MASTER

Argonne National Laboratory

FREQUENCY-RESPONSE ANALYSIS OF STEAM VOIDS
TO SINUSOIDAL POWER MODULATION
IN A THIN-WALLED BOILING WATER
COOLANT CHANNEL

by

Carl C. St. Pierre

RELEASED FOR ANNOUNCEMENT
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ANL-7041

Engineering and Equipment
(TID-4500, 46th Ed.)
AEC Research and
Development Report

ARGONNE NATIONAL LABORATORY
9700 South Cass Avenue
Argonne, Illinois 60440

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Carl C. St. Pierre

Reactor Engineering Division, ANL
and
Associated Midwest Universities

May 1965

Based on a thesis
Submitted to the Faculty of the Graduate School
of Northwestern University
in partial fulfillment of the requirements
for the degree of
Doctor of Philosophy

Operated by The University of Chicago
under
Contract W-31-109-eng-38
with the
U. S. Atomic Energy Commission

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FOREWORD

This report is one of a series that describes heat-transfer and fluid-flow studies performed at Argonne under a program sponsored jointly by the Associated Midwest Universities and the Argonne National Laboratory.

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ABSTRACT

A knowledge of the transient behavior of volume fraction of steam in a coolant channel during power transients is of practical importance in boiler and nuclear-reactor-coolant designs. This is particularly true for boiling water reactors in which the reactivity depends upon the vapor fraction in the channel and the vapor fraction, in turn, depends upon the power level of the reactor. This constitutes a feedback loop which may lead to instability. Due to the large heat-removal rates encountered in these reactors and the feedback nature of the problem, controllability and burnout have become important factors.

The principal objective of this study was to acquire experimental data on power-to-void transfer functions for a single rectangular flow channel at elevated pressures. An "out-of-pile" or boiling loop was operated in forced circulation for various fixed pressures, inlet flow rates, and temperatures, and the power was modulated sinusoidally around a steady-state power level. Then the amplitude and phase of the void response were measured at various axial and transverse positions of the channel, employing a cross-correlation technique over the frequency range from 0.01 to 5 cps. Through an analysis of the data plotted in Bode diagram form, information was obtained as to the propagation velocity of void perturbations, the range of linearity of the void response to power modulation, the shape of the axial variation of transient void amplitudes, and break frequency of the transfer function.

Experimental measurements of power-to-void transfer functions were made in a vertical, rectangular, electrically heated test section in which four main loop parameters were varied: pressure = 200-800 psia; inlet flow rates = 2.52-3.78 ft/sec; heat fluxes = 2.28×10^4 - 9.12×10^4 Btu/hr ft², and inlet subcoolings = 0.8-12.4°F. Data were obtained at three axial positions and also at various transverse positions across the width of the channel for a fixed axial height.

All of the void-amplitude data plotted as a function of frequency exhibited notches at frequencies ranging from 1.2 to 3.7 cps. The notch frequency was highest for those axial positions nearest the test-section

inlet and decreased with increasing axial distance. Transverse measurements showed a maximum value of the notch frequency at the center of the channel, which decreased as the wall was approached, for downstream regions of the channel. The phase measurements also showed an undulation close to the notch frequency.

Good agreement was found between the zero-frequency void-amplitude response obtained from the experimental data and that obtained from steady-state axial void distributions at various power levels. Increased inlet sub-cooling was found to increase the zero-frequency amplitude.

At pressures of 200, 300, and 400 psia the range of linearity of the void response to power modulation was found to extend up to amplitudes of 20% of the average power level.

The axial variation in transient void amplitude was sinusoidal in shape, being of the same form as the power forcing function. As the modulation frequency increased, the period and amplitude of this sinusoidal function decreased until the test-section length corresponded to one-half a wavelength. Then a notch would appear in the void amplitude at the test-section exit.

The transverse void-amplitude distribution data appeared to be of the form $\alpha'/\alpha'_{\max} = y^{1/n}$.

The cross-sectional average propagation velocities were found to be intermediate between the mixture and vapor cross-sectional average velocities at all axial positions downstream of the concave-shaped transverse void distributions. Near the entrance of the test section, the propagation velocity decreased and became more closely related to the cross-sectional average liquid velocity.

At a fixed axial position the local void-propagation velocity was a function of the distance from the wall. In the downstream regions of the channel, the measured propagation velocity profiles exhibited a maximum at the center and decreased near the wall, which is consistent with the convex steady-state velocity profiles which are thought to exist in these regions.

From the Bode diagrams of the data it was found that sharpness of the notch increased with increasing pressure and inlet temperature sub-cooling, but decreased with increasing axial length and increasing distance from the test-section wall. By noting the analogous behavior of slip and the sharpness of the notch, slip was believed to be of importance here in the determination of the break frequency of the power-to-void transfer function.

Steady-state axial and transverse distributions of void fraction were obtained under loop-operating conditions corresponding to those employed in

the measurements of the power-to-void transfer function. The Marchaterre-Hoglund, Griffith, modified Bankoff, and Neal models were all found to predict fairly closely the axial, steady-state, cross-sectional average void distributions.

Concave void distributions were found near the test-section entrance. The degree of concavity and persistence of this profile with axial distance increased with increased inlet temperature subcooling. For all concave-shaped profiles the calculated values of slip were less than or close to unity.

The theoretical models proposed by Zivi and Wright, Christensen, Solberg, and Sanathanan were tested for their ability to predict the power-to-void transfer function data.

A new model was constructed, similar to Solberg's "Kjeller Model," in that Bowring's physical model was employed in the subcooled boiling region. It differed, however, in that the heat capacity of the wall was accounted for in all three regions, the transitional bulk-boiling boundary effects were considered in every section of the subcooled boiling region, and the entire model was programmed for solution on a digital computer employing a step-by-step integration procedure. The steady-state values of $\bar{\alpha}(z)$, $\bar{v}_g(z)$, and $\bar{S}(z)$ were generated for various models and correlations relating quality and void fraction. Fairly close agreement between theory and data was found for the notch frequencies, phase lag, and break frequencies. Appreciable differences between model predictions and data were found for some of the zero-frequency amplitudes.

The ultimate objective of this work and research on boiling channel transfer functions and hydrodynamic instability was to obtain a sufficient understanding of the phenomena to enable prediction of the stability of a reactor system in the design stage.

Dimensionless	<u>Greek Symbols</u>		
Dimensionless	α	Void fraction of cross-sectional area occupied by vapor	Dimensionless
M	α_{avg}	Average void fraction over the whole cross-sectional flow area	Dimensionless
ML/t	α_{y^*}	Average void fraction along transverse position y^*	Dimensionless
ML/t ²	β	Constant defined by Eq. (6.17)	Dimensionless
L	Γ	Number of active nucleating sites	
ML/t ²	γ	Density ratio ρ_g/ρ_l	Dimensionless
Dimensionless	Δa	$a_{N+1} - a_N$, where N and N+1 refer to two control surfaces	
Dimensionless	$\Delta \alpha$	Equivalent void-fraction change of calibration disk	Dimensionless
ML ² /t ³	ΔT_d	Transition-boundary subcooling defined by Eq. (6.18)	T
ML/t ³	ΔT_{scb}	Transition-boundary subcooling defined by Eq. (6.17)	T
	ΔT_{sub}	Inlet subcooling to the test section	T
Dimensionless	ΔT_{TB}	Transition subcooling, least of ΔT_d , ΔT_{scb} , and ΔT_{sub}	T
ML ³	δ	Constant defined in Eq. (9.3)	
ML ³	ϵ	Ratio of agitative to evaporative heat fluxes, q_h/q_e	Dimensionless
Dimensionless	ζ	Bubble diameter at departure	L ²
ML ³	η	Constant defined in Eq. (6.18)	T/t ² /M
ML ³	λ	Latent heat of vaporization	L ² /t ²
Dimensionless	μ	Viscosity	M/Lt
Dimensionless	ν	μ/ρ , kinematic viscosity	L ² /t
Dimensionless	π	3.14159...	
Dimensionless	ρ	Fluid density	M/L ³
Dimensionless	τ	Shear stress tensor	M/t ² L
T	τ	Integration period of wave analyzer	t
t	Φ	Potential energy	ML ² /t ²
t	ϕ	Heat flux	M/t ³
L	ϕ	Phase lag	Degrees
ML ² /t ²	Ψ	Ratio of break frequency to notch frequency	Dimensionless
Dimensionless	Ω	Frequency of bubble departure	t ⁻¹
L ³	ω	Angular velocity	Radians/t

Superscripts

L/t	$\circ$	Denotes dimensionless variable reduced with respect to some characteristic dimension
	$'$	Denotes deviation of quantity from steady state
	c	Denotes complex number whose real and imaginary parts yield the transient response of that variable
L/t	$-$	Denotes steady-state value of variable
L/t	$+$	Denotes quantity averaged with respect to axial distance.

Subscripts

L/t	avg	Denotes average value over cross section
L/t	B	Denotes bulk-boiling boundary
L/t	g	Denotes gaseous or steam phase
	l	Denotes liquid or water phase
	m	Denotes mixture property
ML ² /t ²	N	Denotes control surface at N
M/t	o	Denotes inlet of test section
Dimensionless	S	Denotes thermodynamic saturation conditions
Dimensionless	TB	Denotes transition boundary
	tot	Denotes total quantity in macroscopic system
L	w	Denotes test-section wall
L		

A	Cross-sectional area of channel	L^2	m	Constant for Marchaterre-Hoglund correlation, Eq. (6.53)
a	Constant defined for Eq. (9.12)	t^{-1}	m_N	Constant defined for Eq. (6.33)
$a_1 - a_9$	Constants defined for Eqs. (6.7) to (6.13)	Dimensionless	m_{tot}	Total mass in control volume $A \cdot \Delta z$
a_{10}, a_{11}	Constants defined for Eqs. (6.15) and (6.16)	Dimensionless	P	Momentum
a_{12}, a_{13}, a_{14}	Constants defined for Eqs. (6.29) and (6.30)	Dimensionless	p	Fluid pressure
a_{15}, a_{16}, a_{17}	Constants defined for Eq. (6.33)	Dimensionless	p_c	Perimeter of channel
A_1	Amplitude of the first harmonic of v_R		p_d	Steam-dome pressure
A_N	Amplitude of the Nth harmonic of v_R		$PN+1/2$	Constant defined for Eq. (6.44)
$B_{N+1/2}$	Constant defined for Eq. (6.44)	Dimensionless	$PN+1$	Constant defined for Eq. (6.45)
b	Channel half-width	L	Q	Rate of thermal energy entering system from solid surfaces
b	Constant for Marchaterre-Hoglund correlation, Eq. (6.53)	Dimensionless	Q_{gen}	Rate of energy generation in test-section walls per unit length
b_N	Constant defined for Eq. (6.32)	Dimensionless	Q_h	Perturbation in heat flow
C	Heat capacity at constant pressure per unit mass	L^2/t^2	Q_o	Fractional power variation
$C_{N+1/2}$	Constant defined for Eq. (6.44)	Dimensionless	q_e	Evaporative component of q_{tot}
c_N	Constant defined for Eq. (6.32)	Dimensionless	q_h	Agitative or heating component of q_{tot}
d	Channel diameter	L	q_N	Constant defined for Eq. (6.45)
$D_{N+1/2}$	Constant defined for Eq. (6.44)	Dimensionless	q_{SP}	Single-phase component of q_{tot}
$d_{N+1/2}$	Constant defined for Eq. (6.44)	Dimensionless	q_{tot}	Total energy flux
E	$U + K + \Phi$ = total energy of the system	ML^2/t^2	r	Constant defined for Eq. (6.56)
E_N	Constant defined for Eq. (6.44)	Dimensionless	r_N	Constant defined for Eq. (6.45)
E_c	Wave-analyzer cosine-channel output		S	Velocity ratio (slip) of vapor to liquid, v_g/v_L
E_s	Wave-analyzer sine-channel output		S_N	Constant defined for Eq. (6.37)
$e_{N+1/2}$	Constant defined for Eq. (6.44)	Dimensionless	s_{N+1}	Constant defined for Eq. (6.45)
F	Force of fluid on adjacent solid	ML/t^2	T	Temperature
f_b	Transfer-function break frequency	cycles/t	t	Time
f_N	Constant defined for Eq. (6.44)	Dimensionless	t_p	Transport time
f_n	Lowest node frequency at any position z	cycles/t	t_w	Thickness of test-section wall
G	Volume flow rate defined for Eq. (6.52)	L^3/t	U	Internal energy
G_N	Constant defined for Eq. (6.44)	Dimensionless	U	Constant defined for Eq. (6.47)
G_T	Total gain of wave analyzer		V	Volume
$G(s)$	First-order transfer function, defined for Eq. (9.1)	L/t^2	V_b	Bubble rise velocity with respect to the liquid ahead of the bubble, Eq. (6.52)
g	Gravitational acceleration	Dimensionless	V_a	Void-detector output voltage for void fraction α
g_c	Gravitational conversion factor	Dimensionless	V_E	Void-detector output voltage for empty channel
g_N	Constant defined for Eq. (6.44)	ML^2/t^2	V_F	Void-detector output voltage for full channel
H	$U + pV$ = enthalpy	L	v	Mass-averaged velocity
H	Total channel length	ML^2/t^2	v_E	DC bucking-voltage error component of v_T
H_M	Mixing-cup enthalpy	M/t^3T	$v_{g,m}$	Modified steam velocity defined for Eq. (9.8)
h	Heat transfer coefficient	M/t^3T	v_m	Mixture velocity
h_{SB}	Heat transfer coefficient in subcooled boiling region	M/t^3T	v_{mean}	Mixture velocity
h_{sp}	Single-phase heat transfer coefficient	ML^2/t^2	v_N	Noise component of v_T
K	Kinetic energy	Dimensionless	v_p	Void-perturbation propagation velocity
K_B	Constant for Bankoff Model, Eq. (6.54)	Dimensionless	v_R	Response-signal component of v_T
K_2	Constant defined for Griffith Model, Eq. (6.52)	Dimensionless	v_T	Total signal to wave analyzer
K_3	Constant defined in Eq. (9.5)	Dimensionless	W	Rate of doing work on surroundings
$K_{N+1/2}$	Constant defined for Eq. (6.37)	Dimensionless	w	Mass flow rate
k	Thermal conductivity	ML/t^3T	X	Quality: Ratio of vapor mass flow rate to total mass flow rate
k_1, k_2	Constants defined for steam-dome pressure transfer functions	Dimensionless	Y_N	Constant defined for Eq. (6.37)
k_N	Constant defined for Eq. (6.33)	L	y	Rectangular coordinate in transverse direction
L	Characteristic length	Dimensionless	z	Rectangular coordinate in axial direction
L_N	Constant defined for Eq. (6.31)	Dimensionless		
$L_{N+1/2}$	Constant defined for Eq. (6.37)	Dimensionless		
M_N	Constant defined for Eq. (6.31)	Dimensionless		

I. INTRODUCTION.

Since the advent of the boiling water nuclear reactor (BWR) more than a decade ago, considerable engineering research has been directed to the problem of heat removal from the fuel and the hydrodynamic stability of the reactor. As the reactivity of BWR's depends upon the vapor fraction in the channel and the vapor fraction, in turn, depends upon the power level of the reactor, this constitutes a feedback loop which may lead to instability. Due to the large heat-removal rates encountered in these reactors and the feedback nature of the problem, controllability and burnout have become important factors. Their consideration has led to a greater interest in the time-dependent transient behavior of the two-phase, natural-circulation flow systems.

In order to predict accurately the dynamic performance of the BWR in the design stage, it is necessary to understand the transient behavior of steam bubbles in the reactor core as the heat production in the fuel varies. This constitutes the so-called power-to-void transfer function, which is a concise way of expressing the dynamic relationship between a power disturbance and the ensuing steam-void response. This transfer function shows the variation with frequency of the amplitude and phase shift of the void fraction which results from a sinusoidal modulation of power. A knowledge of the power-to-void transfer function is one requirement along with the frequency response characteristics of the other elements of the system, which are needed in order to predict the stability of the linear feedback loop employing linear-feedback system analysis.

This work was first concerned with the acquisition of experimental data on power-to-void transfer functions for a single rectangular flow channel at elevated pressures. An "out-of-pile" or boiling loop which could be operated at elevated pressures and temperatures in either forced or natural circulation was used. For various fixed pressures, inlet flow rates, and temperatures, the power to the test section was modulated in a sinusoidal manner around a steady-state power level. Then the amplitude and phase of the void response were measured, employing a cross-correlation technique, over the frequency range from 0.01 to 5 cps. These measurements of power-to-void transfer function were made at various axial and transverse positions of the test section. Since the transfer-function technique is valid only for small oscillations under the linear-system assumption, the range of linearity of the void response was also investigated. Various runs were made with low inlet subcooling in order to give a well-defined boiling length over a great portion of the test section. In order to minimize the heat exchange mechanism between coolant and wall, a very thin-walled test section made of stainless steel was employed.

The "zero-frequency" response of the power-to-void transfer function depends upon the distributions for the steady-state void fraction. Thus, in

order to predict correctly the transfer function theoretically, the ability to calculate the distributions of steady-state void fraction becomes essential. Therefore, transverse and axial distributions of void fraction for the steady-state were experimentally obtained to test the capability of various theoretical models to predict the required axial void distributions.

After the transfer-function measurements and steady-state void distributions had been obtained, existing theoretical models were tested to see if they could correctly predict the transient behavior of the steam voids resulting from power modulation.

The ultimate objective of this work and research on boiling channel transfer functions and hydrodynamic instability is to obtain a sufficient understanding of the phenomena to enable prediction of the stability of a reactor system in the design stage.

II. LITERATURE SURVEY

Kramer¹ has given a historical review of the boiling water nuclear reactor up to 1958. Anderson and Lottes² have reviewed the field of boiling stability up to 1961.

A. Early Experimental Stability Investigations Using Nuclear Reactors

The early experimental investigations of reactor stability were usually of two types: (1) those in which the forcing functions were of a step or ramp form of input; (2) those in which the forcing function was oscillatory.

- Among the first stability tests were those named BORAX,³⁻⁸ from Boiling Reactor Experiments, which were of two types: (a) power excursion tests in which, with the reactor running just critical, a large reactivity was inserted and the reactor allowed to run for a predetermined time before shutting down, and (b) steady boiling tests. This type of work was continued in the SPERT⁹⁻¹³ (Special Power Excursion Reactor Test) series of reactors in which the reactivity was increased at a linear rate (ramp forcing function) to a predetermined point and then the subsequent reactor behavior observed. Although a large amount of qualitative information on the dynamic behavior of BWR's resulted from these tests, very little data were obtained that were of help to the model maker. The three most notable findings were the following: (1) the negative effect on reactivity of steam bubbles in the core and their self-regulatory results, (2) pressure increases in all tests resulted in increased stability, and (3) stability increased as the height of water above the core decreased.

The second approach to stability performed in nuclear reactors employed a harmonically oscillated reactivity input and the study of the resultant oscillation in output power. Eriksen¹⁴ was an early investigator who attempted a technique in which void volumes were measured by their reactivity worth. This investigation and Daavettila's¹⁵ indicated that a reactor was a poor detector of voids for studies of dynamic boiling phenomena.

DeShong and Lipinski¹⁶ employed a rod oscillator technique to measure the transfer function of the reactor at several low powers with the Experimental Boiling Water Reactor (EBWR). An analog computer was employed with curve fitting of various time constants in different feedback loops to plot theoretical curves which matched the experimental data and using the values so obtained they extrapolated to obtain transfer functions at higher powers. This technique correctly predicted an instability point at a higher power level and pressure which indicated that it was possible to estimate feedback transfer functions at higher powers and pressures by employing measurements made at lower powers and pressures.

B. Early Experimental Boiling Test Loop Stability Investigations

An economically feasible method of obtaining useful dynamic data for boiling water reactors has been experimental investigations on out-of-pile or boiling loops. This type of experiment started early at Argonne National Laboratory (ANL) and was used to prove the possibility of the BWR and later to furnish data for the BORAX and EBWR reactors.

These loops were normally of the natural-circulation type which could be operated with certain fixed operating conditions such as power, pressure, and make-up water temperature. The power for the loop was furnished by electrically heated test sections. At ANL and many other Laboratories thousands of loop experiments have been conducted, usually in the following manner: for constant pressure and inlet subcooling, the loop power has been gradually increased until the loop became unstable, and the frequency of oscillation and average values of loop parameters were recorded. General conclusions from the natural-circulation runs were the following:² (1) stability increased with increasing pressure, and (2) stability was affected by the geometry of the test section, riser, and downcomer, with increased stability with increased pressure drop in the downcomer, and decreased stability with increased pressure drop in the test section and riser, (3) height of the liquid level above the riser had no effect on stability, and (4) the period of oscillation was quite insensitive to changes in loop parameters.

This loop instability manifested itself in periodic variations of the system parameters. The threshold power level at which disturbances begin to grow can be determined from the linearized equations, using well-known transfer-function techniques.

Wallis and Heasley¹⁷ were early workers who employed the transfer-function technique to study three modes of oscillation in a simple two-phase, natural-circulation system of pentane at atmospheric pressure. They set up the differential equations for the system and solved them in the complex frequency domain for small perturbations of the variables.

The stability of a water loop at 1000 psia has been studied by Beckjord,¹⁸ whose theoretical model incorporated fluid-transportation effects and distributed heat transfer parameters. Essentially, the mathematical model was similar to that of Wallis and Heasley.

The model testing made possible by natural-circulation systems was not precise enough to draw far-reaching conclusions about the validity of the models. Through computer studies it was possible to obtain a precise power at which the system became unstable, but this was, however, generally not the case with the experiments in which it was very difficult to draw the line between "noisy" and "oscillatory" conditions. For this reason, the point

test of the model should be supplemented by comparison with other experimental data such as transfer-function measurements carried out on boiling loops.

C. Power-to-void Transfer Function Measurements

A detailed review of power-to-void transfer function measurements was made by Zivi¹⁹ in 1962, in which the experimental work of Zivi and Wright²⁰⁻²² and Christensen²³ were outlined and interpreted.

1. Measurements at Space Technology Laboratory (STL)

Transfer-function measurements at STL have been carried out with an electrically heated aluminum channel which simulated a SPERT-1A moderator-coolant channel in dimensions and heat capacity. Tests at "boiling conditions" (low subcooling) were run by modulating the channel power about the mean level at a given frequency with small amplitude, usually 10%. Then the in-phase and out-of-phase components of the coherent response of the void fraction were measured at the same frequency. From these measurements the amplitude and phase of the void response were calculated over the full frequency range from zero to 10 cps.

At a frequency of 1/4 cps, the range of linearity of the void response was found to extend up to 20% power modulation, corresponding to a void amplitude of 17%. During the initial power-to-void transfer-function measurements in natural circulation, the flow rate, as well as void fraction, was responding coherently to the power modulation, so that transfer functions of power to flow were also measured. The experimental range of loop variables were as follows: pressure = 1 atm; heat flux = 3,500-28,000 Btu/hr ft², power density = 5 to 50 kW/liter; inlet subcooling = 0 to 3°C; and inlet velocity (natural circulation) $\approx$ 1.50 ft/sec.

2. Measurements at Argonne National Laboratory

Christensen²³ measured the void response at ANL in a manner similar to that at STL while he extended the experimental range of the loop parameters by use of the Small Scale Loop; pressure = 400-1000 psia; heat flux = 70-175 $\times 10^3$ Btu/hr-ft²; power density = 50-110 kW/liter; inlet subcooling = 5-26°F; inlet velocities = 2.5-3.75 fps. Employing a rectangular test section in a forced-circulation loop, he oscillated the power 10% peak-to-peak around the steady-state power level, and measured the amplitude and phase of the void response, employing a cross-correlation technique.

Christensen found that the void response fell off at a frequency that was much lower (0.30-0.60 cps) than that predicted by theoretically derived power-to-void transfer functions used in previous reactor calculations. The most prominent feature of the data was a node

point which was quite sharp and an undulation in the phase measurements at the node frequency. This dip in phase and amplitude at the node point became more pronounced as the entrance of the test section was approached. Christensen's theoretical work and experimental results have been discussed in more detail in a later section.

D. Stability Analyses of Boiling Water Reactor Test Loops

As stated in the introduction, the ultimate objective of research on boiling channel transfer functions and hydrodynamic instability has been the ability to predict the stability of a reactor system in the design stage. To accomplish this goal two main types of approaches have been attempted.

The first employed the fundamental conservation laws of energy, mass, and momentum coupled with the assumption of phase equilibrium. Solution indicated the influence of system parameters on stability.

The second used some controlling phenomenon, viz., the curve of pressure drop vs. flow rate, which was assumed to be the only phenomenon of importance. The influence of the other parameters of the system on this relationship were investigated.

Equations applying the fundamental concepts have been written for a differential volume element of the flow system without much difficulty, but the solution of the equations, which were nonlinear partial differential equations, were difficult because of their nonlinearities and of the empirical and sometimes uncertain value of many of the essential elements of the problem. Thus, in order to obtain solutions several judicious approximations had to be made so that the simplified equations when solved would retain their basic behavioral characteristics. Several authors have employed this procedure.^{17,24,25} Those which have specific applicability to power-to-void transfer functions will be mentioned in a later section.

A review of the second approach to the theoretical prediction of reactor stability was given by Anderson and Lottes.² In the most widely used analysis an instability was considered to occur at conditions accompanying a maximum in the plot of pressure drop vs. mass flow rate. The earliest paper using this method was published by Ledinegg,²⁶ and various other authors^{27,28} have considered this technique. Other approaches for vertical two-phase flow kinematics have been based on the potential energy of the static head in the two-phase region,²⁹ and also the inability of a fluid phase to dissipate at the channel wall all of the shearing energy it received.³⁰⁻³² Another phenomenon which could lead to unstable loop operation was a sonic-velocity choking effect, since the velocity of sound in a two-phase mixture becomes quite low under certain quality and pressure conditions.³³

1. Theoretical Models for the Dynamics of Coolant Channels

Most authors treating the dynamics of coolant channels have used the three fundamental laws for conservation of mass, energy, and momentum. Then different mathematical simplifications and physical assumptions were employed to solve the problem in varying degrees of refinement.

Some of the authors lumped together the coolant-channel parameters, adopting a one-point model. Horning and Corben³⁴ employed a one-point model, writing one differential equation for the total steam volume, and took into account the heat capacity of the fuel and constant frictional rate of removal of heat. Iriarte³⁵ used a one-point model also, but characterized the power-to-void transfer function by two time constants and a time delay, whereas Beckjord³⁶ divided the fuel plates into several regions and derived a transfer function involving three time constants for the power-to-void transfer function, which was derived by a difference method.

The coolant channel was divided by many authors into two sections coinciding with the nonboiling and bulk-boiling regions, but sub-cooled boiling was not included. Thie³⁷⁻³⁹ solved the distributed parameter equations for the heat transport in the fuel and for void transport in the channel. Fleck and Huseby⁴⁰ and Fleck⁴¹ assumed that the conditions in the boiling section could be described by a linear variation with channel height of the following variables: void fraction, water velocity, and steam velocity. Lumped-parameter equations described the heat transfer from fuel plate to liquid, while the distributed parameter equations were written for the transport of disturbances along the coolant channel. Akcazu⁴² expanded the Beckjord model, introducing additional mechanisms which were difficult to evaluate quantitatively from either theory or steady-state measurements.

2. Recent Theoretical Models

a. Space Technology Laboratories

The theoretical work of Zivi and Wright²² was preceded by an experimental investigation with a test loop. This phenomenological model employed the experimentally obtained steady-state steam void distribution and was based on the assumption that changes in the shape of the axial void distribution were much less important than the longitudinal displacement of the axial void distribution during power modulation. To solve for the void response, the movement of the boundary between the nonboiling and boiling sections first had to be computed. Based essentially on Storrer's⁴³ analysis, the movement of the nonboiling length was obtained by adding the cumulative effects of power, flow, and pressure. Then the amplitude of the void response was calculated by multiplying the magnitude

of boundary disturbance by the total derivative of zero-frequency void fraction with respect to axial distance obtained from experimental data. The phase lag was determined as the sum of the moving-boundary phase lag plus that due to the propagation (away from the boundary) of the void disturbance, which was assumed to travel with the liquid velocity. The transfer functions so obtained were combined in a feedback loop with reactor zero-power transfer function to predict instabilities by the Nyquist criterion. This prediction was found to agree with observations with the SPERT-1A reactor in both threshold power level and frequency of oscillation.

b. Christensen

Christensen derived an expression for the power-to-void transfer function which fit his experimental data well by taking into account the pressure changes in the channel caused by the power variations. The break frequency in the amplitude curve was fitted by choosing a constant associated with the completeness of the mixing in the direction perpendicular to flow. Also, the sharpness of the node was adjusted by use of a variable time constant associated with his first-order approximation to the wall-temperature transfer function. Results of Christensen's analysis will be discussed in more detail later, but Christensen claimed that the most important result of his experiments was to show that the void response fell off at a frequency much lower than that predicted by theoretical power-to-void transfer functions (i.e., in fact, the actual propagation velocity of disturbances was found to be lower than either the steam or water velocity).

c. Bankoff et al.

Theoretical papers reviewed as a solution to predicting coolant-channel void response were by Hudson,⁴⁴ Atit,⁴⁵ and Bankoff.⁴⁶ The dynamic behavior of steam voids in a vertical boiling channel resulting from power or inlet flow modulation was studied by analog methods for the case of saturated feed water. The equations of energy, continuity, and momentum for the system were differenced in the height variable and linearized around steady state. Bankoff's⁴⁷ variable-density model was employed to obtain the quality-void fraction relationship, and it was found that a minimum of four differenced sections were necessary to obtain the frequency node in the void-amplitude response. In addition, the propagation velocity of a disturbance was found to be more closely related to the vapor velocity than to the liquid or mixture velocity. Recently, Jain⁴⁸ worked on this approach and obtained solutions with a digital computer by a step-by-step integration procedure.

d. Knolls Atomic Power Laboratory

Jones⁴⁹ and Jones and Dight⁵⁰ have studied the hydrodynamic stability of a boiling channel in a manner similar to that of

Bankoff *et al.* However, an approximate representation of the subcooled boiling region was also included, as was transient energy storage in the fuel element. A modified Bankoff slip ratio correlation was used in the time-linearized conservation equations, which were sectioned in the height variable and solved with a digital computer. Effects of the moving boundary between subcooled boiling and bulk boiling regions were also accounted for, but a basic assumption was made that the bubble lifetime in the subcooled boiling region was zero. One comparison of this method, in closed loop form, agreed quite well with their experimental data points.

e. Kjeller Model

Solberg⁵¹ and Schjetne⁵² have developed and tested, respectively, the Kjeller model for the dynamics of coolant channels in boiling water reactors. This theory was also based upon the linearized and axially sectioned conservation laws but had provision for simultaneous presence of the three regions in the channel: nonboiling, subcooled boiling, and bulk boiling. What made this model distinct was the fact that a mechanism for subcooled boiling was included based upon Bowring's work.⁵³ Semi-empirical steady-state laws relating steam velocity and two-phase friction to local void fraction and water velocity were used, and predictions were compared briefly with the work of Christensen.²³ This model has been treated in detail in a later section.

E. Present Status of Theories of Power-to-void Transfer Functions

The present state of the art in boiling water heat transfer has not yet become sufficiently refined to allow accurate predictions of power-to-void transfer functions. This problem has usually been complicated by the presence of incomplete transverse mixing of the fluid and the presence of subcooled boiling regions along with the bulk-boiling region. When more than one boiling region is present, the effects of the transitional boundaries on the void-phase response must be taken into account.

In those cases for which a subcooled boiling region exists, the theoretical analysis of the problem becomes very difficult. This results from the transverse gradients in void fraction, liquid and vapor velocities, and liquid temperature which exist in that region and change rapidly in the axial direction. Therefore, where incomplete transverse mixing of the fluid exists, the "a priori" theory solution of the problem becomes very difficult. However, for those cases where only the net boiling region is present there is more optimism in the success of theoretical predictions.

An important question which remains to be answered pertains to the detailed mechanisms by which a two-phase boiling mixture is perturbed. Scarcity of experimental data has made this problem even more difficult. The only papers which presented data have been contradictory as to the propagation velocity of a void perturbation. The STL results²⁰ indicate

that disturbances would be propagated with the water velocity for the case of good mixing, whereas Christensen's data²³ suggests a lower velocity for the case of incomplete mixing.

In addition to this work, research on this problem is presently being conducted in other research laboratories: at the Laboratory of Heat Transfer and Reactor Engineering, Technological University of Eindhoven;⁵⁴ Atomic Power Equipment Department of General Electric by Staub and Zuber.⁵⁵

1. PRESSURE	4. SURGE TANK
2. STEAM SEPARATOR LEVEL	5. POWER
3. LINE HEATERS	6. CALROD HEATERS

Fig. 3.2. Control Panel of Small Scale Loop

The heated section contained a rectangular test tube which was electrically heated by passing through it one to two thousand amperes of a.c. current. This test-section assembly will be described in detail later, along with the transition piece, which preceded the test section and provided for a smooth flow transition from the 2-in. pipe of the return leg to the rectangular flow area of the test section. A riser, 2 in. in diameter and 24 in. long, followed the test section.

Above the riser was a steam separator, constructed of 4-in.-diameter, Type 304 stainless steel pipe, 48 in. high. Here the generated steam was bled off and the water returned to the recirculation path. In order to decrease pressure feedback from the steam separator, Anderson⁵⁶ added a steam surge tank to the existing separator by use of a 7-ft length of 2-in.-diameter, schedule 80 stainless steel pipe. This surge tank, which consisted of an internally baffled 6-ft length of 5-in.-ID, schedule 160 stainless steel pipe, reduced the pressure backlash in the system 85% (from 1.5 to 0.2 psi peak-to-peak at an average flow of 1.25 fps with a 0.10 fps peak-to-peak flow variation).

The crossover and downcomer were approximately each 12 ft in length, serving as a recirculation flow path for the water. In this path a

forced-convection, air-cooled heat exchanger could be bypassed or valved into the recirculation loop. Two line heaters were located on the down-comer each having a power output of 2 kW.

In the return leg were located the pump, flow meters, flow-control valves, and additional line heaters for temperature control. For forced-circulation operation, a Westinghouse, Model 100-A, canned rotor pump was employed which had a capacity of 50 gpm at a 100-psi head corrected to 600°F. Flow control was provided by two flow-control valves in parallel: a 2-in. air-operated main flow valve and a 0.5-in., manually operated linear valve. This smaller control valve could be connected to a mechanical oscillator⁵⁶ which allowed periodic variations in the inlet flow if desired. Almost all of the pressure drop around the loop was taken across these valves, and flow remained constant during power oscillations. For natural circulation, the 100-A pump would be turned off, and the main flow and bypass valves opened.

