocities with respect to the wave near $-\omega_c/k$, where ω_c is the electron cyclotron frequency, have a perpendicular velocity $v_{\perp}$ which remains approximately in phase with $B_{\perp}$ as they travel to the left in Fig. 1(a). The $v_{\perp} \times B_{\perp}$ force causes the velocity of these electrons (called resonant electrons) to precess around $B_{\perp}$. Let $\zeta = \phi + kx$ denote the angle between $B_{\perp}$ and $v_{\perp}$, then for small amplitude waves ($\omega_{\perp} k/v_{\perp} \ll 1$ where $\omega_{\perp}$ is the cyclotron frequency associated with $B_{\perp}$) we find

$$\frac{d^2 \zeta}{dt^2} - k\omega_{\perp} v_{\perp} \sin \zeta = 0$$

For $|v'_{\parallel} + \omega_c/k| \lesssim (\omega_{\perp} v_{\perp}/k)^{1/2}$ ($v'_{\parallel}$ parallel velocity with respect to wave) the angle ζ oscillates about the value $-\pi$ with the trapping period

$$t_{\text{trapping}} \approx 2\pi/(k\omega_{\perp} v_{\perp})^{1/2}.$$

Returning to the laboratory system, consider a wavetrain of length ℓ advancing at the group velocity v_g . Resonant electrons which have velocities $v_{\parallel} \approx (\omega - \omega_c)/k$ with respect to the laboratory system travel in the opposite

direction. These electrons encounter the wavetrain, remain in transit during the time $t_{\text{transit}} = \ell / |(\omega - \omega_c)/k - v_g|$ and emerge at the rear of the wavetrain. If the condition $t_{\text{transit}} \gtrsim t_{\text{trapping}}$ is satisfied the resonant electrons undergo at least one trapping oscillation during their transit through the wavetrain and emerge with correlated phase angles ζ . These phase correlated electrons are subject to an instability which is expected to cause the appearance of new waves at the rear of the wavetrain.

A preliminary computer simulation was made with $\omega_c/\omega_p = 0.5$, $kc/\omega_p = 1$ and $n_E/n_o = 0.1$. The energetic electrons had a thermal velocity $v_{th} = c/3$ and the amplitude was such that $B_1/B_o \approx 0.1$. With these parameters, $t_{\text{transit}} \approx 60 \omega_p^{-1}$ and $t_{\text{trapping}} \approx 40 \omega_p^{-1}$. The results of this simulation are given in Fig. 2 which shows the component B_y of the wave magnetic field as a function of position at $t = 5 \omega_p^{-1}$ and $t = 70 \omega_p^{-1}$. At $t = 5 \omega_p^{-1}$ the amplitude is Gaussian and approximately 5 lobes are present. At $t = 70 \omega_p^{-1}$ the wavetrain has advanced in agreement with the group velocity computed from linear theory but we also observe that new lobes have appeared at the rear of the wavetrain. The appearance of these new lobes coincides with the arrival of correlated trapped particle at the rear of the wavetrain.

(a)

(b)

FIG. 1: (a) Whistler magnetic field $\underline{B}_\perp$ and electron perpendicular velocity $\underline{v}_\perp$ with respect to wave coordination system.
 (b) Phase velocity $v_{ph} = \omega/k$ and group velocity $v_g = \partial\omega/\partial k$ for whistler propagation in cold plasma

FIG. 2: Numerical simulation of a whistler wavetrain. Time t is given in units of ω_p^{-1} , position x is given in units of the system's length L , velocity is in units of $L\omega_p$ and B_y is in units of $(m/e)L\omega_p^2$.

APPENDIX C

BERNARD L. BOGEMA, JR.

Born: August 27, 1942

Education: BA Johns Hopkins University, 1965
Ph.D. University of Maryland, 1971

Experience: Teaching and Research Assistant, Department of Physics and
Astronomy, University of Maryland (September 1966-August 1971)
Assistant Professor, Physics Department, Drake University
(August 1971-August 1972)

Publications: Equilibrium and Stability Theory of Nonneutral Plasmas, Ph.D.
Thesis, University of Maryland, August 1971.
Rotor Equilibrium of Nonneutral Plasmas (with R. C. Davidson),
Phys. Fluids 13, 2772 (1970)
Two-Rotating-Stream Instability in a Nonneutral Plasma
(with R. C. Davidson), Phys. Fluids 14, 1456 (1971)