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SOME COMMENTS ON THE
AERODYNAMIC CHARACTERISTICS
OF THE TOMAHAWK SOUNDING ROCKET

Warren H. Curry
and
James C. Usselton

RELEASED FOR ANNOUNCEMENT
IN NUCLEAR SCIENCE ABSTRACTS

Reprint

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SOME COMMENTS ON THE AERODYNAMIC CHARACTERISTICS
OF THE TOMAHAWK SOUNDING ROCKET*

by

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March 1967

This is a reprint of a paper presented at the AIAA Sounding Rocket Vehicle Technology Specialist Conference held in Williamsburg, Virginia on February 27, February 28, and March 1, 1967.

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SOME COMMENTS ON THE AERODYNAMIC CHARACTERISTICS OF THE TOMAHAWK SOUNDING ROCKET*

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ABSTRACT

The Tomahawk sounding rocket, when flown as the second stage of the Nitehawk 9 system, occasionally exhibits a dynamic instability after motor burnout. This instability, which is not related to roll-pitch resonance or roll-pitch lock-in, causes extra-atmospheric coning motions of a magnitude several times that which would result from the normal effect of the dynamic pressure gradient. The research being conducted to explain this phenomenon includes studies of the effects of aeroelasticity, and the cross-coupling moments produced by the Magnus effect and asymmetric lee-side vorticity. Analyses of flight test data have yielded information on the extent of vehicle bending, and on the magnitude of the cross-coupling moments. Wind tunnel tests and theoretical studies have provided further information on the cross-coupling contribution. It has not yet been resolved as to which effect predominates. Although the number of cases of adverse coning motion appears to be greater with structurally more flexible systems, it remains to be determined whether the coning is caused primarily by the relaxing of the static moment during bending or by the aggravation of detrimental cross-coupling moments introduced at burn-out by flexing and other perturbations.

INTRODUCTION

Since 1957 the Sandia Laboratory has flown more than 800 solid-fuel rocket vehicles comprising over 40 separate system designs. The various rocket systems have been required for solar research, metro-studies, upper atmosphere sampling, and component testing. Most flights have been made from Sandia's major test sites at Tonopah, Kauai, and Johnston Island.

Among Sandia's more recent rocket systems is the Nitehawk 9 which is composed of a Nike first stage and a Tomahawk second stage. This system as described by Womack¹ is shown in Figure 1. It is capable of delivering a 9-inch-diameter 125-pound payload to an altitude of about one million feet. To date Sandia has flown over 60 of these systems. On about one-third of the flights, the Tomahawk second stage, after burnout, has exhibited a divergent coning motion which is not attributable to roll-pitch resonance, roll-pitch lock-in, or dynamic pressure gradient. It is the purpose of this paper to describe the aerodynamic development of the Tomahawk, and to outline the research presently being conducted to determine the cause of its occasional instability.

NOMENCLATURE

C_D	drag coefficient, $\frac{\text{drag}}{qS}$.
C_m	pitching moment coefficient, $\frac{M_Y}{qSd}$.
C_{m_α}	static moment coefficient slope, rad^{-1} or deg^{-1} .
$C_{m_q} + C_{m_\alpha}$	damping moment coefficient, $\left[\frac{\partial C_m}{\partial \left(\frac{qd}{V} \right)} + \frac{\partial C_m}{\partial \left(\frac{\dot{\alpha}d}{V} \right)} \right]$, rad^{-1} .
C_n	yawing moment coefficient, $\frac{M_Z}{qSd}$.
C_{n_p}	$\frac{\partial C_n}{\left(\frac{pd}{V} \right)}$.
$C_{n_{p_\alpha}}$	Magnus moment coefficient, $\frac{\partial C_{n_p}}{\partial \alpha}$, rad^{-2} .
C_{N_α}	normal force coefficient slope, rad^{-1} or deg^{-1} .
C_Y	side force coefficient, $\frac{F_Y}{qS}$.
d	diameter of vehicle (also reference length), ft.

*This work was supported by the United States Atomic Energy Commission.

