ANALYSIS OF THE REACTION $K^- p \rightarrow K^- \pi^- \pi^+ p$ at 40 GeV/c

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CERN-IHEP BOSON SPECTROMETER

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Abstract: The reaction $K^- p \rightarrow K^- \pi^- \pi^+ p$ has been measured at 25 and 40 GeV/c at the Serpukhov Proton Accelerator. The production cross-section at 25 and 40 GeV/c as a function of momentum transfer and $K\pi\pi$ mass is presented, and results of the partial wave analysis of the $K\pi\pi$ system yielding information about $\rho(1300)^-$, $K^*(1400)^-$ and $L(1770)^-$ mesons, are discussed.

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1. Introduction

During the period 1970-1972 the CERN-IHEP Boson Spectrometer has been operating in a 25/40 GeV/c negative beam of the Serpukhov Proton Accelerator. Data on the inclusive reactions $\pi^- p \rightarrow X^- p^{1/}$ and $K^- p \rightarrow X_K^- p^{2/}$, as well as particular decay modes of the X systems such as $X^- \rightarrow (3\pi)^- \frac{3,4/}{5/}$ and $X_K^- \rightarrow (K\pi)^- \frac{5/}{5/}$ have been published. Here we present results about the $K^- \pi^- \pi^+$ decay mode for X_K^- masses between 1.0 and 2.0 GeV in the momentum transfer interval between 0.05-0.60 (GeV/c)².

2. Measurement, Selection and Identification of the $K^- p \rightarrow K^- \pi^- \pi^+ p$ Events

Apart from 4 Cherenkov counters in the beam, used for the identification of the incident K-mesons, and a slightly different trigger^{5/}, to select a wider momentum transfer interval to the proton, the apparatus and event selection are identical to those used in the analysis of the reaction $\pi^- p \rightarrow \pi^- \pi^- \pi^+ p$. We therefore discuss only briefly the apparatus here, and refer to previous publications^{3,4/} for details about the experimental setup and a discussion of the experimental resolutions.

The trigger demands an incident K-meson interacting inside the liquid hydrogen target, thereby producing a slow recoil proton detected in the proton telescope. Events that satisfy the following criteria are selected in the off-line analysis:

- (a) an incident particle, identified as a K-meson in the four gas threshold Cherenkov counters and measured by the beam hodoscopes. About 2% of the beam particles are K-mesons; the π -contamination in the selected K-sample is below 2%.
- (b) a proton, momentum analyzed by time-of-flight and range, measured by the two wire chambers of the proton telescope. Events with proton momenta above 220 MeV/c are accepted.

- (c) three tracks measured in the forward spectrometer, originating from the intersection of beam track and proton track inside the liquid hydrogen target. At least two of them have to be measured in the spark chambers behind the magnet and are thus momentum analyzed.
- (d) the three constraints of momentum conservation are used to perform a kinematic fit and test if there are only four particles in the final state. Events with a χ^2 probability exceeding 0.02 are accepted.
- (e) We do not use a Cherenkov counter to identify the outgoing particles. We have examined carefully the problem, introduced by the lack of direct determination of which negative outgoing track is the K^- and

which is the π^- . This two-fold ambiguity can be resolved - in principle - by kinematic fitting. Basically one can choose the correct hypothesis ($A = K^-$ or $B = K^-$) by comparing $\epsilon = (E - p_{\perp})_{out} - (E - p_{\perp})_{in}$ for the two hypotheses, provided the measurement error, $\delta\epsilon$, on ϵ , is small compared to the difference in ϵ , $\Delta\epsilon = |\epsilon_A - \epsilon_B|$, between the two assignments[†]. Fig. 1a shows the distribution in $\delta\epsilon$ in our experiment by displaying the distribution in ϵ for the reaction $\bar{p}p \rightarrow \bar{p}\pi^+\pi^0 p$.^{††} Fig. 1b shows the distribution in $\Delta\epsilon$ for all $K\pi\pi$ events at 40 GeV.

We have adopted the procedure of removing from the analysis events with $\Delta\epsilon < 3$ MeV, and to assign the remaining events to the hypothesis with the smaller value of $|\epsilon|$. The effect of the cut is easily taken into account in fitting the data. (see Section 3). The effect of the remaining misidentified events, is less easy to ascertain. We will

[†] If both particles are relativistic $\Delta\epsilon \doteq \frac{1}{2}(m_K^2 - m_\pi^2) \left| \frac{1}{p_A} - \frac{1}{p_B} \right|$. The error, $\delta\epsilon$, is due essentially to multiple scattering of the recoil proton: $\delta\epsilon \doteq p_R \delta\theta_R$.

^{††} For this reaction, there are practically no ambiguities, because of the large M^2 difference between $\bar{p}$ and π^- . In the case of $\bar{p}p \rightarrow \bar{p}\pi^+\pi^- p$, the ambiguity is small, since $\Delta\epsilon$ is typically 10 MeV.

return to this question in Section 4d.

- (f) For events of the type $KK\bar{K}p$ ($K\bar{K}pp$) interpreted as $K\pi\pi p$ events, the $(E-p_L)$ balance is shifted by at least 11(45) MeV. These events are thus easily removed by the requirement $|\epsilon| < 7.5$ MeV.

The number of events thus selected are shown in Table I.