Inlet temperatures to the test section were controlled by the use of line heaters placed around the loop, the preheater in the surge tank, and the heat exchanger on the return leg of the loop. In order to facilitate operation at inlet temperatures close to saturation, two line heaters were repositioned after the 100-A pump, and two additional bayonet-type Calrod heaters were added before the transition section. These bayonet heaters were 4 ft in length and had a power output of 8 kW, which was controlled by powerstats. Since they contacted the water directly, a rather close temperature control resulted. Also, one line heater was repositioned around the transitional entrance to the test section. This 4-kW heater was powerstat-controlled. The total capability of the line heater totaled 20 kW.

Continuous smooth flow of make-up water to replenish that bled off as steam was provided by a high-pressure feedwater pump used in conjunction with an accumulator system. The gas pressure to the accumulator was automatically regulated at 5 to 15 atm above the loop operating pressure by an on-off controller. Control of the make-up water flow rate was by a globe-type micrometer valve with a 1/8-in. orifice. Make-up water could enter directly into the loop or into a preheater surge tank.

The loop pressure was closely controlled by the use of two steam bleed valves in parallel, in addition to a constant steam bleed rate maintained on the steam surge tank. The 1/2-in. bleed valve was air-operated, while the 1/4-in. bypass was manually controlled. The bled-off steam passed through a water-cooled heat exchanger and then into a water-cooled make-up tank on the suction side of the high-pressure feedwater make-up pump.

The loop normally took 3 to 4 hr after startup to reach operating conditions, whereas 1-2 hr were required for shutdown, depending upon the loop operating pressure.

1. Control of Loop Variables

There were four main variables which had to be maintained constant for satisfactory operation of the loop: pressure, inlet water temperature, average power, and water flow rate.

Of these four the most difficult to obtain was a constant inlet temperature to the test section, especially at temperatures near saturation. This was due to the large heat capacities of the loop structure and also to the danger of obtaining cavitation in the 100-A pump, which tended to place the loop in a seminatural-circulation mode of operation. By approaching this condition of low inlet subcooling slowly, avoiding any sudden changes in loop settings and through careful control of line heaters, the drifts in temperature remained within $\pm 2^{\circ}\text{F}$.

The loop pressure was easy to control with the parallel arrangement of the steam-bleed valves, and care was taken never to make bleed rate changes during the analysis of the void signal. In this manner, pressure fluctuations were maintained to ± 1 psi.

The manual control system for electrical heating power was subject to changes due to fluctuations in power line voltage, and the watt-meter was frequently checked to compensate for these power variations.

At all operating conditions, the flow was easily controlled and maintained at any constant level, with fluctuations being less than ± 0.10 gpm.

B. Loop Power Supplies

During this experiment, two power supplies were employed. The first was a Sciaky-Saturable Reactor Supply which had a power capability of 93 kW at 60 cps, with a series output connection giving 33.9 V at 2740 Amp. The second was a G. E. Ignitron Power Supply with a power capability of 300 kW at 60 cps, with an output of 100 V at 3000 Amp.

The power to the test section could be varied from zero to maximum value by the following arrangement: In series with the primary winding of the output transformer were three saturable reactors capable of handling the full 220-V power of the primary winding. The d.c. control current to the saturable reactors was set by regulating the voltage to a full-wave rectifier system by means of a powerstat located on the control panel.

C. Boiling Test Section

The test-section assembly used in this study has been diagrammed in Fig. 3.3. In Fig. 3.4(A) the test section is shown in place in the loop. Cold-drawn, seamless, stainless steel tubing of Type 304 quality was used in the construction of the rectangular test section, manufactured by Accles and Pollock Ltd., Oldbury, England. For the test tube, the critical dimensions were: flow area, 0.437 in. by 1.750 in.; total axial test section length, 61 in.; heated length, $49\frac{1}{2}$ in.; the wall thickness, 17 mils.

In order to prevent deforming of the thin walls at high pressures the test section had to be backed-up on all four sides. This was accomplished along the 1.750-in. width by the use of $\frac{3}{4}$ -in. thick stainless steel plates and along the 0.437-in. width by employing precision-machined spacers on the bolts holding the back-up plates together. One-sixteenth-inch-thick Durabla provided electrical and thermal insulation between the back-up plates and tube wall; a $\frac{1}{8}$ -in. thick mica mat provided mechanical strength and electrical insulation between the spacers and the section.

New end pieces were also designed to allow the back-up plates and end pieces to be mechanically joined in such a way that the back-up plates took up any stresses due to misalignment or loop shifts resulting at the higher operating temperatures. Provision was made to mount two thermocouples and a pressure tap in each end piece, and the passage of heating current to the tube was facilitated by a 4-in. length of silver solder between the end piece and the section.

In order to provide a smooth transition from the 2-in.-diameter pipe of the loop to the rectangular test section, a transition piece [see Fig. 3.4(B)] was constructed of 37-mil, 304 stainless steel, 25-in. long. The flow areas of the transition section and test tube were identical for most of the transitional length, and, it is believed that this long length was sufficient to eliminate subcooled voids at the entrance which might have originated from the bayonet heaters immediately before the transition piece.

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Fig. 3.3. Diagram of Test-section Assembly

Fig. 3.3 (Contd.)

1. Back-up plates; Type 304 Stainless Steel	12. High-temperature bolt and nut; steel, 1/2-in. diameter
2. Collimator strip, 16 gal; Type 304 Stainless Steel	13. Gasket material insulation, 1/16-in. thickness
3. Lap-joint flange, 2 in.; Type 304 Stainless Steel	14. Void fraction window, 1-1/8 x 0.437 in.
4. Reinforcing spacer blocks; Type 304 Stainless Steel	15. Alignment tubing, 5/8-in. diameter, 1/32-in. wall; Type 304 Stainless Steel
5. Mica-mat insulation, 1/8-in. thickness	16. Insulating cylinder, gasket material
6. Lap-joint stub end, 2 in.; Type 304 Stainless Steel	17. Pressure tap, inlet
7. Thermocouple gland; "Saturation Temperature"	18. Thermocouple gland; inlet temperature
8. Gasket material insulation, 1/16-in. thickness	19. Ring, lap-joint flange; Type 304 Stainless Steel
9. Test section, 17-mil thickness	20. Flow area, 1.750 x 0.437 in.
10. Clamp-on bus bars; copper	21. Butt-weld location
11. End piece; Type 304 Stainless Steel	

(A)

(B)

Fig. 3.4. Test-section Assembly, (A) Test Section in Place and (B) Transition Piece

IV. EXPERIMENTAL ELECTRONIC EQUIPMENT

In this chapter is described the electronic equipment which was required to modulate the power to the test section and to crosscorrelate the resulting transient void-fraction signal.

A. Power Oscillator

The main features of this power oscillator have been described elsewhere,²³ so that only the main characteristics along with changes made to the system have been detailed. Essentially, some of the test element current was bypassed through a shunt of variable resistance which had been connected in parallel with the test section. A test had shown²³ that the percentage change in the test element power was approximately equal to the percentage of the total current which flowed through the shunt resistance and that the response time was less than one cycle of the 60-cycle power.

1. Control Unit for Power Oscillator

The electronic switching control unit is pictured in Fig. 4.1, and a diagram of a control subunit is detailed in Fig. 4.2.

(A) CONTROL CABINET

(B) SCR MODULAR COOLING UNIT

Fig. 4.1. Control Unit for Power Oscillator

Fig. 4.2. Control Subunit for Power Oscillator

The current through each shunt resistor was governed by two controlled silicon rectifiers (SCR's), type Westinghouse Trinistor-2N-1911 in a full-wave arrangement. The rectifiers would not conduct unless trigger pulses were applied at the gate electrodes. Avion Type 410-13 magnetic trigger units were employed for this purpose. By varying the d.c. control current from 0 to 2.5 mA, the trigger pulses could be delayed from 0 to 180°. The control current was supplied by a triode circuit which had the cathode biased at approximately 4-V intervals. Then by applying a sinusoidal control signal to all fifteen grids, the power regulator reproduced a sinusoidal power modulation.

Since the new silicon controlled rectifiers had a current-carrying capability of 70-Amp forward current and the number of shunt resistors in parallel with the test section totaled thirty (15 pairs), the current-carrying capability was a safe 750 Amp which corresponded to power modulation of 25% peak-to-peak at maximum power (94 kW). A forced-convection modular semiconductor cooling unit [see Fig. 4.1(B)] was employed to mount and control the silicon controlled rectifiers.

In order to provide a shunt path for transient overvoltages which could damage the SCR's, three transient voltage-surge suppressors having a 65-Amp maximum discharge current were connected in parallel with the test section and shunt.

The power supply for the control unit of the power oscillator was constructed by the Electronics Group of the Heat Engineering Section at ANL, and the entire unit was cooled by a forced-convection fan mounted in the cabinet [see Fig. 4.1(A)].

2. Variable Resistance Shunt for Power Oscillator

The current shunt, which consisted of fifteen pairs of fixed resistors connected in parallel with the test element, is pictured in Fig. 4.3. The resistors, made of Nichrome strips: 3/16-in. wide, B & S gauge 31, 0.3177 ohm/ft, were 12 ft long, and cooling was provided by two fans in forced air convection. Each pair of resistors was held in tension by a spring turnbuckle arrangement, and all were so arranged as to provide good electrical contact with the traveling busbar. In order to reduce electrical resistance between the busbar and resistors, all Nichrome strips were copper plated and then silver plated.

(A) SIDE VIEW

- 1. RESISTANCE RIBBON
- 2. MOTOR DRIVE

(B) FRONT VIEW

- 3. BUS BAR
- 4. SPRING TENSION CONTROL

Fig. 4.3. Variable Resistance Shunt for Power Oscillator

The solid copper busbar ($1/2 \times 1 \times 30$ in.) was also silver plated and notched in such a manner as to provide maximum area contact with each resistor and also to keep each resistor aligned [see Fig. 4.3(B)]. The vertical traverse was motor driven and could be positioned at any shunt resistance setting between 0.25 and 1.90 ohms by means of a calibrated synchro readout. This arrangement allowed a continuously variable resistance setting which could be changed even while the power oscillator was in operation.

3. Control Signal for Power Oscillator

The sinusoidal control signal to the grids of the triode circuit was provided by a sine potentiometer which was driven over the frequency range from 0.01 to 10 cps by a speed servo which is described later.

This speed servo unit drove at exactly the same speed as the sine potentiometer to provide the sinusoidal forcing function of power, and also the sine-cosine potentiometers in order to crosscorrelate the transient void signal.

B. The Wave Analyzer

In order to allow investigation of signals having low signal-to-noise ratios, a crosscorrelation technique based on Fourier analysis was used to measure the amplitude and phase of the coherent void signal response. This technique has been used by many investigators^{20,23,56} and favored for transfer-function measurements in "out-of-pile" experiments such as those conducted with electrically heated loops.

An alternative method⁵⁷ uses a digital computer to correlate the input and output signals. This system had an inherent accuracy as good as the above method, but the time required to analyze the data was greatly reduced. If a good tape recorder, an analog-digital converter, and a digital computer were employed, it would be possible to record several variables simultaneously which could be individually correlated later with the forcing signal. However, there were three disadvantages to using this method in this experiment: first, the difficulty which would have arisen in assessing whether good data had been taken; second, the high cost of hardware; finally, all of the required equipment for this crosscorrelation analysis was readily available.

1. Description of the Wave Analyzer

In Fig. 4.4 is given a picture of the overall physical layout of equipment for this experiment; in Fig. 4.5, the electronic instrumentation has been block diagrammed. In Fig. 4.6 the wave-analyzer circuitry has been diagrammed in detail.

1. FOUR-CHANNEL RECORDER
2. FLOW FREQUENCY CONVERTER
3. LOW-LEVEL AMPLIFIERS
4. LOW-FLOW POWER TRIP
5. OPERATIONAL AMPLIFIERS
6. POWER SUPPLY (100 mA)
7. POWER SUPPLY (300 mA)
8. LOW-PASS FILTERS
9. TWO-PEN RECORDER
10. POWER: WATT TRANSDUCER
11. WAVE ANALYZER CHASSIS
12. POWER: PRESSURE TRANSDUCER
13. VOID: NEGATIVE POWER SUPPLY
14. VOID: CURRENT AMPLIFIERS
15. VOID: RECORDER INPUT CONTROL
16. CONTROL: POWER SHUNT TRAVERSE
17. CONTROL: VOID TRAVERSE
18. CONTROL: SERVO SYSTEM

Fig. 4.4. Electronic Instrumentation

112-5042

Fig. 4.5. Block Diagram for Electronic Instrumentation

112-5041

Fig. 4.6. Circuit Diagram for Wave Analyzer

Any signal which entered the wave analyzer had its average signal bucked out, leaving only the time-dependent component. The resulting signal now consisted of three main components:

- i. a remaining d.c. level, v_E , due to small errors in bucking voltage and drifts in the average signal during the run;
- ii. a noise voltage, v_N , due to boiling and source noise;
- iii. the response signal, v_R , resulting from power modulation.

Thus, the total signal is given by

$$v_T(t) = v_E(t) + v_N(t) + v_R(t).$$

This signal was then fed to a low-noise, chopper stabilized amplifier which could increase the signal level by factors of 10 to 1000. The amplifier output was filtered by a m-derived filter with a 20-cps cutoff and a zero-frequency d.c. attenuation of 7.87. Then the resulting signal was fed to an

operational amplifier where its amplitude was increased by a factor of 1000. Now the total signal, v_T , together with the inverted signal, was fed to the two sides of a sine-cosine potentiometer, the midpoint of which was grounded. This potentiometer had two wipers spaced 90° apart, and the output from these wiper terminals was $v_T(t) \cos \omega t$ and $v_T(t) \sin \omega t$.

The sine-cosine potentiometer was of the carbon film type which developed negligible noise. It was geared to the servo system which drove the sine potentiometer and triggered the power oscillation. Thus, both the forcing function and sine-cosine potentiometer had exactly the same phase. The wiper terminal outputs were fed to integrators and then the integrated signals were recorded on a $1/4$ -sec. potentiometric recorder. The input and output of the integrating circuit had adjustable coefficient potentiometers to facilitate the variation of the time interval over which the signal was analyzed.

To insure that the steady-state value of the signal had been bucked out and to obtain an indication of system drifts during the run, the signal from the operational amplifier was also fed to a null integrator circuit whose output was recorded on a Sanborn recorder. This wave analyzer had the capability of analyzing two signals simultaneously, viz., void and power, which minimized drift that would have resulted if the signals were analyzed separately in succession and also speeded up the acquisition of data.

2. Operation of Wave Analyzer

With the power oscillator running at the desired preset frequency, both the power and void signals were simultaneously fed into the wave analyzer. Then the bucking voltages were adjusted to obtain null voltages. The phase of the sine-cosine potentiometers were then adjusted by rotating the ganged potentiometers until a zero phase lag was obtained between that of the power signal and the sine-cosine potentiometer. After this procedure, both signals were analyzed by letting the pens deflect over the full ± 5 in. of recorder chart available, and integration over a great number of cycles was insured by varying the input and output potentiometer settings.

3. Calibration of Wave Analyzer

To insure that the gains were the same in all of the four channels of the wave analyzer and the two nulling circuits, an internal calibration was provided. In this calibration, 8.4 V were made input to all of the integrating circuits and the deflection of all recording pens was noted, so that they all deflected equal amounts in the same time interval.

Provision was also made for a nulling arrangement which placed a shorted input into the wave analyzer. This allowed a quick check of the balance of all the amplifiers. Both of the above electronic checks were performed frequently during each run.

A final calibration was made by use of a Micarta disk which was mounted on the gamma-detector carriage and rotated in the gamma beam. This disk was so cut as to provide a sinusoidal oscillation in the void signal which corresponded to the collapse of a certain amount of steam in the test section. A synchro transmitter driven by the servo unit drove a synchro repeater which, in turn, rotated the disk with the same frequency as the sine-cosine potentiometer. The equivalent void change $\Delta\alpha$ caused by the disk was found by a separate calibration. Thus the known signal $\Delta\alpha \sin(\omega t + \phi)$ was fed to the wave analyzer to provide a check on the total gain G_T of the system.

4. Theory of Wave Analyzer

If we assumed that the total gain of a signal was G_T , then the equations for the input voltage to the recorder would be the following (note that in this analysis the phase of the filters was not taken into account since this was corrected for as shown in the next section):

$$\text{Sine channel} = -G_T \int_0^T [v_E(t) + v_N(t) + v_R(t)] (\sin \omega t) dt;$$

$$\text{Cosine channel} = -G_T \int_0^T [v_E(t) + v_N(t) + v_R(t)] (\cos \omega t) dt.$$

If $v_E(t)$ was constant with time, then this term would add a sinusoidally varying signal to the recorder reading. However, if the loop parameters were closely controlled and the bucking voltage was carefully adjusted, then $v_E(t)$ would have been very small and its neglect would result in negligible error. This was insured by use of the null integrator circuit. Since $v_N(t)$ was a random signal with zero average, then when it was multiplied by the sine and cosine and integrated over a great number of cycles, it would still give a zero average. The voltage $v_R(t)$ was a periodic signal with period $2\pi/\omega$ which could be expanded into a Fourier series.

$$v_R(t) = \sum_{N=1}^{\infty} A_N \sin(N\omega t + \phi_N),$$

where

$$\phi_N = \tan^{-1} a_N/b_N.$$

Substitution into the preceding expressions yields (neglecting the contributions of v_E and v_N):

$$\text{Sine channel} = E_S = G_T A_1(\tau/2) \cos \phi_1;$$

$$\text{Cosine channel} = E_C = G_T A_1(\tau/2) \sin \phi_1.$$

Here A_1 was the amplitude and ϕ_1 the phase of the first harmonic of $v_R(t)$. The quantities E_S and E_C were the deflections of the pens after time τ . Solving for A_1 and ϕ_1 , we obtained

$$A_1 = (2G_T/\tau) \sqrt{E_S^2 + E_C^2};$$

$$\phi_1 = \tan^{-1} (E_C/E_S).$$

The value of G_T was determined by a calibration process previously described and also was computed using the actual electronic gains of the components of the wave analyzer.

5. Components of Wave Analyzer

The major components of the wave analyzer were the following: two sine-cosine potentiometers, a wave-analyzer chassis, four m-derived filters, analog computer, and recording equipment.

The two sine-cosine potentiometers were of the carbon film type with a resistance of 16 kohms per quadrant. Both potentiometers were mechanically coupled in phase, and negligible noise was developed in these units.

In order to simplify the operation and to condense the associated equipment required to operate the wave analyzer, a wave-analyzer chassis was built (see Fig. 4.4). This chassis incorporated the bucking voltages, input and output attenuation potentiometers, and the necessary interconnections to the additional equipment.

All four of the filters which were employed in this analysis were two-stage, low-pass, with m-derived end sections. The components used in their construction were all matched to less than $\pm 1\%$ deviation, and their frequency responses have been plotted in Fig. 4.7. There was no measurable deviation in relative amplitude (output over input voltage) between any of the four filters in the frequency range from 0.01 to 5 cps; the phase agreed to $\pm 2\%$ over the same range.

Fig. 4.7. Frequency Responses of m-derived Filters.

The analog computer or operational amplifiers used in the wave analyzer were Philbrick USA-4JX with an open loop gain of one billion and noise of the order of $10\text{ }\mu\text{V}$ a.c. average referred to the summing point at a gain of 100. These units (see Fig. 4.4) were constructed by the Heat Engineering Laboratory Electronics group.

All of the recorders used in the wave analyzer were of the potentiometric type with 1/4-sec full-scale response across 10 in. of chart, except those used for outputs of the null integrating circuits which were recorded on the Sanborn recorder.

C. Speed Servo Unit

The speed servo unit used was identical with that employed by Christensen²³ and consisted essentially of a servo preamplifier, servo amplifier, servo drive motor, feedback tachometer, and a system of gears and clutches to provide a speed ratio which ranged over three decades.

The speed servo served three main purposes:

- i. drove a sine potentiometer over a speed range of 0.01 to 10 cps to provide the control signal for the power oscillator;
- ii. drove both sine-cosine potentiometers at the same speed as the sine potentiometer above;
- iii. drove a synchro transmitter which ran the calibration disk for the wave analyzer.

The control unit for the speed servo is pictured in Fig. 4.4.

V: MEASUREMENT OF LOOP VARIABLES

Five main loop variables had to be accurately measured in this analysis: temperature, inlet flow rate, pressure, heating power, and void fraction.

A. Temperature Measurements

All temperature measurements were made with Chromel-Alumel thermocouples which were previously calibrated on the 2000-psia hypsometer in the Heat Engineering Laboratory.

1. Wall Temperatures

To obtain fast response of the thermocouples mounted on the test section, a bare bead was used. This bead, made from 12-mil wire (30 gauge), was pressed between a 1-mil mica sheet which provided electrical insulation from the test section and a 1/16-in. sheet of Durabla to provide electrical and thermal insulation from the backup plates. Ten thermocouples were installed equidistantly along the test section, and four were continually recorded on a 6-point recorder. For more accurate measurements the Sanborn recorder or student potentiometer was used.

2. Water Temperatures

The most important liquid temperature was that at the inlet of the test section. However, four liquid temperatures were recorded: at the test-section inlet, transition inlet, flowmeter inlet, and test-section exit.

All couples were Chromel-Alumel, with a 60-mil, Inconel sheath diameter, which contained the 30 gauge wire. Thermocouple response was approximately 50-150 msec. These couples were all calibrated on the hypsometer and agreed to $\pm 1^\circ\text{F}$.

In order to obtain accurate inlet temperatures two thermocouples were employed, one at the middle, 0.875 in., and the other at 0.250 in. from the wall, (see Fig. 3.3), and their mean readings noted. The inlet, transitional, and flowmeter temperatures were recorded on a multipoint potentiometric recorder. This provided a valuable check on system drifts, but for accurate readings a portable precision potentiometer was used. By use of the calibration curve the error in the inlet temperature measurement was believed to be less than $\pm 1^\circ\text{F}$.

B. Inlet Flow Rates

The rate of flow of water was measured before the inlet to the test section by the two following methods: a turbine flowmeter and a 0.5791-in. orifice meter.

1. Turbine Flowmeter

A series 5000 Pottermeter, which had a nominal meter size of 3/4 in. and a linear flow-rate range from 2.0 to 35 U.S. gpm, with a response time of less than 5 msec, was used for flow measurement. This meter operated on the following principle: the turbine carried a permanent magnet and variations in magnetic field were sensed by a pick-up coil placed outside the high-pressure region. This sinusoidal voltage had a frequency and amplitude proportional to flow, and by the use of a frequency converter a d.c. signal suitable for recording on the Sanborn was obtained. The accuracy of this meter was guaranteed to be within $\pm 0.02\%$ of the reading in the flow range from 2.0 to 29.0 U.S. gpm, with a linearity of $\pm 1/4\%$ of the reading.

2. Low-flow Trip

In order to prevent burnout of the test section from loss of flow, a low-flow trip unit was designed and built. This safety device used a meter relay to trip a msec relay, which interrupted power to the test section if the output from the turbine flow meter fell below a preset level (see Fig. 4.4).

3. Orifice Meter

Flow was also measured before the Pottermeter by use of a 0.5791-in.-diameter orifice used in conjunction with two high-pressure manometers filled with manometer fluids of specific gravities 1.75 and 2.95. After the flow had been set on the turbine meter, the manometer was used for fine flow adjustment. Because of the large inertial and capacitance lags in the manometers, the turbine meter was more accurate for transient measurements. The orifice meter was calibrated in place in the loop and met standards listed in Fluid Meters.⁵⁸

C. Pressure Measurements

Two kinds of pressure measurements were made: total loop pressure and pressure differentials across the heated test section.

1. Loop Pressure

The saturation pressure in the steam separator was read on a Heise gauge with the dial face graduated in 2-psi increments from 0 to 2000 psig. This gauge was calibrated before, in the middle, and at the end of the experimental runs to ensure an accuracy of ± 1 psi.

2. Pressure Differentials

The pressure drops across both the heated test section and riser were measured with ± 5 -psi strain-gauge differential pressure transducers.

The natural frequency of the transducer and lines was calculated to be 10 cps, and measurements made during steady-state tests indicated a recurrent frequency of about 9 cps which was probably due to resonance in the transducer and lines. To insure reliability, the transducers were calibrated before, during, and at the end of the runs.

Provision was made to record and analyze both signals, but considerable difficulty was encountered when trying to correlate the pressure-drop data. This was believed to have resulted from a low signal-to-noise ratio, and thus no measurements on the pressure-drop response were obtained while the power-to-void transfer function was measured.

D. Measurement of Heating Power

A voltage suitable for recording and for feeding to the wave analyzer, which represented power to the test section, was obtained by use of a watt transducer. The current through the current transformer and the voltage drop across the test section were both fed to a Beckman Watt Transducer whose output was the vector product of load current and load voltage. This transducer operated on the principle of the Hall effect which may be stated simply as: the generation of a voltage across the opposite edges of an electrical conductor carrying current and placed in a magnetic field. The Lorentz force is the basis for this effect, which depends upon the deflection of charged particles moving in a magnetic field. This force is in a direction mutually perpendicular to the path of the particle movement and the direction of the magnetic field.

An output consisting of a double-frequency wave superimposed upon d.c. was obtained from the solid-state Hall multiplier circuit. The double-frequency component was proportional to volt-amperes while the d.c. component was proportional to watts. The watt transducer had four ranges: 0-50, 0-150, 0-250, and 0-500 W, with a linearity of 1% of the total output range. The line frequency could range from 50 to 500 cps, and response time was less than $1/10$ of a cycle. The input wattage range for our operation was from 50 to 150 W, so that the 0-250 scale was used in order to operate in the midscale range. The watt transducer was calibrated in place on the loop against a precision wattmeter, and the error was within that specified for linearity.

Before the signal could be recorded, the a.c. component had to be filtered. A two-stage, low-pass filter (described earlier) was used, and then the filtered signal was fed to the low-level preamplifier of the Sanborn for recording. This same signal was also fed to the wave analyzer in order to obtain the phase and amplitude of the forcing function.

The average loop power was read on a Weston Model 310 indicating wattmeter, for which the main error was due to the current transformer but estimated to be within $\pm 1\%$.

E. Measurement of Void Fraction

The measuring procedure for void fraction was identical with that used by Christensen,²³ which was the gamma-ray attenuation technique developed at the Heat Engineering Laboratory at ANL. This method has been described by Hooker and Popper⁵⁹ and by Petrick and Swanson.⁶⁰ The radiation source was a standard thulium-170 source developed at ANL.⁶¹

The block diagram for this system has been shown in Fig. 4.5 and pictured in Fig. 5.1. The signal from the gamma detector, which consisted of a standard sodium iodide crystal and type EMI-9536S photomultiplier tube, was fed to a Keithly micro-microammeter d.c. current amplifier. This amplifier was accurate to $\pm 2\%$ of full scale, drift was less than 2% of full scale in 24 hr, and at all ranges used the frequency response was better than 1000 cps. The 0-5-V output of the Keithly was filtered and utilized in two ways. For measurements of average void fractions, a portion of the average output of the signal was bucked out and divided to allow recording on a potentiometric recorder. This same signal was also sent to the wave analyzer to be correlated in terms of amplitude and phase.

LEGEND

- 1. VOID FRACTION WINDOW
- 2. CALIBRATION DISC
- 3. PHOTOMULTIPLIER TUBE HOLDER
- 4. LEAD COLLIMATOR
- 5. SOURCE HOLDER
- 6. DETECTOR CARRIAGE

Fig. 5.1. Void Traversing Equipment

On the Small Scale Loop, the void-detector system had provision for both horizontal and vertical traversing of the test section. A fine and coarse synchro indicated the vertical position of the table while the detector carriage (see Fig. 5.1) ran on top of the traverse table between micro switch stops to give the horizontal traverse.

1. Errors in Void Measurement

In this study we were concerned with the accuracy of two different void measurements: the transient void response, and the average void distributions in the transverse and axial directions. An excellent

summary and discussion of the sources of errors, when using the gamma-attenuation technique, has been given by Christensen.²³

The possible sources of error were the following:

i. errors due to drifts in the electrical components of the void detector system. These specifically were: drifts in the negative high-voltage supply to the photomultiplier (PM) tube, changes in gain of the crystal and PM tube due to temperature changes, and drifts in the current amplifier and recorder. The drift errors were reduced by employing quality components which had a high degree of stability and regulation, and by daily calibrations. Errors in gain were reduced by maintaining both the crystal and PM tube at a constant ($\pm 1/2^\circ\text{F}$) low temperature ($40-50^\circ\text{F}$). These errors have a probable cumulative value of approximately 1%;

ii. noise due to thulium-source statistics and boiling. These were eliminated by filtering the output of the current amplifier employing a m-derived low-pass filter;

iii. errors due to distributed source spectrum. Since the spectrum of the source was not monoenergetic, it was possible to have different attenuations of the gamma rays of different energies, but a test has shown that the attenuation could be accounted for by a single absorption coefficient. The errors here have been estimated to be 1%;

iv. detector nonlinearity. Since the output of the void detector can be a noisy signal due to boiling noise, then any averaging procedure used to find a mean signal level would be a source of errors. However, for void variations as large as 20% the maximum error is less than 1%;

v. preferential phase distributions and incomplete traversing. There are two extreme types of phase distributions possible which yield different outputs from the void detector. If there is complete separation of the phases (viz., annular flow) and the gamma beam is parallel to each phase, then

$$\alpha = \frac{V_\alpha - V_F}{V_E - V_F}.$$

However, if the phases are homogeneous (viz., bubble flow) and the gamma beam is perpendicular to the phases, then

$$\alpha = \frac{\ln(V_\alpha/V_F)}{\ln(V_E/V_F)}.$$

Since the void distributions in this study were closer to the latter case, this equation was used in all void calculations. The error here would

have a maximum of 5% at a void fraction of 50%, but would be less than the maximum in this study since it is doubtful that annular flow was ever obtained.⁶⁵

The comparison of void fractions predicted by the "one shot" method and by traversing with a narrow window has been given by Petrick⁶⁶ and Richardson.⁶⁷ It was found that the traversing technique was superior to the "one shot" method, especially for preferential phase distributions. To reduce errors in the measurement of average void fraction, a narrow ($1/32$ in.) window was used, the test section traversed in both directions over the smaller width, and the results integrated. However, for the majority of the transient void measurements the large ($7/8$ in.) window was used, for which there was a greater possibility for errors due to incomplete scanning. Since the region closest to the walls would have the highest probability of being omitted, this would tend to give higher readings than the true in the upper part of test section, where parabolic void profiles are present. The converse case would occur in the lower region, where the void is concentrated close to the wall. The total cumulative magnitude of the above errors in the average void distributions is believed to be less than 10% and approximately the same in the measurements of transient void amplitude.

VI. THEORETICAL DERIVATION OF POWER-TO-VOID TRANSFER FUNCTIONS

In the following derivation of the power-to-void transfer function, the one-dimensional macroscopic conservation laws⁶² were used together with various theoretical models^{47, 63} and correlations,⁶⁴ which related void fraction and quality. The basic equations have been derived previously.^{25, 46, 49} The treatment of the subcooled boiling region was based upon Bowring's work⁵³ and that of the bulk-boiling region upon the paper of Hudson et al.⁴⁶ The solution of the nonlinear differential equations was programmed for a large digital computer (CDC-3600) by first differencing them in the height variable and linearizing around steady state. Then these equations were solved by step-by-step integration, employing complex-variable procedures.

The transitional boundaries between the subcooled and bulk-boiling regions, and subcooled and nonboiling regions were also considered in a manner similar to that in the Kjeller model.^{51, 52}

A. Basic Equations

The macroscopic balances of mass, momentum, and energy for the constant-area flow system and also the energy balance on the channel wall, relating heat storage in the wall to heat transferred into the flowing system, will now be written.

1. Macroscopic Mass Balance:

$$\frac{d}{dt} \left[\alpha \rho_g + (1 - \alpha) \rho_\ell \right] = -\frac{1}{A} \frac{\Delta}{\Delta z} \left[w_\ell + w_g \right], \quad (6.1)$$

where the left-hand side represents the time rate of change of mass content per unit volume and

$$\frac{1}{A} \left[w_\ell + w_g \right] = \left[(1 - \alpha) \rho_\ell v_\ell + \alpha \rho_g v_g \right]$$

is the mass flow rate per unit area.

2. Macroscopic Momentum Balance:

$$\frac{d}{dt} \left[w_\ell + w_g \right] \Delta z = \frac{\Delta}{A} \left[\frac{w_\ell^2}{\rho_\ell (1 - \alpha)} + \frac{w_g^2}{\rho_g \alpha} \right] - A \Delta p_{gc} - \tau_{pc} \Delta z g_c - A g \left[\rho_\ell (1 - \alpha) + \rho_g \alpha \right] \Delta z, \quad (6.2)$$

where the left-hand side represents the time rate of change of the total momentum of the system. The four terms on the right side represent,

respectively, the net rate of momentum flux due to bulk fluid motion; the pressure differential across the control element; the net force of the solid surfaces on the fluid; and, finally, the force of gravity on the total mass of the fluid in the control volume.

