BEYOND THE RISING TOTAL CROSS SECTIONS

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ABSTRACT

The theoretical reasoning that leads to the prediction of, among others, total cross section rising like $(\ln s)^2$ is reviewed. Briefly, relativistic kinematics dictate the dependence on longitudinal momentum of particles produced in the pionization region, and leads to a factor of $\ln s$ in the integrated cross section. Unitarity in the t-channel then amplifies this $\ln s$ to be a power of s , divided possibly by a power of $\ln s$. This power of s is a realization of the strongly absorptive potential with a coupling constant increasing with energy, and thus the Froissart bound for total cross sections is saturated. Many other results, concerning in particular the phases of the scattering amplitudes in the forward direction and differential cross sections for both elastic and inelastic diffractive scattering, are obtained at the same time.

In order to make quantitative comparison with experimental data, past and future, we describe a simple model of hadron scattering. When applied to the elastic scattering of pp , $p\bar{p}$, πp , and K^+p , this model describes reasonably well existing data beyond 20GeV. From the resulting parameters, quantitative predictions about these processes are given in detail up to the highest energy that may become available from accelerators in this century, namely 1.5TeV on 1.5TeV. These considerations are also extended to inelastic diffractive scattering without spin-parity exchange.

A number of problems related to increasing total cross sections, some unsolved, are discussed.

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1. INTRODUCTION

On the basis of quantum field theory, we predicted three years ago that all hadron-hadron total cross sections increase without bound for higher and higher energies.¹ Half a year ago, for the case of proton-proton scattering, this prediction was experimentally confirmed at ISR simultaneously by two groups, one a collaboration between the University of Pisa and the State University of New York at Stony Brook² and the other collaboration between Rome and CERN.³ Furthermore, within a factor of 2, the experimentally observed rise in pp total cross section agrees with the value obtained last year from a very simple phenomenological model.⁴ It is therefore an opportune time to discuss both our theoretical considerations and the phenomenological model.^{4, 5}

One of the starting points of our theoretical work is the following. For many years it has been known that hadronic total cross sections are roughly constant at high energies. Why are they constant? In Regge language, we say that the intercept of the Pomeranchuk trajectory is 1:

$$\alpha_p(0) = 1. \quad (1.1)$$

Next to 0, 1 is the "best" number. In this sense, the 1 of the above equation represents just about the only good number in hadron physics aside from spins. If we believe it is unnatural and ugly to say that this 1 is really 0.93 or something like this, then the question is again why 1. Why?

In order to have some theoretical understanding of this and other problems in connection with hadronic scattering at extremely high energies, we cannot avoid dealing with rather complicated systems. Our basic idea is: Determine the high-energy behavior of relativistic quantum field theories and then apply to the realistic case of the scattering and production of hadrons. The reasoning is as follows. Even in the absence of concrete information about very-high-energy processes, there is very little doubt concerning the validity of relativistic kinematics (we do know kinematics although not dynamics), unitarity (you cannot get something out of nothing), and particle production (what can happen does happen). Much less experimentally established, but nevertheless usually accepted, are analyticity and crossing symmetry. Each of these five properties is "trivial" because of its generality, i.e., independence of the details of the interaction. It is, however, highly non-trivial to incorporate all five into the same theory, and the simplest way is to have a relativistic quantum field theory. We are therefore led to study the high-energy behavior of such theories by our desire not to leave out, on an ad hoc basis, one or more of these basic properties.

Since we do not know the interaction, how can any result from field theory be applied to hadronic processes? It is clearly not profitable to calculate some amplitude from, for example, massive quantum electrodynamics and hope that the result has something to

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do with proton-proton scattering. However, if we calculate various amplitudes from a variety of field theories, i.e., field theories with different interactions, and find that they have the same general feature in the limit of extremely high energies, then it is perhaps not unreasonable to expect this general feature, although not the detailed formulas, to be also valid for processes that occur in nature. This is the procedure that we follow with the extra precaution that a simple physical interpretation is found for each general feature.

Although eminently reasonable, the applicability of the general features of field theories to hadron processes at high energies is nevertheless a pure assumption. The correctness of this assumption can be determined only by experiments at the National Accelerator Laboratory and the Intersecting Storage Rings at CERN. It is for this reason that the recent experimental verifications^{2, 3} mentioned above are so important. We must never forget that physics is basically an experimental science.

In Sec. 2, we give a very brief review of some of the salient points of the theory together with a list of predictions for diffractive processes at infinite energy. Since these predictions are valid only in this remote limit, we must construct phenomenological models before quantitative comparison with experiments is possible. The most simple-minded version of such a model is given in detail in Sec. 3. In Sec. 4, we discuss some more recent theoretical developments, comparison of the present results with other theories, and a general outlook.

2. THEORY

We study the high-energy behavior of scattering amplitudes in various quantum field theories. The limit of interest is the one where the total energy in the center-of-mass system approaches infinity while all transverse momentum transfers are fixed. We reported here⁶ at Stony Brook, four years ago at the third ICHEC, the early developments leading to the impact diagrams. In this section we concentrate on further theoretical considerations since that time.

