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IS-1128

Mathematics and Computers (UC-32)
TID-4500, January 1, 1965

UNITED STATES ATOMIC ENERGY COMMISSION

Research and Development Report

GENERATION AND TESTING OF
RANDOM NUMBER SEQUENCES

by

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March, 1965

Ames Laboratory
at
Iowa State University of Science and Technology
F. H. Spedding, Director
Contract W-7405 eng-82

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GENERATION AND TESTING OF RANDOM NUMBER SEQUENCES

Frank S. Carlsen

ABSTRACT

Two methods of generating pseudo-random numbers in a decimal computer are described. Methods of testing the sequences of generated numbers are discussed and the two methods of generation are compared to determine which method is superior for general usage. A computer routine to generate both uniformly and normally distributed numbers is presented, with a table of 9000 three digit numbers uniformly distributed on the unit (0-1) interval.

INTRODUCTION

In many problems it is often desirable to have available either a table of random numbers or a computer routine to generate such numbers. There are many problems where a solution may be found by using Monte Carlo techniques. Some typical problems are those involving permutations and combinations of variables, evaluation of definite integrals, the solution of ordinary and partial differential equations, solution of integral equations, solution of linear algebraic equations, and simulation studies in nuclear physics.

STATEMENT OF THE PROBLEM

The problem was to provide in subroutine form a program for the IBM 7074 computer, which would generate long sequences of numbers uniformly and normally distributed in a random or nearly as random

sequence as possible. The routine should be fast and the numbers generated should have a long period before the sequence repeats. In addition, statistical tests should be performed to insure that the generated sequences of uniformly distributed numbers meet various criteria for randomness.

SOLUTION OF THE PROBLEM

Several methods of generating random numbers by computers are generally believed to be acceptable. Two such methods are the multiplicative congruential and the mixed congruential methods. In this paper no proofs will be given as to the choice of parameters, proofs are given by the references. A table of 9000 three-digit numbers uniformly distributed on the unit interval is included as an Appendix to the paper.

Method 1:

$$x_{i+1} \equiv \lambda x_i \quad (1)$$

where λ is a fixed odd integer called the multiplier. To start the process we follow the suggestion of [5] and choose x_0 as any integer not divisible by 2 or 5. λ is chosen as the form

$$\lambda = 200t \pm r \quad (2)$$

where t is any integer and r is any of the values 3, 11, 13, 19, 21, etc. A value of λ close to $10^{d/2}$ is desirable. Since $d = 10$ for the IBM 7074 computer, a t of 50 and an r of 11 were chosen giving a λ of 10011, hence

$$x_{i+1} = 10011 x_i \quad (3)$$

It can be proven that this procedure will yield 500,000,000 numbers before repeating.

λx_0 forms a product $2d$ -digits long, the high order d -digits are discarded and the d low-order digits are the value x_1 when properly edited to floating point arithmetic format. The low-order digits are used to produce x_{i+1} using Eq. (1).

Method 2:

The mixed congruential method takes the form

$$x_{i+1} \equiv \lambda x_i + c \quad (4)$$

where λ according to [1], is of the form

$$\lambda = 10^a + 1 \quad (5)$$

Reference [1] further states that c should be odd and $a \geq 2$. Hence for this method λ was chosen as 101 and c as 1 yielding

$$x_{i+1} = 101x_i + 1 \quad (6)$$

The above method should yield 10^{10} numbers before repeating. In a decimal machine, such as the IBM 7074 where multiply time is dependent upon the size of the multiplier, Method 2 should be slightly faster, as well as having a somewhat longer period.

Testing Considerations:

The values of x should be uniformly distributed over the unit interval regardless of the number of values computed. Successive values of x should also be independent.

Many different tests have been used to determine the statistical behavior of the sequences. The testing as reported by references [1], [2], [3], [4], [5], [8], and [9] indicates that the behavior of both methods is acceptable.

It was decided to use the following statistical tests; frequency test, serial correlation test, runs up and down, runs above and below the mean, auto-correlation, digit frequency test, and the mean square difference test. These tests were all performed using various lags. Lag 1 is defined as picking every consecutive generated number as the random quantity, lag 2 as picking every other generated number and so on.

Each of the above tests was applied to 100 blocks of 1000 numbers each, for lags 1 to 10 for each method. It has been shown by [4] that the choice of x_0 for Method 2 makes little or no difference, so most of the testing was done with $x_0 = 7$ which also meets the criteria that x_0 should not be divisible by 2 or 5 for Method 1 as suggested by [5].

Frequency Test:

For the frequency test the 1000 numbers were "binned" into 10 equal subintervals with the expectation that 100 numbers should fall into each bin. The following Chi-squared test was then applied to determine the reasonableness of the results.

Let f_i ($i=1, 2, \dots, 10$) be the number of members x_j satisfying $(i-1)/10 \leq x_j < i/10$, then χ_1^2 was computed

$$\chi_1^2 = \frac{1}{100} \sum_{i=1}^{10} (f_i - 100)^2 \quad (7)$$

Reference [6] states that χ_1^2 is distributed asymptotically as χ^2 with 9 degrees-of-freedom. χ_1^2 was computed for the 100 blocks. Then let F_i be the number of the resulting 100 values of χ_1^2 which were between the $(i-1)$ th and the i th deciles for the χ^2 -distribution with 9 degrees-of-freedom. The following quantity was then computed.

$$\chi_F^2 = \frac{1}{10} \sum_{i=1}^{10} (F_i - 10)^2 \quad (8)$$

Serial Test:

For the serial test the interval $[0, 1]$ was divided into 10 parts, then the numbers were "binned" into a 10×10 matrix. A 1 was added in row i column j , when x_n was in the i th subinterval and followed by x_{n+1} in the j th subinterval. The expected result would be 10 counts in each position of the matrix. The following χ^2 test was then applied

$$\chi_2^2 = \frac{1}{10} \sum_{i,j=1}^{10} (f_{ij} - 10)^2 \quad (9)$$

It has been shown by [8] and [9] that $\chi_2^2 - \chi_1^2$ should be distributed asymptotically as χ^2 with 90 degrees-of-freedom. $\chi_1^2 - \chi_1^2$ was computed for each of the 100 blocks. Then let s_i be the number of the resulting 100 values of $\chi_2^2 - \chi_1^2$ which were between the $(i-1)$ th and the i th deciles for χ^2 with 90 degrees-of-freedom. The following was then computed:

$$\chi_s^2 = \frac{1}{10} \sum_{i=1}^{10} (s_i - 10)^2 \quad (10)$$

Several other interesting statistics were also computed using the 10×10 matrix data. Given the row sums it is equally likely that $1/10$ of the row

sum count would occur in each element of the row. Similar results would be expected for the column sums and the column elements. To test these assumptions a χ^2 test was performed on the composite 10 x 10 matrix obtained by summing the matrix over all 100 blocks.

Auto-Correlation Test:

For the auto-correlation test the auto-correlation coefficient Ch was computed for $h = 0, 1, 2, 3, \dots, 10$ for each of the 100 blocks

$$Ch = \frac{1}{1000} \sum_{i=1}^{1000} x_i x_{i+h} \quad (11)$$

Statistical theory shows that truly random samples give values of Ch for $h = 0$ of .3333 and for $h > 0$ $Ch = .25$.

Runs Test:

To find the runs up and down, a sequence of digits is formed whose n th term is defined as 0 if $x_{n+1} < x_n$ and as 1 if $x_{n+1} > x_n$. A subsequence of k zeros, bracketed by ones at each end forms a run of length k , similarly for runs of ones. The test involves counting the number of occurrences of runs of different lengths and comparing to expected results. The expected results, based on a truly random sample, are: $(2N-1)/3$ runs total or 666.33 for the case at hand. Other expected runs are: 416.75 runs of length 1, 183.1 runs of length 2, 52.65 runs of length 3, 11.4 runs of length 4, and 2.43 runs of length 5 or greater. A χ^2 test was performed on each set of runs up and down for each block of 1000 numbers to check for reasonableness.

To find the runs above and below the mean, a sequence of digits is

formed whose n th term is defined as 0 if $x_n < 1/2$ or 1 if $x_n > 1/2$.

Again runs are counted with the results being checked by a χ^2 test. In this case the expected results are: 500.5 total runs of any length, 250.0 runs of length 1, 125.0 runs of length 2, 62.5 runs of length 3, 31.25 runs of length 4, 15.625 runs of length 5, 7.8 runs of length 6, and 8.325 runs of length 7 or greater.

For both of the run tests above, an average over the 100 blocks is computed for each of the length runs involved in the respective tests.

Mean Square Difference Test:

The mean square difference test involves computing

$$M = \frac{1}{1000} \sum_{i=1}^{1000} (x_i - x_{i+d})^2 \quad (12)$$

where $d = 1, 2, 3, 4, 5, 10, 100, 101$. M has an expected value of .16667.

For this test the M 's obtained from each block of 1000 numbers were averaged over the 100 blocks and this average was tabulated for each value of d .

Digit Frequency Test:

Although the purpose of the computer program under development was to produce random numbers, some information about the distribution of the individual digits making up the numbers is also desirable. For the digit frequency test a count is kept of the occurrence of each digit (0, 1, 2, ..., 9) for the 1000 words. Since each random number in its floating point representation contains only 8 possible digits in each number, there are 8000 possible random digits, with the expectation that each digit should occur

100 times in each digit position. A χ^2 test was used to check the reasonableness of this assumption.

Another possible check is to group the digits together and check for frequency of occurrences of each digit; positions 1, 2, and 3, 300 occurrences; positions 1, 2, 3, and 4, 400 occurrences; ...; 1, 2, 3, 4, 5, 6, 7, and 8, 800 occurrences of each digit. A χ^2 test was then applied to check the results for each block of 1000 numbers. In addition, these two χ^2 for the 100 blocks should be distributed asymptotically as χ^2 with 9 degrees-of-freedom. Following the procedure outlined in the Frequency Test section two additional sets of χ^2 were computed, one for each of the eight individual digit position χ^2 , and one for each the seven grouped digit χ^2 .