Equation (6.2) can also be written in differential form as

$$\begin{aligned} \frac{\partial}{\partial t} \left[(1 - \alpha) \rho_l v_l + \alpha \rho_g v_g \right] + \frac{\partial}{\partial z} \left[(1 - \alpha) \rho_l v_l^2 + \alpha \rho_g v_g^2 \right] \\ = - \frac{\tau_{pc} g_c}{A} - g_c \frac{\partial p}{\partial z} - g \left[\rho_l (1 - \alpha) + \rho_g \alpha \right]. \end{aligned} \quad (6.3)$$

3. Macroscopic Energy Equation

$$\begin{aligned} A \Delta z \frac{d}{dt} \left\{ \alpha \lambda \rho_g + \alpha \rho_g C_g (T - T_0) + (1 - \alpha) \rho_l C_l (T - T_0) \right. \\ \left. + z \left[\rho_g \alpha + (1 - \alpha) \rho_l \right] \frac{g}{g_c} + \frac{1}{2 A^2 g_c} \left[\frac{w_l^2}{\rho_l (1 - \alpha)} + \frac{w_g^2}{\rho_g \alpha} \right] \right\} \\ = - \Delta \left[w_g \lambda + w_g C_g (T - T_0) + w_l C_l (T - T_0) \right] \\ - \Delta \left[\frac{\rho (w_l + w_g)}{\rho_g \alpha + (1 - \alpha) \rho_l} \right] - \frac{1}{2 g_c A^2} \Delta \left[\frac{w_l^3}{\rho_l^2 (1 - \alpha)^2} + \frac{w_g^3}{\rho_g^2 \alpha^2} \right] \\ - \frac{g}{g_c} \Delta \left[z (w_g + w_l) \right] + p_c h (T_w - T) \Delta z, \end{aligned} \quad (6.4)$$

where

$$A [\alpha \lambda \rho_g + \alpha \rho_g C_g (T - T_0) + (1 - \alpha) \rho_l C_l (T - T_0)] \Delta z$$

is the total internal energy of the system, U_{tot} ;

$$A \left[\frac{1}{2 A^2 g_c} \left(\frac{w_l^2}{\rho_l (1 - \alpha)} + \frac{w_g^2}{\rho_g \alpha} \right) \right] \Delta z$$

is the total kinetic energy of the system, K_{tot} ; and

$$A \left[z \rho_g \alpha + z (1 - \alpha) \rho_l \right] \frac{g}{g_c} \Delta z$$

is the total potential energy of the system, Φ_{tot} .

The first four terms on the right side of Eq. (6.4) represent, respectively, the spatial difference of internal energy, the expansion work due to pressure change, the kinetic energy, and potential energy. The final term is the thermal energy entering the fluid via the solid surface. Equation (6.4) can also be written in differential form as

$$\begin{aligned} \frac{\partial}{\partial t} \left\{ \alpha \rho_g \left[\lambda + C_g(T - T_0) + \frac{gz}{g_c} + \frac{v_g^2}{2g_c} \right] + (1 - \alpha) \rho_\ell \left[C_\ell(T - T_0) + \frac{gz}{g_c} + \frac{v_\ell^2}{2g_c} \right] \right\} \\ + \frac{\partial}{\partial z} \left\{ \alpha v_g \rho_g \left[\lambda + C_g(T - T_0) + \frac{gz}{g_c} + \frac{v_g^2}{2g_c} + \frac{p}{\rho_g \alpha + \rho_\ell (1 - \alpha)} \right] \right. \\ \left. + (1 - \alpha) \rho_\ell v_\ell \left[C_\ell(T - T_0) + \frac{gz}{g_c} + \frac{v_\ell^2}{2g_c} + \frac{p}{\rho_g \alpha + \rho_\ell (1 - \alpha)} \right] \right\} = \frac{p_c h (T_w - T)}{A} \end{aligned} \quad (6.5)$$

4. Energy Balance on Channel Wall

There results, from a simple energy balance on an incremental length of channel wall,

$$p_c t_w \rho_w C_w \frac{dT_w}{dt} + p_c h (T_w - T) = Q_{gen}. \quad (6.6)$$

In all of the preceding equations we have assumed that the time-smoothed velocity is parallel to the channel walls and the physical properties do not vary across the cross-sectional flow area.

B. Dimensionless Basic Equations

The following dimensionless variables are introduced:

$$\begin{aligned} w_g^* &= \frac{w_g}{w_{\ell,0}}; & w_\ell^* &= \frac{w_\ell}{w_{\ell,0}}; & T^* &= \frac{C_\ell g_c}{gH} T; \\ p^* &= \frac{\rho_\ell g_c A^2}{w_{\ell,0}^2} p; & t^* &= \frac{w_{\ell,0}}{\rho_\ell A H} t \\ Q^* &= \frac{Q_{gen}}{\bar{Q}_{gen}}; & z^* &= \frac{z}{H}; & \tau^* &= \frac{A p_c H g_c \rho_\ell}{w_{\ell,0}} \tau. \end{aligned}$$

Equations (6.1), (6.2), (6.4), and (6.6) respectively reduce to

$$(1 - \gamma) \frac{d\alpha}{dt^*} = \frac{\Delta}{\Delta z^*} (w_g^* + w_\ell^*); \quad (6.7)$$

$$\frac{d}{dt^*} (w_\ell^* + w_g^*) = -\tau^* - \frac{\Delta p^*}{\Delta z^*} - a_1(1 - \alpha + \alpha\gamma) - \frac{\Delta}{\Delta z^*} \left[\frac{w_\ell^{*2}}{1 - \alpha} + \frac{w_g^{*2}}{\alpha\gamma} \right]; \quad (6.8)$$

$$\begin{aligned} & \frac{d}{dt^*} \left\{ \alpha\gamma \left[1 + a_4(T^* - T_0^*) + a_3z^* + 0.5a_5 \left(\frac{w_g^*}{\alpha\gamma} \right)^2 \right] + (1 - \alpha) \left[a_3(T^* - T_0^*) + a_3z^* + 0.5a_5 \left(\frac{w_\ell^*}{1 - \alpha} \right)^2 \right] \right\} \\ &= -\frac{\Delta}{\Delta z^*} \left\{ w_g^* \left[1 + a_4(T^* - T_0^*) + a_3z^* + 0.5a_5 \left(\frac{w_g^*}{\alpha\gamma} \right)^2 + \frac{a_5 p^*}{1 - \alpha + \alpha\gamma} \right] \right\} \\ &- \frac{\Delta}{\Delta z^*} \left\{ w_\ell^* \left[a_3(T^* - T_0^*) + a_3z^* + 0.5a_5 \left(\frac{w_\ell^*}{1 - \alpha} \right)^2 + \frac{a_5 p^*}{1 - \alpha + \alpha\gamma} \right] \right\} + a_6(T_w^* - T^*) \Delta z^*; \end{aligned} \quad (6.9)$$

$$\frac{dT_w^*}{dt^*} + a_7(T_w^* - T^*) = a_8 Q^*, \quad (6.10)$$

where

$$\begin{aligned} a_1 &= \left(\frac{A\rho_\ell}{w_{\ell,0}} \right)^2 gH; & a_2 &= \frac{a_1}{gH}; & a_3 &= \frac{gH}{g_c \lambda}; & a_4 &= \frac{a_3 C_g}{C_\ell}; & a_5 &= \frac{a_3}{a_1}, \\ a_6 &= \frac{p_c h g H^2}{C_\ell w_{\ell,0} \lambda g_c}; & a_7 &= \frac{A\rho_\ell h H}{\rho_w C_w t_w w_{\ell,0}}; & a_8 &= \frac{\rho_\ell C_\ell A \bar{Q}_g g c}{\rho_w C_w t_w p_c w_{\ell,0} g}; \\ a_9 &= \frac{a_6 a_8}{a_7} \end{aligned}$$

Also, by introducing

$$v_\ell^* = v_\ell / v_{\ell,0}; \quad v_g^* = v_g / v_{\ell,0},$$

we can simplify Eqs. (6.1), (6.3), and (6.5) into

$$(\gamma - 1) \frac{\partial \alpha}{\partial t^*} + \frac{\partial}{\partial z^*} \left[(1 - \alpha) v_\ell^* + \alpha \gamma v_g^* \right] = 0; \quad (6.11)$$

$$\frac{\partial}{\partial t^*} \left[(1 - \alpha) v_\ell^* + \alpha \gamma v_g^* \right] + \frac{\partial}{\partial z^*} \left[(1 - \alpha) v_\ell^{*2} + \alpha \gamma v_g^{*2} \right] = -\tau^* - \frac{\partial p^*}{\partial z^*} - a_1 \left[1 - \alpha + \gamma \alpha \right]; \quad (6.12)$$

$$\begin{aligned}
& \frac{\partial}{\partial t^*} \left\{ \alpha \gamma \left[1 + a_4(T^* - T_\delta^*) + a_3 z^* + 0.5 a_5 v_g^{*2} \right] + (1 - \alpha) \left[a_3(T^* - T_\delta^*) + a_3 z^* + 0.5 a_5 v_\ell^{*2} \right] \right\} \\
& + \frac{\partial}{\partial z^*} \left\{ \alpha \gamma v_g^* \left[1 + a_4(T^* - T_\delta^*) + a_3 z^* + 0.5 a_5 v_g^{*2} + \frac{a_5 p^*}{\alpha \gamma + 1 - \alpha} \right] \right. \\
& \left. + (1 - \alpha) v_\ell^* \left[a_3(T^* - T_\delta^*) + a_3 z^* + 0.5 a_5 v_\ell^{*2} + \frac{a_5 p^*}{\alpha \gamma + 1 - \alpha} \right] \right\} = a_6(T_w^* - T^*).
\end{aligned} \tag{6.13}$$

In obtaining Eqs. (6.7) through (6.13) there have been introduced the additional assumptions that the physical properties of the liquid and vapor are constant in time and space, and are referenced to the total system pressure. These equations represent the dimensionless integral forms of the conservation laws in one dimension for a flow channel of constant area. In Eqs. (6.7) through (6.13) we can make additional assumptions which greatly facilitate their solution without obscuring any essential features. First, a constant pressure drop is assumed across the coolant channel, and pressure variations within the channel are not taken into account. This eliminates the compression-energy terms, and allows the continuity and energy equations to be uncoupled. Secondly, the potential and kinetic energy terms can be neglected, which, physically, is a good assumption in the range of flow rates and pressures covered in this analysis.

The conservation laws will now be linearized around a steady-state value, sectioned in the height variable, and solved by a step-by-step integration employing a complex-variable procedure. The boundary conditions imposed at the inlet are constant flow and temperature while a constant pressure is assumed at the exit to which the physical properties of the fluid are referenced.

C. Nonboiling Region

In the nonboiling region, the subcooling is high and the voidage is due to wall effects, since no free steam bubbles can exist in the liquid. In this case the macroscopic mass balance Eq. (6.7) reduces to

$$\frac{\Delta w_\ell^*}{\Delta z^*} = 0 \tag{6.14}$$

and

$$w_\ell^* = w_{\ell,0}^* = 1.$$

Also, the macroscopic energy Eq. (6.9) simplifies to

$$\frac{dT^*}{dt^*} + w_{\ell,0}^* \frac{\Delta T^*}{\Delta z^*} = \frac{a_{10}}{a_3} (T_w^* - T^*), \tag{6.15}$$

where

$$a_{10} = p_c h_{sp} g H^2 / C_{\ell} w_{\ell,0} \lambda g_c$$

For the steady-state case, Eqs. (6.15) and (6.10) yield the axial temperature distribution:

$$\frac{\Delta T^*}{\Delta z^*} = \frac{a_{10} a_8}{a_3 a_{11}} \frac{Q^*}{w_{\ell,0}^*} \quad (6.16)$$

for the nonboiling region, where

$$a_{11} = A \rho_{\ell} h_{sp} H / \rho_w C_w t_w w_{\ell,0}$$

1. Transition Boundary

The transition boundary, z_{TB}^* , is the point at which the subcooled boiling initiates and a rapid rise in void fraction occurs. Following Bowring's analysis,⁵³ the criterion for location of z_{TB}^* is temperature dependent and the transition subcooling, ΔT_{TB} , at this point is obtained from the least of:

i. ΔT_{sub} , the inlet subcooling;

$$\text{ii. } \Delta T_{scb} = (\phi / h_{sp}) - \beta \phi^{\frac{1}{4}}, \quad (6.17)$$

the Jens-Lottes equation, which gives the condition for initiation of subcooled boiling based upon the wall temperature superheat; and

$$\text{iii. } \Delta T_d = \eta \phi / v_0, \quad (6.18)$$

which relates the subcooling at which bubbles can detach,⁵³ assuming that boiling has started, to the heat flux and inlet velocity.

The axial height at which the transition boundary is located for steady state is easily obtained from Eq. (6.16):

$$\bar{z}_{TB}^* = (\bar{T}_{TB}^* - \bar{T}_0^*) \left(\frac{a_3 a_{11} w_{\ell,0}^*}{a_{10} a_8 Q^*} \right) \quad (6.19)$$

2. Liquid Temperature Response

When the power to the test section is modulated, we have a resulting fluid temperature response in the nonboiling region which can be

obtained from Eqs. (6.10) and (6.15). Here as a first approximation the heat capacity of the wall will be neglected, and then Eq. (6.15) simplifies to

$$\frac{1}{2} \left[\frac{dT'_{N+1}}{dt} + \frac{dT'_N}{dt} \right] + \frac{\Delta T'}{\Delta z} = (a_{10}a_8/a_{31}) Q' \quad (6.20)$$

when sectioned in the height variable z and linearized around steady state.

NOTE: The superscript $*$ will now be eliminated from the dimensionless variables, and the following notation will be adopted for the remainder of the chapter:

$\bar{T}$ denotes the steady-state time-averaged value of T ;

$T = \bar{T} + T'$, where T' represents the deviation of T from steady state; and

$T' = \text{Re}(T^c e^{i\omega t})$, where T^c is a complex quantity whose real and imaginary parts yield the amplitude and phase of the temperature response.

Now the power modulation Q' is of the form $\text{Re}(Q_0^c e^{i\omega t})$, and since we are considering small amplitude perturbations around steady state, where linear behavior is expected, then the temperature response T' will also be of the form $\text{Re}(T^c e^{i\omega t})$. Substitution of the two preceding values into Eq. (6.20) yields

$$T_{N+1}^c = \frac{[(w_{l,0}/\Delta z) - i(\omega/2)]}{[(w_{l,0}/\Delta z)^2 + (\omega/2)^2]} \left\{ \frac{a_{10}a_8Q_0^c}{a_{31}} + [(w_{l,0}/\Delta z) - i(\omega/2)] T_N^c \right\}. \quad (6.21)$$

Equation (6.21) can easily be separated into real and imaginary parts. By application of the boundary condition at the channel entrance: $z = 0$, $N = 0$, and $T_0^c = 0$, we can obtain T_{N+1}^c for all axial positions.

In practice, Eq. (6.18) determines $(\bar{T}_{TB} - \bar{T}_0)$, which in dimensionless form may be written

$$T_S - T_{TB} = C_\ell \eta Q_{gen} g_c / H_{pc} v_{0g}. \quad (6.22)$$

By expanding Eq. (6.22) in a Taylor series expansion around steady state, we obtain

$$T_{TB}^c = -(C_\ell \eta Q_{gen} g_c / H_{pc} v_{0g}) Q_0^c. \quad (6.23)$$

Thus, the total temperature variation contributing to the movement of the transition boundary is the sum of those obtained from Eqs. (6.21), the

water temperature increase from integral heat flux, and Eq. (6.23), which gives the prompt effect of the transition temperature due to the bubble detachment criterion at the transition boundary. The transient height of the transition boundary can be obtained simply from

$$z_{TB}(t) = \bar{z}_{TB} + (\overline{\partial z / \partial T}) T'_{TB}, \quad (6.24)$$

where $(\overline{\partial z / \partial T})$ is obtained from Eq. (6.16) and T'_{TB} is the total temperature response of the fluid at the transition boundary as obtained by integration of Eq. (6.21) up to the location of the steady-state transition point plus that due to bubble detachment effects resulting from power variation. This variation in position of the transition boundary will give variations in the evaporative and heating flow at the boundary.

D. Subcooled Boiling Region

In the subcooled boiling region we have five variables: void fraction $\alpha(t, z)$; liquid velocity $v_l(t, z)$; vapor velocity $v_g(t, z)$; liquid temperature $T(t, z)$; and wall temperature $T_w(t, z)$. Bowring's physical model⁵³ of the steady-state void fraction in this region was based upon experimentally observed detachment of bubbles at low subcooling in a heated channel, and two empirical parameters were employed: one, to relate the ratio of agitative to latent heat flux, and the other to relate subcooling at which bubbles detach to heat flux and inlet velocity. It was also found that the recondensation of free steam bubbles formed by the latent heat flux was negligibly small for a constant-pressure system, and thus it was assumed that negligible energy exchange occurred between the vapor and liquid phases. This allows separate energy equations to be written for each phase. Bowring postulated that the total heat flux could be divided into three separate and distinct parts corresponding to an evaporative flux q_e and agitative flux q_h , and a single-phase heat transfer flux q_{sp} . Also, at the operating conditions of this experiment, q_{sp} is negligibly small compared to q_e and q_h , and the evaporative and convective fluxes are related simply: $q_h/q_e = \epsilon$. Then the total heat flux can be related to the evaporative heat flux by

$$q_e = q_{tot} / (1 + \epsilon) \quad (6.25)$$

and to the convective heat flux by

$$q_h = [\epsilon / (1 + \epsilon)] q_{tot}, \quad (6.26)$$

where ϵ is constant equal to 1.3.

1. Basic Equations

For the subcooled boiling region, the dimensionless macroscopic mass balance Eq. (6.7) becomes

$$\left(\frac{1-\gamma}{2}\right) \left(\frac{d\alpha_N'}{dt} + \frac{d\alpha_{N+1}'}{dt}\right) = \frac{\Delta}{\Delta z} (w_g' + w_l') \quad (6.27)$$

when sectioned in the height variable z and linearized around steady state. The dimensionless macroscopic energy equation for the liquid phase becomes

$$\begin{aligned} \left(\frac{\bar{T}}{2}\right)_{N+\frac{1}{2}} \frac{d}{dt} (-\alpha_{N+1}' - \alpha_N') + \left(\frac{1-\bar{\alpha}}{2}\right)_{N+\frac{1}{2}} \frac{d}{dt} (T_{N+1}' + T_N') \\ = -\frac{\Delta}{\Delta z} (\bar{w}_l T' + \bar{T} w_l') + (a_{12}/a_3)(T_w' - T')_{N+\frac{1}{2}}, \end{aligned} \quad (6.28)$$

where

$$a_{12} = p_c g H^2 h_{SB} \epsilon / C_l w_{l,0} \gamma g_c (1 + \epsilon).$$

Also, the energy equation for the vapor phase simplifies to

$$\left(\frac{\gamma}{2}\right) \frac{d}{dt} (\alpha_{N+1}' + \alpha_N') = -\frac{\Delta w_g'}{\Delta z} + a_{13} (T_w' - T')_{N+\frac{1}{2}}, \quad (6.29)$$

where

$$a_{13} = p_c H^2 h_{SB} g / C_l w_{l,0} \lambda (1 + \epsilon) g_c.$$

The energy balance Eq. (6.6) at the wall becomes

$$\frac{dT_w'}{dt} + a_{14} (T_w' - T') = a_8 Q', \quad (6.30)$$

where

$$a_{14} = A \rho_l h_{SB} H / \rho_w C_w t_w w_{l,0}.$$

Equations (6.27) to (6.30) and an expression relating quality and void fraction:

$$w_{g,N}' = L_N w_{l,N}' + M_N \alpha_N', \quad (6.31)$$

are sufficient to solve for the transient responses of void fraction, liquid and wall temperatures, and liquid and vapor velocities. The values of L_N and M_N in Eq. (6.31) depend upon the particular model or correlation chosen, as will be shown in a later section.

2. Void Fraction and Flow Response

From Eqs. (6.27) and (6.31) we can obtain

$$\frac{d\alpha'_{N+1}}{dt} + \frac{d\alpha'_N}{dt} = \Delta(bw'_\ell) + \Delta(c\alpha'), \quad (6.32)$$

where

$$b_N = 2(1 + L_N)/(1 - \gamma) \Delta z; \quad c_N = 2M_N/(1 - \gamma) \Delta z.$$

If Eqs. (6.32) and (6.31) are substituted into Eq. (6.29) and the terms rearranged, we find

$$\Delta(kw'_\ell) + \Delta(m\alpha') = a_{15}(T'_w - T')_{N+\frac{1}{2}} \quad (6.33)$$

where

$$a_{15} = 2a_{13}/\gamma;$$

$$k_N = b_N + (2L_N/\gamma \Delta z);$$

$$m_N = c_N + (2M_N/\gamma \Delta z).$$

Equation (6.33) can be written in iterative form as

$$w_{\ell, N+1}^c = \frac{1}{k_{N+1}} \left[k_N w_{\ell, N}^c + m_N \alpha_N^c + a_{15}(T_w^c - T^c)_{N+\frac{1}{2}} \right] \div (m_{N+1} \alpha_{N+1}^c / k_{N+1}). \quad (6.34)$$

Equation (6.32) also yields

$$\alpha_{N+1}^c = - \frac{(c_{N+1} + i\omega)}{(c_{N+1}^2 + \omega^2)} \left[- (c_N + i\omega) \alpha_N^c + b_{N+1} w_{\ell, N+1}^c - b_N w_{\ell, N}^c \right]. \quad (6.35)$$

After the expressions for T_w^c and T^c (which are developed in the next section) are substituted into Eq. (6.34), $w_{\ell, N+1}^c$ is substituted into Eq. (6.35) to obtain a recursion formula for α_{N+1}^c . From the real and imaginary parts of α_{N+1}^c the amplitude and phase of the void response can be calculated. Then, knowing α_{N+1}^c , $w_{\ell, N+1}^c$ can be computed by use of Eq. (6.34).

3. Liquid and Wall Temperature Responses

The energy balance at the wall, given by Eq. (6.30), can be solved to obtain

$$T_{w,N}^c = \frac{a_8 Q_0^c + a_{14} T_N^c}{\omega^2 + a_{14}^2} (a_{14} - i\omega). \quad (6.36)$$

The energy Eq. (6.28) for the liquid, the vapor energy Eq. (6.29), and the quality-void fraction correlation (6.31) can be solved to obtain

$$\Delta[(S + i\omega K_{N+\frac{1}{2}} + a_{16}) T^c] + \Delta[(Y - b L_{N+\frac{1}{2}}) w_\ell^c] = +\Delta[(c L_{N+\frac{1}{2}}) \alpha^c] + a_{17} Q_0^c, \quad (6.37)$$

where

$$\begin{aligned} S_N &= (a_3 \bar{w}_\ell / \Delta z)_N; & Y_N &= (a_3 \bar{T} / \Delta z)_N; \\ K_{N+\frac{1}{2}} &= [a_3(1 - \bar{\alpha})/2]_{N+\frac{1}{2}}; & L_{N+\frac{1}{2}} &= (a_3 \bar{T} / 2)_{N+\frac{1}{2}}; \\ a_{16} &= \frac{i\omega a_{12}(a_{14} - i\omega)}{2(a_{14}^2 + \omega^2)}; & a_{17} &= \frac{a_{12} a_8 (a_{11} - i\omega)}{a_{14}^2 + \omega^2}. \end{aligned}$$

Equation (6.37) can be separated into real and imaginary parts to allow computation of the amplitude and phase of the fluid temperature response at every axial section into which the channel has been differenced.

4. Bulk-boiling Boundary

The location of the bulk-boiling boundary is also temperature dependent, and it is the point where the liquid temperature reaches saturation as determined from the steady-state form of the energy equation for the liquid and wall energy balance in the subcooled region:

$$\frac{\Delta}{\Delta z} (w_\ell T) = \frac{a_{12} a_8 \bar{Q}}{a_3 a_{14}}. \quad (6.38)$$

Since $w_\ell = 1 - w_g = 1 - X$, the quality X is $\ll 1$ and changes very slightly in the subcooled region, we assumed w_ℓ to be constant to obtain

$$\frac{\Delta T}{\Delta z} = \frac{a_{12} a_8 \bar{Q}}{a_3 a_{14} w_{\ell,0}}, \quad (6.39)$$

from which we can easily solve for $\bar{z}_B$:

$$\bar{z}_B = \bar{z}_{TB} + \frac{(\bar{T}_S - \bar{T}_{TB}) a_3 a_{14} w_{\ell,0}}{a_{12} a_8 \bar{Q}}. \quad (6.40)$$

For the movement of z_B resulting from the power modulation, we add a first-order perturbation to Eq. (6.40):

$$z_B = \bar{z}_B + z_B^1, \quad (6.41)$$

where

$$z_B = \overline{(\partial z / \partial T)} T' \quad (6.42)$$

Here, $(\partial z / \partial T)$ is obtained from Eq. (6.39) and T' , the liquid temperature response, is calculated by a step-by-step integration of Eq. (6.37). This movement of the bulk-boiling boundary will cause variations in the evaporative and heating flow at the boundary.

5. Step-by-step Integration Procedure

When Eqs. (6.34), (6.35), and (6.37) are to be solved by a step-by-step integration procedure, the boundary conditions at the transition point must first be known. There are two possible cases: firstly, if the nonboiling region is of zero length, then we have the boundary condition at $N = 0$ and $z = 0$, namely, $T'_0 = \alpha'_0 = w'_{\ell,0} = 0$. Secondly, if the non-boiling region has some finite length, then the transition boundary condition gives at $N = N_{TB}$ and $z = z_{TB}$ that $T'_N = T'_{TB}$, $\alpha'_N = \alpha'_{TB}$, and $w'_{\ell,N} = w'_{\ell,TB}$. Now α'_{TB} can be obtained by calculating the quality change $(\partial X / \partial z) z'_{TB}$ associated with the movement of the transition boundary and then employing a quality-void fraction correlation to compute α'_{TB} associated with this quality change.

From a knowledge of the proper boundary condition, α'_{N+1} , $w'_{\ell,N+1}$, and T'_{N+1} can be calculated in their respective order by a stepwise procedure up to the location of the steady-state bulk-boiling boundary.

E. Bulk-boiling Region

The power-to-void transfer function for the bulk-boiling region has been derived previously by Hudson, Atit, and Bankoff⁴⁶ and more recently by Jain,⁴⁸ who took into account the effect of wall heat capacity. Here we have four variables: $\alpha(t, z)$, $v_\ell(t, z)$, $v_g(t, z)$, and $T_w(t)$, which can be solved for employing Eqs. (6.7), (6.9), (6.10), and (6.31).

1. Basic Equations

Proceeding in the same manner as employed in the subcooled boiling region, the macroscopic mass balance (6.7) and the quality-void fraction relationship (6.31) again simplify to

$$\frac{d\alpha'_{N+1}}{dt} + \frac{d\alpha'_N}{dt} = \Delta(bw'_\ell) + \Delta(c\alpha') \quad (6.43)$$

The macroscopic energy equation becomes

$$d_{N+1/2} \left[\frac{d\alpha'_N}{dt} + \frac{d\alpha'_{N+1}}{dt} \right] + e_{N+1/2} \left[\frac{dw'_{\ell,N}}{dt} + \frac{dw'_{\ell,N+1}}{dt} \right] = -\Delta(f\alpha') - \Delta(gw'_\ell) + a_6 T'_w \quad (6.44)$$

when the heat capacity of the wall is taken into account. Here,

$$d_{N+\frac{1}{2}} = B_{N+\frac{1}{2}} + D_{N+\frac{1}{2}} M_{N+\frac{1}{2}};$$

$$e_{N+\frac{1}{2}} = C_{N+\frac{1}{2}} + D_{N+\frac{1}{2}} L_{N+\frac{1}{2}};$$

$$f_N = E_N + G_N M_N;$$

$$g_N = F_N + G_N M_N;$$

$$B_{N+\frac{1}{2}} = \frac{1}{2} \left[\gamma - a_3 z (1 - \gamma) + \frac{a_5 \bar{w}_\ell^2}{2(1 - \bar{\alpha})^2} - \frac{a_5 \bar{w}_g^2}{\bar{\alpha}^2 \gamma} \right]_{N+\frac{1}{2}};$$

$$C_{N+\frac{1}{2}} = \left[(a_5/2) \bar{w}_\ell / (1 - \bar{\alpha}) \right]_{N+\frac{1}{2}};$$

$$D_{N+\frac{1}{2}} = (a_5/2) (\bar{w}_g / \bar{\alpha} \gamma)_{N+\frac{1}{2}};$$

$$E_N = (a_5/\Delta z) \{ -[\bar{w}_\ell / (1 - \bar{\alpha})]^3 - [(\bar{w}_g / \bar{\alpha})^3 \gamma^{-2}] \}_N;$$

$$G_N = (1/\Delta z) \{ 1 + a_3 z + (3a_5/2) (\bar{w}_g / \bar{\alpha} \gamma)^2 \}_N.$$

The coefficient $e_{N+\frac{1}{2}}$ was of the order 10^{-6} and was dropped from Eq. (6.44). Equations (6.43) and (6.44) can be simplified to

$$-p_{N+1} w_{\ell, N+1} + q_N w_{\ell, N} + r_N \alpha'_N - s_{N+1} \alpha'_{N+1} + a_6 T'_w = 0, \quad (6.45)$$

where

$$p_{N+1} = d_{N+\frac{1}{2}} b_{N+1} + g_{N+1};$$

$$q_N = d_{N+\frac{1}{2}} b_N + g_N;$$

$$r_N = d_{N+\frac{1}{2}} c_N + f_N;$$

$$s_{N+1} = d_{N+\frac{1}{2}} c_{N+1} + f_{N+1}.$$

For a sinusoidal power variation Eq. (6.10) yields

$$T'_w = a_8 Q_0 (a_7 - i\omega) e^{i\omega t} / (\omega^2 + a_7^2). \quad (6.46)$$

Substituting (6.46) into (6.45), we obtain

$$-p_{N+1} w_{\ell, N+1}^c + q_N w_{\ell, N}^c + r_N \alpha_N^c - s_{N+1} \alpha_{N+1}^c + U^c e^{i\omega t} = 0, \quad (6.47)$$

where

$$U^C = a_6 a_8 Q_0^C (a_7 - i\omega) / (a_7^2 + \omega^2).$$

2. Void Fraction and Flow Response

The expression for the transient flow response can be obtained from Eq. (6.47):

$$w_{\ell, N+1}^C = [(q_N w_{\ell, N}^C + r_N \alpha_N^C + U^C) / p_{N+1}] - (s_{N+1} \alpha_{N+1}^C / p_{N+1}), \quad (6.48)$$

after the real and imaginary parts have been separated. In order to obtain the general expression for the void response, Eq. (6.43) can be used:

$$\alpha_{N+1}^C = [(C_{N+1} + i\omega) / (C_{N+1}^2 + \omega^2)] [\alpha_N^C (i\omega - c_N) + b_N w_{\ell, N}^C - b_N w_{\ell, N+1}^C]. \quad (6.49)$$

Upon substitution of the expression for $w_{\ell, N+1}^C$ from Eq. (6.48) into Eq. (6.49), we obtain

$$\alpha_{N+1}^C = [C_{N+1} - i\omega - (b_{N+1} s_{N+1} / p_{N+1})]^{-1} \{ \alpha_N^C (i\omega - c_N) + b_N w_{\ell, N}^C - b_{N+1} [(q_N w_{\ell, N}^C + r_N \alpha_N^C + U^C) / p_{N+1}] \}. \quad (6.50)$$

Equation (6.50) can also be separated into real and imaginary parts, and the amplitude and phase of α_{N+1}^C calculated for every axial section into which the channel has been differenced.

3. Steady-state Void Fraction Distribution

In the bulk-boiling region the axial quality distribution can be obtained from the macroscopic energy equation, which for the steady-state case is simply

$$w_g = X = X_B + [a_6 a_8 (z - \bar{z}_B) / a_7], \quad (6.51)$$

where X_B is the axial quality at $\bar{z}_B$, the location of the steady-state bulk-boiling boundary as obtained in the previous section. The void fraction associated with this quality is obtained simply by use of Eq. (6.31) and the dimensionless liquid flow rate: $w_{\ell, N} = 1.0 - w_{g, N}$.

4. Step-by-step Integration Procedure

In order to solve Eqs. (6.48) and (6.50) for the flow and void-fraction response, we must first know the proper boundary conditions. There are two possible boundary conditions at the initiation of the bulk-boiling region: first, if the subcooled boiling region is of zero length, then

we have the bulk-boiling region extending over the entire length of the channel, and at $z = 0$ we have $N = 0$ and $\alpha'_0 = w'_{l,0} = 0$. The second boundary condition results when a subcooled boiling region exists which, at $z = z_B$, gives $N = N_B$, $\alpha'_N = \alpha'_B$, and $w'_{l,N} = w'_{l,B}$. Thus the exit conditions of the subcooled boiling region become the inlet conditions for the bulk-boiling region.

The effects of the temperature response in the subcooled boiling region on the movement of the bulk boiling boundary and the resultant effects on the evaporative and heating fluxes can be determined in two different ways. The fluid temperature response at the bulk-boiling boundary, T'_B , as determined from Eq. (6.37) can be used with Eq. (6.42) to calculate the movement of the boundary. Then the quality change resulting from this movement can be calculated and the void-fraction response determined employing one of the models. This would give an additive component to α'_B due to boundary effects which is in phase with the temperature response.

A second method could be employed in which the transient effects of the transitional boundary is spread over the whole of the subcooled region. Here the temperature response at each axial position is used to calculate a corresponding void fraction response as above, but the additive component is added to the void response determined from Eq. (6.35) in each axial section of the subcooled region. A comparison of these two methods will be shown in a later chapter.

F. Quality-void Fraction Models and Correlations

Three different models and correlations were used to calculate the steady-state axial void distributions and the transient void response which were then compared with the experimental data. A fourth method involved a linear fit of the slip ratio in the subcooled boiling region as a function of enthalpy.

1. Griffith Model

Griffith's model⁶³ for the prediction of low-quality boiling voids was used since the majority of the data obtained should have been in the slug-flow regime,⁶⁵ especially for the runs with low inlet subcooling. However, the validity of employing this model in the higher subcooled runs is doubtful, since in the lower regions of the test section the void fraction is concentrated at the wall and not in the center of the channel, and slug flow would not be obtained until well downstream in the test section.

The average void fraction is given by

$$\bar{\alpha} = G_g / (G_l + G_g + V_b A), \quad (6.52)$$

and the expression relating quality and void fraction in the form of Eq. (6.31) is

$$w'_{g,N} = L_N w'_{\ell,N} + M_N \alpha'_N,$$

where

$$L_N = \left\{ \frac{\gamma \bar{\alpha}}{1 - \bar{\alpha}} \left[1 + \frac{K_2(\gamma - 1)}{\gamma} \right] \right\}_N;$$

$$M_N = \left[\frac{\gamma \bar{w}_\ell}{(1 - \bar{\alpha})^2 G_\ell} (G_\ell + V_b A) \right]_N.$$

2. Marchaterre-Hoglund Correlation

The second method used in the prediction of the quality-void fraction data was a two-phase flow correlation⁶⁴ based upon the theory of similarity in which the Froude number, velocity ratio, and volumetric flow rate ratio were the groups picked to have a significant effect.