A. Pionization

For the sake of definiteness, let us use temporarily the language of massive quantum electrodynamics, where electrons interact with a massive photon (or gluon). Consider a definite process such as

$$e^- + e^- + e^- + e^- + e^- + e^+ \quad (2.1)$$

In such a process, a produced particle (such as the e^+ in the above reaction) may be relatively slow in the center-of-mass system. Such slow particles are referred to as pionization product.^{7, 8} Because of invariance under a Lorentz transformation in the

direction z of the incident particles, we find that the distribution of pionization product is given by⁹

$$E^{-1} f(p_x, p_y) d^3p \quad (2.2)$$

where E and $\vec{p}$ are the energy and momentum of the produced particle. For the reaction (2.1), (2.2) is also explicitly verified for massive quantum electrodynamics, and the important diagrams are the two shown in Fig. 1. Since d^3p/E is the invariant phase-space volume, it is perhaps not difficult to believe (2.2).

This simple result (2.2) has far-reaching consequences. If (2.2) is integrated over all p_z , we get a factor

$$\ln s \quad (2.3)$$

when s , the c.m. energy squared, is large. To understand this factor, we consider first a similar prediction of Feynman¹⁰ at nearly the same time. Aside from differences due to off-shell or on-shell, partons or physical particles, and bremsstrahlung or pionization diagrams, the major issue is: Feynman's formula refers to inclusive one-particle distribution, while ours is for a specific channel such as (2.1). Therefore, Feynman is able to, and does, attribute this $\ln s$ to multiplicity, but we must face squarely these logarithmic factors. The multiplicity for the reaction (2.1) is clearly 4, not $\ln s$.

B. Black Disk

Explicit computation from the two diagrams of Fig. 1 shows that the resulting $f(p_x, p_y)$ does not depend on s .^{9, 11} Therefore, to this lowest order, the integrated cross section for the reaction (2.1) increases like $\ln s$ when $s \rightarrow \infty$ and the photon has a non-zero mass. [For the actual case of massless photon, the increase is $(\ln s)^3$. We shall not consider any further this massless case, because there is no massless hadron.] This fact in particular implies that the total cross section for electron-electron scattering is also unbounded as $s \rightarrow \infty$. Something interesting must be going on.

Since the diagrams of Fig. 1 are of order e^4 , the $\ln s$ contribution to the total cross section is of order e^8 . The diagrams that contribute to this $\ln s$ term must be obtained from two copies of those in Fig. 1, and are shown explicitly in Fig. 2(a) and 2(b). For reasons of gauge invariance, we include also that of Fig. 2(c).

Once we understand the origin of this $\ln s$, it is straightforward to get powers of $\ln s$ by introducing more fermion loops. The one-tower diagrams with n loops, as shown in Fig. 3, are the lowest order diagrams which contribute $(\ln s)^n$ to the total cross section. Summing up these cross sections over n , we get a total cross section of the order of $s^a (\ln s)^{-2}$, where $a = 11\alpha^2 \pi / 32$.^{12, 13} The fact that this result is larger than any power of $\ln s$ cannot be blamed on the process of summing only the leading terms. This is because the cross section for n -pair production is always positive.

By choosing n sufficiently large, the corresponding n -pair production cross section is already larger than any given power of $\ln s$. Thus the sum must also be larger than any power of $\ln s$.

This result $s^a/(\ln s)^2$ is not to be interpreted as a violation of the Froissart bound.¹⁴ Rather, it ought to be regarded as a realization of the strongly absorptive "potential" with a coupling constant increasing with energy, as conceived in Froissart's original paper. Thus, in a two-particle scattering process, if the interaction takes place at a sufficiently close transverse distance from the center of the target, the incident particle creates slow particles in the c.m. system and is lost to the beam. If the transverse distance involved is large, the incident particle does not necessarily create pionization products and may survive. Since the exponent a is positive, the radius R of the central region of nearly complete absorption increases indefinitely as $s \rightarrow \infty$. Since the interaction is Yukawa at large distances, R is asymptotically proportional to $\ln s$. Thus the Froissart bound¹⁴ on total cross section is saturated. Of course, Froissart's other bounds¹⁴ cannot be saturated.

We recapitulate briefly the above description of the arguments leading to increasing cross sections. Since particle production is possible, some of the produced particles are slow in the c.m. system. Relativistic kinematics then dictate the dependence of the distribution of such slow particles on the longitudinal momentum. Since the integration of this distribution gives a factor of $\ln s$, t -channel unitarity amplifies this $\ln s$ to be a power of s (divided possibly by a power of $\ln s$). Finally, by s -channel unitarity this power of s means a black disk of increasing radius.¹ Thus (1.1) is the result of satisfying both the s -channel unitarity and t -channel unitarity.

It is thus clear that the detailed nature of the underlying interaction is not important. What is needed is a sufficiently strong interaction that attempts to push some Regge singularity to the right of 1. For example, in the approximation of keeping only ladder diagrams, a black disk is also formed in ϕ^3 theory when the coupling constant is sufficiently strong.^{15, 16}

The black region is surrounded by a gray region (i.e., partially absorbing region) of a radius of order 1, as shown in Fig. 4. Inelastic diffraction dissociation occurs in this gray region.¹⁷ Since the area of this gray region is of order $\ln s$, integrated cross sections for inelastic diffraction dissociation increases as $\ln s$ for $s \rightarrow \infty$.

These increasing cross sections in turn modify pionization distributions so that the $f(p_x, p_y)$ depends also on s . However, we shall not discuss here this point any further.

C. Predictions for Infinite Energy

For brevity, we refer to the above physical picture for high-energy scattering and production processes as the impact picture. We now apply the impact picture to hadrons.

