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THE TEMPERATURE OF THIN FOILS IN ION BEAMS

J. O. Liljenzin

Lawrence Berkeley Laboratory
University of California
Berkeley, California 94720

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ABSTRACT

The effect of beam-shape, pulsed operation and ion type on the temperature of thin foils in ion beams is discussed for cases where the cooling is by heat radiation, heat conduction, or convection in an external medium. Analytic solutions are given for various combinations of beam shape and cooling conditions. Nomographs are given for the estimation of the maximum temperature of thin circular foils when the cooling is either by radiation to the surroundings or by conduction to the edge of the foil. A program is described which can be used to calculate the temperature distribution at equilibrium conditions or as a function of time for arbitrarily shaped non-isotropic and non-homogeneous bodies heated internally and cooled by any combination of radiation, conduction, or convection of heat in an external gaseous medium. The heat conductivity, heat capacity, heat generation, and grayness can vary within the body; the heat conductivity and heat capacity can vary with temperature; and the heat-generation can be pulsed. Cooling by a liquid streaming on the backside of the foil is generally most effective. Radiative cooling is also dominant for very thin foils, and conduction cooling is dominant for thick foils. At a given diameter, this leads to a maximum temperature for foils with a thickness a little below the range of the ions.

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Thin foils are used in ion beams for stripping of the ions, degradation of beam energy, as vacuum seals, and as beam-transparent targets. Homogeneous foils, made from a single substance, are used for all of the mentioned purposes. Targets are, however, often made in two layers, a backing which gives mechanical strength and impermeability for gases and a layer of the target substance which can be much thinner than the backing and lack mechanical strength.

When the beam passes through the foil it loses part of its energy by interaction with the atoms in the foil. The major part of the energy transferred from the beam to the foil occurs as heat, which must be removed to prevent the target from melting or evaporating (see Fig. 12).

1. CALCULATION OF THE TOTAL AMOUNT OF BEAM ENERGY LOST IN A FOIL

For sufficiently thin foils the electronic stopping power, $^1 -dE/dl$, can be assumed constant. The energy transferred to the foil per unit time, P_{tot} , can then be calculated from the relation,

$$P_{tot} \approx 2 \left(-\frac{dE}{dl} \right) I / Z_{eff}, \quad (1)$$

where l is the foil thickness, I the electric beam current and Z_{eff} the mean electric charge of the ions.

The electronic stopping power is no longer constant for thicker foils. The energy transferred to the foil must then be calculated from the range variation with energy.¹ Let E_0 and $R(E_0)$ be the energy and range in the foil material of the incident ions, E_l and $R(E_l)$ the energy and range of the

ions after passing a layer of foil material of thickness l . The residual range, $R(E_f)$ is computed from

$$R(E_f) = R(E_0) - l, \quad (2)$$

which gives E_f , and then P_{tot} can be calculated from

$$P_{tot} = (E_0 - E_f) \cdot I / Z_{eff}, \quad (3)$$

where I/Z_{eff} is the "particle current."

Equation (2) is most conveniently solved in a graph of R vs E , where E_f then can be read off on the energy scale.

2. COOLING METHODS

The heat evolved in the target can be removed by many different mechanisms, some of which are probably without interest in practical applications, i.e., removal by vaporization of the foil material, or by melting of the foil. The most useful cooling methods are cooling by heat radiation, cooling by heat conduction, and cooling by convection in an external fluid medium.

2.1. Radiation Cooling

When the foil surface is at a temperature, T_{surf} , which is higher than the surroundings (which are at a temperature T_{sur}), the foil will radiate away heat with a rate per unit surface area, P_{rad} , given by Stefan's law.

$$P_{rad} = g \delta (T_{surf}^4 - T_{sur}^4), \quad (4)$$

where T_{surf} and T_{sur} are in degrees K, δ has the value 5.70×10^{-12} ($\text{W cm}^{-2} \text{ degree}^{-4}$) and g is the grayness factor as compared to a black body ($0 < g \leq 1$). The grayness factors for some common materials are listed in Table I.

2.2. Conduction Cooling

When an internal volume element of the foil is at a different temperature than its surroundings, heat will flow in a direction as to eliminate the temperature differences.

$$P_{\text{cond}} = \left\{ \frac{d(\lambda_x \frac{dT}{dx})}{dx} + \frac{d(\lambda_y \frac{dT}{dy})}{dy} + \frac{d(\lambda_l \frac{dT}{dl})}{dl} \right\} dV, \quad (5)$$

where P_{cond} is the heat conduction rate; λ_x , λ_y , and λ_l are the heat conductivities in the x , y , and l direction respectively; dV is the volume element $dx dy dl$. Constant values for the heat conductivities (λ) can only be used in a small temperature range. For larger temperature ranges an approximation with the following equation can be used:

$$\lambda_i = a_i - b_i \cdot T + c_i \cdot T^{-2}. \quad (6)$$

Fortunately most materials are isotropic, i. e., $\lambda_x = \lambda_y = \lambda_l$. Values of a , b , and c in Eq. 6 are given in Table II for some common materials.

2.3. Convection Cooling

The effect of convection cooling is highly dependent on the heat transfer between the foil and the cooling medium. This is usually expressed as a film-transfer coefficient h defined by

$$P_{\text{conv}} = h (T_{\text{surf}} - T_{\text{fluid}}), \quad (7)$$

where P_{conv} is the heat transfer rate per unit surface (wall area), h is the film transfer coefficient, T_{surf} is the foil surface temperature, and T_{fluid} is the fluid phase (gas or liquid) bulk temperature.

The calculation of the heat removed by convection cooling is now transformed to a problem to calculate the film-transfer coefficient (h). This is usually solved by the application of dimensional theory to relate the actual problem to another well-investigated case for which an empirical equation is available.

The dimensionless numbers of interest when scaling and comparing film-transfer coefficients are: Reynolds number (Re), Nusselts number (Nu), and Grashofs number (Gr).² For large streaming velocities ($Re \geq 2100$) with turbulent flow the value of Gr is no longer important for calculating the film-transfer coefficient.

The main problem is to find the comparable case. As a rough approximation, the foil can be thought of as a part of the wall in a circular tube. Then the following equation can be used² at low velocities and laminar flow

(Re < 2100):

$$Nu = 1.62 \sqrt[3]{\mu/\mu_f} (1 + 0.015 \sqrt[3]{Gr}) \sqrt[3]{\frac{4 w C_p}{\pi \lambda_{\text{fluid}} D}}, \quad (8)$$

where Nu and Gr are

$$Nu = h \cdot D / \lambda_{\text{fluid}}, \quad (8a)$$

$$Gr = D^3 \rho^2 \beta g (T_{\text{surf}} - T_{\text{fluid}}), \quad (8b)$$

where μ is the viscosity of fluid at the bulk temperature, μ_f is the viscosity of the fluid at the film mean temperature, and w is the flow rate (mass per time). D is a characteristic length (the diameter), λ_{fluid} is the heat conductivity of the fluid, ρ is the density of the fluid, β is the volume expansion coefficient, g the gravity constant, and C_p is the heat capacity of the fluid. The Grashof number reflects the increased flow due to the heating of the fluid at the surface.

For water streaming upwards, the simpler equation

$$h = (2.685 T_{\text{water}} + 51.62) \sqrt[3]{T_{\text{surf}} - T_{\text{water}}} \cdot 10^{-4} \quad (9)$$

can be used instead of (8).² Here T is in degrees centigrade and h is in $W/cm^2 \cdot ^\circ C$.

When the streaming velocity of the fluid is so high that turbulent flow occurs, $Re > 10\,000$, the following equation is valid for all fluids streaming upwards in a pipe:²

$$Nu = 0.023 (Re)^{0.8} \cdot (Pr)^{0.4}. \quad (10)$$

For liquids with $\mu \leq 2$ cP and for gases with $w \geq 600$ p^{2/3} (where p is in atm), Eq. (10) can be used almost down to $Re = 2100$; w should be calculated as kg/hour. In case of a downward flow, the corresponding empirical equation is²

$$Nu = 0.023 (Re)^{0.8} \cdot (Pr)^{0.3}, \quad (11)$$

where $Pr = C_p \mu / \lambda$.

