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UNITED STATES ATOMIC ENERGY COMMISSION

TEMPERATURE DISTRIBUTION IN THIN WALLED HEAT-EXCHANGERS HAVING NON-CIRCULAR FLOW PASSAGES

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SUMMARY

In heat exchangers, in which the walls are heated by internal heat sources, it is possible for wall temperatures greater than the mean to occur in the corners of noncircular flow passages. Thus in a square or triangular passage low velocities occur in the corners, and the resulting decrease in the heat transfer coefficient produces higher temperatures in the walls at these locations

A generalized analysis is presented, taking into account the variation in the heat transfer coefficient along the surface, by which it is possible to compute local temperatures in the walls of noncircular flow passages in the vicinity of the corners. Computations have been made for a representative component (Figure I) composed of a honey comb of rectangular passages. The resulting temperature at the hottest point is approximately 125°F greater than the uniform plate temperature. This analysis is based on a 90° angle between the plate and retaining plate, and a uniform source distribution throughout all plates. Had the angle between the primary plate and retaining plate been much less than 90° on both sides, then the resulting local temperature rise would be several times the above value. If the primary plate and retaining plate are "dead" at junctions and corners then of course a "hot spot" will not occur at these locations.

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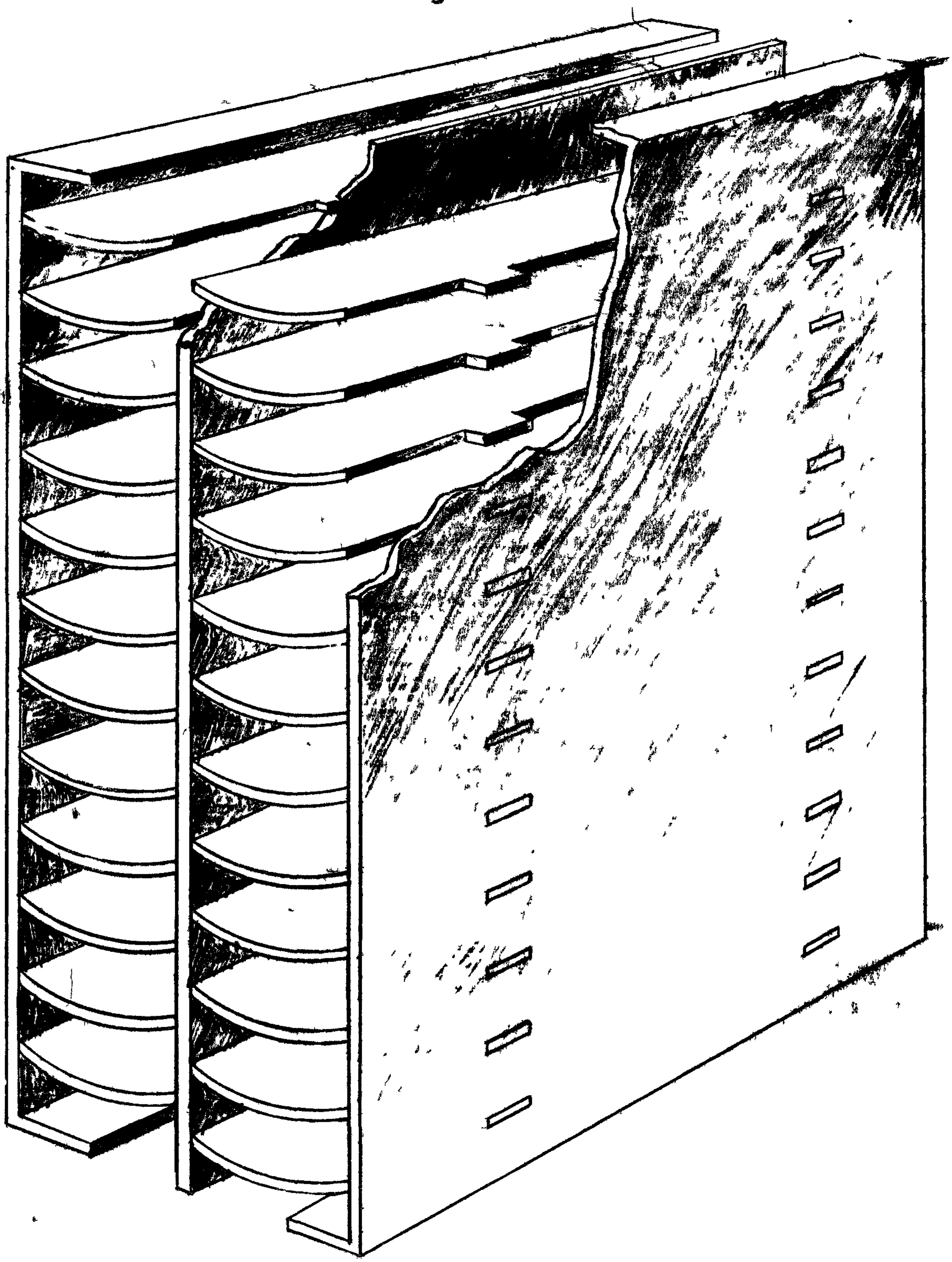

