DYNAMICAL SYMMETRY BREAKING

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I. INTRODUCTION

An attractive idea for field theoretic models involving massive vector mesons is that this mass is not a parameter in the Lagrangian, but arises spontaneously from a breakdown of gauge symmetry. The reason why this circumstance is desirable lies in the fact that the free meson propagator vanishes at high energy in the latter case, but not the former. The free propagator for a massive vector meson is $O(1)$ at large energies.¹

$$D_0^{\mu\nu}(k) = -i(g^{\mu\nu} - \frac{k^\mu k^\nu}{k^2}) \frac{1}{k^2 - m^2} \xrightarrow{k \rightarrow \infty} \frac{1}{m^2} \frac{k^\mu k^\nu}{k^2}$$

If there is no mass term in the Lagrangian, the free propagator is $O(k^{-2})$ in the same limit.

$$D_0^{\mu\nu}(k) = -i(g^{\mu\nu} - \frac{k^\mu k^\nu}{k^2}) \frac{1}{k^2}$$

The decrease of the free propagator at high energies in the massless case allows for a consistent perturbative expansion of the theory. This technique is not in general available, if the propagator remains constant in the high energy region, because of well-known non-renormalizable divergences. (At present it is still an open question whether non-renormalizable vector-meson theories with mass parameters in the Lagrangian are intrinsically inconsistent, or whether only perturbation theory is inapplicable.) Currently, the great activity in the theory of weak interactions is centered about just such a possibility of spontaneous mass

generation for the various vector mesons that are thought to carry weak forces.²

The fundamental reason why an apparently massless vector meson acquires a mass has been given a decade ago by Schwinger.³ The reason is that the vacuum polarization tensor, $\Pi^{\mu\nu}$ -- the proper two point correlation function of the conserved current J^μ to which the meson couples with strength g -- acquires a pole at zero momentum transfer. To see this explicitly, consider the complete vector meson propagator, $D^{\mu\nu}(q)$.

$$D^{\mu\nu}(q) = -i(g^{\mu\nu} - q^\mu q^\nu / q^2) D(q^2) \\ = -i(g^{\mu\nu} - q^\mu q^\nu / q^2) \frac{1}{q^2 - q^2 \Pi(q^2)}$$

$$\Pi^{\mu\nu}(q) = (q^\mu q^\nu - q^2 g^{\mu\nu}) \Pi(q^2)$$

(We insist that $\Pi^{\mu\nu}(q)$ be transverse, since the current is conserved; this entails a judicious choice of seagulls.) Clearly, if $\Pi(q^2)$ has a pole at $q^2=0$, the vector meson is massive, even though it is massless in the absence of interactions ($g=0$, $\Pi=0$). We shall call this the Schwinger mechanism.

In these lectures I shall discuss three different examples of the Schwinger mechanism; that is I shall present three different circumstances which lead to a pole in $\Pi(q^2)$. Two are well known, and I shall be brief. They are Schwinger's original example of 2-dimensional, massless spinor electrodynamics³ and the currently popular "Higgs mechanism"⁴. The third example, which will be treated at length, is given in a recent paper by K.

II. MASSLESS SPINOR ELECTRODYNAMICS IN TWO DIMENSIONS

$$L = i\bar{\psi}\not{\partial}\psi - \frac{1}{2}F^{\mu\nu}F_{\mu\nu} + \psi\gamma^{\mu}\partial_{\mu}\psi$$

$$F^{\mu\nu} = \partial^{\mu}A^{\nu} - \partial^{\nu}A^{\mu}$$

$$J^{\mu} = \bar{\psi}\gamma^{\mu}\psi$$
$$\Pi^{\mu\nu} = -e^2 \text{ (loop diagram)} \quad (2.2)$$
$$\begin{aligned} \Pi^{\mu\nu}(q) &= \frac{ie^2}{\pi q^2} (g^{\mu\nu} q^2 - q^\mu q^\nu) \\ \Pi(q^2) &= \frac{e^2}{2q^2} \end{aligned} \quad (2.3)$$
$$v_A^2 = \frac{c_A^2}{2} \quad (2.4)$$

The conservation of J^μ , an essential ingredient for the unambiguous evaluation of $\Pi^{\mu\nu}$, is a consequence of the fact that we are dealing with a gauge theory, where one equation of motion is

$$\partial_\nu F^{\mu\nu} = eJ^\mu \quad (2.5)$$

Since $F^{\mu\nu}$ is antisymmetric, consistency of the theory requires that

$$\partial_\mu J^\mu = \frac{1}{e} \partial_\mu \partial_\nu F^{\mu\nu} = 0 \quad (2.6)$$

There exists also a symmetry principle which by Noether's theorem implies the conservation of J^μ : the transformation law $\delta\psi = i\psi$ leaves the Lagrangian invariant and leads to $\partial_\mu (\bar{\psi}\gamma^\mu\psi) = 0$. However we prefer not to invoke this property of the theory, since it requires taking quite literally the canonical formalism, the form of the Lagrangian, and the formula for the current $J^\mu = \bar{\psi}\gamma^\mu\psi$. All these may be beset by anomalies, and may not survive in the solutions to the theory. Of course in the case of the vector current, as long as the theory exists, i.e., provided equation (2.5) is true, we know that (2.6) must hold; there are no anomalies. Thus we view current conservation not as a consequence of a symmetry principle, but as a consequence of gauge properties.

Indeed L apparently is also left invariant by chiral transformations, $\delta\psi = \gamma^5\psi$ ($\gamma^5 = i\gamma^0\gamma^1$); this leads to the conservation of the axial vector current: $\partial_\mu (i\bar{\psi}\gamma^\mu\gamma^5\psi) = 0$. However, the conservation law is not maintained in the solutions to the theory;⁸ there is an anomaly of the axial vector current, quite analogous

to the well known triangle anomaly of 4-dimensional physics.⁹ This anomaly is also responsible for the spontaneous generation of the mass. We now show this.

