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Studies of High-Energy Nuclear Reactions

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Analysis of Thick-Target Recoil Studies of High Energy Nuclear Reactions*

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The effect of broad velocity distributions on the vector model analysis of average projected ranges obtained in thick-target experiments has been explored. The Monte Carlo technique was used to construct laboratory velocity distributions on the basis of impact ($v_{\parallel}$) and breakup (V) velocity distributions inferred from differential recoil studies. A range-velocity relation for fission products in uranium was constructed and used to convert the laboratory velocities to average projected ranges. The latter were then analyzed by means of equations based on the vector model to yield values of $\langle v_{\parallel} \rangle$ and $\langle V \rangle$ which could be compared with the respective average values of the input velocity distributions.

The calculation was performed for Mo^{99} , Pd^{103} , and Ba^{131} formed in the high-energy fission of uranium as well as for several simple velocity distributions. The effects tend to be rather small (0-20%) although in most cases the vector model analysis does tend to overestimate the value of $\langle V \rangle$ and underestimate that of $\langle v_{\parallel} \rangle$.

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I. Introduction

The study of high-energy nuclear reactions by means of thick-target average range measurements has been of great importance in the determination of the mechanism of these reactions. The technique has been applied to fission, spallation, and to reactions involving the emission of light fragments.¹⁻⁴ The analysis of these experiments is based on the assumption that the reaction can be separated into two distinct steps: a prompt knock-out cascade followed by a slower deexcitation step involving fission, evaporation, or a combination of these processes. The first of these steps imparts to the residual nucleus a forwardly directed impact velocity, v , which can be decomposed into parallel ($v_{\parallel}$) and perpendicular ($v_{\perp}$) components with respect to the beam, and the second, a breakup velocity, V , whose angular distribution is symmetric about an axis at 90° to the beam direction.

The experimental recoil ranges have usually been analyzed⁵⁻⁷ to yield the values of $v_{\parallel}$ and V for a particular reaction. These quantities have then been used to derive further information about the reaction, such as kinetic and deposition energies. The analysis⁵⁻⁷ is based on a velocity vector-model and the use of a particular range-velocity relation for the product of interest. The relation

$$R = k V_L^N \quad (1)$$

has been commonly used. This expression relates the range of the product, R , to its velocity in the laboratory frame, V_L , by means of the constants k and N .

In addition to the basic assumption of the separability of the laboratory velocity into components v and V the analysis assumes that $v_{\parallel} < V$ and that these velocities have unique values. In the more realistic case where $v_{\parallel}$ and V have reasonably narrow distributions the analysis determines the average values

of these quantities, $\langle v_{//} \rangle$ and $\langle V \rangle$. The particular weighting factors associated with these averages have been discussed by Winsberg.⁸

Recent differential range measurements of high-energy fission products⁹ indicate that, at least for the neutron deficient products, the distribution of recoil velocities is rather broad. This fact introduces several possibly important complications into the analysis of average range data. First, it is unreasonable to expect that Eq. (1) will be valid for the same values of k and N over a broad range of velocities. Since the value of N enters directly into the analysis, the choice of a constant N to represent a quantity that is, in fact, a function of V_L introduces an unknown source of error in the values of $v_{//}$ and V . Second, if the distributions of $v_{//}$ and V are sufficiently broad there may be some interactions for which $v_{//} > V$, even though $\langle v_{//} \rangle < \langle V \rangle$. Third, a broad distribution of velocities can lead to erroneous values of $\langle V \rangle$ and $\langle v_{//} \rangle$ due to the nature of the averaging process. For instance, the analysis of data for products obeying Eq. 1 requires the assumption that $\langle V \rangle = \langle v_{//}^N \rangle^{1/N}$ which is generally not valid for broad distributions. Finally, in a given interaction there may be a correlation between $v_{//}$ and V whose effect on the analysis of integral range data remains to be assessed.

The purpose of the present calculation is to determine the effect of broad, and possibly correlated distributions of $v_{//}$ and V on the conventional vector-model analysis of thick-target integral range experiments. The Monte Carlo technique is used in conjunction with range-energy data to calculate values of the average projected ranges on the basis of assumed distributions of $v_{//}$ and V . These ranges are then treated as "experimental" values and analyzed in the conventional way to yield mean values of $v_{//}$ and V . The latter are then

compared with the input values in order to determine the validity of the conventional analysis.