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D	damping parameter,
	$\frac{S}{2m} \left[C_{N_\alpha} - 2C_D - k_t^{-2} (C_{m_q} + C_{m_{\dot{\alpha}}}) \right]$
F _Y	side force, lbs.
G	gyroscopic parameter, $\frac{p}{V} \frac{I_x}{I}$.
h	altitude, ft.
i	$\sqrt{-1}$.
I	transverse moment of inertia, slug-ft ² .
I _x	roll moment of inertia, slug-ft ² .
k	density gradient parameter, $\hat{\beta} \sin \gamma$.
k _a	nondimensional axial radius of gyration, $\sqrt{\frac{I_x}{md^2}}$.
k _t	nondimensional transverse radius of gyration, $\sqrt{\frac{I}{md^2}}$.
K _j	constant, j = 1, 2.
ℓ	length of vehicle, ft.
m	mass of vehicle, slugs.
M	static moment parameter, $\frac{SdC_{m_\alpha}}{2I}$.
M _Y	pitching moment about center of gravity, ft-lbs.
M _Z	yawing moment about center of gravity, ft-lbs.
M _∞	Mach number.
p	roll rate, rad/sec.
q	dynamic pressure, $\frac{1}{2} \rho V^2$.
R _e	free-stream Reynolds number, $\frac{\rho V \ell}{\mu}$.
s	distance traveled, ft.
S	vehicle reference area, $\frac{\pi d^2}{4}$.
T	Magnus moment parameter,
	$\frac{S}{2m} \left(C_{N_\alpha} - C_D + k_a^{-2} C_{n_{p_\alpha}} \right)$
v	$\approx \sqrt{\frac{G^2}{4} - \rho M}$.

V	velocity, ft/sec.
w	$\approx \frac{\rho G \left(T - \frac{D}{2} \right)}{2 \sqrt{\frac{G^2}{4} - \rho M}}$
x _{cp}	distance of center of pressure aft of vehicle nose, ft.
α	angle of attack, rad or deg.
β	angle of sideslip, rad or deg.
$\hat{\beta}$	exponent in density-altitude function $\rho = \rho_0 e^{-\hat{\beta} h}$; $\hat{\beta} = \frac{1}{22,000} \text{ ft}^{-1}$.
γ	flight path angle, deg.
θ	phase shift function, $\tan^{-1} \left(-\frac{w}{v} \right)$.
λ _j	damping factor, ft ⁻¹ , j = 1, 2.
μ	free-stream viscosity.
ξ	complex angle of yaw (β + iα).
ρ	density, slugs/ft ³ .
ω _j	frequency factor, ft ⁻¹ , j = 1, 2.

BASIC AERODYNAMIC CHARACTERISTICS

The basic aerodynamic characteristics of the Tomahawk vehicle are shown in Figures 2-6. For the coasting phase of flight, the variation of the drag coefficient at zero angle of attack with Mach number is shown in Figure 2. This drag curve is based primarily on accelerometer and trajectory data from full scale free flight tests.²

The variations of C_{N_α} , C_{m_α} , and $\frac{x_{op}}{d}$, at zero angle of attack, with Mach number are given in Figures 3-5. These data summarize the results derived from theory,³⁻⁴ wind tunnel tests,⁵⁻⁸ and flight tests.^{2,9} The variation of C_{m_q} at zero angle of attack with Mach number is shown in Figure 6. These values of C_{m_q} were derived by assuming all the damping was in the fins, and calculating fin lift by employing Luther's⁴ semiempirical method A which includes the effect of the body on the fins. For a comprehensive treatment of the Tomahawk's basic aerodynamic characteristics, the reader is referred to the work of Stone.¹⁰

PERFORMANCE

An extensive analysis of the Tomahawk's performance when flown as the second stage of the

Nitehawk 9 system has been made by Stone and Connell.¹¹ Trajectory data* for an 84-degree launch angle and a 16-second Tomahawk ignition time are presented in Figures 7-8. Altitude versus range is shown in Figure 7; and the variations of altitude, range, Mach number, Reynolds number, and dynamic pressure with time for the first 40 seconds after launch are presented in Figures 8a and b. At motor burnout, the Tomahawk has reached an altitude of about 65,000 feet and is flying at Mach 8. The maximum Reynolds number attained during exit is about 165 million based on vehicle length; and the maximum dynamic pressure during exit is 7400 pounds per square foot. The total flight time is almost 9 minutes.