Consider the complete two current correlation function.

$$\begin{aligned} T^{\mu\nu}(q) &= \int d^2x e^{iqx} \langle 0 | T^* J^\mu(x) J^\nu(0) | 0 \rangle \\ &= iT(q^2) (q^\mu q^\nu - q^\mu q^\nu) \end{aligned} \quad (2.7)$$

$\Gamma(q^2) = \eta(q^2) [1 - \eta(q^2)]^{-1}$
In 2 dimensions, the axial vector current, $J_5^\mu = i\bar{\psi}\gamma^\mu\gamma^5\psi$, is related to the vector current by

$$J_5^\mu = \epsilon^{\mu\nu} J_\nu \quad (2.8)$$

since $i\gamma^\mu\gamma^5 = \epsilon^{\mu\nu}\gamma_\nu$. Hence one can compute matrix elements of J_5^μ from those of J^μ . Specifically

$$\begin{aligned} T_5^{\mu\nu}(q) &= \int d^2x e^{iqx} \langle 0 | T^* J_5^\mu(x) J^\nu(0) | 0 \rangle \\ &= \epsilon^{\mu\mu'} T_{\mu'}^{\nu}(q) \end{aligned} \quad (2.9)$$

Now observe that

$$q_\mu T_5^{\mu\nu}(q) = iq_\mu \epsilon^{\mu\nu} q^2 T(q^2) \neq 0 \quad (2.10)$$

Thus, in spite of the formal symmetry of the Lagrangian, the axial vector current is not conserved, and the non-conservation can be summarized by¹⁰

$$\partial_\mu J_5^\mu = \frac{e}{2\pi} \epsilon_{\mu\nu} F^{\mu\nu} \quad (2.11)$$

It is easy to see that (2.11) implies the existence of a massive particle in the theory. From (2.5) and (2.8) it follows that

$$\partial^\nu \epsilon^{\mu\nu} F_{\mu\nu} = \epsilon J_5^\alpha \quad (2.12a)$$

while (2.11) further yields

$$\partial_\alpha \partial^\nu \epsilon^{\mu\nu} F_{\mu\nu} = \frac{e^2}{2\pi} \epsilon_{\mu\nu} F^{\mu\nu} \quad (2.12b)$$

In 2 dimensions, $F^{\mu\nu} = \epsilon^{\mu\nu} F$, hence (2.12b) is equivalent to

$$\left(\square + \frac{e^2}{\pi}\right) F = 0 \quad (2.12c)$$

which shows that $F^{\mu\nu}$ is a free field with mass $e/\sqrt{\pi}$.¹¹

There are two lessons to be learned from this simple exercise. Firstly, Noether symmetry currents, as opposed to gauge currents, need not be conserved. Secondly, the spontaneous generation of a vector meson mass is associated with the breaking of an apparent invariance of the theory; in the present case it is chiral symmetry that is broken by the anomaly. Finally note that the complete meson propagator is transverse and well behaved at high energy, in spite of the fact that it describes massive vector particles.

$$D^{\mu\nu}(q) = -i \left(g^{\mu\nu} - \frac{q^\mu q^\nu}{q^2} \right) \frac{1}{q^2 - \frac{e^2}{\pi}} \quad (2.13)$$

III. THE HIGGS MECHANISM

The second example is the currently popular Higgs mechanism.⁴ Since this subject is so much discussed these days, I shall not repeat familiar material. Nevertheless I want to show how a pole in the vacuum polarization tensor arises in this context.

A Lagrangian which exhibits the Higgs mechanism involves scalar mesons with an imaginary mass parameter, e.g.,

$$L = (\partial^\mu - ieA^\mu)\phi^* (\partial^\mu + ieA_\mu)\phi - \frac{1}{2}F_{\mu\nu}^2 + \mu^2 \phi^* \phi - g(\phi^* \phi)^2 \quad (3.1)$$

Since μ^2 occurs with the wrong sign, the only way to avoid a nonsensical theory is to assume that the vacuum expectation value of ϕ is nonvanishing.

$$\langle 0 | \phi(0) | 0 \rangle = \langle 0 | \phi^*(0) | 0 \rangle = \lambda \neq 0 \quad (3.2)$$

Consider now the complete vector meson propagator, to lowest order perturbation theory.

$$\begin{aligned} D^{\mu\nu}(q) &= i \int d^4x e^{iqx} \langle 0 | T A^\mu(x) A^\nu(0) | 0 \rangle \\ &+ i e^2 / d^4x d^4z e^{iqx} \langle 0 | T A^\mu(x) A^\nu(0) \phi^*(z) \phi(z) A^\alpha(z) A_\alpha(z) | 0 \rangle \\ &= D_0^{\mu\nu}(q) + 2i e^2 D_0^{\mu\alpha}(q) D_0^{\beta\nu}(q) g_{\alpha\beta} \langle 0 | \phi^*(0) \phi(0) | 0 \rangle \end{aligned} \quad (3.3)$$

(In Eq. (3.3) all fields are free.) The contribution of the vacuum expectation value of ϕ to $\langle 0 | \phi^*(0) \phi(0) | 0 \rangle$ is λ^2 . Since the free propagator can be chosen to be transverse, the contribution of λ^2 to (3.3) is

$$2ie^2 \lambda^2 D_0^{\mu\alpha}(q) \frac{(g_{\alpha\beta} q^2 - q_\alpha q_\beta)}{q^2} D_0^{\alpha\nu}(q)$$

Thus the vacuum polarization tensor is seen to be, to this order,

$$\Pi^{\mu\nu}(q) = i \frac{2e^2 \lambda^2}{q^2} (g^{\mu\nu} q^2 - q^\mu q^\nu) \quad (3.4)$$

A pole is indeed generated, which then leads to a vector meson mass μ_λ .

$$\mu_\lambda^2 = 2\lambda^2 e^2. \quad (3.5)$$

Of course the same result (3.5) is obtained by the conventional method of shifting ϕ by λ . The usual method also exposes massless excitations, which however decouple.