Figure I

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ASSUMPTIONS

A generalized analysis of this problem has been made without imposing limitations on the nature of the fluid employed. However, to obtain numerical results a knowledge of the local velocity or temperature profile throughout the noncircular passage is required.

Thus in obtaining the generalized solution the following assumptions are made in order to obtain an analytical result.

(1) The plate forms a fin of infinite length in the direction of fluid motion and of uniform thickness.

(2) The thermal conductivity of the wall is constant over the range of temperatures present in the plate and is assumed infinite across the thickness. This reduces the analysis to one dimension.

(3) The internal heat source in the wall is assumed to be uniformly distributed throughout the wall material.

(4) The heat transfer coefficient in the corner of the passage must be capable of being approximated as a linear function of the distance from the corner.

In order to solve the generalized equations for the particular conditions involved in the assembly, additional assumptions had to be made.

(1) The Prandtl number of the coolant is taken as 1. A condition fulfilled by air.

(2) A fully developed velocity profile is assumed.

(3) The turbulent diffusivity of heat and momentum are equal.

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If these latter assumptions are fulfilled, then one can employ data for local heat transfer coefficients in noncircular flow passages. It can be seen by examining Figure 7 on page 32 of their report that the local heat transfer coefficient can be approximated by a linear function of the distance from the corner, thus satisfying a previous assumption.

GENERALIZED SOLUTION

Let us consider one wall of a rectangular passage. The temperature of the wall will be symmetrical about a plane midway between the two corners or a half width of the wall, e. The wall or plate fin has a shape as shown below.