2.4. Combined Cooling

In practice, the cooling method used is some combination of radiation cooling, conduction cooling, and convection cooling. Sometimes, depending on geometry, surface area, gas pressure, and foil temperature, one of these processes may dominate over the others. To be able to treat realistic cases, all the mentioned cooling processes must be included in the final differential equation. Assuming that heat radiation is only lost in the l direction and that the eventual surrounding fluid passes by the foil in an x - y plane, the rate of heat loss by cooling, P_{cool} , can be written

$$P_{cool} = dV \left[\frac{ng\delta}{dl} (T^4 - T_{sur}^4) + \frac{d(\lambda_x \frac{dT}{dx})}{dx} + \frac{d(\lambda_y \frac{dT}{dy})}{dy} + \frac{d(\lambda_l \frac{dT}{dl})}{dl} + \frac{nh}{dl} (T - T_{fluid}) \right]. \quad (12)$$

Some of the "constants" (i.e., λ 's and h) are in turn functions of T and some (n and g) may be functions of x , y , and l .

3. BEAM SHAPE AND TIME FUNCTION

The shape of the beam, i.e., the variation in beam intensity with x and y on the foil, is of great importance for the temperature distribution over the foil and thus for the maximum temperature that will occur at a given total beam current. The variation of beam current with time is also important when the cooling time-constant is less than or comparable to the frequency of the beam-current variations. Of the many possible variations of beam current with time, only two will be discussed: continuous beam and pulsed beam with fixed frequency. All beams will be assumed to be circular symmetric.

3.1. Uniform Beam Shape

The beam is hitting the foil with the same intensity at all points of the surface. The power released on an area dA ($= dx dy$) is then

$$P_{beam} = P_{tot} dA/A, \quad (13)$$

where A is the total exposed area of the foil.

When the beam is pulsed with a frequency f and a duty cycle of C percent,

$$t_{\text{on}} = 0.01 C/f, \quad (14)$$

$$t_{\text{off}} = (1-0.01C)/f, \quad (15)$$

where t_{on} is the ontime and t_{off} is the off time of each cycle.

The current during t_{on} must now be larger than in the stationary case to give the same mean current, I :

$$I_{\text{on}} = 100 I/C, \quad (16a)$$

$$I_{\text{off}} = 0.0. \quad (16b)$$

Thus in a pulsed beam,

$$P_{\text{beam, on}} = 100 P_{\text{tot}} dA/(AC), \quad (17a)$$

$$P_{\text{beam, off}} = 0.0. \quad (17a)$$

Tilting the foil against the beam will increase the total area A with a factor $1/\sin(\phi)$, where ϕ is the angle between the foil and the beam, and thus decrease P_{beam} .

3.2. Gaussian Beam Shape

The beam is hitting the foil with an intensity distribution (current distribution)

$$p(r) = k \cdot e^{-(r/s)^2}, \quad (18)$$

where r is the distance from the center of the beam, k is a proportionality constant and s is a shape factor: larger s , more diffuse beam; smaller s , more focused beam.

The total power dissipated within a radius R is $P(R)$

$$P(R) = 2\pi k \int_{r=0}^R r e^{-(r/s)^2} dr = \pi k s^2 [1 - e^{-(R/s)^2}], \quad (19)$$

where k is a proportionality constant between surface and power. The total power of the beam must be dissipated for $R = \infty$, thus

$$P(\infty) = P_{\text{tot}} = \pi k s^2 \quad (20)$$

and hence

$$P(R) = P_{\text{tot}} [1 - e^{-(R/s)^2}]. \quad (21)$$

When a fraction, α , of the total beam is known to hit the foil within a certain radius, R_a , s^2 can be obtained from the equation

$$s^2 = - \frac{R_a^2}{\ln(1-\alpha)}. \quad (22)$$

Thus the Gaussian beam is defined by its total energy, P_{tot} , and a fraction, α , of P_{tot} hitting within a radius R_a .

The proportionality constant k is then

$$k = - \frac{P_{\text{tot}} \ln(1-\alpha)}{\pi R_a^2}. \quad (23)$$

The beam hitting a unit surface area is now

$$p_{\text{beam}}(r) = - \frac{P_{\text{tot}} \ln(1-\alpha)}{\pi R_a^2} e^{-r^2 \sqrt{-\ln(1-\alpha)}/R_a^2}. \quad (24)$$

In the case of a pulsed beam defined by Eqs. (16a) and (16b) we obtain

$$p(r)_{\text{beam, on}} = - \frac{100 P_{\text{tot}} \ln(1-\alpha)}{\pi C R_a^2} e^{-r^2 \sqrt{-\ln(1-\alpha)}/R_a^2}, \quad (25a)$$

$$p(r)_{\text{beam, off}} = 0.0. \quad (25b)$$

When the beam passing through the foil, P_{tr} , is used instead of P_{tot} , the term $-P_{\text{tot}} \ln(1-\alpha)$ should be replaced by $-P_{\text{tr}} \ln(1-\alpha)/\alpha$ in all equations where it occurs.

3.3. Double-Gaussian Beam Shape

The beam is hitting the foil with an intensity distribution (current distribution)

$$p(r) = k \cdot e^{-\left(\frac{r-\bar{R}}{s}\right)^2} \quad (26)$$

where k is a proportionality constant as before and $\bar{R}$ is the radius at which the beam intensity is highest.

The total power dissipated within a radius R is $P(R)$:

$$P(R) = 2\pi k \int_0^R r e^{-\left(\frac{r-\bar{R}}{s}\right)^2} dr = \pi k s^2 \left[e^{-\left(\frac{R}{s}\right)^2} - e^{-\left(\frac{R-\bar{R}}{s}\right)^2} \right] + \pi^{3/2} k s \bar{R} \left[\operatorname{erf}\left(\frac{R}{s}\right) + \operatorname{erf}\left(\frac{R-\bar{R}}{s}\right) \right], \quad (27)$$

where erf is the error function. As before, the total power of the beam must be dissipated for $R = \infty$, thus

$$P(\infty) = P_{\text{tot}} = \pi k s^2 e^{-\left(\frac{\bar{R}}{s}\right)^2} + \pi^{3/2} k s \bar{R} \left[\operatorname{erf}\left(\frac{\bar{R}}{s}\right) + 1 \right] \quad (28)$$

and thus

$$P(R) = \frac{P_{\text{tot}}}{s \cdot e^{-\left(\frac{\bar{R}}{s}\right)^2} + \sqrt{\pi} [\operatorname{erf}\left(\frac{\bar{R}}{s}\right) + 1]} \left\{ s \left[e^{-\left(\frac{\bar{R}}{s}\right)^2} - e^{-\left(\frac{R-\bar{R}}{s}\right)^2} \right] + \sqrt{\pi} \bar{R} \left[\operatorname{erf}\left(\frac{\bar{R}}{s}\right) - \operatorname{erf}\left(\frac{R-\bar{R}}{s}\right) \right] \right\} \quad (29)$$

is obtained with

$$k = \frac{P_{\text{tot}}}{\pi s^2 e^{-\left(\frac{\bar{R}}{s}\right)^2} + \pi^{3/2} s \bar{R} [\operatorname{erf}\left(\frac{\bar{R}}{s}\right) + 1]} \quad (30)$$

With this value for k the beam shape is

$$p(r)_{\text{beam}} = \frac{P_{\text{tot}}}{\pi s^2 e^{-\left(\frac{\bar{R}}{s}\right)^2} + \pi^{3/2} s \bar{R} [\text{erf}(\frac{\bar{R}}{s}) + 1]} e^{-\left(\frac{r-\bar{R}}{s}\right)^2} \quad (31)$$

To determine s and $\bar{R}$, two further values of $P(R)$ must be given, but $\bar{R}$ will probably be defined by other conditions and hence already be known. The spread s can then be obtained from any other known condition, e.g., $P(0)$ known or $P(\bar{R})$ known. Transcendental equations are generally obtained of such a form that they must be solved by numerical methods.

As before, the beam at pulsed operation is given by the two states:

$$p(r)_{\text{beam, on}} = \frac{100 P_{\text{tot}}}{C \pi s (s e^{-\left(\frac{\bar{R}}{s}\right)^2} + \sqrt{\pi} \bar{R} [\text{erf}(\frac{\bar{R}}{s}) + 1])} e^{-\left(\frac{r-\bar{R}}{s}\right)^2}, \quad (32a)$$

$$p(r)_{\text{beam, off}} = 0.0. \quad (32b)$$

4. THE FOIL TEMPERATURE

The foil temperature is given by the differential equation

$$P_{\text{beam}} - P_{\text{cool}} = \rho \cdot dV \cdot C_p \frac{dT}{dt}, \quad (33)$$

where p_{beam} is given by any suitable beam shape and time function, and P_{cool} is given by Eq. (12).

No general solution to Eq. (33) is known, but for some combination of beam shape and cooling type special solutions can be obtained.