The characteristic feature of the Higgs phenomenon is the presence of scalar fields in the Lagrangian, which acquire a symmetry breaking vacuum expectation value. For two reasons these scalar fields are an unattractive aspect of the model. Firstly, there do not appear to exist any physical particles which would correspond to the scalar fields. Secondly, it has been observed that renormalizable non-Abelian vector meson theories can become free at high energies, when there are no scalar fields with quartic interactions.¹² When scalar fields are present, it is difficult to implement this very desirable feature. Also it should be remarked that the Ansatz $\lambda \neq 0$ is an assumption

whose validity can only be checked in low orders of perturbation theory. By no means is it obvious that the complete theory possesses solutions with this property.

It is therefore useful to inquire whether a vector meson mass can be generated in a 4-dimensional model without invoking canonical scalar fields. The exploration of such a possibility will concern us for the remainder of these lectures.

IV. THE BOUND-STATE MODEL

Imagine a theory with a massless Fermion field ψ and a neutral vector meson field A^μ , interacting through a conserved current J^μ . Masslessness of the Fermion insures formal chiral symmetry in this theory. However, we assume that the Schwinger-Dyson equation for the Fermion mass operator $\Sigma(p)$ has a symmetry breaking solution, $\{\gamma^5 \Sigma(p)\} \neq 0$. It is well known that this can happen if there is a massless, bound excitation in the Fermion anti-Fermion channel. Since a Fermion mass is being generated, this zero-mass excitation couples to $\bar{\psi}\psi$ or $\bar{\psi}\gamma^5\psi$.

Ultimately we shall want the zero-mass state to combine with the massless vector meson field and generate a massive meson. This can only happen if a transition between J^μ and $\bar{\psi}\psi$ or $\bar{\psi}\gamma^5\psi$ is allowed. Therefore, as long as charge conjugation invariance is not spontaneously broken, J^μ must contain the axial vector current.

We are thus led to consider a theory described by the Lagrangian

$$\begin{aligned} L &= i\bar{\psi}\not{\partial}\psi - \frac{1}{4}F_{\mu\nu}^2 + gJ_5^\mu A_\mu \\ J^\mu &= i\bar{\psi}\gamma^\mu\gamma^5\psi \\ F^{\mu\nu} &= \partial^\mu A^\nu - \partial^\nu A^\mu \end{aligned} \quad (4.1)$$

The axial-vector anomaly occurs in the above theory by virtue of the axial vector coupling.⁹ For the time being, I shall ignore this; the anomaly can be eliminated by introducing additional

Fermions.¹³ Indeed the anomaly must be removed; otherwise the equation of motion

$$\partial_\nu F^{\mu\nu} = gJ_5^\mu \quad (4.2)$$

is not consistent with the antisymmetry of $F^{\mu\nu}$. (It is interesting that the axial-vector current, together with the anomaly -- which to be sure must be removed -- is central to the present discussion, just as it is central in Schwinger's 2-dimensional model.) I shall return later to questions associated with the anomaly.

The theory (4.1) is chirally invariant and renormalizable, in the sense that off-mass-shell Green's functions can be computed in perturbation theory. The normalization point will not be taken at $k^2=0$ because of possible infrared divergences; rather an arbitrary value $k^2=k_0^2$ will be chosen.

The proper vertex function $\Gamma_5^\mu(p,p')$ associated with J_5^μ satisfies a Ward-Takahashi identity

$$q_\mu \Gamma_5^\mu(p,p+q) = \gamma^5 G^{-1}(p+q) + G^{-1}(p) \gamma^5 \quad (4.3)$$

There must not be any anomalous exceptions to this equation, since the current, a source of the gauge field, must be always conserved; see (4.2). In (4.3), $G(p)$ is the complete Fermion Green's function, which is given by the following Schwinger-Dyson equation.

$$G^{-1}(p) = -i(\not{p} - \Sigma(p))$$

$$\Sigma(p) = -ig^2$$

$$\text{wavy line } q = D^{\mu\nu}(q)$$

$$\text{solid line } p = G(p)$$

$$\text{vertex } = i\gamma^\mu\gamma^5$$

$$\text{blob } A = \Gamma_5^\mu(p, p+q)$$

$$(4.4)$$

We assume a chiral symmetry breaking solution for Eq. (4.4) exists such that $\{\gamma^5 \Gamma(p)\}$ is not zero. Then (4.3) implies that $\Gamma_5^\mu(p, p+q)$ has a pole at $q=0$ with residue $i\Gamma_5(p)$, where

$$\Gamma_5(p) = \{\gamma^5 \Gamma(p)\} \neq 0 \quad (4.5)$$

Now it can happen that $\Gamma_5^\mu(p, p+q)$ has a pole at $q=0$, which as I have shown guarantees a non-vanishing mass for the Fermion. To see this, observe that the proper vertex function is also related to a portion of the Fermion-Fermion scattering amplitude

$$(4.6)$$

T' is the "one vector meson irreducible" Fermion-Fermion scattering amplitude, i.e. the full scattering amplitude T is given by

$$(4.7)$$

The reason why T' rather than T appears in (4.6) is that Γ_5^μ is the proper vertex function and does not contain graphs like

Our principal assumption is that the pole in $\Gamma_5^\mu(p, p+q)$ is to be attributed to a pole in T' . That is, we assume that there exists a zero mass bound state in the Fermion anti-Fermion channel. In a sense this is the analog in the present theory of the assumption, made in the conventional approach to spontaneous symmetry breaking, that the vacuum expectation value of the scalar field is non-zero. First I shall work out the consequences of this zero mass bound state; then I shall turn to the much more difficult question whether or not such a bound state can exist in the theory.

Represent T' by a pole term plus a regular term R .

$$(4.8)$$

$$\Pi_{\text{pole}}^{\mu\nu} = g^2 \text{ [diagram: two circles labeled P connected by a dashed line, with wavy lines on the left and right]} \quad (4.12a)$$

The integrals occurring in (4.12a) are already introduced in (4.9b). We find

$$\begin{aligned} \Pi_{\text{pole}}^{\mu\nu}(q) &= g^2 [q^\mu I(q^2)] \frac{i}{q^2} (-q^\nu I(q^2)) \\ &\rightarrow -ig^2 \lambda^2 q^\mu q^\nu / q^2 \end{aligned} \quad (4.12b)$$

(The minus sign in the last factor in the first equation above arises as follows. The second pseudoscalar vertex in (4.12a) occurs with arguments $P(r, r+q)$, rather than $P(r+q, r)$ as in the definition $q^\nu I(q^2)$ in (4.9b). To make contact with (4.9b) the integration variable r is shifted to $r-q$, and q is replaced by $-q$.) Thus $\Pi(q^2)$, defined in (1.1) has the pole $\Pi_{\text{pole}}(q^2) = \frac{g^2 \lambda^2}{q^2}$, which indicates that the vector meson acquires a mass μ . In the approximation where only the pole term is kept, $\mu = g\lambda$. An exact theorem is $D^{-1}(0) = -g^2 \lambda^2$.

