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Parametric Instability and Anomalous Backscattering of Electromagnetic Waves in Underdense Plasmas

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The conditions for stimulated backscattering of an electromagnetic wave by plasma waves in an underdense plasma ($\omega_0 \gg \omega_p$), where the Debye length is smaller than the incident laser wavelength, are derived. In an inhomogeneous plasma with density scale length L , significant amplification is attained if $\epsilon_0^2/4\pi > nmc^4\omega_0/2\omega_p^2 L$. Application to laser-pellet irradiation is considered.

In considering the possibility of fusion by intense laser irradiation on a solid deuterium pellet, it is important that a significant fraction of light is absorbed by the plasma. It is well-known that, near the region of critical density where the local plasma frequency ω_p is equal to the laser frequency ω_0 , the parametric decay of the EM wave into electron plasma waves and ion waves constitutes an important process for the anomalous absorption of the laser light. As the laser light is nearly normally incident on a plasma profile of increasing density, it must pass through the underdense region, where the local plasma frequency ω_p is much lower

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than the laser frequency ω_0 , to reach the region of critical density. Though the underdense region is transparent to the EM wave according to the linear theory, it is shown here that an important backscattering instability exists if the local Debye length $\lambda_D = (KT/4\pi n e^2)^{1/2}$ is smaller than the laser wavelength, whereby the laser light decays into a backward scattered EM wave and a forward propagating electron plasma wave (plasmon). A major fraction of the incident energy is therefore "anomalously backscattered" in the underdense region once certain threshold power is exceeded, thus preventing the light wave from reaching the critical density where it can be anomalously absorbed.

In an inhomogeneous plasma, this threshold power for the stimulated backscattering is mainly due to the propagation of the light wave out of the interaction region, usually higher than that for the anomalous absorption. Hence there is an optimum laser power for the absorption of light and the heating of the plasma. This threshold power for backscattering, however, is accessible in laser-plasma experiments and strong backward scattering has been observed experimentally with laser-pellet irradiation.

Consider the linearly polarized EM wave $\epsilon^{(0)} = 2\epsilon_0 \hat{y} \sin(k_0 x - \omega_0 t)$ incident on an underdense region of a plasma where $\omega_p = (4\pi n_0 e^2/m)^{1/2} < \omega_0$. In the following discussion, we assume $\alpha = (\epsilon_0/m\omega_0 c) \ll 1$. The excited waves are assumed to be propagating along x with the vector potential

of the EM wave $\vec{A}$ along $\hat{y}$. The electron equation of motion is

$$nm\left(\frac{\partial \vec{v}}{\partial t} + \vec{v} \cdot \nabla \vec{v}\right) = -\nabla p + ne\left(\vec{E} + \frac{1}{c}\vec{v} \times \vec{B}\right) - nm\nu\vec{v} \quad (1)$$

where p is the electron pressure, $\nu = \omega_p (2/\pi)^{1/2} \ln \Lambda / \Lambda$ is the electron-ion collision frequency and $\Lambda = 4\pi n \lambda_D^3$. The y -component velocity, v_y , can simply be solved from Eq. (1) as $v_{0y} = ier_0/m\omega_0$ and $v_{ly} = eA_l/mc$ due to their high-frequency $\omega \gg \omega_p$. The x -component of the $\vec{v} \times \vec{B}$ force due to the beating of the incident and secondary EM waves can then drive the plasma oscillation if the frequency and wave number matching conditions are satisfied, thus serving as the principal nonlinear source term in Eq. (1).

With Poisson's equation: $\partial E / \partial x = 4\pi n_1 e$ and the continuity equation: $\partial n_1 / \partial t + n_0 \partial v_x / \partial x = 0$, the x -component of Eq. (1) can be written

$$\begin{aligned} \text{as} \\ -\frac{\partial^2 n_1}{\partial t^2} + 3v_e^2 \frac{\partial^2 n_1}{\partial x^2} - \nu \frac{\partial n_1}{\partial t} - \omega_p^2 n_1 = -2\left(\frac{e}{mc}\right)^2 \epsilon_0 \left\{ \frac{c}{\omega_0} \frac{\partial}{\partial x} \left[\cos(k_0 x - \omega_0 t) \frac{\partial A}{\partial x} \right] \right. \\ \left. - \frac{\partial}{\partial x} [\sin(k_0 x - \omega_0 t) A] \right\}. \end{aligned} \quad (2)$$

The density fluctuation of the plasma wave, on the other hand, can couple with v_{0y} to serve as the nonlinear current source in the Maxwell equation to drive the secondary EM wave:

$$\begin{aligned} c^2 \frac{\partial^2 A}{\partial x^2} - \frac{\partial^2 A}{\partial t^2} + \omega_p^2 A - \left\{ \nu \omega_p^2 \left[\exp(-\nu t) \right] \int_0^t dt' \left[\exp(\nu t') \right] A(t') \right\} \\ = \frac{2\omega_p^2 c}{n_0 \omega_0} n_1 \epsilon_0 \cos(k_0 x - \omega_0 t). \end{aligned} \quad (3)$$