In addition to the above tests the mean and variance of all numbers comprising a block were computed using:

$$\bar{x} = \frac{1}{1000} \sum_{i=1}^{1000} x_i \quad (13)$$

$$v = \frac{1}{1000} \left(\sum_{i=1}^{1000} x_i^2 - \frac{\left(\sum_{i=1}^{1000} x_i \right)^2}{1000} \right) \quad (14)$$

The expected values of these quantities are 0.50000 and 0.08333 respectively. The following was then computed using the entire 100 blocks

$$XM = \frac{1}{100} \sum_{j=1}^{100} \bar{x}_j \quad (15)$$

$$VM = \frac{1}{100} \sum_{j=1}^{100} v_j \quad (16)$$

In the tables that follow these definitions apply. Let k be the number of times χ^2_1 for the frequency test exceeds 21.666, the 99% level for χ^2 with 9 degrees-of-freedom. In a similar manner let l and m be the number of times χ^2 for runs up and down and runs above and below the mean exceed 21.666. n is the number of times χ^2 for the digit frequency tests exceeded the value for the 99% level.

Results for Method 1:

TABLE 1A

XM	VM	k	χ^2_F	χ^2_S	l	m	Lag
.49809	.08384	1	6.4	4.0	0	0	1
.49889	.08376	1	6.4	5.2	0	0	2
.49919	.08372	1	9.2	6.8	0	0	3
.49876	.08366	3	8.0	16.8	0	1	4
.49923	.08314	2	5.6	9.2	0	1	5
.49837	.08362	1	7.0	14.2	0	0	6
.49921	.08353	2	7.2	10.2	0	0	7
.49871	.08383	2	12.4	11.2	1	0	8
.49848	.08372	3	21.6	6.6	0	0	9
.49868	.08288	1	6.8	9.4	0	0	10

TABLE 1B

Auto-Correlation based on 100 sets of 1000

h =	0	1	2	3	4	5	6	7	8	9	10	Lag
Observed	.33243	.24853	.24843	.24847	.24860	.24846	.24823	.24860	.24880	.24855	.24865	1
	.33268	.24901	.24871	.24896	.24929	.24860	.24910	.24891	.24904	.24859	.24895	2
	.33266	.24864	.24882	.24918	.24878	.24946	.24865	.24887	.24913	.24891	.24887	3
	.33260	.24906	.24904	.24857	.24912	.24885	.24893	.24900	.24923	.24885	.24860	4
	.33268	.24891	.24969	.24994	.24946	.24933	.24933	.24895	.24924	.24956	.24970	5
	.33159	.24746	.24741	.24805	.24793	.24791	.24771	.24813	.24805	.24761	.24755	6
	.33269	.24911	.24923	.24898	.24913	.24917	.24892	.24905	.24917	.24914	.24937	7
	.33248	.24877	.24893	.24892	.24900	.24879	.24829	.24849	.24854	.24869	.24890	8
	.33214	.24914	.24842	.24827	.24803	.24881	.24840	.24824	.24833	.24802	.24830	9
	.33189	.24914	.24867	.24929	.24906	.24908	.24884	.24882	.24864	.24918	.24923	10
Expected	.33333					.25000						All

TABLE 1C

Mean Square Difference based on 100 sets of 1000

$\overline{(x_i - x_{i+d})^2}$									
d =	1	2	3	4	5	10	100	101	LAG
OBSERVED	.16780	.16800	.16793	.16766	.16745	.16756	.16722	.16734	1
	.16733	.16797	.16745	.16679	.16817	.16747	.16758	.16729	2
	.16803	.16767	.16696	.16774	.16639	.16756	.16748	.16871	3
	.16708	.16711	.16805	.16695	.16749	.16800	.16722	.16749	4
	.16753	.16597	.16548	.16642	.16668	.16592	.16666	.16684	5
	.16825	.16836	.16708	.16731	.16735	.16807	.16760	.16899	6
	.16715	.16691	.16741	.16712	.16703	.16665	.16764	.16724	7
	.16742	.16709	.16712	.16695	.16738	.16714	.16828	.16819	8
	.16598	.16742	.16772	.16820	.16664	.16767	.16731	.16806	9
	.16552	.16644	.16520	.16565	.16562	.16534	.16673	.16639	10
EXPECTED	.16667								ALL

TABLE 1D

Runs above and below the mean based on 100 sets of 1000

Length of run	1	2	3	4	5	6	≥ 7	ANY	LAG
Observed ave. no. of runs	250.98	125.11	62.37	31.36	14.95	7.80	7.87	500.44	1
	248.66	123.86	64.07	30.74	15.09	7.79	8.14	498.35	2
	249.08	126.44	62.44	31.09	15.10	7.60	7.97	499.92	3
	249.38	123.48	62.71	31.51	16.04	7.67	7.68	498.47	4
	252.07	124.98	61.40	30.90	15.76	7.60	8.02	500.73	5
	251.10	127.42	62.59	30.53	14.94	7.71	7.78	502.07	6
	250.11	125.65	61.82	31.28	16.05	7.46	7.73	500.10	7
	250.63	125.51	62.71	30.35	15.32	8.02	7.80	500.34	8
	245.30	124.04	62.30	32.23	15.15	7.89	8.23	495.14	9
	248.22	125.73	62.56	30.74	15.12	7.73	8.29	498.39	10
Expected ave. no. of runs	250.00	125.00	62.50	31.25	15.625	7.80	8.325	500.50	ALL

TABLE 1E

Runs up and down based on 100 sets of 1000

Length of run	1	2	3	4	≥5	ANY	LAG
Observed ave. no. of runs	415.33	184.64	52.16	11.57	2.14	665.84	1
	415.29	183.22	53.12	11.44	2.19	665.26	2
	415.78	183.04	52.37	11.48	2.61	665.28	3
	417.16	182.56	53.49	10.97	2.26	666.44	4
	420.47	182.07	52.72	10.87	2.34	668.47	5
	415.68	183.58	51.82	12.32	2.07	665.47	6
	416.87	182.05	53.09	11.11	2.63	665.75	7
	414.40	182.70	53.75	11.29	2.37	664.51	8
	412.84	182.99	53.04	12.00	2.36	663.23	9
	415.06	184.14	52.33	11.84	2.09	665.46	10
Expected ave. no. of runs	416.75	183.10	52.65	11.40	2.43	666.33	ALL

For the digit frequency tests it is convenient to tabulate the number of times χ^2 exceeds the χ^2 value of 21.666 for the 99% level. It is expected that χ^2 should exceed this value once in every 100 samples.

TABLE 1F

Digit frequency test for 100 sets of 1000

Digit position	1	2	3	4	5	6	7	8	LAG
1	2	0	0	2	4	1	0	1	
1	0	0	2	1	1	0	4	2	
1	0	1	2	0	4	0	0	3	
3	1	0	1	1	1	0	0	4	
2	0	2	1	1	4	1	0	5	
1	2	1	1	0	1	0	0	6	
2	1	1	1	2	0	2	2	7	
2	2	0	2	0	1	0	4	8	
3	1	1	1	2	3	2	0	9	
1	0	0	2	0	0	0	0	10	

TABLE 1G
n for Grouped Digit Frequency Test based on 100 sets of 1000

Digit position	1-2	1-3	1-4	1-5	1-6	1-7	1-8	LAG
	0	0	1	6	17	22	14	1
	0	1	1	1	1	1	1	2
	0	1	0	0	0	0	1	3
	0	4	2	3	3	3	1	4
	1	0	1	1	2	1	2	5
	2	0	1	0	0	0	0	6
	1	2	2	1	0	1	2	7
	3	2	2	1	0	1	0	8
	2	1	1	0	0	2	1	9
	2	1	1	0	0	1	0	10

TABLE 1H
 χ^2 for individual digit positions based on 100 sets of 1000

Digit position	1	2	3	4	5	6	7	8	LAG
	6.4	9.0	13.4	14.2	16.0	10.2	3.8	9.6	1
	6.4	9.2	16.8	9.6	16.4	13.6	8.0	16.8	2
	9.2	12.8	12.0	6.0	9.6	6.0	8.2	24.8	3
	8.0	10.2	15.2	6.4	7.4	5.0	9.8	26.4	4
	5.6	7.4	11.6	12.2	3.6	8.8	23.0	100.0	5
	7.0	6.0	22.6	11.4	9.8	9.8	6.8	45.6	6
	7.2	5.4	18.4	6.0	19.2	7.2	18.2	16.0	7
	12.4	13.2	6.6	19.0	13.2	8.6	8.2	32.8	8
	21.6	10.2	7.0	7.8	7.4	8.8	6.8	16.0	9
	6.8	7.4	13.0	3.0	6.4	6.2	25.6	340.0	10

TABLE 11
 χ^2 for grouped digit positions based on 100 sets of 1000

Digit positions	1-2	1-3	1-4	1-5	1-6	1-7	1-8	LAG
	15.0	9.8	7.2	83.0	184.6	149.4	159.6	1
	22.8	13.0	5.2	11.2	5.0	5.8	5.8	2
	5.2	9.0	11.4	7.4	6.0	3.2	7.6	3
	10.6	7.2	2.8	9.0	10.8	11.2	7.0	4
	5.4	5.8	7.4	4.4	18.2	17.4	5.8	5
	7.0	12.6	12.8	6.2	6.6	13.6	6.4	6
	9.8	7.4	13.8	9.0	12.4	10.8	12.6	7
	8.2	4.8	6.4	4.4	6.8	7.8	1.2	8
	13.2	12.0	12.6	14.2	7.0	11.8	7.6	9
	16.6	5.4	9.0	10.6	4.8	11.8	13.4	10

TABLE 11
 Frequency Distribution Among 100,000 numbers paired for Lag 1

j \ i	0	1	2	3	4	5	6	7	8	9	SUM	χ^2 ROWS
0	1107	933	1001	1024	1011	1056	1036	1008	1108	1028	10312	25.124
1	995	1020	1016	986	989	949	1033	986	1002	965	9941	5.799
2	1080	0043	991	1008	966	989	982	974	965	1015	10011	12.239
3	1022	990	1002	996	1000	985	971	967	996	1017	9946	2.787
4	977	980	1016	993	950	1017	1003	991	988	1012	9927	3.897
5	1051	1008	976	985	955	1020	972	995	1025	1014	10001	7.520
6	1049	1034	1057	966	979	1018	965	930	980	986	9964	15.374
7	1003	948	993	992	997	1018	974	964	976	985	9850	3.738
8	985	1006	981	1037	1017	1003	1027	1016	997	973	10042	3.820
9	1044	979	978	959	1063	946	1000	1019	1007	1011	10006	11.947
SUM	10313	9941	10011	9946	9927	10001	9963	9850	10042	10006	100000	
χ^2 col.	15.769	11.358	5.299	5.132	10.344	10.344	6.861	6.796	14.618	4.128		

For the remaining lags only the row and column χ^2 will be presented for the composite matrix.