The final task is to show that the zero mass pole decouples from the theory. The amplitude T' contains a massless pole; nevertheless, it is true that the full on-mass-shell scattering amplitude T does not possess this pole. To establish this important result, we combine (4.7) and (4.8).

$$\begin{aligned} \text{[diagram: box T with four external lines]} &= \text{[diagram: box R with four external lines]} \\ &- \text{[diagram: two circles labeled P connected by a dashed line, with four external lines]} - g^2 \text{ [diagram: two circles labeled A connected by a wavy line, with four external lines]} \end{aligned} \quad (4.13)$$

Only the last two terms in (4.13) contain poles at $q^2=0$. The external Fermions are on-mass-shell; thus the $q^\mu q^\nu$ term of the vector meson propagator in the last term in (4.13) gives zero contribution since the on-mass-shell vertex is transverse. Consequently $D^{\mu\nu}(q)$ may be set equal to $-ig^{\mu\nu} D(q^2)$. Furthermore if the left-most vertex function in the last term in (4.13) is decomposed into a regular piece and the pole term (4.9c), one sees that the pole term does not contribute: the pole is proportional to q^μ , which contracts with the right-most vertex function and annihilates it. Thus the terms in (4.13), which potentially contain a zero-mass pole, are represented by

$$T_{\text{pole}} = - \text{[diagram: two circles connected by a dashed line, with four external lines]} - g^2 \text{ [diagram: circle A-tilde connected by a wavy line to circle A_P, with four external lines]} \quad (4.14)$$

Here $\hat{\Gamma}_5^\mu$ is the regular part of the axial vector current,
and $\Gamma_{5\text{pole}}^\mu$ is the pole part given by (4.9).

We concentrate on the second term in (4.14), and set
 $\hat{\Gamma}_5^\mu = \Gamma_5^\mu - \Gamma_{5\text{pole}}^\mu$. Since the remaining part of the graph is
proportional to q^μ , Γ_5^μ does not contribute and there remains

(4.15a)

When (4.9a) is inserted in (4.15a) for Γ_5^μ , the following sequence
of equations now shows that the pole term of the above cancels
against the first term in (4.14).

(4.15b)

The above is equal to the negative of the first term in (4.14),
and T on mass shell is free from poles at $q^2=0$. This completes
the discussion of our model.

V. A MASS SUM RULE

An interesting sum rule for the vector meson mass may be derived in our model. Let us begin with (4.9b)

$$\begin{aligned} q^\mu I(q^2) &= \text{Tr} \int \frac{d^4x}{(2\pi)^4} \{G(x) i \gamma^\mu \gamma^5 G(x+q) P(x+q, x)\} \\ &= -\text{Tr} \int \frac{d^4x}{(2\pi)^4} \{G(x+q) i \gamma^\mu \gamma^5 G(x) P(x, x+q)\} \end{aligned} \quad (5.1a)$$

$$\begin{aligned} g^{\mu\nu}_\lambda &= -\text{Tr} \int \frac{d^4x}{(2\pi)^4} \{ \partial^\nu G(x) i \gamma^\mu \gamma^5 G(x) P(x, x) \\ &\quad + G(x) i \gamma^\mu \gamma^5 G(x) \frac{\partial}{\partial q_\nu} P(x, x+q) |_{q=0} \} \end{aligned} \quad (5.1b)$$

We also know from (4.3) and (4.9c) that

$$q_\mu \tilde{\Gamma}_S^\mu(p, p+q) = \gamma^5 G^{-1}(p+q) + G^{-1}(p) \gamma^5 - i P(p, p+q) \lambda \quad (5.2)$$

where $\tilde{\Gamma}_S^\mu(p, p+q)$ is the vertex function, without its pole at $q^2=0$. Hence it is true that

$$i \lambda P(p, p) = \{\gamma^5, G^{-1}(p)\} \quad (5.3)$$

$$i \lambda \frac{\partial}{\partial q_\nu} P(p, p+q) |_{q=0} = \gamma^5 \partial^\nu G^{-1}(p) - \tilde{\Gamma}_S^\nu(p, p)$$

Inserting (5.3) in (5.1b) gives

$$\begin{aligned} g^{\mu\nu}_\lambda &= -\text{Tr} \int \frac{d^4x}{(2\pi)^4} \{ G(x) \partial^\nu G^{-1}(x) G(x) \gamma^\mu \\ &\quad - G(x) \partial^\nu G^{-1}(x) G(x) \gamma^\mu \gamma^5 G(x) \gamma^5 G^{-1}(x) \\ &\quad + G(x) \gamma^\mu \gamma^5 G(x) \gamma^5 \partial^\nu G^{-1}(x) \\ &\quad - G(x) \gamma^\mu \gamma^5 G(x) \tilde{\Gamma}_S^\nu(x, x) \} \end{aligned} \quad (5.4a)$$

The middle two terms cancel against each other, while $\partial^\nu G^{-1}(x)$ may be related to the vertex function Γ^ν of the vector current.

$$\partial^\nu G^{-1}(x) = -i \Gamma^\nu(x, x)$$

Hence

$$\begin{aligned} g^{\mu\nu}_\lambda &= i \text{Tr} \int \frac{d^4x}{(2\pi)^4} \{ G(x) \Gamma^\nu(x, x) G(x) \gamma^\mu \\ &\quad - G(x) \tilde{\Gamma}_S^\nu(x, x) G(x) i \gamma^\mu \gamma^5 \} \end{aligned} \quad (5.4b)$$

Recalling the representation of the vacuum polarization tensor by the Schwinger-Dyson equation (4.11), we recognize that (5.4b) is equivalent to

$$g^{\mu\nu}_\lambda = -i [\tilde{\Pi}_A^{\mu\nu}(0) - \Pi_V^{\mu\nu}(0)] \quad (5.5)$$

$\tilde{\Pi}_A^{\mu\nu}$ is the vacuum polarization tensor associated with the non-singular, non-conserved axial vector current, while $\Pi_V^{\mu\nu}$ is the vacuum polarization of the conserved vector current, apart from seaquarks.

