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AN EVALUATION OF RECENT STUDIES OF HIGH-FLUX HEAT TRANSFER TO GASES

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W. R. Gambill

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ABSTRACT

A review was made of the literature of the past decade in order to collect quantitative information relevant to the transmission of heat to flowing gases at high heat fluxes (defined here as a maximum experimental flux of at least 0.5×10^6 Btu/hr·ft²). The available data, obtained mostly under NASA and AEC programs, include heat-transfer-coefficient determinations for He, H₂, and N₂ in tubular, parallel-plate, and wire-mesh geometries for pressures of 15 to 1511 psia, Reynolds numbers of 2.0×10^2 to 1.5×10^6 , maximum heat fluxes of 0.58×10^6 to 7.68×10^6 Btu/hr·ft², and maximum surface and bulk coolant temperatures of 5440°F and 4180°F, respectively.

The correlations proposed for heat-transfer coefficients are reviewed and a recommendation made for turbulent internal duct flow. Graphical results are given which directly relate surface temperature to heat flux for helium flowing through a 0.5-in.-diam tube at mass velocities of 10^4 to 10^8 lb/hr·ft² and at bulk temperatures of 200 to 4000°R.

Friction data are also reviewed and a correlation recommended for the computation of friction factors up to a temperature ratio of ~8.

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SCOPE

Before an experimental study was begun at the Oak Ridge National Laboratory to determine the high-flux thermal characteristics of non-standard, gas-cooled geometries of potentially improved performance, a review was made of the literature, with emphasis on that of the past decade, which deals with heat and momentum transfer for flowing gases under high heat flux conditions. In this context, a high-heat-flux study is arbitrarily taken to mean one encompassing a maximum experimental flux of at least 0.5×10^6 Btu/hr·ft². Although the modification of constant-property correlations necessitated by the effect of a high Mach number is essentially a problem involving the temperature dependence of the pertinent physical properties, the work reviewed relates generally to flows exhibiting negligible viscous dissipation. The effects associated with extreme gas temperatures, such as dissociation, ionization, and surface chemical reaction, are excluded.

BACKGROUND

At small heat fluxes, the radial contrast of coolant temperature is also small, and there is no problem of accounting for the nonisothermality of the coolant stream. With increasing heat flux, however, the temperature gradient established by the heat transmission steepens, and since the viscosity (as well as other physical properties) is temperature dependent, the velocity profile distorts, or deviates from its isothermal shape. In this fashion, the temperature field feeds back into the velocity distribution and alters values of heat-transfer coefficient based solely on evaluation of the relevant properties at the bulk (mixed mean) coolant temperature. To account in a rigorous fashion for such interdependencies over property variations of many fold would require a form of correlation more complicated than any yet proposed; but over moderate property ranges, relatively simple corrections to the low flux, constant-property solutions have been developed.

Some of the earliest proposals recognized gross differences between the heating and cooling cases, and in correlations such as those of Dittus-Boelter and of Kraussold,¹ a Prandtl-number exponent of 0.37 to 0.40

was recommended for fluid heating and a value of 0.30 for cooling. The recognition has since evolved that variable-property effects for gases and for liquids may be handled separately in different ways and in a more generalized manner than by simply varying the exponent on the Prandtl modulus.

For liquids, the temperature dependencies of heat capacity, density, and thermal conductivity (at temperatures significantly below the thermodynamic critical temperature) are generally quite small compared to that of viscosity. For six typical nonmetallic liquids over the temperature range of 10 to 100°C, e.g.,² the density varies by 4 to 12%, the heat capacity by 0 to 30%, and the thermal conductivity by 4 to 17%, but the viscosity varies by 3 to 300 fold (200 to almost 30,000%). It is thus reasonable with liquids to assign most of the variable-property effect to viscosity variation, and to modify a constant-property correlation of the form:

$$(Nu_b)_o = K Re_b^a Pr_b^b \psi(L/D) \quad (1)$$

to:

$$Nu_b = K Re_b^a Pr_b^b \psi(L/D) (\mu_b/\mu_w)^c, \quad (2)$$

in which equations common values of a , b , and c are 0.8, 0.4, and 1/9 to 1/4, respectively.* The liquid-metal group is a special case, for which it is feasible, in practice, to neglect property variability.

