FEB 18 1963

HH/HJH-2

BNL 6726

Facsimile Price \$ 2.60

Microfilm Price \$.89

Available from the
Office of Technical Services
Department of Commerce
Washington 25, D. C.

BROOKHAVEN NATIONAL LABORATORY
Associated Universities, Inc.
Upton, L.I., N.Y.

ACCELERATOR DEPARTMENT
(AGS)
Internal Report

MASTER

MODE IDENTIFICATION IN THE IRIS-LOADED WAVEGUIDE OF A RF PARTICLE SEPARATOR

H. Hahn and H. J. Halama

January 14, 1963

LEGAL NOTICE

This report was prepared as an account of Government sponsored work. Neither the United States, nor the Commission, nor any person acting on behalf of the Commission:

A. Makes any warranty or representation, expressed or implied, with respect to the accuracy, completeness, or usefulness of the information contained in this report, or that the use of any information, apparatus, method, or process disclosed in this report may not infringe privately owned rights; or

B. Assumes any liabilities with respect to the use of, or for damages resulting from the use of any information, apparatus, method, or process disclosed in this report.

As used in the above, "person acting on behalf of the Commission" includes any employee or contractor of the Commission, or employee of such contractor, to the extent that such employee or contractor of the Commission, or employee of such contractor prepares, disseminates, or provides access to, any information pursuant to his employment or contract with the Commission, or his employment with such contractor.

I. Introduction

The theoretical and practical aspects of the accelerating mode (TM_{01}) in iris-loaded waveguides have been covered extensively in many reports and publications. The pulse shortening observed in linacs and the possible application of an iris-loaded waveguide as the deflecting structure for rf particle separators stimulated the interest in the nature of higher order modes.

Some experimental results on higher order modes in iris-loaded waveguides are available in references^{1,2,3,4}. Results of studies done at CERN are not yet published; they were however communicated^{5,6,7} to the authors and represented the basis of work done at Brookhaven. A preliminary account of BNL results was given in reference⁸.

The purpose of this report is to describe the model measurements which were necessary to determine the geometry of a brazed or electroformed prototype for a deflecting waveguide. At the same time, a systematic investigation of other than the deflecting mode was done to ensure a non-degenerate field solution at the operating frequency. The measurements

DISCLAIMER

This report was prepared as an account of work sponsored by an agency of the United States Government. Neither the United States Government nor any agency Thereof, nor any of their employees, makes any warranty, express or implied, or assumes any legal liability or responsibility for the accuracy, completeness, or usefulness of any information, apparatus, product, or process disclosed, or represents that its use would not infringe privately owned rights. Reference herein to any specific commercial product, process, or service by trade name, trademark, manufacturer, or otherwise does not necessarily constitute or imply its endorsement, recommendation, or favoring by the United States Government or any agency thereof. The views and opinions of authors expressed herein do not necessarily state or reflect those of the United States Government or any agency thereof.

DISCLAIMER

Portions of this document may be illegible in electronic image products. Images are produced from the best available original document.

are compared with computational results obtained on the IBM 7090 at BNL.

Throughout this report a coherent rationalized unit system with four basic units (length m^* , time s^* , current A^* , voltage V^*) will be used.

The new units are defined in terms of MKSA units by:

$$c = 1 m^*/s^* = 2.997925 \times 10^8 \text{ m/s}$$

$$\lambda_0 = 2\pi m^*$$

$$\mu_0 = 1 V^* s^*/A^* m^* = 4\pi \times 10^{-7} \text{ Vs/Am}$$

$$eV = 1 V^* A^* s^* = 1.602 \times 10^{-19} \text{ VAs}$$

The deflecting waveguide will work with four irises per wavelength ($\pi/2$ mode). The spacing of the irises w (pitch) has therefore the numerical value:

$$w = \pi/2$$

II. The Model Measurements

All tests were done at L-band frequencies ($\lambda_0 = 22.75 \text{ cm}$) because of the immediate availability of measuring equipment. The results, however, are all scaled to S-band ($\lambda_0 = 10.5 \text{ cm}$).

The models consisted of spacers and irises, which were stacked on each other and then pressed together (Fig. 1). The first model had grooves for spring-rings to ensure a good contact. In the later models no spring-rings were used, resulting in slightly lower Q values.

The waveguide was transformed into a cavity by use of two end plates placed in the center of a spacer. Thus a symmetrical cavity of variable length was obtained. A longer cavity has a higher Q due to the smaller relative losses in the end plates. Since a higher Q gives better defined resonant frequencies, most experiments were done on cavities with eight

or ten irises.