i=j=	0	1	2	3	4	5	6	7	8	9	LAC
χ^2	7.328	10.644	10.779	6.152	8.314	5.298	12.663	7.491	5.527	6.509	2
rows	4.860	4.463	4.987	20.301	5.469	9.420	4.836	4.127	4.996	11.949	3
	9.840	4.134	4.270	5.418	8.201	4.536	8.100	14.173	9.624	11.208	4
	8.912	9.425	9.946	6.812	8.544	12.398	6.441	15.900	19.892	10.826	5
	16.063	4.828	5.195	4.459	15.680	11.813	12.292	11.043	16.157	11.388	6
	5.289	11.882	5.998	5.615	16.910	7.321	9.369	6.313	7.201	9.336	7
	3.197	20.293	10.604	16.418	8.491	15.211	5.982	11.455	8.076	10.442	8
	19.307	7.962	6.154	8.611	18.542	16.380	2.534	5.273	12.624	9.294	9
	13.166	11.596	8.852	3.487	20.339	13.306	4.419	4.576	4.331	8.839	10
χ^2	4.017	9.939	9.464	9.478	7.210	16.40	3.801	8.907	2.894	8.613	2
columns	7.662	7.587	9.035	1.801	6.513	4.921	6.006	8.992	3.478	19.201	3
	7.958	10.901	4.914	10.349	13.292	2.524	10.388	8.265	5.071	5.797	4
	12.483	11.379	9.493	13.704	6.046	7.119	8.906	9.060	12.821	17.899	5
	5.311	16.595	4.215	6.206	9.664	17.354	11.005	18.413	8.775	11.774	6
	9.166	11.458	7.057	10.803	14.090	6.223	4.122	6.420	6.452	9.374	7
	14.159	9.478	10.453	12.516	9.264	25.633	6.516	8.488	10.778	2.847	8
	14.295	12.802	10.920	11.916	9.585	11.329	4.983	15.206	7.088	8.984	9
	6.323	17.673	7.917	4.107	9.012	6.553	9.106	8.477	9.061	15.073	10

Results for Method 2:

TABLE 2A

XM	VM	k	χ_F^2	χ_S^2	l	m	LAC
.49526	.08469	0	13.0	18.2	1	1	1
.49592	.08487	1	8.4	8.4	0	1	2
.49301	.08494	1	4.6	15.2	0	0	3
.49664	.08479	0	7.4	18.6	0	0	4
.49610	.08459	4	9.2	4.6	2	1	5
.49516	.08483	1	7.6	6.8	0	0	6
.49523	.08489	2	15.4	5.6	0	0	7
.49592	.08482	1	12.4	6.6	0	0	8
.49767	.08491	3	20.0	12.2	0	1	9
.49767	.08491	5	13.2	11.4	0	3	10

TABLE 2B
Auto-Correlation based on 100 sets of 1000

	h=0	1	2	3	4	5	6	7	8	9	10	LAG
Observed	.33035	.24655	.24568	.24585	.24585	.24565	.24597	.24556	.24552	.24550	.24579	1
	.33101	.24587	.24577	.24641	.24600	.24648	.24569	.24626	.24608	.24572	.24633	2
	.32820	.24342	.24344	.24298	.24293	.24326	.24328	.24374	.24354	.24339	.24282	3
	.33135	.24618	.24598	.24642	.24659	.24709	.24646	.24638	.24672	.24662	.24720	4
	.33090	.24626	.24667	.24600	.24594	.24606	.24546	.24660	.24801	.24665	.24575	5
	.33022	.24540	.24484	.24504	.24560	.24510	.24532	.24561	.24492	.24573	.24485	6
	.33034	.24559	.24567	.24493	.24539	.24546	.24505	.24508	.24540	.24558	.24510	7
	.33110	.24611	.24658	.24608	.24630	.24663	.24586	.24612	.24697	.24595	.24569	8
	.33022	.24517	.24535	.24533	.24542	.24562	.24513	.24541	.24538	.24503	.24533	9
	.33303	.24825	.24643	.24813	.25059	.24813	.24745	.24789	.24810	.24844	.24792	10
Expected	.33333				.25000							ALL

TABLE 2C
Mean Square Difference based on 100 sets of 1000

$$(x_i - x_{i+d})^2$$

d =	1	2	3	4	5	10	100	101	LAG
Observed	.16762	.16935	.16902	.16902	.16943	.16913	.16979	.16913	1
	.17028	.17048	.16919	.17001	.16907	.16937	.16986	.17027	2
	.16957	.16952	.17044	.17054	.16988	.17076	.17135	.17049	3
	.17035	.17074	.16987	.16954	.16854	.16831	.16859	.16895	4
	.16928	.16845	.16981	.16992	.16968	.17031	.17028	.16954	5
	.16964	.17077	.17036	.16924	.17025	.17075	.16927	.16958	6
	.16949	.16933	.17082	.16991	.16975	.17047	.17024	.17033	7
	.16997	.16904	.17005	.16960	.16894	.17079	.17053	.17036	8
	.17009	.16975	.16979	.16961	.16920	.16981	.17024	.17088	9
	.16956	.17321	.16980	.16488	.16981	.17023	.17018	.16956	10
Expected				.16667					ALL

TABLE 2D
Runs above and below the Mean based on 100 sets of 1000

Length of Run	1	2	3	4	5	6	≥ 7	ANY	LAG
Observed Average	251.76	123.65	62.37	31.69	16.31	7.35	7.47	500.60	1
Number of Runs	248.63	126.20	61.89	32.27	14.90	7.34	7.99	499.22	2
	248.71	124.17	63.30	31.46	15.29	7.70	7.92	498.55	3
	249.01	125.95	63.43	31.40	14.63	7.76	7.72	499.90	4
	252.20	123.93	61.72	31.45	15.00	8.36	7.82	500.48	5
	249.52	126.30	62.98	30.77	15.06	7.79	7.86	500.08	6
	249.37	122.37	63.34	31.27	15.77	7.56	8.09	497.77	7
	252.95	123.03	62.87	31.55	14.85	7.56	8.13	500.94	8
	251.42	124.08	63.52	30.46	14.96	7.81	8.15	500.40	9
	240.14	127.93	71.21	30.54	13.37	8.70	5.99	497.88	10
Expected Average	250.00	125.00	62.50	31.25	15.625	7.80	8.325	500.50	ALL

TABLE 2E
Runs up and down based on 100 sets of 1000

Length of Run	1	2	3	4	≥ 5	ANY	LAG
Observed Average	410.64	182.93	52.78	12.23	2.76	661.34	1
Number of Runs	414.16	183.32	53.40	11.31	2.34	664.53	2
	416.19	182.52	53.63	11.24	2.17	665.75	3
	414.00	182.35	53.06	12.07	2.40	663.88	4
	417.71	181.71	53.78	10.61	2.58	666.39	5
	413.38	183.78	53.23	11.23	2.59	664.11	6
	416.98	182.60	52.64	11.56	2.29	666.07	7
	418.85	182.57	53.38	10.99	2.01	667.80	8
	416.23	182.86	53.18	11.34	2.24	665.85	9
	405.38	176.71	59.88	12.84	1.67	656.48	10
Expected Average	416.75	183.10	52.65	11.40	2.43	666.33	ALL

TABLE 2F

n for Digit Frequency Test based on 100 sets of 1000

Digit Position	1	2	3	4	5	6	7	8	LAG
	0	2	1	3	1	2	3	0	1
	1	1	0	2	0	0	0	0	2
	1	1	1	0	0	3	0	0	3
	0	2	0	0	1	0	0	0	4
	4	6	0	1	0	2	0	0	5
	1	6	1	1	2	5	0	0	6
	2	4	0	0	2	1	1	0	7
	1	1	2	0	1	0	0	60	8
	3	3	0	3	1	1	0	0	9
	5	5	2	0	2	0	0	0	10

TABLE 2G

n for Grouped Digit Frequency Test based on 100 sets of 1000

Digit Positions	1-2	1-3	1-4	1-5	1-6	1-7	1-8	LAG
	3	18	22	17	12	10	9	1
	3	0	1	0	0	1	0	2
	4	0	0	1	1	1	1	3
	3	3	1	0	1	0	1	4
	5	6	5	7	7	5	2	5
	2	1	0	1	3	1	1	6
	4	3	0	1	4	3	1	7
	0	1	0	0	0	3	4	8
	7	4	3	3	2	3	1	9
	4	4	3	3	1	0	0	10

TABLE 2H

 χ^2 for individual digit positions based on 100 sets of 1000

Digit Position	1	2	3	4	5	6	7	8	LAG
	13.0	10.0	7.0	7.6	10.8	7.0	33.8	120.0	1
	8.4	12.0	10.4	14.2	4.2	13.6	16.8	900.0	2
	4.6	10.8	9.8	3.2	8.2	8.0	13.4	200.0	3
	7.4	3.0	4.4	8.2	12.8	7.0	157.6	180.0	4
	9.2	26.8	5.6	4.2	9.4	43.2	45.0	900.0	5
	7.6	18.0	4.4	12.6	8.8	11.0	12.8	900.0	6
	15.4	21.8	10.4	14.0	5.2	9.0	11.4	440.0	7
	12.4	7.4	11.2	6.4	8.8	11.4	80.8	420.0	8
	20.0	6.4	10.0	11.8	6.8	6.4	16.2	180.0	9
	13.2	17.2	3.6	8.2	23.6	35.0	140.0	900.0	10

TABLE 2I

 χ^2 for grouped digit positions based on 100 sets of 1000

Digit Positions	1-2	1-3	1-4	1-5	1-6	1-7	1-8	LAG
	27.2	154.0	187.6	101.8	93.8	75.0	61.8	1
	16.4	8.0	6.4	11.6	16.6	8.0	8.6	2
	12.4	9.6	12.4	13.4	5.8	6.6	7.0	3
	8.6	14.6	4.6	14.4	5.0	8.2	13.0	4
	38.0	21.6	18.2	23.2	17.4	10.8	13.0	5
	23.6	8.8	5.6	17.2	5.2	8.0	14.6	6
	18.4	12.2	12.6	3.0	8.8	7.2	5.4	7
	7.4	12.2	5.2	8.2	9.8	8.4	23.8	8
	37.2	23.8	31.0	31.0	18.8	10.6	15.0	9
	39.2	33.2	27.4	20.6	10.6	8.8	4.6	10

TABLE 2J
Frequency Distribution Among 100,000 numbers paired for Lag 1

i \ j	0	1	2	3	4	5	6	7	8	9	SUM	χ^2 rows
0	1166	1005	1114	1068	1071	1089	1093	1069	1055	1087	10817	14.254
1	1081	929	962	996	925	1044	950	981	986	958	9812	22.329
2	1062	1028	1014	945	966	927	1041	1014	971	997	9965	16.998
3	1075	991	993	995	952	992	986	989	943	938	9854	13.625
4	1068	1022	990	953	986	1040	960	948	995	1010	9972	13.602
5	1079	983	993	1017	1052	1018	979	985	969	970	10045	12.166
6	1087	989	967	966	1014	1005	971	944	984	974	9901	14.266
7	1091	925	944	950	1014	926	969	954	1014	979	9816	22.863
8	1043	973	965	975	979	1023	981	977	998	976	9888	5.823
9	1066	967	975	989	1013	981	971	955	972	1041	9930	11.644
SUM	10818	9812	9965	9854	9972	10045	9901	9816	9887	9930	100000	
χ^2 col.	8.868	10.939	17.893	12.643	18.000	23.258	17.262	13.028	8.355	17.190		

For the remaining lags only the row and column χ^2 will be presented for the composite matrix.