3. The Partial Wave Method

The partial-wave method is used to extract the physical information from the data. The method is described in detail elsewhere^{6/}; here we only show Fig. 2 as a pictorial description of the method. The unknown parameters of our model are the elements of the $K\pi\pi$ density matrix, which measure the intensities of and interferences between different J^P_M states of the $K\pi\pi$ system (spin, parity, 3rd component of spin relative to Gottfried Jackson axes), and the complex coupling constants of the different decay modes of the $K\pi\pi$ states. The $(K\pi)^0$ systems of spin S_1 and the $(\pi\pi)^0$ systems of spin S_2 are parametrized by the known resonances (see Table II). Interactions between the K^- and the π^- are neglected. We use the maximum likelihood method to determine the unknown parameters for the different s , t and $m_{K\pi\pi}$ -bins from the data, taking into account both the acceptance and inefficiency of the spectrometer as well as the "inefficiency" due to the removal of ambiguous events. This is done by calculating the normalization integral of the likelihood function by Monte Carlo techniques, weighting every event by the acceptance of the apparatus and removing events with a ΔE (see 2e) of less than 3 MeV. By maximizing the likelihood function we obtain the best estimates of the model as well as the acceptance for the s , t , and $m_{K\pi\pi}$ intervals investigated. Fig. 3 shows the acceptance of the spectrometer and the ambiguity-cut as a function of $K\pi\pi$ mass at 40 GeV/c.

4. Results

(a) Effective mass spectra:

We will show only histograms of the 40 GeV/c data, the 25 GeV/c data

being of low statistical significance. Fig. 4 shows the measured $K\pi\pi$ mass distribution for all events, and the "unambiguous" events (i.e. $\Delta E > 3 \text{ MeV}$) in the momentum transfer interval $0.05 - 0.60 \text{ (GeV/c)}^2$. The 1.2-1.4 GeV region is dominated by a broad bump known as the Q-bump^{8/}; around 1.75 there is indication for the L-bump^{8/}. The $K^-\pi^+$ spectrum of Fig. 5a) is dominated by the $K^*(890)$ and shows indication of a weak $K^{**}(1420)$. There is a significant, but weak ρ^0 signal in the $\pi^+\pi^-$ spectrum of Fig. 5b). Comparing the $\pi^+\pi^-$ distribution for ambiguous and unambiguous events we see that the $K\pi$ ambiguity is most important in the ρ region: misidentified events with $(K\pi)^0$ mass in the K^* region generates a peak in the $(\pi\pi)^0$ system in the ρ region.

(b) Cross section for the reaction $K^-p \rightarrow K^-\pi^+\pi^-p$ vs. momentum transfer and $K\pi\pi$ mass.

In the momentum transfer interval accepted by the apparatus ($0.05 < |t| < 0.60 \text{ (GeV/c)}^2$) the differential cross section is well described by the function:

$$\frac{d\sigma}{d|t|} (m_{K\pi\pi}, t) = A(m_{K\pi\pi}) \exp(b(m_{K\pi\pi})|t|)$$

Table III shows the dependence of the slope parameter b on $K\pi\pi$ mass for the data at 25 and 40 GeV/c. The value of b decreases smoothly from about 14 (GeV/c)^{-2} at 1.2 GeV to about 7 (GeV/c)^{-2} at 2.2 GeV. The values found at 25 and 40 GeV/c are consistent within their measurement error.

Assuming that the $\exp(bt)$ dependence can be extrapolated, we obtain the integrated cross section $\sigma(m_{K\pi\pi})$ which is shown in Table III.

(c) Partial wave decomposition of the $K\pi\pi$ system

The major problem of the partial wave analysis of our data is that there are many unknown parameters to fit a rather modest number of events.

Having found a remarkable similarity between our $K\pi\pi p$ data and our $\pi\pi\pi p$ data^{3,4/}, we profit from our experience of the analysis of the 3π

system when choosing the states which we include in the partial wave analysis. Table IV lists the hypothesis used.

We include in our analysis a non-interfering phase space term ("FLAT") hoping that it might indicate the omission of important states in the analysis. The two S-wave decay amplitudes of the 0^- state, $0^-(S \rightarrow K\epsilon)$ and $0^-(S \rightarrow K\pi)$, are so similar in all five decay variables, (i.e. essentially flat) that they cannot be determined separately from the data. We therefore exclude the wave $0^-(S \rightarrow K\pi)$ from the hypothesis.

Fig. 6 shows the $m_{K\pi\pi}$ dependence of the different spin parity states. The error bars reflect not only the statistical uncertainty but also the effect on the solution due to the inclusion of additional states in the analysis. The phase space term "FLAT" is consistent with 0 in all $m_{K\pi\pi}$ intervals. The Q-region (1.2-1.5 GeV) is dominated by the 1^+ state, which is mainly $M = 0$ in the Gottfried Jackson system. The 2^+ , $M = 1$ state shows a small enhancement between 1.4 and 1.5 GeV - an indication for production of the $K^{**}(1420)$ meson. In the same mass bin the 0^- state shows an enhancement. The big error bars on the 0^- intensity reflect the variation of the number of events in this state when additional states are allowed in the analysis. Above 1.5 GeV the 2^- state shows the biggest enhancement, suggesting that a 2^- wave is responsible for the L-enhancement. However, we do not have sufficient statistics in this mass region to investigate the influence of higher partial waves.