The construction used does not permit to evacuate the cavity. If resonant frequencies in vacuo are desired, a correction must be applied according to:

$$f_{\text{vacuum}} \approx 1.0003 f_{\text{air}}$$

Most waveguide characteristics for a given mode can be obtained by standard cavity tests:

1. Q Measurement

The Q factor of the cavity, resonating in the deflecting mode with $\beta = 1$, is determined in the transmission method by the upper and lower 3 dB - frequencies. The use of a precision 3 dB - attenuator makes the results independent of the diode characteristics. Since the generator is a varactor multiplier, the frequency measurement can be done very accurately (10^{-6}) with a counter.

2. The Dispersion Diagram (k - β curve)

In a cavity with N irises the only possible values for the propagation constant $\beta = \omega/v_p$ are:

$$\beta = n\pi/Nw \quad n = 0, 1, 2, \dots, N$$

The corresponding circular wave numbers $k = \omega/c$ are points on the dispersion diagram of a given mode. The frequencies are measured with a counter or to a less good accuracy with a cavity frequency meter (± 1 Mc).

The z - dependence (the β - value) may be obtained by bead pulling. For $\beta > 1.4$ this method is not reliable as the dips on the indicating meter are not pronounced. It is then very helpful to excite the cavity and to detect in such a way to limit the number of possible β - values.

3. The θ - dependence was checked by placing the detecting loop at an angle of 0° , 90° , 135° , 180° with respect to the exciting loop.

4. Field Distribution

Relative field measurements can be done by bead pulling and measuring the resulting frequency shift. The experimental results and the interpretation will be given in a forthcoming report.

It is very difficult to decide exclusively by experiments in what mode a measured resonance is oscillating. It is more effective to compute approximately the dispersion diagram of an expected mode and to check experimentally the corresponding resonances.

The possible modes in circular iris-loaded waveguides can be classified into groups according to the θ - periodicity $\exp(-j\nu\theta)$ of their field expressions, designated by a first index ($\nu = 0, 1, 2, \dots$). Each group consists of an infinity of passbands designated by a second index ($m = 1, 2, \dots$).

1. The group with $\nu = 0$: TM_{0m} TE_{0m}

The TM_0 modes are pure transverse modes ($H_z = 0$). The lowest passband TM_{01} is used as accelerating mode in linacs. Very little is known of the higher passbands (TM_{02} etc.), though TM_{02} is generally considered as being a backward wave (e.g. 9).

Nothing seems to be known about the existence or non-existence of TE_0 modes.

2. The group with $\nu = 1$: HEM_{1m}

In iris-loaded waveguides a coupled TM_1 - TE_1 mode only is able to satisfy the boundary conditions. It is proposed to call it a "hybrid electromagnetic" HEM_1 mode. At Stanford and CERN it is referred to as

"TM₁ - like" mode¹, and in France as EH₁ mode¹¹. The lowest passband HEM₁₁ is held responsible for the pulse shortening and can be used as deflecting mode in rf particle separators.

3. The group with $\nu = 2$: HEM_{2m}

This group has equally hybrid character and therefore transverse deflecting properties.

4. The groups with $\nu > 2$ were not investigated, since their passbands are well above the interesting frequency range.

III. Hybrid Electromagnetic Mode HEM₁

The theory of hybrid modes in an iris-loaded waveguide was developed in a previous report¹². An approximate expression for the frequency condition of the HEM₁ mode was found in the form:

$$ka \frac{J_1(\gamma a)}{\gamma a} J_1'(\gamma a) Z_1'(ka) + \frac{d}{w} C_{oo}^2 \left[(\beta a)^2 \frac{J_2(\gamma a)}{(\gamma a)^2} J_0(\gamma a) - J_1'^2(\gamma a) \right] Z_1(ka) = 0 \quad (3.1)$$

where:

$$k = \omega/c; \quad \beta = \omega/v_p; \quad \gamma^2 = k^2 - \beta^2$$

$$Z_\nu(ka) = J_\nu(ka) Y_\nu(kb) - J_\nu(kb) Y_\nu(ka)$$

$$Z_\nu'(ka) = J_\nu'(ka) Y_\nu(kb) - J_\nu(kb) Y_\nu'(ka)$$

$$C_{oo} = \frac{\sin(\beta d/2)}{(\beta d/2)}$$

Equation (3.1) gives the dispersion diagram $k = k(\beta)$. Phase velocity v_p and group velocity v_g are then defined as:

$$v_p = c \frac{k}{\beta} \text{ and } v_g = c \frac{\delta k}{\delta \beta}$$

The group velocity at $\beta = n \frac{\pi}{Nw}$ is obtained from experimental data according to:

$$v_g \approx c \frac{k([n + 1] \pi/Nw) - k([n - 1] \pi/Nw)}{2\pi/Nw}$$

FIRST PASSBAND: HEM_{11} (deflecting mode)

The first model tested was a copy of the CERN prototype with square irises and no flats. It had the following dimensions:

$$a = 1.645, \quad b = 3.459, \quad w = \pi/2, \quad d = 1.257$$

The measured dispersion diagram is plotted in Fig. 2. We find for $\beta = 1$:

$$k = 1.0003, \quad v_g = 0.0173, \quad Q = 6800$$

The k - β curve is double valued for $k = 1$, indicating a degeneracy of modes. The computed k - β curve (Fig. 3) shows the same qualitative behavior.