Here the average void fraction in the case of steady state is given by

$$\bar{\alpha} = \bar{w}_g / [\bar{S}\gamma + \bar{w}_g(1 - \bar{S}\gamma)], \quad (6.53)$$

and L_N and M_N for Eq. (6.31) becomes

$$L_N = \gamma [(1 - \bar{\alpha})/b\bar{\alpha}]_N^{1/(m-1)};$$

$$M_N = \gamma \left\{ \frac{\bar{w}_\ell}{(1 - m) b \bar{\alpha}^2} \left[\frac{1 - \bar{\alpha}}{b \bar{\alpha}} \right]^{[1/(m-1)]-1} \right\}_N.$$

3. Modified Variable-density Single-fluid Model

Bankoff's⁴⁷ variable-density model which has been modified by Jones,^{49, 50} was a third model tested. In this model, the mixture flows as a suspension of bubbles in the liquid; radial gradients in the bubble concentration exist, and power-law distributions for both velocity and void fraction were assumed to hold

$$\bar{\alpha} = \frac{K_B X}{\gamma + (1 - \gamma) X}, \quad (6.54)$$

for the steady state.

Since the slip expression

$$S = \frac{1 - \bar{\alpha}}{K_B - \bar{\alpha}} \quad (6.55)$$

had a discontinuity when $K_B = \alpha$, Jones modified the constant K_B to

$$K_B \rightarrow K_B + (1 - K_B) \bar{\alpha}^r, \quad (6.56)$$

and the corresponding expression for average void fraction became

$$\bar{\alpha} = \frac{K_B X}{\gamma + (1 - \gamma) X - (1 - K_B) X \bar{\alpha}^{r-1}} \quad (6.57)$$

Equation (6.57) was used in all calculations for this model, and the values of L_N and M_N in accordance with Eq. (6.31) were

$$L_N = \{\gamma \bar{\alpha} / [K_B - \bar{\alpha} + \bar{\alpha}^r (1 - K_B)]\}_N;$$

$$M_N = \{\bar{w}_\ell \gamma [K_B + \bar{\alpha}^r (1 - K_B)(1 - r)] / [K_B - \bar{\alpha} + \bar{\alpha}^r (1 - K_B)]^2\}_N.$$

4. Linear Variation of Slip in the Subcooled Boiling Region

Since none of the preceding models were developed to cover the case of developing flow in the subcooled region, where the velocity and transverse void profiles are rapidly changing, close agreement between theory and data was not expected. A fourth model assumed the slip in the subcooled boiling region to be a linear function of the mixing cup enthalpy:

$$S = S_{TB} + \{(S_B - S_{TB})[(H_M - H_{M,TB}) / (H_{M,B} - H_{M,TB})]\}, \quad (6.58)$$

where H_M is the mixing-cup enthalpy. The enthalpies were calculated according to Bowring's model; the value of slip at the bulk-boiling boundary was computed using the particular model incorporated in the bulk-boiling region. The value of slip at the transition boundary, S_{TB} , was determined by comparison with the corresponding experimental slip ratios calculated for this region. Again, the expressions for L_N and M_N are

$$L_N = [\gamma \bar{S} \bar{\alpha} / (1 - \bar{\alpha})]_N;$$

$$M_N = [\bar{w}_\ell \bar{S} \gamma / (1 - \bar{\alpha})^2]_N,$$

and the expression for average void fraction in the case of steady state is

$$\bar{\alpha} = \bar{w}_g / [\gamma \bar{S} + \bar{w}_g (1 - \gamma \bar{S})]. \quad (6.59)$$

VII. STEADY-STATE MEASUREMENTS

A knowledge of vapor volume fractions or void fractions for the steady-state case is of importance in predictions of the zero-frequency transient void amplitudes. Therefore, experimental data were acquired on the transverse shape of the void distributions and the axial variation of cross-section-averaged void fractions. These data were also required to calculate average vapor and liquid velocities needed for transient data analysis at experimental operating conditions corresponding to power-to-void transfer-function measurements. The transverse measurements provided valuable information on the changing shape of the void profiles during developing flow.

There has been an increasing interest in void fraction data relating to the distribution of the gas and liquid phases in a flowing two-phase system. Methods for predicting quality-void fraction relationships have been both empirical^{64,68} and theoretical^{47,69,70} in nature. Predictions of vapor volume fractions are complicated by the existence of slip (that is, of relative velocities between phases). Furthermore, slip can be a function of the shape and distribution of the gas and liquid phases as well as the existence of fully developed or developing flow. In this chapter one correlation⁶⁴ and two theoretical models^{47,63} are tested as to their ability to predict correctly cross-section-averaged, steady-state void fractions. Two additional models^{69,70} are tested in Appendix IV.

Steady-state axial and transverse void fraction distributions were obtained under loop-operating conditions corresponding to those employed in the power-to-void transfer-function measurements. The steady-state values of the loop parameters for each run have been summarized in Table 7.1, and all figures in this chapter have been referred to this table.

Table 7.1
SUMMARY OF STEADY-STATE VOID FRACTION RUNS

Run Number	Pressure p (psia)	Inlet Velocity v_0 (ft/sec)	Heat Flux $\phi \times 10^4$ [Btu/(hr)(ft ²)]	Inlet Subcooling ΔT_{sub} (°F)
1	200	3.78	2.28	0.5
2 & 3	300	2.52	2.28	1.8
4 & 5	400	2.52	4.56	1.3
6	400	2.52	4.56	9.5
7	400	3.78	6.84	1.2
8	400	3.78	6.84	11.1
9	600	3.78	9.12	4.2
10	600	3.78	9.12	12.6
12	800	2.52	4.56	4.9
13	800	3.78	9.12	4.7

Note that the steady-state runs did not have inlet subcoolings corresponding exactly to those of the power-to-void transfer-function measurements. The equipment employed in these measurements has been described in Ch. V. Measurements were made at thirteen locations spaced equidistantly along the test-channel length, and the gamma beam in all cases passed through the 1.750-in. depth of the channel. A narrow (1/32 in.) void window was used for all transverse measurements. Cross-section-averaged and transverse void distribution data have been tabulated in Appendix III for all runs.

A. Axial Void Fraction Distributions

1. Measurement

Cross-section-averaged void fractions at each axial position were obtained from the transverse measurements by integrating the void distribution across the narrow width of the channel. Due to small misalignments of the test section with respect to the narrow void window, as well as the limited resolution of the γ -attenuation measurement technique, only 80% of the cross section could be accurately traversed. This would tend to give higher void fractions in the experimental measurements for the downstream portion of the test section, where convex void distributions were present, and lower near the inlet, where concave distributions were found. The magnitude of this error has been discussed in Ch. V. The axial variation of the cross-section-averaged void fractions for all runs are shown in Figs. 7.1 and 7.2. The locations of the transition, z_T^* , and Bowring

Fig. 7.1. Steady-state Axial Void Fraction Distributions for Runs 1-7

Fig. 7.2. Steady-state, Axial Void Fraction Distributions for Runs 8-13

bulk-boiling boundaries, z_B^* , as calculated by Bowring's analysis, are also shown. The thermodynamic boiling boundary, z_S^* , which results from a simple energy balance and the assumption of thermal equilibrium, has also been denoted. All theoretical predictions of the axial, cross-section-averaged void fraction distribution have been indicated by the solid and dashed lines on the figures.

2. Theoretical Predictions

In order to predict transfer functions "a priori," the axial steady-state void fraction distributions must first be calculable. The Marchaterre-Hoglund correlation, the modified Bankoff model, and the Griffith prediction, which relate quality and void fraction, were tested as to their capability of predicting the steady-state axial void fraction distribution.

The quality was calculated by the method of Bowring, which will be detailed in the next section. All three quality-void fraction relationships were described in Ch. VI, and their predictions are summarized in Figs. 7.1 and 7.2.

There was good agreement between the experimental data and all three models. Good agreement between models and data was also found for those runs (6, 8, and 10) in which an appreciable subcooled boiling length existed. Here the agreement might well be considered fortuitous, on account of the sensitivity of the prediction of the inception of subcooled boiling to the calculated local quality, and also the inapplicability of the bulk-boiling models in this region. However, two significant points may account for this fine agreement between theory and experiment in the subcooled boiling region. Firstly, Bowring in the development of his physical model used extensive subcooled void data obtained from rectangular channels similar in flow area and length to the test section employed in this study. Secondly, the data from these test sections were analyzed on the assumption that a constant slip existed throughout the subcooled boiling regions, taken to be equal in value to that calculated at the intersection of the bulk-boiling and subcooled boiling void-fraction curves, z_B .

For the range of qualities, pressures, and flow rates covered in this study any one of the three models or correlations could have been used successfully to predict the axial, steady-state, cross-section-averaged void distribution.

A fourth method involved an assumption that slip is a linear function of enthalpy in the subcooled boiling region. This did not markedly improve the theoretical predictions shown in Figs. 7.1 and 7.2, and hence has not been included.

B. Cross-section-averaged Velocity and Slip Distributions

In order to compare void propagation velocities (obtained from transfer-function data) with the liquid and vapor velocities, the axial variation of these velocities were calculated from the experimental void-fraction measurements, and qualities were calculated after the method of Bowring. By Bowring's model the axial variation of quality can be calculated even for those runs with appreciable subcooled boiling. In the subcooled region the total heat flux was divided into two main parts: an evaporative component q_e and a heating component q_h . These heat fluxes were found to be related simply by $q_h/q_e = \epsilon$, and their relation to the total heat flux was given by

$$q_h = \frac{\epsilon}{1 + \epsilon} q_{tot}; \quad q_e = \frac{1}{1 + \epsilon} q_{tot},$$

where ϵ is an empirical constant equal to 1.3 for all runs here.

The heating component of total heat flux was used to calculate the axial position z_B at which the bulk fluid temperature equals the saturation temperature. The evaporative component was employed to calculate the axial quality distribution. In the region beyond z_B (bulk-boiling region), $\epsilon = 0$ and $q_e = q_{tot}$. Knowing the quality, the vapor and liquid velocities were calculated by use of the equations

$$v_g(z) = X(z)v_0/\alpha(z)\gamma; \quad v_l(z) = v_0[1 - X(z)]/[1 - \alpha(z)].$$

Then these data were plotted for all cases in a manner similar to that in Fig. 7.3. Smooth curves, which best fit the data, were drawn through the points. Next, these curves were used to calculate the axial variation of slip and the mixture velocity v_m :

$$v_m(z) = \alpha(z)v_g(z) + [1 - \alpha(z)]v_l(z).$$

All the smoothed data for cross-section-averaged velocities, slip, and mixture velocity have been tabulated in Appendix I.

The cross-section-averaged liquid velocities in all cases could be well represented as a linear function of axial distance. Also, in most cases the cross-section-averaged vapor velocities could be approximated as a linear function of axial distance, except for those regions near the test section inlet.

The axial variation of slip was found to increase with axial distance and increasing quality. Slips with values less than unity were calculated for axial positions near the test-section entrance for runs with both high and low inlet subcooling. However, due to the combined uncertainties in

Fig. 7.3. Steady-state Axial Velocity and Slip Distributions

calculating the location of the transition boundary, the reliability of Bowring's analysis, and the experimental accuracy in the void measurements in this region, these results must be carefully weighed.

C. Transverse Void Fraction Distributions

As mentioned previously, a 1/32-in. collimator window was used in all transverse void measurements across the 0.437-in. width of the channel. For this rectangular channel it was assumed that the flow was essentially two-dimensional and the density traverse across the narrow dimension of the channel was equivalent to the phase distribution (i.e., a flat void distribution was assumed to exist across the 1.750-in. channel depth).

The axial development of the transverse void fraction distributions has been shown for two typical cases in Figs. 7.4 and 7.5. Figure 7.4 illustrates the development of transverse void distributions for a case with low inlet subcooling (1.2°F), whereas Fig. 7.5 is for a case with an appreciable subcooled boiling length ($\Delta T_{\text{sub}} = 11.1^{\circ}\text{F}$). The profiles are seen to change from concave to convex with increasing axial distance, but the concave profiles persist for a much longer axial distance with increased subcooling (see Fig. 7.5).

There is a scarcity of data on transverse void profiles for developing two-phase flow. However, the axial development of the transverse void profiles was similar to that found by Christensen²³ under similar conditions, and by Zivi and Wright^{20,22} and Kagawa and Hada⁷² at atmospheric pressure. The concave-shaped transverse void profiles have also been obtained in a forced-circulation air-water loop.⁷¹ A porous bronze tube was used as a test section, and air was bubbled into the water or air-water mixture flowing inside the tube continuously along the length of the test section. Recently, Staub and Zuber⁷³ have obtained similar transverse void profiles at elevated pressures employing Refrigerant 22 as the working fluid.

In acquiring the axial, cross-section-averaged void distributions, a large amount of data was obtained on the axial development of transverse void distributions. This information should be of considerable interest to workers investigating the prediction of, or effects of, local void fraction in two-phase flow systems. For this reason all the transverse and axial void fraction data have been tabulated in Appendix III.

In order to obtain quantitative information on the transverse void profiles a detailed treatment of the data would be required. However, several qualitative observations can be made which are of interest. As mentioned previously, concave void distributions were found near the test-section entrance for most of the runs. The degree of concavity and the persistence of this profile with axial distance was found to increase with increased inlet temperature subcooling. Also, as the shape of the profile

Fig. 7.4. Transverse Void Fraction Distributions as a Function of Axial Distance for Run No. 7

Fig. 7.5. Transverse Void Fraction Distributions as a Function of Axial Distance for Run No. 8

passed from concave to convex, points of inflection occurred in the curves. This suggests that for developing flows of this type an expression more complicated than a power-law model would be required to describe these profiles near the test-section entrance. However, with increasing axial distance downstream of the point where the convex profile had developed, a power-law model could describe the profile. There was parallel behavior in the variation of slip with axial distance as well as of the relative change in void profiles with axial distance; that is, the greater the degree of change in the transverse void profiles with axial distance, the greater was the change in slip for the same axial distance. For all concave-shaped profiles the calculated values of slip were found to be less than or approximately equal to unity. This suggests that the transverse shape of void fraction across the test section is an important parameter affecting slip in developing flow.

VIII. POWER-TO-VOID TRANSFER FUNCTION MEASUREMENTS

The transient void-amplitude and phase-response data have been presented in Bode-diagram form in Figs. 8.1 to 8.9. Four main loop parameters were varied in this study: pressure, power, inlet flow, and subcooling. Runs were made at five different pressures ranging from 200 to 800 psia. Two different inlet flow rates were employed: 2.52 and 3.78 ft/sec. The heat flux was varied from 2.28×10^4 to 9.12×10^4 Btu/(hr)(ft²). The majority of the fourteen runs were made with low inlet temperature subcooling in order to obtain a well-defined boiling length over the major portion of the test section. However, four runs (6, 8, 10, and 14) had an appreciable subcooled boiling length. The experimental values of the loop variables for all runs have been summarized in Table 8.1. Also included are the calculated values denoting the steady-state location of the transition, thermodynamic, and Bowring bulk-boiling[†] boundaries.

Measurements were made at three axial positions: $z^* = 0.834$, 0.467, and 0.226. The "one-shot" runs were made using a large lead collimator window ($7/8 \times 1\frac{1}{4}$ in.) so that the steel liners and test-section walls collimated the gamma beam. This minimized any error which would have arisen due to small misalignments between the detector carriage and test section. The transverse measurements (Runs 3 and 4) were made with the narrow ($1/32$ in.) window described in the previous chapter. Three transverse positions were employed: $y^* = 0.0$ (at channel mid-point), 0.50, and 0.80. In all measurements the gamma beam passed through the $1\frac{3}{4}$ -in. depth of the channel.

A. Experimental Results

Every run was made in the following manner. For a chosen set of loop-operating variables (pressure, heat flux, inlet subcooling, and inlet flow rate), transient measurements were made at various axial positions over the full frequency range from 0.01 to 5 cps. The frequencies at which the measurements were made were staggered so that all of the high-frequency and low-frequency data were not taken together. Rather, the frequencies were chosen in such a manner that high-frequency measurements were staggered between low-frequency measurements, and the low- or zero-frequency measurements were taken at various intervals during the run. This provided a check on system drifts and gave an indication as to the reproducibility of the data. A run as defined here could be spread over 2-3 days, as was the case for Run 4, and the reproducibility of the data from day to day was good.

[†]The height at which the bulk temperature of the liquid reaches the saturation temperature as calculated with Bowring's analysis. Here the heat flux q_e which raises the temperature of the fluid to the saturation temperature was equal to $q_{tot}/(1 + \epsilon)$, where q_{tot} is the total heat flux and ϵ is an empirical constant.

RUN NO. 1
 $z^* = 0.834$
 $\alpha_{AVG} = 46.4\%$

RUN NO. 1
 $z^* = 0.467$
 $\alpha_{AVG} = 34.5\%$

RUN NO. 2
 $z^* = 0.834$
 $\alpha_{AVG} = 52.6\%$

112-5104

Fig. 8.1. Power-to-void Transfer-function Measurements (Runs 1 and 2)

RUN NO. 3
 $z^* = 0.834$
 $\gamma^* = 0.0$
 $\alpha_{v^*} = 54.1\%$

RUN NO. 3
 $z^* = 0.834$
 $\gamma^* = 0.5$
 $\alpha_{v^*} = 38.5\%$

RUN NO. 3
 $z^* = 0.834$
 $\gamma^* = 0.8$
 $\alpha_{v^*} = 45.2\%$

112-5111

Fig. 8.2. Power-to-void Transfer-function Measurements (Run 3)

112-5098

Fig. 8.3. Power-to-void Transfer-function Measurements (Run 4)

RUN NO. 5
 $z^* = 0.834$
 $\alpha_{AVG} = 53.6\%$

RUN NO. 5
 $z^* = 0.467$
 $\alpha_{AVG} = 41.8\%$

RUN NO. 5
 $z^* = 0.226$
 $\alpha_{AVG} = 29.7\%$

112-5112

Fig. 8.4. Power-to-void Transfer-function Measurements (Run 5)

112-5102

Fig. 8.5. Power-to-void Transfer-function Measurements (Run 6)

RUN NO. 7
 $z^* = 0.834$
 $\alpha_{AVG} = 53.1\%$

RUN NO. 7
 $z^* = 0.467$
 $\alpha_{AVG} = 32.7\%$

RUN NO. 7
 $z^* = 0.226$
 $\alpha_{AVG} = 15.4\%$

112-5113

Fig. 8.6. Power-to-void Transfer-function Measurements (Run 7)

112-5100

Fig. 8.7. Power-to-void Transfer-function Measurements
(Runs 8 and 9)

112-5101

Fig. 8.8. Power-to-void Transfer-function Measurements
(Runs 10 and 11)

112-5099

Fig. 8.9. Power-to-void Transfer-function Measurements
(Runs 12, 13, and 14)

Table 8.1
EXPERIMENTAL CONDITIONS OF POWER-TO-VOID
TRANSFER-FUNCTION MEASUREMENTS

Run No.	Pressure (psia)	Heat Flux $\times 10^{-4}$ (Btu/hr-ft ²)	Inlet Velocity (ft/sec)	Inlet Subcooling (°F)	Transition Boundary z_{TB}^*	Thermodynamic Boiling Boundary z_S^*	Bowring Bulk-boiling Boundary z_B^*
1	200	2.28	3.78	0.8	0.00	0.098	0.18
2	300	2.28	2.52	1.3	0.00	0.106	0.20
3	300	2.28	2.52	1.8	0.00	0.147	0.26
4	400	4.56	2.52	4.8	0.00	0.195	0.36
5	400	4.56	2.52	4.0	0.00	0.162	0.30
6	400	4.56	2.52	9.2	0.155	0.374	0.54
7	400	6.84	3.78	1.1	0.00	0.045	0.10
8	400	6.84	3.78	11.3	0.241	0.460	0.64
9	600	9.12	3.78	3.9	0.00	0.120	0.22
10	600	9.12	3.78	12.4	0.183	0.384	0.54
11	600	4.56	2.52	3.2	0.00	0.148	0.26
12	800	4.56	2.52	3.6	0.00	0.039	0.30
13	800	9.12	3.78	4.9	0.00	0.151	0.28
14	800	9.12	3.78	11.2	0.00	0.345	0.64

The following qualitative observations were made during the experimental runs. The lowest pressures (200 and 300 psia) gave the most difficulty during the power-to-void transfer-function measurements. The loop was more sensitive to slight drifts in pressure or average power, and the void signal to the wave analyzer was "noisiest" at these pressures. As the loop pressure was increased, the data scatter diminished and the reproducibility of the data improved.

In the Bode diagrams the data points are circled, whereas the solid lines represented curves which were believed to fit the data best. For each axial position in the "one-shot" runs a pair of plots have been given presenting the void amplitude and phase response. In the transverse runs the void amplitude and phase response were plotted for each transverse position.

It should be noted, since all the power amplitudes were not of equal magnitude, that all void responses have been normalized corresponding to a 10% power modulation amplitude peak-to-peak. This could be done since the void response was found to be linear with respect to power modulation. All of the transfer function data taken have been tabulated in Appendix II.

1. Measurements of Void Amplitude

All of the void-amplitude data plotted as a function of frequency exhibited notches or nulls at frequencies ranging from 1.2 to 3.7 cps. This null in the void-amplitude response was associated with the transportation delay of a void perturbation propagating downstream through the test section.

The frequency at which this null first occurred has been called the notch frequency and will be used later to calculate propagation velocities of the void perturbations.

The notch frequency was highest for those axial positions nearer the test-section inlet and decreased with axial distance. From the transverse measurements, it can be seen that the notch frequency was highest at the middle of the channel and decreased as the test-section wall was approached. The notch became more distinct and sharper as the pressure was increased and, at a particular pressure, as the test-section inlet was approached. Increased subcooling has two observable effects: (1) to decrease the frequency at which the notch appeared; (2) to make the notch more distinct. These characteristics, and in particular, the perturbation velocities obtained from the notch location will be discussed in a later section.

2. Measurements of Void Phase

The phase measurements also showed a dip or undulation close to the notch frequency. This dip in the phase measurements paralleled the behavior of the amplitude notch. In some of the phase measurements near the notch frequency, it was difficult to ascertain whether the phase lag increased through 180° or decreased to a lower value. This was due to the loss in definition of the in-phase and out-of-phase signal components as both amplitudes tended toward zero. In these phase measurements there was less scatter of the data than for the corresponding void amplitudes, indicating that the void amplitude was more sensitive to noise and drifts in the settings of the loop-operating variables.

There was a small phase lead in the void signal at the lowest frequencies. The magnitude of this phase lead was temperature dependent, and also varied with the pressure and axial position. This effect, which is thought to be due to a slight pressure feedback from the steam separator, will be discussed in a later section.

B. Analysis of Results

Through an analysis of the power-to-void transfer-function measurements it was possible to obtain information regarding the range of linearity of the void response, the axial variation of void amplitude and phase lag as a function of frequency, and propagation velocities of the void disturbance. An analysis of the propagation velocities follows.

1. Verification of Zero-frequency Void Amplitude

The zero-frequency void-amplitude response is intimately related to the steady-state axial void distribution. A comparison of the zero-frequency void amplitude obtained in the dynamic analysis for Run 9 was

made with that obtained from steady-state axial void distributions in the following manner. Three steady-state axial void distribution runs were made in which all loop parameters were held constant except the steady-state power. For each of the three power levels the transverse void distributions at various axial positions were obtained. Then these distributions were integrated to yield the axial void distributions plotted in Fig. 8.10.

Fig. 8.10
Zero-frequency Void
Amplitude Verification
for Run No. 9

112-5110

At any axial position, the change in void fraction between any two curves yields the zero-frequency transient void amplitude which results from a power modulation amplitude equal to the difference in the two power levels. At an axial height of $z^* = 0.834$, we see from Fig. 8.10 that a 10% power modulation amplitude yielded a 3.8% void fraction amplitude. This agreed closely with the zero-frequency amplitude of 4.0% obtained for Run 9 at the same position. There was also good agreement at $z^* = 0.467$, where an amplitude of 3.5% was obtained for Run 9 as compared to 3.2% obtained from Fig. 8.10.

2. Range of Linearity of Void Response

Since the theoretical analysis was based upon the assumption of linearity in the void response to small power perturbations, the range of linearity of the void response to power modulation was investigated. It was found to extend up to amplitudes of 20% of the average power level in all runs at 200, 300, and 400 psia (see Fig. 8.11). Also, the void phase response remained essentially constant at any one frequency for increasing power modulation amplitudes up to 20%.

At the higher powers and pressures, due to an internal feedback in the loop power supply, the magnitude of the power oscillation amplitude was limited. However, in the power amplitude range covered (4-12%), there was no evidence of any nonlinear behavior in the runs at 600 and 800 psia.

PARAMETER	RUN NO. 1	RUN NO. 2	RUN NO. 7	RUN NO. 5
p , psia	200	300	400	400
v_0 , ft/sec	3.78	2.52	3.78	2.52
ϕ , Btu/(hr)(ft ²)	2.28×10^4	2.28×10^4	6.84×10^4	4.56×10^4
ΔT_{SUB} , °F	0.8	1.3	1.1	4.0
z^*	0.467	0.834	0.834	0.834
α_{AVG} , %	34.5	52.6	53.1	53.6

112-5114

Fig. 8.11. Range of Linearity of Void Response to Power Modulation

3. Axial Variation of Transient Void Amplitude and Phase as a Function of Frequency

The axial variation in amplitude and phase lag of the transient void response is shown in Fig. 8.12 for cases with and without appreciable

subcooling. In both runs, the void amplitude behavior was similar. The axial variation in void amplitude was sinusoidal in shape and of the same form as the power forcing function. As the modulation frequency increased, the period and amplitude of this sinusoidal function decreased until the test-section length corresponded to one-half wave length. Then a null or notch would appear in the void amplitude at the test-section exit. If the frequency were increased further, the period would further decrease and the notch would move to a new axial position nearer the inlet.

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Fig. 8.12. Axial Variation of Transient Void Amplitude as a Function of Frequency

The void phase lag appeared to increase linearly with axial distance. As the modulation frequency was increased, the phase lag per unit length increased, which was consistent with the phase-lag behavior associated with a transportation delay.

4. Zero-frequency Void Amplitude Response

The low-frequency (0.01 cps) or zero-frequency void amplitudes were found to increase with axial distance in all cases studied. From transverse measurements with the narrow void window, the zero-frequency amplitudes were found to decrease as the test-section wall was approached. This two-dimensional phenomenon had a more pronounced effect in the downstream regions of the channel where convex transverse void distributions were present. At the lower regions of the test section the amplitude was essentially constant across the channel.

For Runs 2 and 3, the experimental setting of the loop variables were essentially identical. However, in Run 3 the narrow void window was employed for transverse measurements. There was good agreement in the zero-frequency amplitudes between these two runs if the transverse amplitudes across the channel from Run 3 were plotted, integrated, and averaged. This was also found to be true for Runs 4 and 5 at axial heights of $z^* = 0.834$ and 0.467 .

Although there were not sufficient data available to obtain a correlation, it appeared that the void amplitude distribution could be of the form

$$\alpha' / \alpha'_{\max} = y^{1/n}.$$

Increased inlet subcooling was found to increase the zero-frequency amplitude. This effect was noted by examining runs with identical loop-operating conditions except for inlet subcooling. At a fixed axial position, the run with higher subcooling yielded the higher zero-frequency amplitude. This is thought to result from the additive effects of time-dependent transitional boundaries which will have a void amplitude response associated with them; and at a fixed axial position for the case of the higher subcooling, the change in cross-section-averaged void fraction with axial distance will now be greater than for the case of low inlet subcooling.

C. Velocity of Propagation of Void Perturbation

The appearance of the notch or null in the void amplitude response is a transport effect associated with the downstream propagation of the void perturbation. Christensen has derived an expression which allows the velocity of void propagation to be found from the notch frequency. For a constant propagation velocity, v_p , of a void fraction disturbance the amplitude of the power-steam mass flow rate is given by

$$\left| \frac{W'_g(z,t)}{Q'(t)} \right| = \left| \frac{1}{\lambda} \frac{v_p}{i2\pi f} \right| \left(1 - e^{-2\pi i f z / v_p} \right). \quad (8.1)$$

The response vanishes whenever fz/v_p is an integer, and thus for the assumption of constant propagation velocity

$$v_p = f_n z, \quad (8.2)$$

where f_n is the lowest node frequency at any position z . Equation (8.2) was used to find the average propagation velocities through use of the measured transfer functions. The null frequency in the void amplitude as well as the frequency at which an undulation appears in the phase lag were both used to estimate the correct notch frequency. If the notch was not well-defined in either the amplitude- or phase-response data, it was taken to be at the mean of an estimated upper and lower bound.

The power-to-void transfer-function measurements have been summarized in Table 8.2

Table 8.2
SUMMARY OF POWER-TO-VOID TRANSFER-FUNCTION
MEASUREMENTS

Run No.	y^*	z^*	α_{avg} (%)	Zero-frequency Peak-to-peak Amplitude (%)	Notch Frequency (cps)	Propagation Velocity (ft/sec)	Frequency where Phase Lag Equals 180° (cps)	Break Frequency (cps)
1	-	0.834	46.4	2.25	1.6	6.3	~1.3	0.70
1	-	0.467	34.5	1.80	2.3	4.4	~1.7	0.95
2	-	0.834	52.6	1.75	~1.6	5.5	1.0	0.83
3	0.00	0.834	54.1	3.0	1.9	6.6	0.85	0.44
3	0.50	0.834	38.5	1.8	1.2-2.2	4.0-7.6	0.84	0.53
3	0.80	0.834	45.2	1.6	1.4-2.0	4.8-6.9	0.88	0.68
4	0.00	0.834	61.6	3.5	2.0	6.9	0.85	0.42
4	0.50	0.834	48.6	2.5	1.1-1.8	3.8-6.2	0.85	0.70
4	0.80	0.834	39.8	2.2	1.3	4.5	0.95	0.68
4	0.00	0.467	53.2	2.4	3.2	6.1	1.3	1.4
4	0.80	0.467	33.4	2.3	2.7-3.2	5.2-6.2	1.2	1.2
5	-	0.834	53.6	2.4	1.2-1.8	4.2-6.2	0.80	0.80
5	-	0.467	41.8	2.3	-	-	0.70	1.30
5	-	0.226	29.7	1.7	3.7	3.4	0.80	1.50
6	-	0.834	37.4	4.0	1.3	4.5	0.72	0.44
6	-	0.467	19.4	3.2	1.9	3.7	1.1	0.54
6	-	0.226	12.3	1.9	2.1	2.0	~1.9	1.1
7	-	0.834	53.1	3.4	1.9	6.6	1.4	0.96
7	-	0.467	32.7	3.3	~3.0	5.7	2.2	1.4
7	-	0.226	15.4	2.7	>3.0	>2.8	~3.0	1.7
8	-	0.834	32.8	5.4	1.5	5.2	1.2	0.73
8	-	0.467	14.0	5.1	~2.4	~4.6	~2.2	0.72
9	-	0.834	46.2	4.0	2.1	7.3	1.5	0.85
9	-	0.467	33.0	3.5	>2.6	>5.0	2.2	1.30
10	-	0.834	31.6	4.6	1.5	5.2	1.2	0.80
11	-	0.834	36.5	3.8	1.2	4.2	0.88	0.53
11	-	0.467	19.5	3.0	1.8	3.5	1.5	0.73
11	-	0.225	8.6	2.0	3.3	3.1	2.6	1.70
12	-	0.834	31.3	3.8	1.3	4.5	0.95	0.51
12	-	0.467	15.6	2.6	2.1	4.0	1.6	0.85
13	-	0.834	34.5	3.8	1.9	6.6	1.5	0.90
14	-	0.834	26.5	4.5	1.6	5.5	1.5	0.86

1. One-shot Measurements

From the one-shot measurements, the propagation velocity calculated by use of Eq. (8.2) will be a value averaged with respect to both the whole cross-sectional flow area and the axial length from the point of initial void response up to the point of measurement. We will denote this axial length average of any quantity v_p as $v_p^\dagger$. In order to investigate the relationship of this propagation velocity to liquid and vapor velocities present in the channel, the cross-section-averaged liquid, mixture, and vapor velocities were calculated as a function of z and plotted as shown in Ch. VII. The propagation velocities were obtained from the graphs at an axial distance midway between the channel entrance and the point of measurement. These results have been tabulated in Table 8.3.

Table 8.3

SUMMARY OF VOID PROPAGATION VELOCITIES AND RELATIONSHIP TO LIQUID, MEAN, AND GAS VELOCITIES

Run No.	z^*	y^*	$(v_p/v_l)^\dagger$	$(v_p/v_m)^\dagger$	$(v_p/v_g)^\dagger$	Ψ
1	0.834	one-shot	1.30	1.10	0.84	0.44
1	0.467	one-shot	1.00	0.87	0.70	0.41
2	0.834	one-shot	1.65	1.57	1.16	0.52
3	0.834	0.00	1.88	1.71	1.25	0.23
3	0.834	0.50	1.66	1.49	1.10	0.31
3	0.834	0.80	1.65	1.46	1.08	0.43
4	0.834	0.00	1.89	1.53	1.07	0.42
4	0.834	0.50	1.37	1.11	0.78	0.59
4	0.834	0.80	1.23	1.00	0.71	0.73
4	0.467	0.00	1.77	1.58	1.18	0.41
4	0.467	0.80	1.65	1.48	1.10	0.40
5	0.834	one-shot	1.43	1.15	0.80	0.53
5	0.467	one-shot	-	-	-	-
5	0.226	one-shot	1.19	1.15	0.97	0.22
6	0.834	one-shot	1.52	1.52	1.00	0.55
6	0.467	one-shot	1.30	1.30	1.20	0.58
6	0.226	one-shot	0.76	0.76	-	0.90
7	0.834	one-shot	1.25	0.99	0.65	0.73
7	0.467	one-shot	1.22	1.07	0.69	0.73
7	0.226	one-shot	-	-	-	-
8	0.834	one-shot	1.21	1.17	0.91	0.80
8	0.467	one-shot	1.18	1.18	-	0.91
9	0.834	one-shot	1.45	1.22	0.80	0.71
9	0.467	one-shot	-	-	-	-
10	0.834	one-shot	1.17	1.15	0.91	0.80
11	0.834	one-shot	1.38	1.22	0.77	0.73
11	0.467	one-shot	1.30	1.18	0.79	0.83
11	0.225	one-shot	1.17	1.11	0.81	0.79
12	0.834	one-shot	1.47	1.36	0.95	0.73
12	0.467	one-shot	1.43	1.40	1.12	0.76
13	0.834	one-shot	1.28	1.24	1.15	0.79
14	0.834	one-shot	-	-	-	0.94

The propagation velocities were found to be intermediate between the mixture and vapor cross-section-averaged velocities for all axial positions downstream of the concave-shaped transverse void distributions. As the entrance of the test section was approached, the propagation velocity decreased and became more closely related to the cross-section-averaged liquid velocity. For those cases with higher inlet subcooling, the propagation velocity was more closely related to the liquid velocity over a greater portion of the test section.