(d) The Q-system

Next we investigate the 1^+ state below 1.5 GeV. In the partial wave fit we allow for two different polarizations and four different decay modes of the 1^+ state (see Table IV). Their intensities (for $M = 0$) as a function of $K\pi\pi$ mass are shown in Fig. 7. The $K^*\pi$ S-wave decay mode dominates the Q-region. We remark that the relative intensity $1^+(S \rightarrow K\rho)/1^+(S \rightarrow K^*\pi)$ is quite sensitive to the removal of ambiguous events. In the $K\pi\pi$ mass interval 1.2-1.4 GeV we find a ratio of

0.25±0.05 if none of the ambiguous events are removed and 0.13±0.05 if all events with $\Delta E < 3$ MeV (see 2e) are removed.

Due to the limited number of measured events we cannot afford a wider ΔE -cut. To investigate the influence of the ambiguous events further we generated Monte Carlo events according to the results of the likelihood fit, faked the experimental resolution, and assigned the $K\pi\pi$ hypothesis to the Monte Carlo events in the same way as described in 2(a) for the measured events. The partial wave analysis of the Monte Carlo data in the mass interval $m_{K\pi\pi} = 1.2-1.4$ GeV with a ΔE cut of 3 MeV yields 0.21±0.03 for the intensity ratio $1^+(S \rightarrow K\rho)/1^+(S \rightarrow K^*\pi)$. This is significantly bigger than the value of 0.13 used as input for the Monte Carlo generation. We thus estimate a branching ratio

$$BR \left(\frac{1^+(S \rightarrow K\rho)}{1^+(S \rightarrow K^*\pi)} \right) = 0.05 \pm 0.10$$

for the reaction $K^- p \rightarrow K^- \pi^- \pi^+ p$ at 40 GeV/c in the $K\pi\pi$ -interval 1.2-1.4 GeV. We thus conclude that our data are compatible with zero genuine $1^+(S \rightarrow K\rho)$ events, and that the fraction of $1^+(S \rightarrow K\rho)$ events obtained from the partial wave analysis can be explained by ambiguous events with $\Delta E > 3$ MeV. We were surprised to find within 4° the same relative phase between $1^+(S \rightarrow K^*\pi)$ and $1^+(S \rightarrow K\rho)$ waves from the partial wave analysis of the real data and the Monte Carlo events. This suggests that this relative phase, which is well determined by the likelihood fit, just results from the wrongly assigned $K\pi\pi$ events.

In order to find out if the $1^+(S \rightarrow K^*\pi)$ peak corresponds to a resonance we investigate its interference phase with several other partial waves as a function of $K\pi\pi$ mass. In Fig. 8 we have selected interference phases which seem to be determined with reasonable accuracy and do not change significantly when other small states are included in the hypothesis.

For comparison with other experiments, we also included the interference phase with the $1^+(S \rightarrow K\rho)$ wave although we attach no physical meaning to it. For a $1^+(S \rightarrow K^*\pi)$ resonance we expect all interference phases to increase by 90° over the width of the $1^+(S \rightarrow K^*\pi)$ peak. As all the phases are constant within the big experimental uncertainties we conclude that the $1^+(S \rightarrow K^*\pi)$ wave does not "resonate" in the Q-region. We have also measured the polarization density matrix of the 1^+ state[†] in the Gottfried-Jackson system for the mass region 1.1-1.5 GeV and the momentum transfer interval $0.05-0.6 \text{ (GeV/c)}^2$. The results are shown in Table V for four mass bins between 1.1 and 1.5 GeV. The $M = 1$ component is small but significant and within the errors independent of $K\pi\pi$ mass. The table also lists the values of the density matrix element ρ_{01} in the Gottfried-Jackson system assuming helicity conservation in the s-channel system. The polarization density matrix is very similar to that of the 1^+ state measured at the same energy in the reaction $\pi p \rightarrow \pi^-\pi^-\pi^+ p$ ^{3,4/1}.

We finally compare in Figs. 9 and 10 several distributions of the data in the mass interval 1.2-1.4 GeV with the result of the likelihood fit. The agreement is satisfactory. For the $1^+(S \rightarrow K^*\pi)$ wave in the interval 1.2-1.4 GeV and the momentum transfer interval $0.05-0.60 \text{ (GeV/c)}^2$ we measure a cross section

$$\Delta\sigma = 26 \pm 3 \text{ (5)} \mu\text{b} \quad \text{at } 40 \text{ GeV/c.}$$

The estimated systematic error is given in parentheses.

(e) The $K^{**}(1420)$ -system

The 2^+ wave^{††} shows a narrow peak around 1.4 GeV (see Fig. 6) which we

[†] The same polarization is assumed for all decay modes of the 1^+ state in this analysis.

^{††} We have assumed that the 2^+ -wave is in a state $|2^+1\rangle + |2^+-1\rangle$ similar to the 2^+ -wave (A_2) in the reaction $\pi p \rightarrow \pi\pi\pi p$ at 25 and 40 GeV/c.^{3,4/} [See Table IV]

interpret as $K^{**}(1420)$ signal. In Fig. 11 we show the mass dependence of the two decay modes of the 2^+ state which we assumed in the partial wave fit ($2^+(D \rightarrow K^*\pi)$, $2^+(D \rightarrow K\rho)$). The entire signal is due to the $(K^*\pi)$ decay mode.