Let us discuss first the condition for pionization to occur. For the reaction

$$a + b \rightarrow c + x \quad (2.4)$$

where x is a set of particles, at high energies c can be a pionization product with a distribution of the form (2.2) if and only if x can be separated into three subsets x_1 , x_2 , and x_3 such that Pomeranchons may be exchanged between $a-x_1$ and x_2 , and between x_2 and $b-x_3$. More precisely, a diagram of the form of Fig. 5 is possible. The Pomeranchons are $+$ under charge conjugation, carry no charge, no isospin, no baryon number, and no strangeness, but may carry spin-parity.

Let us give a few examples. First, in

$$p + p \rightarrow p + p + \pi^+ + \pi^-, \quad (2.5)$$

The π^+ satisfies (2.2) at high energies but not p . Here $x_1 = p$, $x_2 = \pi^+$, and $x_3 = p$. In

$$p + p \rightarrow n + p + \pi^+ \quad (2.6)$$

or

$$p + p \rightarrow p + p + \pi^0, \quad (2.7)$$

none of the three particles is distributed according to (2.2). Finally in

$$p + p \rightarrow \Lambda + p^* (1400) + K^+ + K^+ + K^-, \quad (2.8)$$

the K 's satisfy (2.2) at high energies. Evidently, in (2.2) the $f(p_x, p_y)$ is the same for particle and antiparticle.

We list some of the immediate predictions of this impact picture for hadron interactions at extremely high energies.

a) Hadrons that are slow in the cm system are produced according to the distribution^{9, 10} (2.2), where f may also depend on s .

Clear evidence for the experimental detection of these hadrons has been found at both the PS¹⁸ and the Intersecting Storage Rings.^{19, 20}

b) The distribution (2.1) holds not only for the inclusive process but also for each individual exclusive process with suitable quantum numbers, as discussed above.

c) All total cross sections rise indefinitely with energy.¹

d) All total elastic cross sections rise indefinitely with energy. Furthermore, also as a consequence of scattering by a large totally absorbing disk, we have¹

$$\lim_{s \rightarrow \infty} \frac{\sigma_{el}}{\sigma_{total}} = \frac{1}{2} \quad (2.9)$$

e) At extremely high energies, all hadron total cross sections are the same. More precisely, consider two hadron processes

$h_1 + h_2$ and $h_3 + h_4$, then

$$\lim_{s \rightarrow \infty} \frac{\sigma_{\text{total}}(h_1 h_2)}{\sigma_{\text{total}}(h_3 h_4)} = 1 \quad (2.10)$$

In particular,

$$\lim_{s \rightarrow \infty} \frac{\sigma_{\text{total}}(p)}{\sigma_{\text{total}}(pp)} = \lim_{s \rightarrow \infty} \frac{\sigma_{\text{total}}(Kp)}{\sigma_{\text{total}}(pp)} = 1 \quad (2.11)$$

The Pomeranchuk theorem²¹ is a special case of (2.10).

f) The diffraction peak in elastic scattering shrinks indefinitely with increasing energy.

g) In the elastic scattering cross section, a dip exists and moves to arbitrarily small values of momentum transfer with increasing energy. More precisely, let r be the value of $|t|$ at which the first dip occurs, then¹

$$\lim_{s \rightarrow \infty} r \sigma_{\text{tot}} = 35.92 \text{ mb (GeV/c)}^2. \quad (2.12)$$

h) For elastic scattering, the ratio α of the real to imaginary parts of the forward scattering amplitude is positive for sufficiently high energy and approaches zero as $s \rightarrow \infty$.¹

i) The total cross sections for diffractive inelastic processes rise indefinitely with energy. The width of the peak for inelastic cross section near the forward direction also shrink with increasing energy. Furthermore, when there is no spin-parity exchange, the dip in the differential cross section occurs at a momentum transfer significantly less than the corresponding dip position for elastic scattering.¹⁷

3. PHENOMENOLOGY

A. Motivation

Of the partial list of impact-picture predictions (a) - (i) given above, only the first item has experimental support.¹⁸⁻²⁰ Although not yet carried out, there is no difficulty in performing the necessary experiments for the second item. These two items, (a) and (b), have the common feature that they do not depend critically on the presence of the black core.

All the other impact-picture predictions, (c) - (i), depend not only on the existence of the black core, but also on its becoming arbitrarily large and hence dominating at extremely high energies. The analysis of Chou and Yang²² shows that, at about 30 GeV, there is no evidence for a black disk. It is therefore unreasonable to expect this logarithmically expanding black disk not only to appear but also to dominate even at ISABELLE energies. As an example, at 55 BeV, the total pp and K^+p cross sections have been measured to be 38.4 mb and 18.2 mb respectively by Denisov et al.,²³ and hence at this energy

$$\sigma_{\text{total}}(K^+p)/\sigma_{\text{total}}(pp) = 0.472 \quad (3.1)$$

which is less than half of the value predicted by (2.11). In order to have meaningful comparisons with experiments at the National Accelerator Laboratory and at the Intersecting Storage Rings, we must find a way of deciding how this ratio of total cross sections increases from the experimental value of (3.1) to the theoretical limit of (2.11). What is needed is therefore an analysis to correlate the existing experimental data at moderate energies with the impact-picture predictions for extremely high energies so that theoretical results may be obtained for the energies that are beginning to be available.