4.1. Radiation Cooling Dominates

Introducing polar coordinates, we obtain the equation

$$P_{\text{beam}} = n \sigma [T_{(r,t)}^4 - T_{\text{sur}}^4] = d \ell \rho C_p \frac{dT_{(r,t)}}{dt}. \quad (34)$$

4.1.1. Uniform beam shape

In this case Eqs. (34) and (13) give

$$\frac{P_{\text{tot}}}{\pi R^2} - n\delta g T(r, t)^4 + n\delta g T_{\text{sur}}^4 = d\ell \rho C_p \frac{dT(r, t)}{dt}. \quad (35)$$

The steady-state solution is obtained when $dT(r, t)/dt = 0$,

$$T(r, \infty) = \sqrt[4]{T_{\text{sur}}^4 + \frac{P_{\text{tot}}}{n\delta g \pi R^2}} \quad (36)$$

where n is the number of radiating surfaces. As can be seen, $T(r, \infty)$ in Eq. (36) is independent of r , so the surface will have the same temperature all over.³

When the beam is pulsed, however, the heat capacity of the foil will decrease the maximum temperature and increase the minimum temperature. The two differential equations valid during heating (t_{on}) and cooling (t_{off}) are

$$\frac{100 P_{\text{tot}}}{C \pi R^2} + n\delta g T_{\text{sur}}^4 - n\delta g T(r, t)^4 = d\ell \rho C_p \frac{dT(r, t)}{dt} \quad (37a)$$

$$\text{for } \frac{m}{f} \leq t \leq \frac{m}{f} + t_{\text{on}} \quad \text{where } m = 0, 1, 2, 3, \text{ etc.}$$

and

$$n\delta g T_{\text{sur}}^4 - n\delta g T(r, t)^4 = d\ell \rho C_p \frac{dT(r, t)}{dt} \quad (37b)$$

$$\text{for } \frac{m}{f} + t_{\text{on}} \leq t \leq \frac{m+1}{f}.$$

As C_p is a function of T very similar to Eq. (6),

$$C_p \approx a + bT + cT^{-2}. \quad (38)$$

The exact solutions of Eqs. (37a) and (37b) are quite complicated functions of T . However, the third term in Eq. (38) is only important at very low temperatures where the radiation cooling is negligible in comparison with conduction and convection. Thus a linear dependence of C_p on T is sufficient. The solution is independent of r and can be written as follows:

$$t - t_0 = \frac{1}{4B^3} \left\{ A \ln \frac{[B+T(t)][B-T(t_0)]}{[B-T(t)][B+T(t_0)]} + 2A \arctan \frac{B[T(t) - T(t_0)]}{B^2 + T(t)T(t_0)} \right. \\ \left. + D \ln \frac{[B^2 + T(t)^2][B^2 - T(t_0)^2]}{[B^2 - T(t)^2][B^2 + T(t_0)^2]} \right\}, \quad (39)$$

where $B = \sqrt[4]{\frac{100 P_{\text{tot}}}{C_p R_a^2 n \delta g} + T_{\text{sur}}^4}$ with $P_{\text{tot}} = 0$ for

$$\frac{m}{f} + t_{\text{on}} \leq t \leq \frac{m+1}{f}, \quad A = d l \rho a / (n \delta g), \quad \text{and}$$

$D = d l \rho b / (n \delta g)$. Figure 1 shows the temperature calculated for a frequency of 40 Hz and a typical foil. Similar results are given in Refs. 3 and 4, which however, consider C_p as a constant independent of T .

4.1.2. Gaussian beam shape

Equations (34) and (24) give

$$-\frac{P_{\text{tot}} \ln(1-a)}{\pi R_a^2} e^{-\frac{r^2 \sqrt{-\ln(1-a)}}{R_a^2}} - n \delta g T(r, t)^4 + n \delta g T_{\text{sur}}^4 = d l \rho C_p \frac{dT(r, t)}{dt}. \quad (40)$$

Obviously the temperature in the center of the beam will determine the behavior of the foil. A specialization of Eq. (40) to the hottest spot on the target gives for a continuous beam

$$-\frac{P_{\text{tot}} \ln(1-a)}{\pi R_a^2} - n \delta g T_{\text{max}}^4 + n \delta g T_{\text{sur}}^4 = 0. \quad (41)$$

The solution of Eq. (41) can be written as follows:

$$T_{\text{max}} = \sqrt[4]{-\frac{P_{\text{tot}} \ln(1-a)}{\pi R_a^2 n \delta g} + T_{\text{sur}}^4}. \quad (42)$$

A comparison with Eq. (36), valid for uniform beam shape, shows that $T_{\max}$ for the Gaussian beam shape is higher. When T_{sur}^4 is negligible, $T_{\max}/T(r, \infty) \approx \sqrt[4]{-\ln(1-a)}$ is obtained. So, the smaller the utilized part of the Gaussian beam is, the cooler is the foil, approaching $T(r, \infty)$ as a goes to zero. Figure 2 shows the variation of $T_{\max}/T(r, \infty)$ with a at equal transmitted beam intensities.

When a pulsed beam is used, the equation for the highest temperature $T(0, t)$ is

$$-\frac{100 P_{\text{tot}} \ln(1-a)}{C \pi R_a^2} - n \delta g T(0, t)^4 + n \delta g T_{\text{sur}}^4 = d \ell \rho C_p \frac{dT(0, t)}{dt}. \quad (43)$$

Equation (39) is a solution to Eq. 43 with

$$B = \sqrt[4]{-\frac{100 P_{\text{tot}} \ln(1-a)}{C \pi R_a^2 n \delta g} + T_{\text{sur}}^4} \quad \text{for} \quad \frac{m}{f} \leq t \leq \frac{m}{f} + t_{\text{on}},$$

$$B = 0 \quad \text{for} \quad \frac{m}{f} + t_{\text{on}} \leq t \leq \frac{m+1}{f}, \quad A = d \ell \rho a / (n \delta g) \quad \text{and}$$

$D = d \ell \rho b / n \delta g$. About the same shapes as shown in Fig. 1 are thus obtained. Some deviation occur at beam intensities where T_{sur}^4 is important in the calculation of B .

4.1.3. Double-Gaussian beam shape

The differential equation for radiation cooling at a double-Gaussian beam shape is obtained from Eqs. (34) and (31).

$$\frac{P_{\text{tot}} e^{-\frac{(r-\bar{R})^2}{s}}}{\pi s^2 e^{-\frac{(\bar{R})^2}{s}} + \pi^{3/2} s \bar{R} [\text{erf}(\frac{\bar{R}}{s}) + 1]} - n \delta g [T(r, t)^4 - T_{\text{sur}}^4] = d \ell \rho C_p \frac{dT(r, t)}{dt}. \quad (44)$$

The steady-state solution to Eq. (44) is

$$T(r, \infty) = \sqrt[4]{T_{\text{sur}}^4 + \frac{P_{\text{tot}} e^{-\frac{(r-\bar{R})^2}{s}}}{n\delta g(\pi s^2 e^{-\frac{(\bar{R})^2}{s}} + \pi^{3/2} \bar{R} [\text{erf}(\frac{\bar{R}}{s}) + 1])}} \quad (45)$$

In this case the highest temperature will occur at $r = \bar{R}$.

$$T(\bar{R}, \infty) = \sqrt[4]{T_{\text{sur}}^4 + \frac{P_{\text{tot}}}{n\delta g(\pi s^2 e^{-\frac{(\bar{R})^2}{s}} + \pi^{3/2} \bar{R} [\text{erf}(\frac{\bar{R}}{s}) + 1])}} \quad (46)$$

Numerical calculations showed that $T_{\text{max}} < T(\bar{R}, \infty) < T(0, \infty)$ holds for all investigated cases, as can intuitively be understood from the fact that the maximal intensity for a given beam is less for a double-Gaussian shape than for a Gaussian shape, but larger than for a uniform beam.

4.1.4. Calculation of the maximum temperature

Determination of the maximum temperature when radiation cooling dominated is a worst-case calculation, as there always will be some conduction cooling and often even some convection cooling. The result is presented as a nomogram, see Fig. 3, from which T_{max} can be estimated for various foil-thicknesses, ions, foil materials, foil diameters, beam shapes, beam current, and duty-cycles, with the assumption of 7.5 MeV/amu initial beam energy and a 40-Hz repetition rate. It should be observed that the evaporation rate at reduced pressure, rather than the melting point, limits the highest temperature at which the foils can be operated in vacuum. As the variation of T_{max} with the duty cycle is moderate, only 0.63% and 100% duty-cycle is shown in Fig. 3, and all other values fall in between these limits.

4.2. Conduction Cooling Dominates

The equation for heat conduction in a thin circular disc is, expressed in polar coordinates with constant λ ,⁵

$$\frac{d^2 T(r, t)}{dr^2} + \frac{1}{r} \frac{dT(r, t)}{dr} - \frac{\rho C_p}{\lambda} \frac{dT(r, t)}{dt} + \frac{P_{\text{beam}}}{\lambda d l} = 0, \quad (47)$$

where $d l$ is the thickness of the foil and P_{beam} is any beam-shape function.