The calculation has been performed for the case of various products from the high-energy fission of uranium. Detailed differential recoil measurements have been performed in this case and information on the distributions of V and $v_{//}$ and their correlation is available.⁹ The results, however, are qualitatively applicable to other reactions for which the velocity distributions may be broad, such as the $\text{Al}^{27}(\text{p}, 3\text{pn})$ reaction¹⁰ and the production of Tb^{149} from heavy elements.¹¹ In addition, calculations have been performed for a number of simple velocity distributions in order to unravel the combined effects of the various factors mentioned above.

The method used in this calculation is outlined in Section II and the results are presented and discussed in Section III.

II. Method of Calculation

A. The Monte Carlo Calculation

The calculation was performed for Mo^{99} , Pd^{103} , and Ba^{131} formed in the interaction of uranium with 2.2 GeV protons. The distribution of V values was obtained from the differential measurements by Crespo et al.⁹ Several of these distributions are shown in Fig. 1.

Little is known about the distribution of $v_{//}$ values. Even the differential experiments only yield the average values of this quantity. Monte Carlo cascade calculations suggest that a fairly broad range of $v_{//}$ is associated with a particular value of the deposition energy and hence with a specific reaction.^{12,13} The present calculation has been performed for unique values of $v_{//}$, rectangular

distributions of $v_{//}$, and for values of $v_{//}$ correlated with those of V for the iteration in question. The average values of $v_{//}$ were taken from the differential recoil studies.⁹

The calculation was programmed for the Purdue CDC 6500 computer and proceeded in the following fashion. The spectra of breakup velocities were stored in the form of cumulative probability distributions and, for a given iteration, a random number was used to select a particular value of V by the equation

$$V = [V(N) - V(N-1)] \frac{[SPV(N) - RN]}{[SPV(N) - SPV(N-1)]}. \quad (2)$$

Here $V(N-1)$ and $V(N)$ are two consecutive entries in the table of V values and $SPV(N-1)$ and $SPV(N)$ are their respective cumulative probabilities. The choice of a particular $V(N)$ is dictated by the condition that

$$SPV(N-1) \leq RN \leq SPV(N) \quad (3)$$

where RN is a random number between 0 and 1.

The direction of V relative to the beam axis was determined by random choice of the cosine of the emission angle between -1 and +1, corresponding to isotropic emission in the moving system.

The value of $v_{//}$ for the iteration in question was either the unique input value, a value chosen randomly from a rectangular distribution of $v_{//}$, or a value correlated with the particular choice of V , $v_{//} = kV^n$. The correlation exponent n was obtained from the differential experiments⁹ and the constant k was chosen to maintain the correct relationship between $\langle v_{//} \rangle$ and $\langle V \rangle$,

$$k = \langle v_{//} \rangle / \langle V \rangle^n \quad (4)$$

The velocity of the recoil product in the laboratory system, V_L , was obtained from the vectorial addition of $v_{//}$ and V . It was assumed that the transverse component of the impact velocity, $v_{\perp}$, was equal to zero.

B. Range-Velocity Relation

The velocity of the fragment in the laboratory frame was converted to its

range in uranium by means of an appropriate range-velocity relation.

The relation used in this calculation was based on the energy loss measurements of Br and I ions in UF_4 by Bridwell and Moak.¹⁴ These workers correlated their data by means of the following relation valid for Br and I in uranium over the energy range of 10-100 MeV:

$$R(V_L) - R(X \text{ MeV}) = A_2 Z_2^{-1/2} \left[0.0685 Z_1^{1/2} V_L - 0.182 \right] \text{mg/cm}^2 \quad (5)$$

where X is approximately 10 MeV and is obtained by setting the quantity in brackets equal to zero, A_2 and Z_2 are the mass and atomic numbers of the stopping medium, Z_1 is that of the recoil, and V_L is expressed in $(\text{MeV}/\text{amu})^{1/2}$.