2. Transverse Measurements

From the measurements with the narrow void window, the propagation velocity calculated by means of Eq. (8.2) will be a value averaged with respect to both a narrow transverse portion of the cross-sectional flow area and the axial length from the point of initial void response up to the point of measurement. These propagation velocities were then compared to the cross-section-averaged velocities as in the one-shot measurements.

The propagation velocities were found to be a function of the distance from the wall. At a fixed axial position the propagation velocity decreased as the test-section wall was approached. This effect was more pronounced in the downstream regions of the test section where convex void distributions existed. On account of the limited data available and the sensitivity of the propagation velocity on the notch frequency, which was sometimes difficult to ascertain accurately, an empirical correlation is not presented for this transverse effect.

The local propagation velocity for a void fraction disturbance should depend strongly upon the local liquid, gas, and mixture velocities in the steady state. The fact that the measured propagation velocity profiles exhibit a maximum at the center, and decrease near the wall, in the downstream channel regions is consistent with the convex steady-state velocity profiles which are thought to exist in these regions. The latter quantities have not been measured, to our knowledge, in developing steam-water flow, but the present data tend to confirm the existence of convex steady-state velocity profiles. A recent report⁷⁴ presented phase and velocity distribution profiles relevant to annular-dispersed flow obtained using an "isokinetic probe." Here both the vapor and liquid profiles were found to be convex in shape for the downstream regions of the channel.

D. Transfer-function Break Frequency

In the log-log plots of void amplitude vs. frequency presented in Figs. 8.1 to 8.9, it was found that sharpness of the notch varied with pressure, axial distance from the inlet, inlet subcooling, and at a fixed

axial position with distance from the wall. Therefore, a common basis for comparing these effects was sought, and the ratio $f_b/f_n = \Psi$ was chosen, where f_b is the break frequency (or corner frequency) normally obtained as the intersection of lines asymptotic to the zero-frequency response and high-frequency response on the transfer function log-magnitude vs. log-frequency plot. Here, the break frequency was arbitrarily chosen as that frequency at which the amplitude response had decreased to $1/\sqrt{2}$ of its zero-frequency value (actually the break frequency for a first-order system), and f_n is the frequency at which the first notch appeared in the void amplitude response.

The break frequency in the transient void amplitude response is believed to be influenced mainly by heat conduction effects at the wall and the delay associated with void-perturbation transportation. Christensen in his analysis found that a first-order system was adequate to describe the surface temperature response to power modulation. In this study the time constant for the test section has been calculated to be approximately 0.04 sec. This corresponds to a break frequency of 4 cps and therefore heat conduction effects at the wall should have a small effect here on the break frequencies, which ranged from 0.4 to 2.7 cps.

The value of Ψ would be maximized if void perturbations were to propagate downstream in plug form at constant velocity. Conversely, if the void perturbation were to be completely mixed or "spread out" during passage through the test section, then Ψ would be minimized. Therefore, Ψ can be considered as a measure of the axial diffusion or mixing of a void perturbation as it is propagated through the test section. The lower the value of Ψ , the greater is the degree to which the void signal has "spread out" in an axial direction. All values were previously tabulated in Table 8.3 and have been plotted in Fig. 8.13(A) as a function of pressure for constant axial height. The effects of pressure and inlet subcooling are evident, but there was insufficient data to obtain a correlation. No flow effects were evident.

As the pressure and inlet subcooling increased, Ψ approached unity (which is the maximum for Ψ as defined here). The value of Ψ also increased with decreasing axial length, but decreased with separation from the test-section wall as shown in Fig. 8.13(B).

Christensen also noted that his data exhibited increasing sharpness in the node, especially in the phase angle, as the test-section inlet was approached. He inferred that this might be due to the effect of axial variation of the propagation velocity. Another factor which would affect the sharpness of the node would be the degree of axial mixing of the void perturbation as it propagates downstream. In two-phase steam-water systems, the existence of slip would result in increased axial mixing. The greater the deviation of slip from unity, the greater the degree of axial mixing.

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Fig. 8.13. The Ψ Factor at $z^* = 0.834$, (A) as a Function of Pressure and (B) as a Function of Transverse Position

It is known that slip decreases with increasing pressure and with increased inlet subcooling. Also, average slip over the whole flow area increases with increasing axial distance or quality. The analogous behavior of slip and Ψ is evident, and it appears that slip is an important factor in the determination of the break frequency of the power-to-void transfer function.

E. Pressure Variations in the Steam Separator

As mentioned previously, there was a phase lead in the void response at frequencies ranging from 0.01 to 0.10 cps. The magnitude of the phase lead was found to be a function of the axial position, loop-operating pressure, and inlet subcooling, as well as of the modulation frequency. This phase lead is thought to be due to slight pressure variations ($< \pm 1$ psi) in the steam separator and steam surge tank, resulting from their finite capacitance. This capacitance effect would be expected to be more important at the lower (0.01 to 0.10 cps) frequencies.

1. Effect of Pressure Variations on Transfer-function Measurements

The transfer function from steam production, $w'_g(s)$, to steam dome pressure $p'_d(s)$ has been derived by Christensen⁷⁵ and is of the form

$$\frac{p'_d(s)}{w'_g(s)} = \frac{k_1}{s}$$

for low frequencies when acoustical resonance effects are absent. At very low frequencies the steam production will essentially be in phase with the power modulation, and we obtain

$$\frac{p'_d(s)}{Q'(s)} = \frac{k_2}{s}$$

Steam-separator pressure variations have a feedback effect on the void response as diagrammed in Fig. 8.14. If we designate the

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Fig. 8.14. Block Diagram Showing the Effect of Pressure Variations in the Steam Dome

pressure-to-void transfer function as $\alpha'_p(z,s)$, the total void response is given by

$$\alpha'(z,s) = \frac{k_2}{s} \alpha'_p(z,s).$$

At high modulation frequencies, this expression approaches $\alpha'(z,s)$, and the zero-frequency limit is of the form

$$\alpha'(z,i\omega) = \frac{ik_2}{\omega} \alpha'_p(z,i\omega).$$

As ω approaches zero, the phase angle tends to $+90^\circ$. The major effect of this pressure-to-void transfer function was felt in the void phase response with little effect on the void amplitude.

The phase lead was found to be largest near the inlet of the channel and to increase with increased inlet subcooling. The effect of increased loop pressure was to decrease rapidly the magnitude of this phase lead. (This loop-operating pressure effect will be discussed in the next section.) In Fig. 8.15, the void phase response at a frequency of 0.01 cps has been plotted as a function of pressure with axial height and inlet subcooling as parameters.

112-5103

Fig. 8.15. Variation of Void Phase Lead with Pressure at a Frequency of 0.01 cps

It is doubtful that this phase lead could be accounted for purely by pressure feedback affecting the gas density, and thus the pressure feedback effect on nucleation centers appears to be significant. If pressure feedback was affecting nucleation sites, then the effect would be most important at the test-section entrance near the point of incipient boiling.

A factor which could cause the phase lead to be less at the channel entrance than further downstream is the angle at which the pressure effect is added vectorially to the larger power term. The pressure effect will add vectorially at right angles to the power response at the test section exit. Since the sonic velocity is estimated⁷⁶ to be 30-100 ft/sec in this system, then just a very small fraction of one wavelength of the standing wave will be contained in the channel. Therefore, the pressure effects will be felt almost instantaneously over all parts of the channel. However, there is an increasing phase lag between a point at the channel entrance and one at the exit. This means that the pressure effect will be added vectorially to the power term at angles less than 90° near the test-section inlet.

A third factor which would tend to give larger void phase leads near the entrance results from the magnitude of the power term being less near the entrance than at the exit. This would allow the pressure term to have a greater relative effect near the test-section inlet.

Since the phase lead was greatest nearer the test-section inlet, this would imply that the nucleation-site response and relative magnitude of the pressure term to the power term were of the main importance. This effect of phase lead on the transient void phase-lag response is evident in Fig. 8.12 where the phase lag per unit length was found to be greater at 0.02 than at 0.08 cps.

2. Pressure Effect on Magnitude of Pressure Variations in the Steam Dome

From Fig. 8.15 for a fixed axial height, essentially constant inlet subcooling, and with transient steam productions $w'_g(s)$ of the same order of magnitude, the void phase lead is seen to diminish rapidly with increasing pressure. In fact, the magnitude of the pressure-feedback effect was inversely proportional to the loop-operating pressure. One simple explanation for this behavior is that the pressure increase effectively increases the capacity of the steam separator and surge tank. Then for steam productions of the same order of magnitude, the volume of vapor associated with this production decreases rapidly with increasing pressure (inversely proportional by the ideal gas law).

It was also of interest to investigate the effect of pressure on the mechanics of vaporization from nucleation sites. Zysina-Molozhen and Kutateladze⁷⁷ have reported experimental data relating the effect of pressure

on the number of active nucleating sites, Γ ; the frequency of bubble departure, Ω ; and the bubble diameter at departure, ξ , for pressures of 1 to 5 atm. In this pressure range, they found that (Γ/Γ_0) , (Ω/Ω_0) , and (ξ/ξ_0) were of the form

$$(\xi/\xi_0) = p^{-b_1}, \quad (\Omega/\Omega_0) = p^{-b_2}, \quad (\Gamma/\Gamma_0) = p^{b_3},$$

where the subscript zero refers to pressure of 1 atm. Now the volumetric rate of vapor production is proportional to $\xi^3 \Omega \Gamma$, which can be expressed as

$$\xi^3 \Omega \Gamma / \xi_0^3 \Omega_0 \Gamma_0 = p^{-3b_1 - b_2 + b_3}.$$

The values of b_1 , b_2 , and b_3 , respectively, were 0.230, 0.724, and 0.410, which gives

$$\xi^3 \Omega \Gamma / \xi_0^3 \Omega_0 \Gamma_0 = p^{-1.00}.$$

By this analysis we see that the number of voids associated with nucleation sites would also be inversely proportional to the pressure, which agrees with the results in Fig. 8.15 and with loop-pressure effects on the effective steam-dome capacitance.

IX. COMPARISON OF THEORETICAL MODELS WITH EXPERIMENTAL RESULTS

In this chapter various theoretical models will be tested as to their ability of theoretically predicting the power-to-void transfer functions obtained experimentally.

A. Comparison with Present Model

The theoretical model for predicting power-to-void transfer functions to be discussed here has been developed previously in Ch. VI. This model was similar to the "Kjeller Model"^{51,52} in that Bowring's physical model⁵³ was employed in the subcooled boiling region and the transitional boundaries separating the non-, subcooled- and bulk-boiling regions were included in the transient analysis.

However, this model was distinct from the Kjeller model because of the following changes. The heat capacity of the wall was accounted for in all three regions: non-, subcooled, and bulk boiling. The component of the transient void response due to movement of the bulk-boiling boundary was considered for every section in the subcooled boiling region instead of being concentrated solely at the steady-state location of the bulk-boiling boundary $\bar{z}_B$.

Also, the entire model was programmed for solution on a digital computer and solved employing a step-by-step integration procedure. The steady-state constants $\bar{\alpha}(z)$, $\bar{v}_g(z)$, and $\bar{S}(z)$ required for the transient solution were generated by use of various models and correlations relating quality and void fraction.

1. Comparison of Analog and Digital Computer Solutions

The conservation equations (less the wall-energy balance) used in the bulk-boiling region, for the power-to-void transfer function, were identical to those used by Hudson, Atit, and Bankoff⁴⁶ in an analog computer solution of the same problem. A digital computer solution was obtained for conditions corresponding to those reported for Fig. 1 in Ref. 46. The two solutions have been compared in Fig. 9.1.

There was excellent agreement between the analog and digital predictions of the void phase lag for the full frequency range covered. Also, the zero-frequency amplitudes and notch-frequency locations agreed very well. The only significant difference between the analog and digital solutions was in the sharpness of the notches. The analog solution showed a much more pronounced notch than the digital solution. However, the sharpness of this notch was a function of the number of axial sections employed as was evident in the results of Hudson et al., and also reported by Schjetne.⁵²

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Fig. 9.1. Comparison of Analog and Digital Computer Predictions of Power-to-void Transfer Functions

The results reported by Hudson *et al.*, exhibited a maximum sharpness with six axial sections, whereas Schjetne found that the peaks and notches in the transfer function curves above 10 rad/sec were smaller in amplitude for the 10-section than in the 6-section system.

2. Bulk-boiling Boundary

As indicated in Ch. VI, due to power modulation the bulk-boiling boundary z_B oscillated around its steady-state location, $\bar{z}_B$, with amplitude z_B' , given by $(\partial z / \partial T) T'$, where $(\partial z / \partial T)$ was obtained from Eq. (6.39). The temperature response of the liquid was calculated by a step-by-step integration of Eq. (6.37). This movement of z_B gave an additional component to the transient void response. Initially this additional component was added to the transient void response, obtained by the step-by-step integration procedure, at the first axial section in the bulk-boiling region (as in the Kjeller Model). This gave results similar to those shown in Fig. 9.2, in which there is a discontinuous jump in the void response at $\bar{z}_B$. In Fig. 8.12, the void amplitude data as a function of z was found to be continuous and had a constantly increasing amplitude at low frequencies. This indicated that the effects of z_B' were not concentrated at $\bar{z}_B$.

112-5106

Fig. 9.2. Axial Variation of Transient Void Amplitude at Low Frequencies (0.01-0.10 cps) for Run 10, (A) Unmodified Model, (B) Present Model and (C) Data

The effects of z_B' on the transient void response were then assumed to be spread over the whole subcooled boiling length and were

calculated in the following manner. The amplitude of z'_B at an axial position $N+1$ in the subcooled boiling region was given by

$$z'_{B,N+1} = \overline{(\partial z / \partial T)} (T'_{N+1} - T'_N),$$

and the phase was taken to be the same as that of T'_{N+1} . Since the temperature response at each axial position was known, z'_B could be readily calculated. The quality change associated with this axial translation was then calculated and the void fraction response determined by employing one of the models. A comparison with the previous method is presented in Fig. 9.2.

3. Theoretical Predictions

In Figs. 9.3 and 9.4 theoretical predictions obtained with this model are compared with the experimental results. There were no significant differences in the predicted zero-frequency amplitude response, frequency notches, phase lag, and break frequencies as given by the Marchaterre-Hoglund correlation,⁶⁴ modified Bankoff model,⁵⁰ and Griffith's low-quality boiling voids model.⁶³ The Marchaterre-Hoglund correlation gave slightly higher zero-frequency amplitudes (0.5%) than the other two models; the Griffith model predicted the lowest zero-frequency amplitudes. The void phase lags predicted by the model are in excellent agreement with the experimental data (if the phase leads at low frequencies are neglected) up to the vicinity of the notch frequency. At higher frequencies the predicted phase lag increases continuously with increasing frequency while undulations still appear in the experimental results.

Generally, the predicted notch frequencies agrees well with the experimental values. The experimental notch frequency is normally higher than that predicted, and the agreement is best at the higher axial positions. Better agreement was also found for those cases with low inlet-temperature subcooling, where the effects of the transition boundaries were at a minimum.

The break frequencies were difficult to compare unless the notch frequency had been predicted accurately. For those cases where the notch frequency was correctly predicted the break frequency was in good agreement with the experimental results.

Appreciable differences were found between the model and data in some of the zero-frequency amplitudes. Although there was excellent agreement for some runs, there were also cases with appreciable discrepancies. Normally, the predicted zero-frequency amplitudes would be lower than the experimental, but in some of the upstream regions of the channel the converse was found.

The model was also used to predict the power-to-void transfer functions for conditions corresponding to Christensen's test runs. The

Fig. 9.3. Power-to-void Transfer Function: Comparison with Theory for Runs 1, 9, and 14

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Fig. 9.4. Power-to-void Transfer Function: Comparison with Theory for Run 11

predicted zero-frequency amplitudes were found to be one-half to one-third of the experimental values, and the experimental notch frequency occurred at frequencies much lower than the predicted (approximately one-half).

Schjetne in his testing of the Kjeller model obtained similar results when studying Christensen's experimental data. He offered three possible explanations for the zero-frequency differences: the steam-dome pressure cannot be regarded as constant, the wrong values of slip may have been employed, and the omission of riser effects in the theoretical model were believed to be of importance. With the use of the identical riser in this study no discernible effects were found attributable to the riser which would cause the zero-frequency amplitudes to be far greater than those predicted from steady-state correlations.

It is also doubtful that the errors in slip could be large enough to account for this discrepancy. Further, it is difficult to see how pressure variations in the steam dome could result in amplitudes differing by a factor of two. For the above reasons no logical explanation can be given for the wide disagreement between Christensen's zero-frequency amplitude and those predicted from steady-state arguments. However, it could be possible for pressure variations in the steam dome to give rise to nulls in the void amplitude response at frequencies lower than those due to the transportation delay of the void perturbation.

There was one significant point where this model predicted qualitative results differing from the experimental data. In runs having an appreciable subcooled boiling length, the model predicted at low frequencies a maximum void amplitude at the bulk-boiling boundary $\bar{z}_B$. This was not in agreement with the experimental results in which the void amplitude was seen to increase continuously with axial distance for low frequencies in all cases.

B. Christensen's Theoretical Analysis

Christensen derived a theoretical model for predicting power-to-void transfer functions which fit his experimental data very well. First, he found that the wall-temperature transfer function, which related perturbation in heat flow into the coolant channel, $Q'(s)$, to the perturbation in total heating power to the test section, $Q_W^1(s)$, could be expressed simply as

$$Q'(s)/Q_W^1(s) = G(s), \quad (9.1)$$

where $G(s)$ is a first-order transfer function.

Next, a power-to-outlet-temperature transfer function was derived for the case with no boiling. The rate of heat transfer from wall to coolant

was a function of the water temperature, and the transfer function for perturbation in heat flow, $Q_h'(z, s)$ at the point z , was given as:

$$\frac{Q_h'(z, s)}{Q_w'(s)} = \frac{z}{H} G(s) \frac{1 - \exp \{-[1 + \delta G(s)]\} st_p}{[1 + \delta G(s)] st_p}, \quad (9.2)$$

where

$$t_p = z/v_p; \quad \delta = \frac{p_c t_c}{A} \frac{C_w \rho_w t_w}{C_\ell \rho_\ell t_c}, \quad (9.3)$$

and v_p is the propagation velocity of the heat-flow perturbation. A complete expression for power-to-void transfer functions was also derived assuming negligible response of the channel pressure due to power modulation. This was found to give propagation velocities roughly twice as high as those experimentally measured. Thus a power-to-void transfer function was derived which accounted for pressure variations within the channel. The heat flux into the channel became a function of the time-varying saturation temperature, and the expression relating heat flow into the coolant was analogous to that used in deriving Eq. (9.2). The coolant in the channel was considered as a hypothetical fluid consisting of various fractions of saturated steam, saturated water, and subcooled water; expressions for densities and heat capacities were derived for this coolant mixture. All equations to be solved for the transfer-function perturbation in heat flow were now analogous to those used in deriving Eq. (9.2):

$$\frac{Q_h'(z, s)}{Q_w'(s)} = \frac{z}{H} G(s) \frac{1 - \exp \{-[1 + \delta' G(s)]\} st_p}{[1 + \delta' G(s)] st_p}, \quad (9.4)$$

where now

$$\delta' = K_3 \frac{p_c t_c}{A} \frac{C_w \rho_w t_w}{C_m \rho_m t_c} \quad (9.5)$$

is a parameter associated with the completeness of mixing and

$$t_p = \int_0^z \frac{dz}{v_p(z)} \quad (9.6)$$

the transport time associated with heat perturbation. K_3 is an additional constant which is a function of loop pressure and average power, and C_m and ρ_m are, respectively, the specific heat capacity and density of the mixture.

The perturbation in the steam mass flow rate was

$$w'_g(z,s) = Q'_h(z,s)/\lambda, \quad (9.7)$$

in which it is assumed that the total heat-flow perturbation leads to the formation of steam. The transient void response was obtained from

$$w'_g(z,S) = \alpha'(z,S) \rho_g A \bar{v}_{g,m}(z), \quad (9.8)$$

where $\bar{v}_{g,m}(z)$ was a modified steam velocity. Combination of Eqs. (9.1), (9.4), (9.7), and (9.8) leads to the complete power-to-void transfer function:

$$\frac{\alpha'(z,s)}{Q'_w(s)} = \frac{z}{A \rho_g \lambda \bar{v}_{g,m}(z)} \frac{1}{\frac{\tau}{1+\delta'} s + 1} \frac{1 - \exp \left\{ - \left[\frac{\tau}{1+\delta'} s + 1 \right] [s(1+\delta') t_p] \right\}}{s(1+\delta') t_p} \quad (9.9)$$

This equation was used in generating all theoretical power-to-void transfer functions. The parameter δ' was chosen in order to fit properly the break frequency in the amplitude curve, and the sharpness of the node was determined by the time constant τ of $G(s)$. In practice $v_p(z)$ for Eq. (9.6) was taken as

$$v_p(z) = \bar{v}_\ell(0) + az, \quad (9.10)$$

and "a" was obtained from $\partial \bar{v}_\ell(z)/\partial z$ in the steady-state measurements. Also, $\bar{v}_{g,m}(z)$ was taken to be equal to $v_p(z)$ in Eq. (9.9) according to Appendix IV.²³

In order to test the validity of the assumptions Christensen made in his model, Christensen's Run 9 was compared with Run 5 of this report. Data were obtained at the same pressure, inlet flow rate, and inlet subcooling. The only differences were in the average power (30 kW for Christensen and 20 kW for this report) and the thickness of the test channel (35 mils for Christensen). The values of δ' and τ listed for Christensen's Run 9 were corrected for the power and wall thickness differences, and Eq. (9.9) was used to generate the power-to-void transfer function.

The zero-frequency response predicted for the present Run 5 was found to be twice as large as the experimental, and the theoretical amplitude notch was at a frequency one-third of that actually measured. The parameter δ' which was used to give the correct notch frequency was approximately one-third to one-sixth of the value used by Christensen, and the value of "a"

used in obtaining the linear variation of $v_p(z)$ with axial distance was obtained from $\partial \bar{v}_g(z)/\partial z$ rather than $\partial \bar{v}_l(z)/\partial z$. Therefore, the frequency notch was predicted correctly by the use of a propagation velocity slightly less than the gas velocity and with much lower values of δ' , indicating more complete transverse mixing. However, the predicted zero-frequency response remained much higher than that obtained experimentally. Christensen assumed that the heat-flow rate into the coolant was same in all parts of the channel and that all this heat flow went to form steam. By means of Bowring's analysis for the subcooled boiling region Eq. (9.7) was modified to

$$w'_g(z,s) = Q'_h(z,S)/\lambda(1+\epsilon)$$

for all axial positions in the subcooled boiling region. This modification brought the predicted zero-frequency response into closer agreement with the experimental data, as shown in Fig. 9.5.

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Fig. 9.5. Comparison of Modified Christensen Predictions with Data for (A) Run 5, and for (B) Run 9

A comparison was also made between Christensen's Run 13 and Run 9 of this report. The same modifications were required to predict correctly the zero-frequency response and notch frequency.

C. Space Technology Laboratories

A phenomenological model of power-to-void and power-to-flow transfer functions developed by Morse, Wright, and Zivi²² has been described briefly in Ch. II. This model was used successfully to predict power-to-void transfer functions which agreed with their experimental results.

In the atmospheric pressure runs made at STL, the void-fraction response to small-amplitude power modulation was found to vary proportionally with the local slope of the steady-state void-fraction curve, $\bar{\alpha}(z)$. From an analysis of steady-state void-fraction data obtained at various power levels, the $\bar{\alpha}(z)$ curves were found to translate essentially, as the power was modulated, with negligible changes in the shape of the curves. For this reason, the transient void amplitudes were calculated by use of

$$|\alpha'(z)|_z = |z'_S| \left(\frac{d\bar{\alpha}(z)}{dz} \right)_z,$$

where $|z'_S|$ is the amplitude of the boiling-boundary translation due to power modulation. Since the magnitude of $d\bar{\alpha}(z)/dz$ decreases with increasing z , decreasing void amplitudes will be predicted with increasing axial height. This behavior was not found here, as has been shown in Fig. 8.12, where the void amplitude at low frequency continually increases with increasing z .

For the experimental conditions used in this work the shape of the $\bar{\alpha}(z)$ curves changed with power modulation. Therefore this model could not be expected to predict correctly the transient void amplitude data reported here.

However, this model introduced a method of treating that contribution to the total void response due to boiling-boundary movement. This is essentially the same method with which the boiling-boundary effects are treated in the Kjeller Model and the model introduced here.

D. Sanathanan's Theoretical Analysis

Another analytical method which can be used to predict the space-time-dependent transient steam void fraction in forced- or natural-circulation systems has been developed recently by Sanathanan.⁷⁸ In this method the one-dimensional conservation equations describing the flow are

written after the method of Meyer⁷⁹ and Laplace transformed with respect to time. Then the dependent variables are written in the form

$$\alpha'(z,t) = b_0 P_0(z) + b_1 P_1(z) + b_2 P_2(z) + \dots,$$

where the coefficients b_0, b_1 , etc., are unknown functions of time only, and P_0, P_1, P_2 , and so forth are a complete set of orthogonal eigenfunctions in the interval $0 \leq z \leq H$. These coefficients are evaluated by the application of the orthogonality conditions and the existing boundary conditions.

The Legendre polynomials were chosen as the most suitable eigenfunctions. To facilitate their use, a linear transformation of the space variable from z to x was performed such that x varied from -1 (at the inlet) to $+1$ (at the exit).

The steady-state quantities were expressed as

$$\bar{v}_\ell(x) = A_0 + A_1 P_1(x) + A_2 P_2(x) + \dots;$$

$$\bar{\alpha}(x) = B_0 + B_1 P_1(x) + B_2 P_2(x) + \dots;$$

$$S(x) = S_0 + S_1 P_1(x) + S_2 P_2(x) + \dots,$$

where the coefficients are obtained through a curve-fitting procedure applied to the steady-state spatial variations in $\bar{v}_\ell, \bar{\alpha}$, and S . The above expressions for the steady-state quantities were then substituted into the transformed equations of change, the equations were multiplied by $P_0(x), P_1(x), P_2(x)$, etc., respectively, and integrated over x between the limits -1 and $+1$ (orthogonality conditions). For a third-degree approximation in x for α' and a second-degree approximation for the steady-state terms, nine unknowns result. Six independent equations result from the application of orthogonality conditions plus two boundary conditions, which enables transfer functions of the form $a_0(s)/Q'(s), a_1(s)/Q'(s)$, etc., to be calculated. These six equations plus the two boundary conditions are expressed in matrix equation form and the transfer functions $a_0(s)/Q'(s), a_1(s)/Q'(s)$, etc., obtained.

This method was applied⁸⁰ to Run 9 of this experiment by means of a two-term Legendre polynomial approximation in α' . The prediction of the zero-frequency response and break frequency of the power-to-void transfer function was encouraging and indicated that a more detailed comparison should be attempted. An approximation of much higher degree in α' would be required to predict correctly the experimentally measured notches (zeros) and peaks (poles). This analysis should also be extended to include the sub-cooled boiling region, and for these reasons a more extensive analysis could not be completed at this time.

E. Kagawa and Hada

Another investigation⁷² in which power-to-void transfer functions were experimentally measured was performed at atmospheric pressure with a forced-circulation steam-water loop. The flow cross section consisted of two equal rectangular areas, 0.394×0.290 in., separated by an aluminum heating plate, 0.0098 in. thick. The effective heating length was 25.6 in. and for the one set of measurements reported the experimental settings of the loop variables were: mean heat input = 795 W, inlet velocity = 1.11 ft/sec, and inlet temperature = 212.9°F. The axial location of the measurement was at $z^* = 0.892$, and the power modulation amplitude was 22.6%.

The transient void measurements were made in a direction perpendicular to the heating plate by use of X rays, with a collimator window 0.197×0.0079 in. Measurements were made at three transverse positions: $y^* = 0.0$, $y^* = 0.64$, and $y^* = 0.84$. The transient void amplitude was found to be a maximum at the center ($y^* = 0.0$) and to decrease as the wall was approached. For a 10% power-modulation amplitude the mean zero-frequency void response was approximately 2.6%. Since the cross-section-averaged void fractions were not tabulated in the report (the transverse distributions were shown but in a very small graph), the axial distribution of the liquid and vapor velocities could not be accurately ascertained. From the Bode diagram of the amplitude response, notches are found in the frequency range from 0.70 to 0.90 cps. The notch frequency was lowest for measurements at the channel midpoint and increased as the wall was approached. However, the opposite response was reported for the phase lag, in that the notch frequency was lowest at points nearest the wall. For this reason it was difficult to determine correctly where the propagation velocity was maximum. An approximate mean propagation velocity equaled 1.9 ft/sec, and an estimated mean liquid velocity over the same axial distance equaled 1.8 ft/sec.

Since the transverse shape of the void profiles were concave in shape over a majority of the test-section length, these results thus appear to agree with the experimental results reported here.

The theoretical analyses of Kagawa and Hada consisted of a Laplace transform solution of the continuity and energy equations in which the potential, kinetic, and expansion energy terms were neglected. The test section was divided into two parts corresponding to a nonboiling and boiling region separated by the time-dependent location of the thermodynamic boiling point, z_g . They reported good agreement between predictions of the theoretical model and experimental data, but since the analysis does not specifically take into account the subcooled boiling region, it is not treated in detail here.

X. CONCLUSIONS

Experimental measurements of power-to-void transfer functions were made in a vertical, rectangular, electrically heated test section in which four main loop parameters were varied: pressure from 200 to 800 psia; inlet flow rates from 2.52 to 3.78 ft/sec; heat fluxes from 2.28×10^4 to 9.12×10^4 Btu/(hr)(ft²) and inlet subcoolings from 0.8 to 12.4°F. Data were obtained at three axial positions and also at various transverse positions across the width of the channel for a fixed axial height.

All of the void-amplitude data plotted as a function of frequency exhibited notches at frequencies ranging from 1.2 to 3.7 cps. The notch frequency was highest for those axial positions nearer the test-section inlet and decreased with increasing axial distance. Transverse measurements showed a maximum value of the notch frequency at the center of the channel, which decreased as the wall was approached, for downstream regions of the channel. The phase measurements also showed an undulation close to the notch frequency.

Good agreement was found between the zero-frequency void amplitude response obtained from the experimental data and that obtained from steady-state axial void distributions at various power levels. Increased inlet subcooling was found to increase the zero-frequency amplitude.

At pressures of 200, 300, and 400 psia the range of linearity of the void response to power modulation was found to extend up to amplitudes of 20% of the average power level. There was no evidence of any nonlinear behavior at 600 and 800 psia for power amplitudes up to 12%.

The axial variation in transient void amplitude was sinusoidal in shape, being of the same form as the power-forcing function. As the modulation frequency increased, the period and amplitude of this sinusoidal function decreased until the test-section length corresponded to one-half a wavelength. Then a notch would appear in the void amplitude at the test-section exit.

The transverse void amplitude distribution data appeared to be of the form $\alpha'/\alpha'_{\max} = y^{1/n}$.

The cross-section-averaged propagation velocities were found to be intermediate between the mixture and vapor cross-section-averaged velocities at all axial positions downstream of the concave-shaped transverse void distributions. Near the entrance of the test section, the propagation velocity decreased and became more closely related to the cross-section-averaged liquid velocity.

At a fixed axial position the local void-propagation velocity was a function of the distance from the wall. In the downstream regions of the

channel the measured propagation velocity profiles exhibited a maximum at the center and decreased near the wall, which is consistent with the convex steady-state velocity profiles which are thought to exist in these regions.

From the Bode diagrams of the data it was found that sharpness of the notch increased with increasing pressure and inlet temperature subcooling, but decreased with increasing axial length and increasing distance from the test section wall. By noting the analogous behavior of slip and the sharpness of the notch, slip was believed to be of importance here in the determination of the break frequency of the power-to-void transfer function.

Steady-state axial and transverse void fraction distributions were obtained under loop-operating conditions corresponding to those employed in the power-to-void transfer-function measurements. The Marchaterre-Hoglund, Griffith, modified Bankoff, and Neal models were all found to predict fairly closely the axial, steady-state, cross-section-averaged void distributions. The Neal model appeared to be best at high void fractions.

Concave void distributions were found near the test-section entrance. The degree of concavity and persistence of this profile with axial distance increased with increased inlet-temperature subcooling. For all concave-shaped profiles the calculated values of slip were less than or close to unity.

The theoretical models proposed by Zivi and Wright (at STL), Christensen, Solberg, and Sanathanan were tested as to their ability to predict power-to-void transfer-function data reported here. The STL model predicted decreasing transient void amplitudes with increasing axial distance at zero frequency, whereas the experimental data exhibited the opposite behavior. Zivi and Wright's model predicted transient void amplitudes by considering only the translation of the $\bar{\alpha}(z)$ curve. However, in this study the shape of the $\bar{\alpha}(z)$ curves was found to change with power modulation as well, and for this reason the STL model could not be expected to predict correctly the transient void amplitude data reported here.

Christensen's model gave zero-frequency amplitudes approximately twice as large as the experimental, and the predicted theoretical notch frequencies were about one-third of that actually measured here. However, when the void-propagation velocities were taken to be equal to the gas velocity and the parameter associated with the completeness of mixing (δ') was taken to be one-third to one-sixth the value used by Christensen, the notch frequency was predicted correctly by Christensen's model. Also, the addition of Bowring's physical model for the subcooled boiling region brought the predicted zero-frequency response into closer agreement with the experimental data.

The theoretical analysis of Sanathanan with a two-term Legendre polynomial approximation in α' predicted a zero-frequency response and

break frequency of the power-to-void transfer function which was in fair agreement with the data. However, an approximation of much higher degree in α' would be required to predict correctly the experimentally measured notches (zeros) and peaks (poles), and an extension to include the subcooled boiling region should be made. An extensive analysis could not be completed at this time.

