✓

This is an informal report intended primarily for internal or limited external distribution. The opinions and conclusions stated are those of the author and may or may not be those of the laboratory.

UCID - 16643

LAWRENCE LIVERMORE LABORATORY

University of California/Livermore, California

LASER-TARGET PLASMA MODEL

A. H. Futch

November 25, 1974

NOTICE

This report was prepared as an account of work sponsored by the United States Government. Neither the United States nor the United States Atomic Energy Commission, nor any of their employees, nor any of their contractors, subcontractors, or their employees, makes any warranty, express or implied, or assumes any legal liability or responsibility for the accuracy, completeness or usefulness of any information, apparatus, product or process disclosed, or represents that its use would not infringe privately owned rights.

MASTER

Prepared for U. S. Atomic Energy Commission under contract no. W-7405-Eng-48

DISTRIBUTION OF THIS DOCUMENT IS UNLIMITED

fy

DISCLAIMER

This report was prepared as an account of work sponsored by an agency of the United States Government. Neither the United States Government nor any agency Thereof, nor any of their employees, makes any warranty, express or implied, or assumes any legal liability or responsibility for the accuracy, completeness, or usefulness of any information, apparatus, product, or process disclosed, or represents that its use would not infringe privately owned rights. Reference herein to any specific commercial product, process, or service by trade name, trademark, manufacturer, or otherwise does not necessarily constitute or imply its endorsement, recommendation, or favoring by the United States Government or any agency thereof. The views and opinions of authors expressed herein do not necessarily state or reflect those of the United States Government or any agency thereof.

DISCLAIMER

Portions of this document may be illegible in electronic image products. Images are produced from the best available original document.

LASER-TARGET PLASMA MODEL

ABSTRACT

A mathematical model describing the trapping of an energetic neutral beam on a decaying laser-target plasma has been formulated. Fokker-Planck equations for the ionized species are coupled to rate equations for the neutral gas species. The plasma radius is a variable that changes with time to maintain an equilibrium between trapping and charge-exchange losses at the plasma surface.

INTRODUCTION

A mathematical model to describe the trapping of an energetic neutral beam on a decaying laser-target plasma has been formulated. The electron and ion species in both the target plasma and the energetic trapped plasma are described by one-dimensional Fokker-Planck equations. The Livermore multispecies code has been modified¹ to include various collisional trapping terms in addition to both atomic and molecular charge-exchange terms. Separate equations are used to describe the thermal neutrals and Franck-Condon neutrals. These equations which are coupled to the Fokker-Planck equations describing the ionized species are based on a three-regional model for the neutral gas densities.

A key feature of this model is the inclusion of Fowler's criterion for charge exchange at the surface of a mirror-confined plasma which is maintained by neutral injection.² The plasma radius is regarded as a variable that changes with time in an attempt to maintain an equilibrium between trapping and charge-exchange losses at the plasma surface. The model does not assume that the dimensions of the neutral beam are small compared with plasma dimensions.

Other features of the model are similar to the United Aircraft model.³ Both molecular and atomic charge-exchange rates are computed by averaging over the ion distribution function. Other rate parameters are approximated by their average values.

DISCUSSION OF EQUATIONS

The three-region model is illustrated schematically by Fig. 1. Region I contains the plasma of density n_+ . It has a volume V , a surface area S , and a mean chord length L . We assume Region I is cylindrical. The thermal and Franck-Condon neutral densities are given by n_0 and n_{fc} , respectively. Region II is that part of the vacuum system within the baseball coil minus the plasma volume V . Region II has a volume

Fig. 1. Schematic diagram of three-region model.

V_1 , and with a wall surface area S_w and a vacuum surface area S_1 . The thermal and Franck-Condon neutral densities in Region II are given by N_0 and N_{fc} , respectively. Region III includes the volume within the magnet chamber but outside the magnetic coil. Region III has a volume V_v , and a surface area S_v . The neutral gas density in this region is assumed to consist only of cold H_2 molecules having a density N_v .

The differential equations describing the neutral gas densities in Region I are

$$\frac{dn_0}{dt} = \frac{v_0' S}{4V} (N_0 - n_0) - 2 n_0 \sum_j n_j \overline{\sigma_{ij} v_j} - n_0 \sum_j K_j \int_0^\infty F_j \sigma_{cx}^{mj} x^3 dx + S_R/V$$

$$\frac{dn_{fc}}{dt} = \frac{v_{fc} S}{4V} (N_{fc} - n_{fc}) - n_{fc} \sum_j n_j \overline{\sigma_{ij} v_j} - n_{fc} \sum_j K_j \int_0^\infty F_j \sigma_{cx}^{Aj} x^3 dx,$$

where

$x = v/v_0$, v_0 is a normalizing velocity,

v_0' = velocity of the thermal molecules,

v_{fc} = velocity of the Franck-Condon neutrals,

v_j = velocity of the ions of species j ,

σ_{ij} = cross section for ionization of neutral atoms by ions of species j ,

σ_{cx}^{mj} = cross section for charge exchange of ions of species j with thermal molecules,

σ_{cx}^{Aj} = cross section for charge exchange of ions of species j with atomic atoms,

K_j = normalizing constant described in Livermore multispecies (MS) code,

S_R = direct streaming of gas from the neutralizer,

P = constant in cross section for σ_{cx}^{Aj} .