Such an analysis cannot be model independent. We shall here try to present the most simple-minded model consistent with the impact picture. Before describing the details of our model, we discuss first a point of paramount importance. In obtaining the logarithmically increasing radius of the black core as discussed in Section 2, we must have, at least roughly, a Yukawa interaction at large impact distances.¹ More precisely, the important factor is $e^{-\lambda r}$ in the Yukawa form $e^{-\lambda r}/r$, and reflects the absence of a massless hadron. For the impact picture, the other factor $\frac{1}{r}$ does not play any significant role and is indeed absent in the case of massive quantum electrodynamics. It is thus necessary to incorporate in the model a dependence of the form $e^{-\lambda r}$ for large r ; this factor cannot be replaced by either a power r^{-n} or a Gaussian e^{-r^2} .

B. Description of the Model

We first describe our model for elastic scattering. Let j denote an elastic channel such as pp , $\bar{p}p$, π^+p , π^-p , K^+p , or K^-p , and write the amplitude at high energies in the form of the impact distance representation

$$M_j(s, \vec{\Delta}) = \frac{is}{2\pi} \int d\vec{x}_\perp e^{-i\vec{\Delta} \cdot \vec{x}_\perp} D_j(s, \vec{x}_\perp), \quad (3.2)$$

where $\vec{\Delta}$ is the momentum transfer and all spin variables have been omitted. The simplest form that we may use for the opacity $D_j(s, \vec{x})$ is¹

$$D_j(s, \vec{x}_\perp) = 1 - \exp[-S(s) F_j(x_\perp^2)]. \quad (3.3)$$

Here S is a function of s and is characteristic of the Pomeron being exchanged. Since the same Pomeron is responsible for all diffractive processes, S is the same for all channels. On the other hand, all information about the distribution of bare particles resides in F_j , and hence F_j does not depend on s . Because experimentally the differential cross sections for hadronic elastic scattering decrease rapidly for increasing momentum transfers,^{24, 25}

we want $F_j(x_\perp^2)$ to be an analytic function of $\vec{x}$.²⁶

What is the simplest function F_j that we may use? In the drop-let model of Chou and Yang,^{22, 27} sums of Gaussian functions are used. However, as discussed in Section A above, a Gaussian is inconsistent with the impact picture. Thus, in addition to being an analytic function of $\vec{x}$, F_j must be asymptotically proportional to $e^{-\lambda|\vec{x}_\perp|}$ for large $\vec{x}_\perp$. By far the simplest function with both of these properties is

$$F_j(x_\perp^2) = f_j \exp[-\lambda(x_\perp^2 + x_{oj}^2) \frac{1}{2}] \quad (3.4)$$

which involves the three parameters f_j , x_{oj} , and λ . Although (3.4) will be used throughout this section, we emphasize that it is chosen entirely on the basis of simplicity, and certainly needs modification when we want to account for fine details of the differential cross sections.

Let us explain why, of these parameters, two of them depend on the channel j , but λ does not. For extremely high energies, by (3.4) and (3.3), the radius of the black core is given roughly by

$$R_j \sim \lambda^{-1} \ln |S|. \quad (3.5)$$

In order to satisfy (2.3), R_j , to this approximation, must be independent of j . Hence λ does not depend on j .

It remains to choose S as a function of s . From our knowledge of electrodynamics,^{1, 12, 13} we use

$$S = \frac{s^c}{(\ln s)^{c'}} + \frac{u^c}{(\ln u)^{c'}}, \quad (3.6)$$

where u is the third Mandelstam variable. Clearly, crossing symmetry is satisfied when (3.6) is used. Thus the elastic scattering amplitudes $M_j(s, \vec{\Delta})$ is given in terms of five real parameters, of which two, f_j and x_{oj} , vary from channel to channel, while the other three, λ , c , and c' , are the same for all channels.

The differential cross section for elastic scattering in channel j is given by

$$\frac{d\sigma(j)}{dt} = \pi \left| \frac{M_j(s, \vec{\Delta})}{s} \right|^2 \quad (3.7)$$

where $t = -\vec{\Delta}^2$. The total elastic cross section is then of course

$$\sigma_{el}(j) = \int_0^\infty \frac{d\sigma(j)}{dt} d|t|. \quad (3.8)$$

If $M_j(s, \vec{\Delta})$ is taken literally as the elastic scattering amplitude, then the total cross section must be proportional to $\text{Im } M_j(s, 0)$. But, for our model, this imaginary part $\text{Im } M_j(s, 0)$ is an increasing function of s , and thus cannot possibly fit the experimental data from Serpukhov. We are therefore forced to introduce a background term, and we again use the simplest reasonable choice

$$\sigma_{\text{total}}(j) = A_j s^{-1/2} + \frac{4\pi}{s} \text{Im } M_j(s, 0). \quad (3.9)$$

[Remember that to convert from $(\text{GeV})^{-2}$ to mb, a factor of $\frac{4.893}{4\pi}$ is needed.] In this way, we introduce one more real parameter A_j for each channel.

In order to satisfy the optical theorem, this parameter A_j should appear also in the formula for $d\sigma/dt$. However, we shall see below that this extra term $A_j s^{-1/2}$ is comparatively the smaller term on the right-hand side of (3.9). Therefore, in view of the extreme crudeness of the present model, we deem it not justified to consider such corrections. We shall therefore use (3.9) together with (3.7).