The above relation has an unspecified constant, namely the range of a 10 MeV recoil. This quantity was determined by adjusting the calculated ranges to fit the experimental average ranges in uranium of various products from the fission of uranium by thermal neutrons as measured by Niday¹⁵. In order to perform this calculation it was first necessary to determine the most probable nuclear charge and kinetic energy of the products arising from thermal neutron fission. The values of Z_p were obtained from various published reports.¹⁶⁻¹⁹ The kinetic energies of the primary products were obtained from the study by Schmitt et al.²⁰ and a correction for neutron emission from the primary fragments was applied on the basis of the neutron measurements by Appalin et al.²¹

The results of this calculation are summarized in Table I. It is seen that the ranges of 10 MeV products, obtained as the difference between the experimental ranges and the values based on Eq. (5) do not uniformly show the expected inverse dependence on mass. The observed fluctuations are a measure of the uncertainties inherent in the direct application of Eq. (5) to a variety

of fission products, the choice of kinetic energy values, and the use of a single Z value to average over the actual charge distribution. The adjustment of the constant to fit the measured ranges should put Eq. (5) on a reasonably sound basis for the analysis of the high-energy data.

In order to apply this relation to the fragments of interest it was necessary to determine the effective nuclear charge with which they are produced at GeV energies. This information was obtained from charge dispersion measurements²²⁻²⁴ and is summarized in Table I. The ranges of ~ 10 MeV fragments were interpolated from the values derived for the thermal neutron fission products. The ranges were then obtained by means of Eq. (5). For energies below ~ 10 MeV a linear range-energy relation was assumed, $R = kE$, with the proportionality constant adjusted to fit the listed ranges of ~ 10 MeV fragments. This is a highly approximate procedure since the 10 MeV fragment ranges bear the entire uncertainty inherent in the use of Eq. 5. It is nonetheless considered to be satisfactory in view of the fact that the probability of emitting fragments with such low energies is small.

The ranges obtained in this fashion from the calculated V_L values are directed at an angle θ_L with respect to the beam axis. In order to duplicate the results of thick-target recoil studies the calculated ranges were projected along this axis. For a single iteration the ranges projected at 0° and 180° to the beam axis, denoted by fW and bW , respectively, are given by the relations:²⁵

$$\begin{aligned} fW &= R \cos \theta_L & 0 \leq \theta_L \leq \pi/2 & \quad (6) \\ &= 0 & \pi/2 \leq \theta_L \leq \pi \end{aligned}$$

$$\begin{aligned} bW &= 0 & 0 \leq \theta_L \leq \pi/2 & \quad (7) \\ &= |R \cos \theta_L| & \pi/2 \leq \theta_L \leq \pi \end{aligned}$$

The average projected ranges, FW and BW, are obtained by averaging the individual values over all N iterations performed:

$$FW = \frac{1}{N} \sum_N fW \quad (8)$$

A similar equation defines BW.

C. Vector-Model Analysis

The values of FW and BW were usually obtained on the basis of 50,000 iterations. These values were then treated as "experimental" quantities and analyzed to yield the values of $\langle v_{//} \rangle$ and $\langle V \rangle$. The analysis was performed by means of the equations derived by Sugarman:⁷

$$2W(F + B) = R_0 \left[1 + \eta_{//}^2 (N + 1)^2 / 4 \right] \quad (9)$$

$$W(F - B) = R_0 \eta_{//} (N + 2) / 3 \quad (10)$$

The quantity $\eta_{//}$ is defined as $\eta_{//} = v_{//} / V$ and the range, R_0 , is that of the fragment in the moving frame, i.e., corresponding to the velocity V . In these equations N is the range-velocity exponent defined in Eq. 1. The desired values of N for the various fission products were determined by fitting Eq. (1) to the range-energy relation given by Eq. (5) in the vicinity of the mean breakup velocity of the fragment, $\langle V \rangle$. These values of N are summarized in Table I. It should be stated that the above equations neglect higher order terms in an expansion and so are only strictly valid when $\eta \ll 1$ for each interaction.

The analysis thus involves solving Eqs. 9 and 10 for R_0 and $\eta_{//}$, obtaining V from R_0 by means of Eq. 1, and then $v_{//}$ from the relation $v_{//} = \eta_{//} V$. Keeping in the spirit of the model, it is then assumed that these values of V and $v_{//}$ are in fact the true average values of these quantities, $\langle V \rangle$ and $\langle v_{//} \rangle$.

It should be noted that Eqs. 9 and 10 assume that the emission of fragments is isotropic in the moving frame and that $v_{\perp} = 0$. These assumptions are consistent with the model on whose basis the projected ranges were calculated. Since the effects of anisotropy and of $v_{\perp}$ on the calculated values of $\langle V \rangle$ and $\langle v_{\parallel} \rangle$ are known to be very small, these parameters and their possible distributions and correlations were not included in the model used in this analysis.