Taking the Fourier transform one obtains

$$\left[\omega^2 - (\omega_p^2 + 3k^2 v_e^2) + i\omega\nu \right] n(k, \omega) = n_0 \left(\frac{e}{m}\right)^2 \frac{\epsilon_0 k^2}{c\omega_0} \left[A(k-k_0, \omega-\omega_0) + A(k+k_0, \omega+\omega_0) \right] \quad (4)$$

$$\left[\omega^2 - (\omega_p^2 + k^2 c^2) + i\omega_p^2 \frac{\nu}{\omega} \right] A(k, \omega) = \frac{c\omega_p^2 \epsilon_0}{n_0 \omega_0} [n(k+k_0, \omega-\omega_0) + n(k+k_0, \omega+\omega_0)] \quad (5)$$

where $\nu \ll \omega$ is assumed in arriving at Eq. (5). These sets of Eqs.

[(2), (3), (4) and (5)] are now in the canonical form for the parametric coupling and Nishikawa's general theory³ for parametric processes is applicable.

Assuming that the frequency and wave-number matching conditions $\omega_0 = \omega_1 + \omega_2$ and $k_0 = k_1 + k_2$ are approximately satisfied where $\omega_1^2 = \omega_p^2 + 3k_1^2 v_e^2$, $\omega_2^2 = \omega_p^2 + k_2^2 c^2$ and $\omega \approx \omega_1$, $k \approx k_1$ then one can neglect the off-resonance high-frequency response $n(\omega \pm 2\omega_0, k \pm 2k_0)$. In this way, Eqs. (4) and (5) are reduced to a set of linear homogeneous equations for $n(\omega, k)$, $A(\omega-\omega_0, k-k_0)$ and $A(\omega+\omega_0, k+k_0)$ and the vanishing of the determinant then yields the eigenvalue equation for the frequency and growth rate:

$$1 = \frac{\lambda \mu \epsilon_0^2}{D_0} \left[\frac{1}{D_-} + \frac{1}{D_+} \right] \quad (6)$$

where $D_0 = \omega^2 - \omega_1^2 + i\omega\nu$, $D_{\pm} = (\omega \pm \omega_0)^2 - \omega_{\pm}^2 + i\omega_p^2 \nu / (\omega \pm \omega_0)$, $\omega_{\pm}^2 = \omega_p^2 + (k \pm k_0)^2 c^2$, and $\lambda = (e/m)^2 k^2 n_0 / c\omega_0$, $\mu = \omega_p^2 c / n_0 \omega_0$. For backscattering in general or forward scattering with the excited waves near their resonant frequencies, we can neglect the off-resonance term $1/D_+$ in Eq. (6) and obtain the following dispersion relation with the resonance approximation $\omega \approx \omega_1$ and $\omega_0 - \omega_1 \approx \omega_-$:

$$(\omega - \omega_1 + i\nu/2)(\omega - \omega_0 + \omega_- + i\nu\omega_p^2/2\omega_-^2) = -\mu \lambda \epsilon_0^2 / 4\omega_1 \omega_- \quad (7)$$

The threshold power for the instability is

$$I = \frac{\nu^2}{k_1^2 c^2} \left[\frac{\omega_0^2}{\omega_p (\omega_0 - \omega_p)} \right] n m c^3 \quad (8)$$

in agreement with previous work⁴. Due to the high frequency of the wave, the frequency matching condition implies that $k_1 \approx \omega_p/c$ for the forward scattering. For the backward scattering, however, the maximum k_1 consistent with the frequency-matching condition is approximately $2k_0$. The threshold power for backscattering, including the Landau damping, with damping rate $\gamma_L = (\pi/2)^{1/2} (\omega_p/k_1^3 \lambda_D^3) \exp(-1/2 k_1^2 \lambda_D^2)$ is

$$P = \frac{\nu(\nu + 2\gamma_L)}{4(\omega_0 - \omega_p)\omega_p} n m c^3, \quad (9)$$

which is $(\omega_p/2\omega_0)^2$ lower than that for the forward scattering. The maximum growth rate of this backscattering instability as $\nu \rightarrow 0$ can be obtained from Eq. (7),

$$\gamma = \frac{e^2 \omega_0}{m \omega_0 c} (\omega_p \omega_0)^{1/2}, \quad (10)$$