This completes the description of our simple model for elastic processes. This model can be generalized in a natural way to the inelastic diffraction scattering of hadrons without spin-parity exchange. Let $M_{jj'}$ be the matrix element of such a process, where j and j' denote respectively the incoming and the outgoing channels. Like (3.3), we write

$$M_{jj'}(s, \vec{\Delta}) = \frac{is}{2\pi} \int d\vec{x}_\perp e^{-i\vec{\Delta} \cdot \vec{x}_\perp} D_{jj'}(s, \vec{x}_\perp); \quad (3.10)$$

unlike (3.3), the opacity $D_{jj'}$ cannot contain the term 1, and hence the simplest choice is

$$D_{jj'}(s, \vec{x}_\perp) = C_{jj'} S F_{jj'}(x_\perp^2) \exp[-S F_{jj'}(x_\perp^2)], \quad (3.11)$$

with

$$F_{jj'}(x_\perp^2) = f_{jj'} \exp[-\lambda(x_\perp^2 + x_{ojj'}^2)^{1/2}]. \quad (3.12)$$

In (3.11) $C_{jj'}$ is an overall constant.

In the remainder of this section we discuss some simple properties of this model.

1) Let h and h' be two hadrons. Are the parameters for the channel hh' related to those for $\bar{h}h'$? The answer is that

$$f(hh') = f(\bar{h}h') \quad (3.13)$$

and

$$x_o(hh') = x_o(\bar{h}h'), \quad (3.14)$$

but $A(hh') \neq A(\bar{h}h')$. (3.15)

2) With (3.4), the integration in (3.2) can be carried out to yield

$$M_j(s, \Delta) = i s \lambda \sum_{n=1}^{\infty} \frac{(-1)^{n-1} (f_j S)^n}{(n-1)! j!} [1 + x_{oj} (\Delta^2 + n^2 \lambda^2)^{1/2}] (\Delta^2 + n^2 \lambda^2)^{-3/2} \exp[-x_{oj} (\Delta^2 + n^2 \lambda^2)^{1/2}]. \quad (3.16)$$

3) Similarly, (3.10) gives

$$M_{jj}(s, \Delta) = i s \lambda C_{jj} \sum_{n=1}^{\infty} \frac{n(-1)^{n-1} (f_{jj} S)^n}{(n-1)! j!} [1 + x_{ojj} (\Delta^2 + n^2 \lambda^2)^{1/2}] (\Delta^2 + n^2 \lambda^2)^{-3/2} \exp[-x_{ojj} (\Delta^2 + n^2 \lambda^2)^{1/2}]. \quad (3.17)$$

C. Values of Parameters

We want to use this phenomenological model to describe the elastic scattering of pp, $\bar{p}p$, π^+p , and K^+p . For this purpose, fifteen parameters are needed: c , c' , λ , $x(pp)$, $x(\pi p)$, $x(Kp)$, $f(pp)$, $f(\pi p)$, $f(Kp)$, $A(pp)$, $A(\bar{p}p)$, $A(\pi^+p)$, $A(\pi^-p)$, $A(K^+p)$ and $A(K^-p)$. Of these, the first nine are needed for very high energies, and the remaining six are introduced to fit, for example, Serpukhov data. The asymptotic energy scale of hadron interactions is controlled by the parameter c .

We present here three fits.

Zeroth Fit. This fit was carried out⁴ before the recent data from ISR.^{2, 3} Here c' was taken to be zero and c is determined from Serpukhov data under the assumption $A(K^+p) = 0$. The resulting pp total cross section is shown in Fig. 6, and, compared with the experimental data, the total rise from the minimum to $s = (53\text{GeV})^2$ is too low by not quite a factor of 2. We shall not discuss this fit any more.²⁸

First Fit. In this and the next fits, the ISR data^{2, 3} are used to determine c . The difference is that in this fit we use $c' = 0$ but in the next fit we use $c' = 1$. This leaves fourteen other parameters to be determined. Specially these fourteen parameters are determined from the following fourteen pieces of data,⁵ where E is the energy of the incident particle in the laboratory system:

- 1) At $E = 20$ GeV, the pp total cross section is 39.0 mb.
- 2) At $E = 60$ GeV, the pp total cross section is 38.4 mb.
- 3) At $E = 1485$ GeV, the pp total cross section is 43.2 mb.^{2, 3}
- 4) At $E = 60$ GeV, the dip in the pp differential cross section occurs at $|t| = 1.8(\text{GeV}/c)^2$.²⁹

5) At $E = 505$ GeV, the average slope, between $|t|$ of 0.14 and $0.24(\text{GeV}/c)^2$, for $\ln d\sigma/dt$ is $11.5(\text{GeV}/c)^{-2}$ for pp elastic scattering.³⁰

- 6) At $E = 20$ GeV, the total K^+p cross section is 17.5 mb.
- 7) At $E = 60$ GeV, the total K^+p cross section is 18.3 mb.
- 8) At $E = 60$ GeV, the average slope, between $|t|$ of 0.1 and $0.28(\text{GeV}/c)^2$, for $\ln d\sigma/dt$ is $7.4(\text{GeV}/c)^{-2}$ for K^+p elastic scattering.
- 9) At $E = 47.5$ GeV, the pp total cross section is 44.1 mb.
- 10) At $E = 52.5$ GeV, the K^-p total cross section is 20.3 mb.
- 11) At $E = 20$ GeV, the π^+p total cross section is 23.5 mb.
- 12) At $E = 60$ GeV, the π^+p total cross section is 23.15 mb.
- 13) At $E = 54.2$ GeV, the average slope, between $|t|$ of 0.05 and $0.53(\text{GeV}/c)^2$, for $\ln d\sigma/dt$ is $9.5(\text{GeV}/c)^{-2}$ for π^+p elastic scattering.
- 14) At $E = 60$ GeV, the π^-p total cross section is 24.2 mb.