With gases, however, the situation is different, since the temperature dependence of physical properties is about the same for all dilute (low-density) species. For this case, Prandtl moduli vary⁴ only between 2/3 and 1.1, and the heat capacity is also relatively temperature independent. The density, ρ , varies as T^{-1} , and both μ and k vary approximately as $T^{3/4}$. For atmospheric-pressure air over the temperature range of 600 to 2400°F; e.g.,⁵ $k \propto T^{0.85}$, $\mu \propto T^{0.70}$, $C_p \propto T^{0.19}$, and $\rho \propto T^{-1}$, whence $Pr \propto T^{0.04}$ (about a standard-temperature value of 0.71). Since the largest

*In the Russian literature,³ a ratio of Prandtl moduli, $(Pr_b/Pr_w)^c$, is often used for the liquid case, in which $c = 0.06$ for heating ($t_w > t_b$) and 0.25 for cooling ($t_w < t_b$).

property variations other than of ρ (which combines with V in the mass velocity $G = \rho V$) are those of k and μ , and generally $\mu \propto k$, the following modification to Eq. (1) may be made:

$$Nu_b = K Re_b^a Pr_b^b \psi(L/D) (T_w/T_b)^n. \quad (3)$$

The property-ratio approach outlined above leads to a viscosity ratio for the liquid case and to an absolute-temperature ratio in the gas case. Another method commonly proposed⁶ has been use of the correlation form of Eq. (1), but with all properties evaluated at a reference temperature other than t_b ; e.g., at the wall (surface) temperature t_w or at the arithmetic-average film temperature $t_f = (t_w + t_b)/2$. The latter approach is more inconvenient in that four properties (C_p , ρ , k , and μ) must be evaluated at an initially unknown temperature, as opposed to the evaluation of only μ_w in Eq. (2) or T_w in Eq. (3). The reference-temperature formulation may ultimately prove to be more general, but the convenient property-ratio method seems more commensurate with the present level of data and correlation development.

PAST WORK

Before describing the more recent high-flux experimental studies, a summary will be given of some general recommendations which have been made. Kays reviewed much of the literature and concluded⁵ that for gas flows the best values of the exponents in the correction equations

$$\frac{Nu}{Nu_o} = \left(\frac{T_w}{T_b} \right)^n \quad (4)$$

and

$$\frac{f}{f_o} = \left(\frac{T_w}{T_b} \right)^m \quad (5)$$

are as follows:

Case	Condition	n	m
Laminar tube flow	---	0	1
Laminar external flow	---	0	0
Turbulent tube flow	$T_w > T_b$	-0.5	-0.1
	$T_w < T_b$	0	-0.1
Turbulent external flow	$T_w > T_\infty$	-0.5	-0.5
	$T_w < T_\infty$	-0.25	-0.25

The turbulent tube-flow values are for the constant heat flux case and are probably applicable to noncircular ducts as well. All physical properties involved in the calculation of h or f are evaluated at T_b for internal flow and at the free-stream static temperature T_∞ for external (immersed body) flow.

Kutateladze³ recommends that average gas-flow Nusselt moduli be calculated with the relation:

$$Nu_{b,a} = 0.023 Re_b^{0.8} Pr_b^{0.4} \left(1.27 - 0.27 \frac{T_w}{T_b} \right) \quad (6)$$

for $0.5 < (T_w/T_b) < 1.0$, and from:

$$Nu_{b,a} = 0.023 Re_b^{0.8} Pr_b^{0.4} (T_w/T_b)^{-0.55} \quad (7)$$

for $1.0 \leq (T_w/T_b) < 3.5$. The same investigator has presented² an interesting derivation of the effect of the temperature factor T_w/T_b on friction factors. For the limiting case of $f_o = 0$ (approached by flow at high Reynolds numbers over smooth surfaces), an explicit asymptotic solution was found:

$$\frac{f}{f_o} = \left(\frac{2}{\sqrt{T_w/T_b} + 1} \right)^2 \quad (8)$$

For T_w/T_b near unity, Eq. (8) corresponds² to:

$$\frac{f}{f_o} \approx \left\{ \frac{T_w}{T_b} \right\}^{-0.5} \approx \frac{2}{(T_w/T_b) + 1}, \quad (9)$$

and for $T_w/T_b \rightarrow \infty$, to $f/f_o \rightarrow 4/(T_w/T_b)$. Kutateladze also suggests that the approximate similarity of velocity and temperature profiles in subsonic gas flows ($Pr \approx 1$) allows one to use $h/h_o \approx f/f_o$ if h_o is evaluated at the given Pr for $Pr \neq 1$. In ref. 2, a plot is given which compares Eq. (8) with ~170 data points from the Russian, German, and American literature. For T_w/T_b between 0.5 and 2.6, the data generally fall within 10 to 20% of the prediction and exhibit a maximum deviation of $\sim \pm 30\%$.