The model presented in this work was similar to Solberg's "Kjeller Model" in that Bowring's physical model was employed in the subcooled boiling region. Here the heat capacity of the wall was accounted for in all three regions, the transitional bulk-boiling boundary effects were considered in every section of the subcooled boiling region, and the entire model was programmed for solution on a digital computer by a step-by-step integration procedure. The steady-state values of $\bar{\alpha}(z)$, $\bar{v}_g(z)$, and $\bar{S}(z)$ were generated from various models and correlations relating quality and void fraction. Fairly close agreement between theory and data was found for the notch frequencies, phase lag, and break frequencies. Appreciable differences between model predictions and data were found for some of the zero-frequency amplitudes. In general, the overall agreement between theory and data was best for these runs with low inlet-temperature subcooling. For runs having an appreciable subcooled boiling length, the model predicted at low frequencies a maximum void amplitude at the bulk-boiling boundary $\bar{z}_B$ which was not in agreement with the experimental results. This was believed to be due to an overestimation of the effects of the transitional bulk-boiling boundary z_B^1 on the transient void response.

The above conclusions regarding the experimental data were drawn from information obtained by use of a test section with a rectangular flow area. In future experimental work an investigation as to effects, if any, of different geometries and flow areas would be of practical interest. Also, more information is required about the transverse behavior of transient void amplitude and local void propagation velocities in order to draw more definite conclusions. A more detailed study of the relationship between slip and break frequency of the transfer function would also be of practical importance.

In order to test in detail the various models used to relate quality and void fraction, data are required about the local liquid velocities as well as the transverse void distribution.

More theoretical work and experimental data are required in order to allow accurate predictions of the power-to-void transfer functions, especially for those cases with appreciable subcooled boiling where the transitional boundaries must be taken into account.

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APPENDIX I

Cross-section-averaged, Steady-
state Values of Void Fraction,
Velocities, and Slip

Table I.1

RUN NO. 1

z^*	0.104	0.164	0.225	0.285	0.346	0.406	0.467	0.528	0.588	0.649	0.710	0.770	0.831
$X \cdot 10^{+3}$	0.625	1.24	1.78	2.40	3.12	3.65	4.28	4.91	5.53	6.15	6.79	7.41	8.04
$\bar{a}_{avg}$	6.5	11.4	14.7	18.1	21.6	23.3	25.4	27.2	28.6	29.9	31.2	32.3	33.2
$\bar{v}_g$	4.50	5.10	5.70	6.25	6.80	7.35	7.95	8.50	9.10	9.70	10.25	10.8	11.4
$\bar{v}_g$	4.00	4.15	4.35	4.55	4.70	4.90	5.10	5.20	5.40	5.55	5.70	5.90	6.10
Slip	1.12	1.23	1.31	1.37	1.45	1.50	1.56	1.63	1.69	1.75	1.80	1.83	1.87
$\bar{v}_{mean}$	4.04	4.26	4.54	4.86	5.15	5.47	5.82	6.10	6.45	6.79	7.12	7.48	7.86

Table I.2

RUNS NO. 2 & 3

z^*	0.104	0.164	0.225	0.285	0.346	0.406	0.467	0.528	0.588	0.649	0.710	0.770	0.831
$X \cdot 10^{+3}$	0.96	1.42	1.71	2.49	3.54	4.85	5.53	6.53	7.52	8.55	9.56	10.6	11.6
$\bar{a}_{avg}$	5.7	7.7	8.7	12.1	16.4	21.6	23.6	26.9	29.8	32.5	35.2	37.8	39.8
$\bar{v}_g$	3.55	3.85	4.12	4.32	4.52	4.72	4.91	5.10	5.30	5.52	5.70	5.90	6.12
$\bar{v}_g$	2.63	2.70	2.80	2.90	3.00	3.13	3.27	3.42	3.57	3.70	3.87	4.03	4.20
Slip	1.35	1.42	1.47	1.49	1.51	1.51	1.51	1.49	1.48	1.49	1.48	1.47	1.46
$\bar{v}_{mean}$	2.78	2.79	2.92	3.07	3.25	3.48	3.66	3.87	4.08	4.29	4.52	4.74	4.95

Table I.3

RUNS NO. 4 & 5

z^*	0.104	0.164	0.225	0.285	0.346	0.406	0.467	0.528	0.588	0.649	0.710	0.770	0.831
$X \cdot 10^{+3}$	1.94	4.04	6.22	8.28	10.4	12.5	14.6	16.8	19.0	21.0	23.2	25.2	27.4
$\bar{a}_{avg}$	8.8	15.6	21.2	24.9	28.3	30.8	32.8	34.7	36.6	37.4	39.1	39.8	40.8
$\bar{v}_g$	3.40	4.00	4.55	5.15	5.70	6.30	6.90	7.50	8.05	8.70	9.20	9.80	10.4
$\bar{v}_g$	2.80	2.95	3.10	3.30	3.45	3.60	3.75	3.90	4.05	4.20	4.35	4.50	4.65
Slip	1.21	1.35	1.47	1.56	1.65	1.85	1.84	1.92	1.98	2.07	2.12	2.18	2.24
$\bar{v}_{mean}$	2.85	3.11	3.41	3.76	4.08	4.43	4.78	5.14	5.51	5.89	6.25	6.61	7.00

Table I.4

RUN NO. 6

z^*	0.104	0.164	0.225	0.285	0.346	0.406	0.467	0.528	0.588	0.649	0.710	0.770	0.831
$X \cdot 10^{+3}$	-	-	1.32	2.24	3.20	4.12	5.00	5.94	7.70	9.83	12.1	14.0	16.2
$\bar{a}_{avg}$	-	-	6.8	9.4	11.8	14.5	16.7	18.8	23.4	27.7	31.8	31.9	31.8
$\bar{v}_g$	[†] 1.40	2.25	3.00	3.70	4.20	4.40	4.65	4.90	5.10	5.50	5.90	6.80	7.90
$\bar{v}_g$	2.52	2.60	2.70	2.80	2.90	3.00	3.10	3.20	3.25	3.35	3.50	3.55	3.75
Slip	0.55	0.87	1.11	1.32	1.45	1.47	1.50	1.53	1.57	1.64	1.68	1.92	2.10
$\bar{v}_{mean}$	2.52	2.60	2.74	2.89	3.06	3.20	3.36	3.52	3.68	3.94	4.26	4.57	5.14

[†] Denotes extrapolated values.

Table I.5

RUN NO. 7

z^*	0.104	0.164	0.225	0.285	0.346	0.406	0.467	0.528	0.588	0.649	0.710	0.770	0.831
$X \cdot 10^{+3}$	2.48	4.65	6.68	8.78	10.9	12.9	15.0	17.2	19.3	21.4	23.6	25.6	27.8
$\bar{a}_{avg}$	8.4	13.8	18.3	22.5	26.1	29.1	32.0	35.0	37.3	39.0	41.1	42.6	44.6
$\bar{v}_g$	6.70	7.70	8.30	8.90	9.50	10.1	10.7	11.2	11.8	12.5	13.1	13.7	14.2
$\bar{v}_g$	4.10	4.35	4.55	4.80	5.00	5.25	5.50	5.70	5.90	6.10	6.30	6.55	6.75
Slip	1.63	1.77	1.82	1.85	1.90	1.92	1.94	1.97	2.00	2.05	2.08	2.09	2.10
$\bar{v}_{mean}$	4.32	4.81	5.24	5.72	6.18	6.66	7.16	7.63	8.10	8.47	9.10	9.58	10.1

Table I.6
RUN NO. 8

z°	0.104	0.164	0.225	0.285	0.346	0.406	0.467	0.528	0.588	0.649	0.710	0.770	0.831
$X \cdot 10^{+3}$	0.0	0.0	0.0	0.98	1.89	2.70	3.61	4.52	5.43	6.80	8.95	11.0	13.0
$\bar{a}_{avg}$	-	-	-	5.9	8.8	11.1	13.3	14.9	16.5	18.9	23.2	26.5	29.3
$\bar{v}_g$	0.0	0.0	2.30	3.80	4.90	5.55	6.20	6.90	7.50	8.20	8.80	9.45	10.1
$\bar{v}_L$	3.78	3.78	3.85	4.00	4.10	4.30	4.40	4.55	4.70	4.92	5.00	5.15	5.30
Slip	-	-	0.60	0.95	1.20	1.29	1.41	1.51	1.60	1.67	1.76	1.83	1.91
$\bar{v}_{mean}$	3.78	3.78	3.78	3.99	4.17	4.44	4.64	4.90	5.16	5.53	5.88	6.19	6.70

Table I.7
RUN NO. 9

z°	0.104	0.164	0.225	0.285	0.346	0.406	0.467	0.528	0.588	0.649	0.710	0.770	0.831
$X \cdot 10^{+3}$	2.70	4.09	5.63	8.70	11.8	16.0	18.1	21.2	24.4	27.6	30.8	33.9	37.1
$\bar{a}_{avg}$	13.0	12.6	13.3	17.5	21.0	25.8	27.0	29.6	32.2	34.5	36.6	38.7	40.8
$\bar{v}_g$	3.00	4.70	6.15	7.20	8.15	9.00	9.70	10.40	11.0	11.6	12.2	12.7	13.2
$\bar{v}_L$	4.10	4.30	4.45	4.65	4.80	4.95	5.10	5.30	5.50	5.70	5.90	6.05	6.25
Slip	0.73	1.09	1.37	1.55	1.70	1.81	1.90	1.96	2.00	2.04	2.07	2.10	2.11
$\bar{v}_{mean}$	3.96	4.35	4.68	5.10	5.50	5.99	6.34	6.82	7.27	7.74	8.21	8.63	9.13

Table I.8
RUN NO. 10

z°	0.104	0.164	0.225	0.285	0.346	0.406	0.467	0.528	0.588	0.649	0.710	0.770	0.831
$X \cdot 10^{+3}$	0.0	0.0	0.76	2.14	3.43	4.84	6.14	7.54	10.3	13.2	16.5	19.8	22.7
$\bar{a}_{avg}$	-	-	5.24	9.12	10.8	12.8	13.8	15.3	18.7	21.8	24.9	27.6	29.4
$\bar{v}_g$	0.0	0.0	2.10	3.40	4.60	5.50	6.45	7.10	8.00	8.80	9.60	10.4	11.2
$\bar{v}_L$	3.90	4.00	4.10	4.20	4.30	4.40	4.50	4.60	4.70	4.80	4.90	5.00	5.10
Slip	-	-	0.51	0.81	1.07	1.25	1.43	1.54	1.70	1.83	1.96	2.08	2.20
$\bar{v}_{mean}$	3.90	4.00	4.00	4.13	4.34	4.53	4.77	4.98	5.32	5.67	6.07	6.49	6.89

Table I.9
RUN NO. 12

z°	0.104	0.164	0.225	0.285	0.346	0.406	0.467	0.528	0.588	0.649	0.710	0.770	0.831
$X \cdot 10^{+2}$	0.233	0.344	0.454	0.562	0.680	0.930	1.18	1.45	1.70	1.95	2.20	2.44	2.69
$\bar{a}_{avg}$	6.1	7.7	9.0	9.9	10.8	13.6	16.4	19.1	21.3	23.5	25.8	28.0	30.4
$\bar{v}_g$	2.62	3.05	3.47	3.90	4.30	4.70	4.95	5.20	5.48	5.70	5.84	5.98	6.08
$\bar{v}_L$	2.62	2.70	2.78	2.86	2.94	3.02	3.08	3.17	3.24	3.32	3.39	3.47	3.56
Slip	1.0	1.13	1.25	1.36	1.46	1.55	1.61	1.64	1.69	1.72	1.73	1.73	1.71
$\bar{v}_{mean}$	2.62	2.72	2.84	2.97	3.08	3.25	3.38	3.56	3.71	3.88	4.02	4.18	4.33

Table I.10
RUN NO. 13

z°	0.104	0.164	0.225	0.285	0.346	0.406	0.467	0.528	0.588	0.649	0.710	0.770	0.831
$X \cdot 10^{+2}$	-	0.446	0.592	0.752	1.08	1.41	1.79	2.08	2.33	2.66	3.10	3.43	3.76
$\bar{a}_{avg}$	-	16.4	16.6	18.0	22.4	25.6	29.2	31.1	32.2	34.5	38.4	40.8	43.6
$\bar{v}_g$	-	2.8	3.66	4.30	4.97	5.68	6.32	6.88	7.45	7.95	8.31	8.66	8.88
$\bar{v}_L$	-	4.35	4.50	4.70	4.90	5.10	5.30	5.55	5.75	5.95	6.15	6.35	6.55
Slip	-	0.64	0.81	0.92	1.01	1.11	1.19	1.24	1.30	1.34	1.35	1.37	1.40
$\bar{v}_{mean}$	-	3.90	4.36	4.63	4.91	5.24	5.59	5.96	6.30	6.64	6.97	7.29	7.57

Table II.1A

 RUN NO. 1
 ONE-SHOT METHOD

 $p = 200 \text{ psia}$ $z^* = 0.834$
 $\Delta T_{\text{sub}} = 0.8^\circ\text{F}$ $\phi = 2.28 \times 10^4 \text{ Btu/hr-ft}^2$
 $v_0 = 3.78 \text{ ft/sec}$ $\alpha_{\text{avg}} = 46.4\%$

Frequency (cps)	Power Amplitude (%)	Void Amplitude (%)	Normal Void Amplitude (%)	Void Phase (deg)	Frequency (cps)	Power Amplitude (%)	Void Amplitude (%)	Normal Void Amplitude (%)	Void Phase (deg)
0.025	11.6	1.89	1.63	+17	0.29	10.2	2.36	2.30	-54
0.040	11.6	2.76	2.39	+7	2.85	10.8	0.968	0.90	-149
0.058	11.2	2.58	2.29	+2	3.80	12.2	0.185	0.15	-183
0.078	13.5	3.09	2.29	-3	1.18	13.5	0.577	0.43	-165
0.10	13.2	3.26	2.46	-12	1.45	13.6	0.986	0.72	-114
0.048	15.7	3.30	2.09	+10	2.20	12.8	0.521	0.41	-128
0.50	10.9	2.23	2.04	-84	0.25	10.7	2.62	2.45	-38
0.68	12.0	2.28	1.90	-115	0.10	17.1	2.89	1.69	-10
0.85	11.2	1.88	1.67	-142	0.075	16.2	2.62	1.62	0
1.00	11.3	1.30	1.15	-159	0.060	14.1	3.00	2.13	0
0.78	12.1	1.64	1.36	-119	0.048	18.3	3.18	1.74	+11
0.60	11.8	2.15	1.81	-99	0.012	18.0	4.36	2.41	+27
0.38	9.47	2.52	2.66	-66	0.028	11.7	2.04	1.74	+17

Table II.1B

 RUN NO. 1
 ONE-SHOT METHOD

 $p = 200 \text{ psia}$ $z^* = 0.467$
 $\Delta T_{\text{sub}} = 0.8^\circ\text{F}$ $\phi = 2.28 \times 10^4 \text{ Btu/hr-ft}^2$
 $v_0 = 3.78 \text{ ft/sec}$ $\alpha_{\text{avg}} = 34.5\%$

Frequency (cps)	Power Amplitude (%)	Void Amplitude (%)	Normal Void Amplitude (%)	Void Phase (deg)	Frequency (cps)	Power Amplitude (%)	Void Amplitude (%)	Normal Void Amplitude (%)	Void Phase (deg)
0.015	17.0	2.08	1.22	+51	0.790	9.04	1.46	1.62	-98
0.030	11.7	1.76	1.50	+37	0.590	9.37	1.54	1.64	-74
0.050	11.4	1.89	1.66	+15	0.370	9.92	1.76	1.77	-42
0.070	10.6	1.98	1.87	+10	0.180	11.02	2.30	2.08	-7
0.095	11.3	2.04	1.80	+6	1.08	8.40	0.788	0.94	-148
0.100	13.2	2.48	1.88	+6	1.55	9.23	0.843	0.91	-146
0.055	15.2	2.39	1.58	+22	1.95	9.59	0.506	0.53	-213
0.038	16.3	2.98	1.84	+21	2.28	9.87	0.450	0.46	-251
0.015	19.4	2.34	1.20	+44	3.00	9.84	0.562	0.57	-195
0.160	11.9	2.06	1.74	0	4.60	9.62	0.393	0.41	-405
0.295	11.0	2.06	1.86	-31	1.19	10.2	1.01	0.99	-126
0.470	10.5	1.70	1.62	-64	0.058	15.5	2.54	1.64	+10
0.700	10.4	1.46	1.40	-85	0.032	19.2	2.24	1.16	+34
0.875	9.43	1.35	1.43	-118	0.012	26.3	3.02	1.15	+41
0.975	9.01	1.82	2.01	-109					

Table II.2

 RUN NO. 2
 ONE-SHOT METHOD

 $p = 300 \text{ psia}$ $z^* = 0.834$
 $\Delta T_{\text{sub}} = 1.3^\circ\text{F}$ $\phi = 2.28 \times 10^4 \text{ Btu/hr-ft}^2$
 $v_0 = 2.52 \text{ ft/sec}$ $\alpha_{\text{avg}} = 52.6\%$

Frequency (cps)	Power Amplitude (%)	Void Amplitude (%)	Normal Void Amplitude (%)	Void Phase (deg)	Frequency (cps)	Power Amplitude (%)	Void Amplitude (%)	Normal Void Amplitude (%)	Void Phase (deg)
0.100	8.26	1.70	2.06	-7	2.00	9.92	0.658	0.66	-264
0.010	13.4	2.54	1.89	+37	3.00	10.0	0.711	0.71	-288
0.051	10.3	1.83	1.77	-25	4.00	9.56	0.750	0.78	-337
0.305	10.4	1.38	1.33	-47	2.50	9.80	0.638	0.65	-376
0.510	7.42	0.860	1.16	-102	0.011	7.48	1.09	1.45	+11
0.720	6.24	0.786	1.26	-113	0.011	14.6	2.48	1.70	+20
0.885	6.95	0.673	0.97	-147	0.011	20.7	3.14	1.52	+25
1.04	4.99	0.579	1.16	-216	0.011	22.0	3.24	1.47	+19
0.415	8.91	1.46	1.63	-65	0.031	19.3	3.33	1.72	+2
0.620	8.55	0.935	1.09	-101	0.051	18.1	3.24	1.79	+2
0.03	5.94	0.998	1.68	+20	0.071	14.9	3.06	2.05	-3
0.06	6.24	0.976	1.56	0	0.091	15.4	2.59	1.68	-11
0.10	5.52	1.55	2.81	-9	0.105	15.0	2.76	1.84	-18
0.30	6.42	1.65	2.57	-56	0.320	13.4	2.35	1.76	-59
0.50	7.54	0.732	0.97	-46	0.520	12.7	1.33	1.05	-88
0.75	7.01	0.806	1.15	-146	0.715	11.3	1.87	1.66	-157
0.85	7.36	1.20	1.63	-148	0.912	10.2	1.01	0.99	-146
1.00	7.19	0.900	1.25	-217	0.812	10.6	1.33	1.25	-138
1.50	7.42	0.470	0.63	-298	1.19	10.1	1.05	1.04	-221

Table II.3A

RUN NO. 3
NARROW VOID WINDOW

$p = 300$ psia $z^* = 0.834$
 $\Delta T_{sub} = 1.8^\circ\text{F}$ $\phi = 2.28 \times 10^4$ Btu/hr-ft²
 $v_0 = 2.52$ ft/sec $\alpha_{y^*} = 54.1\%$
 $y^* = 0.00$

Frequency (cps)	Power Amplitude (%)	Void Amplitude (%)	Normal Void Amplitude (%)	Void Phase (deg)	Frequency (cps)	Power Amplitude (%)	Void Amplitude (%)	Normal Void Amplitude (%)	Void Phase (deg)
0.925	10.8	1.49	1.39	-188	3.20	9.60	0.460	0.48	-267
0.012	6.11	1.96	3.21	0	5.20	9.40	0.200	0.21	-375
0.021	9.09	2.64	2.90	+21	0.021	8.14	2.96	3.64	+15
0.042	10.3	3.34	3.23	+1	0.042	7.72	2.30	2.98	+8
0.060	14.2	3.86	2.72	-11	0.061	8.38	2.80	3.34	+12
0.080	15.3	4.77	3.12	-21	0.081	11.4	2.46	2.16	-9
0.110	15.3	4.70	3.07	-30	0.098	15.3	3.92	2.57	-9
0.220	13.4	2.78	2.07	-63	0.307	12.4	3.48	2.82	-85
0.430	12.1	2.43	2.01	-126	0.510	11.9	1.76	1.48	-124
0.620	11.4	1.68	1.47	-154	0.705	11.3	1.61	1.42	-197
0.830	11.3	0.98	0.87	-181	0.900	10.1	0.600	0.59	-154
1.00	10.2	1.55	1.52	-226	1.41	9.38	0.640	0.68	-269
1.55	8.80	0.737	0.84	-242	1.30	9.32	0.640	0.69	-242
2.35	7.60	0.627	0.82	-240	1.89	9.90	0.240	0.25	299

Table II.3B

RUN NO. 3
NARROW VOID WINDOW

$p = 300$ psia $z^* = 0.834$
 $\Delta T_{sub} = 1.3^\circ\text{F}$ $\phi = 2.28 \times 10^4$ Btu/hr-ft²
 $v_0 = 2.52$ ft/sec $\alpha_{y^*} = 38.5\%$

Frequency (cps)	Power Amplitude (%)	Void Amplitude (%)	Normal Void Amplitude (%)	Void Phase (deg)	Frequency (cps)	Power Amplitude (%)	Void Amplitude (%)	Normal Void Amplitude (%)	Void Phase (deg)
$y^* = 0.500$					$y^* = 0.500$				
0.011	9.10	1.96	2.15	+1	0.600	11.0	0.78	0.70	-120
0.031	7.80	1.30	1.67	+15	0.806	9.32	1.06	1.14	-221
0.031	11.6	1.69	1.46	+5	0.806	9.50	0.341	0.36	-238
0.051	11.7	1.96	1.68	-1	0.840	9.09	0.531	0.58	-202
0.072	11.6	1.84	1.59	+1	2.26	10.8	0.208	0.19	-214
0.092	14.4	2.24	1.56	-9	3.10	10.4	0.208	0.20	-290
0.150	13.6	1.90	1.40	-32	3.72	10.3	0.131	0.13	-401
0.310	12.5	2.30	1.84	-92	5.00	9.50	0.264	0.28	-266
0.513	12.2	1.24	1.02	-103	5.98	9.50	0.170	0.18	-367
0.800	10.3	0.865	0.84	-171	0.774	10.2	0.855	0.84	-244
0.913	9.10	0.670	0.74	-230	1.02	9.92	0.208	0.21	-285
0.725	9.40	1.08	1.15	-137	$y^* = 0.250$ $\alpha_{avg} = 48.2\%$				
0.026	8.67	2.38	2.74	-28	0.315	11.0	2.08	1.89	-103
0.041	9.80	2.22	2.26	-14	0.206	11.9	2.28	1.92	-71
0.060	11.4	2.02	1.86	-21	0.082	15.9	3.64	2.30	-24
0.081	16.2	3.16	1.39	-28	0.063	15.0	3.92	2.61	-21
0.162	12.8	2.69	1.95	-49	0.046	15.2	3.11	2.04	-38
0.225	12.2	2.02	1.61	-55	0.022	12.9	3.23	2.72	+15
0.420	11.0	1.81	1.64	-82					

Table II.3C

RUN NO. 3
NARROW VOID WINDOW

$p = 300$ psia $\phi = 2.28 \times 10^4$ Btu/hr-ft²
 $\Delta T_{sub} = 1.5^\circ\text{F}$ $y^* = 0.800$
 $v_0 = 2.52$ ft/sec $\alpha_{y^*} = 45.2\%$
 $z^* = 0.834$

Frequency (cps)	Power Amplitude (%)	Void Amplitude (%)	Normal Void Amplitude (%)	Void Phase (deg)	Frequency (cps)	Power Amplitude (%)	Void Amplitude (%)	Normal Void Amplitude (%)	Void Phase (deg)
0.015	7.38	1.02	1.38	-9	0.712	10.2	1.27	1.24	-146
0.015	7.81	1.48	1.90	+17	0.925	7.44	0.60	0.82	-198
0.015	1.71	1.47	1.47	-13	1.02	12.3	0.652	0.53	-196
0.015	23.9	3.67	1.53	+36	0.812	11.8	1.86	1.58	-183
0.035	21.8	3.05	1.40	-7	0.615	12.2	0.877	0.72	-134
0.055	18.9	3.58	1.90	-2	0.410	13.2	1.31	0.99	-117
0.075	15.8	3.08	1.96	+4	0.210	12.6	2.30	1.82	-47
0.090	16.2	3.67	2.27	-14	2.08	10.6	0.147	0.14	-336
0.080	16.4	3.04	1.86	-12	3.15	11.1	0.765	0.69	-293
0.058	17.8	3.15	1.77	+10	5.10	10.2	0.450	0.44	-352
0.046	19.6	3.51	1.79	-5	4.15	11.2	0.296	0.26	-226
0.025	22.1	3.46	1.56	+26	2.15	12.1	0.293	0.24	-368
0.345	13.5	1.12	0.83	-91	1.26	12.8	0.536	0.42	-220
0.525	12.2	1.37	1.12	-114	1.25	12.2	0.735	0.60	-220

Table II.4A
RUN NO. 4
NARROW VOID WINDOW

$p = 400$ psia $\phi = 4.56 \times 10^4$ Btu/hr-ft²
 $\Delta T_{\text{sub}} = 4.8^\circ\text{F}$ $y^* = 0.00$
 $v_0 = 2.52$ ft/sec $\alpha = 61.6\%$
 $z^* = 0.834$

Frequency (cps)	Power Amplitude (%)	Void Amplitude (%)	Normal Void Amplitude (%)	Void Phase (deg)	Frequency (cps)	Power Amplitude (%)	Void Amplitude (%)	Normal Void Amplitude (%)	Void Phase (deg)
0.022	8.60	2.76	3.21	+9	5.00	8.40	0.180	0.214	-350
0.040	9.30	3.16	3.40	+5	0.510	9.00	1.97	2.19	-119
0.060	8.60	3.77	4.38	-15	0.700	7.70	1.39	1.80	-138
0.080	9.50	2.96	3.12	-14	0.844	6.95	0.840	1.21	-151
0.110	9.05	2.55	2.82	-42	0.881	6.75	1.31	1.94	-202
0.300	9.05	2.20	2.43	-83	0.925	7.40	1.10	1.49	-196
0.422	9.05	1.20	1.32	-111	0.962	7.70	1.04	1.35	-164
0.600	8.45	1.45	1.72	-117	0.290	9.95	2.38	2.39	-80
0.788	7.95	0.830	1.04	-130	0.180	10.8	3.18	2.96	-39
0.900	7.95	1.08	1.36	-197	0.090	14.7	4.20	2.87	-5
1.40	7.15	0.307	0.43	-363	0.072	13.5	3.34	2.47	-10
2.12	7.05	0.095	0.135	-135	0.050	14.1	4.97	3.52	-11
2.80	7.25	0.384	0.530	-246	0.030	17.4	6.05	3.48	+4
3.35	8.85	0.288	0.325	-268	0.011	12.2	4.60	3.77	+13
4.20	8.70	0.164	0.188	-304					

Table II.4B
RUN NO. 4
NARROW VOID WINDOW

$p = 400$ psia $\phi = 4.56 \times 10^4$ Btu/hr-ft²
 $\Delta T_{\text{sub}} = 5.1^\circ\text{F}$ $y^* = 0.500$
 $v_0 = 2.52$ ft/sec $\alpha_{y^*} = 48.6\%$
 $z^* = 0.834$

Frequency (cps)	Power Amplitude (%)	Void Amplitude (%)	Normal Void Amplitude (%)	Void Phase (deg)	Frequency (cps)	Power Amplitude (%)	Void Amplitude (%)	Normal Void Amplitude (%)	Void Phase (deg)
0.014	17.2	5.53	3.10	+17	0.320	8.63	2.46	2.85	-63
0.024	16.4	3.98	2.44	+19	0.530	8.38	1.47	1.75	-127
0.034	15.1	3.34	2.22	+3	0.710	7.81	1.44	1.84	-82
0.044	15.0	4.04	2.69	+8	0.825	7.81	1.20	1.54	-194
0.052	14.5	3.54	2.44	0	0.910	7.43	1.04	1.40	-209
0.062	13.1	3.52	2.68	-19	0.994	7.81	0.560	0.717	-136
0.072	12.2	2.92	2.40	-7	1.975	7.37	0.512	0.695	-245
0.082	12.0	2.92	2.44	-11	3.10	5.00	0.416	0.834	-238
0.092	11.8	3.15	2.68	-22	4.25	5.70	0.468	0.826	-267
0.099	11.2	2.82	2.52	-30	5.15	5.76	0.356	0.619	-288
0.145	9.45	2.88	3.05	-51					

Table II.4C
RUN NO. 4
NARROW VOID WINDOW

$p = 400$ psia $\phi = 4.56 \times 10^4$ Btu/hr-ft²
 $\Delta T_{\text{sub}} = 4.6^\circ\text{F}$ $y^* = 0.800$
 $v_0 = 2.52$ ft/sec $\alpha_{y^*} = 39.8\%$
 $z^* = 0.834$

Frequency (cps)	Power Amplitude (%)	Void Amplitude (%)	Normal Void Amplitude (%)	Void Phase (deg)	Frequency (cps)	Power Amplitude (%)	Void Amplitude (%)	Normal Void Amplitude (%)	Void Phase (deg)
0.020	5.50	1.53	2.78	+30	1.86	7.56	0.701	0.92	-237
0.020	6.80	1.22	1.79	+23	2.02	5.53	0.380	0.69	-255
0.020	9.07	1.72	1.90	+18	2.45	5.34	0.283	0.52	-219
0.020	11.5	3.54	3.09	+20	3.05	6.25	0.132	0.21	-310
0.040	14.0	2.96	2.12	-7	4.08	6.30	0.064	0.10	-233
0.060	16.5	3.40	2.06	-15	5.20	6.25	0.160	0.256	-288
0.080	13.9	3.04	2.19	-10	0.512	7.24	1.10	1.52	-161
0.095	12.0	2.82	2.34	-26	0.700	7.24	1.27	1.75	-120
0.191	9.89	2.10	2.13	-42	0.310	6.87	1.43	2.08	-87
0.395	8.82	1.73	1.96	-95	0.142	9.07	1.81	1.99	-69
0.590	8.25	1.16	1.40	-123	0.071	11.6	2.24	1.93	-23
0.800	7.56	0.960	1.27	-127	0.051	14.1	3.92	2.78	-5
0.962	7.81	1.12	1.30	-202	0.031	16.8	4.00	2.39	-3
1.38	7.18	-	-	-215	0.011	18.0	4.40	2.44	+25

Table II.4D
RUN NO. 4
NARROW VOID WINDOW

$p = 400 \text{ psia}$ $\phi = 4.56 \times 10^4 \text{ Btu/hr-ft}^2$
 $\Delta T_{\text{sub}} = 7.4^\circ\text{F}$ $y^* = 0.00$
 $v_0 = 2.52 \text{ ft/sec}$ $\alpha_{y^*} = 53.2\%$
 $z^* = 0.467$

Frequency (cps)	Power Amplitude (%)	Void Amplitude (%)	Normal Void Amplitude (%)	Void Phase (deg)	Frequency (cps)	Power Amplitude (%)	Void Amplitude (%)	Normal Void Amplitude (%)	Void Phase (deg)
0.011	9.72	2.49	2.56	+46	1.00	8.69	1.94	2.23	-141
0.011	14.3	2.49	1.74	+46	0.812	8.56	1.86	2.17	-134
0.011	16.5	3.89	2.35	+17	0.625	9.47	2.24	2.36	-139
0.011	15.2	3.42	2.25	+19	0.438	9.81	2.88	2.94	-51
0.035	13.4	2.40	1.79	+14	2.25	7.34	0.74	1.01	-232
0.055	15.5	3.32	2.14	+1	3.35	11.1	0.04	0.04	-335
0.072	14.2	3.70	2.60	+5	5.50	11.6	0.46	0.40	-280
0.092	15.9	3.89	2.45	-6	4.35	12.7	0.33	0.26	-237
0.100	17.0	4.29	2.53	-1	0.800	12.3	2.11	1.71	-127
0.082	19.0	4.04	2.12	+7	0.875	11.4	2.11	1.85	-152
0.065	20.9	4.33	2.07	+18	0.225	13.5	2.96	2.19	-40
0.045	17.8	4.62	2.60	+19	0.440	12.0	3.28	2.73	-118
0.025	14.8	2.57	1.74	-12	0.662	11.5	2.29	1.99	-119
0.160	12.8	3.18	2.48	-51	0.825	11.3	2.15	1.91	-158
0.335	12.8	2.67	2.08	-96	0.085	18.7	4.48	2.40	-15
0.545	11.2	2.57	2.28	-147	0.065	21.9	4.21	1.92	+3
0.725	8.25	2.74	3.32	-129	0.045	24.8	5.33	2.11	+18
0.950	8.34	2.02	2.42	-166					

Table II.4E
RUN NO. 4
NARROW VOID WINDOW

$p = 400 \text{ psia}$ $z^* = 0.467$
 $\Delta T_{\text{sub}} = 6.5^\circ\text{F}$ $\phi = 4.56 \times 10^4 \text{ Btu/hr-ft}^2$
 $v_0 = 2.52 \text{ ft/sec}$

Frequency (cps)	Power Amplitude (%)	Void Amplitude (%)	Normal Void Amplitude (%)	Void Phase (deg)	Frequency (cps)	Power Amplitude (%)	Void Amplitude (%)	Normal Void Amplitude (%)	Void Phase (deg)
$y^* = 0.50$ $\alpha_{y^*} = 45.0\%$					$y^* = 0.800$ $\alpha_{y^*} = 33.4\%$				
0.450	14.7	2.88	1.96	-103	0.070	12.6	3.01	2.40	+16
0.662	12.4	2.04	1.64	-139	0.055	14.1	3.15	1.99	+6
0.825	10.9	3.43	3.15	-172	0.032	16.8	2.38	1.42	+20
0.925	12.1	1.39	1.15	-138	0.265	15.2	1.93	1.27	-30
0.725	13.3	2.23	1.67	-84	0.402	14.1	3.01	2.14	-92
0.190	15.6	2.61	1.67	-27	0.630	13.6	1.70	1.24	-116
0.042	25.4	4.63	1.82	+6	0.800	9.34	1.96	2.10	-141
0.075	21.8	4.58	2.07	0	0.812	6.16	1.41	2.29	-112
0.025	20.6	3.99	1.94	+26	0.988	6.25	1.10	1.75	-181
$y^* = 0.800$ $\alpha_{y^*} = 33.4\%$					0.712	7.44	1.20	1.61	-100
0.024	9.47	2.19	2.31	+10	0.512	7.97	2.50	3.13	-86
0.025	9.97	3.49	3.50	+11	0.320	9.97	3.36	3.37	-57
0.025	13.4	3.15	2.35	+4	0.220	15.5	3.28	2.12	-41
0.042	13.4	2.77	2.07	+8	2.25	6.84	0.58	0.85	-265
0.062	13.2	2.94	2.23	-3	2.10	9.72	1.00	1.03	-249
0.082	11.9	3.07	2.58	-8	2.75	9.00	0.44	0.49	-338
0.100	11.2	2.84	2.52	-14	3.80	11.7	0.58	0.49	-251
0.092	11.7	2.61	2.23	-4	4.62	12.7	0.46	0.36	-271