The nominal variations of roll rate and natural pitch frequency during exit are shown in Figure 9. It is seen that the roll rate passes from below to above the pitch frequency during Tomahawk burning. At motor burnout the ratio of roll rate to pitch frequency is about 3.

FLIGHT TEST PROGRAM

Over 60 Nitehawk 9 systems have been flown since the first flight in June 1963. These systems have been used to acquire information on upper atmospheric density, radiation, and particulate matter. They have also been used in the development of high resolution telemetry and payload attitude control devices.

Instrumentation for the ballistic development flights, and for some of the special purpose flights, included the Miniature Attitude Reference System (MARS)¹² which is a roll-stabilized free gyro platform. Most of the angular position data Sandia has obtained on the Tomahawk has been acquired with the MARS package. It has been estimated that the MARS can define the motion of the vehicle to within about 0.1 degree.

The basic types of motions encountered during flights of the Tomahawk are shown in Figure 10 where yaw gyro traces are shown for various flights. During normal flights, depicted by Figure 10a, the complex angle of yaw does not exceed approximately 2 degrees at that time during burning when the roll rate crosses the nutational frequency; and the extra-atmospheric coning is less than 5 degrees.** A case of roll-pitch lock-in that occurred near resonance is shown in Figure 10b. Lock-in

has occurred on only two flights to date so has not been considered a serious problem. In one case the tolerances on fin construction and alignment were marginal. In the other, a payload section with seven joints had been used, and the accuracy of payload alignment is questioned. Mayo¹³ of the NASA has run six-degree-of-freedom computer studies on resonance and conditions for lock-in; however his input values were entered in an additive manner which in general would not occur in practice.

In Figure 10c is shown an example of the severe coning motion which about one-third of the Tomahawks have exhibited upon leaving the sensible atmosphere. It can be seen that the instability is initiated immediately after burnout and is not of the type which would result from the dynamic pressure gradient. Stone¹⁴ has calculated the dynamic pressure gradient effect using the basic aerodynamic characteristics, and has shown that any angular perturbation at burnout will first begin to damp and then develop into an extra-atmospheric coning of only about six times its initial value. For the flight shown in Figure 10c, the perturbation at burnout was at the most about one degree; no initial damping was evident; and the resultant maximum coning was about 35 degrees. There has been at least one flight in which the coning was greater than 60 degrees. The level of pitch damping does not seem to have a marked effect. On a flight with an over 30-percent increase in fin planform area, coning comparable to that shown in Figure 10c was obtained. It should be pointed out that in all cases the roll rate during coning was in the neighborhood of its design value which is well above the natural pitch frequency as was shown in Figure 9.

RESEARCH RELATED TO THE CONING PROBLEM

In an effort to explain the coning phenomenon, Sandia initiated studies in the areas of aeroelasticity and higher order aerodynamics. The higher order effects considered were Magnus, asymmetric lee-side vorticity, and nonlinearities in the basic damping parameter $C_{m_q} + C_{m_{\dot{\alpha}}}$.

Aeroelasticity

Reis and Sundberg¹⁵ in a paper being presented in the Aeroelasticity Session of this meeting show that coning occurred more frequently with the structurally more flexible systems. In their analysis they hypothesized that flexing between the nose and tail, in the plane containing the average body centerline and the velocity vector, may be sufficient to cause decrements in static moment which would adversely affect the stability characteristics. They formulated a mathematical model in which the payload and motor were treated as two separate rigid bodies hinged at the payload-motor joint. They then calculated the aeroelastic bending

*Data presented are for the old TE-406 motor used on the first ten flights. Most of the detailed analyses referenced in this paper were made on these flights. Present systems use the TE-416 motor with about 6 percent greater total impulse.