In Region II, the equations describing the neutral gas densities are

$$\frac{dN_a}{dt} = C_1 [N_0(0) - N_0(t)] - \frac{v_0' S}{4V_1} (N_0 - n_0) + \frac{v_{fc} S_w}{8V_1} N_{fc} + \frac{I}{2V_1} \sum_j \frac{C_{0f} \sigma_{cx}^{Aj}}{\sigma_{cx}^{Aj} + \sigma_{ij}} \times \left[1 - e^{-L \sum_j (\sigma_{cx}^{Aj} + \sigma_{ij}) n_j} \right] \quad \text{(Equation continues on next page.)}$$

$$+ \frac{C_{of}}{2} \sum_j \left(n_0 K_j \int_0^\infty F_j \sigma_{cx}^{m_j} x^3 dx + n_{fc} K_j \int_0^\infty F_j \sigma_{cx}^{A_j} x^3 dx \right) \frac{V}{V_1} \\ + \frac{a_0 f \mathcal{L}_c}{V_1} + \frac{S_1 v_0'}{4 V_1} (N_v - N_0)$$

$$\frac{dN_{fc}}{dt} = \frac{V}{V_1} \left(2 n_0 \sum_j n_j \overline{\sigma_{ij} v_j} + n_0 \sum_j K_j \int_0^\infty F_j \sigma_{cx}^{m_j} x^3 dx \right) (IP) - \frac{v_{fc} S_w}{4 V_1} N_{fc} \\ - \frac{V}{V_1} \left(n_{fc} \sum_j n_j \overline{\sigma_{ij} v_j} + n_{fc} \sum_j K_j \int_0^\infty F_j \sigma_{cx}^{A_j} x^3 dx \right),$$

where

(IP) = ionization probability for Franck-Condon neutrals

$$= (1 + P_{fc})/2$$

$$= 1/2 \left\{ 1 + \left[1 + \frac{4 V}{v_{fc} S} \left(\sum_j n_j \overline{\sigma_{ij} v_j} + \sum_j K_j \int_0^\infty F_j \sigma_{cx}^{A_j} x^3 dx \right) \right]^{-1} \right\},$$

a_0 = knock-off probability produced by plasma ion in Region II,

$$c_1 = b_0 \frac{v_0' S_w}{4 V_1},$$

f = fraction of loss-cone ions intercepted by the coil,

S_w = wall area of the coil seen by the plasma,

$$\mathcal{L}_c = V \left[n_0 \sum_j n_j \overline{\sigma_{ij} v_j} + n_0 \sum_j K_j \int_0^\infty F_j \sigma_{cx}^{m_j} x^3 dx + n_{fc} \sum_j n_j \overline{\sigma_{ij} v_j} \right. \\ \left. + n_{fc} \sum_j K_j \int_0^\infty F_j \sigma_{cx}^{A_j} x^3 dx + \frac{n^2}{6.3 \times 10^9 \left(\frac{E}{10^3} \right)^{3/2}} \right].$$

The thermal gas density in Region III is given by

$$\frac{dN_v}{dt} = C_2 (N_v(0) - N_v) - C_3 (N_v - N_0) + \frac{a_1 (1-f) \mathcal{L}_c}{V_v} + \frac{C_4}{V_v} \left(1 - e^{-t/\tau_p}\right),$$

where

$$C_2 = \frac{b_v v_0'}{4} \frac{S_v}{V_v},$$

$$C_3 = \frac{v_0'}{4} \frac{S_1}{V_v},$$

$$C_4 = n_{0s}(t = \infty) C_s = \text{input number},$$

a_1 = knock-off probability produced by plasma ions in Region III,

τ_p = time constant for pumping in the source tank.