Formally, if one ignores seagulls, $\tilde{\Pi}_V^{\mu\nu}(0) = 0$, since the vector current is conserved. This is also seen from (5.4a) where the integrand in the first term on the right-hand side is a total divergence: $-\partial^\nu G(r) \gamma^\mu$. However, if $G(r) \sim \frac{1}{r}$ for large r , $\tilde{\Pi}_V^{\mu\nu}(0)$ is in fact, a quadratically divergent constant which cancels a similar quadratic divergence in $\tilde{\Pi}_A^{\mu\nu}(0)$. In other words, $\tilde{\Pi}_V^{\mu\nu}(0)$ is the seagull, which is necessary to convert the formal Feynman-Dyson equation for $\tilde{\Pi}_A^{\mu\nu}$ into a correct expression. Thus we may replace (5.5) by

$$g^{\mu\nu} g^2 \lambda^2 = -i \tilde{\Pi}_A^{\mu\nu}(0) \quad (5.6)$$

where $\tilde{\Pi}_A^{\mu\nu}$ is now understood to include the requisite seagull.

The formula (5.6) may also be understood from (1.1) and (4.12b). We have

$$\tilde{\Pi}^{\mu\nu}(q) = (g^{\mu\nu} q^2 - q^\mu q^\nu) (i \tilde{\Pi}(q^2) + i \frac{g^2 \lambda^2}{q^2}) \quad (5.7)$$

where $\tilde{\Pi}(q^2)$ is by definition regular. It is natural to identify $\tilde{\Pi}_A^{\mu\nu}$ with the regular part of (5.7). Thus the following, non-transverse expression is found

$$\tilde{\Pi}_A^{\mu\nu}(q) = i g^2 \lambda^2 g^{\mu\nu} + (g^{\mu\nu} q^2 - q^\mu q^\nu) i \tilde{\Pi}(q^2) \quad (5.8)$$

and $-i \tilde{\Pi}_A^{\mu\nu}(0) = g^{\mu\nu} g^2 \lambda^2$, which agrees with (5.6). Note also that $\tilde{\Pi}_A^{\mu\nu}(q)$, though non-conserved has a conserved absorptive part.

Eq. (5.5) is also related to Weinberg's first sum rule.¹⁴ If we identify $i \tilde{\Pi}_V^{\mu\nu}(0)$ with $g^2 g^{\mu\nu}$ times the Schwinger term S_V of the vector current commutator, and similarly for the Schwinger term

S_A of the regular part of the axial vector current, then (5.5) reads $\lambda^2 + S_A = S_V$. This is recognized as Weinberg's first sum rule, when it is recalled that the Schwinger term of the total axial vector current differs from that of the regular part by the current-pseudoscalar coupling, which in our case is λ .

VI. PROBLEMS WITH THE MODEL

The specific model which I have discussed presents some problems which should be exposed. Some of these are specific to the example that I have chosen, and are trivially remedied. Others are more general.

The Lagrangian (4.1) is formally scale invariant. Consequently the emergence of mass terms appears to violate that symmetry and to lead to further massless excitations. It is clear that these potential Goldstone Bosons are spin-zero, positive parity objects. They could couple to the energy-momentum tensor, but not to a vector or axial vector meson.

However, the above circumstances very likely do not occur, since scale invariance does not appear to be realized in the solutions of a field theory, in spite of the formal symmetry of the Lagrangian. The reason is not spontaneous violation, but rather the presence of anomalies, analogous to those of the axial vector current.¹⁵ Thus we shall ignore any considerations of scale invariance.

Indeed it would appear that scalar massless mesons coupled to the energy-momentum tensor must be avoided. Such mesons would lead to a pole in the vacuum polarization tensor of two energy-momentum tensors. If one then imagines coupling the energy-momentum tensor to a gravitational gauge field, the pole would produce a mass for the graviton, in a fashion entirely analogous to our previous discussions. Since massive gravity apparently is not realized in Nature, this state of affairs must be avoided.

Our model possesses two other currents which are formally conserved. These are $\bar{\psi}\gamma^\mu\gamma^5\psi$ and its Hermitian conjugate. Here ψ^C is the charge conjugate field $\psi_i^C = C_{ij}\bar{\psi}_j$; C is the charge conjugation matrix $i\gamma^2\gamma^0$ which satisfies $C = -C^T = -C^{-1} = -C^\dagger$, and which transposes the gamma matrices: $C^{-1}\gamma^\mu C = -\gamma^\mu$, $C^{-1}\gamma^5 C = \gamma^5$.

If these two currents are conserved in the solutions of the theory, one obtains Ward identities which at zero momentum transfer require that $G(p)\gamma^5 C + \gamma^5 C G(-p) = 0$. By charge conjugation invariance of the theory, it is also true that $C G(-p) = G(p)C$. Hence the conservation of these odd currents requires massless Fermions: $\{\gamma^5, E(p)\} = 0$.

Clearly this result is unacceptable, nor is its evasion with Goldstone particles satisfactory. We do not wish to deal with a theory which possesses massless, doubly charged scalars! The additional, unwanted symmetries may be disposed of by one of two devices.