From a fit in the mass interval 1.30-1.54 GeV and the momentum transfer interval 0.05-0.55 (GeV/c)² we obtain the number of 2^+ events displayed in Table VI. The corresponding cross sections, as well as the cross section for $K^{**}(1420)$ production^{5/} in the reaction $K^-p \rightarrow (K\pi)^-p$ at the same energy and momentum transfer interval, are also given. From this we derive the branching ratios:

$$BR\left(\frac{K^{**}(1420)^- \rightarrow (K^*\pi)^-}{K^{**}(1420)^- \rightarrow (K\pi)^-}\right) = 0.65 \pm 0.13 \quad (0.12)$$

$$BR\left(\frac{K^{**}(1420)^- \rightarrow (K\rho)^-}{K^{**}(1420)^- \rightarrow (K^*\pi)^-}\right) = 0.03 \pm 0.03$$

The errors in parentheses are the estimated systematic errors, the corresponding values from the particle data table^{8/} are 0.54 ± 0.06 and 0.31 ± 0.10 . The small number of events does not permit a determination of the polarization density matrix, nor a measurement of the mass dependence of the phase of the $K^{**}(1420)$ propagator. For the relative phase of $K^{**}(1420)$ wave to $1^+(S \rightarrow K^*\pi)$ wave we find $(+10 \pm 35)^\circ$ at $m_{K\pi\pi} = 1420$ MeV.

(f) The L-system

The L appears as a shoulder at about 1.8 GeV in the $K\pi\pi$ mass spectrum of Fig. 4. Selecting events[†] in the $K\pi\pi$ mass interval 1.6-1.9 GeV yields the $m_{K^-\pi^+}$ and $m_{\pi^+\pi^-}$ spectra of Fig. 12. There is evidence for strong $K^*(890)$ and $K^{**}(1420)$ production as well as for possible $\rho(765)$ production. Fig. 13

[†] Here we do not cut out ambiguous events, we just choose the hypotheses which yield the smaller δE .

shows the $K^{**}(1420) \pi^-$ effective mass spectrum ($1.32 < m_{K\pi} < 1.52$ GeV). A prominent enhancement is seen, centered at about 1.75 GeV with a width of about 0.3 GeV. Fig. 14 shows for events in the L-region ($m_{K\pi\pi} = 1.6-1.9$ GeV) the distribution of $\cos\theta_{\pi^+}$, where θ_{π^+} is the polar angle of the π^+ from the decay of the $K^{**}(1420)$. The angular distribution of Fig. 14 suggests that the L enhancement is mainly spin parity 2^- with spin projection $M = 0$ in the Gottfried-Jackson system and S-wave decay into $(K^{**} \pi^-)$. The angle is measured in the $K^{**}(1420)$ -frame from the $K\pi\pi$ -Gottfried-Jackson axes. For a S-wave decay of the $K\pi\pi$ system into $K^{**}\pi$ with polarization $M = 0$, we expect the distribution $P_2(\cos\theta_{\pi^+})^2$. The data follow such a distribution approximately.

Similar results are obtained from the partial wave analysis. Fig. 6 suggests that the 2^- state is responsible for the L-enhancement. Fig. 15 shows different decay modes of the 2^- state. The $(K^{**}\pi)$ -decay mode peaks in the L-region, but with the available statistics it is impossible to prove or disprove other decay modes, nor to measure polarization or phase of the propagator of the 2^- state.

This analysis yields a cross-section of

$$\Delta\sigma = 6 \pm 1 (1) \mu\text{b}$$

for the reaction $K^- p \rightarrow L^- p$

at 40 GeV/c for the mass interval 1.6-1.9 GeV and the momentum transfer interval $0.05-0.60$ (GeV/c)², where the L is defined as the 2^- state.

5. Discussion of the Results and Conclusions

(a) Comparison with the reaction $\pi^- p \rightarrow \pi^- \pi^- \pi^+ p$ at 40 GeV/c

The low mass 3π system produced at 40 GeV/c in $\pi^- p \rightarrow \pi^- \pi^- \pi^+ p$ has been measured in the same experiment^{3,4/}. The reactions $Kp \rightarrow K\pi\pi p$ and

$\pi p \rightarrow \pi\pi\pi p$ show many similarities, which we will discuss in this section: Apart from a mass shift of the order of $K^*(890)$ and $\rho(765)$ mass difference, 3π and $K\pi\pi$ mass spectra look very similar: practically no events below $\rho\pi(K^*\pi)$ threshold, a sharp rise to a peak about 300 MeV above this threshold with a width of 300 MeV, then another, less important, peak about 200 MeV above $f(1265)\pi$ ($K^{**}(1420)\pi$) threshold with 300 MeV width. Both reactions are dominated by resonances (ρ , f and K^* , K^{**}) in the neutral two meson systems.[†] The close similarity between the two reactions extends to the decay distribution of the three meson system. Thus it is not too surprising that the partial wave analysis yields very similar results for both reactions: the low mass peak has mainly spin parity 1^+ with S-wave decay into $\rho\pi(K^*\pi)$ and polarization $M = 0$ in the Gottfried-Jackson system. Other states like 0^- and 2^- are clearly present, but significantly weaker. In both channels there is good evidence that the 1^+ states do not have the rapid phase-change characteristic of a Breit Wigner resonance. At the upper edge of the 1^+ peak a resonance is found in the 2^+ wave ($A_2(1315)$ and $K^{**}(1420)$). The main decay mode of the A_2 is D-wave into $\rho\pi$. The K^{**} has other dominant decay modes besides D-wave into $K^*\pi$. Therefore the 2^+ resonance appears stronger in the 3π system than in the $K\pi\pi$ system. The Breit Wigner resonance behavior of the A_2 is well established from measurements of the A_2 propagator phase in the 3π decay mode and the appearance of an A_2 signal in several different decay modes; for the K^{**} the statistics of this experiment are too low for measuring the propagator phase, and its resonance character is