We assume that all cross sections are for either atomic or molecular hydrogen gas or an isotope of H^0 or H_2^0 . The molecular charge-exchange cross section is given by

$$\sigma_{cx}^m = \frac{1 - e^{-0.3491 \times 10^{-30} v^4}}{\frac{1}{9} \times 10^{16} + 0.0135067 \times 10^{16} \left| \ln \frac{v}{v_0''} \right| + 0.050987 (v - v_0'')^2 + dv^8} \text{ cm}^2,$$

where

$$d = 2.09231 \times 10^{-53}$$

$$v_0'' = 1 \times 10^8$$

$$\sigma_{cx}^{Aj} = \frac{0.6937 \times 10^{-14} \left[1 - 0.155 \log_{10} \left(\frac{E}{P} \right) \right]^2}{1 + 0.1112 \times 10^{-14} \left(\frac{E}{P} \right)^{3.3}} \text{ cm}^2 \quad (E \text{ in eV}),$$

where $P = 1$ for H and $P = 2$ for D. If the reaction involves both H and D nuclei, then a value of P appropriate to the species having the greater velocity should be used.

So far, the mathematical model is essentially the same as that discussed in a previous report⁴ with the exception that the radically dependent build-up equation for the ions has been replaced with Fokker-Planck equations describing each ionized species. These equations assume that both the ionized and neutral gas species are spatially constant within the plasma volume. The neutral gas densities are also assumed to be spatially constant in the region outside the plasma.

The significant modification of the previous model which we now add is to regard the plasma radius as a time-dependent variable. This requires that the plasma volume

V, the plasma surface S, and the path length L, are also time dependent. If Fowler's criterion for charge exchange at the surface is not met, then the radius is decreased until either the required condition is met or the radius is equal to the mixing length l over which rapid radial mixing of the ions occurs. If trapping does not exceed charge exchange in this minimum volume, then the plasma decays away.

A time-dependent formulation of Fowler's criterion may be written as follows

$$\frac{dN_s}{dt} = \sum_j \iint n_j(r) J(r, \theta) \sigma_{ij} r d\theta dr - \sum_j \iint n_j(r) n_0(r) \sigma_{cx}^m v r d\theta dr,$$

where N_s is the total number of ions in the volume between cylinders of radius R and (R - l) and having unit length in the axial direction. The cross section σ_{ij} is the sum of all cross sections that contribute to beam ionization. Assuming that

$$n_j(r, t) = n_j(0, t) \left(1 - \frac{r^2}{R^2}\right)$$

and

$$n_0(r) = N_0 e^{-(R-r)/\lambda_0},$$

where

$$\lambda_0^{-1} = \sum_j n_j (\sigma_{ij} + \sigma_{cx}^m) 4 \frac{v_j}{v_0}.$$

Substituting into the foregoing equation, but dropping the subscripts and summation signs for simplicity, we obtain

$$\begin{aligned} \frac{dN_s(t)}{dt} = & \iint n_+(0, t) \left(1 - \frac{r^2}{R^2}\right) J(n, \theta) \sigma_t r d\theta dr \\ & - 2\pi \int n_+(t) \left(1 - \frac{r^2}{R^2}\right) N_0 e^{-(R-r)/\lambda_0} \sigma_{cx}^m v r dr. \end{aligned}$$

Dividing by $n_+(0, t)$ gives

$$\begin{aligned} \frac{1}{n_+(0, t)} \frac{dN_s(t)}{dt} = & \int_{R-l}^R \left(1 - \frac{r^2}{R^2}\right) \left[\sigma_t \left(\int_0^{\theta_1} J d\theta + \int_{\theta_2}^{\pi} J d\theta \right) \right. \\ & \left. - 2\pi N_0 \sigma_{cx}^m v e^{-(R-r)/\lambda_0} \right] r dr, \end{aligned}$$

where

$$\theta_1 = \begin{cases} \sin^{-1} d/2r & r > d/2 \\ \pi/2 & r \leq d/2 \end{cases},$$

$$\theta_2 = \begin{cases} \pi - \sin^{-1} d/2r & r > d/2 \\ \pi/2 & r \leq d/2 \end{cases}.$$

Evaluating the last integral gives

$$\frac{1}{n_+(0,t)} \frac{dN_s(t)}{dt} = \sigma_t \int_{R-\ell}^R \left(1 - \frac{r^2}{R^2}\right) \left(\int_0^{\theta_1} J d\theta + \int_{\theta_2}^{\pi} J d\theta \right) r dr - N_0 \sigma_{cx}^m v K(R, \ell, \lambda_0),$$

where

$$K(R, \ell, \lambda_0) = 2\pi \lambda_0^2 \left\{ \left(\frac{R}{\lambda_0} - 1 \right) + \frac{\lambda_0^2}{R^2} \left(\frac{R^3}{\lambda_0^3} - \frac{3R^2}{\lambda_0^2} + \frac{6R}{\lambda_0} - 6 \right) \right. \\ \left. - e^{-\ell/\lambda_0} \left[\frac{R-\ell}{\lambda_0} - 1 + \frac{\lambda_0}{R^2} \left(\frac{(R-\ell)^3}{\lambda_0^3} - \frac{3(R-\ell)^2}{\lambda_0^2} + \frac{6(R-\ell)}{\lambda_0} - 6 \right) \right] \right\}.$$