Firstly, one may argue that in the solutions of the theory the currents are not conserved, again because of anomalies. The envisioned situation is analogous to the discussion of massless electrodynamics, where the electron acquires a mass spontaneously. The axial-vector current is formally conserved in that model, yet one can show that it is consistent that the equations determining the matrix elements of the axial vector current and of its divergence, admit a non-conserved solution. Unlike the anomalies of the triangle graph and of scale invariance which occur in a low order of perturbation theory, examples of this mechanism require

infinite summation of graphs. I shall later show explicitly how the same mechanism which leads to a massless bound state can also generate this type of anomalous symmetry breaking.¹⁶

The second way of destroying the additional symmetry is by introducing vector couplings into the Lagrangian (4.1). This vector coupling can be a gauge coupling to the vector field A^μ : the interaction term in (4.1) is replaced by $\bar{\Psi}\gamma^\mu(g' + ig\gamma^5)\Psi A_\mu$. This leads to a parity violating theory. If one wishes to maintain parity, one can introduce an additional, massive vector field B_μ coupled to $\bar{\Psi}\gamma^\mu\Psi$. In the presence of a vector interaction, $\bar{\Psi}\gamma^\mu\gamma^5\Psi$ is no longer conserved. This modification has the additional attraction that scale invariance becomes explicitly broken, and one need not invoke anomalies to evade scalar Goldstone particles. Moreover, it will also be demonstrated that a vector interaction is required to produce a zero mass bound state in the weak coupling limit.

The last problem has already been mentioned: the theory is inconsistent because of the triangle anomaly.⁹ In order to remove this contradiction, the number of Fermion fields must be increased.^{13,17} We introduce a multiplet of n fields $\psi = \begin{pmatrix} \psi_1 \\ \vdots \\ \psi_n \end{pmatrix}$, coupled to the axial vector field by the interaction $i\bar{\psi}\gamma^\mu\gamma^5\mathbf{g}\psi\mathbf{A}$. Provided the $n \times n$ Hermitian matrix $\mathbf{g}$ satisfies $\text{Tr } \mathbf{g}^3 = 0$, the anomaly is absent. We shall limit the discussion to the simplest, two-Fermion case:

$$\psi = \begin{pmatrix} \psi_1 \\ \psi_2 \end{pmatrix} \quad \mathbf{g} = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix} \quad (6.1)$$

The theory now possesses many conserved currents. Two, given by $\bar{\Psi}\gamma^\mu\mathbf{g}\Psi$, and $\bar{\Psi}\gamma^\mu\Psi$, or $\bar{\psi}_i\gamma^\mu\psi_i$, $i=1,2$, insure the conservation of the individual species of Fermions. Fortunately, this continuous symmetry is not broken even if we allow each of the Fermions to acquire a different mass, and a different coupling to the massless bound state. In addition to the Fermion number currents, the model also possesses doubly charged conserved currents as discussed in the previous subsection. These must be disposed of by one of the two methods mentioned above. Finally the axial vector current $\bar{\Psi}\gamma^\mu\gamma^5\Psi$, though beset by the triangle anomaly, can also lead to Goldstone Bosons. The reason is that the anomaly does not contribute to the Ward identity at zero momentum; while it is only the zero momentum Ward identity that is required to establish the existence of massless excitations. Thus conservation of this current must be broken by the first method discussed above, just as in massless quantum electrodynamics.

VII. EXISTENCE OF THE BOUND STATE

Now I must examine the most important question about our theory. Can the zero mass bound state, which is responsible both for the Fermion mass and for the vector meson mass, actually occur? To see what must transpire so that this bound state is present, let me return to the equation satisfied by the proper vertex function (4.6), and re-express it in Bethe-Salpeter form.

$$\text{Diagram (7.1): } A = \text{tree-level} + \text{loop diagram with } A \text{ and } K'$$

Here K' is the Bethe-Salpeter kernel, with one vector meson line deleted (compare (4.7)). Decomposing Γ_5^μ into its regular and pole parts, we have, according to (4.9c),

$$\text{Diagram (7.2): } A = \tilde{A} + \frac{i g^4 \lambda}{q^2} P$$

If (7.2) is substituted into (7.1), we find, by equating pole terms

$$\text{Diagram (7.3): } P = \text{loop diagram with } P \text{ and } K' + O(q^2)$$

As is to be expected, the existence of the bound state at $q^2=0$, requires the solution of a homogeneous Bethe-Salpeter equation. The important feature of this equation is that the kernel is not Fredholm, since the theory is renormalizable and not super-renormalizable.¹⁸ This has far reaching consequences. In particular, we shall argue that (7.3) is not an eigenvalue equation; it can be solved for arbitrary coupling constant. Consequently the solutions of (7.1) are not unique; one may always add a solution of the homogeneous equation. Moreover, (7.1) and (7.3) do not determine the normalization of P ; thus the Fermion mass is an arbitrary parameter in the theory.

Properties of the homogeneous Bethe-Salpeter equation may also be used to show the possibility of anomalous symmetry breaking, not by individual graphs as with the triangle anomaly and scale invariance, but rather by infinite sums of graphs summarized by the equation. (This is the symmetry breaking envisioned in the Johnson, Baker, Willey theory of massless electrodynamics.⁶ It is also the mechanism which I mentioned in connection with the problems of additional, unwanted symmetries in our theory.)

Consider for example, the proper vertex function Γ^μ associated with some current, which is formally conserved in the theory, as well as in any finite order of perturbation theory. The naive Ward identity is schematically given by

$$q_\mu \Gamma^\mu = \delta G^{-1} \quad (7.4)$$

and F^μ satisfies a Bethe-Salpeter equation of the form

$$F^\mu = F_0^\mu + g^2 / F^\mu K \quad (7.5)$$

[The inhomogeneous term is the lowest order contribution to F^μ .] We inquire whether F^μ may have the property that (7.4) is not true; rather an anomalous Ward identity holds, which means that the current is not conserved in the non-perturbative solutions of the theory.

$$q_\mu F^\mu = X + \delta G^{-1} \quad (7.6)$$

By contracting (7.5) with q_μ , we get

$$X + \delta G^{-1} = \delta G_0^{-1} + g^2 / \delta G^{-1} X + g^2 / X K \quad (7.7)$$

The inhomogeneous term in (7.7) is the lowest order expression for δG^{-1} ; it contains no anomalies since the current is formally conserved. δG^{-1} satisfies its own Bethe-Salpeter equation, hence (7.7) implies that the anomaly X satisfies a homogeneous equation

$$X = g^2 / X K \quad (7.8)$$

Thus we see that whenever the homogeneous Bethe-Salpeter equation possesses solutions; and furthermore when these solutions are not of the eigenvalue type but are always possible, which can happen when the kernel is not Fredholm, a formal symmetry can be broken. Since the normalization of X is not determined, these symmetry breaking solutions introduce new arbitrary parameters.