Current NASA design practice for heat transfer in regeneratively cooled rocket nozzles (simultaneous transmission of heat from the hot gas within the nozzle to a countercurrent coolant stream in flow channels located in the nozzle wall) has been summarized in refs. 7 and 8. In this case, compressibility effects in the high-velocity hot-gas flow are quite significant, and only the coolant-side correlation will be described:

$$Nu_F = 0.0208 Re_F^{0.8} Pr_F^{0.4} C_2 C_3. \quad (10)$$

In Eq. (10), all properties are evaluated at a reference film temperature defined by:

$$t_F = \frac{t_w + (t_b)_t}{2}. \quad (11)$$

The multiplier, C_2 , is a tube-curvature factor which is taken as unity for $[Re_F(r_t/r_c)^2] < 6$. When the subject group ≥ 6 , C_2 is obtained from:

$$C_2 = [Re_F(r_t/r_c)^2]^{0.5}. \quad (12)$$

The factor, C_3 , which accounts for property variability and is based on data for H_2 only, may be written as:

$$C_3 = 1.0 + 0.01457 \left[\frac{v_w}{(v_b)_{st}} \right] \quad (13)$$

The preceding summary indicates the considerable variety of correlation forms in current use.

HIGH-FLUX EXPERIMENTAL STUDIES

Heat Transfer. In Table 1, the experimental parameters of seven studies are summarized. Remarkable maximum values have been attained; e.g., a 5440°F wall temperature, 4180°F bulk temperature, temperature factor of 11.1, and heat flux of 7.7×10^6 Btu/hr·ft². Additional studies which encompassed heat fluxes insufficiently high to qualify for inclusion in Table 1, or which were disclosed more fully in a later report, or for which insufficient information was reported, are described in refs. 17 through 23. The early NACA work¹⁷ attained maximum values of ϕ and T_w/T_b of 0.15×10^6 Btu/hr·ft² and 3.5, respectively. The Rocketdyne study¹⁸ of H₂ is disclosed in more detail in ref. 9. The maximum T_w/T_b ratio in ref. 20 is 2.5. Weiland²¹ measured local heat-transfer coefficients for He and H₂ in a tube of $L/D = 250$ at inlet pressures of 250 to 1000 psia, wall temperatures to 2300°R and temperature factors to 4.5. Newman's unpublished data²² relate to He and H₂ at inlet pressures of 110 to 850 psia in a tube of $L/D = 23.2$; the maximum wall temperature was 4600°R and the peak temperature factor 4.52.

The correlations proposed by the authors of the seven investigations of Table 1 are summarized in Table 2. Equations (14) and (19) use the temperature ratio; Eqs. (16), (17), (18), and (23) are based on a reference temperature other than t_b , and Eqs. (15), (20), and (21) combine the two methods. Equation (21) is particularly unusual in that the temperature ratio T_f/T_b (rather than T_w/T_b) is mixed with a film reference temperature. The fully developed laminar-flow Nusselt number of 9.0 given by Eq. (22) was not attained experimentally¹⁴ until x reached $\sim 0.96 L$; in this study both t_w and ϕ increased in the axial direction and the velocity distribution was variable. The limited data obtained¹⁴ indicate that the L/D at which an asymptotic, laminar-flow Nu is reached may be approximated by:

Table 1. Recent High-Flux Experimental Studies of Heat Transfer to Gases^a

Study No.	Coolant	Test Section		Resistance Heating	P ₁ (psia)	Re	Maximum Value of				Ref.
		Material	Geometry				t _w (°F)	t _b (°F)	T _w /T _b	10 ⁻⁶ q̇ (Btu/hr·ft ²)	
1	He, H ₂	321 SS, A-nickel	Tubes 0.194 < ID < 0.430 in. 21 < L/D < 67	dc	32 - 1354	4.0 × 10 ³ - 1.5 × 10 ⁶	1780 (avg)	470 ^b	11.1	7.68	9
2 ^c	H ₂ , N ₂	W	Wire Mesh 0.64 < ε < 0.72 0.020 < D _w < 0.035 in.	ac	15 - 24	2.0 × 10 ² - 6.0 × 10 ³	4740	1940	5.7	4.30	10
3	He	Graphite	Tube 1/16 in. ID × 23 in. L L/D = 368	ac	506 - 1511	1.0 × 10 ⁴ - 1.35 × 10 ⁶	4580	4180	1.4	3.92	11
4	He, H ₂	W	Tube ID = 0.115 in., L/D = 77	ac	37 - 93	5.7 × 10 ³ - 4.84 × 10 ⁴	4840	1980	8.0	2.37	12
5	He, H ₂	W	Tube ID = 0.116 in. L/D = 77	ac	40 - 100	7.6 × 10 ³ - 3.95 × 10 ⁴	5140	2370	5.6	1.70	13
6	H ₂	W	5 parallel plates Gap = 0.060 in. L/D _e = 74	ac	20 - 46	1.26 × 10 ³ - 1.25 × 10 ⁴	4540	3040	3.0	1.43	14
7	He	W, Mo, Inconel	Tubes 0.124 < ID < 0.191 in. L/D = 60, 92	ac	17 - 40	3.23 × 10 ³ - 6.0 × 10 ⁴	5440	2260	3.9	0.58	15

^aAll with axial gas flow. Friction as well as heat-transfer data included in all studies except No. 3.

^bInlet T_b as low as 135°R was used.

^cRe based on an equivalent diameter equal to four times the void volume divided by the surface area.

Table 2. Proposed Correlations for Nusselt Number

Study No.	Eq. No.	Correlation	Deviation ^a	Comments	Ref.
1	14	$(Nu_b)_a = 0.048 Re_b^{0.8} Fr_b^{0.4} \left(\frac{T_w}{T_b}\right)^{-0.55} \left(\frac{L}{D}\right)^{-0.15}$	±8% avg	Without acoustical resonance	9
	15	$(Nu_w)_a \approx 0.045 Re_w^{0.8} Fr_w^{0.4} (T_b/T_w)^{0.8}$	Rough correlation only	With acoustical resonance ^b	
2	16	$(Nu_w)_a = 0.462 Re_w^{0.53} Pr_w^{0.40}$	±14% maximum	For six wire meshes	10
3	17	$(Nu_f)_x = 0.036 (Re_f)_x^{0.8} (Pr_f)_x^{0.4} (x/D)^{-0.1}$	80% of data within ±10%	For $Re_f > 10^4$ Eqs. are same as those for air in Ref. 17	11
	18	$(Nu_f)_a = 0.034 Re_f^{0.8} Pr_f^{0.4} (L/D)^{-0.1}$	< ±10% avg		
4	19	$(Nu_b)_x = \frac{0.021 (Re_b)_x^{0.8} (Pr_b)_x^{0.4}}{(T_w/T_b)^{0.29} + 0.0019 (x/D)}$	±10% max for ~85% of data from Refs. 12, 13, and 15	Recommended tube correlation. Coeff. = 0.019 gives min. Nu	12,16, 23
5	20	$(Nu_w)_x = 0.0255 Re_w^{0.8} Pr_w^{0.4} (T_b/T_w)^{0.8}$	~±30% max	Coefficient is mean value for He and H ₂ . Almost as good with properties at t_f & coeff.=0.021	13
6	21	$(Nu_f)_x = 0.023 Re_f^{0.8} Fr_f^{0.4} (T_b/T_f)^{0.8}$	16% max, 70% within ±6%, largest deviation at $L/D < 20$	Turbulent flow ($Re_f > 3000$)	14
	22	$(Nu)_{lam.,\infty} = 9.0$	~6%	Fully developed laminar flow	
7	23	$(Nu_f)_a = 0.021 Re_f^{0.8} Fr_f^{0.4} [1 + (L/D)^{-0.7}]$	±10% max	For $Re_f \geq 10^4$	15

^aUnless otherwise noted, the deviation quoted refers only to the data of the subject study.

^bA longitudinal acoustic vibration obtained with some combinations of ϕ and G .

$$(L/D)_{\text{development}} \approx 0.037 \text{ Re}_f, \quad (24)$$

the coefficient of which is in reasonable agreement with literature values, which range from 0.029 to 0.063 depending on the boundary conditions and reference temperature.