The total number of ions in the cylindrical shell $N_s(t)$, is

$$N_s(t) = n_+(0,t) \int_0^{2\pi} \int_{R_1}^{R(t)} \left[1 - \frac{r^2}{R^2(t)} \right] r dr = 2\pi n_+(0,t) \left[\frac{R^2(t)}{4} - \frac{R_1^2}{2} + \frac{R_1^4}{4 R(t)^2} \right].$$

At any given time, $N_s(t)$ may change due to the time dependence of either $n_+(0,t)$ or $R(t)$. The time derivative of $N_s(t)$ with $n_+(0,t)$ held constant is

$$\frac{dN_s(t)}{dt} = 2\pi n_+(0,t) \left[\frac{2 R(t)}{4} \frac{dR(t)}{dt} - \frac{2 R_1^4}{4 R(t)^3} \frac{dR(t)}{dt} \right]$$

or

$$\frac{1}{n_+(0,t)} \frac{dN_s(t)}{dt} = \pi \left[R(t) - \frac{R_1^4}{R(t)^3} \right] \frac{dR(t)}{dt}.$$

If the charge exchange rate in a volume shell of thickness ℓ at the plasma surface exceeds the trapping rate, then the plasma radius must decrease until either an equilibrium radius or a minimum radius equal to the mixing length is reached. The plasma radius changes at a rate given by

$$\frac{dR}{dt} = \frac{1}{\pi n_+(0,t) \left[R - \frac{R_1^4}{R^3(t)} \right]} \frac{dN_s(t)}{dt}$$

or

$$\frac{dR}{dt} = \frac{1}{\pi \left(R - \frac{R_1^4}{R(t)^3} \right)} \left[\sigma_t \int_{R-\ell}^R \left(1 - \frac{r^2}{R^2} \right) r dr \left(\int_0^{\theta_1} J d\theta + \int_{\theta_2}^{\pi} J d\theta \right) - N_0 \sigma_{cx}^m v K(R, \ell, \lambda_0) \right].$$

If

$$\frac{1}{n_+(0,t)} \frac{dN_s(t)}{dt} > 0,$$

then the plasma may expand by radial diffusion to some maximum radius, $R_{\max}$. Assuming that each trapping or charge-exchange event moves the radius an average distance ρ , the expansion velocity is

$$v = \frac{\rho}{\tau} = \frac{\rho}{n} \frac{dn}{dt} = \frac{\rho}{(2\pi R)I} \frac{1}{n_+(0,t)} \frac{dN_s(t)}{dt} = \frac{1}{2\pi R} \frac{1}{n_+(0,t)} \frac{dN_s(t)}{dt}$$

assuming $\rho = \ell$

$$v = \frac{1}{2\pi R} \left[\sigma_t \int_{R-\ell}^R \left(1 - \frac{r^2}{R^2} \right) r dr \left(\int_0^{\theta_1} J d\theta + \int_{\theta_2}^{\pi} J d\theta \right) - N_0 \sigma_{cx}^m v K(R, \ell, \lambda_0) \right].$$

The current density is given by

$$J(r, \theta) = \alpha J e^{-\Delta z / \lambda},$$

where

$$\lambda^{-1} = \sum_j n_j (\sigma_{ij} + \sigma_{cx}^a),$$

$$\Delta z = \sqrt{R^2 - r^2 \sin^2 \theta} - r \cos \theta,$$

and α is the ratio of the beam length to the plasma length in a direction perpendicular to the polar coordinates.

REFERENCES

1. A. H. Futch, Jr., J. P. Holdren, J. Killeen, and A. A. Mirin, Plasma Phys. **14**, 211 (1972).
2. T. K. Fowler, Charge Exchange at the Surface of Mirror-Confined Plasmas, Lawrence Livermore Laboratory, Rept. UCRL-75802 (1974).
3. A. F. Haught, D. H. Polk, J. T. Woo, W. J. Fader, R. G. Tomlinson, R. A. Jong, and W. B. Ard, United Aircraft Semiannual Report COO-2227-2, E. Hartford, Conn. (July 1973).
4. A. H. Futch, Jr., W. Heckrotte, C. C. Damm, J. Killeen, and L. E. Mish, Phys. Fluids **5**, 1277 (1962).

NOTICE

"This report was prepared as an account of work sponsored by the United States Government. Neither the United States nor the United States Atomic Energy Commission, nor any of their employees, nor any of their contractors, subcontractors, or their employees, makes any warranty, express or implied, or assumes any legal liability or responsibility for the accuracy, completeness or usefulness of any information, apparatus, product or process disclosed, or represents that its use would not infringe privately-owned rights."