**In this paper, the quoted magnitudes of coning are the half-angle values.

encountered in one of the large coning flights of the Tomahawk (152-9). Their results are shown in Figure 11. Near burnout, the angle between the nose and the tail increased to about three-quarters of a degree in a time span of about 2 seconds. This angle then decreased to less than two-tenths of a degree in the subsequent three seconds.

As a by-product of their study, Reis and Sundberg also derived external moments transverse to their plane of bending. These are shown in coefficient form in Figure 12. Any higher order aerodynamic effects, such as Magnus and asymmetric lee-side vorticity, would be included in these transverse moments. It should be emphasized that both the aforementioned flexing and the presence of these transverse moments must be present to satisfy the equations of motion which Reis and Sundberg used to define the motion of this particular flight (152-9).

Magnus and Asymmetric Lee-Side Vorticity

The importance of the higher order aerodynamic effects has been emphasized by Murphy¹⁶ and Nicolaides.¹⁷ The initial study of Magnus effects in connection with the Tomahawk was conducted by Stone.¹⁴ Stone hypothesized that the large coning motion was caused by Magnus moments, and used his approximate solution¹⁸ to the rigid body equations of motion to extract Magnus moment coefficients from four full scale flight tests. He found these coefficients to be nonlinear and sensitive to Reynolds number. His results for a Reynolds number of 39 million, which occurred near an altitude of 80,000 feet, are shown in Figure 13. It can be seen that in this case the Magnus moment approximates a cubic function of angle of attack.

Stone's work prompted an experimental investigation by Curry, Reed, and Useton^{19, 8} in which side forces and moments were measured on a two-ninths scale model of the Tomahawk, with and without spin in a supersonic wind tunnel. The results showed that, in addition to the side loads induced by spin, the model experienced significant side forces and moments at angles of attack when it was oriented at zero roll in the no-spin condition. This can be seen in Figure 14. The side loads measured without spin are attributed to a steady-state asymmetric lee-side vortex pattern. This phenomenon is not new. Several investigators,²⁰⁻²⁵ by use of schlieren, pressure distribution, and vapor screen methods, have shown steady-state asymmetric vorticity to exist on symmetrical models under some conditions. In addition, a theoretical explanation has been set forth by Thomson and Morrison²⁶ of the Australian Weapons Research Laboratory. It can also be seen in Figure 14 that the asymmetric vortex pattern at zero spin influenced the data when the model was spinning so that for equal and opposite spin rates the side force and moment curves were not mirror images of each

other on opposite sides of the abscissa. Moreover, there is some slight indication that the initial establishment of spin causes a "jump" in the coefficients, and that any further increase in spin, once it has been established, results in a lesser and perhaps linear increase in the coefficients. Side moments were calculated^{8, 19} using the theories of Kelly and Thacker²⁷ for the body contribution and Benton²⁸ for the fin cant contribution, and, as expected, were found to be considerably lower than the experimental values.

Further results from the wind tunnel tests indicate significant Mach number effects on the side loads over the Mach number 3-5 range investigated. Reducing the Mach number at constant Reynolds number increased the side loads caused by the asymmetric vortex pattern. An effect of Reynolds number at constant Mach number was not readily apparent. However, caution should be exercised here, since as pointed out by Platou,²⁹ Magnus characteristics have indeed been shown to be Reynolds number sensitive.

At this point it may be interesting to examine the relative importance of the various aerodynamic coefficients upon the damping of the Tomahawk at that instant of time immediately after burnout. For an ascending hypersonic rigid body, the complex angle of yaw has been shown by Stone¹⁸ to be

$$\xi \approx \frac{e^{\left(\frac{\rho D}{2k} - i \frac{\theta}{2}\right)}}{\left[v^2 + w^2\right]^{\frac{1}{4}}} \left\{ K_1 e^{\left[\int w ds + i \left(\frac{Gs}{2} + \int v ds\right)\right]} + K_2 e^{\left[-\int w ds + i \left(\frac{Gs}{2} - \int v ds\right)\right]} \right\}.$$

In the Appendix, this equation is changed to the form

$$\xi \approx K_j e^{\int (\lambda_j + i\omega_j) ds} \quad j = 1, 2$$

where the λ_j represent the damping factors.