which is attained at $k_1 \approx 2k_0$. The excited EM wave is thus backscattered with wave-number $-k_0$ and frequency $\omega_0 - \omega_p$. The ratio of the amplitudes of the electric field of the backscattered EM wave to that of the plasmon is $(\omega_0/\omega_p)^{1/2}$, the well-known Manley-Rowe relation. For the plasmon to be weakly damped, $2k_0 \lambda_D \ll 0.3$ is required, which then sets the maximum temperature for backscattering of an EM wave of frequency ω_0 by plasmons: $c/v_e \gtrsim 5\omega_0/\omega_p \gg 5$. Stimulated backscattering is still possible even for $2k_0 \lambda_D \approx 1$ by heavily damped quasi-modes, but a kinetic-theoretical

treatment is necessary in this region. For very high incident power i. e. $\alpha(\omega_0/\omega_p)^{1/2} > 1$, the frequency-shift of the excited plasma wave is higher than ω_p and the resonance approximation Eq. (7) breaks down and Eq. (6) must be solved for the growth rate. Assuming $\omega_0 \gg \omega \gg \omega_p$ and the damping rates of the free oscillations, we obtain the frequency and the growth rate for the plasma wave in the case of backscattering:

$$\omega = (eE_0/m\omega_0 c)^{2/3} (\omega_p^2 \omega_0/4)^{1/3} [\sqrt{3} + i] : \quad (11)$$

In an inhomogeneous plasma, the phase-matching conditions hold only locally and the propagation of the unstable waves out of the resonantly unstable region thus provides an additional threshold for the instability. We consider the effect of inhomogeneity on the decay instability with a slab model: $n(x) = n_0(1+x/L)$. The incident and the backward scattered EM waves are assumed to be propagating along the density gradient with linear polarization along y in the WKB approximation, with $\omega_0 = \omega_1 + \omega_2$ and $k_0 = k_1 - k_2$ at $x = 0$ and ω_1, ω_2 are the resonant frequencies. For $\omega_0, \omega_2 \gg \omega_p$, the spatial variation of k_0 and k_2 are negligible and the phase mismatch is primarily due to the spatial variation of the plasmon wave-number k_1 . Neglecting the damping and off-resonance terms, we obtain the following equations for the slowly varying amplitudes:

$$\left(\frac{\partial}{\partial t} + v_g \frac{\partial}{\partial x}\right) n_{kl, \omega_1}(x, t) = -i \left(\frac{e}{m}\right)^2 \frac{k_1^2 \epsilon_0 n_0}{2\omega_1 \omega_0 c} A_{k2, \omega_2}^*(x, t) \exp(i \int^x \Delta k dx') \quad (12)$$

$$\left(\frac{\partial}{\partial t} + c \frac{\partial}{\partial x}\right) A_{k2, \omega_2}^* = i \frac{c \omega_p^2 \epsilon_0}{2n_0 \omega_0 \omega_2} n_{kl, \omega_1}(x, t) \exp(-i \int^x \Delta k dx') \quad (13)$$

where $v_g = 3v_e^2 k / \omega_p$ is the group velocity of the plasmon. Clearly, in the absence of inhomogeneity, one recovers from Eqs.(12) and(13) the growth rate given by Eq. (10). Applying Rosenbluth's general theory of parametric instability in inhomogeneous media,⁵ we find that for the linear density profile or $\Delta k(x) = k(0) + [d(\Delta k)/dx] x$, there is no temporal growth but only spatial amplification between $x = 0$ and the turning point, $x_T = 2\gamma_0 / |k' v_g c|^{1/2}$, with peak intensity

$$I = I_0 \exp(2\pi\gamma_0^2 / |c v_g k'|), \quad (14)$$

where I_0 is the thermal source intensity, γ_0 is given by Eq. (10), and $k' = d\Delta k/dx = (-1/2L)(\omega_p^2 / 3k v_e^2) = (-\omega_p^2 / 2L v_g)$. The effective amplification then requires that $\gamma_0^2 / |c v_g k'| > 1$ or, expressed in terms of the laser wavelength, λ_0 ,

$$c \frac{\epsilon_0^2}{4\pi} > \frac{m^2 c^5}{4\lambda_0^2 e^2 L}, \quad (15)$$

which is to say that, for significant amplification, the wave must grow substantially before it propagates to the region of significant phase mismatch. Note that this condition is independent of the density and temperature, although the local Debye length of the plasma at $x = 0$ must be small compared with wavelength so that the k-matching for backscattering can be satisfied. With power P expressed in the unit of 10^{14} watts/cm² sec and L in cm, the above condition for excitation becomes $50 LP > 1$ for 10μ -CO₂ laser and $5 LP > 1$ for 1.06μ -Nd glass laser realized in many laser-pellet experiments. Once this condition is satisfied, a major

fraction of the incident energy is backscattered, significantly reducing the anomalous absorption at the critical density, thus constituting an important process in the laser-pellet irradiation for fusion ignition. However, this threshold for backscattering due to light propagation out of the resonant region usually exceeds the threshold for the decay into electron and ion waves near the critical density where $\omega_p = \omega_0$. Thus the backscattering threshold sets a maximum incident power for optimum anomalous absorption.

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