4.2.1. Uniform beam shape

The introduction of the uniform beam shape, as given by Eq. (13), into Eq. (47) gives

$$\frac{d^2 T(r, t)}{dr^2} + \frac{1}{r} \frac{dT(r, t)}{dr} - \frac{\rho C_p}{\lambda} \frac{dT(r, t)}{dt} + \frac{P_{tot}}{\pi R^2 \lambda d f} = 0. \quad (48)$$

The steady-state solution, $T(r, \infty)$, is obtained by setting the time derivative to zero and solving the resulting differential equation with the boundary conditions $dT(0, \infty)/dr = 0$ and $T(a, \infty) = T_{sur}$. Assuming λ to be constant, the solution is then

$$T(r, \infty) = T_{sur} + \frac{P_{tot}}{4\pi \lambda d f} \left[1 - \left(\frac{r}{R} \right)^2 \right], \quad (49)$$

where R is the radius of the foil and T_{sur} is the temperature at the cooled edge. It is evident from Eq. (49) that the temperature of a foil of given thickness is not dependent on the size of the foil. This is easily understood as the nearest distance from a point on the foil to the edge increases linearly with R and at the same time the cross section increases linearly with R . These two effects compensate each other, which makes the temperature independent of the absolute size.

As often high and low temperatures occur on the same foil, the heat conductivity cannot be assumed to be constant. The most common approximation for $\lambda(T)$ is Eq. (6), where a , b , and c are constants obtained by fitting Eq. (6) to experimental data.

When the heat conductivity varies it must be included in the second derivative of T . This gives Eq. (48) the following form:⁵

$$\frac{1}{r} \frac{\partial}{\partial r} \left\{ \lambda [T(r, t)] r \frac{\partial T(r, t)}{\partial r} \right\} - \rho C_p \frac{\partial T(r, t)}{\partial t} + \frac{P_{tot}}{\pi R^2 d f} = 0. \quad (50)$$

Introduction of Eq. (6) into Eq. (50) and taking the partial derivatives as indicated yields

$$\begin{aligned} \frac{\partial^2 T(r, t)}{\partial r^2} + \frac{1}{r} \frac{\partial T(r, t)}{\partial r} - \frac{b+2cT(r, t)^{-3}}{a-bT(r, t)+cT(r, t)^2} \left[\frac{\partial T(r, t)}{\partial r} \right]^2 \\ - \rho C_p \frac{\partial T(r, t)}{\partial t} + \frac{P_{tot}}{\pi R^2 d f} = 0. \end{aligned} \quad (51)$$

The solution at 100% duty cycle and long times is obtained by setting the time derivative to zero in Eq. (51). The resulting differential equation was solved with a fourth-order Runge-Kutta method. The result is shown in Fig. 4 as a nomograph for Al, Be, and C foils. As can be expected from Eq. (49), the temperature in the center of the foil is independent of the radius of the foil. This was tested by integrating Eq. (51) to various radii. The results showed that the temperature at the center was independent of R.

When a pulsed beam is used the solution to Eq. (48) during the first beam pulse is

$$T(r,t) = T_{\text{sur}} + \frac{100 P_{\text{tot}}}{4\pi\lambda d f C} \left[1 - \left(\frac{r}{R}\right)^2 \right] - \frac{200 P_{\text{tot}}}{4\pi\lambda d f C} \sum_{n=1}^{\infty} e^{-\frac{\beta_n^2 \lambda}{R^2 \rho C_p} t} \cdot \frac{J_0\left(\frac{r}{R} \cdot \beta_n\right)}{\beta_n^3 J_1(\beta_n)} \quad (52)$$

where J_0 and J_1 are Bessel functions and β_n are the roots of the equation $J_0(\beta_n) = 0$. The highest temperature will as usual occur at the center of the foil. Equation (52) can then be somewhat simplified as some r terms disappear:

$$T_{\text{max}} = T_{\text{sur}} + \frac{100 P_{\text{tot}}}{4\pi\lambda d f C} - \frac{200 P_{\text{tot}}}{\pi\lambda d f C} \sum_{n=1}^{\infty} e^{-\frac{\beta_n^2 \lambda}{R^2 \rho C_p} t_{\text{on}}} \frac{1}{\beta_n^3 J_1(\beta_n)} \quad (53)$$

Equation (53) shows that the time constant for the heating cycle is determined by $\lambda/(R^2 \rho C_p)$. Thus a smaller foil tends to follow the variations in the beam to a larger extent than a bigger foil. Taking only the first term in the sum in Eq. (53) into account and using $C/100 f$ for t_{on} , we obtain

$$T_{\text{max}} \approx T_{\text{sur}} + \frac{100 P_{\text{tot}}}{4\pi\lambda d f C} \left(1 - e^{-\frac{5.783 \lambda C}{R^2 \rho C_p 100 f}} \right) \quad (54)$$

When $100 f R^2 \rho C_p / \lambda$ is smaller than 1.25 times C , the foil will already reach its final maximum temperature during the first cycle. This condition has been used to obtain the worst-case temperatures for pulsed beam given in Fig. 4. For larger diameters the value of T_{max} will approach T_{max} for $C = 100\%$. When the beam intensity varies with time only, the following solution to Eq. (47) is obtained, using Laplace transforms and the "faltung" theorem^{5,6}

$$T(r, t) = T_{\text{sur}} + \frac{2\lambda}{\rho C_p R} \int_0^t Q(\tau) \sum_{n=1}^{\infty} e^{-\frac{\lambda \beta_n^2 (t - \tau)}{\rho C_p}} \cdot \frac{J_0(r \beta_n)}{a_n J_1(R \beta_n)} d\tau, \quad (55)$$

where $Q(\tau)$ is the beam intensity as a function of time and β_n are the roots to the Eq. $J_0(\beta_n) = 0$.

When the beam is pulsed [Eqs. (17a) and (17b)] the integrand is zero during the "off" part of the beam cycle and $Q(\tau)$ is constant during the "on" part of the beam cycle. Thus the integral in Eq. (55) can be replaced by a sum giving the solution

$$T(r, t) = T_{\text{sur}} + \frac{200 P_{\text{tot}}}{\pi C \lambda d f} \sum_{n=1}^{\infty} \frac{J_0(r \beta_n)}{\beta_n^3 J_1(\beta_n)} \left\{ e^{-\frac{\beta_n^2 \lambda t}{\rho C_p R^2}} \left(e^{\frac{\beta_n^2 C \lambda}{100 \rho C_p f R^2}} - 1 \right) \sum_{m=0}^M \frac{\beta_n^2 \lambda m}{\rho C_p f R^2} \right. \\ \left. + \delta \left[1 - e^{-\frac{\beta_n^2 \lambda}{\rho C_p R^2} (t - \frac{M+1}{f})} \right] \right\}, \quad (56)$$

where β_n is the solutions to the equation $J_0(\beta_n) = 0$, M is the integer number of complete beam cycles contained within $t - (C/100f)$, δ is a delta function which has the value 0 when t corresponds to the beam-off part and the value 1 when t corresponds to the beam-on part of the beam cycle. Equation (56) transforms to Eq. (53) for the first beam pulse ($M=0$, $\delta=0$) and to Eq. (49) for $C = 100$.

It is seen from Eq. (56) that the cooling-off time constants are $\beta_n^2 \lambda / \rho C_p R^2$ and the same as the heating time constants. Figure 5 shows the central temperature as a function of time for some values of C when the time constant is comparable to the frequency. It is obvious that the large increase in λ obtained by cooling the edge to low temperatures will result in more rapid (and larger) temperature fluctuations. The increase in λ can to some degree be compensated by increasing the foil diameter, but this leads easily to such large diameters that the beam shape no longer is uniform. The increase in λ can also be compensated by using a higher frequency. The effect of foil diameter on the center temperature is shown in Fig. 6.