III. Results

A. Analysis of High-Energy Fission Data

The results of the calculation are summarized in Table II. Listed for each product are the input values of the mean velocities, $\langle v_{\parallel} \rangle_i$ and $\langle V \rangle_i$, a brief statement concerning the distributions of $v_{\parallel}$ and V , the values of $2W(F+B)$ and $W(F-B)$ obtained from the Monte Carlo calculation, the mean values of the velocities obtained by use of Eqs. 9 and 10, $\langle v_{\parallel} \rangle_o$ and $\langle V \rangle_o$, and the percentage difference between the output and input velocities, discussed in more detail below.

The first entry for each product is the result of a calculation in which unique input values of $v_{\parallel}$ and V , equal to the average values of the respective distributions, $\langle v_{\parallel} \rangle_i$ and $\langle V \rangle_i$, were used. This calculation served several purposes. For unique values of $v_{\parallel}$ and V the conventional analysis is exact and so the output values of $v_{\parallel}$ and V should be identical to the corresponding input values for a sufficiently large number of iterations. This particular calculation thus serves as an overall check on the method. The analysis of the 2.2 GeV data shows that, on the average, the values of V_o agree to within 0.2% with the values of V_i while the values of $v_{\parallel i}$ and $v_{\parallel o}$ agree to within 2%. This agreement

is consistent with the statistical uncertainty expected for 50,000 iterations. The discrepancy is somewhat larger for Ba^{131} at 6 GeV because the assumption that $n_{//} \ll 1$ is no longer strictly valid.

The use of the various velocity distributions often led to differences between the input and output velocities that were of comparable magnitude to the statistical uncertainty. The calculation with unique input velocities made a reduction of this uncertainty possible. This improvement was accomplished by the use of identical random number sequences for all calculations involving a particular product. In those computations where fewer random numbers were required, additional ones were generated but not used at the appropriate steps in the program. This procedure insured the congruence of the random numbers used at equivalent steps in the various computations. In order to take advantage of this increase in relative accuracy the output values of $\langle v_{//} \rangle$ and $\langle V \rangle$ were in all cases compared with the corresponding output values of the calculation with unique values of $v_{//i}$ and V_i instead of with the actual input velocities. The differences between these values are summarized in Columns 10 and 11 of Table II. This procedure also reduced the error inherent in the use of the vector-model equations in situations where the condition $n_{//} \ll 1$ is no longer met.

The first set of entries in Table II refers to Mo^{99} . The spectrum of V values shown in Fig. 1 indicates that in this case there is no overlap between the v and V distributions. It is seen that the conventional analysis predicts $\langle V \rangle_0$ values that are in virtually perfect agreement with $\langle V \rangle_i$. On the other hand the values of $\langle v_{//} \rangle_0$ are some 2.5% larger than the input values irrespective of whether the latter are unique, uniformly distributed over a range of values, or correlated with V .

The next set of entries is for Pd^{103} . The distribution of breakup velocities is intermediate to those of Mo^{99} and Ba^{131} and the overlap between $v_{//}$ and the V spectrum is less than 1%. The analysis of the calculated projected ranges on the basis of the vector model now underestimates the value of $\langle v_{//} \rangle$ by about 1%, again irrespective of the details of the $v_{//}$ distribution. On the other hand the analysis overestimates the values of $\langle V \rangle$ by approximately 0.5%.

The distribution of V values for Ba^{131} is fairly broad and extends to values corresponding to energies well below 10 MeV. The input value of $v_{//}$ is in this case larger than about 1% of the V values. The results in Table II indicate that the vector model underestimates the value of $\langle v_{//} \rangle$ by 4-5% and overestimates that of $\langle V \rangle$ by just over 1%.

Also included in Table II are some entries for Ba^{131} for bombarding energies in the neighborhood of 6 GeV and above. It is believed that at these energies the spectrum of breakup velocities will be considerably broader than at 2 GeV and may look approximately like the dashed curve in Fig. 1. The basis of this expectation is the recent finding²⁶ that the average range of Ba^{131} decreases by about 30% between 2 and 6 GeV. Crespo et al.⁹ have decomposed their spectrum of breakup velocities into a low-velocity component associated with spallation and a higher velocity component due to fission. The spectrum shown in Fig. 1 for Ba^{131} formed at 6 GeV results from the assumption that the decrease in range between 2 and 6 GeV is ascribable to an increase in the relative contribution from spallation. The resultant spectrum is then obtained by combining the spallation and fission spectra for 2 GeV protons in the correct proportions,

two to one, to yield a mean velocity corresponding to the experimental average range at 6 GeV.