TABLE 2K

i=j=	0	1	2	3	4	5	6	7	8	9	LAC
χ^2 rows	13.913	11.966	13.022	2.418	30.794	14.866	13.970	12.401	19.894	14.054	2
	23.852	33.261	14.458	20.936	16.341	10.618	23.796	18.472	11.099	20.535	3
	18.682	15.087	10.417	28.537	14.461	26.768	20.723	21.481	14.738	18.320	4
	23.991	13.636	21.886	16.145	16.928	13.606	12.557	23.462	20.453	13.500	5
	29.287	13.772	31.331	6.048	28.329	7.271	9.472	17.800	31.110	16.995	6
	28.637	17.402	12.652	22.786	14.408	23.180	10.864	16.211	22.658	6.870	7
	18.077	29.114	15.821	13.945	15.198	8.233	58.478	10.883	12.639	13.008	8
	31.968	22.100	14.320	25.982	23.032	9.027	15.657	21.006	15.676	6.688	9
	6.084	28.400	11.329	43.546	24.211	20.061	13.536	9.774	12.041	9.715	10

TABLE 2K

$i=j=$	0	1	2	3	4	5	6	7	8	9	LAG
χ^2	20.619	8.836	16.136	9.035	16.873	20.240	14.990	13.524	18.492	8.072	2
columns	18.300	22.904	10.695	16.494	14.217	25.622	17.350	21.704	37.493	9.315	3
	13.329	9.288	43.073	10.901	17.215	10.989	24.331	11.773	21.095	27.180	4
	16.043	9.431	25.511	14.742	8.851	38.690	19.157	13.884	16.200	13.929	5
	33.967	15.813	20.568	10.116	15.338	25.792	33.926	13.439	7.391	14.353	6
	24.965	10.870	13.307	13.500	20.587	11.408	7.017	29.815	19.131	25.140	7
	30.717	12.409	20.742	16.815	30.656	12.995	19.812	15.903	23.742	10.999	8
	32.324	9.798	20.415	22.185	19.679	11.594	9.810	15.340	22.157	22.030	9
	21.378	20.221	14.948	21.557	17.653	16.668	21.799	14.464	13.891	14.203	10

INTERPRETATION OF RESULTS

It is convenient to analyse the results according to the manner in which the tables are arranged, comparing the two methods for each test.

For Tables 1A and 2A we would like our generators to yield values of χ_F^2 and χ_S^2 that are not large and not more than about 1% in excess of 21.666. In addition, we would desire that χ_1^2 not exceed 21.666 more than about 1% of the time or more than about once in every 100 samples (tabulated as k). Furthermore the quantities l and m should not exceed one.

Inspection of Table 1A shows that χ_F^2 and χ_S^2 are well behaved. l and m have reasonable values for all lags. k has good values for lags 1, 2, 3, 6, and 10 whereas for lags 4, 5, 7, 8 and 9, k is somewhat large. The values of VM are acceptable, however, XM for all lags is somewhat low. Although not tabulated, an examination of $\bar{x}$ for lag 1 revealed that 59 values of $\bar{x}$ are less than .5 while 41 values are greater than .5. It should be noted that the tests involving k, χ_F^2 , χ_S^2 , l and m are really tests of the most significant digit of each number, while the mean and variance use all eight digits. In this event we can conclude that probably only the first 2 or 3 significant digits of each number are really random. In view of this, XM and VM are reasonable and, based on these tests, we conclude that Method 1 is quite acceptable.

Referring to Table 2A, we see again that χ_F^2 and χ_S^2 are reasonable for all lags. The quantity l is slightly large for lag 5 while m is too large for lag 10. The quantity k is acceptable for lags 1, 2, 3, 4, 6, and 8, somewhat large for lags 7 and 9, and possibly unacceptable for lags 5

and 10. We also notice that for all lags Method 2 yields values of VM that are consistently larger than the expected value of .08333. As with Method 1 we notice that XM is lower than the expected value of .5000, only in this case to a more pronounced degree than with Method 1. Based on these tests we conclude that while Method 2 is probably acceptable, Method 1 is preferred.

Considering Tables 1B and 2B, the tabulation of the Auto-Correlation tests for various values of h, both methods approach the expected values; however, again Method 1 yields more acceptable results for all lags.

The results of the Mean Square Difference Tests as tabulated in Tables 1C and 2C again show that Method 1 is superior to Method 2. The results for Method 1 for all lags agree quite well to the expected value of .16667, while the results for Method 2 are all higher than the expected value.

Considering the results of the Runs Test both for runs up and down and runs above and below the mean, both methods appear to be acceptable. The noteworthy exception being Method 2 for lag 10 where, for runs above and below the mean, the observed average number of runs of lengths 1 and 3 differ considerably from the expected values. Again for Method 2, for runs up and down of lengths 1 and 2, the observed average number of runs differs significantly from the expected results. This fact is verified for runs above and below the mean in Table 2A, where $k = 3$ for lag 10, indicating excessive failures of the χ^2 test for reasonableness for the number of runs above and below the mean.

A comparison of the two methods, as given by the digit frequency tests, is one of the more graphic illustrations of the superiority of Method 1. Tables 1H and 1I illustrate that the numbers generated by Method 1 have good individual digit distribution through the sixth significant digit position and, with the exception of lag 1, this is also true when the digits are grouped. Tables 2H and 2I illustrate that this is definitely not true for Method 2 where lag 1 fails for all digit groupings. In this instance only lags 2, 3, 4, 7, and 8 pass this test. For the individual digit positions for Method 2 all lags are acceptable for the most part through the fifth significant digit position.

Using the information created by the Serial Test where numbers are binned into a 10 x 10 matrix as previously described, we again conclude that Method 1 is to be preferred over Method 2. Inspection of Tables 1J, 1K, 2J, and 2K reveals that the matrix row and column elements resulting from using Method 2 fail the χ^2 reasonableness test with great regularity, whereas the results obtained from Method 1 are most satisfactory.

FINDINGS

A particular multiplier λ may be considered as defining a group of number generators. Hence before a particular λ is judged to be satisfactory, many such generators should be tested for acceptability by using different x_0 values to start the sequence. Although not tabulated here, five different values of x_0 were tested at selected lags for each generator. These results together with those presented above indicate that Method 1 is the superior method. It should, however, be used with the understanding that probably

only the first three significant digits are random to the desired extent.

NORMAL DISTRIBUTION

The methods just described produce random numbers uniformly distributed on the unit interval; however, often numbers drawn from a normal distribution are really what is desired. By applying the Central Limit Theorem it is possible to obtain normally distributed numbers from the uniform distribution. [6] The Central Limit Theorem states that if a population has a finite variance σ_x^2 and mean $\tilde{x}$; then as the sample size n increases, the distribution of the sample mean $\bar{x}$ approaches the normal distribution with mean $\tilde{x}$ and variance σ_x^2/n . For the uniform distribution, $\tilde{x}$ is .5 and σ_x^2 is $1/12$ or .08333, hence

$$\mu_{\bar{x}} = \tilde{x} = .5, \text{ and} \quad (17)$$

$$\sigma_{\bar{x}}^2 = \frac{\sigma_x^2}{n} = \frac{1}{12n} \quad (18)$$

The random variable

$$y = \sqrt{12n} (\bar{x} - .5) \quad (19)$$

is then normally distributed with a mean of 0.0 and a variance of 1. Picking n as 15 which should be satisfactory considering reference [5] the following results

$$y = 13.416408 \left[\left(\sum_{i=1}^{15} x_i \right) / 15 - 1/2 \right] \quad (20)$$

Eqns. (3) and (20) were programmed in FORTRAN compatible subroutine form for the IBM 7074 computer. This routine is available in the Ames Laboratory Computer Library under the name RANFCT, distribution number 707401632300009.

PROGRAM LISTING

RANFCT	XU	11, SAVIW
	XU	12, SAVIW+1
	XU	13, SAVIW+2
	BLX	94, 0+X94
	ZS1	0+X13
	BM1	NORMAL
UNIFORM	BLX	13, GENER
	ZST1	0+X12
EXIT	XL	11, SAVIW
	XL	12, SAVIW+1
	XL	13, SAVIW+2
	B	0+X94
GENER	ZA3	0+X11
	M	CONST(5, 9)
	ZST2	0+X11
	SR2	2
	ZA3	CONST(0, 1)
	ZST2	WORK
	STD3	WORK(0, 1)
	FZA	WORK
	B	0+X13
NORMAL	ZA1	CONST(1)
	ZST1	WORK+1
	XL	98, LOOP
L1	BLX	13, GENER
	FA	WORK+1
	ZST1	WORK+1
	BIX	98, L1
	FM	F15
	FS	FP5
	FM	FETC
	B	EXIT-1
	DC	
SAVIW		+0+0+0
WORK		+0+0
CONST		15000010011
LOOP		+10015
F15		+4966666667
FP5		+5050000000
FETC		+5213416408
END	CNTRL	RANFCT

REFERENCES

1. Rotenburg, A., A new psuedo-random number generator, J. Assoc. Comp. Mach. 7(1960), 75-77.
2. Coveyou, R. R., Serial correlation in the generation of pseudo-random numbers, J. Assoc. Comp. Mach. 7(1960), 72-74.
3. Pathria, R. K., A statistical study of randomness among the first 10,000 digits of π , Math. Comput. 16 April (1962), 188-197.
4. Allard, J. L.; A. R. Dobell; and T. E. Hull, Mixed congruential random number generators for decimal machines, J. Assoc. Comp. Mach. 10(1963), 131-141.
5. Random Number Generation and Testing: IBM Reference Manual C20-8011, 1959.
6. Burington, R. S. and D. C. May, "Handbook of Probability and Statistics", Handbook Publishers, Inc. 1953.
7. Greeburger, M., Notes on a new pseudo-random number generator, J. Assoc. Comp. Mach. 8(1961), 163-167.
8. Good, I. J., the serial test for sampling numbers and other tests of randomness, Proc. Camb. Phil. Soc. 49(1953), 276-284.
9. Good, I. J., On the serial test for random sequences. Ann. Math. Stat. 28(1957), 262-264.
10. Kronmal, R., Evaluation of a pseudo-random normal number generator, J. Assoc. Comp. Mach. 11(1964), 357-363.