4.2.2. Gaussian beam shape

When the beam has any non-uniform shape, p_{beam} in Eq. (47) will be dependent on r . In the case of a Gaussian beam shape we obtain

$$\frac{d^2 T(r, t)}{dr^2} + \frac{1}{r} \frac{dT(r, t)}{dr} - \frac{\rho C_p}{\lambda} \frac{dT(r, t)}{dt} - \frac{P_{\text{tot}} \ln(1-\alpha)}{\pi R_a^2 \lambda d l} e^{-\left(\frac{r}{R_a}\right)^2 \sqrt{-\ln(1-\alpha)}} = 0. \quad (57)$$

The steady-state solution, $T(r, \infty)$, is obtained by setting the time derivative to zero and solving the resulting differential equation with the boundary conditions $dT(0, \infty)/dr = 0$ and $T(R, \infty) = T_{\text{sur}}$. Assuming λ to be constant, the solution is

$$T(r, \infty) = T_{\text{sur}} + \frac{P_{\text{tot}} \sqrt{-\ln(1-\alpha)}}{4\pi \lambda d l} \sum_{n=1}^{\infty} \frac{(-1)^{n-1} \frac{[-\ln(1-\alpha)]^{n/2} [1 - (\frac{r}{R})^2]}{n \cdot n!}}{n \cdot n!}, \quad (58)$$

where α is the fraction of the total beam current hitting the foil and P_{tot} is the total power released by the beam. As usual, the highest temperature will occur at the center of the foil ($r = 0$). By introducing $r = 0$ in Eq. (58) we obtain the equation for T_{max} ,

$$T_{\text{max}} = T_{\text{sur}} + \frac{P_{\text{tot}} \sqrt{-\ln(1-\alpha)}}{4\pi \lambda d l} \sum_{n=1}^{\infty} \frac{(-1)^{n-1} \frac{[-\ln(1-\alpha)]^{n/2}}{n \cdot n!}}{n \cdot n!}, \quad (59)$$

which is very similar to Eq. (49) with $r = 0$. Figure 7 shows the shape factor $\psi(\alpha)$ as a function of the transmission factor α .

$$\psi(\alpha) = \sqrt{-\ln(1-\alpha)} \sum_{n=1}^{\infty} \frac{(-1)^{n-1} \frac{[-\ln(1-\alpha)]^{n/2}}{n \cdot n!}}{n \cdot n!}. \quad (60)$$

Using ψ , equation (59) can be written as

$$T_{\text{max}} = T_{\text{sur}} + \frac{P_{\text{tot}}}{4\pi \lambda d l} \cdot \psi(\alpha). \quad (61)$$

If the transmitted beam power is used instead of the total beam power, Eq. (61) changes to

$$T_{\text{max}} = T_{\text{sur}} + \frac{P_{\text{tr}}}{4\pi \lambda d l} \cdot \frac{\psi(\alpha)}{\alpha}. \quad (62)$$

The function $\psi(\alpha)/\alpha$ vs α is shown in Fig. 8. A comparison of Figs. 2 and 8 shows that the temperature rises more rapidly with α for conduction cooling than for radiation cooling.

The complete time-dependent solution of Eq. (57) was not obtained because of difficulties with the solution of the new differential equation obtained after Laplace transformation of Eq. (57). An analytical solution for a pulsed Gaussian beam was not obtained for the same reasons. A numerical solution is shown in Fig. 10.

4.2.3. Favorable beam shapes

The uniform beam can be regarded as one in a series of simple r -dependent beam shapes of the general type

$$P_{\text{beam}} = k \cdot r^n, \quad (63)$$

where the uniform beam corresponds to $n = 0$. The total beam power is then given by

$$P_{\text{tot}} = \int_0^R 2\pi k r^{n+1} dr, \quad (64)$$

from which k can be determined:

$$k = \frac{(n+2) P_{\text{tot}}}{2\pi R^{n+2}}, \quad (65)$$

The steady-state solution of the heat-conduction equation is then

$$T(r, \infty) = T_0 + \frac{P_{\text{tot}}}{2(n+2)\pi\lambda df} \left[1 - \left(\frac{r}{R}\right)^{2+n} \right], \quad (66)$$

where $n = 0, 1, 2, 3$ etc.

It can be seen from Eq. (66) that the maximum temperature decreases as n increases. When T_0 can be neglected, the following relations are obtained between the temperatures:

$$T_{\text{max}}(n) = \frac{2}{2+n} T_{\text{max}}(\text{even-beam}). \quad (67)$$

And thus lower temperature can be obtained by choosing a beam shape which corresponds to a higher value of n in Eq. (63), i.e., by concentrating the beam to a small area near the rim.

4.3. Convection-Cooling Dominates

When convection cooling dominates over radiation cooling and conduction cooling, the amount of heat generated by the beam is equal to the amount of heat carried away by convection from any point on the foil. Thus we obtain the relation (one-sided cooling)

$$P_{\text{beam}} = h [T(r, t) - T_{\text{fluid}}] + \rho C_p d \frac{dT(r, t)}{dt}, \quad (68)$$

where h is a function of $T(r, t)$, T_{fluid} , w , etc., as given by Eqs. (8), (9), (10), or (14). The resistance to heat flow through the thin foil is neglected. T_{fluid} can be calculated with sufficient accuracy from P_{tot} , w , $(C_p)_{\text{fluid}}$, and the entrance temperature of the fluid, T_{fluid}^0 :

$$T_{\text{fluid}} \approx T_{\text{fluid}}^0 + \frac{P_{\text{tot}}}{w (C_p)_{\text{fluid}}}, \quad (69)$$

where P_{tot} is the total beam power released in the foil and fluid. Analytic solutions to Eq. (68) can usually not be obtained because h is often given by a function containing fractional powers of $T(r, t)$. The stationary state at constant beam can, however, be expressed analytically in some cases.

4.3.1. Water cooling

In the case of laminar flow of water, h is given by Eq. (9). For a uniform beam shape the temperature is almost constant on the foil and is given by the equation

$$T(r, \infty) = \sqrt[3]{\left\{ \frac{P_{\text{tot}} \cdot 10^4}{\pi R^2 (2.685 T_w + 51.62)} \right\}^4} + T_w, \quad (70)$$

where T_w is the water mean temperature. At sufficiently large flow rates $T_w \approx T_w^0$, where T_w^0 is the temperature of the incoming water. As can be seen from Eq. (70), the foil temperature is independent of r . The temperature increases roughly with the 1.33 power of P_{tot} and decreases roughly with the 2.66 power of R .

When the beam shape is Gaussian the temperature is given instead by

$$T(r, \infty) = e^{-\frac{r^2}{R_a^2} \cdot \frac{4}{3} \sqrt{-\ln(1-\alpha)}} \sqrt[3]{\frac{-P_{\text{tot}} \ln(1-\alpha) \cdot 10^4}{\pi R_a^2 (2.685 T_w + 51.62)}}^4 + T_w \quad (71)$$

and $T(r, \infty)$ varies with radius like $a \cdot e^{-br^2} + c$, i.e., in the same manner as the beam shape.

The water cooling is very effective in comparison with radiation cooling or conduction cooling. At a mean water temperature of 300°K, $P_{\text{tot}}/\pi R^2 \approx 3.8$ ($\kappa W/cm^2$) is needed to increase the foil temperature to 373°K. In comparison, a foil temperature of 373°K at pure radiation-cooling is reached for $P_{\text{tot}}/\pi R^2 \leq 0.13 (W/cm^2)$. Conduction cooling is more difficult to compare with, as it varies with P_{tot}/dl instead of $P_{\text{tot}}/\pi R^2$, but it is usually intermediate in efficiency. At a water temperature of 20°C and a surface temperature of 100°C the heat resistance at the metal-water interface corresponds to 84 cm copper.

4.3.2. Gas cooling

The heat-carrying capacity of gases is less than for water. Hence it is necessary to use large flow rates to obtain a good cooling with gases. When turbulent flow exists, Eq. (10) can be used to calculate the film coefficient, h . Then the foil temperature at uniform beam shape is given by

$$T(r, \infty) = T_g^0 + \frac{P_{\text{tot}} + P_g}{w(C_p)_g} + \frac{P_{\text{tot}}}{\pi R^2} \cdot \frac{D}{0.023 \lambda_g} \cdot \left(\frac{DV \rho_g}{\mu} \right)^{-0.8} \left(\frac{(C_p)_g \mu}{\lambda_g} \right)^{-0.4} \quad (72)$$

where subscript g refers to the gas, V is the linear gas velocity (cm/sec), D is the diameter of the gas channel, and w is the gas flow rate (g/sec). It can be seen that the foil temperature is very nearly inversely proportional to the gas flow rate. The term P_g accounts for the heat generated in the gas during passage of the cooling channel.

5. NUMERICAL SOLUTION OF THE GENERAL CASE

In practice, the observed beam functions and foil arrangements often deviate from the simple shapes and forms discussed above. It is then necessary to solve Eq. (33) numerically for arbitrary beam shapes, foil geometries, and cooling arrangements. At the same time it is necessary to use temperature-dependent heat conductivities and heat capacities to make a realistic calculation.

A program was thus developed where the only assumptions are: beam-power is independent of temperature, radiation can only escape in the l -direction, convection cooling is only possible in the x - y plane, the grayness factors are temperature-independent, and λ and C_p values can be expressed with sufficient accuracy as second-degree polynomials in T .