In performing this calculation it was somewhat arbitrarily assumed that $\langle v_{//} \rangle$ increases by a factor of two between 2 and 6 GeV, a value that is not unreasonable if the relation between $\langle v_{//} \rangle$ and the average deposition energy obtained at lower energies¹² continues to hold at 6 GeV. It is seen that the discrepancy between the velocity values obtained from the vector model analysis and the actual average input values is now quite substantial. The value of $\langle v_{//} \rangle$ is underestimated by some 23% while $\langle V \rangle$ is overestimated by 3-5%. As before there is little difference between a unique value of $v_{//}$ and a reasonable range of equiprobable values.

It may be concluded from these results that in most instances the conventional analysis of thick-target recoil data yields values of $\langle v_{//} \rangle$ and $\langle V \rangle$ that agree to within a few percent with the actual average values of these quantities. The only sizeable effect occurs in the case of Ba^{131} at 6 GeV where the distribution of breakup velocities is very broad, spans a region where the functional form of the range-energy relation changes, and overlaps $v_{//}$ to a significant extent. As indicated above, even in this case the discrepancy is a moderate 20%.

It should be stated that the approximate validity of the vector-model analysis of thick target recoil data implied by the present calculation refers only to the values of $\langle v_{//} \rangle$ and $\langle V \rangle$. The subsequent use of $\langle v_{//} \rangle$ to obtain the average deposition energy depends on considerations outside the scope of the present calculation. Some comments can be made about the use of $\langle V \rangle$ to obtain

the average kinetic energy of the fission fragments by the relation $\langle T \rangle = \frac{1}{2} M \langle V \rangle^2$. This procedure requires that $\langle V \rangle^2 = \langle V^2 \rangle$ which is in error when the V distribution is broad. For instance, the use of $\langle V \rangle^2$ instead of $\langle V^2 \rangle$ underestimates the kinetic energy of Mo^{99} at 2.2 GeV by just over 1% and that of Ba^{131} at the same bombarding energy by about 10%. However, since the calculation overestimates $\langle V \rangle$ in the first place, the net effect on the kinetic energy is smaller than indicated.

B. Results for Simple Velocity Distributions

In order to obtain further insight into the individual effect of the various factors affecting the analysis of thick target recoil data a number of calculations designed to isolate these effects were performed. All the calculations were done for a fragment with $A = 100$ using unique values of $v_{//}$ and rectangular distributions of V with a mean value of $1.0 \text{ (MeV/amu)}^{1/2}$.

The effect of an overlap between $v_{//}$ and V is shown in Fig. 2. The V distribution was assumed to be uniform between 0 and $2 \text{ (MeV/amu)}^{1/2}$ and the percentage overlap is defined as $(v_{//}/2.0) \times 100$. The calculation was performed for several range-velocity relations. The calculation for $R = kV_L$ with $\langle V \rangle$ obtained from the conventional analysis with $N = 1$ gives the clearest evidence on the effect of an overlap between $v_{//}$ and V since in this case the vector model equations are exact and valid up to $\eta_{//} = 1$. It is seen that the conventional analysis increasingly overestimates $\langle V \rangle$ as the overlap increases. On the other hand, the overlap has virtually no effect on the value of $\langle v_{//} \rangle$.

Also shown in Fig. 2 are curves based on $R = kV_L^{1.5}$ and $R = kV_L - \Delta$ (i.e. Eq. 5) for the Monte Carlo calculations and a vector model analysis employing the relation $\langle V \rangle = (\langle R_O \rangle / k)^{1/1.5}$. It can be seen that in both instances

the vector model overestimates $\langle V \rangle$ and also underestimates $\langle v_{//} \rangle$. It should be noted, however, that in these cases there are factors besides the overlap effect contributing to the discrepancy. This can be seen from the fact that $\langle V \rangle_o / \langle V \rangle_i$ is already larger than unity even when there is no overlap.