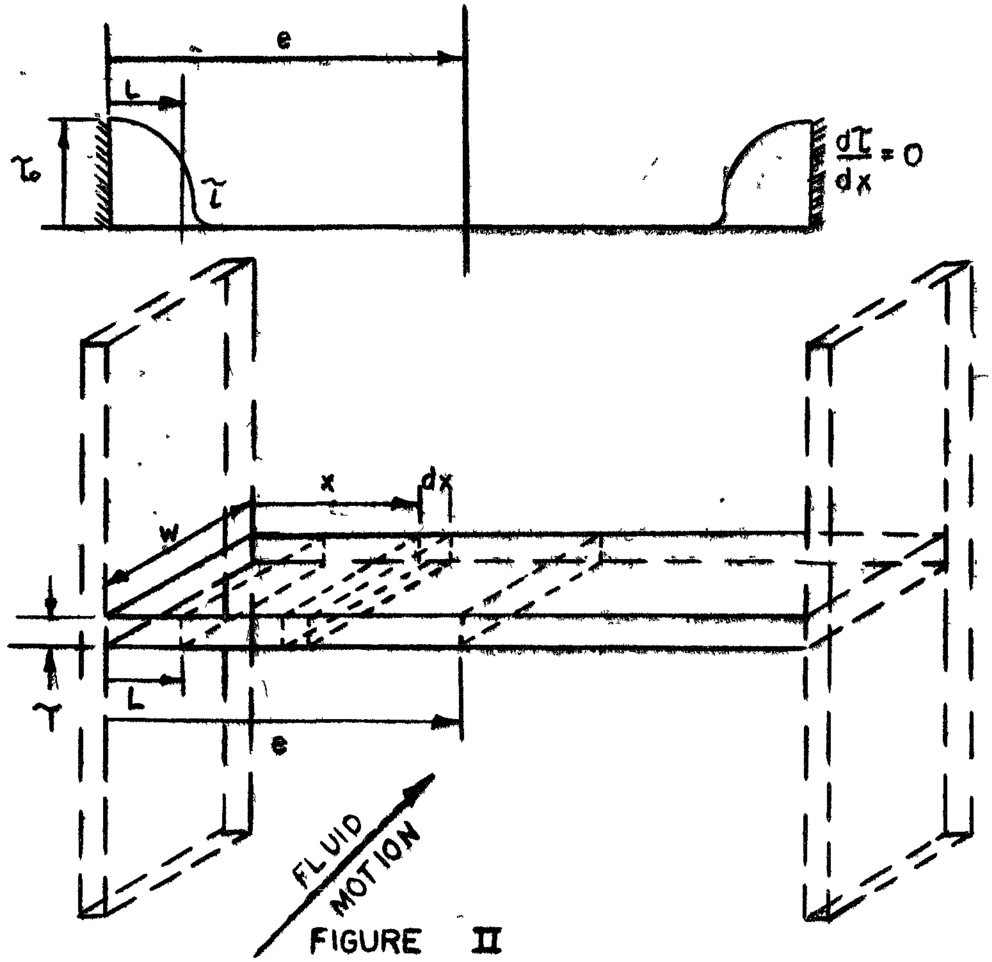

FIGURE II

TWO REGION FIN WITH A VARIABLE HEAT TRANSFER COEFFICIENT ON THE SURFACE OF THE CORNER REGION

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where

T	-	fin thickness
W	-	fin width (assumed infinite)
x	-	distance from the base of the fin
k	-	thermal conductivity of fin material
h	-	uniform heat transfer coefficient for the passage, at ℄
h_x	-	local heat transfer coefficient
D	-	hydraulic diameter
Q	-	heat generation per unit volume
t	-	local plate or fin temperature
$t_{\text{℄}}$	-	fin temperature midway between the retaining plates
L	-	fin length at which $h_x = h$
t_a	-	mixed mean air temperature
τ	-	$(t - t_a)$
X	-	x/L
E	-	℄$/L$

It is necessary to divide the fin into two regions. The first has a variable heat transfer coefficient and extends a distance L from the corner. The second extends from L to ℄ and is one over which the heat transfer coefficient is constant. The fin or wall is thin and hence the flow of heat is treated in one dimension.

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Considering the region at the corner first, let us represent our local heat transfer coefficient as a linear function of x for the upper fin surface. Thus:

$$h_x/h = ax$$

Similarly for the lower surface

$$h_x/h = bx$$

A differential equation can be written for the fin by means of a heat balance across the increment dx.

(1) Heat into dx, at x, by conduction:

$$q_1 = -kWT\left(\frac{dt}{dx}\right)$$

(2) Heat out of dx, at x + dx, by conduction:

$$q_2 = -kWT\left(\frac{dt}{dx} + \frac{d^2t}{dx^2}\,dx\right)$$

(3) Heat lost to the surroundings by convection:

$$q_3 = (ahx + bhx)(W\tau dx) = hWf\tau xdx$$

Letting $f = a + b$

(4) Heat generated by internal sources:

$$q_4 = QWT\,dx$$

Combining these heat terms and simplifying we obtain the general differential equation:

$$\frac{d^2t}{dx^2} - \frac{hf}{kT}\,x\,\tau = -\frac{Q}{k}$$

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On changing variables we obtain:

$$\frac{d^2 \tau}{d X^2} - \frac{hfL^3}{kT} X \tau = - \frac{QL^2}{k}$$

This is a linear second order differential equation. The solution of the homogeneous equation can be found by the method of integration by series. Having determined the solution of the homogeneous equation, it is then possible to obtain the particular solution to the complete differential equation using Picards iterative method. The sum of the homogeneous and particular solution forms the complete solution as follows:

$$\begin{aligned} \tau = {} & A_o \left(1 + \frac{hfL^3}{kT} \frac{X^3}{2 \cdot 3} + \left[\frac{hfL^3}{kT} \right]^2 \frac{X^6}{2 \cdot 3 \cdot 5 \cdot 6} + \cdots \right) \\ & + A_1 \left(X + \frac{hfL^3}{kT} \frac{X^4}{3 \cdot 4} + \left[\frac{hfL^3}{kT} \right]^2 \frac{X^7}{3 \cdot 4 \cdot 6 \cdot 7} + \cdots \right) \\ & - \frac{QL^2}{k} \left(\frac{X^2}{2} + \frac{hfL^3}{kT} \frac{X^5}{2 \cdot 4 \cdot 5} + \left[\frac{hfL^3}{kT} \right]^2 \frac{X^8}{2 \cdot 4 \cdot 5 \cdot 7 \cdot 8} + \cdots \right) \end{aligned}$$

where A_o and A_1 are constants whose value depends on the boundary conditions.