Let us now return to the problem of determining P ,

Eq. (7.3). We show that to lowest order in the coupling, solutions of (7.3) exist, for (almost) arbitrary coupling. For reasons that will emerge presently, we allow for massive vector mesons in our model, with vector coupling. (Recall that we found previously that vector couplings are needed to eliminate explicitly undesirable additional symmetries.) Thus,

$$K' = (ig_A)^2 \text{ [diagram]} + (ig_v)^2 \text{ [diagram]} \quad (7.9)$$

(The axial vector coupling constant has been called g_A ; the vector, g_v .) For the propagators we take lowest order expressions in the Landau gauge. However, it will be seen that the spontaneously induced masses are independent of coupling. Hence all mass terms are kept; they will not influence the result. We set $P(p,p) = \gamma^5 P(p^2)$, and $P(p^2)$ is found to satisfy the equation

$$P(p^2) = -3i \int \frac{d^4 r}{(2\pi)^4} \frac{P(r^2)}{r^2 - m^2} \times \left[\frac{g_v^2}{(p-r)^2 - \mu_v^2} - \frac{g_A^2}{(p-r)^2 - \mu_A^2} \right] \quad (7.10)$$

The solution of this equation is known for large p^2 where μ_v and μ_A may be ignored.¹⁹

$$P(p^2) \underset{p^2 \rightarrow \infty}{\sim} (-p^2)^{-\epsilon} \quad (7.11)$$

$$\epsilon = \frac{3}{16\pi^2} (g_V^2 - g_A^2) \quad (7.12)$$

If $\epsilon < 0$, then the integral (7.10) would diverge and no solutions are possible. Clearly the only condition is that binding occurs $\epsilon > 0$ or $g_V^2 > g_A^2$; once this is achieved no further eigenvalue conditions are necessary.

The induced mass for the axial vector meson may also be computed. From the mass sum rule (5.4b) it follows that

$$g^{\mu\nu} \mu_A^2 = i g_A^2 \int \frac{d^4 x}{(2\pi)^4} \text{Tr} [G(x) \Gamma^\nu(x, x) G(x) \gamma^\mu - G(x) \tilde{\Gamma}_5^\nu(x, x) G(x) i \gamma^\mu \gamma^5] \quad (7.13)$$

Consistent with our lowest order approximation, we set

$$\Gamma^\nu(x, x) = \gamma^\nu, \quad \tilde{\Gamma}_5^\nu(x, x) = i \gamma^\nu \gamma^5 \quad (7.14)$$

For $G(x)$ we must leave a mass term, otherwise the integral in (7.13) vanishes identically

$$G(x) = \frac{i}{\not{x} - i} \quad (7.15)$$

However, it would be wrong to set $\tilde{\Gamma}(x) = m$, because the bare mass of the Fermion is zero,

$$\lim_{\epsilon \rightarrow 0} \langle \gamma^5, \tilde{\Gamma}(x) \rangle = 0 \quad (7.16)$$

The correct formula for $\tilde{\Gamma}(x)$, consistent with (7.11), (7.12) and (7.14), is

$$\tilde{\Gamma}(p) = m \left(\frac{-\not{p}}{2} \right)^{-1} \quad (7.17)$$

The integral (7.13) may now be evaluated. We find, for small g_V^2 and g_A^2 ,

$$\mu_A^2 = \frac{4}{3} m^2 \frac{1}{g_V^2/g_A^2 - 1} \quad (7.18)$$

Thus to lowest order in the coupling, our theory successfully generates masses for the Fermion and the axial vector meson. The former is an arbitrary parameter, which also determines the latter. Note μ_A^2 can become arbitrarily large as g_V^2 approaches g_A^2 .

Beyond lowest order, it is of course, very difficult to discuss solutions to (7.3). However, a general analysis due to Johnson can be given,^{2D} which sheds some light on the possibility of solving that equation. Let us represent (7.3) schematically by

$$P(p^2) = g^2 \int d^4 x K(p, x) P(x^2) \quad (7.19)$$

The reason that a Fredholm kernel requires an eigenvalue condition, can be understood heuristically: the validity of (7.19) requires that in some sense $g^2/K=1$. For arbitrary g^2 this will not happen; a quantization condition on g^2 is necessary. However if g^2/K is divergent, but g^2/KV is convergent then factor of g^2 can be cancelled and plays no role in the problem (e.g.,

$$g^2 \int_1^\infty \frac{dx}{x} = g^2 \ln \infty, \quad g^2 \int_1^\infty \frac{dx}{x} x^{-g^2} = 1).$$

Let us assume that the dominant contributions in (7.19)

come from large r , because the kernel remains large in that region, and let us see whether an asymptotic solution to (7.19) for large p can be found. Upon continuing (7.19) to Euclidean space and averaging over angles, (7.19) becomes

$$\begin{aligned} P(-p^2) &= g^2 \int_0^\infty dx x^3 \bar{K}(p, x) P(-x^2) \\ &= g^2 \int_0^\infty dx x^3 p^4 \bar{K}(p, xp) P(-x^2 p^2) \end{aligned} \quad (7.20)$$

where the second equation follows from the first by a change of variable $r \rightarrow xp$. In the asymptotic domain of large p , $P(p^2)$ is replaced by $P_\infty(p^2)$. Also $P(-x^2 p^2)$ may be set equal to $P_\infty(-x^2 p^2)$, provided we assume that the region of small x^2/p is unimportant. Finally we assume that the kernel has the property that

$$\lim_{p \rightarrow \infty} p^4 \bar{K}(p, xp) = f(x) \quad (7.21)$$

Then (7.20) becomes, in the asymptotic region

$$P_\infty(-p^2) = g^2 \int_0^\infty dx x^3 f(x) P_\infty(px^2 p^2) \quad (7.22)$$

A solution of this equation can always be found

$$P_\infty(-p^2) = (-p^2)^{-\epsilon} \quad (7.23)$$

provided that

$$1 = g^2 \int_0^\infty dx f(x) x^{3-2\epsilon} \quad (7.24)$$

This is not an eigenvalue condition; rather it is an evaluation of ϵ in terms of g and the parameters present in $f(x)$. The only requirement is that the answer for ϵ be such that the integral (7.24) converges.