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APPENDIX

A Table of Uniformly Distributed
Random Numbers

A Table of Uniformly Distributed Random Numbers

875	634	815	243	073	948	948	447	768	681	230	072	587	295	788	302	272	762	477	203	947	781	422	912	387	868	210	916	301	674
595	017	143	165	672	455	660	775	753	736	224	318	657	009	123	352	798	282	227	710	362	501	946	274	902	327	081	464	994	597
019	662	078	675	848	630	246	425	208	077	682	178	195	804	483	989	309	349	902	511	321	706	523	134	008	614	567	501	987	940
384	183	367	018	302	192	598	227	258	085	800	825	112	688	929	892	257	951	387	346	979	130	222	257	127	365	682	177	926	805
808	874	243	718	817	737	290	491	309	222	657	113	798	945	602	618	777	558	848	037	374	185	593	689	760	098	120	122	736	373
289	707	413	583	254	743	576	007	046	029	209	575	122	141	724	605	682	120	731	486	425	967	655	754	490	210	254	444	623	906
704	338	067	697	555	569	389	044	436	128	295	426	433	121	597	169	465	066	761	675	928	247	948	060	979	980	138	733	072	746
811	099	876	426	740	450	344	352	723	222	632	557	562	012	602	509	300	306	944	664	561	480	431	491	566	565	505	060	850	313
247	003	988	415	912	506	733	160	433	090	818	544	818	218	202	702	042	229	565	596	883	799	585	213	275	002	769	973	001	111
530	746	034	591	255	734	530	177	372	596	330	640	990	426	936	706	222	706	191	692	329	019	358	671	806	209	023	503	851	720
058	699	123	159	023	010	389	595	625	680	525	778	349	601	428	359	056	085	666	253	219	632	181	592	542	984	042	159	006	801
108	918	844	603	565	094	643	081	558	364	642	572	643	989	379	378	436	196	116	662	749	812	942	980	544	004	278	930	351	097
839	134	267	691	300	622	306	786	817	750	799	316	102	115	570	360	937	350	947	380	998	413	092	122	953	827	866	286	052	430
286	763	942	468	730	112	072	339	327	019	991	983	436	784	863	783	811	335	844	949	923	968	440	583	991	890	619	177	555	700
369	897	898	259	437	961	314	851	294	433	099	226	835	083	199	006	992	421	773	989	361	692	356	209	960	512	031	032	586	889
884	311	645	669	081	448	086	909	204	334	877	380	968	377	601	265	095	357	980	204	030	477	668	125	242	889	274	761	149	060
510	457	172	585	404	730	121	585	821	143	966	458	984	312	169	677	412	374	991	373	527	897	688	737	297	099	204	754	530	353
803	469	950	173	229	844	633	428	192	361	881	153	514	813	086	242	917	181	610	360	331	205	205	730	668	100	352	879	295	990
202	162	927	876	457	709	316	466	642	571	021	840	666	085	612	836	264	968	623	105	799	337	487	070	976	730	934	488	291	273
024	028	534	422	069	121	343	211	777	434	664	572	032	615	091	216	786	403	297	630	168	904	285	002	923	706	842	408	641	583
466	241	679	816	126	758	367	651	483	691	966	081	679	168	942	021	498	637	376	037	557	200	829	053	290	625	651	951	752	382
607	655	753	343	772	178	523	257	924	165	966	783	158	790	668	769	092	300	086	508	962	200	827	028	939	966	613	905	309	211
404	804	625	569	226	818	624	977	547	756	580	771	498	806	850	855	942	500	633	303	261	557	470	012	812	085	663	540	279	693
694	900	645	340	791	997	154	241	078	447	608	817	209	822	150	559	102	828	502	765	212	604	427	371	929	221	413	606	906	528
195	838	642	781	349	911	315	959	521	298	725	375	281	723	310	038	305	353	460	315	452	355	847	751	394	491	158	332	716	499
505	191	926	297	860	639	933	521	162	452	491	582	182	675	151	330	965	624	551	454	500	504	361	077	386	893	872	428	515	467
101	213	287	575	367	137	550	795	568	130	342	248	864	124	575	351	176	672	101	765	752	423	079	555	169	304	207	083	389	373
341	838	993	581	991	244	379	133	583	634	596	867	755	795	563	900	711	006	717	909	487	168	587	670	083	481	497	968	702	239
462	678	795	558	434	677	316	363	333	345	450	613	764	693	176	654	023	615	282	628	861	471	958	188	551	064	757	231	102	167
582	028	923	034	477	033	932	794	224	725	983	340	282	106	558	172	248	970	964	742	913	746	741	155	013	568	679	502	512	339
699	861	085	814	983	791	443	217	942	316	152	582	178	597	928	889	197	019	207	155	560	086	966	896	231	392	637	892	139	015
691	830	472	983	691	307	900	901	452	739	794	551	802	013	590	125	364	192	737	846	599	266	142	016	687	813	685	989	469	538
314	271	753	908	225	820	799	596	999	695	628	142	983	479	323	076	924	399	560	879	709	738	257	403	183	989	556	863	266	329
206	195	078	232	086	447	472	532	109	967	249	929	031	400	391	821	730	030	962	394	446	637	782	220	540	957	664	064	577	890
886	296	076	883	554	186	894	417	588	416	138	164	736	464	534	316	315	954	508	613	249	776	668	914	660	636	103	621	728	802
750	949	857	066	193	914	717	441	522	983	650	781	367	634	974	400	892	521	043	838	435	648	343	210	524	822	646	043	323	727
077	207	010	267	043	389	216	275	276	691	023	394	673	157	413	790	357	559	693	450	202	428	716	114	194	193	747	322	249	407
023	803	605	250	627	248	563	067	495	640	376	297	884	558	633	083	282	380	865	912	627	968	179	911	812	308	539	925	670	662
627	151	192	062	445	010	323	448	105	013	705	463	711	643	494	758	920	772	242	763	668	802	595	167	598	602	837	804	034	396
806	344	801	028	080	071	900	526	313	070	888	546	342	497	938	172	206	006	792	627	162	145	327	728	856	395	132	386	064	588
358	157	940	754	192	709	336	892	602	154	683	880	447	486	988	562	753	830	759	205	828	889	192	719	965	883	600	582	767	302
960	043	600	125	524	082	375	615	740	686	728	477	176	256	745	046	854	474	670	278	262	608	504	546	389	145	094	782	429	678
169	134	250	307	897	227	430	245	771	168	540	332	158	733	390	622	484	648	329	708	942	557	053	895	668	138	147	855	615	939
424	246	840	746	213	063	598	811	022	181	516	318	503	120	185	167	295	542	823	436	226	667	108	730	425	699	973	151	170	385
043	872	798	167	453	386	650	823	097	387	935	188	800	906	471	439	621	824	516	485	352	554	418	298	360	546	466	499	221	398
221	184	036	576	679	875	033	271	883	528	953	577	120	854	671	075	477	646	056	955	436	510	214	124	255	276	198	208	172	440
039	038	942	259	033	086	899	623	545	425	609	498	011	011	285	594	555	636	366	029	477	509	204	014	972	368	425	070	710	053
451	965	387	780	736	458	044	831	528	979	820	045	504	702	896	392	230	903	654	967	353	205	579	053	453	179	079	352	800	858
298	181	719	986	089	308	963	322	559	173	384	491	107	533	164	747	554	038	404	112	820	931	009	608	719	946	774	805	687	557
295	578	768	002	475	833	844	008	642	068	978	790	611	390	832	818	261	111	381	886	517	701	641	323	872	788	803	659	898	931