Since backgrounds are not properly incorporated in this simple model, we have taken the liberty of altering somewhat the various values to improve the overall fit. However, this alteration has not been carried out systematically. The resulting parameters must, therefore, be considered to be preliminary, and may be improved as more experimental data become available.

Second Fit. Here $c' = 1$ and the above 14 pieces of data are also used except that 4) is replaced by

4') At $E = 60$ GeV, the dip in the pp differential cross section occurs at $|t| = 1.6(\text{GeV}/c)^2$.

The fifteen parameters for both the first fit and the second fit are given in Table I. All the A 's are in units of mb-GeV, while all the other parameters are in units of some power of GeV. The last place of each value is unreliable. The values of f are not the same as those of Ref. 5 because S is defined slightly different. (See table on next page.)

D. Numerical Results

We present below a number of curves from the first fit (solid lines) and the second fit (dotted lines).

Total Cross Sections. In Fig. 7 we show the comparison between the first fit and the experimental data from Serpukhov,²³ NAL,³¹⁻³⁴ and ISR.^{2, 3, 35} Because λ , c , and c' pertain to the Pomeron trajectory and are common to all reactions, the increase in pp total cross section determines the increase in Kp and πp cross sections. During the Aspen study this summer, TeV colliding beams were seriously considered. We, therefore, give in Fig. 8 a comparison of these two fits with each other over a much large energy range, up to $s = 10^7(\text{GeV})^2$. At this extremely high energy, all total cross sections nearly double. It is also seen from Fig. 8 that up to $s = 10^4(\text{GeV})^2$, the differences in total cross sections between these two fits are quite small.

Elastic Scattering. In Figs. 9-13, we show the squares of the matrix elements, which are essentially the differential cross

TABLE I Parameters for the First Fit and the Second Fit

Parameter	First Fit	Second Fit
c	0.082925	0.20193
c'	0.	1.
λ	0.60071	0.59582
$x_0(pp)$	3.8750	3.7508
$f(pp)$	3.1499	9.4966
$A(pp)$	47.372	9.4468
$A(\bar{p}p)$	101.81	63.772
$x_0(Kp)$	1.4295	1.2355
$f(Kp)$	0.64384	1.9597
$A(K^+p)$	8.2543	-11.731
$A(K^-p)$	29.384	9.3388
$x_0(\pi p)$	3.8457	3.6757
$f(\pi p)$	1.7396	5.1270
$A(\pi^+p)$	29.220	5.4329
$A(\pi^-p)$	40.416	16.619

sections, for pp, πp , and Kp elastic scatterings. Comparisons of the pp differential cross section with the experimental data from ISR³⁰ may be found in Ref. 5 and are not repeated here. Note that, in Figs. 9-13, for large $|t|$ the Kp cross sections are always larger. For the present phenomenological model, this is due to the relatively small size of K, as given by $x_0(Kp)$. These curves show many interesting features, but we shall discuss only one or two. In Figs. 9-13, the details of the pp differential cross sections for small $|t|$ cannot be seen clearly. However, a careful examination shows that the curves for small s are comparatively less straight than those for large s. In Fig. 14 we show the average slope in the two ranges $0.05 \leq |t| \leq 0.15$ and $0.15 \leq |t| \leq 0.35$, and the observation of Carrigan³⁶ on two slopes is clearly present. We emphasize that the existence of two slopes is not at all present in the list of fourteen pieces of experimental data, but is rather a

prediction of the model. The model further implies that this difference of slopes reduces for high energies and is absent for the second fit near $s = 10^7$ (GeV)². It is also interesting to note that the πp differential cross section shows some structure for $|t|$ between 2 and 3 when $s = 10^3 - 10^4$ (GeV)². This is presumably the same as the one observed by Orear et. al.²⁵ at much lower energies. To reach experimentally such extremely high energies for πp and Kp scattering, single-pass colliding beams seem necessary.^{37, 38}

Ratio of Real to Imaginary Parts. Since the scattering amplitude is an analytic function of s, the real part is determined by the imaginary part. Therefore we get from the optical theorem that, for example for pp elastic scattering in the forward direction, the ratio of real to imaginary parts

$$\alpha = \frac{-A(\bar{p}p) s^{1/2} + 4 \pi \operatorname{Re} M(s, 0)}{A(pp) s^{1/2} + 4 \pi \operatorname{Im} M(s, 0)} \quad (3.18)$$

No new parameter is needed. In Figs. 15-17 we plot this ratio for pp, $\bar{p}p$, K^+p , and π^+p forward scattering. For the pp case, the result here is in agreement with the ISR data.³⁹ Note that even at $s = 10^7$ (GeV)², α is not asymptotic yet in the usual sense.