Let us take for boundary conditions $\tau = \tau_o$ and $\frac{d\tau}{dX} = 0$ at $X = 0$. This is equivalent to assuming an insulated boundary or a point of symmetry at $X = 0$. Then $A_o = \tau_o$ and $A_1 = 0$.

Over the second region of our fin the heat transfer coefficient is a constant value on both surfaces. For this region the differential equation is

$$\frac{d^2 \tau}{dX^2} - \frac{2hL^2}{kT} \tau = - \frac{QL^2}{k}$$

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This is a second order linear differential equation with constant coefficients. The solution is readily found to be:

$$\tau = A_2 e^{+\sqrt{\frac{2h}{kT}}\,L\,X} + A_3 e^{-\sqrt{\frac{2h}{kT}}\,L\,X} + \frac{QT}{2h}$$

The boundary conditions are given by $\tau = \tau_E$ at $X = E$ and that the temperature gradient and temperature at the junction of the two regions must be equal. Designating the temperature at the junction where $X = 1$ $(x = L)$, as τ_1 the arbitrary constants A_2 and A_3 above can be evaluated. The solution then becomes:

$$\tau_{I} = \frac{\left.\frac{d\tau}{dX}\right|_{\tau_1} \cdot \sqrt{\frac{kT}{2h}}\ \frac{1}{L}}{1 + e^{+2\sqrt{\frac{2h}{kT}}\,(LE-L)}}\, e^{+\sqrt{\frac{2h}{kT}}(L\,X-L)} - \frac{\left.\frac{d\tau}{d\,X}\right|_{\tau_1} \cdot \sqrt{\frac{kT}{2h}}\ \frac{1}{L}}{1 + e^{2\sqrt{\frac{2h}{kT}}\,(L-LE)}}\, e^{-\sqrt{\frac{2h}{kT}}(L\,X-L)} + \frac{QT}{2h}$$

A solution for the corner temperature can now be found by equating temperatures for both regions at $X = 1$ and substituting for the gradient above that derived from the expression for the region near the corner.

Thus let us use the solution for the first region for an insulated base or zero gradient at $X = 0$. On equating temperatures for both regions at $X = 1$ the following expression containing the "hot spot" or corner temperature τ_o results:

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$$\tau_o = \left(1 + \frac{hfL^3}{kT}\,\frac{1}{2\cdot 3} + \left[\frac{hfL^3}{kT}\right]^2 \frac{1}{2\cdot 3\cdot 5\cdot 6} + \cdots\right)$$

$$- \frac{QL^2}{k}\left(\frac{1}{2} + \frac{hfL^3}{kT}\,\frac{1}{2\cdot 4\cdot 5} + \left[\frac{hfL^3}{kT}\right]^2 \frac{1}{2\cdot 4\cdot 5\cdot 7\cdot 8} + \cdots\right)$$

$$= \frac{\left.\frac{d\tau}{dX}\right|_{\tau_1}\cdot\sqrt{\frac{kT}{2h}}\,\frac{1}{L}}{1 + e^{+2\sqrt{\frac{2h}{kT}}(LE-L)}} - \frac{\left.\frac{d\tau}{dX}\right|_{\tau_1}\cdot\sqrt{\frac{kT}{2h}}\,\frac{1}{L}}{1 + e^{+2\sqrt{\frac{2h}{kT}}(L-LE)}} + \frac{QT}{2h}$$

where

$$\left.\frac{d\tau}{dX}\right|_{\tau_1} = \tau_o\left(\frac{hfL^3}{kT}\,\frac{1}{2} + \left[\frac{hfL^3}{kT}\right]^2 \frac{1}{2\cdot 3\cdot 5} + \cdots\right)$$

$$- \frac{QL^2}{k}\left(1 + \frac{hfL^3}{kT}\,\frac{1}{2\cdot 4} + \left[\frac{hfL^3}{kT}\right]^2 \frac{1}{2\cdot 4\cdot 5\cdot 7} + \cdots\right)$$

In order to complete our analysis we must examine the vertical wall or retaining plate to ascertain the local temperature rise due to the insulating effect of the horizontal wall. We shall treat it as a plate with an insulated strip and shape as follows:

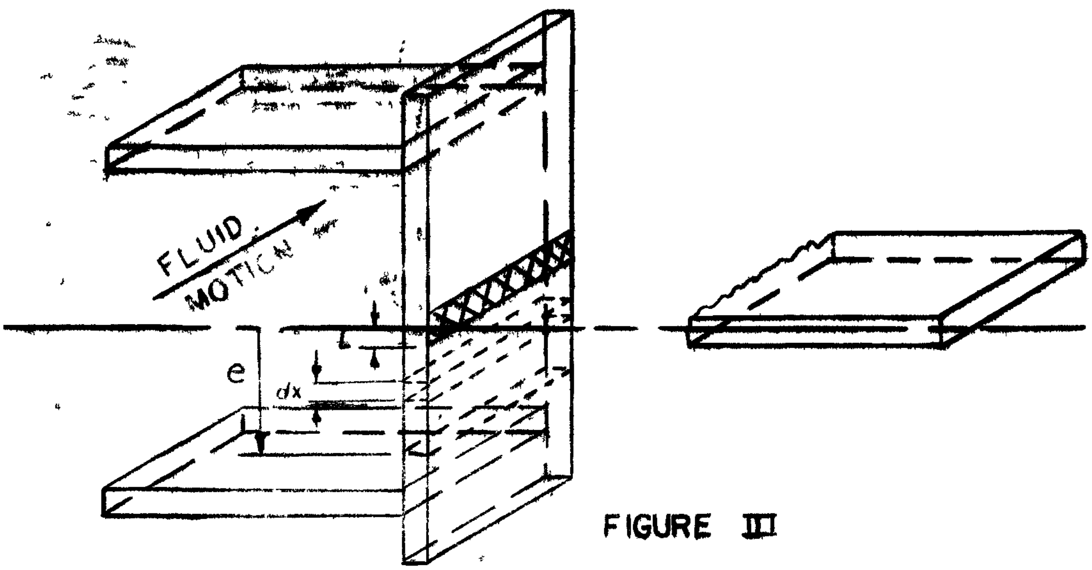

FIGURE III

TWO REGION FIN WITH AN INSULATED STRIP ON THE SURFACE OF ONE REGION

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The symbols to be employed have the same designation as in the previous problem. The plate temperature is symmetrical about a plane through the middle of the insulated section. Again we must divide the resulting fin below the plane of symmetry into two regions. The first region extends a distance L from the symmetrical temperature plane. Over this region, one side of the plate has a uniform h and the other side is insulated. The second region extends from L to e and the h is uniform on both sides.

The equation giving the wall temperature over the region with a constant heat transfer coefficient on one side and insulated boundary on the other, is given by:

$$\tau = C_1 e^{+\sqrt{\frac{h}{kT}} L X} + C_2 e^{-\sqrt{\frac{h}{kT}} L X} + \frac{QT}{h}$$

where C_1 and C_2 are arbitrary constants. The point X = 0 has a temperature τ_o and a temperature gradient of 0. Hence, the constants are evaluated as:

$$C_1 = C_2 = \frac{\tau_o}{2} - \frac{QT}{2h}$$

The wall temperature over the second region in which the heat transfer coefficient is a constant on both sides is given by:

$$\tau = C_3 e^{\sqrt{\frac{2h}{kT}} L X} + C_4 e^{-\sqrt{\frac{2h}{kT}} L X} + \frac{QT}{2h}$$

Again equating temperatures and gradients at the junction of the two regions (at X = L), using the boundary condition $\tau = \tau_E$ at X = E, we arrive at an expression containing the "hot spot" or corner temperature τ_o.

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$$\left(\frac{\tau_o}{2} - \frac{QT}{2h}\right)\left(e^{\sqrt{\frac{h}{kT}}L} + e^{-\sqrt{\frac{h}{kT}}L}\right) + \frac{QT}{2h} =$$

$$\frac{1}{\sqrt{2}}\left(\frac{\tau_o}{2} - \frac{QT}{2h}\right)\left(e^{\sqrt{\frac{h}{kT}}L} - e^{-\sqrt{\frac{h}{kT}}L}\right)\left(\frac{1}{1+e^{+2\frac{2h}{kT}(LE-L)}} - \frac{1}{1+e^{+2\frac{2h}{kT}(L-LE)}}\right)$$

Numerical results are presented for the assembly (See Figure I) listed as first preference.

The junction of the horizontal plate with the retaining plate can be approximated by our two previous fin solutions. The fin to wall angle is taken as 90^o on each side. The physical dimensions and heat transfer properties of the assembly are taken from the above mentioned memo.

of the assembly are taken from the above mentioned memo.

k = 15 Btu/Hr. Ft.^{2} ^{o}F/ft.