The second model was therefore built with smaller aperture a , the other dimensions remained unchanged:

$$a = 1.441$$

From the measured dispersion diagram which is plotted in Fig. 2 we find for $\beta = 1$:

$$k = 1.00955, \quad v_g = -0.0246, \quad Q = 3500$$

As expected from the computed curve (Fig. 3), the degeneracy was now removed. The negative group velocity indicates a backward wave. The Q value is considerably smaller, in part due to the elimination of spring-rings and the less good surface finish of the second model.

To prevent electrical breakdown, the iris edges have to be rounded. The rounding is done in a similar way as at Stanford (Fig. 4), and the geometrical aperture a remains therefore unchanged. The electric effect is an increased equivalent aperture, resulting in a higher group velocity. The Q -value is equally increased. The measured dispersion diagrams for square and rounded irises are plotted in Fig. 5. From the curve for rounded irises we find for $\beta = 1$:

$$k = 1.00852, \quad v_g = -0.0184, \quad Q = 4900$$

The use of related narrow band equipment requires $k = 1$ at $\beta = 1$. The simplest way to achieve this is by changing the outer radius b . The frequency at which $\beta = 1$ as a function of radius b is depicted in Fig. 6. The measured slope is:

$$\partial k / \partial b = -0.26$$

The dimensions of the final model with rounded edges are thus:

$$a = 1.441, \quad b = 3.491, \quad w = \pi/2, \quad d = 1.257$$

We measured for $\beta = 1$:

$$k = 1.00011, \quad v_g = -0.0203, \quad Q = 5400$$

The error of the frequency measurement introduced by the location of feeding and detecting loops was investigated. The coupling loop is a half circle at the end of a RG-9A/U cable. The loops are always at a location, where they couple to the H_θ - component. The resonant frequencies are obtained by averaging the upper and lower 3 dB points. Measured frequencies are given below for three different cases:

- Feeding and detection between 2. and 3. iris of a 10 iris cavity:

$$k = 1.00010$$

- Feeding and detection in the same end plate:

$$k = 1.00008$$

-Feeding and detection in opposite end plates:

$$k = 1.00008$$

The deflecting mode in a perfect cylindrical waveguide is degenerate with respect to θ . Perturbations are therefore likely to rotate the plane of polarization of the mode. It was proposed at CERN to use small lateral flats as mode stabilizers (Fig. 7). The flats remove the degeneracy and produce a splitting of the k - β curve, depending on the plane of excitation (Fig. 8). The repercussion on the phase velocity of the wanted, lower mode is negligible: The necessary change in frequency to maintain $\beta = 1$ is $\Delta k = -4 \times 10^{-5}$. It is seen from Fig. 8 that the splitting is sufficient to suppress completely the unwanted rotated mode.

Waveguides with flats are easily produced by electro-forming. The brazing technique, preferred in U.S.A., requires different mode stabilizers. The use of grooves instead of flats is possible, but the wanted lower mode becomes very sensitive to the geometrical tolerances of the grooves. Another possible solution using coupling holes in the irises was proposed at the E.N.S., Orsay (Fig. 9). The frequencies at which $\beta = 1$ are plotted in Fig. 10 for the wanted and rotated modes as a function of the hole diameter. The maximum frequency difference obtained with holes is insufficient for a complete suppression of the rotated mode; holes are therefore not recommended as mode stabilizers.

Perfect suppression of the rotated mode is combined with simple fabrication technique by use of rods as shown in Fig. 11. The rods shift the frequency for $\beta = 1$ by a small amount $\Delta k = +7 \times 10^{-5}$. The π - cutoff frequency $k = 1.012$ of the rotated mode is well above the operating frequency $k = 1$.

SECOND PASSBAND: HEM_{12}

The equation (3.1) has a second solution HEM_{12} at higher frequencies. The experiments confirmed the existence of a passband immediately above the deflecting mode. The measured and computed dispersion diagrams of the HEM_{11} and HEM_{12} modes are plotted in Fig. 12. The curves are given for a model with the following dimensions:

$$a = 1.447, \quad b = 3.459, \quad w = \pi/2, \quad d = 1.257$$

The second passband of the hybrid modes must have equally deflecting properties and A. W. Maschke suggested to use it as a deflecting mode, since a waveguide designed for the HEM_{12} mode is likely to have an increased aperture.