Equation (19) of Table 2, the relation recommended here for turbulent internal duct flow, is a slightly modified version of a correlation proposed by Dalle-Donne and Bowditch,²³ who studied the heating of subsonic air flows in tubes of 0.25- and 0.50-in. internal diameter at wall temperatures up to 2300°R. Their data show that the value of n in Eq. (3), when $\psi(L/D)$ is excluded, is a function of L/D , and all their tests could be correlated within $\pm 10\%$, for $\text{Re}_b \geq 10^4$, by:

$$(\text{Nu}_b)_x = \frac{0.024 (\text{Re}_b)_x^{0.8} (\text{Pr}_b)_x^{0.4}}{(T_w/T_b)^{0.29 + 0.0056(x/D)}} \quad (25)$$

A plausible hypothesis is described²³ which relates the correlation form of the temperature factor in turbulent external flow (free stream geometry) with that in developed duct flow; the quantitative expression of the hypothesis is given by the equation:

$$\left(\frac{T_w}{T_\infty} \right)^{-0.32}_{\text{external}} = \left(\frac{T_w}{T_b} \right)^{-[0.29 + 0.0056(x/D)]}_{\text{duct}} \quad (26)$$

The modified version of Eq. (25) which best fits^{12,16} the available high-flux He and H₂ data^{12,13,15} is:

$$(\text{Nu}_b)_x = \frac{0.021 (\text{Re}_b)_x^{0.8} (\text{Pr}_b)_x^{0.4}}{(T_w/T_b)^{0.29 + 0.0019(x/D)}} \quad (19)$$

This form has been applied with excellent results to data obtained with $10 < (L/D) < 240$, $200 < T_b < 2800^\circ\text{R}$, and $1.1 < (T_w/T_b) < 8.0$. With increasing L/D , $(T_w/T_b) \rightarrow 1$, and the uncertainty of the exponent on the

temperature factor in Eq. (19) increases, since the Nusselt modulus is based on a value of $(T_w - T_b)$ which is increasingly the difference between two large numbers.

In a correlation study limited to low-temperature H_2 , Williamson and Bartlit²⁴ obtained the best degree of correlation by using a coefficient half the normal value in the Dittus-Boelter equation:

$$(Nu_b)_a = 0.0115 Re_b^{0.8} Pr_b^{0.4} \quad (27)$$

For the range $100 < T_b < 500^\circ R$, Eq. (27) correlated 70% of the data within $\pm 20\%$. The data used in the comparison included Reynolds numbers to 1.59×10^7 and heat fluxes to 4.18×10^6 Btu/hr·ft². Comparisons were made²⁴ with eight other correlations, and it was found that the modified Dalle-Donne relation, Eq. (19), gave the second best fit for the restricted data range considered.

In Fig. 1, seven equations from Table 2 and Eq. (27) are compared for the case $T_w/T_b \rightarrow 1$, $Pr = 0.7$, and $L/D = 72$. Because of the different forms of the correlations, an unequivocal comparison can be made only for the low-flux case ($T_w \approx T_b$). The recommended Eq. (19), Curve D, lies $\sim 20\%$ below the highest of the six correlations represented by the group of parallel lines A through D. Equation (16), Curve F, is based on wire-mesh data and is not directly comparable with the others.

Parametric Curves for Helium. To illustrate the interactions of heat flux, mass velocity, and bulk temperature in determining surface temperature, Fig. 2 was prepared, which shows the dependence of wall temperature on heat flux for He flowing in a heated tube of 0.5-in. diam and 3-ft length. The mass velocity G was varied between 10^4 and 10^8 lb/hr·ft², and T_b between 200 and $4000^\circ R$. For various values of wall temperature, heat fluxes were calculated from $\phi = h(T_w - T_b)$; h was derived from Eq. (19). The transport and thermodynamic properties of He were taken from the NASA curves.¹³

The properties C_p , μ , k , and Pr for helium vary only $\sim \pm 5\%$ above the mean for pressures up to ~ 6000 psia at $T_b \geq 500^\circ R$, but there are several limitations associated with Fig. 2 which should be recognized. The lower limit on G corresponds to the laminar-turbulent transition ($Re \approx 2300$);

Fig. 1. Comparison of Nu Correlations.