A mesh system is set up in the x - y plane which is then reproduced as layers in the l direction. At each mesh-point, the beam power, starting temperature, grayness factor, geometrical connection to adjacent points in the positive coordinate directions, type of polynomials for λ and C_p , and type of point—i.e., internal or boundary—is specified. The mesh width is prescribed for each coordinate separately.

Starting at a corner and proceeding through the whole mesh system, excluding boundary points, the temperature-heat conductivity product is interpolated from the surrounding mesh points by using a second-degree interpolation formula. This results in almost as many linear equations as there are mesh-points. The large number of mesh-points desirable makes normal solution by matrix-inversion impractical. Therefore the Gauss-Siedel iteration method was employed, as then only one equation had to be evaluated at a time. The coefficients were recalculated every time, thus avoiding the storage of large sparse matrices, of the size of about 250 000 elements. The Gauss-Siedel iteration is convergent, as the main diagonal elements are larger than the other elements in each row and the matrix is positive definite.⁶

The power lost by radiation and convection is calculated from the new set of temperatures in the mesh-points and subtracted from the power released from the beam. Then the whole iteration cycle is repeated until a new iteration changes all temperatures with less than a prescribed value.

The calculation can be performed to obtain a steady-state solution by setting all C_p values to zero or to obtain snapshots at given time intervals with a continuous or pulsed beam.

Symmetry axes can be used freely to reduce the problem as long as the λ values, C_p values, and grayness factors for mesh-points at a symmetry cut are divided by the appropriate number of used symmetry axes passing through the mesh-point. Values beyond a symmetry axis will then be mirror images of the values before the axis.

For very thin foils, the heat conduction in the z direction is much larger than in the x and y directions. This may cause numerical difficulties which can give a premature halt in the calculations. The simplest cure is to treat such foils as 2-dimensional structures, which is equivalent to neglecting the resistance to heat flow in the z direction as compared to the resistances in the x - y plane. The heat conductivity and heat capacity are automatically multiplied by the actual thickness in a 2-dimensional treatment and by the product of thickness and length in a 1-dimensional case.

For cases where the beam power is small compared to the heat-capacity, it may be necessary to decrease the value of the allowed difference between two successive iterations. On the other hand, high values for the grayness factors seems to increase the convergence rate.

A complete listing of the program is given in Appendix 2. Appendix 3 gives a sample of input data and results. The organization and format of the input data is shown in Appendix 1. The algorithm is slightly modified, as the number of subscripts must be limited to three to be able to use the Fortran compilers available for the CDC-6600 and CDC-7600 computers. This is the reason for the split of the temperature array into two arrays (T and TG) and of the coupling-constant array into three arrays (CX, CY, and CZ) etc.

5.1. Combined Radiation and Conduction Cooling

The variation of temperature with radius and time can be obtained for any given foil, foil-material, ion, beam-shape, frequency, duty-cycle, and mean current with the aid of the program given in Appendix 2. To illustrate the effect of beam-pulsing and beam shape on foil temperature, a half-mil-thick gold foil of 1 cm diameter was chosen. The edge of the foil is clamped to 20°C and the surroundings are assumed to be at 20°C. The foil is surrounded on both sides by vacuum. The ion chosen was $^{40}\text{Ar}^{13+}$ of

7.2 MeV/amu, which is typical for an Ar beam from the Berkeley SuperHILAC. These ions will release $10.9 \text{ W}/\mu\text{A}$ ($^{40}\text{Ar}^{13+}$) during their passage through a half-mil gold foil.

Figure 9 shows the variation of temperature with radius and time for an incident beam with a uniform shape. The steep increase of temperature near the edge of the foil is typical for the uniform beam shape. Figure 10 shows the variation of temperature with radius and time for a Gaussian beam shape. Here the temperature increases more rapidly towards the center, leading to higher maximum temperature. Figure 11 shows the effect of a double-Gaussian beam shape. Typical for this case is the delayed heating of the central part of the foil, leading to a lower maximum temperature than for a simple Gaussian beam shape.

When the foil thickness is decreased, the differential energy-loss (dE/dl) of the beam becomes constant. For very thin foils the equilibrium temperature is determined by the radiation cooling, and the foil temperature falls approximately with the fourth root of the thickness, l , [see Eq. (36)], as P_{tot} is a linear function of l . For thicker foils, where dE/dl still is constant, the temperature becomes independent of the foil thickness, as the cooling is mainly by conduction. In this case both the beam power and the conduction area vary linearly with the thickness [see Eq. (47)]. When the foil thickness approaches the range of the ions in the foil material, dE/dl increases, which leads to an increase in the foil temperature with thickness. When the foil is thicker than the range of the ions, the temperature drops again with increasing thickness—almost linearly at first, as long as the thickness is small compared to the diameter, and then the temperature levels off to a constant value, determined by conduction in the l direction. This temperature is mainly independent of the type of ion and depends almost only on the energy per charge unit, the beam current, and the foil diameter.

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Appendix 1.

Input data for HEAT v4m1

(Always punch the decimal point!)

1. HEADING-CARD (15A4)

Column 1 to 60: Alphanumeric text, which will be used as a heading on printout.

2. BEAM-CARD (8F10.0)

(A blank BEAM-CARD defines END-OF-DATA.)

- 2.1 Beam-Current in μA . Normally as mean particle current.
- 2.2 Beam-Energy-loss in MeV/Grid-unit. (See Grid-point cards!)
- 2.3 Beam-pulse on-time in seconds. (Only for pulsed beam, else 1.0)
- 2.4 Beam-pulse off-time in seconds. (Only for pulsed beam, else 0.0)
- 2.5 Melting temperature in $^{\circ}\text{C}$. (Default is 10 000 $^{\circ}\text{C}$. Melted material is removed)
- 2.6 Temperature offset in $^{\circ}\text{C}$. (The offset is added to all grid-point-temperatures before calculation).
- 2.7 Surrounding radiation temperature in $^{\circ}\text{C}$. (Default is -273.16 $^{\circ}\text{C}$)
- 2.8 Grayness factor (Default is 1.0)

3. GRID-DIMENSION-CARD (8F10.0)

- 3.1 Grid-spacing, X-direction, in cm.
- 3.2 Grid-spacing, Y-direction, in cm.
- 3.3 Grid-spacing, Z-direction, in cm. (Z is parallel to the beam)
- 3.4 Grid-spacing, time-direction, in seconds. (Zero implies a steady-state condition)
- 3.5 Time before full printout, in seconds. (Neglected for steady state)
- 3.6 Time when calculation stops, in seconds. (Neglected for steady state)

4. COOLING-TYPE-CARD (I5)

- 4.1 COOLING TYPE $\left\{ \begin{array}{l} 0=\text{VACUUM ON BOTH SIDES IN x-y plane} \\ 1=\text{GAS FLOWS ON ONE SIDE, PARALLEL} \\ \text{TO THE x-y plane.} \end{array} \right.$

5. GAS-DATA-CARDS (8F10.0)

(Only when cooling-type is 1)

5.1 Hydraulic diameter of cooling channel, in cm.

5.2 Heat conductivity of gas expressed as a Sutherland approximation

$$(\lambda_{\text{gas}} = a T \sqrt{T} / (b + T), \text{ with } T \text{ in } ^\circ\text{K}), \text{ in W/cm degree.}$$

5.2.1 a.

5.2.2 b.

5.3 Viscosity of gas expressed as a Sutherland approximation

$$[\mu_{\text{gas}} = a T \sqrt{T} / (b + T), \text{ with } T \text{ in } ^\circ\text{K}]. \text{ in poise.}$$

5.3.1 a.

5.3.2 b.

5.4 Heat capacity of gas expressed as a polynomial

$$(C_{p\text{gas}} = a + bT + c/T^2, \text{ with } T \text{ in } ^\circ\text{K}), \text{ in J/g, degree.}$$

5.4.1 a.

5.4.2 b

5.4.3 c.

5.5 Volume expansion coefficient of gas.

5.6 Flow-rate of gas, in g/sec.

5.7 Entering temperature of gas, in $^{\circ}\text{C}$.

5.8 Gas pressure in atm.

5.9 Molecular weight of gas.

(The gas-cooling utilizes several similarity rules and rough approximations, hence the result should not be regarded as very accurate!)

6. λ_x -NUMBER CARD (15)

6.1 Number of heat-conductivities. (≤ 10 , ≥ 1)

7. λ_x -FUNCTION-CARDS (3F10.0)

One card for each heat-conductivity type. The heat conductivity is expressed as $\lambda_x = a + bT + c/T^2$, where T is in °K and λ_x in W/cm degree.

7.1 a.