Inelastic Diffractive Scattering. Since the present phenomenological model applies only when there is no spin-parity exchange, we restrict ourselves to the following three processes:

$$p + p \rightarrow p + N^* \quad (1470), \quad (3.19)$$

$$K + p \rightarrow K + N^* \quad (1470), \quad (3.20)$$

$$\text{and} \quad \pi + p \rightarrow \pi + N^* \quad (1470). \quad (3.21)$$

For these three processes, how can we determine the values of the parameters? We shall assume that the parameters of Table I can be used without modification. This cannot be strictly correct, but may not be a bad approximation. Furthermore, the normalization constant C is not known, and we take it to be one. In other words, the scale for differential cross section is really arbitrary. In Fig. 18-22 we plot the results for $s = 10^3, 10^4, 10^5, 10^6$, and 10^7 (GeV)². A comparison with the corresponding figures for elastic scattering shows that 1) of Section 2C about the dip position holds well even at relatively low energies. Comparison with the experimental data for the reaction

$$p + p \rightarrow p + N^* \quad (1400)$$

will be given elsewhere. Finally, in Fig. 23, we show the details of the differential cross section of (3.19) for $|t| < 0.3$. The phenomenon of larger slope for smaller $|t|$, as shown in Fig. 14 for elastic scattering, does not occur for this inelastic diffractive process.

We believe that the numerical results presented here are

reasonable in spite of the crude nature of the model. We wish to emphasize again that (3.4) is chosen entirely on the basis of simplicity. Since all the numerical work was carried out on programmable desk calculators with rather inefficient programs written mostly by one of us (TTW), more sophisticated numerical procedures are needed for any modification.

4. DISCUSSIONS

(A) As discussed in Sec. 1, we extract from relativistic quantum field theories the general features in the limit of extremely high energies, and apply them to hadron physics. Even with the help of physical interpretations, the concept of a "general feature" cannot be rigorously, or even sharply, defined. The procedure we follow is accordingly not entirely deductive, and subjective judgment cannot be avoided. It is our feeling that at present the theory has not yet reached a stage where discussions of mathematical rigor is useful.

(B) In describing the present theory in Sec. 2, we have emphasized the physical arguments at the expense of mathematical manipulations. It is thus perhaps worthwhile to add a few words here about one possible calculation that leads to the black disk as shown in Fig. 4. The sum of the one-tower diagrams of Fig. 2 and Fig. 3 gives a strongly absorptive potential of the form $s^a/(\ln s)^2$. In order to get a scattering amplitude from such a strong potential, we have to allow this potential to act an arbitrary number of times. In diagrammatic language, we must include s-channel iterations of the one-tower diagrams. This is however not enough, because gauge invariance must be preserved. For this purpose, it is sufficient to allow all permutations of the appropriate photon lines. The resulting diagrams are referred to as multi-tower diagrams, and a typical example is shown in Fig. 24. If the leading contributions of the one-tower diagrams, the two-tower diagrams, the three-tower diagrams, etc. are summed, we get the expanding black disk with gray fringe, as shown in Fig. 4.

(C) The strongly absorptive potential comes from the one-tower diagram, which in turn is closely linked to two-particle interaction. In Sec. 2 we have followed a procedure of summing leading terms; if we are not allowed to sum only the leading terms, then it is not clear which diagrams should be taken into account. Such a procedure of summing leading terms is valid only when the coupling constant is not too large, and it is thus not surprising that two-particle interactions dominate. On the other hand, for hadron physics it is not inconceivable that three-particle, or even four-particle, interactions are equally or more important. If so, the strongly absorptive potential comes from these multi-particle interactions, but the picture of the black disk still holds. The physical picture may be different if infinite-particle interactions dominate, a possibility that cannot be excluded mathematically but can perhaps be considered unlikely.

(D) The possibility of an increasing total cross section has

been discussed before 1970, and it is interesting to compare the present theory with these earlier theories. The main distinction is that, contrary to c) of Sec. 2C here, the total cross section approaches a finite limit as $s \rightarrow \infty$ in the earlier theories of Frautschi and Margolis,⁴⁰ Gribov,⁴¹ and Dean.⁴² In these theories, the total cross section is increasing at very high energies according to

$$\sigma_{\text{tot}} \sim \sigma_{\infty} - \frac{C}{\ln s}. \quad (4.1)$$

Although this form is not inconsistent with the recent data^{2,3} on total cross sections, the main objection is that in these theories the total elastic cross section behaves, at high energies, as

$$\sigma_{\text{el}} \sim \frac{C'}{\ln s}, \quad (4.2)$$

which is decreasing, while an increasing trend is observed by the Rome-CERN group.³

(E) Barger and Phillips⁴³ discussed in 1970 the Regge dipole which also leads to an infinitely rising total cross sections. However, they considered the Regge dipole as one of many possibilities, and, in contrast with the present theory, did not attempt to give a sequence of arguments that lead to the Regge dipole. Some time after the publication of reference 1, Finkelstein and Zachariasen⁴⁴ gave a self-consistent argument that reproduces some of the predictions of the present theory. The basic physical picture is essentially the same.

(F) One of the triumphs of the Regge-pole theory is factorization. Since in the present theory the Pomeron is a branch cut, factorization cannot possibly hold exactly. From the result of Sec. 3, we can find out numerically how well factorization holds by comparing the elastic cross sections with the diffractive cross sections for the processes (3.19)-(3.21). From Fig. 18 for example, the differential cross section for (3.19) has a dip at $|t| \sim 0.6$. At this point, factorization must be badly violated. It therefore is very important to determine experimentally whether this dip at $|t| \sim 0.6$ exists for the process (3.19). On the other hand, the dips for (3.20) and (3.21) occur at much larger values of $|t|$. Indeed, a comparison of (3.20) and (3.21) with the corresponding elastic processes shows that, up to $|t| = 0.6$, factorization holds to an accuracy of about 10% at $s = 10^3 \text{ GeV}^2$.