In a previous report⁸, this mode was identified with the lowest passband of the TM_1 - mode ($H_z = 0$). In view of the sufficient agreement between computational and experimental results presented here and the theory given in reference¹² it was inferred that TM_1 modes do not propagate in an iris-loaded waveguide. A similar conclusion was reached earlier at CERN¹⁰. The propagation of the TE_1 - modes was already questioned in reference¹.

IV. Transverse Magnetic Mode TM_0

The approximate frequency condition for the TM_0 mode is known to be (13):

$$J_0(\gamma a) Z'_0(ka) - \frac{d}{w} C_{00}^2 ka \frac{J'_0(\gamma a)}{\gamma a} Z_0(ka) = 0 \quad (4.1)$$

The first passband TM_{01} is used as accelerating mode. The computed and experimental dispersion diagrams are plotted in Fig. 13 for a model with the following dimensions:

$$a = 1.447, \quad b = 3.491, \quad w = \pi/2, \quad d = 1.257$$

The frequency $k = 1$ is in the center of a stopband and no propagation is possible.

It is generally assumed that the second passband TM_{02} has to be a backward wave. Our computational and experimental results show clearly a forward wave. In reference¹, a second "Brillouin-zone" different from TM_{02} and with opposite slope was mentioned. The complete theory for TM_0 modes¹⁴, however, requires that they are identical.

V. Hybrid Electromagnetic Mode HEM_2

The approximate frequency condition for the HEM_2 mode is found to be:

$$ka \frac{J_2(\gamma a)}{(\gamma a)^2} \frac{J_2'(\gamma a)}{\gamma a} Z_2'(ka) + \frac{d}{w} C_{oo}^2 \left[(\beta a)^2 \frac{J_3(\gamma a)}{(\gamma a)^3} \frac{J_1(\gamma a)}{(\gamma a)} - \left[\frac{J_2'(\gamma a)}{\gamma a} \right]^2 \right] Z_2(ka) = 0 \quad (5.1)$$

The computed dispersion diagram of the lowest passband is plotted in Fig. 14 for a model with the following dimensions:

$$a = 1.447, \quad b = 3.459, \quad w = \pi/2, \quad d = 1.257$$

The bead pulling failed to determine the β - value of the resonances observed. The measured bandwidth for the HEM_{21} mode was 1.385 to 1.450 which is in good agreement with the computational values. It is safe to conclude that the HEM_{21} mode is equally a backward wave.

Acknowledgements

The constant interest of Dr. A. W. Maschke and his various contributions were appreciated. The mechanical problems of the models were handled by R. Damm. The experimental data was obtained with the assistance of P. A. Montemurro.

References

1. M. L. Report 581, Stanford University (1959).
2. M. C. Crowley-Milling et al., Nature vol. 191, no. 4787, p. 483 (1961).
3. K. L. Brown et al., Proc. International Conf. High Energy Accelerators, Brookhaven, p. 79 (1961).
4. J. E. Leiss and R. A. Schrack, Internal Report NBS, p. 47 (1962).
5. H. G. Hereward, B. W. Montague, private communications.
6. H. G. Hereward, Minutes CERN-BNL Meeting, Brookhaven, p. 94 (1962).
7. J. W. Bittner, Internal Report BNL-AGS, JWB-1 (1962).
8. H. Hahn, Minutes CERN-BNL Meeting, Brookhaven, p. 103 (1962).
9. J. C. Slater, Microwave Electronics, p. 174 (Van Nostrand 1950).
10. B. W. Montague, private communication.
11. Y. Garault, Comptes Rendus Paris, vol. 255, no. 22, p. 2920 (1962).
12. H. Hahn, Internal Report BNL-AGS, HH-5 (1962).
13. K. Poeschl, Mathematische Methoden in der HF Technik, p. 274 (Springer 1956).
14. W. Walkinshaw, J. S. Bell, AERE report G/R 675 (1951).

HH/HJH/eb

1/14/63

Distribution: AD: B1, B2.

Fig. 1 The Model

Fig. 2 Measured Dispersion Diagram of HEM_{11} mode

Fig. 3 Computed Dispersion of HEM_{11} - mode

Fig. 4 Rounded iris edges

Fig. 7 Flats

Fig. 5 Square and rounded iris edges

Fig. 6

Fig. 8 Mode splitting due to flats

Fig. 9 Holes as mode stabilizers

Fig. 11 Rods as mode stabilizers

Fig. 10 Mode splitting by holes

Fig. 12 1. and 2. passband of HEM₁ mode

Fig. 13. 1. and 2. passband of TM_0 mode

Fig. 14. Computed HEM_{21} mode