The additional factor affecting the calculation for $R = kV_L^{1.5}$ is the effect of the averaging process on the calculation of the mean range and the subsequent evaluation of $\langle V \rangle$. The averaging process leads to correct results provided that $\langle V \rangle = \langle V^N \rangle^{1/N}$. This is not the case when $N \neq 1$ and the distribution is broad. The effect of this factor is shown in Fig. 3 where the ratio of average output to input velocities is plotted as a function of $\Delta V / \langle V \rangle$. As before a rectangular distribution whose width is ΔV , having a mean value $\langle V \rangle = 1.0 \text{ (MeV/amu)}^{1/2}$, was assumed. The ratio of calculated to input $\langle V \rangle$ values shows the expected increase both with the width of the V distribution and the value of N . The effect of the averaging process on the value of $\langle v_{//} \rangle$ is smaller and $\langle v_{//} \rangle_o / \langle v_{//} \rangle_i$ is seen to be slightly larger than unity for moderately wide V distributions and to decrease below unity for the broadest distributions.

The calculation in which the vector model is used to analyze ranges that cannot be described by the relation $R = kV_L^N$ over a broad range of V_L values is subject to the error inherent in this approximation. The magnitude of this effect is shown in Fig. 3 for a calculation in which the ranges were obtained from the laboratory velocities by means of Eq. 5 and then analyzed in the conventional way with $N = 1.5$. It is seen that this procedure has a negligible effect on the value of $\langle V \rangle$ but leads to a substantial underestimation of $\langle v_{//} \rangle$ in the case of broad velocity distributions.

The effect of correlations between $v_{//}$ and V was investigated for rectangular distributions of V , with $v_{//}$ being obtained for a given iteration as $v_{//} = kV^n$. The effect of this correlation was investigated for n values ranging from -3 to $+3$. It was found in all cases that the effects of the correlation on $\langle v_{//} \rangle$ and $\langle V \rangle$ were negligible.

In summary, the use of some simple velocity distributions shows that there are a number of factors affecting the values of $\langle V \rangle$ and $\langle v_{//} \rangle$ obtained by the vector-model analysis of thick-target recoil data. If the distribution of velocities is broad the analysis may overestimate $\langle V \rangle$. The principal factors contributing to this effect are the overlap between $v_{//}$ and V and the difference between $\langle V \rangle$ and $\langle V^N \rangle^{1/N}$ when the range-velocity exponent, N , is not unity. Under the same conditions, the analysis underestimates the value of $\langle v \rangle$ both because of the overlap between $v_{//}$ and V and the use of a range-velocity relation outside of its region of validity. These conclusions are in general agreement with the results obtained with the more realistic velocity distributions for high-energy fission fragments. However, the principal conclusion of our calculation is that in most realistic situations these effects are small. Only in the most unfavorable cases, corresponding to neutron deficient products formed at bombarding energies above 4-6 GeV, do the errors become as large as 25%. The validity of the analysis of thick-target recoil data of high-energy nuclear reactions proceeding via a two-step mechanism is thereby confirmed.

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Table I

Range-Energy Data for Products from the Fission of Uranium

Observed Product	Z ^p ^c	Kinetic Energy ^e (MeV)	Average Range in Uranium ^f (mg/cm ²)	R-R(XMeV) ^g (mg/cm ²)	R(XMeV) ^h (mg/cm ²)	X (MeV)	N ^j
^a As ⁷⁷	30.60	98.8	12.3	10.60	1.70	8.9	
Sr ⁸⁹	35.37	100.6	11.05	10.68	0.37	8.9	
Zr ⁹⁷	39.14	100.4	10.84	10.78	0.06	8.8	
Ru ¹⁰³	41.42	99.3	10.72	10.67	0.05	8.8	
Ag ¹¹¹	43.98	81.1	9.38	9.11	0.27	8.9	
Sb ¹²⁷	49.5	79.8	9.30	8.89	0.41	9.1	
Te ¹³²	51.07	79.0	9.35	8.77	0.58	9.1	
Ba ¹⁴⁰	54.39	68.1	8.50	7.85	0.65	9.1	
Sm ¹⁵³	60.1	50.5	7.21	6.19	1.04	9.0	
^b Mo ⁹⁹	40.6 ^d				0.05 ⁱ	7.8	1.50
Pd ¹⁰³	46.7				0.34	8.6	1.44
Ba ¹³¹	57.1				0.88	8.1	1.58
							1.97 ^k

a. Thermal neutron fission.

b. High-energy fission.

c. Based on refs. 16-19.

d. Based on refs. 22-24.

e. Based on refs. 20 and 21.

f. From reference 15.

g. Calculated by Eq. 5.

h. Difference between columns 4 and 5.

i. Interpolated from thermal neutron data.

j. Constant in range-velocity relation, Eqs. 9 and 10.

k. For analysis of 6 GeV results.