[†] As there are two identical π mesons in the 3π reaction, there are two $\pi^+\pi^-$ mass combinations per event. The dominance of the ρ is therefore not immediately obvious from the $\pi^+\pi^-$ mass distributions.

deduced from the presence of different decay modes. The peaks at higher masses than the 2^+ resonances have spin parity 2^- and are called $A_3(1640)$ and $L(1770)$ respectively. In the 3π experiment we have sufficient events to establish that the A_3 is a non-resonating $f\pi$ threshold effect, with lower statistics the data in the $K\pi\pi$ channel suggest that the L might be similarly a $K^{**}\pi$ threshold effect.

The strong correlation between slope of the differential cross-section and 3-meson mass is another similarity between production of 3π and $K\pi\pi$ systems at high energy. At $\rho\pi(K^*\pi)$ threshold a slope of about $14(\text{GeV}/c)^{-2}$, significantly steeper than the slope of elastic scattering is found, at higher masses the slope decreases towards a value of 5 to 7 $(\text{GeV}/c)^{-2}$.

Finally we remark that the production cross-sections for low mass 3π and $K\pi\pi$ systems are of the same order of magnitude. More quantitatively, at 40 GeV/c we find an integrated production cross-section for a 200 MeV mass band at the $A_1(Q)$ peak of:[†]

$$\begin{array}{ll} m_{3\pi} = 1.1-1.3 \text{ GeV} & \sigma_{3\pi} = 128 \pm 3 \text{ (13)} \mu\text{b} \\ m_{K\pi\pi} = 1.2-1.4 \text{ GeV} & \sigma_{K\pi\pi} = 97 \pm 4 \text{ (10)} \mu\text{b} \end{array}$$

The ratio $\sigma_{3\pi}/\sigma_{K\pi\pi} = 1.3 \pm 0.2$ is consistent with the ratio of the total inelastic cross-sections^{9/} measured at 40 GeV/c.

$$\frac{\sigma_{\pi p}(\text{inelastic})}{\sigma_{Kp}(\text{inelastic})} = \frac{(24.36 \pm 0.07 - 3.32 \pm 0.06) \text{ mb}}{(20.49 \pm 0.08 - 2.33 \pm 0.03) \text{ mb}} = 1.16 \pm 0.01$$

(b) Comparison with results from other experiments on $Kp \rightarrow K\pi\pi p$

In this section we compare our results with results from bubble chamber experiments^{7,10,11/} on the reactions $K^\pm p \rightarrow K^\pm \pi^+ \pi^- p$ and $K^\pm p \rightarrow K^0 \pi^\pm \pi^0 p$.

Only experiments at high energy ($p_{\text{inc}} \gtrsim 9 \text{ GeV}/c$) and with high statistics are considered.

Shape of $K\pi\pi$ mass spectrum

All high statistics experiments agree that the $K\pi\pi$ spectrum in the Q

[†] Both statistical and systematic error (in parentheses) are quoted.

region (1.2-1.5 GeV) cannot be described by a single Breit Wigner resonance above a smooth background. This is also clear from the flat top of the Q-peak in the $K\pi\pi$ spectrum of this experiment (Fig. 4).

There are experiments which find a more or less significant substructure in the Q-peak, others have a smooth shape similar to the mass spectrum of this experiment. Two Breit Wigner resonances with parameters $m_1 = 1250$ MeV, $\Gamma_1 \approx 100$ MeV and $m_2 = 1400$ MeV and $\Gamma \approx 140$ MeV above a smooth background seem to fit all spectra well^{12/}.

There is also general agreement on a peak in the L-region ($m_{K\pi\pi} = 1.6-1.9$ GeV). Mass values around 1760 MeV and values for the width between 60 and 250 MeV are quoted^{8/}. Our experiment favors a large width, about 300 MeV.

Spin Parity of the Q-region

Most of the analyses done so far investigated either moments of the distribution in the decay angles of the $K\pi\pi$ system or the distribution of events over the $K\pi\pi$ Dalitz plot. There is general agreement that spin parity 1^+ with polarization $M = 0$ in the Gottfried Jackson system is dominant. In addition, there is evidence for $M \neq 0$ components of the 1^+ state and possibly for other states like 0^- , 2^- and 2^+ . The authors of Ref. 7 made a complete partial wave analysis of the $K\pi\pi$ system fitting simultaneously the distribution of the events in all five decay variables. Their results on $K^-p \rightarrow K^- \pi^- \pi^+ p$ at 10 and 16 GeV/c are very similar to our results at 40 GeV/c: the $1^+ M = 0$ state dominates; next in importance is the 0^- state which seems to peak around 1.4 GeV and the 2^- state. The $M = 1$ component of the 1^+ state is weak but significant (see Table V). Around 1.4 GeV this experiment finds an indication of the $K^{**}(1420)$.