(G) One may ask which one of the two sets of parameters given in Table I is better. We believe that there are several reasons to prefer the second fit.

a) The background term, $A(pp)$, for pp scattering is smaller in the second fit. Although by no means ruled out, a value of 47 from the first fit is uncomfortably large.

b) No field theory considered so far gives a c' of zero. For massive quantum electrodynamics, $c' = 2$ as given in Sec. 2.

c) If the amplitudes are analytically continued to unphysical values of t , the resulting cut for $c' = 1$ is quite soft and satisfies the condition of Bronzan and Jones.⁴⁵

(H) Is the black core really black? If we follow the analysis of (E) above for electron-electron scattering, the core is really totally absorbing and hence black. But if we attempt to repeat the analysis for electron-photon scattering, the core is not totally absorbing and hence not black. The reason is that a physical photon is sometimes a bare photon, which goes through the electron without interacting. What is the situation with hadron physics? The answer is not known. It is quite possible, and indeed most interesting, that there may be hadronic states that do not couple to the Pomeron. (These hadronic states are of course not physical particles.)

In the model analysis of Sec. 3, it is a pure assumption that such states are absent. Modification of the model will be necessary if this assumption turns out to be not true. The result of Chou and Yang²² shows that the proton does not have a very strong overlap with such states. As seen in the example of massive quantum electrodynamics, even the total absence of any overlap in one case (electron) does not exclude the presence of such states (photon).

(I) Assuming that future experiments confirm the various predictions here so that the impact picture indeed gives an adequate understanding of two-body diffractive processes, we may ask what we can look forward to. First, we must search for further experimental effects. For example, one interesting result will be discussed by Chou and Yang⁴⁶ tomorrow. Secondly, some further understanding about multi-particle production processes and correlation effects is essential.⁴⁷ Much more work still needs to be done in this direction. Thirdly, extension to processes involving the exchange of quantum numbers, such as charge or hypercharge, is a necessary prerequisite to a complete understanding of high-energy processes.

Let us discuss the last point in some more detail. Independent of the details of how to exchange quantum numbers, such processes must occur in the gray region, not in the black core. In particular, dip structures must be present at roughly the same values of t as for inelastic diffractive processes with the same spin-parity exchange. In order to get further details, we must turn once more to relativistic quantum field theories. In particular, a most important and urgent problem is to study the high-energy behavior of electron-exchange processes in massive quantum electrodynamics. This problem was analyzed nearly a decade ago by Gell-Mann, Goldberger, Low, Marx, and Zachariasen⁴⁸ up to the fourth order, and later by us to the sixth order.⁴⁹ Our analysis is unsatisfactory for a number of reasons. Aside from the undesirable restriction to sixth order, these diagrams under consideration have Reggeization and exponentiation⁵⁰ mixed together and must be disentangled. Furthermore, at least for one of the trajectories, very extensive cancellation is essential, and hence summation of only the leading terms is certainly not sufficient. The trajectories without this extensive cancellation have rather peculiar behaviors,⁴⁸ particularly if exponentiation is taken into account. In connection with these points, the diagrams of Fig. 25 are particularly interesting. These and other diagrams are being studied in collaboration with Barry McCoy.

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Fig. 1. Feynman diagrams for pionization in quantum electrodynamics.

Fig. 2. Lowest-order diagrams that lead to an increasing total cross section.

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Fig. 3. The one-tower diagrams.

Fig. 4. The black disk with gray fringe.

Fig. 5. Schematic diagram for pionization.

Fig. 6. An early rough prediction of rising total cross sections.

Fig. 7. Comparison of the first fit with experimental data from Serpukhov, NAL, and ISR.

Fig. 8. Predictions of rising total cross sections for extremely high energies.

Fig. 9. Theoretical elastic differential cross sections for $s = 10^3 GeV^2$.

Fig. 10. Theoretical elastic differential cross sections for $s = 10^4 \text{ GeV}^2$.

Fig. 11. Theoretical elastic differential cross sections for $s = 10^5 \text{ GeV}^2$.

Fig. 12. Theoretical elastic differential cross sections for $s = 10^6 \text{ GeV}^2$.

Fig. 13. Theoretical elastic differential cross sections for $s = 10^7 \text{ GeV}^2$.

Fig. 14. Slope parameters.

Fig. 15. Ratio of real to imaginary parts of the pp and $\bar{p}p$ forward scattering amplitudes.

Fig. 16. Ratio of the real to imaginary parts of the K^+p and K^-p forward scattering amplitudes.

Fig. 17. Ratio of the real to imaginary parts of the π^+p and π^-p forward scattering amplitudes.

Fig. 18. Theoretical differential cross sections at $s = 10^3 \text{ GeV}^2$ for the processes (3.19)-(3.21).

Fig. 19. Theoretical differential cross sections at $s = 10^4 \text{ GeV}^2$ for the processes (3.19)-(3.21).

Fig. 20. Theoretical differential cross sections at $s = 10^5$ GeV² for the processes (3.19)-(3.21).

Fig. 21. Theoretical differential cross sections at $s = 10^6$ GeV² for the processes (3.19)-(3.21).

Fig. 22. Theoretical differential cross sections at $s = 10^7 \text{ GeV}^2$ for the processes (3.19)-(3.21).

Fig. 23. Theoretical differential cross sections at small momentum transfers for the process $p + p \rightarrow p + N^* (1470)$.

Fig. 24. An example of multi-tower diagrams.

Fig. 25. Some interesting diagrams for fermion exchange.