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339	468	016	781	622	913	799	347	502	888	929	499	320	683	684	093	207	584	351	947	562	282	966	229	512	165	710	940	395	836	
807	072	779	493	904	938	203	280	147	196	711	407	665	426	343	701	865	550	000	348	805	104	606	861	989	222	152	773	227	292	
098	259	669	512	683	751	800	056	981	710	076	249	998	971	946	016	827	582	294	962	711	730	818	711	979	226	529	616	080	089	
450	641	434	084	661	070	484	504	131	811	222	197	231	324	555	716	547	669	119	530	436	895	673	543	901	814	856	617	598	327	
981	513	305	734	620	492	827	933	001	962	226	106	752	770	312	359	158	282	640	875	019	012	719	404	258	504	825	407	708	188	
690	848	991	268	420	496	085	135	278	702	046	510	431	875	437	383	474	369	302	898	377	175	986	619	632	694	810	095	614	935	
452	300	681	771	004	439	190	377	926	655	519	621	618	485	234	105	989	360	832	579	308	157	985	794	683	662	865	270	803	908	
026	202	045	610	393	558	755	410	193	522	364	335	143	725	083	722	876	166	234	982	489	411	704	814	154	564	724	002	039	529	
050	568	232	428	688	971	076	463	602	084	177	223	314	001	445	313	989	174	795	247	477	072	613	845	866	438	799	841	370	300	
041	092	873	154	072	676	124	246	960	203	436	540	921	999	863	834	861	256	079	597	711	953	661	333	721	203	185	816	119	802	
396	147	077	991	806	550	555	949	353	820	499	220	235	683	959	124	706	761	934	332	509	547	279	004	700	655	655	437	894	697	
394	787	434	425	231	351	700	241	146	958	604	875	005	300	433	900	417	518	483	834	066	029	376	862	865	473	663	693	578	727	
191	745	012	222	770	716	575	271	984	717	867	800	459	375	067	760	569	837	134	565	463	251	341	193	358	213	343	055	339	713	
826	435	363	428	923	164	872	670	794	280	287	968	309	714	724	181	415	507	570	067	654	748	044	563	400	314	507	471	621	556	
216	951	514	369	273	091	965	547	780	907	742	033	743	403	343	521	888	798	759	961	480	733	835	818	292	093	650	372	151	239	
159	065	977	649	480	775	909	491	429	941	988	329	431	806	948	017	601	461	945	949	656	099	544	084	417	747	945	666	933	823	
332	233	740	154	287	374	435	572	506	803	664	868	523	570	640	787	850	723	655	812	781	420	479	765	236	355	245	744	254	449	
293	587	273	050	516	926	959	340	056	995	287	345	648	620	600	829	606	295	693	412	333	950	431	547	290	873	086	475	679	339	
479	941	525	783	067	348	573	824	406	099	254	133	916	162	090	020	525	367	146	691	668	623	669	396	202	141	679	209	054	796	
208	788	927	079	922	437	051	534	160	775	844	286	917	682	452	118	939	608	379	669	025	052	942	559	672	874	918	775	503	199	
678	303	388	942	143	872	847	460	205	766	214	537	719	944	106	761	861	167	038	450	520	530	481	559	482	670	378	483	434	012	
966	339	297	658	872	209	094	070	706	893	401	300	874	995	556	465	987	675	047	213	153	031	995	203	495	008	312	122	530	775	
030	428	525	794	330	887	607	316	108	057	323	669	409	714	381	630	688	241	614	222	799	209	673	576	651	245	653	963	759	110	
708	786	420	194	818	223	878	625	138	241	778	417	836	046	245	531	019	454	223	817	218	398	185	045	972	619	015	754	366	718	
716	306	813	984	719	414	081	909	801	506	443	997	143	536	888	327	714	385	640	420	046	610	681	254	560	246	691	725	876	383	
654	560	012	570	494	539	070	556	381	993	876	544	800	798	132	056	186	582	910	533	801	540	790	129	596	125	656	587	094	964	
997	803	808	638	684	555	378	436	446	925	514	872	757	276	879	635	528	075	360	738	880	561	623	877	342	133	563	528	107	403	
104	969	470	152	912	299	219	399	841	602	675	472	443	696	111	861	514	374	594	695	562	727	767	981	139	028	745	217	280	723	
213	671	747	360	878	489	487	774	691	407	557	520	768	065	888	413	598	469	499	147	004	771	294	209	410	448	215	806	259	025	
441	202	870	785	365	723	755	371	401	800	238	868	121	666	353	848	914	829	240	915	243	108	752	605	656	910	669	923	968	489	
785	536	548	234	233	478	277	480	658	324	674	051	373	067	727	603	274	403	263	901	197	830	172	495	457	811	478	678	615	379	
124	328	970	793	425	112	986	369	427	601	703	282	872	112	312	996	173	622	293	086	664	711	063	486	477	429	698	660	684	035	
215	909	807	826	963	863	496	790	954	331	044	455	449	927	489	224	784	394	335	532	321	204	414	461	456	923	060	940	941	880	
695	295	207	981	946	847	100	970	764	296	293	142	412	919	720	366	961	111	676	380	726	444	695	588	217	329	980	066	431	414	
082	178	800	181	559	063	773	621	662	358	929	599	552	771	546	379	238	640	881	853	317	245	856	311	660	565	550	068	481	220	
773	932	696	633	061	389	168	931	735	459	308	758	138	451	590	100	827	332	796	251	410	098	326	356	767	428	545	456	696	958	
047	611	485	823	795	581	618	570	348	620	669	232	920	203	552	248	623	016	546	957	204	179	015	730	601	597	417	220	961	371	
061	948	236	516	181	278	137	687	147	942	129	317	127	554	215	419	199	002	536	432	777	341	313	716	302	629	301	829	442	280	
852	358	498	757	723	996	418	913	057	608	686	984	469	308	435	091	809	080	453	217	085	117	088	882	092	961	009	233	585	587	
339	933	302	872	001	135	836	357	283	879	216	888	135	552	168	622	386	518	263	934	967	721	692	072	273	911	036	861	115	273	
318	447	157	468	677	970	443	609	313	096	479	363	796	650	421	249	544	068	209	285	139	046	952	413	226	677	556	622	037	400	
467	354	052	428	492	661	974	737	910	682	111	422	600	250	301	091	577	957	819	051	201	667	180	309	413	336	421	908	638	110	
344	787	457	920	269	244	842	292	120	137	630	758	177	275	989	144	458	897	898	095	628	837	164	447	375	846	165	586	484	623	
387	560	322	389	908	637	140	304	270	044	434	745	638	933	748	287	841	076	531	357	780	489	174	792	735	044	003	007	418	243	
912	167	077	559	391	638	643	281	965	063	800	437	570	707	917	276	059	164	084	859	893	236	960	589	194	648	827	000	568	350	
118	781	630	437	781	923	803	214	090	937	886	568	045	365	920	750	126	311	203	702	085	374	751	365	532	257	211	875	339	405	
083	256	372	309	218	052	755	572	811	487	730	550	612	951	258	226	735	146	813	069	353	874	257	924	613	346	409	422	416	952	
763	125	171	740	924	461	312	305	574	615	248	478	300	792	512	101	261	779	120	221	576	392	668	353	377	274	354	910	765	610	
997	602	379	575	201	469	967	842	104	303	239	126	619	492	344	654	757	800	609	511	260	652	017	847	279	660	088	632	082		
502	581	824	941	430	272	894	093	407	611	380	946	558	938	495	042	956	277	046	326	795	491	360	561	123	477	621	187	542	150	

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433	232	447	849	001	C09	081	978	267	424	259	857	283	601	411	014	573	136	255	021	808	713	741	077	466	592	639	694	519	328
648	057	646	350	966	551	177	516	571	986	829	426	902	373	393	179	362	547	804	315	878	636	496	978	907	246	653	813	491	318
806	785	491	879	349	G01	785	417	784	673	101	249	032	668	240	172	776	545	717	743	731	684	344	817	780	968	922	461	203	567
146	029	731	680	850	C77	800	510	033	868	274	499	580	490	012	670	267	118	881	045	524	591	353	359	506	393	460	788	300	179
785	082	597	280	252	377	795	105	723	032	426	029	936	126	852	230	969	735	461	043	294	769	075	649	586	041	960	423	709	334
721	919	798	483	416	379	024	992	539	706	513	374	971	140	981	291	697	348	903	639	960	313	538	014	603	309	797	476	635	294
830	192	524	376	283	439	420	440	447	512	373	747	033	380	701	171	943	385	932	814	318	996	252	058	218	473	023	536	563	400
872	236	443	323	875	795	597	198	693	152	725	042	953	969	393	065	881	756	554	630	046	272	206	668	172	693	373	674	259	075
483	063	706	972	294	566	849	497	801	407	165	832	039	314	519	053	365	850	054	229	702	274	870	008	288	005	260	438	769	819
181	368	943	246	721	748	149	045	579	140	172	370	083	101	621	092	929	537	998	831	724	817	194	526	466	327	777	858	418	215
364	525	262	353	419	219	150	033	111	290	103	591	352	295	319	020	785	167	232	740	428	393	607	945	689	456	699	445	811	923
309	586	254	776	727	738	138	131	763	881	195	108	596	141	317	553	828	570	881	336	013	176	020	273	017	071	478	187	836	686
174	285	988	283	069	941	274	486	181	014	566	214	047	166	394	290	632	054	350	080	557	019	820	793	593	729	249	554	656	325
997	037	014	319	946	345	102	731	290	870	213	884	412	174	414	709	448	409	326	516	016	457	979	073	638	867	824	495	717	742
694	934	361	008	939	350	047	973	296	711	015	770	881	252	317	166	213	906	775	263	229	703	545	958	454	803	699	441	746	918
064	750	538	158	710	232	162	802	684	879	729	207	125	765	126	899	538	294	940	024	913	651	549	572	422	735	045	301	748	916
784	939	537	252	001	148	179	289	220	794	992	208	292	358	941	027	718	802	349	581	665	874	500	323	005	740	717	464	008	875
413	634	825	458	632	137	514	982	950	960	322	466	012	958	944	546	726	139	807	794	964	626	387	894	742	776	249	801	093	019
386	650	353	221	506	116	260	912	199	956	117	356	396	769	398	335	216	496	400	606	167	840	080	253	257	681	853	660	847	649
023	479	551	265	605	843	189	587	573	446	655	930	032	278	643	150	521	543	492	038	511	130	829	645	251	171	424	871	396	146
520	295	327	598	990	155	757	998	957	170	091	922	990	250	101	630	655	428	730	192	114	789	363	594	505	846	536	745	145	972
956	952	072	504	802	370	096	614	518	950	468	747	820	731	274	292	312	781	039	249	974	792	392	908	880	181	441	070	782	668
287	983	654	549	263	085	019	385	699	688	692	497	551	045	743	534	865	712	625	471	968	791	106	671	320	535	073	115	270	856
352	602	425	578	675	579	022	740	228	366	576	946	693	300	170	633	367	811	973	199	853	121	174	249	844	146	047	632	856	239
868	703	213	718	419	048	276	589	110	044	790	549	235	380	296	746	553	146	849	856	269	795	745	287	555	131	654	849	065	596
433	859	328	373	958	570	637	321	631	865	890	437	648	451	944	912	836	269	299	943	731	506	450	712	635	487	871	476	703	791
523	323	559	230	831	103	636	807	355	050	316	425	880	958	014	049	310	207	648	040	638	629	397	729	344	093	348	702	855	764
498	031	176	254	662	465	489	395	129	900	385	307	361	128	489	953	748	472	502	811	267	216	177	823	025	705	422	197	888	537
594	594	929	691	152	432	088	916	078	798	294	357	002	966	430	302	603	052	746	996	777	002	859	760	100	962	104	111	446	212
930	306	047	066	082	544	007	678	608	205	122	327	190	757	978	654	010	417	547	418	203	400	593	585	068	381	089	074	455	970
501	142	241	184	314	296	500	472	404	663	826	451	797	067	355	446	782	517	349	977	465	503	608	625	514	360	750	194	121	073
187	755	698	132	789	047	501	567	433	792	243	943	172	741	864	996	412	781	878	655	359	085	214	485	097	176	141	062	929	863
745	478	090	275	530	035	622	713	939	296	391	697	144	906	884	503	075	119	140	515	564	600	801	050	559	987	995	746	645	761
813	324	566	259	637	377	159	139	448	954	367	286	023	967	878	042	623	921	420	697	636	182	838	486	723	830	726	798	315	268
907	989	754	009	293	071	083	555	766	630	350	964	598	609	388	572	591	056	285	423	014	644	274	238	490	623	427	246	265	967
426	845	766	730	759	993	049	150	978	264	597	664	140	798	035	931	192	873	579	994	015	479	539	033	841	164	873	599	099	519
648	945	190	939	376	903	390	594	451	878	245	000	397	779	521	836	319	203	428	794	836	861	544	876	837	131	517	849	703	665
729	024	096	312	566	437	120	038	829	574	513	266	599	079	627	884	547	354	238	282	556	644	677	052	622	080	492	463	244	228
708	495	034	932	831	113	933	109	038	534	597	435	456	503	215	554	128	117	695	000	132	361	808	127	415	449	612	392	166	108
501	451	034	247	752	323	202	918	284	018	175	161	158	136	227	202	996	761	763	571	401	227	287	947	519	557	370	065	195	287
908	667	604	712	991	362	981	055	053	369	405	777	374	344	683	067	766	036	689	695	081	134	943	637	316	600	941	392	337	828
604	595	734	262	290	370	004	589	109	008	124	297	254	773	687	266	730	171	999	155	770	656	036	602	266	656	178	763	878	871
149	369	895	063	470	398	683	070	499	436	351	414	427	348	419	797	862	876	497	812	945	047	505	530	912	684	613	047	382	638
979	804	035	561	433	341	113	527	549	236	982	503	002	275	140	536	817	340	269	607	964	240	471	384	876	519	462	593	696	431
412	392	585	481	160	014	066	469	863	069	196	616	571	039	193	243	926	234	682	562	064	849	732	410	857	881	618	232	946	631
645	308	454	829	713	102	998	888	327	677	449	488	201	406	000	554	204	706	380	779	363	168	518	135	640	367	653	272	536	700
757	404	031	890	233	148	041	251	108	880	480	532	443	422	061	988	345	387	289	440	859	170	540	364	084	453	823	505	152	180
705	214	186	230	943	595	009	509	650	581	106	842	326	412	958	940	723	385	615	805	430	509	986	182	131	499	059	198	761	692
326	953	269	695	143	760	396	091	680	762	124	190	361	932	354	691	390	291	844	217	832	518	526	956	804	741	977	102	607	937
338	513	109	410	216	840	375	908	202	484	413	032	535	017	988	396	080	174	741	097	703	211	309	329	204	297	869	446	217	698