Table II.5A

RUN NO. 5
ONE-SHOT METHOD

$p = 400$ psia $z^* = 0.834$
 $\Delta T_{\text{sub}} = 4.0^\circ\text{F}$ $\phi = 4.56 \times 10^4$ Btu/hr-ft²
 $v_0 = 2.52$ ft/sec $\alpha_{\text{avg}} = 53.6\%$

Frequency (cps)	Power Amplitude (%)	Void Amplitude (%)	Normal Void Amplitude (%)	Void Phase (deg)	Frequency (cps)	Power Amplitude (%)	Void Amplitude (%)	Normal Void Amplitude (%)	Void Phase (deg)
0.025	8.34	2.17	2.60	+7	0.700	13.0	2.33	1.80	-160
0.055	6.96	1.74	2.50	-6	0.900	9.97	1.62	1.62	-112
0.098	5.44	1.46	2.68	-15	1.00	10.2	1.30	1.28	-214
0.020	12.4	2.82	2.27	+8	0.825	8.70	1.95	2.24	-192
0.050	9.72	2.29	2.36	+6	0.688	7.90	2.03	2.57	-169
0.098	8.16	2.23	2.73	-12	0.500	11.9	2.48	2.08	-155
0.029	16.0	3.05	1.91	+10	0.265	10.70	2.52	2.36	-90
0.045	12.4	3.05	2.46	-4	3.40	7.99	0.73	0.91	-269
0.060	14.3	3.13	2.19	-8	2.15	9.46	0.39	0.41	-203
0.080	16.5	3.97	2.41	-12	2.65	11.3	0.35	0.31	-208
0.085	16.1	3.93	2.44	-8	2.78	12.4	0.16	0.13	-241
0.070	17.5	3.97	2.27	-9	3.90	11.3	0.41	0.36	-264
0.052	19.2	4.26	2.22	-1	3.10	12.2	0.43	0.35	-267
0.040	21.0	4.55	2.16	-6	3.85	12.6	0.30	0.24	-278
0.016	22.8	4.72	2.07	+4	5.25	12.0	0.14	0.12	-319
0.230	13.0	2.95	2.26	-46	0.048	22.8	5.52	2.42	+2
0.150	14.1	3.89	2.77	-31	0.10	19.1	4.47	2.34	-12
0.300	15.0	2.62	1.74	-68	0.018	29.6	5.96	2.01	+20
0.500	14.5	2.34	1.61	-127					

Table II.5B

RUN NO. 5
ONE-SHOT METHOD

$p = 400$ psia $z^* = 0.467$
 $\Delta T_{\text{sub}} = 4.4^\circ\text{F}$ $\phi = 4.56 \times 10^4$ Btu/hr-ft²
 $v_0 = 2.52$ ft/sec $\alpha_{\text{avg}} = 41.8\%$

Frequency (cps)	Power Amplitude (%)	Void Amplitude (%)	Normal Void Amplitude (%)	Void Phase (deg)	Frequency (cps)	Power Amplitude (%)	Void Amplitude (%)	Normal Void Amplitude (%)	Void Phase (deg)
0.012	11.9	2.81	2.36	+14	0.688	14.4	1.99	1.38	-122
0.012	14.9	2.95	1.98	+61	0.912	12.6	1.36	1.08	-127
0.022	16.5	3.49	2.11	+24	1.00	7.98	1.30	1.63	-160
0.042	18.9	4.38	2.32	+7	0.800	11.67	1.91	1.64	-167
0.065	20.1	4.89	2.44	+2	0.600	10.6	1.89	1.79	-126
0.080	19.4	4.31	2.22	+6	0.395	11.2	2.75	2.46	-101
0.100	18.1	3.88	2.14	-4	0.310	12.6	3.08	2.44	-66
0.090	19.0	4.66	2.45	-4	0.130	13.0	4.13	3.16	-19
0.070	20.5	4.81	2.34	+4	1.15	10.2	0.83	0.81	-124
0.050	22.8	4.69	2.06	+4	1.975	9.40	0.65	0.69	-251
0.032	26.2	5.81	2.21	+9	2.75	11.2	0.33	0.30	-270
0.320	14.9	3.14	2.11	-68	3.25	12.3	0.12	0.10	-183
0.495	15.8	2.81	1.78	-92	4.7	11.7	0.08	0.07	-313

Table II.5C

RUN NO. 5
ONE-SHOT METHOD

$p = 400$ psia $z^* = 0.226$
 $\Delta T_{\text{sub}} = 4.4^\circ\text{F}$ $\phi = 4.56 \times 10^4$ Btu/hr-ft²
 $v_0 = 2.52$ ft/sec $\alpha_{\text{avg}} = 29.7\%$

Frequency (cps)	Power Amplitude (%)	Void Amplitude (%)	Normal Void Amplitude (%)	Void Phase (deg)	Frequency (cps)	Power Amplitude (%)	Void Amplitude (%)	Normal Void Amplitude (%)	Void Phase (deg)
0.460	11.6	1.69	1.46	-81	0.032	24.4	3.92	1.61	+31
0.600	11.3	1.93	1.71	-80	0.048	22.3	4.17	1.87	+20
0.800	11.3	0.99	0.87	-115	0.065	19.5	2.93	1.50	+10
1.00	11.2	1.12	1.00	-129	0.075	18.4	2.89	1.57	+10
0.838	12.4	2.01	1.62	-133	0.100	17.3	3.36	1.95	+5
0.725	12.7	0.89	0.70	-101	2.52	10.8	0.35	0.32	-245
0.500	12.8	1.40	1.09	-95	3.70	10.9	0.02	0.02	-358
0.280	13.2	3.28	2.49	-34	4.00	11.6	0.10	0.07	-356
0.022	26.2	4.42	1.68	+38					

Table II.6A

RUN NO. 6
ONE-SHOT METHOD

$p = 400$ psia $z^* = 0.834$
 $\Delta T_{sub} = 9.2^\circ F$ $\phi = 4.56 \times 10^4$ Btu/hr-ft²
 $v_0 = 2.52$ ft/sec $\alpha_{avg} = 37.4\%$

Frequency (cps)	Power Amplitude (%)	Void Amplitude (%)	Normal Void Amplitude (%)	Void Phase (deg)	Frequency (cps)	Power Amplitude (%)	Void Amplitude (%)	Normal Void Amplitude (%)	Void Phase (deg)
0.022	10.9	5.06	4.65	+14	0.875	10.3	0.73	0.70	-207
0.044	14.9	5.76	3.86	-3	0.750	10.2	1.51	1.48	-196
0.065	13.1	4.93	3.76	-14	0.680	9.50	1.65	1.74	-190
0.088	13.4	5.43	4.05	-21	0.493	8.62	2.39	2.77	-149
0.100	12.3	5.06	4.11	-26	0.302	10.2	3.40	3.05	-98
0.075	13.8	5.43	3.94	-15	0.208	12.0	4.39	3.65	-58
0.055	15.7	6.27	4.00	-8	2.08	8.40	0.49	0.58	-235
0.020	19.7	7.47	3.79	0	2.85	9.22	0.63	0.68	-251
0.200	14.9	5.76	3.87	-56	3.75	9.06	0.45	0.50	-263
0.380	13.9	3.94	2.84	-100	4.20	10.2	0.18	0.18	-265
0.570	12.5	2.61	2.08	-146	2.65	10.7	0.27	0.25	-238
0.681	9.06	1.51	1.67	-170	3.85	10.7	0.27	0.25	-229
0.748	9.06	1.92	2.12	-199	0.688	12.7	2.26	1.78	-122
1.00	9.12	0.71	0.79	-223	1.35	11.6	0.53	0.46	-154

Table II.6B

RUN NO. 6
ONE-SHOT METHOD

$p = 400$ psia $z^* = 0.467$
 $\Delta T_{sub} = 9.4^\circ F$ $\phi = 4.56 \times 10^4$ Btu/hr-ft²
 $v_0 = 2.52$ ft/sec $\alpha_{avg} = 19.4\%$

Frequency (cps)	Power Amplitude (%)	Void Amplitude (%)	Normal Void Amplitude (%)	Void Phase (deg)	Frequency (cps)	Power Amplitude (%)	Void Amplitude (%)	Normal Void Amplitude (%)	Void Phase (deg)
0.202	13.8	4.93	3.58	-25	0.100	10.6	3.25	3.07	-16
0.265	12.5	3.87	3.10	-56	0.182	12.2	3.62	2.97	-37
0.482	12.2	2.74	2.25	-91	0.380	12.0	3.14	2.61	-71
0.700	9.97	1.94	1.94	-131	0.580	11.8	2.51	2.14	-111
0.850	9.97	1.63	1.63	-172	0.745	10.9	1.83	1.68	-133
0.975	9.90	1.51	1.52	-183	0.950	10.3	0.94	0.91	-160
0.725	12.1	1.88	1.55	-129	0.700	10.2	1.98	1.94	-142
0.580	12.4	2.04	1.64	-119	0.460	10.3	2.51	2.44	-94
0.270	13.8	3.29	2.39	-49	0.180	13.3	4.16	3.12	-35
0.170	14.8	4.06	2.75	-31	1.75	10.1	0.45	0.45	-256
1.72	9.78	0.57	0.58	-256	1.75	10.7	0.47	0.44	-237
2.25	10.9	0.75	0.69	-268	2.40	11.2	0.29	0.26	-229
3.30	10.2	0.14	0.14	-288	3.10	10.7	0.31	0.29	-293
0.075	18.5	6.27	3.39	-10	3.80	10.8	0.24	0.22	-270
0.045	21.2	6.84	3.18	-4	4.70	10.6	0.22	0.21	-262
0.030	19.7	6.29	3.19	+6	1.30	11.5	0.45	0.39	-175
0.035	9.50	3.07	3.23	+11	1.95	11.2	0.35	0.31	-163
0.040	9.34	2.90	3.10	+7	2.65	10.9	0.33	0.30	-233
0.055	9.00	2.76	3.07	-5	3.88	10.2	0.31	0.30	-276
0.078	10.7	3.23	3.03	-9					

Table II.6C

RUN NO. 6
ONE-SHOT METHOD

$p = 400$ psia $z^* = 0.226$
 $\Delta T_{sub} = 9.0^\circ F$ $\phi = 4.56 \times 10^4$ Btu/hr-ft²
 $v_0 = 2.52$ ft/sec $\alpha_{avg} = 12.3\%$

Frequency (cps)	Power Amplitude (%)	Void Amplitude (%)	Normal Void Amplitude (%)	Void Phase (deg)	Frequency (cps)	Power Amplitude (%)	Void Amplitude (%)	Normal Void Amplitude (%)	Void Phase (deg)
0.34	11.2	1.59	1.42	-31	3.00	6.40	0.27	0.42	-159
0.475	9.06	1.85	2.04	-50	4.10	8.22	0.21	0.26	-134
0.675	10.9	0.85	0.78	-59	4.80	9.06	0.14	0.15	-104
0.850	9.94	1.55	1.56	-84	3.60	9.00	0.25	0.28	-115
1.00	10.2	1.42	1.40	-110	2.10	9.50	0.25	0.26	-127
0.790	11.6	1.52	1.31	-77	1.40	8.69	0.94	1.08	-100
0.702	11.2	2.10	1.87	-63	0.095	15.3	3.46	2.26	-9
0.570	8.19	1.89	2.31	-52	0.080	17.0	3.56	2.09	-7
0.360	9.06	1.95	2.15	-35	0.060	19.6	3.35	1.71	+1
0.162	11.3	2.39	2.11	-14	0.020	22.2	3.62	1.63	+5
1.65	7.25	1.36	1.88	-157	0.010	24.9	4.20	1.68	+16
2.22	7.03	0.66	0.94	-136					

Table II.7A

RUN NO. 7

ONE-SHOT METHOD

$p = 400 \text{ psia}$ $z^* = 0.834$
 $\Delta T_{\text{sub}} = 1.1^\circ\text{F}$ $\phi = 6.84 \times 10^4 \text{ Btu/hr-ft}^2$
 $v_0 = 3.78 \text{ ft/sec}$ $\alpha_{\text{avg}} = 53.1\%$

Frequency (cps)	Power Amplitude (%)	Void Amplitude (%)	Normal Void Amplitude (%)	Void Phase (deg)	Frequency (cps)	Power Amplitude (%)	Void Amplitude (%)	Normal Void Amplitude (%)	Void Phase (deg)
0.025	4.34	1.57	3.62	+11	0.320	6.77	2.03	3.00	-47
0.048	4.24	1.70	4.02	+6	1.90	5.05	0.15	0.30	-262
0.098	3.19	1.15	3.61	-11	2.65	5.06	0.15	0.30	-122
0.088	4.10	1.24	3.04	-11	3.85	5.18	0.21	0.40	-201
0.038	5.55	1.70	3.07	+16	4.70	5.33	0.06	0.10	-180
0.021	5.93	1.93	3.26	+22	1.25	5.64	0.83	1.46	-156
0.285	4.42	1.26	2.86	-39	0.975	5.57	1.15	2.07	-117
0.500	5.83	1.69	2.89	-70	0.195	6.94	2.14	3.09	-30
0.740	5.03	1.38	2.74	-101	0.080	8.91	3.13	3.51	-4
0.380	5.10	1.61	3.16	-61	0.070	9.54	2.94	3.08	-3
0.190	5.71	2.01	3.52	-34	0.060	10.4	3.30	3.19	+1
0.620	5.35	2.05	3.83	-74	0.032	13.0	4.06	3.12	+11
0.775	5.45	1.38	2.53	-91	0.022	12.5	4.02	3.21	+20
0.900	4.73	1.38	2.92	-116	0.170	7.37	2.41	3.27	-19
1.02	5.23	1.19	2.27	-126	2.18	4.36	0.248	0.57	-174

Table II.7B

RUN NO. 7

ONE-SHOT METHOD

$p = 400 \text{ psia}$ $z^* = 0.467$
 $\Delta T_{\text{sub}} = 1.1^\circ\text{F}$ $\phi = 6.84 \times 10^4 \text{ Btu/hr-ft}^2$
 $v_0 = 3.78 \text{ ft/sec}$ $\alpha_{\text{avg}} = 32.7\%$

Frequency (cps)	Power Amplitude (%)	Void Amplitude (%)	Normal Void Amplitude (%)	Void Phase (deg)	Frequency (cps)	Power Amplitude (%)	Void Amplitude (%)	Normal Void Amplitude (%)	Void Phase (deg)
0.028	9.09	3.09	3.40	+8	0.270	4.49	1.72	3.84	-36
0.042	9.80	3.28	3.34	+7	0.115	6.59	2.14	3.26	-8
0.060	9.80	3.24	3.31	+6	1.10	4.16	1.22	2.94	-111
0.080	8.63	2.93	3.40	-3	1.42	4.59	0.93	2.03	-118
0.098	7.70	2.61	3.38	-3	1.62	4.19	0.68	1.62	-149
0.225	7.22	2.34	3.24	-17	1.95	3.79	0.39	1.02	-161
0.365	6.18	2.05	3.32	-43	2.30	4.23	0.33	0.78	-201
0.580	6.08	1.64	2.70	-58	3.05	4.53	0.13	0.30	-217
0.788	5.47	1.38	2.51	-84	4.20	4.24	0.19	0.45	-127
0.975	4.85	1.20	2.48	-111	0.075	9.19	3.91	3.17	+3
0.862	5.05	1.36	2.68	-86	0.068	9.64	3.22	3.34	+4
0.680	4.34	1.63	3.75	-71	0.025	12.8	3.51	2.73	+19
0.470	3.94	1.59	4.03	-62	0.012	12.8	3.95	3.08	+37

Table II.7C

RUN NO. 7

ONE-SHOT METHOD

$p = 400 \text{ psia}$ $z^* = 0.226$
 $\Delta T_{\text{sub}} = 1.1^\circ\text{F}$ $\phi = 6.84 \times 10^4 \text{ Btu/hr-ft}^2$
 $v_0 = 3.78 \text{ ft/sec}$ $\alpha_{\text{avg}} = 15.4\%$

Frequency (cps)	Power Amplitude (%)	Void Amplitude (%)	Normal Void Amplitude (%)	Void Phase (deg)	Frequency (cps)	Power Amplitude (%)	Void Amplitude (%)	Normal Void Amplitude (%)	Void Phase (deg)
0.015	13.5	3.32	2.46	+56	0.525	6.50	1.80	2.77	-42
0.030	12.4	3.56	2.87	+28	0.630	6.24	1.16	1.86	-38
0.048	11.3	3.07	2.71	+12	0.700	5.86	1.41	2.41	-53
0.068	10.1	2.74	2.71	+10	0.775	5.00	1.68	3.37	-48
0.090	9.29	2.59	2.79	+3	0.875	5.20	1.39	2.68	-61
0.098	9.06	2.66	2.94	0	1.00	4.49	0.97	2.16	-100
0.158	8.31	2.34	2.81	-7	1.98	3.36	0.87	2.59	-134
0.255	7.17	1.88	2.62	-10	3.20	3.99	0.27	0.68	-193

Table II.8A

RUN NO. 8
ONE-SHOT METHOD

$p = 400$ psia $z^* = 0.834$
 $\Delta T_{\text{sub}} = 11.3^\circ\text{F}$ $\phi = 6.84 \times 10^4$ Btu/hr-ft²
 $v_0 = 3.78$ ft/sec $\alpha_{\text{avg}} = 32.8\%$

Frequency (cps)	Power Amplitude (%)	Void Amplitude (%)	Normal Void Amplitude (%)	Void Phase (deg)	Frequency (cps)	Power Amplitude (%)	Void Amplitude (%)	Normal Void Amplitude (%)	Void Phase (deg)
0.022	5.59	2.81	5.03	+13	0.410	4.70	3.00	6.38	-83
0.032	7.08	3.47	4.90	+10	0.225	6.53	3.75	5.74	-46
0.055	6.76	3.88	5.74	-2	0.220	6.68	4.10	6.13	-42
0.068	7.33	3.84	5.24	-3	2.15	4.36	0.45	1.03	-130
0.085	7.79	4.35	5.58	-11	1.75	5.03	0.69	1.37	-133
0.100	7.10	3.94	5.55	-13	2.70	4.83	0.10	0.20	-192
0.075	8.17	4.25	5.20	-12	3.80	5.49	0.20	0.36	-141
0.058	8.93	4.90	5.49	-5	2.40	5.60	0.14	0.25	-214
0.038	10.2	5.15	5.03	+5	2.00	5.94	0.52	0.88	-147
0.030	11.6	5.90	5.10	+4	1.25	6.30	0.42	0.67	-183
0.165	5.44	3.22	5.92	-28	0.975	5.78	1.33	2.30	-148
0.260	5.94	3.32	5.59	-46	0.105	9.16	4.51	4.92	-14
0.480	5.29	2.61	4.93	-87	0.850	4.95	1.76	3.56	-138
0.660	5.78	2.16	3.74	-111	0.950	5.31	1.27	2.39	-154
0.860	5.64	1.49	2.64	-148	0.098	9.48	5.09	5.37	-13
0.960	5.08	0.97	1.91	-147	0.050	12.6	6.62	5.27	+1
0.800	4.72	1.70	3.60	-115	0.018	15.4	7.23	4.68	+18
0.620	4.55	2.57	5.65	-100					

Table II.8B

RUN NO. 8
ONE-SHOT METHOD

$p = 400$ psia $z^* = 0.467$
 $\Delta T_{\text{sub}} = 11.3^\circ\text{F}$ $\phi = 6.84 \times 10^4$ Btu/hr-ft²
 $v_0 = 3.78$ ft/sec $\alpha_{\text{avg}} = 14.0\%$

Frequency (cps)	Power Amplitude (%)	Void Amplitude (%)	Normal Void Amplitude (%)	Void Phase (deg)	Frequency (cps)	Power Amplitude (%)	Void Amplitude (%)	Normal Void Amplitude (%)	Void Phase (deg)
0.028	7.80	4.10	5.26	+18	0.780	5.20	1.75	3.36	-83
0.048	6.73	3.24	4.81	+4	0.560	4.93	2.27	4.60	-61
0.068	6.96	3.63	5.22	0	0.360	7.00	3.52	5.03	-43
0.085	7.16	3.63	5.07	-7	0.160	8.86	4.49	5.07	-16
0.100	6.86	3.73	5.44	-11	1.60	5.60	0.85	1.51	-129
0.080	7.66	3.91	5.10	-2	2.10	4.97	0.52	1.05	-168
0.055	8.60	4.44	5.14	+5	3.20	4.17	0.26	0.62	-123
0.035	9.73	5.13	5.27	+7	3.60	4.73	0.18	0.38	-183
0.020	10.3	4.85	4.71	+18	4.60	4.93	0.08	0.16	-240
0.015	9.80	5.33	5.44	+25	1.98	4.73	0.59	1.25	-150
0.095	9.06	4.85	5.35	-6	2.30	4.93	0.52	1.05	-151
0.310	6.40	2.94	4.59	-36	0.52	6.60	3.12	4.73	-63
0.465	7.43	3.12	4.20	-51	0.72	6.13	3.04	4.96	-79
0.680	6.93	2.68	3.87	-73	0.94	5.26	1.15	2.18	-106
0.860	5.60	1.58	2.82	-96	0.98	4.47	1.02	2.28	-108
1.05	5.53	1.58	2.86	-113	0.10	9.62	4.65	5.02	-6

Table II.9A

RUN NO. 9
ONE-SHOT METHOD

$p = 600$ psia $z^* = 0.834$
 $\Delta T_{\text{sub}} = 3.9^\circ\text{F}$ $\phi = 9.12 \times 10^4$ Btu/hr-ft²
 $v_0 = 3.78$ ft/sec $\alpha_{\text{avg}} = 46.2\%$

Frequency (cps)	Power Amplitude (%)	Void Amplitude (%)	Normal Void Amplitude (%)	Void Phase (deg)	Frequency (cps)	Power Amplitude (%)	Void Amplitude (%)	Normal Void Amplitude (%)	Void Phase (deg)
0.030	4.20	1.80	4.28	0	0.840	3.53	0.94	2.66	-116
0.050	4.98	2.03	4.08	+3	0.660	3.50	1.05	3.00	-94
0.070	6.13	2.30	3.75	-1	0.455	3.62	1.15	3.18	-67
0.088	5.69	2.15	3.78	-4	0.365	3.62	1.53	4.23	-47
0.100	5.44	2.13	3.92	-5	0.270	3.80	1.70	4.47	-32
0.082	5.51	2.13	3.86	-6	0.105	4.95	1.86	3.76	-9
0.058	5.84	2.11	3.61	0	1.05	2.62	0.80	3.05	-129
0.038	6.16	2.20	3.57	+2	1.62	2.67	0.29	1.09	-197
0.018	6.74	2.57	3.81	+31	3.15	2.86	0.27	0.94	-130
0.162	5.98	2.30	3.85	-16	4.80	2.90	0.14	0.48	-169
0.270	4.62	1.86	4.02	-38	4.30	3.89	0.13	0.33	-119
0.480	3.62	1.55	4.28	-70	3.20	3.41	0.13	0.38	-180
0.590	3.90	1.20	3.08	-92	2.50	3.02	0.12	0.40	-108
0.800	3.65	0.98	2.68	-129	2.30	3.48	0.15	0.43	-200
1.00	3.42	0.71	2.08	-149	0.98	3.71	1.03	2.78	-124

Table II.9B

RUN NO. 9

ONE-SHOT METHOD

$p = 600 \text{ psia}$ $z^\circ = 0.467$
 $\Delta T_{\text{sub}} = 3.7^\circ\text{F}$ $\phi = 9.12 \times 10^4 \text{ Btu/hr-ft}^2$
 $v_0 = 3.78 \text{ ft/sec}$ $\alpha_{\text{avg}} = 33.0\%$

Frequency (cps)	Power Amplitude (%)	Void Amplitude (%)	Normal Void Amplitude (%)	Void Phase (deg)	Frequency (cps)	Power Amplitude (%)	Void Amplitude (%)	Normal Void Amplitude (%)	Void Phase (deg)
0.015	6.20	1.70	2.74	+18	0.680	4.10	1.25	3.05	-65
0.028	6.38	1.60	2.51	+32	0.880	3.59	1.07	2.98	-84
0.048	6.92	2.74	3.96	+10	1.02	2.99	1.02	3.41	-93
0.070	6.25	2.23	3.57	+3	0.930	3.35	1.00	2.98	-90
0.090	5.92	2.11	3.56	-3	0.780	3.06	1.23	4.02	-73
0.100	5.71	2.17	3.80	-4	0.560	3.28	1.15	3.51	-57
0.060	6.38	1.99	3.12	+15	0.360	3.80	1.42	3.76	-38
0.040	6.65	2.46	3.70	+12	0.160	4.64	1.86	4.01	-15
0.022	6.80	2.62	3.85	+21	1.60	2.46	0.68	2.76	-151
0.220	5.76	2.09	3.63	-19	2.00	2.03	0.26	1.28	-144
0.350	5.22	1.72	3.30	-37	2.55	2.61	0.24	0.92	-203
0.500	4.84	1.58	3.26	-48					

Table II.10A

RUN NO. 10

ONE-SHOT METHOD

$p = 600 \text{ psia}$ $z^\circ = 0.834$
 $\Delta T_{\text{sub}} = 12.4^\circ\text{F}$ $\phi = 9.12 \times 10^4 \text{ Btu/hr-ft}^2$
 $v_0 = 3.78 \text{ ft/sec}$ $\alpha_{\text{avg}} = 31.6\%$

Frequency (cps)	Power Amplitude (%)	Void Amplitude (%)	Normal Void Amplitude (%)	Void Phase (deg)	Frequency (cps)	Power Amplitude (%)	Void Amplitude (%)	Normal Void Amplitude (%)	Void Phase (deg)
0.020	7.18	2.59	3.60	-3	0.560	2.54	1.09	4.29	-93
0.025	6.94	2.94	4.24	+3	0.361	2.34	1.50	6.41	-68
0.048	6.40	2.81	4.39	+1	0.169	3.96	1.73	4.37	-28
0.068	6.96	3.28	4.71	-5	1.40	2.30	0.09	0.39	-205
0.088	6.67	3.22	4.83	-11	2.22	2.13	0.05	0.23	-141
0.100	6.60	3.00	4.54	-10	2.05	2.48	0.13	0.52	-119
0.078	6.78	3.17	4.68	-3	3.00	2.59	0.05	0.19	-225
0.058	7.28	3.28	4.50	+2	3.95	2.67	0.11	0.41	-176
0.038	5.56	3.35	6.02	+4	4.60	2.70	0.09	0.33	-180
0.012	8.05	3.48	4.32	+21	3.10	2.65	0.07	0.26	-121
0.130	6.94	3.35	4.83	-19	2.35	2.93	0.22	0.75	-150
0.265	5.26	2.24	4.26	-38	1.95	2.88	0.13	0.45	-122
0.490	4.10	1.71	4.17	-77	1.55	2.81	0.07	0.25	-151
0.680	3.41	1.28	3.75	-106	1.24	2.46	0.33	1.34	-197
0.880	2.55	0.83	3.25	-122	0.990	2.42	0.52	2.15	-153
0.960	2.35	0.56	2.38	-145	0.100	5.11	2.68	5.24	-11
0.922	2.39	0.58	2.43	-140	0.045	6.13	3.41	5.56	+3
0.758	2.42	0.90	3.72	-113					

Table II.10B
 RUN NO. 10
 ONE-SHOT METHOD

$p = 600$ psia $v_0 = 3.78$ ft/sec
 $\Delta T_{\text{sub}} = 13.4^\circ\text{F}$ $\phi = 9.12 \times 10^4$ Btu/hr-ft²

Frequency (cps)	Power Amplitude (%)	Void Amplitude (%)	Normal Void Amplitude (%)	Void Phase (deg)	Frequency (cps)	Power Amplitude (%)	Void Amplitude (%)	Normal Void Amplitude (%)	Void Phase (deg)
$z^\circ = 0.834$ $\alpha_{\text{avg}} = 34.0\%$					$z^\circ = 0.103$ $\alpha_{\text{avg}} = 5.0\%$				
0.02	5.40	2.79	5.17	+12	0.020	7.68	0.62	0.81	+5
0.225	5.47	2.83	5.17	-38	0.205	5.06	0.39	0.78	-19
0.078	6.98	3.84	5.50	-5	0.080	6.74	0.58	0.86	-13
0.760	4.44	1.49	3.36	-110	$z^\circ = 0.164$ $\alpha_{\text{avg}} = 8.1\%$				
0.960	3.32	0.685	2.06	-125	0.080	6.98	1.04	1.49	+6
$z^\circ = 0.588$ $\alpha_{\text{avg}} = 24.1\%$					0.760	3.31	0.56	1.69	-53
0.078	6.56	3.08	4.70	0	0.210	4.80	0.79	1.64	-15
0.780	4.40	1.27	2.89	-82	0.020	7.52	1.37	1.82	+27
0.980	3.54	0.91	2.58	-100	$z^\circ = 0.285$ $\alpha_{\text{avg}} = 9.3\%$				
0.205	4.24	2.36	5.57	-26	0.020	7.64	2.15	2.81	+19
0.020	8.01	3.53	4.41	+19	0.210	5.26	1.47	2.79	-16
$z^\circ = 0.346$ $\alpha_{\text{avg}} = 17.2\%$					0.740	3.40	0.87	2.57	-64
0.020	7.47	2.42	3.24	+24	0.960	3.31	0.79	2.38	-66
0.205	4.93	1.74	3.53	-21	0.080	6.74	2.09	3.10	-1
0.080	7.34	2.50	3.40	-3	$z^\circ = 0.467$ $\alpha_{\text{avg}} = 18.9\%$				
0.780	2.97	1.08	3.64	-69	0.080	6.69	2.73	4.08	-1
0.960	2.86	0.77	2.69	-90	0.820	2.98	1.06	3.68	-86
$z^\circ = 0.225$ $\alpha_{\text{avg}} = 8.1\%$					0.980	2.94	0.91	3.10	-90
0.960	2.63	0.50	1.89	-54	0.200	3.99	1.66	4.16	-25
0.760	2.96	0.64	2.18	-45	0.020	7.69	2.83	3.68	-23
0.080	6.05	1.45	2.40	+3	$z^\circ = 0.710$ $\alpha_{\text{avg}} = 30.2\%$				
0.020	7.76	1.72	2.22	+26	0.020	8.26	3.33	4.03	+9
0.200	4.80	1.02	2.12	-15	0.210	4.86	2.24	4.61	-33

Table II.11A
 RUN NO.-11
 ONE-SHOT METHOD

$p = 600$ psia $z^\circ = 0.834$
 $\Delta T_{\text{sub}} = 3.2^\circ\text{F}$ $\phi = 4.56 \times 10^4$ Btu/hr-ft²
 $v_0 = 2.52$ ft/sec $\alpha_{\text{avg}} = 36.5\%$

Frequency (cps)	Power Amplitude (%)	Void Amplitude (%)	Normal Void Amplitude (%)	Void Phase (deg)	Frequency (cps)	Power Amplitude (%)	Void Amplitude (%)	Normal Void Amplitude (%)	Void Phase (deg)
0.021	10.8	3.96	3.67	+6	0.700	9.00	1.91	2.12	-122
0.042	10.5	4.28	4.08	-8	0.500	9.25	2.98	3.22	-108
0.065	10.5	3.85	3.67	-11	0.290	9.62	3.53	3.67	-69
0.082	10.4	4.23	4.07	-18	1.35	7.31	0.52	0.71	-78
0.103	11.2	4.26	3.80	-19	1.71	8.06	0.61	0.76	-111
0.056	12.8	4.32	3.38	-3	2.08	8.25	0.30	0.36	-155
0.032	12.8	5.30	4.14	0	2.58	8.69	0.37	0.42	-105
0.208	13.2	4.56	3.45	-50	3.10	9.00	0.30	0.33	-193
0.445	11.3	3.10	2.74	-108	4.10	8.75	0.38	0.43	-175
0.655	9.62	1.89	1.96	-117	2.25	9.38	0.57	0.61	-143
0.800	10.5	1.63	1.55	-171	1.68	9.69	0.41	0.42	-142
1.00	8.69	0.35	0.40	-214	1.20	9.94	0.09	0.09	-115
0.901	7.75	0.81	1.04	-157	0.910	9.38	0.47	0.50	-193

Table II.11B

RUN NO. 11
ONE-SHOT METHOD

$p = 600$ psia $z^* = 0.467$
 $\Delta T_{\text{sub}} = 3.2^\circ\text{F}$ $\phi = 4.56 \times 10^4$ Btu/hr-ft²
 $v_0 = 2.52$ ft/sec $\alpha_{\text{avg}} = 19.5\%$

Frequency (cps)	Power Amplitude (%)	Void Amplitude (%)	Normal Void Amplitude (%)	Void Phase (deg)	Frequency (cps)	Power Amplitude (%)	Void Amplitude (%)	Normal Void Amplitude (%)	Void Phase (deg)
0.022	12.5	3.90	3.12	+13	0.400	10.9	3.16	2.90	-63
0.040	12.5	3.94	3.15	+1	0.195	11.0	3.16	2.87	-33
0.060	12.7	3.78	2.98	-2	1.14	8.70	0.64	0.74	-132
0.080	12.2	3.94	3.23	-8	1.56	9.25	0.59	0.64	-171
0.100	12.6	3.80	3.02	-12	1.77	8.55	0.32	0.37	-169
0.070	12.5	3.96	3.17	-3	2.05	9.60	0.46	0.48	-149
0.050	12.8	3.86	3.02	+5	2.65	9.80	0.41	0.42	-178
0.029	12.9	4.06	3.15	+7	3.20	10.3	0.32	0.31	-257
0.298	12.6	3.68	3.92	-45	4.20	10.0	0.18	0.18	-164
0.510	12.7	2.76	2.17	-73	1.61	11.6	0.69	0.59	-166
0.720	11.0	1.96	1.78	-100	1.74	11.8	0.45	0.38	-162
0.950	9.00	1.39	1.54	-135	1.95	11.3	0.38	0.34	-147
1.01	8.25	1.72	2.08	-170	1.24	10.6	1.38	1.30	-149
0.805	8.90	2.06	2.31	-104	1.41	10.3	0.87	0.84	-155
0.600	9.60	2.48	2.58	-85	1.51	10.1	0.45	0.44	-188