The results of the partial wave analysis thus suggest that the flat-top shape of the $K\pi\pi$ mass spectrum is due to a 1^+ peak ($m \approx 1300$, $\Gamma \approx 300$ MeV), a 0^- peak ($m \approx 1450$ MeV, $\Gamma \approx 100$ MeV) and the 2^+ peak of the $K^{**}(1420)$ meson and not due to a double structure of the 1^+ state.

Decay analysis of the 1^+ state in the Q-region

Previous analysis of the $K\pi\pi$ final state show a wide range of results for the branching ratio for $1^+(S \rightarrow K\rho)/1^+(S \rightarrow K^*\pi)$ in the final state $K^\pm \pi^\mp \pi^\pm$. We believe that this is due in part to the effect of misidentified events ($K \rightarrow \pi$ ambiguity) and in part to simplifying assumptions about spin-parity states and decay modes contributing to the Q-region. If one disregards the $K^\pm \pi^\mp \pi^\pm$ data in these analyses and infers the ratio in the $K^\pm \pi^\mp \pi^\pm$ channel from the results obtained by the same authors from the $K^0 \pi^0 \pi^\pm$ channel, which is free from the $K\pi$ ambiguity, one concludes that a $(K\rho/K^*\pi)$ intensity ratio of 0.15 ± 0.05 is in reasonable agreement with the data. Our results at 40 GeV/c suggest a somewhat lower intensity ratio (0.05 ± 0.10), and are compatible with zero $K^0 \rho^0$ contribution to the 1^+ state. In view of the above we suggest that present data, including our own, are insufficient to conclude very much about a possible sub-structure of the 1^+ state in the Q-region from the variation of the $(K\rho/K^*\pi)$ -ratio and even less from the $(K\rho/K^*\pi)$ relative phase. In particular, we see no experimental evidence for the existence of two 1^+ resonances in the Q-region^{13/}.

In addition to interference phases between partial waves with the same spin parity, the partial wave analysis also allows one to measure interference phases between states of different spin parity. The interference phase of the $1^+(S \rightarrow K^*\pi)$ state, with the $2^-(P \rightarrow K^*\pi)$ wave, which is reasonably well determined experimentally, is independent of $K\pi\pi$ mass within experimental errors. This suggests that the $1^+(S \rightarrow K^*\pi)$ peak in the Q-region is not a simple Breit Wigner resonance.

$K^{**}(1420)$ meson

This experiment has determined the branching ratio[†]

[†] The errors given the statistical and the systematic in parentheses.

$$\frac{K^{**}(1420) \rightarrow (K^* \pi)^-}{K^{**}(1420) \rightarrow (K\pi)^-} = 0.65 \pm 0.13 \text{ } (\pm 0.12) \text{ at } 40 \text{ GeV/c.}$$

This value agrees with the branching ratio measured at lower energy^{8/}, thus confirming the results of Ref. 5 that the production cross section $\sigma(K^- p \rightarrow K^{**}(1420)p)$ decreases like $p_{inc}^{-(0.8 \pm 0.2)}$. No evidence for the decay $K^{**}(1420) \rightarrow K_0$ has been found.

L(1770) region

The main conclusion about the $1.6-1.9 \text{ m}_{K\pi\pi}$ interval which can be drawn from this experiment is that many partial spin parity states with several decay modes contribute significantly, and that an order of magnitude more statistics is needed to disentangle the different partial waves. Within statistics, the data are compatible with spin parity 2^- and dominant decay mode $K^{**}(1420)\pi$ S-wave for the peak in the L-region, in analogy to the $f\pi$ threshold enhancement in the 3π reaction (A_3).

(c) Conclusions

The spin parity analysis of the $K\pi\pi$ system measured in the reaction $K^- p \rightarrow K^- \pi^- \pi^+ p$ at 40 GeV/c yields the following results:

- (i) The Q region is dominated by a single 1^+ peak with $K^*\pi$ S-wave decay, no evidence for the K_0 decay mode is found.
- (ii) The phase of the $1^+(S \rightarrow K^*\pi)$ wave in the Q-region is independent of $K\pi\pi$ mass, suggesting that the Q is no ordinary Breit Wigner resonance.
- (iii) The polarization of the 1^+ state in the Q-region determined in the Gottfried Jackson system is mainly $M = 0$, the $M = 1$ component being small but significant.
- (iv) Evidence for the $K^*\pi$ decay mode of the $K^{**}(1420)$ meson is found at 40 GeV/c, confirming the $p_{inc}^{-0.8 \pm 0.2}$ decrease of the production cross section with beam momentum.

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Table I.

Statistics of the events used in the analysis

Running Period	Oct. 1971	Oct. 1971	Jan 1972	April 1972	April 1972
Incident momentum [GeV/c]	40	25	40	40	25
Number of raw triggers	81,500	21,000	120,000	180,000	100,000
Number of $K\pi\pi$ events	1,662	485	2,489	3,345	2,196
Number of inc. K-mesons	$1.59 \cdot 10^8$	$0.42 \cdot 10^8$	$2.34 \cdot 10^8$	$3.92 \cdot 10^8$	$1.91 \cdot 10^8$