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157	926	107	447	209	235	793	338	061	291	219	144	475	049	967	066	369	418	189	425	683	074	745	455	250	149	999	178	672	150
119	972	742	738	858	294	580	482	024	106	576	375	370	850	273	788	289	275	637	613	581	099	616	625	655	401	584	582	784	673
145	242	542	776	844	381	200	508	617	966	756	179	295	894	363	458	154	238	170	854	382	569	099	454	411	840	744	036	355	376
472	347	258	806	869	215	546	247	965	254	956	730	158	186	299	753	417	296	674	889	242	216	264	705	940	103	493	688	032	361
219	581	120	355	714	393	192	007	475	431	255	882	350	011	222	231	211	919	313	541	200	456	661	424	744	708	525	168	741	810
382	919	836	228	241	391	393	579	830	039	610	169	740	935	354	330	350	057	534	710	172	382	319	938	404	053	213	587	686	983
839	014	597	510	392	569	080	232	997	699	858	803	678	804	998	366	327	139	062	378	958	766	748	852	583	415	611	532	353	223
348	199	072	567	188	163	869	716	223	112	717	679	993	743	533	539	317	076	903	608	233	063	938	051	020	952	452	075	509	951
546	488	410	045	730	291	052	260	031	061	785	370	995	157	423	924	172	255	343	540	557	407	358	701	539	701	151	765	198	668
952	575	242	869	201	950	603	574	228	407	539	130	474	734	208	480	427	548	946	396	366	611	957	248	041	580	799	631	746	957
962	832	677	245	861	019	176	848	500	091	337	893	698	437	510	045	295	303	560	478	978	168	166	417	507	387	172	183	759	479
855	315	304	658	054	255	105	750	411	135	416	271	418	513	031	336	669	351	308	167	590	252	893	214	999	800	723	410	123	976
787	756	193	874	199	295	402	432	408	641	894	559	864	487	552	950	124	001	597	925	281	717	529	924	658	375	585	782	002	268
797	569	895	939	799	657	762	521	817	791	769	730	744	164	935	366	912	042	113	294	008	097	943	113	707	551	572	249	843	258
802	847	437	178	435	739	558	129	842	845	919	437	249	632	122	941	968	744	820	895	608	604	485	627	446	644	178	241	372	928
599	365	330	197	770	818	844	843	969	146	100	016	048	254	133	912	905	857	965	429	799	133	984	591	258	854	576	666	593	339
867	575	564	881	544	052	354	735	965	115	950	478	291	677	072	397	016	610	074	679	179	258	751	411	603	256	619	915	793	632
162	612	608	396	579	479	500	354	874	254	988	517	607	826	119	394	275	713	951	506	225	231	573	772	024	809	842	857	536	029
923	289	411	188	776	015	377	729	823	144	610	508	106	906	535	780	337	356	683	851	295	986	722	640	143	351	458	841	670	832
467	099	404	982	118	459	758	370	816	447	095	504	377	405	664	312	533	207	634	737	627	137	946	261	660	599	360	698	318	422
992	217	496	784	666	489	097	266	739	904	599	238	490	086	925	630	879	418	452	264	337	978	476	160	357	151	123	737	887	261
575	495	076	879	562	661	527	888	359	338	161	124	996	996	821	249	067	616	060	614	425	482	020	143	096	484	000	747	063	891
174	467	014	833	027	414	862	184	320	649	698	255	922	460	934	568	471	913	665	050	766	303	769	295	819	955	923	998	811	933
626	348	660	492	362	064	596	585	349	820	008	406	779	084	924	864	145	897	753	912	119	775	392	983	547	803	508	240	376	089
650	030	843	982	950	811	902	999	450	912	768	030	557	753	534	295	823	638	089	622	122	910	039	851	381	146	048	702	284	140
842	087	873	706	251	731	633	817	910	066	535	259	725	634	585	899	917	686	718	682	292	403	339	825	504	979	515	095	342	948
680	774	540	353	809	781	324	908	294	504	748	909	233	171	979	585	311	891	986	144	392	068	249	789	983	240	744	031	632	472
522	022	113	886	243	800	188	621	447	528	725	473	510	090	697	174	411	300	440	257	603	635	817	194	742	512	529	907	525	681
605	447	341	552	256	505	119	787	496	520	661	123	465	397	802	320	038	365	024	176	179	162	655	840	619	607	423	147	663	729
048	341	455	876	629	494	690	715	845	940	636	715	490	377	433	590	536	736	883	251	793	621	464	095	311	983	939	954	971	781
847	678	163	664	225	244	154	194	181	331	607	780	452	596	814	419	720	361	465	383	362	106	275	285	400	039	451	440	656	097
881	112	656	002	984	112	446	495	468	314	412	534	702	900	245	127	081	671	493	555	684	390	596	014	547	053	013	193	204	020
906	976	604	254	928	338	179	366	954	591	767	869	069	415	109	911	795	574	975	828	436	927	418	168	493	181	359	283	380	971
561	284	155	067	158	037	647	037	162	943	271	360	864	545	867	848	715	461	195	343	176	849	210	914	060	462	901	260	229	452
363	730	939	366	857	208	822	219	900	232	402	259	874	976	060	895	374	201	718	322	341	972	922	696	150	812	735	153	078	045
709	687	066	357	286	728	360	099	252	399	516	501	371	675	311	891	986	144	392	068	249	789	983	240	744	031	632	472	532	410
878	209	125	526	787	355	592	349	584	467	852	698	104	886	320	553	445	119	341	960	098	473	303	552	904	794	047	207	475	290
027	029	187	637	780	727	534	118	541	536	526	145	301	135	869	479	324	436	970	461	965	877	271	917	772	661	113	827	075	505
193	562	800	737	769	360	878	035	050	789	538	816	674	228	821	146	877	884	966	113	808	536	757	902	569	068	645	281	777	959
294	899	995	152	333	653	999	209	315	487	763	363	411	251	116	912	037	734	294	538	465	663	112	351	596	333	134	000	305	632
128	816	280	486	136	884	949	232	823	971	961	120	183	568	776	014	418	734	199	435	652	151	163	390	236	653	757	892	667	586
372	766	646	625	918	209	463	171	339	663	768	102	138	826	903	571	313	115	208	589	969	575	221	425	950	107	364	348	147	962
584	882	563	736	502	667	953	577	908	065	702	000	906	951	678	579	697	585	125	859	891	187	076	142	280	652	492	237	311	983
201	978	978	263	788	175	515	479	857	759	160	191	597	147	363	916	222	335	037	187	777	922	997	506	846	125	352	909	004	949
541	548	323	932	758	530	920	387	791	405	421	726	801	900	300	340	223	034	309	596	865	392	734	762	351	244	839	193	353	243
802	764	507	750	474	411	623	291	595	746	642	339	586	977	909	489	713	831	587	187	272	892	516	436	577	607	526	399	763	325
062	482	920	000	078	374	758	660	435	603	860	444	504	422	694	879	386	357	796	140	995	396	052	334	384	691	666	317	919	738
277	234	430	250	791	858	137	443	756	878	993	135	583	561	235	910	614	721	141	719	913	556	534	542	715	854	195	216	787	104
285	234	389	344	915	180	255	071	285	552	839	185	332	001	193	857	452	513	109	264	783	707	629	424	592	334	724	845	612	122
805	775	624	408	831	538	285	453	027	687	075	048	742	625	919	880	634	801	465	198	242	861	488	628	115	247	547	434	921	577

A Table of Uniformly Distributed Random Numbers (Cont.)