Q = 329 x 10^6 Btu/hr.ft.^{3} (at the back of the first row of plates)

T = 0.001 ft. for the plates

= 0.00125 ft. for the retaining plate

h = 181 Btu/hr.ft.^{2} ^{o}F

t_a = 658^oF

t_m = 1573^oF

D = 0.0368 ft.

$a = b = 638$ hence $f = 1276$

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Let us consider first a horizontal plate using our first generalized solution for a variable heat transfer coefficient along the surface. For the assembly shown in Figure I, a zero temperature gradient at the fin base or at X = 0 is a satisfactory first assumption. For the assembly shown, L = 0.001565 ft. and E = 42.5. Substituting the above values into the generalized expression, an expression is obtained containing the "hot spot' temperature τ_0.

$$\tau_o \left(1 + \frac{0.059}{6} + \ldots\right) - 54\left(0.5 + \frac{0.059}{40} + \ldots\right) =$$

$$\left\{\tau_o\left(\frac{0.059}{2} + \frac{0.059^2}{30} + \ldots\right) - 54\left(1 + \frac{0.059}{8} + \ldots\right)\right\} \cdot \left\{\frac{1}{0.243}\right\}.$$

$$\left\{\frac{1}{1 + e^{311 \times 42.5}} - \frac{1}{1 + e^{-311 \times 0.01565 \times 4.25}}\right\} + 915$$

or

$$\tau_o\ (1.001) - 54\ (0.5015) = -\frac{0.0295\,\tau_o - 1.0074 \times 54}{0.243} + 915$$

Hence

$$\tau_o = 1037^\circ F$$

and

$$\tau_o - \tau_m = 1037 - 915 = 122^\circ F \text{ (Hot spot rise)}$$

Next let us examine the vertical retaining plate for a hot spot at the junction with the plate using our second generalized fin solution for an insulated border. The insulated border can be taken as equal to the sum of the width of a plate and the distance over which h_x is less than h. Hence for

38

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the assembly shown, L = 0.002065 ft. and E = 32.2. Substituting the above values into the generalized solution for this situation, the following equation containing the "hot spot" temperature results.

$$\left(\frac{\Upsilon_o}{2} - 915\right)\left(e^{0.203} + e^{-0.203}\right) + 915 =$$

$$\frac{1}{1.415}\left(\frac{\Upsilon_o}{2} - 915\right)\left(e^{0.203} - e^{-0.203}\right)\left\{\frac{1}{1 + e^{0.574 \times 31.2}} - \frac{1}{1 + e^{-0.574 \times 31.2}}\right\}$$

or

$$\left(\frac{\Upsilon_o}{2} - 915\right) 2.040 + 915 = -\left(\frac{\Upsilon_{x=0}}{2} - 915\right)\frac{0.410}{1.415}$$

Hence

$$\Upsilon_o = 1044^{\circ}F$$

and

$$\Upsilon_o - \Upsilon_m = 1044 - 915 = 129^{\circ}F \text{ (Hot spot rise)}$$

Since both temperatures (Υ_o) are approximately the same, the assumption that the base of the plate is insulated is satisfactory.

DISCUSSION

The preceding analysis has been made in a general fashion to facilitate computations on various design proposals for the assembly. The equations are applicable to other primary to retaining plate angles besides the 90° angle used in the numerical calculation above. It is interesting to note that a change from 90° to 60° on both sides of the primary plate in our previous problem would increase the "hot spot" temperature rise from 125°F to approxi-

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mately 250°F. Hence, it is desirable to keep angles between plates as large as possible.

If in our first generalized solution the base of the fin is not insulated, then A_1 is not zero. Expressing the temperature gradient at $X = 0$ by $\frac{d\tau}{dX} = -\frac{QL}{TkW}$, then $A_1 = -\frac{QL}{TkW} = \left.\frac{d\tau}{dX}\right|_{\tau_0}$. In most problems the corner will be a point of symmetry for the temperature pattern and hence the temperature gradient at that point will be zero and consequently A_1 will be zero. We have also assumed the midpoint temperature to be the average temperature. Since the local temperature rise occurs only in that small part of the plate at the corner this assumption is sufficiently accurate for computational purposes.

When the included angle between the primary plate and adjacent plate or retaining plate is different on both sides, then the first generalized analysis will involve a three region solution. The method to be employed will however be identical to that for the two region problem presented herein.

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