Table II

Parametrization of $K^+\pi^-$ ($\pi^+\pi^-$) amplitudes

$$A_{S_j}(s_j) = (m_{S_j}^2 - s_j - im_{S_j}\gamma_{S_j})^{-1}$$

$$\gamma_{S_j}(s_j) = \Gamma_{S_j} \left(\frac{q_j}{q_0} \right)^{2S_j+1} \frac{m_{S_j}}{\sqrt{s_j}}$$

j	particle system	S_j	name of resonance	m_{S_j} [MeV]	Γ_{S_j} [MeV]
1	$(K\pi)^0$	0	κ	1320	600
		1	$K^*(890)$	892	50
		2	$K^{**}(1420)$	1420	110
2	$(\pi\pi)^0$	0	ϵ	765	400
		1	ρ	765	135
		2	f	1269	154

$\sqrt{s_j}$ mass of the particle system j

q_j center-of-mass momentum of the decay particles of system j in the j-center-of-mass frame

q_0 q_j for $\sqrt{s_j} = m_j$

Table III

Slope b of differential cross section and integrated cross section σ for $K^- p \rightarrow K^- \pi^- \pi^+ p$; systematic errors are given in parenthesis

$m_{K\pi\pi}$ [GeV]	<u>25 GeV/c</u>		<u>40 GeV/c</u>	
	b [(GeV/c) ⁻²]	$\sigma^\dagger$ [μ b]	b [(GeV/c) ⁻²]	σ [μ b]
1.0-1.2	15.3 $\pm$ 1.2	-	13.2 $\pm$ 0.6	30 $\pm$ 2(3)
1.2-1.4	10.4 $\pm$ 0.4	119 $\pm$ 5(12)	10.7 $\pm$ 0.3	97 $\pm$ 4(10)
1.4-1.6	8.0 $\pm$ 0.5	69 $\pm$ 4(7)	8.8 $\pm$ 0.4	61 $\pm$ 3(6)
1.6-2.0	8.4 $\pm$ 0.5	-	7.8 $\pm$ 0.5	65 $\pm$ 3(7)
2.0-2.4	5.7 $\pm$ 1.2	-	8.2 $\pm$ 0.5	40 $\pm$ 3(8)

[†] In some of the intervals the statistics are too low to perform the partial wave analysis and determine the acceptance factor.

Table IV

Partial waves included in the analysis of the
K $\pi\pi$ system

State $J^P(l)M$	$m_{K\pi\pi}$	interval
	1.1-1.6	1.6-2.0
FLAT	X	X
$0^- (S \rightarrow K\epsilon / P \rightarrow K^*\pi / P \rightarrow K\rho) 0$	X	X
$1^+ (S \rightarrow K^*\pi / S \rightarrow K\rho / P \rightarrow K\pi / P \rightarrow K\epsilon) 0$	X	X
$1^+ (S \rightarrow K^*\pi / S \rightarrow K\rho / P \rightarrow K\pi / P \rightarrow K\epsilon) 1$	X	
$2^- (P \rightarrow K^*\pi / P \rightarrow K\rho) 0$	X	X
$2^- (S \rightarrow K^*\pi / S \rightarrow Kf / P \rightarrow K^*\pi / P \rightarrow K\rho) 0$		X
$2^+ (D \rightarrow K^*\pi / D \rightarrow K\rho) 1$	X	X

Partial waves in the same line are assumed to be coherent; the 3rd component of spin M refers to Gottfried Jackson axes; for the $M = 1$ states only the natural parity exchange contribution^{3/} is considered.

Table V

Polarization density matrix of the 1^+ state in the
Gottfried Jackson system at 40 GeV/c ($\Delta t = 0.05-0.60 \text{ (GeV/c)}^2$)

		$m_{K\pi\pi} \text{ [GeV]}$			
		1.1-1.2	1.2-1.3	1.3-1.4	1.4-1.5
Natural	ρ_{00}	0.90 ± 0.06	0.84 ± 0.04	0.90 ± 0.04	0.90 ± 0.05
Parity	$\rho_{11} - \rho_{1-1}$	0.09 ± 0.06	0.15 ± 0.04	0.08 ± 0.04	0.10 ± 0.05
Exchange	$\text{Re}(\rho_{01})$	-0.14 ± 0.04	-0.19 ± 0.03	-0.14 ± 0.03	-0.12 ± 0.03
	$\text{Im}(\rho_{01})$	-0.05 ± 0.15	-0.08 ± 0.10	0.06 ± 0.06	0.01 ± 0.10
Unnatural					
Parity	$\rho_{11} + \rho_{1-1}$	0.01 ± 0.06	0.01 ± 0.03	0.02 ± 0.03	0.00 ± 0.05
Exchange					
Value of $\text{Re}(\rho_{01})$ expected for helicity conservation in the s-channel		-0.35	-0.35	-0.34	-0.33

Table VI

Cross section and number of events for the $K\pi\pi$ decay modes of the $K^{**}(1420)$ measured at 40 GeV/c and $|\Delta t| = 0.05-0.55 (\text{GeV}/c)^2$ for $1.30 < m_{K\pi\pi} < 1.54 \text{ GeV}$.