Also in the Appendix, the λ_j were evaluated at Tomahawk burnout (TBO), in terms of conventional aerodynamic coefficients, with the following result:

$$\lambda_{j_{TBO}} \times 10^6 \approx -1.68 C_{N_\alpha} + 3.4 C_D + 11.4 + 0.024 \left(C_{m_q} + C_{m_{\dot{\alpha}}} \right) \pm 0.069 C_{n_{p_\alpha}}.$$

Using the values shown in Figure 3, 2, and 6, respectively, for $C_{N\alpha}$, and C_D , and $C_{m\dot{\alpha}}$ ($C_{m\dot{\alpha}}$ is neglected), the significance of the various terms can be seen:

$$\lambda_{jTBO} \times 10^6 \approx -15.1 \text{ which is the normal force effect}$$

$$+1.1 \text{ which is the drag effect}$$

$$+11.4 \text{ which is the density gradient effect}$$

$$-14.4 \text{ which is the pitch damping effect}$$

$$\pm 0.069 C_{n_{p\alpha}} \text{ which is the Magnus effect.}$$

For zero damping of the nutational motion (K_1 arm), $C_{n_{p\alpha}} \approx +250$ per radian²; and for zero damping of the precessional motion (K_2 arm), $C_{n_{p\alpha}} \approx -250$ per radian². Using a nominal $\frac{pd}{V}$ of 0.0032 radian for the full scale Tomahawk after burnout, the respective $C_{n_{p\alpha}}$ would become about ± 0.014 per degree. The wind tunnel tests (Figure 14) produced results of this order of magnitude. Moreover, Reis and Sundberg (Figure 12) in their aeroelastic analysis obtained values of C_n of about 0.1 within 2 seconds after burn-out, and the complex angle of yaw had reached only about 3 degrees. It is therefore apparent that the cross-coupling effects of Magnus and asymmetric lee-side vorticity cannot be ignored.

Nonlinearity of the Basic Damping Parameter $C_{m_q} + C_{m_{\dot{\alpha}}}$

Besides the lack of adequate data on the cross-coupling terms in the equations of motion, the basic damping parameter $C_{m_q} + C_{m_{\dot{\alpha}}}$ also may not be sufficiently well defined. As mentioned previously, only theoretical values have been used. Nicolaides, Eikenberry, and Ingram³⁰ however, in their analysis of the flight test results from a modified Tomahawk configuration, point out the possibility of a marked dependency of $C_{m_q} + C_{m_{\dot{\alpha}}}$ on angle of attack.

An attempt was therefore made to determine $C_{m_q} + C_{m_{\dot{\alpha}}}$ from a test in Sandia's 18-inch Hypersonic Wind Tunnel. This is not easily done with models of such high fineness ratio. The transverse support approach was used. However, to minimize support interference, it was made extremely thin (0.02 body diameter) and placed in tension in the pitch plane of the model as shown in Figure 15. After the Mach 7 flow in the tunnel was established, the model was displaced to angles of attack up to 15 degrees then released, by a slide-wire and burn-wire arrangement attached to the model base. The

model oscillated about a ball bearing pivot at the center of gravity. In some cases a small amount of lateral motion was also observed. In general, however, the rig appeared to work well. At the time of this writing, data reduction had not been completed. The results will be presented in a forthcoming report by Stone and Millard.³¹ It is anticipated that some comment on the results can be made at the Conference during the presentation of this paper.

CONCLUDING REMARKS

The configuration of the Tomahawk being flown as the second stage of Sandia's Nitehawk 9 sounding system has been described. A brief history of Tomahawk aerodynamics has been presented. It was seen that the flight characteristics of the majority of the vehicles have been satisfactory. In these cases the complex yaw amplitudes at resonance were only about 2 degrees, and extra-atmospheric coning motions were less than 5 degrees. It was also seen that roll-pitch coupling has not been a serious problem.