7.2 b.

7.3 c.

The first card will be type 1, the 2nd type 2, etc. See 15.

8. λ_y -NUMBER CARD (I5)

8.1 Number of heat conductivities. (≤ 10 , ≥ 1)

For a one-dimensional case, only one dummy λ_y is needed.

9. λ_y -FUNCTION CARDS (3F10.0)

Same as 7, but for conduction in y-direction.

10. λ_z -NUMBER CARD (I5)

10.1 Number of heat-conductivities. (≤ 10 , ≥ 1).

For a one- or two-dimensional case, only one dummy λ_z is needed.

11. λ_z -FUNCTION CARDS (3F10.0)

Same as 7, but for conduction in z direction.

12. C_p -NUMBER CARD (I5)

12.1 Number of C_p -types (≥ 1 , ≤ 10).

13. C_p-FUNCTION CARD (3F10.0)

One card for each C_p type. First card will be type 1, second type 2, etc.

C_p is expressed as a polynomial $C_p = a + bT + c/T^2$, where T is in °K and C_p in J/ml degree. (OBS! not J/g degree)

13.1 a.

13.2 b.

13.3 c.

14. GRID-NUMBER CARD (3I5)

14.1 Number of grid lines crossing x coordinate (≥ 1 , ≤ 10)

14.2 Number of grid lines crossing y coordinate (≥ 1 , ≤ 10)

14.3 Number of grid lines crossing z coordinate (≥ 1 , ≤ 5)

15. GRID-POINT CARDS (6I3, 2X, 6F10.0)

Grid-point cards are needed to specify points deviating from the default assumption: outside beam area, clamped at 0°C and without connection to surrounding points. (A point is the crossing of three grid lines). One card per grid point.

15.1 Number of grid lines from origin along x axis. (> 0 , ≤ 10)

15.2 Number of grid lines from origin along y axis. (> 0 , ≤ 10)

15.3 Number of grid lines from origin along z axis. (> 0 , ≤ 5)

A card with 0 grid-lines from origin terminates the reading of grid-point cards. Normally grid-line no. 1 goes through origin.

15.4 C_p-function type for this point (≥ 1 , ≤ 10)

15.5 λ -function type for this point (≥ 1 , ≤ 10)

The type selected is common for λ_x , λ_y , and λ_z . The λ_x is considered

to be for conduction between this point and the next along the x axis. The same holds for λ_y and λ_z .

15.6 Gas contact. (0 = no gas contact, 1 = gas contact.)

15.7 Temperature in degrees centigrade. (Starting value includes temp. offset.)

15.8 Fraction of beam power released at the point. (Beam power-loss must be normalized to this value.)

15.9 Point type. 1.0 means that temperature is clamped to its initial value. 0.0 means that no radiation is lost from this point. When heat radiation can escape in the z direction, $-G$ is specified, where G is the surface grayness factor $0 < G < 1.0$. For 2-dimensional foils G is the sum of the grayness factors for both surfaces.

15.10 Fractional area coupling to next point in x direction.

15.11 Fractional area coupling to next point in y direction.

15.12 Fractional area coupling to next point in z direction.

The fractional areas are especially useful when symmetry properties are used to reduce the size of a problem.

The test example supplied with the program is for 1 quadrant of an approximately circular foil with no radiation or gas cooling. It shows that the coupling areas (15.10 - 15.12) are not reduced along the symmetry axes, but how instead three different λ and C_p sets are used and the fractional beam (15.8) is used to adjust the conditions along the symmetry lines. Thus functions and coupling areas can be traded against each other to simplify a problem.

Arrangement of data-deck for HEAT v4m1

APPENDIX 2. Program HEAT v4m1

HEAT

PROGRAM HEAT(INPUT,OUTPUT,TAPE5=INPUT,TAPE6=OUTPUT)

PROGRAM FOR SOLVING DYNAMIC AND STATIC 3-D HEAT-TRANSFER WITH
POWER-SOURCES, RADIATION COOLING, HEAT-CONDUCTION IN NONISOTROPIC
CASES. RADIATION SOURCES ARE TREATED AS BLACK BODIES. HEAT-TRANS
FER COEFFICIENTS AND HEAT-CAPACITIES CAN BE LINEARLY DEPENDENT ON
TEMPERATURE. THE POWER SOURCES MAY BE IN TWO STATES FOR GIVEN
TIME INTERVALS.

J.O.LILJENZIN 29.9.1972

VERSION 4 USES POLYNOMIALS $A+B*T+C/T**2$ FOR HEAT-CONDUCTIVITIES
TO ALLOW A LARGER TEMPERATURE RANGE TO BE COVERED
WHEN THIS IS NOT NECESSARY, USE V3M0 WHICH IS FASTER.

UA=BEAM CURRENT IN UA(DC)

PA=BEAM-POWER LOST IN MEV/POINT

TON=TIME INTERVAL WITH BEAM ON

TOFF=TIME-INTERVAL WITH BEAM OFF

UA.LE.O. STOPS PROGRAM

DX=CM/GRID-UNIT IN X-DIRECTION

DY=CM/GRID-UNIT IN Y-DIRECTION

DZ=CM/GRID-UNIT IN Z-DIRECTION

DT=TIME-STEP IN SECONDS

IF DT=0. THE STEADY-STATE IS CALCULATED

TMAX=TIME-LIMIT IN SECONDS (STOPS THE PRESENT CASE)

TMIN IS THE RUN-IN TIME BEFORE FIRST COMPLETE PRINT-OUT

MX=TOTAL NO. OF RX-FUNCTIONS

AX(I) AND BX(I) ARE THE COEFFICIENTS IN THE EXPRESSION

$RX(I,J,K)=CX(I,J,K)*(AX(I)+BX(I)*T(I,J,K))*XM$ (WHERE $L=NR(I,J,K)$)

THE AX(I).S ARE IN W/CM DEG. AT 0 DEG.K (NOT DEG C)

THE BX IS IN W/CM(DEG)**2

MY, AY(I) AND BY(I) ARE THE SAME FOR RY

MZ, AZ(I) AND BZ(I) ARE THE SAME FOR RZ

MC, AC(I) AND BC(I) ARE THE CORRESPONDING DATA FOR THE HEAT-

CAPACITY. AC(I).S IN J/MLDEG AT 0 DEG. K

BC.S IN J/ML(DEG)**2

IM=NUMBER OF GRID-POINTS IN X-DIRECTION

JM=NUMBER OF POINTS IN Y-DIRECTION

KM=NUMBER OF POINTS IN Z-DIRECTION

L=DIMENSIONALITY OF ACTUAL CASE

I=NO. OF DX TO A POINT -1

J=NO. OF DY TO A POINT -1

K=NO. OF DZ TO A POINT -1

T(I,J,K)=TEMPERATURE IN DEG.C AT POINT I,J,K AND TIME

Q(I,J,K)=FRACTION OF POWER AT POINT I,J,K

F(I,J,K)=1. MEANS T(I,J,K)=CONSTANT (BORDER)

=0. MEANS T(I,J,K)=VARIABLE

=-V. MEANS T(I,J,K)=VARIABLE AND RADIATION FROM V

SURFACES IN THE X-Y-PLANE (EACH AREA IS DY*DX)