Reaction	No. of events	cross section [†] (μb)
$K^- p \rightarrow K^{*0} \pi^- p$ $\downarrow$ $\rightarrow K^- \pi^+$	222 $\pm$ 40	4.1 $\pm$ 0.7 (+0.4)
$K^- p \rightarrow K^- \rho^0 p$ $\downarrow$ $\rightarrow \pi^+ \pi^-$	10 $\pm$ 10	0.1 $\pm$ 0.1
$K^- p \rightarrow (K\pi)^- \bar{p}$	See Ref. [5]	14.2 $\pm$ 1.4 (2.1)
[†] The systematic errors are given in parentheses		

Figure Captions

- Fig. 1. a) Distribution of the $(E-p_L)$ balance for events $\bar{p}p \rightarrow \bar{p}\pi^-\pi^+p$ at 40 GeV/c.
The width of ± 3 MeV measures the experimental resolution.
- b) Absolute value of the difference in $(E-p_L)$ balance for the two hypotheses $K^-p \rightarrow K^-\pi^-\pi^+p$ and $K^-p \rightarrow \pi^-K^-\pi^+p$ at 40 GeV/c.
- Fig. 2. Pictorial description for the model used for the partial wave analysis of the $K\pi\pi$ system. A sum over different values of S, ℓ , is implied.
- Fig. 3. Acceptance of magnetic spectrometer for the reaction $K^-p \rightarrow (K\pi\pi)^-p$ at 40 GeV/c.
- Fig. 4. The measured $m_{K\pi\pi}$ spectrum for ambiguous and unambiguous events at 40 GeV/c.
- Fig. 5. a) The measured $m_{K^-\pi^+}$ spectrum for ambiguous and unambiguous events at 40 GeV/c.
- b) The $m_{\pi^+\pi^-}$ spectrum.
- Fig. 6. Intensities of the different spin parity states for the reaction $K^-p \rightarrow K^-\pi^-\pi^+p$ at 40 GeV/c. The notation is $J^P M^\eta$ (spin parity, 3rd component of spin in Gottfried Jackson system, exchanged spin parity).
- Fig. 7. Intensities of the different decay modes of the 1^+ state.
- Fig. 8. Interference phase of the $1^+(S \rightarrow K^*\pi)$ wave with other partial waves. See text, regarding phase relative to $1^+(S \rightarrow K\rho)$.
- Fig. 9. Comparison of the measured data (histogram) with the result of the partial wave fit in the interval $m_{K\pi\pi} = 1.2-1.4$ GeV.
- a) $m_{K^-\pi^+}$ spectrum
- b) $m_{\pi^+\pi^-}$ spectrum
- c) $m_{K^-\pi^-}$ spectrum
- Fig. 10. Comparison of the measured data with the result of the partial wave fit in the interval $m_{K\pi\pi} = 1.2-1.4$ GeV.
- a) α , azimuth of the π^+ in the 3π frame, measured from Gottfried Jackson axes.
- b) $\cos\beta$, polar angle of π^+ in the same system.
- c) γ , angle between the decay plane of the $(K\pi)^-$ system and the plane formed by K_{in}^- and π_{out}^+ .

Fig. 11. Intensities of the decay mode of the 2^+ wave.

Fig. 12. Dimeson masses for events with $m_{K\pi\pi} = 1.6-1.9$ GeV (L-region)

a) $m_{K^-\pi^+}$

b) $m_{\pi^+\pi^-}$

Fig. 13. $K^{**}(1420)\pi^-$ effective mass ($m_{K^-\pi^+} = 1.32-1.52$ GeV).

Fig. 14. $\cos\theta_{\pi^+}$, polar angle of the π^+ from the $K^{**}(1420)$ decay in the $K^{**}(1420)$ rest system relative to Gottfried Jackson axes for the decay chain

($m_L = 1.6-1.9$ GeV, $m_{K^{**}} = 1.32-1.52$ GeV)

Fig. 15. Intensities of the decay modes of the 2^- state of the $K\pi\pi$ system.

Figure 1

$$W(\text{stm}_{K\pi\pi} s_1 s_2 \alpha \beta \gamma) =$$

$$= \left| \sum_{J^P M} \left(\begin{array}{c} K^- \quad (K\pi\pi)^- \quad (K\pi)^0 \quad K^- \\ \quad \quad J^P M \\ \xrightarrow{s} \quad \quad \quad \ell_1 \quad \quad S_1 \\ \quad \quad \quad \pi^- \quad \quad \pi^+ \\ p \quad \uparrow \quad p \end{array} + \begin{array}{c} K^- \quad (K\pi\pi)^- \quad (\pi\pi)^0 \quad \pi^- \\ \quad \quad J^P M \\ \xrightarrow{s} \quad \quad \quad \ell_2 \quad \quad S_2 \\ \quad \quad \quad K^- \quad \quad \pi^+ \\ p \quad \uparrow \quad p \end{array} \right) \right|^2$$

Figure 2

Figure 3

Figure 4

Figure 5

Figure 6

$K^-p \rightarrow K^- \pi^- \pi^+ p$ 40 GeV/c

Figure 7

$K^-p \rightarrow K^- \pi^- \pi^+ p$ 40 GeV/c
 Phase of $I^+(S \rightarrow K^* \pi)$

Figure 8

Figure 9

$K^- p \rightarrow K^- \pi^- \pi^+ p$ 40 GeV/c

$1.2 < m_{K\pi\pi} < 1.4$ GeV

$\Delta E > 3$ MeV

Figure 10

$$K^- p \rightarrow K^- \pi^- \pi^+ p \quad 40 \text{ GeV}/c$$

Figure 11

Figure 12

$K^-p \rightarrow K^- \pi^- \pi^+ p$ 40 GeV/c

$1.32 < m_{K^- \pi^+} < 1.52$ GeV

Figure 13

Figure 14

$K^- p \rightarrow K^- \pi^- \pi^+ p$ 40 GeV/c

Figure 15