On about one-third of the flights, however, a dynamic instability has occurred after motor burnout, resulting in extremely large extra-atmospheric coning motions. Possible causes of this instability include aeroelasticity, Magnus effects, and asymmetric lee-side vorticity.

The frequency of coning has been greater with the more flexible systems. The question remains however whether the predominant effect of flexing is the relaxation of the static moment or an enhancement of the perturbation required to introduce the detrimental transverse moments. This question must eventually be answered. In the one case, an increase in the vehicle's rigidity may be sufficient to eliminate the coning problem. In the other, increased rigidity may help, but it may not be sufficient; changes to the basic aerodynamic shape may also be required.

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APPENDIX

Evaluation of Damping at Tomahawk Burnout

Stone¹⁸ has shown that an approximate solution to the linearized equations of motion of an ascending hypersonic vehicle can be written as:

$$\xi \approx \frac{e^{\left(\frac{\rho D}{2k} - i\frac{\theta}{2} - \frac{1}{2} \ln v\right)}}{\left[v^2 + w^2\right]^{\frac{1}{4}}} \left\{ K_1 e^{\left[\int w ds + i\left(\frac{Gs}{2} + \int v ds\right)\right]} + K_2 e^{\left[-\int w ds + i\left(\frac{Gs}{2} - \int v ds\right)\right]} \right\} \quad (1)$$

where

$$v \approx \sqrt{\frac{G^2}{4} - \rho M} \quad (2)$$

$$w \approx \frac{\rho G \left(T - \frac{D}{2}\right)}{2 \sqrt{\frac{G^2}{4} - \rho M}} \quad (3)$$

and

$$\begin{aligned} \theta &= \text{the phase shift function} \\ &= \tan^{-1} \left(-\frac{w}{v} \right). \end{aligned} \quad (4)$$

Stone also points out that $|w| \ll |v|$, so the denominator on the right hand side of equation (1) can be written:

$$\left[v^2 + w^2\right]^{\frac{1}{4}} \approx v^{\frac{1}{2}} = e^{\frac{1}{2} \ln v} = e^{\frac{1}{2} \ln v}. \quad (5)$$

Substituting (5) into (1) yields:

$$\xi \approx e^{\left(\frac{\rho D}{2k} - i\frac{\theta}{2} - \frac{1}{2} \ln v\right)} \left\{ K_1 e^{\left[\int w ds + i\left(\frac{Gs}{2} + \int v ds\right)\right]} + K_2 e^{\left[-\int w ds + i\left(\frac{Gs}{2} - \int v ds\right)\right]} \right\}, \quad (6)$$

which is of the form:

$$\xi \approx K_j e^{\int (\lambda_j + i\omega_j) ds} \quad j = 1, 2 \quad (7)$$

where

$$\int \lambda_j ds = \frac{\rho D}{2k} - \frac{1}{2} \ln v \pm \int w ds \quad (8)$$

and, dropping the phase shift function,

$$\int \omega_j ds = \frac{Gs}{2} \pm \int v ds. \quad (9)$$

But

$$\rho = \rho_0 e^{-ks} = -k \int \rho ds, \quad (10)$$

so equation (8) becomes:

$$\int \lambda_j ds = -\frac{D}{2} \int \rho ds - \frac{1}{2} \ln v \pm \int w ds. \quad (11)$$