Fig. 2. Variation of Wall Temperature with Heat Flux for He at Various Mass Velocities and Bulk Temperatures.

or, for best applicability of Eq. (19), to $Re_b \geq 10^4$. For the pressure and temperature ranges of the calculation (i.e., $1 < P < 400$ atm and $200 < T_b < 4000^\circ R$), the minimum G at the lower limit is $\sim 10^3$. The upper limit on G is related to the sonic velocity; i.e., $G_{\max} = \rho V_s$. Though V_s for He varies sharply with T_b and only by $\sim \pm 5\%$ about the mean for $P = 1 - 400$ atm,²⁵ the associated density variation causes $G_{\max}$ to vary greatly with P and T_b . The maximum G value of 10^8 in Fig. 2 corresponds to He at the extreme conditions of 400 atm and $200^\circ R$, but the minimum value may be as small as $\sim 10^5$ within the $P - T_b$ domain considered. The third restriction on the curve shown is related to viscous dissipation as $G \rightarrow G_{\max}$ ($Ma \rightarrow 1$). This compressibility effect may be taken into account by using as the temperature potential the difference between the wall and adiabatic-wall temperatures; i.e.,

$$\phi = h(T_w - T_{aw}) \neq h(T_w - T_b) . \quad (28)$$

In addition to the alteration of the driving force as $Ma \rightarrow 1$, there is a small direct effect of Mach number on Nu and f which is best correlated²³ by Voronin's factor:²⁶

$$\beta = \left[1 + \frac{\gamma - 1}{2} Ma^2 \right]^{0.47} . \quad (29)$$

The factor β is defined by:

$$\beta \equiv \frac{(Nu \text{ or } f)_{\text{incompressible}}}{(Nu \text{ or } f)_{\text{compressible}}} . \quad (30)$$

Though developed theoretically and confirmed experimentally²⁶ for the frictional effect for gas flow in tubes, Eq. (29) has been found also to apply to heat-transfer results and many different external-flow geometries.²³

Since in Fig. 2 only the minimum lower and maximum upper limits are imposed on G , and since viscous-dissipation effects are not included, the $P - T_b - Ma$ conditions for any particular case should be reviewed to insure that $Re_b \geq 10^4$, $G < G_{\max}$, and that compressibility effects are included for Mach moduli exceeding ~ 0.5 .

Friction Factors. For adiabatic tests ($\phi = 0$) in the tubular geometries of Table 1, average friction-factor data are in good to excellent agreement with the von Kármán-Nikuradse formula²⁷ for turbulent flow in a smooth tube:

$$\frac{1}{\sqrt{f_o}} = 4 \log(Re_o \sqrt{f_o}) - 0.40, \quad (31)$$

for Reynolds numbers exceeding ~ 6000 . The NASA data from high-flux diabatic tests, however, generally fall above the predictions of Eq. (31), whether evaluated as f_b versus Re_b or f_f versus Re_f . The deviation of these data generally decreases with increasing Re — typically, from $\sim +40\%$ at $Re = 10^4$ to $\sim +10\%$ at $Re = 10^5$. This experimental finding is in contrast not only with the variation of Nu with T_w/T_b , but also with Kutateladze's correlation, Eq. (8), which predicts that $f/f_o < 1$ for $T_w/T_b > 1$. In the light of the following discussion, it appears that these friction data, taken under difficult conditions of extreme temperature, may be inaccurate. Friction data reported by Dalle-Donne and Bowditch²³ for moderate values of T_w/T_b (up to 2.5) were correlated within $\sim \pm 25\%$ for $Re_w > 4 \times 10^3$ by simply relating βf_b to Re_w :

$$f_b = \frac{0.079}{\beta Re_w^{1/4}}. \quad (32)$$

Average friction coefficients for He and H₂ at T_w/T_b to 7.5 were reported but not correlated by Rocketdyne.⁹ These diabatic data generally decreased as the temperature factor increased, a trend opposite to that shown by most of the NASA data, but in accord with the variation of Nu with T_w/T_b and with the prediction of Eq. (8). In addition, a considerable quantity of local friction data for N₂ at T_w/T_b to 7.5 which vary in the same general manner as the Rocketdyne data have been reported by Perkins and Worsoe-Schmidt.²⁸ These authors point out that their data fall well above the prediction of Eq. (8):

$$\frac{f}{f_o} = \left[\frac{2}{(T_w/T_b)^{1/2} + 1} \right]^2 \quad (8)$$

if f_o is based on Re_b , which is the manner in which Kutateladze apparently applied the correlation.²