Table II.11C

RUN NO. 11
ONE-SHOT METHOD

$p = 600$ psia $z^* = 0.285$
 $\Delta T_{\text{sub}} = 3.2^\circ\text{F}$ $\phi = 4.56 \times 10^4$ Btu/hr-ft²
 $v_0 = 2.52$ ft/sec $\alpha_{\text{avg}} = 8.6\%$

Frequency (cps)	Power Amplitude (%)	Void Amplitude (%)	Normal Void Amplitude (%)	Void Phase (deg)	Frequency (cps)	Power Amplitude (%)	Void Amplitude (%)	Normal Void Amplitude (%)	Void Phase (deg)
0.163	11.8	1.55	1.31	-26	0.022	13.1	2.36	1.80	+18
0.303	9.55	1.96	2.05	-43	2.02	7.45	0.65	0.87	-140
0.483	8.90	1.79	2.01	-66	1.51	8.45	1.38	1.63	-143
0.712	7.60	1.75	2.30	-74	2.63	6.90	0.40	0.58	-174
0.900	8.40	1.68	2.00	-117	3.05	7.40	0.20	0.27	-214
0.983	9.45	1.84	1.95	-108	3.40	7.55	-0	-0	-225
0.082	12.4	2.04	1.64	-9	3.95	8.70	0.103	0.12	-127
0.052	12.5	2.20	1.76	+2					

Table II.12A

RUN NO. 12
ONE-SHOT METHOD

$p = 800$ psia $z^* = 0.834$
 $\Delta T_{\text{sub}} = 3.6^\circ\text{F}$ $\phi = 4.56 \times 10^4$ Btu/hr-ft²
 $v_0 = 2.52$ ft/sec $\alpha_{\text{avg}} = 31.3\%$

Frequency (cps)	Power Amplitude (%)	Void Amplitude (%)	Normal Void Amplitude (%)	Void Phase (deg)	Frequency (cps)	Power Amplitude (%)	Void Amplitude (%)	Normal Void Amplitude (%)	Void Phase (deg)
0.023	11.0	4.06	3.69	+4	0.614	7.77	1.86	2.39	-117
0.041	10.8	4.18	3.87	-2	0.387	8.90	3.21	3.61	-80
0.062	11.3	3.78	3.34	-12	0.185	10.9	3.55	3.26	-47
0.083	11.3	4.04	3.58	-16	1.23	9.25	0.27	0.29	-130
0.100	11.1	4.10	3.69	-24	1.42	9.25	0.56	0.60	-110
0.090	11.1	4.10	3.69	-15	1.65	9.00	0.49	0.54	-124
0.069	11.2	3.86	3.45	-7	1.85	9.25	0.51	0.55	-153
0.050	11.2	4.16	3.71	-7	2.05	9.25	0.31	0.34	-161
0.029	11.4	3.90	3.42	+1	2.61	9.55	0.33	0.34	-176
0.285	10.9	3.41	3.13	-66	3.10	9.55	0.14	0.15	-173
0.500	10.1	2.68	2.65	-102	4.65	10.6	0.19	0.18	-184
0.700	10.2	1.64	1.61	-151	4.10	10.4	0.19	0.18	-123
0.900	9.40	0.89	0.95	-183	1.02	10.2	0.59	0.58	-206
1.00	9.10	0.37	0.41	-195	0.888	9.10	0.90	0.99	-169
0.975	8.95	0.52	0.58	-172	0.850	8.95	1.22	1.36	-166
0.800	7.80	1.04	1.33	-173	0.082	10.2	3.72	3.65	-14

Table II.12B
RUN NO. 12
ONE-SHOT METHOD

$p = 800$ psia $z^* = 0.467$
 $\Delta T_{\text{sub}} = 3.6^\circ\text{F}$ $\rho = 4.56 \times 10^4$ Btu/hr-ft²
 $v_0 = 2.52$ ft/sec $\alpha_{\text{avg}} = 15.6\%$

Frequency (cps)	Power Amplitude (%)	Void Amplitude (%)	Normal Void Amplitude (%)	Void Phase (deg)	Frequency (cps)	Power Amplitude (%)	Void Amplitude (%)	Normal Void Amplitude (%)	Void Phase (deg)
0.022	11.4	3.30	2.89	+1	3.05	9.60	0.312	0.32	-210
0.046	11.8	3.12	2.64	-7	5.20	9.25	0.136	0.15	-138
0.093	11.8	3.10	2.63	-11	1.90	9.25	0.371	0.40	-172
0.180	11.6	2.89	2.49	-29	1.70	9.25	0.462	0.50	-192
0.417	10.8	2.71	2.51	-59	1.95	9.10	0.326	0.36	-154
0.600	10.2	1.88	1.84	-90	1.90	9.60	0.408	0.42	-159
0.800	9.00	1.72	1.91	-109	0.900	9.05	1.79	1.98	-113
0.930	8.60	1.18	1.37	-113	1.00	9.45	1.53	1.62	-122
1.42	7.80	0.652	0.84	-162	0.720	9.25	2.26	2.44	-95
2.56	6.30	0.326	0.52	-170	0.500	10.4	2.58	2.48	-81
2.50	6.40	0.398	0.62	-164	0.421	10.8	2.89	2.68	-66
2.65	8.00	0.416	0.52	-145	0.070	12.5	3.00	2.40	-5
3.10	8.95	0.250	0.28	-194	0.059	12.3	2.93	2.38	-3
3.70	9.25	0.250	0.27	-167	0.038	11.0	2.91	2.64	+3
2.10	10.5	0.336	0.32	-150	0.026	11.0	2.61	2.37	+7

Table II.13
RUN NO. 13
ONE-SHOT METHOD

$p = 800$ psia $z^* = 0.834$
 $\Delta T_{\text{sub}} = 4.9^\circ\text{F}$ $\rho = 9.12 \times 10^4$ Btu/hr-ft²
 $v_0 = 3.78$ ft/sec $\alpha_{\text{avg}} = 34.5\%$

Frequency (cps)	Power Amplitude (%)	Void Amplitude (%)	Normal Void Amplitude (%)	Void Phase (deg)	Frequency (cps)	Power Amplitude (%)	Void Amplitude (%)	Normal Void Amplitude (%)	Void Phase (deg)
0.040	11.5	4.30	3.74	+4	1.40	7.25	0.86	1.19	-170
0.060	11.4	4.07	3.57	-4	1.63	6.88	0.51	0.74	-188
0.080	11.0	4.18	3.80	-11	1.85	7.20	0.22	0.30	-173
0.100	11.2	4.42	3.95	-11	2.10	7.20	0.41	0.57	-130
0.070	11.3	4.57	4.04	-7	2.28	7.25	0.47	0.65	-123
0.022	10.7	4.48	4.19	+8	2.40	7.30	0.61	0.84	-140
0.330	11.0	4.35	3.95	-48	2.75	7.62	0.59	0.77	-159
0.184	11.0	4.35	3.95	-24	3.05	7.38	0.42	0.57	-180
0.420	10.9	3.72	3.41	-59	3.34	7.45	0.26	0.35	-186
0.602	10.9	3.16	2.90	-88	3.70	7.80	0.24	0.31	-178
0.800	10.3	2.83	2.75	-110	4.60	7.70	0.46	0.60	-175
1.00	9.30	2.18	2.34	-135	1.75	8.25	0.46	0.56	-206
0.900	9.30	2.70	2.90	-119	1.90	7.80	0.18	0.23	-167
0.688	9.55	3.04	3.18	-101	2.10	7.30	0.23	0.32	-146
0.490	9.05	3.37	3.72	-76	2.38	7.40	0.63	0.85	-131
1.20	7.30	1.44	1.97	-148					

Table II.14
RUN NO. 14
ONE-SHOT METHOD

$p = 800$ psia $z^* = 0.834$
 $\Delta T_{\text{sub}} = 11.2^\circ\text{F}$ $\rho = 9.14 \times 10^4$ Btu/hr-ft²
 $v_0 = 3.78$ ft/sec $\alpha_{\text{avg}} = 26.5\%$

Frequency (cps)	Power Amplitude (%)	Void Amplitude (%)	Normal Void Amplitude (%)	Void Phase (deg)	Frequency (cps)	Power Amplitude (%)	Void Amplitude (%)	Normal Void Amplitude (%)	Void Phase (deg)
0.022	12.0	4.73	3.94	+10	0.386	9.45	4.59	4.86	-55
0.040	11.6	5.12	4.41	+1	0.193	9.50	4.81	5.06	-34
0.060	11.4	4.88	4.28	-3	1.70	6.98	0.40	0.57	-132
0.080	11.5	5.12	4.45	-10	1.38	7.25	0.78	1.08	-162
0.100	11.6	5.17	4.46	-15	1.20	7.62	1.37	1.80	-153
0.070	11.6	4.32	4.59	-4	1.45	7.40	0.50	0.68	-168
0.050	11.5	5.23	4.55	-2	1.88	6.90	0.55	0.80	-106
0.030	11.7	5.03	4.30	+7	1.92	6.98	0.66	0.94	-108
0.350	11.1	4.85	4.37	-63	2.32	7.28	0.66	0.91	-136
0.490	10.7	4.44	4.15	-76	2.70	7.80	0.55	0.70	-165
0.760	8.12	3.58	4.40	-104	3.05	8.00	0.38	0.48	-180
0.940	9.90	2.61	2.64	-129	3.50	8.25	0.44	0.53	-135
1.00	9.02	2.10	2.33	-139	3.70	8.55	0.45	0.53	-154
0.810	8.45	2.91	3.44	-108	4.60	8.25	0.34	0.41	-165
0.600	8.62	3.67	4.26	-97	1.65	8.12	0.39	0.48	-162

APPENDIX III

Steady-state Axial and Transverse Void Distributions

Table III.1

RUN NO. 1

$p = 200 \text{ psia}$ $\phi = 2.28 \times 10^4 \text{ Btu/hr-ft}^2$
 $v_0 = 3.78 \text{ ft/sec}$ $\Delta T_{\text{sub}} = 0.5^\circ\text{F}$

z^*	α_{avg}	α_{y^*}								
		$y^* = 0.80$	0.75	0.50	0.25	0.0	0.25	0.50	0.75	0.80
0.104	6.3	9.4	0	0	0.8	9.5	9.0	7.7	5.0	15.6
0.164	8.6	12.2	2.0	7.7	10.4	10.8	10.4	9.0	7.7	17.1
0.225	11.0	11.9	6.5	8.8	9.5	9.9	9.5	9.5	7.2	16.5
0.285	18.6	12.2	16.5	20.8	22.6	21.2	19.3	20.8	17.8	15.9
0.346	23.4	17.4	21.3	25.3	28.1	28.2	27.3	24.8	20.6	17.9
0.406	24.6	17.1	21.1	24.8	28.6	30.8	30.6	28.1	22.8	17.9
0.467	25.4	16.0	23.4	28.0	32.3	32.8	30.3	25.7	22.4	18.0
0.528	24.0	12.2	18.3	26.8	30.6	32.4	30.2	25.8	25.4	13.9
0.588	27.4	17.9	24.4	31.0	37.2	39.2	37.1	32.7	26.0	21.0
0.649	35.3	22.3	31.8	39.5	44.0	45.5	43.3	37.6	29.5	23.1
0.710	30.0	16.7	25.2	33.7	39.0	39.4	37.7	32.6	24.1	21.9
0.770	31.3	18.1	26.8	38.7	40.7	42.3	39.7	33.1	24.5	17.5
0.831	38.8	26.3	34.6	43.8	48.6	49.1	44.2	41.6	34.6	26.5

Table III.2

RUNS NO. 2 AND 3

$p = 300 \text{ psia}$ $\phi = 2.28 \times 10^4 \text{ Btu/hr-ft}^2$
 $v_0 = 2.52 \text{ ft/sec}$ $\Delta T_{\text{sub}} = 1.8^\circ\text{F}$

z^*	α_{avg}	α_{y^*}								
		$y^* = 0.80$	0.75	0.50	0.25	0.0	0.25	0.50	0.75	0.80
0.104	4.2	2.2	4.6	3.8	3.8	4.3	5.4	4.9	3.4	5.1
0.164	4.8	8.0	4.9	3.8	3.8	4.3	3.8	4.7	4.7	6.0
0.225	8.7	7.0	9.1	9.7	10.2	10.4	10.6	10.4	10.4	7.9
0.285	12.7	-	12.6	12.6	13.4	15.0	13.0	12.6	9.5	-
0.346	17.4	11.0	16.7	19.0	22.5	23.6	21.4	18.3	13.4	10.5
0.406	19.1	13.7	16.5	19.7	21.4	24.7	22.7	19.8	16.1	17.9
0.467	22.7	12.3	19.0	24.9	27.8	29.7	29.1	26.2	18.4	18.7
0.528	28.7	22.1	27.7	31.8	34.2	34.7	34.6	30.1	23.6	20.1
0.588	32.5	26.4	31.2	34.7	37.6	38.4	36.6	33.8	28.4	25.6
0.649	38.1	23.4	27.8	33.5	35.7	36.6	35.3	32.0	25.0	21.3
0.710	37.4	25.3	35.7	40.7	43.8	45.4	44.0	40.6	34.0	27.1
0.770	34.9	24.7	32.9	39.3	42.0	42.6	41.3	36.8	27.1	26.4
0.831	42.6	31.2	41.1	47.0	50.4	51.4	49.7	44.2	37.4	30.8

Table III.3
RUNS NO. 4 AND 5

$p = 400$ psia $\phi = 4.56 \times 10^4$ Btu/hr-ft²
 $v_0 = 2.52$ ft/sec $\Delta T_{\text{sub}} = 1.3^\circ\text{F}$

z^*	α_{avg}	α_{y^*}								
		$y^* = 0.80$	0.75	0.50	0.25	0.0	0.25	0.50	0.75	0.80
0.104	7.9	18.4	4.2	1.8	0.9	0	0.7	4.8	9.9	20.8
0.164	19.4	28.8	18.9	16.7	14.6	12.2	14.4	18.3	22.1	28.6
0.225	21.4	27.5	19.4	20.2	19.8	19.8	19.8	20.6	21.4	24.0
0.285	24.7	24.8	19.8	25.1	28.4	28.0	26.4	24.9	21.1	24.1
0.346	27.8	23.8	21.6	28.4	32.8	34.2	33.0	28.6	24.5	23.0
0.406	30.3	26.2	25.7	29.7	32.5	37.8	35.4	32.0	27.1	26.6
0.467	29.9	24.9	26.6	32.6	35.7	34.6	33.4	29.7	26.4	24.9
0.528	34.7	27.0	30.7	36.5	40.7	41.3	40.2	36.1	32.2	27.6
0.588	38.6	31.0	34.2	40.0	43.5	45.0	43.8	40.9	35.7	33.4
0.649	42.7	33.2	38.8	45.6	49.4	49.8	48.4	44.8	38.7	35.2
0.710	40.0	32.6	35.2	42.1	46.0	46.9	46.0	42.0	35.7	34.0
0.770	39.2	28.6	34.4	42.7	47.1	47.7	46.7	41.5	35.4	28.6
0.831	43.2	35.1	39.4	45.4	50.2	51.0	46.3	44.8	39.6	36.9
0.831	41.2	33.2	36.1	44.2	49.0	49.4	45.2	42.5	37.1	34.4

Table III.4
RUN NO. 6

$p = 400$ psia $\phi = 4.56 \times 10^4$ Btu/hr-ft²
 $v_0 = 2.52$ ft/sec $\Delta T_{\text{sub}} = 9.5^\circ\text{F}$

z^*	α_{avg}	α_{y^*}								
		$y^* = 0.80$	0.75	0.50	0.25	0.0	0.25	0.50	0.75	0.80
0.104	2.3	0.7	0.6	1.8	2.5	2.2	3.6	4.0	4.0	1.3
0.164	8.0	10.5	4.6	5.7	6.3	6.9	6.3	7.2	8.9	15.7
0.225	7.8	12.4	6.8	5.1	7.5	3.2	3.1	5.7	9.2	17.3
0.285	8.4	12.3	7.0	9.8	5.4	2.4	3.0	7.7	11.2	17.3
0.346	12.7	14.8	10.5	10.4	11.7	10.8	11.0	12.0	13.0	20.3
0.406	12.7	14.6	10.4	10.3	9.8	12.0	11.7	12.0	13.9	19.4
0.467	15.2	16.8	13.8	15.0	15.4	14.4	12.6	12.6	16.3	19.7
0.528	19.8	18.4	17.5	20.4	20.2	19.3	20.0	19.9	19.4	22.8
0.588	21.9	20.2	18.6	20.1	22.5	24.6	23.6	22.9	22.5	22.3
0.649	29.4	23.8	27.2	31.2	33.8	34.3	31.5	30.0	26.6	26.5
0.710	31.6	26.2	30.0	33.3	35.0	35.2	34.8	32.2	28.9	28.6
0.770	31.9	24.8	29.1	34.0	38.7	38.9	36.6	32.4	27.8	24.6
0.831	28.8	20.2	25.1	31.8	36.8	37.3	31.1	29.3	27.0	21.1
0.831	28.4	20.2	24.7	31.0	35.2	35.1	29.5	27.8	26.2	24.1

Table III.5

RUN NO. 7

$$p = 400 \text{ psia} \quad \phi = 6.84 \times 10^4 \text{ Btu/hr-ft}^2$$

$$v_0 = 3.78 \text{ ft/sec} \quad \Delta T_{\text{sub}} = 1.2^\circ\text{F}$$

z^*	α_{avg}	α_{y^*}								
		$y^* = 0.80$	0.75	0.50	0.25	0.0	0.25	0.50	0.75	0.80
0.104	5.0	9.02	4.36	0.38	0.60	0.82	0.19	3.60	9.72	16.1
0.164	14.5	16.7	15.9	12.8	10.5	9.40	11.1	14.8	18.5	20.8
0.225	18.6	15.5	17.6	18.5	19.5	19.7	18.8	19.7	18.2	19.7
0.285	18.3	9.66	13.8	18.8	23.5	23.0	22.2	20.6	15.6	17.4
0.346	25.0	16.3	20.0	26.1	32.7	35.2	32.5	26.8	20.7	14.6
0.406	29.4	15.9	21.3	28.3	33.7	38.7	36.9	31.5	23.4	15.4
0.467	31.9	20.8	28.2	35.8	40.9	42.0	38.2	33.6	26.7	20.4
0.528	37.2	25.6	32.2	41.4	46.9	48.5	45.5	40.2	32.3	22.4
0.588	35.7	25.0	32.0	37.7	42.9	46.0	43.6	38.4	30.7	24.4
0.649	43.7	33.0	38.6	47.6	53.0	54.8	51.7	46.2	37.4	27.2
0.710	41.3	27.3	36.9	44.2	51.2	53.5	50.2	45.7	36.3	27.0
0.770	40.8	28.9	36.2	44.4	50.1	51.7	50.1	44.1	35.2	27.0
0.831	46.2	32.8	41.8	50.0	54.4	56.2	50.1	46.4	39.1	11.0

Table III.6

RUN NO. 8

$$p = 400 \text{ psia} \quad \phi = 6.84 \times 10^4 \text{ Btu/hr-ft}^2$$

$$v_0 = 3.78 \text{ ft/sec} \quad \Delta T_{\text{sub}} = 8.1^\circ\text{F}$$

z^*	α_{avg}	α_{y^*}								
		$y^* = 0.80$	0.75	0.50	0.25	0.0	0.25	0.50	0.75	0.80
0.104	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0
0.164	3.2	11.2	0.4	0.2	0.0	0.0	0.2	1.5	4.0	11.7
0.225	3.6	13.0	2.3	1.0	0.0	0.0	0.0	0.0	3.8	12.3
0.285	7.1	11.6	5.9	5.7	4.4	2.7	1.7	6.7	8.4	16.6
0.346	6.8	13.4	4.0	4.6	5.2	5.0	4.6	5.2	7.1	11.7
0.406	12.3	16.0	9.6	9.8	9.8	13.3	11.9	12.7	12.1	15.3
0.467	10.1	14.0	7.4	10.2	11.0	10.0	9.2	8.4	9.0	11.9
0.528	14.0	20.0	11.7	13.9	16.3	18.1	15.9	13.1	9.6	7.6
0.588	17.4	16.9	12.7	16.3	19.9	20.8	20.2	18.3	15.1	16.8
0.649	26.0	23.4	22.2	27.8	32.1	32.9	30.0	24.5	21.2	20.1
0.710	24.1	21.4	18.7	25.3	29.1	30.6	28.6	24.5	19.1	20.1
0.770	28.0	25.5	24.0	30.0	34.4	34.9	33.4	27.8	22.4	20.1
0.831	26.0	21.5	22.0	28.6	32.9	33.4	27.6	25.7	22.0	20.0
0.831	28.6	24.2	24.5	31.0	35.3	36.2	33.8	29.3	23.8	19.6

Table III.7

RUN NO. 9

$$p = 600 \text{ psia} \quad \phi = 9.12 \times 10^4 \text{ Btu/hr-ft}^2$$

$$v_0 = 3.78 \text{ ft/sec} \quad \Delta T_{\text{sub}} = 4.2^\circ\text{F}$$

z^*	α_{avg}	α_{y^*}								
		$y^* = 0.80$	0.75	0.50	0.25	0.0	0.25	0.50	0.75	0.80
0.104	5.2	13.3	4.0	2.1	0.2	0.0	1.0	3.3	8.7	12.3
0.164	15.0	22.0	16.3	13.9	11.7	9.5	11.1	15.1	19.2	16.3
0.225	17.7	19.8	17.1	18.2	15.9	16.5	17.1	18.2	18.8	17.6
0.285	18.2	16.8	14.5	18.8	21.7	22.7	19.4	19.4	17.6	12.7
0.346	21.8	16.0	16.9	22.5	28.8	30.7	28.2	22.7	18.0	12.1
0.406	24.1	19.0	18.4	24.2	31.0	34.3	32.3	26.9	20.9	9.9
0.467	25.2	15.0	21.4	28.9	34.8	36.8	31.5	26.7	21.0	11.1
0.528	28.3	20.8	24.8	31.2	37.8	38.8	36.3	31.9	23.3	15.3
0.588	31.2	21.6	27.1	33.4	38.5	41.4	39.7	32.8	28.2	18.4
0.649	37.8	27.3	35.5	41.3	48.4	47.7	45.7	40.0	32.8	21.2
0.710	37.8	27.0	35.0	41.6	47.4	48.4	46.7	43.2	27.4	24.0
0.770	38.8	27.3	33.0	42.5	48.8	49.8	48.3	42.2	35.2	22.1
0.831	40.1	24.4	31.2	41.8	56.5	57.2	52.4	40.4	34.7	22.7
0.831	41.4	28.9	37.6	47.2	51.2	52.2	48.0	43.5	39.7	23.9

Table III.8

RUN NO. 10

$$p = 600 \text{ psia} \quad \phi = 9.12 \times 10^4 \text{ Btu/hr-ft}^2$$

$$v_0 = 3.78 \text{ ft/sec} \quad \Delta T_{\text{sub}} = 12.6^\circ\text{F}$$

z^*	α_{avg}	α_{y^*}								
		$y^* = 0.80$	0.75	0.50	0.25	0.0	0.25	0.50	0.75	0.80
0.104	1.3	4.9	0.0	0.0	0.0	0.0	0.0	1.0	1.7	3.8
0.164	3.6	10.4	4.2	1.2	0.0	0.0	0.0	2.3	5.2	9.0
0.225	5.4	12.9	7.0	4.6	2.5	1.2	1.2	3.1	7.2	8.7
0.285	9.3	12.1	8.7	9.3	9.3	7.2	7.2	10.1	11.1	9.1
0.346	8.3	9.5	5.5	7.9	8.3	9.1	8.7	8.7	9.5	7.9
0.406	13.0	13.1	10.7	11.3	13.1	16.2	16.6	14.7	13.1	8.4
0.467	13.9	13.6	11.7	15.5	17.8	17.5	15.5	13.1	11.5	9.3
0.528	17.5	15.2	12.3	19.0	22.1	22.9	21.4	18.0	15.9	11.1
0.588	22.6	17.8	17.8	22.5	28.3	29.9	29.3	23.6	20.4	14.3
0.649	23.1	17.8	20.0	26.9	31.0	31.0	29.5	22.7	19.0	11.9
0.710	25.6	19.0	20.9	27.4	32.6	33.2	31.7	27.8	22.3	15.7
0.770	28.9	14.3	25.9	33.9	38.1	39.1	38.6	30.4	24.6	14.7
0.831	24.7	11.7	23.6	29.9	33.4	33.5	28.6	25.2	22.5	13.9
0.831	27.6	14.3	24.4	33.0	37.0	36.6	31.2	28.6	25.6	17.5

Table III.9
RUN NO. 12

$$p = 800 \text{ psia} \quad \phi = 4.56 \times 10^4 \text{ Btu/hr-ft}^2$$

$$v_0 = 2.52 \text{ ft/sec} \quad \Delta T_{\text{sub}} = 4.9^\circ\text{F}$$

z^*	α_{avg}	α_{y^*}								
		$y^* = 0.80$	0.75	0.50	0.25	0.0	0.25	0.50	0.75	0.80
0.164	7.1	11.5	14.8	2.9	0.7	0.6	0.9	1.3	10.8	20.4
0.225	9.8	19.8	14.4	7.2	2.1	2.1	3.3	5.9	16.4	17.3
0.285	9.4	16.4	14.4	7.7	4.6	0.9	4.2	5.6	13.9	16.9
0.346	9.5	15.4	13.0	5.9	5.9	6.1	5.6	5.4	11.7	16.4
0.406	14.3	11.1	12.6	16.1	17.8	18.0	14.0	13.6	15.3	13.1
0.467	11.7	-	6.6	11.5	11.1	13.8	14.7	12.2	12.2	-
0.528	11.3	-	10.3	11.9	15.0	16.6	16.1	11.9	10.5	9.8
0.649	21.0	17.9	15.1	20.9	24.3	27.3	27.0	22.3	15.1	18.7
0.710	26.9	28.6	24.5	25.6	29.5	30.0	29.5	24.4	23.0	27.9
0.770	30.0	21.1	24.1	28.0	32.9	34.7	34.8	30.6	21.1	21.9
0.831	31.4	18.2	35.6	39.1	40.3	41.9	42.5	39.6	32.4	13.4

Table III.10
RUN NO. 13

$$p = 800 \text{ psia} \quad \phi = 9.12 \times 10^4 \text{ Btu/hr-ft}^2$$

$$v_0 = 3.78 \text{ ft/sec} \quad \Delta T_{\text{sub}} = 4.7^\circ\text{F}$$

z^*	α_{avg}	α_{y^*}								
		$y^* = 0.80$	0.75	0.50	0.25	0.0	0.25	0.50	0.75	0.80
0.164	16.2	32.3	28.6	8.7	6.0	5.2	6.5	7.6	22.5	28.7
0.225	17.0	24.3	26.7	12.6	8.7	7.6	9.4	12.1	27.1	24.1
0.285	20.4	26.9	26.8	18.0	15.6	13.2	15.5	16.1	28.0	23.6
0.346	25.5	27.5	32.0	19.6	24.4	26.2	26.2	19.3	23.1	31.5
0.406	24.9	22.4	30.3	25.2	25.8	26.2	23.1	21.7	27.8	22.0
0.467	32.7	26.4	27.8	30.9	37.9	41.0	41.3	33.2	27.8	28.0
0.528	27.0	35.4	25.1	21.8	26.4	28.9	27.6	23.1	25.1	30.0
0.588	30.6	-	27.8	28.4	36.5	37.7	36.0	29.4	24.0	24.5
0.649	32.5	-	22.9	30.9	36.5	38.8	37.9	30.7	29.6	-
0.710	43.8	32.5	41.9	47.8	54.0	55.6	51.8	44.8	32.2	33.4
0.770	39.4	26.9	30.5	41.7	49.5	51.3	50.9	44.8	35.6	23.5
0.831	43.9	32.5	32.3	47.6	51.2	55.6	54.8	47.8	37.8	32.5
0.831	39.6	14.8	31.4	45.2	49.1	51.8	52.0	48.0	44.4	19.3

APPENDIX IV

Neal and Zuber Models

In this appendix two additional theoretical models relating quality and void fraction are compared with the steady-state data.

A. Neal Model

Neal⁶⁹ derived his for the bubble- and slug-flow regimes, beginning with the idea that slip in two-phase (gas-liquid) flow is a result of two effects: the difference in the gravitational forces acting upon the phases, and the existence of radial gradients of void fraction and stream velocity. Two flow parameters were introduced; the first was a correlation coefficient between void fraction and mixture velocity, and the second was a measure of the resistance to gas motion with respect to the liquid.

The bubble-rise velocity with respect to the cross-section-averaged mixture velocity, due to buoyancy, was given as $C\sqrt{gd(\rho_l - \rho_g)/\rho_l}$, where C is a parameter which is a measure of the resistance to bubble motion with respect to the liquid and depends upon the inertial, viscous, and surface forces, and d is the pipe diameter.

For nonzero liquid flow rates an additional component for the vapor velocity was obtained from the liquid motion. This component is greater than the flow-area-averaged stream velocity when the bubble concentration is highest at the center of the channel where the velocity has a maximum. A spatial correlation factor between α and $v_{m,avg}$ was defined:

$$C_0 \equiv \frac{\int_0^A \alpha v_m dA}{\left[\int_0^A v_m dA \right] \left[\int_0^A \alpha dA \right]}, \quad (IV.1)$$

where v_m is the local mixture velocity, defined by

$$v_m = \alpha v_g + (1 - \alpha) v_l. \quad (IV.2)$$

The cross-section-averaged gas velocity with respect to laboratory coordinates was then obtained from the two additive components:

$$v_{g,avg} = Q_g / A \alpha_{avg} = C_0 v_{m,avg} + C \sqrt{gd(\rho_l / \rho_g) / \rho_l}, \quad (IV.3)$$

where Q_g is the total volumetric gas flow rate and A is the cross-sectional flow area of the channel.

Then an expression for slip was derived:

$$S_{avg} = v_{g,avg}/v_{l,avg} = \frac{1 - \alpha_{avg}}{(1/C_0) - \alpha_{avg}} \left\{ 1 + \frac{C}{C_0} \left[\frac{(\rho_l - \rho_g)gd}{\rho_l v_0^2} \right]^{\frac{1}{2}} \right\} \quad (IV.4)$$

For steam-water flow the values of C_0 and C were determined by a least-squares analysis which minimized the error of calculating slip from available data. The average best values for steam-water systems were given as $C_0 = 1.25$ and $C = 0.60$. These values when used in this model were found to give good agreement between theory and the data reported here (see Fig. IV.1).

Fig. IV.1. Steady-state Axial Void Fraction Distributions

B. Zuber and Findlay Model

Zuber and Findlay⁷⁰ performed a similar analysis to that of Neal but extended the consideration of local slip to regimes other than slug flow. They obtained a modified Eq. (IV.3):

$$\frac{Q_g}{A\alpha_{avg}} = C_0 v_{m,avg} + \frac{(\alpha v_{vg})_{avg}}{\alpha_{avg}}, \quad (IV.5)$$

where v_{vg} was defined as the gas "drift velocity" with respect to the volumetric flux density of the mixture, v_m . Zuber and Findlay theorized that a plot of $Q_g/A\alpha_{avg}$ vs. $v_{m,avg}$ will yield the value of C_0 as the slope and the weighted average drift velocity as the intercept, when the local slip does not depend on concentration.

For those cases where the local slip is very small, the second term of the right-hand side of Eq. (IV.5) may be neglected and $C_0 \approx 1/K_B$, where K_B is the constant obtained in Bankoff's analysis.⁴⁷

Zuber and Findlay's expressions for the weighted average drift velocity in the slug-flow regime differed from Neal's in that it had a slightly lower coefficient. For the bubbly churn-turbulent regime the weighted average drift velocity was given in terms of the Stoke's terminal velocity multiplied by a coefficient which is a function of void fraction.

Thus, in order to compare this model with the experimental data obtained here, the flow regimes would have to be first determined.

Figure IV.2 presents all of the void fraction data obtained here on a plot of $Q_g/A\alpha_{avg} = v_{g,avg}$ vs. $v_{m,avg}$. The circled data points represent for each run the mixture velocity at which a convex void-fraction profile was first obtained; for all downstream points, corresponding to higher mixture velocities, the void profiles became more fully developed.

Two lines, one with a slope of $C_0 = 1.2$ and the second with $C_0 = 1.4$, were drawn through the data at the higher values of $v_{m,avg}$. The second line gives slightly better agreement with the data, but the calculated liquid velocity profiles are considerably more peaked. It should be noted that the individual curves are all slightly convex. This might result from a decrease in C_0 , corresponding to flatter profiles and/or a decrease in local drift velocity at higher mixture velocities. Both trends might be expected as the mixture velocity increases, in view of the increased wall shear stress.

Fig. IV.2. Cross-section-averaged Vapor Velocity as a Function of Mixture Velocity

ACKNOWLEDGMENTS

The author gratefully acknowledges the assistance of the many individuals who have contributed to the success of this thesis and to the author's professional development. First and foremost to Professor S. G. Bankoff, under whose direction this work was performed, for his suggestion of the problem, and for his constant aid and able direction throughout the study. Next, to Dr. Michael Petrick for his technical advice and administrative direction of the combined Associated Midwest Universities-Argonne National Laboratory program under whose auspices this study was performed.

This work was performed at Argonne National Laboratory in the D-11 Heat Engineering Section where all of the personnel, including my fellow AMU students, contributed in some way to this work. The following have provided valuable aid in a special way: Matthew Gats and Elmer Gunchin for their technical aid in the initial experimental design changes; to Joseph Kemp and Louis Bova for their aid in the design, construction, installation, and servicing of all the electronic equipment; to James O'Grady for operating the loop and assistance in data reduction.

While at Northwestern University financial assistance was provided by a Dow Chemical Company Fellowship and the Ford Foundation. In addition to George Nassos, a fellow student with whom I shared an office, for his helpful discussions and unselfish aid.