The weak coupling approximation exhibits all the features which we have listed above: the kernel is non-Fredholm; (7.21) is satisfied; the infra-red region is unimportant; the significant contribution to the integral comes from the ultraviolet. Moreover, an analysis has been given of the complete Bethe-Salpeter kernel for vector meson theories without vacuum polarization.²¹ All our hypotheses are valid. Thus there is some hope the requisite bound state survives beyond the weak coupling limit.

VIII. PHENOMENOLOGICAL LAGRANGIANS

The mechanism which we have discussed is attractive in that it dispenses with fundamental scalar fields as the agents responsible for a pole in the vacuum polarization tensor. Furthermore, since the vector meson propagator has the form $-i(g^{\mu\nu} - \frac{q^\mu q^\nu}{q^2}) [q^2 - q^2 \Pi(q^2)]^{-1}$ its high energy behavior is no more divergent than that of the free propagator, and the theory should be renormalizable.

An obvious shortcoming of the theory, as developed so far, is that no effective method of computation has been found. Ordinary perturbation theory will never expose a bound state: to any finite order in g we shall always have massless fermions and vector mesons.

One possible, though incomplete, approach is to describe the physical system by an effective Lagrangian. The effective Lagrangian, in tree approximation, should reproduce some of the dynamics of the complete theory. The description, necessarily limited to a low energy domain, should take into account the following features of the complete theory: (1) the excitation spectrum; (2) couplings of the states to each other; and (3) the symmetries of the problem.

In the model considered by us, the excitation spectrum consists of a massive fermion, massive axial vector meson and massless pseudoscalar meson, which, however, decouples from the theory. There may be other, massive bound states in the theory. Presumably these are important only at higher energies, and for a low energy phenomenology may be ignored.

The interactions of the theory involve fermion-vector meson couplings, fermion-pseudoscalar couplings, and vector meson-pseudoscalar couplings. The latter two, however, are removable by an appropriate choice of fields.

Finally, the symmetries which are to be maintained are vector and axial vector current conservation, and parity conservation. The axial current should be related to a gauge symmetry.

The above requirements lead to the following Lagrangian.

$$\begin{aligned} \mathcal{L} = & i\bar{\psi}\not{\partial}\psi + \frac{1}{2}\partial_\mu\phi\partial^\mu\phi - \frac{1}{4}F^{\mu\nu}F_{\mu\nu} + \frac{1}{2}\mu^2 A^2 \\ & + ig\bar{\psi}\gamma^\mu\gamma^5\phi A_\mu - m\bar{\psi}(\exp\frac{2ig}{\mu}\gamma^5\phi)\psi - \mu A^\mu\partial_\mu\phi \end{aligned} \quad (8.1)$$

It possesses a gauge symmetry

$$\begin{aligned} \delta\psi &= \gamma^5\psi\theta \\ \delta\bar{\psi} &= \bar{\psi}\gamma^5\theta \\ \delta\phi &= -\frac{\mu}{g}\theta \\ \delta A^\mu &= -\frac{1}{g}\partial^\mu\theta \end{aligned} \quad (8.2)$$

It is not difficult to see that the massless field ϕ , present in (8.1), in fact decouples.²² Change variables in (8.1)

$$\psi + e^{-\gamma^5 \frac{g}{\mu}} \phi$$

$$\bar{\psi} + \bar{\psi} e^{-\gamma^5 \frac{g}{\mu}} \phi$$

$$A^\mu \rightarrow A^\mu + \frac{1}{\mu} \partial^\mu \phi \quad (8.3)$$

Then

$$L = i\bar{\psi}\not{\partial}\psi - m\bar{\psi}\psi - \frac{1}{4}F^{\mu\nu}F_{\mu\nu} + \frac{1}{2}\mu^2 A^2 + ig\bar{\psi}\gamma^\mu\gamma^5 A_\mu \quad (8.4)$$

This Lagrangian exhibits the physical spectrum of excitations: a massive Fermion and vector meson interacting through the axial vector current.

The phenomenological description of our theory does not lead to a renormalizable Lagrangian. This is seen either from (8.1), where the non-renormalizability resides in the non-polynomial Fermion-pseudoscalar interaction, or from (8.4) where it is the non-conservation of $i\bar{\psi}\gamma^\mu\gamma^5\psi$ that is responsible for the divergent, presumably unphysical, high-energy behavior of the theory. We do not consider the rapid growth at high energy as a defect of the fundamental theory. The phenomenological description is not meant to extend to high energies; in particular one does not expect to use the Lagrangians (8.1) or (8.4) for higher-order calculations.

IX. CONCLUSION

The interesting aspect of the present investigation is the demonstration, that the vector particles can acquire a mass from a bound state mechanism, rather than from a vacuum expectation value of a canonical scalar field.

The present work should be extended to include internal symmetry. More importantly, an effective computational method should be developed, which bypasses ordinary perturbation theory, yet maintains renormalizability.

Especially provocative is the view, to which we have been led, that the theory is not uniquely determined by the Lagrangian. Rather, solutions of homogeneous integral equations with arbitrary constants, provide various inequivalent physical alternatives: generation of Fermion mass, anomalous non-perturbative violation of symmetry. It would obviously be most important to study this possibility further.

The physical relevance of our example is not apparent at the present time. Clearly a non-Abelian version may be used for weak interaction model building. More interesting is the possibility that the Abelian model may be relevant to pure strong interactions. A world with massless quarks and gluons which acquire their mass spontaneously without Goldstone Bosons is attractive for its economy. Moreover, we see in this picture the possibility of avoiding the problem of too much symmetry in the quark model. If chiral $U(3)$ is spontaneously broken in the fashion described here, then the troublesome ninth Goldstone Boson disappears from the theory.

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