Perkins and Worsoe-Schmidt found good correlation of their own data²⁸ and other literature data^{29,30} for air and He when f_o in Eq. (8) was evaluated at a modified wall Reynolds number defined by:

$$Re_{wm} \equiv \frac{DG}{\mu_w} \left(\frac{T_b}{T_w} \right) \quad , \quad (33)$$

which reduces to DV_b/ν_w for an ideal gas. This Re , which reflects the relatively viscous nature of the important wall flow in a heated gas stream, is considerably smaller than Re_b ; and is related to the latter, if it is assumed that $\mu \propto T^{3/4}$, by:

$$Re_{wm} = Re_b (T_b/T_w)^{7/4} \quad . \quad (34)$$

These investigators, after applying an entry correction (which is ≤ 1.1 for $x/D \geq 50$), obtained surprisingly good results using this approach for values of Re_{wm} as small as 5000. The modified expression may be written as:

$$f = (f_o)_{Re_{wm}} \left[\frac{2}{(T_w/T_b)^{1/2} + 1} \right]^2 \quad , \quad (35)$$

and may be approximated²⁸ for temperature factors between 1 and 8.0 by:

$$\frac{f}{(f_o)_{Re_{wm}}} \approx \left(\frac{T_w}{T_b} \right)^{-0.6} \quad . \quad (36)$$

The writer applied Eq. (35) to the Rocketdyne data⁹ with good results. The original data, plotted as f_b versus Re_b , fell below the isothermal friction-factor curve by an average of 35.7%. When Eq. (8) was applied, with f_o based on Re_b , the data were overcorrected and fell above the curve

by 45.3% (the same behavior shown by the data of ref. 28). Application of Eq. (35), however, brought the data into reasonably good agreement with the isothermal curve, as shown in Fig. 3, in which the average deviation of the sixteen data points about the curve is 14.2%. The insert shows the 87 points of local friction data of Perkins and Worsoe-Schmidt²⁸ correlated in the same manner.

Conclusions. For turbulent internal duct flow at large heat fluxes and temperature ratios, heat-transfer coefficients may be calculated with good accuracy by Eq. (19). For wire-mesh and parallel-plate geometries, Eqs. (16) and (21), respectively, are the most appropriate available. Estimates of heat-transfer rates for He in turbulent tube flow may be made with Fig. 2, if they are subjected to the restrictions described in the text.

Certain anomalies appear in the NASA friction data for large T_w/T_b and small turbulent Re which are in contrast with other high-temperature data and with theory. Equation (35) is recommended for computing diabatic friction factors up to a T_w/T_b ratio of at least 8.0.

Fig. 3. Correlation of High-Flux Friction Data with EQ. (35).

NOMENCLATURE

a, b, c, m, n	exponents
A	cross-sectional flow area
C_p	specific heat at constant pressure
C_v	specific heat at constant volume
C_2, C_3	multipliers in Eq. (10)
D	inside tube diameter
D_e	minimum equivalent diameter (at spacer cross section, e.g.)
D_w	wire diameter
f	friction factor, dimensionless
G	coolant mass velocity (ρV or W/A), $\text{lb/hr}\cdot\text{ft}^2$
h	heat-transfer coefficient
k	coolant thermal conductivity
K	constant coefficient
L	total heated length
Ma	Mach modulus (V/V_s), dimensionless
Nu	Nusselt modulus (hD/k), dimensionless
P_1	inlet gas pressure, psia
Pr	Prandtl modulus ($C_p\mu/k$), dimensionless
Re	Reynolds modulus (DG/μ), dimensionless
r_c	radius of curvature of centerline of tube
r_t	inside tube radius
t	temperature
T	absolute temperature
V	linear coolant velocity
W	weight flow rate
x	local length
β	Voronin factor [see Eqs. (29) and (30)]
γ	specific-heat ratio (C_p/C_v)
ϵ	porosity (void fraction) of wire mesh
μ	dynamic viscosity of coolant
ν	kinematic viscosity (μ/ρ) of coolant
ρ	coolant density
ϕ	heat flux to coolant
ψ	denotes functional dependence

NOMENCLATURE, continued

Subscripts

a	average
aw	adiabatic wall
b	bulk
f	film
lam	laminar
m	modified
max	maximum
o	constant-property (isothermal) case
Re _w	at the same modified wall Re
s	sonic
st	static
t	total
w	wall (surface)
x	local
∞	free-stream value; for fully-developed flow

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