495	103	119	233	117	076	507	249	739	013	028	509	550	534	570	559	250	173	147	223	324	587	444	800	309	654	007	336	070	784
952	593	624	103	133	170	676	603	021	335	960	063	996	395	523	396	433	920	394	198	877	815	545	510	640	100	735	311	460	374
713	945	335	163	325	532	319	881	373	543	955	830	512	938	099	421	201	814	166	066	572	127	299	316	163	974	678	375	750	984
015	773	002	643	377	881	328	910	920	018	210	984	006	168	362	311	006	843	348	568	568	257	775	986	299	912	851	678	586	716
976	370	854	097	129	813	277	001	068	221	804	379	869	372	452	623	983	477	106	526	900	620	523	565	550	432	945	849	893	751
594	711	601	303	383	806	420	944	502	696	693	548	970	115	591	797	945	666	886	842	487	308	572	378	497	931	336	999	877	351
746	448	433	369	180	219	691	008	189	062	716	705	658	242	081	148	385	839	413	497	128	095	452	030	802	688	077	715	024	857
189	916	019	645	382	287	702	943	373	022	590	744	764	880	304	875	478	906	693	554	498	436	092	408	207	860	901	069	098	692
563	128	508	782	092	130	748	977	580	358	913	238	597	434	720	055	076	256	011	153	156	462	023	677	532	484	222	609	146	356
383	777	531	704	649	745	802	822	616	930	161	441	946	589	872	646	714	760	934	515	540	988	173	282	681	478	133	366	493	432
C48	238	196	619	237	008	518	667	566	682	694	286	081	310	381	486	604	766	306	944	967	508	972	950	633	640	409	849	745	581
E35	563	094	011	196	679	229	180	796	633	748	387	752	845	948	292	642	104	253	820	634	196	331	686	452	647	503	047	787	544
502	487	295	646	948	394	950	512	952	886	441	037	189	718	356	661	399	085	181	605	620	904	618	775	278	057	548	430	786	144
287	423	346	569	995	671	372	292	959	622	771	211	100	734	466	071	130	497	775	841	882	166	763	783	334	307	357	948	186	281
907	464	280	167	918	908	871	631	023	103	615	562	675	980	220	881	769	610	002	148	257	196	855	555	920	716	426	030	713	937
559	384	604	865	379	382	500	116	630	671	731	423	585	821	638	327	928	173	106	229	463	888	345	218	418	480	926	586	373	174
921	637	308	728	120	250	992	819	544	746	757	808	979	903	824	527	902	417	613	866	097	815	942	177	291	677	713	006	451	134
552	357	863	862	922	552	761	289	812	832	209	410	485	151	957	478	664	051	329	920	638	231	635	117	079	266	319	158	514	038
888	357	217	712	866	203	901	554	759	509	486	604	215	771	301	059	868	265	339	332	443	069	655	004	404	082	957	394	406	188
247	130	801	005	635	002	185	126	991	439	866	442	757	248	196	026	848	727	010	124	749	943	870	084	967	845	523	541	253	965
826	850	523	746	510	626	066	993	394	363	505	659	182	348	064	019	616	588	986	398	675	147	590	881	551	151	588	911	461	831
704	371	774	220	512	633	679	626	132	104	442	668	037	116	407	553	867	478	193	335	217	654	966	203	017	479	407	292	715	328
838	226	423	993	044	461	837	972	652	562	593	562	355	879	807	026	974	505	837	198	253	118	785	134	307	185	914	954	982	077
065	630	820	911	226	426	034	464	680	719	758	116	643	242	924	718	991	260	403	326	541	872	519	040	442	508	721	647	506	779
103	475	794	099	500	727	443	009	220	638	612	782	891	090	501	783	651	812	658	143	718	930	517	567	523	565	124	600	813	217
550	335	655	963	329	442	917	998	558	459	715	694	570	589	358	262	368	710	646	150	303	985	808	640	733	354	094	523	709	252
883	464	192	188	253	527	991	299	261	404	503	667	629	185	399	070	236	985	993	928	692	412	101	465	333	752	286	605	280	825
459	795	676	740	894	821	878	264	173	775	294	193	496	603	604	006	028	146	406	139	164	263	188	527	564	517	034	517	890	958
518	942	855	865	954	041	471	720	420	953	285	447	083	849	035	747	198	182	668	524	875	273	536	592	148	286	350	407	187	753
175	199	959	088	214	784	345	979	408	400	555	281	778	208	833	852	879	564	647	906	864	855	696	706	287	577	929	905	094	392
429	539	699	214	536	529	752	829	823	658	061	229	451	246	221	756	885	240	787	185	047	103	297	194	563	788	144	122	818	135
158	616	263	331	862	632	627	540	630	348	640	506	452	809	499	779	709	640	814	344	224	789	650	662	336	196	049	646	845	325
118	763	321	802	212	332	582	862	074	171	010	004	610	022	050	117	526	674	734	444	071	369	342	995	849	959	377	547	939	383
947	994	022	274	690	747	913	024	681	910	769	297	235	291	334	849	189	732	833	626	145	974	645	359	223	114	543	375	148	811
164	003	997	672	476	872	591	737	257	426	394	639	117	301	895	931	231	683	676	112	885	420	508	200	960	580	639	159	795	190
165	163	355	202	832	587	271	188	888	660	243	964	524	018	352	202	866	876	108	204	608	200	555	243	041	153	439	408	487	182
229	527	686	343	099	649	287	049	939	635	554	884	208	688	408	379	987	142	255	283	512	486	510	494	528	511	396	114	110	528
512	830	058	873	699	695	651	468	055	451	443	694	396	836	844	060	169	790	524	811	674	133	145	238	562	213	646	744	828	051
053	485	022	833	135	244	359	076	162	290	910	367	800	268	522	721	664	609	598	330	052	675	347	042	366	694	000	249	089	651
759	586	608	551	985	692	882	981	467	415	830	557	608	069	384	722	407	869	445	461	483	324	451	749	240	274	953	059	616	311
458	906	324	624	915	317	175	774	453	166	962	597	490	606	451	299	011	320	309	906	686	976	294	488	567	150	677	082	417	092
737	898	161	942	664	277	571	524	888	965	944	501	595	523	824	570	769	192	717	446	257	202	183	662	308	398	028	709	775	135
343	697	588	672	055	610	785	781	816	314	293	963	555	746	686	533	656	193	473	943	674	257	908	957	504	978	538	809	258	662
535	116	554	253	987	232	509	574	564	794	406	356	477	481	298	065	324	514	664	338	295	075	740	339	277	726	421	732	710	975
691	648	490	426	445	942	317	413	736	067	562	734	951	213	002	924	108	823	655	654	357	268	428	054	329	360	571	307	258	456
006	468	305	182	483	418	163	288	219	875	917	830	048	709	218	748	020	272	091	991	979	131	200	627	942	477	561	843	306	565
560	428	388	811	253	216	160	467	178	038	510	058	316	013	450	053	756	489	898	531	157	637	768	864	476	556	644	132	541	865
310	063	610	880	973	775	582	502	058	460	258	512	785	451	277	238	687	583												

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511	549	998	575	248	590	935	429	865	487	809	362	898	027	354	247	594	430	153	159	189	366	979	362	864	837	788	392	856	302
221	818	068	655	580	167	413	178	508	555	953	254	068	858	983	527	646	733	500	241	914	492	965	803	232	686	944	082	107	358
114	628	824	203	170	892	044	329	480	589	040	039	788	651	316	874	963	661	653	295	662	383	084	620	872	842	996	340	538	515
428	595	015	463	718	483	722	713	928	971	268	892	967	412	415	967	997	204	496	063	590	772	405	580	205	942	957	318	794	873
282	011	872	961	007	538	020	639	256	161	714	666	995	426	422	362	882	833	195	368	735	073	477	730	732	505	520	644	802	615
673	450	104	503	897	534	191	897	151	702	336	893	740	260	558	498	432	496	195	111	015	380	330	393	036	927	060	428	766	002
477	766	900	175	331	829	170	755	578	215	972	789	553	758	125	692	141	624	223	273	228	466	475	176	777	486	629	259	173	375
454	093	930	341	330	660	570	964	783	402	338	247	264	046	503	142	182	126	283	916	051	784	900	965	698	341	962	207	788	156
240	843	344	648	996	145	685	754	290	044	034	839	181	531	156	925	409	391	662	181	042	469	075	924	621	527	471	822	657	847
352	711	771	022	510	282	488	833	987	004	535	820	095	896	625	998	355	289	925	291	638	335	950	500	446	964	251	133	104	030
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030	156	629	995	158	779	961	107	665	598	853	833	911	337	539	741	046	678	874	100	636	345	468	584	553	151	351	741	379	541
227	031	005	489	482	099	852	042	373	041	955	505	307	684	286	154	806	404	929	402	630	666	238	683	605	364	750	734	000	105
539	691	743	878	762	258	500	015	418	309	452	035	847	541	756	846	153	312	896	856	617	435	655	826	268	615	087	770	027	268
764	910	352	766	659	535	957	615	235	743	101	754	202	273	971	054	701	150	020	164	316	546	550	140	923	102	536	761	468	092
579	141	821	039	917	086	958	912	740	766	539	678	520	525	033	897	744	149	828	108	323	571	231	029	032	902	951	095	607	718
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061	287	704	775	310	704	957	139	820	110	375	441	109	157	602	974	522	860	358	718	605	955	563	944	037	290	871	286	534	013
881	246	649	570	375	021	060	042	219	597	810	012	033	346	057	015	428	833	167	751	482	229	458	467	204	602	589	444	011	435
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247	229	983	943	190	708	580	389	943	571	658	820	159	358	003	469	903	409	331	685	212	253	680	451	969	381	171	912	910	576
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823	143	316	674	965	321	238	647	987	329	019	040	336	656	102	523	087	936	611	119	890	639	171	474	116	849	583	416	222	451
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697	405	347	763	155	770	166	749	999	216	074	972	225	453	165	449	946	759	840	471	938	183	265	817	256	505	153	550	145	512
939	174	973	405	509	827	565	666	907	261	205	363	934	653	686	783	237	732	999	319	345	447	813	871	226	223	507	452	174	966
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739	403	290	672	028	937	225	834	146	386	388	326	041	522	949	401	001	766	283	457	990	014	291	758	801	588	007	295	278	244
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717	278	186	586	745	108	225	230	376	286	982	495	895	895	635	752	507	437	854	834	159	029	167	285	503	238	329	664	446	307
978	896	187	790	111	535	347	138	623	903	646	394	221	322	698	092	499	162	739	506	854	098	613	674	987	836	002	122	027	674
769	867	789	519	376	178	352	483	599	649	041	836	252	409	949	006	326	788	447	352	798	423	587	778	464	688	981	664	446	384
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053	082	151	495	973	138	347	162	196	611	968	808	422	549	416	426	058	059	836	098	221	533	754	157	647	860	672	955	342	797
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875	445	739	523	517	731	544	781	920	671	584	110	534	550	863	154	945	464	404	642	363	825	963	423	209	056	900	318	647	638
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