Table II

Results of Monte Carlo and Vector-Model Calculations

Product	$\langle v_{//i} \rangle_i$ (MeV/amu) ^{1/2}	Distribution of $v_{//i}$	$\langle V \rangle_i$ (MeV/amu) ^{1/2}	Distribution of V_i	$2W(F+B)$ (mg/cm ²)	$W(F-B)$ (mg/cm ²)	$\langle v_{//o} \rangle_o$ (MeV/amu) ^{1/2}	$\langle V \rangle_o$ (MeV/amu) ^{1/2}
Mo ⁹⁹	0.05	unique value	1.241	unique value	8.988	0.4141	0.0490	1.240
	0.05	unique value	1.241	from ref. 9	8.993	0.4236	0.0502	1.241
	0.05	$0 \leq v_{//i} < 0.1^b$	1.241	from ref. 9	9.001	0.4240	0.0503	1.241
	0.05	$v_{//i} = kV_i^{-0.25^c}$	1.241	from ref. 9	8.993	0.4236	0.0503	1.241
Pd ¹⁰³	0.101	unique value	0.994	unique value	7.469	0.8837	0.1040	0.9922
	0.101	unique value	0.994	from ref. 9	7.509	0.8753	0.1029	0.9959
	0.101	$0 \leq v_{//i} < 0.202$	0.994	from ref. 9	7.549	0.8755	0.1027	0.9995
	0.101	$v_{//i} = kV_i^{-0.25}$	0.994	from ref. 9	7.515	0.8752	0.1028	0.9964
Ba ¹³¹	0.08	unique value	0.724	unique value	5.758	0.7543	0.0810	0.7222
	0.08	unique value	0.724	from ref. 9	5.852	0.7268	0.0774	0.7304
	0.08	$0 \leq v_{//i} < 0.16$	0.724	from ref. 9	5.891	0.7275	0.0773	0.7334
	0.08	$v_{//i} = kV_i^{-0.25}$	0.724	from ref. 9	5.863	0.7249	0.0771	0.7313
Ba ^{131a}	0.16	unique value	0.526	unique value	3.651	1.3139	0.1742	0.5090
	0.16	unique value	0.526	see Fig. 1	3.527	1.0483	0.1346	0.5227
	0.16	$0 \leq v_{//i} < 0.32$	0.526	see Fig. 1	3.705	1.0774	0.1342	0.5366

a. entries for Ba¹³¹ at 6 GeV. b. rectangular distribution of $v_{//i}$ c. $v_{//i}$ correlated with V_i

Figures

Figure 1 - Distribution of breakup velocities for products from the 2.2 GeV proton fission of uranium.⁹ --- Mo⁹⁹; — Ba¹³¹; ---- Ba¹³¹ at 6 GeV.

Figure 2-Effect of overlap between $v_{//}$ and the V distribution on the values of $\langle v_{//} \rangle$ (closed symbols) and $\langle V \rangle$ (open symbols) calculated by the vector-model equations. O - based on $R = kV_L$ and $\langle V \rangle = \langle R_O \rangle / k$. $\square$ - based on $R = kV_L - \Delta$ and $\langle V \rangle = (\langle R_O \rangle / k)^{1/1.5}$; Δ - based on $R = kV_L^{1.5}$ and $\langle V \rangle = (\langle R_O \rangle / k)^{1/1.5}$.

Figure 3 - Dependence of calculated values of $\langle v_{//} \rangle$ (closed symbols) and $\langle V \rangle$ (open symbols) on the width of the V distribution for various range-velocity relations. Δ - based on $R_L = kV_L - \Delta$ and $\langle V \rangle = (\langle R_O \rangle / k)^{1/1.5}$; O - $R = kV_L^{1.5}$ and $\langle V \rangle = (\langle R_O \rangle / k)^{1/1.5}$; $\square$ - $R = kV_L^2$ and $\langle V \rangle = (\langle R_O \rangle / k)^{1/2}$.

Fig. 1

Fig. 2