Differentiating (11) yields

$$\lambda_j = -\frac{D}{2} \rho - \frac{1}{2v} \frac{dv}{ds} \pm w, \quad (12)$$

but

$$\frac{dv}{ds} = \frac{d}{ds} \left[\left(\frac{G^2}{4} - \rho M \right)^{\frac{1}{2}} \right] = \frac{Mk\rho}{2v}; \quad (13)$$

so,

$$\lambda_j = -\frac{D}{2}\rho - \frac{1}{4v^2}Mk\rho \pm w. \quad (14)$$

Differentiating (9) yields:

$$\omega_j = \frac{G}{2} \pm v. \quad (15)$$

Substituting (2) and (3) into (14) and (15) gives

$$\lambda_j = -\left(1 \pm \frac{G}{2\sqrt{\frac{G^2}{4} - \rho M}}\right)\frac{\rho D}{2} - \frac{\rho Mk}{4\left(\frac{G^2}{4} - \rho M\right)} \pm \frac{\rho GT}{2\sqrt{\frac{G^2}{4} - \rho M}} \quad (16)$$

and

$$\omega_j = \frac{G}{2} \pm \sqrt{\frac{G^2}{4} - \rho M}. \quad (17)$$

For the particular case of the Tomahawk at burnout (designated by TBO):

$$\rho_{TBO} \approx 1.26 \times 10^{-4},$$

and since

$$G \approx 2.5 \times 10^{-5}$$

and

$$M \approx -1.6 \times 10^{-2},$$

it follows that

$$\left(\frac{G^2}{4} - \rho M\right) \approx -\rho M$$

and

$$\left|\frac{G}{2\sqrt{-\rho M}}\right| \ll 1.$$

Equation (16) then reduces to:

$$\lambda_{jTBO} = -\frac{\rho D}{2} + \frac{k}{4} \pm \frac{\rho GT}{2\sqrt{-\rho M}}.$$

Substituting the numerical values for G and M , and the expressions for D and T , the following equation is obtained for λ_{jTBO} in terms of

conventional aerodynamic coefficients:

$$\lambda_{jTBO} \times 10^6 \approx -1.68 C_{N_\alpha} + 3.4 C_D + 11.4 + 0.024(C_{m_q} + C_{m_{\dot{\alpha}}}) \pm 0.069 C_{n_{p_\alpha}}.$$

Using the design values of $C_{N_\alpha} = 9.0$, $C_D = 0.33$, and $C_{m_q} + C_{m_{\dot{\alpha}}} = -600$, the significance of the various terms can be evaluated:

$\lambda_{jTBO} \times 10^6 = -15.1$	normal force effect
+1.1	drag effect
+11.4	density gradient effect
-14.4	pitch damping effect
$\pm 0.069 C_{n_{p_\alpha}}$	Magnus effect.

Figure 1. Nitehawk 9 (Reference 1)

Figure 2. Drag Coefficient Variation with Mach Number for the Coasting Tomahawk

Figure 3. Tomahawk Normal Force Coefficient Slope Variation with Mach Number

Figure 4. Tomahawk Static Moment Coefficient Slope Variation with Mach Number

Figure 5. Tomahawk Center of Pressure Variation with Mach Number

Figure 6. Tomahawk Pitch Damping Variation with Mach Number

Figure 7. Tomahawk Trajectory for an 84 Degree Launch of a 125 Pound Payload (TE-406 Motor)

Figure 8a. Nitehawk 9 Altitude, Mach Number, and Reynolds Number Versus Time

Figure 8b. Nitehawk 9 Altitude, Range, and Dynamic Pressure Versus Time

Figure 9. Tomahawk Roll Rate and Natural Pitch Frequency During Exit for a 125 Pound Payload

Figure 10. Yaw Gyro Traces from Tomahawk Flights Showing Normal, Locked-In, and Coning Motion

Figure 11. Angle Between Tomahawk Payload and Motor Case Versus Time for Flight 152-9 as Calculated by Reis and Sundberg in Reference 15

Figure 12. Average Angle of Attack and Yawing Moment Coefficient Derived from Reis and Sundberg's Data in Reference 15

Figure 13. Magnus Moment Characteristics of the Nitehawk 9 Second Stage at an Altitude of 80,000 Ft. MSL as Derived by Stone in Reference 14

Figure 14. Effect of Spin Magnitude on Yawing Force and Moment Coefficients on the Tomahawk with Asymmetric Lee-Side Vorticity as Determined from Wind Tunnel Test (Reference 19)

Figure 15. Tomahawk Wind Tunnel Model Installation for Determining Pitch Damping Characteristics