CX(I,J,K)=COUPLING-CONSTANT BETWEEN I AND I+1

CY(I,J,K)=DITO FOR J AND J+1

CZ(I,J,K)=DITO FOR K AND K+1

NR(I,J,K) SELECTS THE APPROPRIATE RX-, RY-, AND RZ-FUNCTIONS

NC(I,J,K) SELECTS THE APPROPRIATE C-FUNCTION

ITER IS LOOP-COUNTER

HEAT

```

C      SUM IS CONVERGENCE-TEST AGAINST RKT
C
C      OBSERVE THE POSSIBILITY TO UTILIZE SYMMETRIES TO REDUCE A GIVEN
C      PROBLEM.
C      WHEN SYMMETRY IS USED THE Q, C AND V.S FOR A POINT MUST BE
C      DIVIDED BY THE NUMBER OF USED SYMMETRY AXES PASSING THRU THIS
C      POINT.
C      OBSERVE ALSO THAT THE RZ-S OF A HIGHER LAYER (LARGER K) AFFECTS
C      THE RZ-S OF THE NEXT LOWER LAYER. THIS IS ALSO TRUE FOR MATERIAL
C      CHANGES IN THE X- AND Z-DIRECTIONS.
C      TSUR IS SURROUNDING TEMPERATURE TO BE USED FOR CALC OF RAD COOL
C
C      FOR CYCLIC PHENOMENA IT IS ADVISED TO START BY OBTAINING YOUR
C      STEADY-STATE CONDITIONS AT THE MEAN-POWER USED AND THEN USE THIS
C      AS THE STARTING POINT FOR THE T(X,Y,Z,T) CALCULATIONS.
C      TADD IS ADDED TO ALL POINT-TEMPERATURES
C
C      VERSION 3 ALLOWS GAS-COOLING ON RADIATING SURFACES
C
C      ITYPE=0 FOR NO GAS-COOLING (=VACUUM)
C      1 FOR GAS-COOLING
C
C      CD IS CHARACTERISTIC DIAMETER (CM)
C      CK=GAS HEAT CONDUCTIVITY (W/CM.DEG)
C      CKA AND CKB ARE CONSTANTS IN  $CK=CKA*T**1.5/(T+CKB)$ 
C      GMU IS GAS BULK VISCOSITY
C      GBUF IS GAS VISCOSITY IN BOUNDARY LAYER
C      CMA AND CMB ARE CONSTANTS IN  $MU=CMA*T**1.5/(T+CMB)$ 
C      CP IS GAS HEAT CAPACITY (J/G DEG.)
C      CPA,CPB AND CPC ARE COEFFICIENTS IN  $CP=CPA+CPB*T+CPC/T**2$ 
C      CB IS VOLUME EXPANSION COEFFICIENT (1/DEG.)
C      CV IS GAS MASS FLOW (G/S)
C      CTC IS GAS TEMPERATURE AT INLET (DEG. C)
C      CPRES IS GAS PRESSURE AT TARGET SURFACE (ATA)
C      CMV IS THE MOLECULAR WEIGHT OF THE GAS
C
C      CR IS GAS DENSITY (G/ML) FROM THE GAS LAW
C      CONA AND CONB ARE CONSTANTS FOR GIVEN GAS, BULK TEMP AND PRESSURE
C      GT IS CALCULATED MEAN TEMPERATURE OF GAS (DEG.K)
C      NG(I,J,K) IS 1 IF GAS IS IN CONTACT WITH THIS POINT
C      0 IF NO GAS-CONTACT
C      QTG IS TOTAL HEAT TRANSFERRED TO GAS-PHASE
C
C      DIMENSION F(10,10,5),T(10,10,5),TG(10,10,5),Q(10,10,5),AX(10),
C      LBX(10),AY(10),BY(10),AZ(10),BZ(10),AC(10),BC(10),PR(10),PT(15),
C      ZCX(10,10,5),CY(10,10,5),CZ(10,10,5),DC(10),EX(10),EY(10),EZ(10),
C      3NR(10,10,5),NC(10,10,5),NG(10,10,5)
C
C      STATEMENT FUNCTIONS
C      SUD EVALUATES SUTHERLAND EQN AND POL POLYNOMIAL
C       $SUD(A,B,C)=A*C**1.5/(B+C)$ 
C       $POL(A,B,C,D)=A+B*D+C/D**2$ 
C
30      IVERS=4
31      MODIF=1
31      1000  TIME=0.

```

```

HEAT
32      ITER=-1
33      RKT=0.
34      DO 1 I=1,10
35      DO 1 J=1,10
36      DO 1 K=1,5
50      F(I,J,K)=1.
51      T(I,J,K)=0.
51      TG(I,J,K)=0.
52      CX(I,J,K)=0.
53      CY(I,J,K)=0.
53      CZ(I,J,K)=0.
54      NR(I,J,K)=0
54      NC(I,J,K)=0
55      NG(I,J,K)=0
55      1  Q(I,J,K)=0.
      C
63      READ(5,4){PT(I),I=1,15}
      4    FORMAT(15A4)
70      WRITE(6,5){PT(I),I=1,15},IVERS,MODIF
      5    FORMAT(1H1,15A4,9HHEAT VERS,13,5H MOD,13)
      C
102     READ(5,2)UA,PA,TON,TOFF,TMLT,TADD,TSUR,GSIGE
      2    FORMAT(8F10.0)
126     IF(UA.LE.0.) STOP
132     IF(TMLT.LE.0.) TMLT=10000.
135     IF(TSUR.EQ.0.) TSUR=-273.16
137     READ(5,2)DX,DY,DZ,DT,TMAX,TMIN
157     IF(DT)200,200,201
      C    SET CONDITIONS FOR A STEADY-STATE CASE
161     200 TON=1.
162     TOFF=0.
163     TMAX=0.
164     201 CONTINUE
164     WRITE(6,3)UA,PA,TON,TOFF,TMLT
      3    FORMAT(6H0BEAM=F10.3,10HUA, POWER=,F10.3,9HMEV/PDINT,5H TON=,
164     IF10.4,8HS, TOFF=F10.4,14HS, MELTS AT T=F10.2)
202     TMTT=TMLT+273.16
      C
204     WRITE(6,6)DX,DY,DZ,DT,TMAX,TMIN
      6    FORMAT(14HDDX=,F10.5,7HCM, DY=,F10.5,7HCM, DZ=,F10.5,7HCM, DT=,
204     IF10.5,10HSEC, TMAX=,F10.5,10HSEC, TMIN=,F10.5,3HSEC)
224     IF(TMIN.GE.TMAX.AND.DT.NE.0.) GO TO 1000
234     WRITE(6,444)TSUR
      444   FORMAT(6HOTSUR=,F10.2,3HDEG)
241     TSUR=TSUR+273.16
      C
243     SIG=5.70E-12*DX*DY
246     READ(5,300)ITYPE
      300   FORMAT(15)
253     IF(ITYPE.LE.0) GO TO 301
      C    OBS NEXT STATEMENT READS TWO CARDS
255     READ(5,2)CD,CKA,CKB,CMA,CMB,CPA,CPB,CPC,CB,CW,CTC,CPRES,CNV
312     WRITE(6,302)ITYPE,
      1     CD,CKA,CKB,CMA,CMB,CPA,CPB,CPC,CB,CW,CTC,CPRES,CNV
302     FORMAT(6H0TYPE=,12,12H GAS-COOLING/
302     16H DIAM=,9X,F10.2,2HCM/

```

HEAT

212H HEAT COND.=,8X,2E10.3,9HW/CM.DEG./
 311H VISCOSITY=,9X,2E10.3,4HPOIS/
 44H CP=,16X,3E10.3,8HJ/G.DEG./15H VOL.EXP.COEF.=,5X,E10.3/
 511H FLOW-RATE=,9X,E10.3,5HG/SEC/
 615H ENTERING TEMP=,F10.2,5HDEG.C/
 710H PRESSURE=,7X,F10.4,3HATA/
 812H MOL.WEIGHT=,4X,F10.3)

353 CTC=CTC+273.16

354 GT=CTC

355 GO TO 303

356 301 WRITE(6,304) I TYPE

304 FORMAT(6H0TYPE=,I2,7H VACUUM)

364 303 CONTINUE

C

364 READ(5,7)MX,(AX(I),BX(I),EX(I),I=1,MX)

FORMAT(15/(3F10.0))

410 WRITE(6,8)(I,AX(I),BX(I),EX(I),I=1,MX)

FORMAT(8HORX-FUNC/(1X,I3,2F10.5,F11.2))

C

434 READ(5,7)MY,(AY(I),BY(I),EY(I),I=1,MY)

460 WRITE(6,9)(I,AY(I),BY(I),EY(I),I=1,MY)

FORMAT(8HORY-FUNC/(1X,I3,2F10.5,F11.2))

C

504 READ(5,7)MZ,(AZ(I),BZ(I),EZ(I),I=1,MZ)

530 WRITE(6,10)(I,AZ(I),BZ(I),EZ(I),I=1,MZ)

FORMAT(8HORZ-FUNC/(1X,I3,2F10.5,F11.2))

C

554 READ(5,70)MC,(AC(I),BC(I),DC(I),I=1,MC)

FORMAT(15/(3F10.0))

600 WRITE(6,11)(I,AC(I),BC(I),DC(I),I=1,MC)

FORMAT(8H0 C-FUNC/(1X,I3,2F10.5,F11.2))

C

624 READ(5,12)IM,JM,KM

FORMAT(3I5)

636 LM=3

637 IF(KM.EQ.1) LM=LM-1

642 IF(JM.EQ.1) LM=LM-1

645 IF(IM.EQ.1) LM=LM-1

650 WRITE(6,13)IM,JM,KM,LM

C

FORMAT(1H0,2(I3,2H X),I3,2H =,I2,12H DIMENSIONAL)

664 XM=DY*DZ/DX

666 YM=DX*DZ/DY

667 ZM=DX*DY/DZ

670 VM=0.

671 IF(DT.GT.0.) VM=DX*DY*DZ/DT

676 TIME=0.

677 POINTS=0.

700 14 READ(5,15)I,J,K,NCIJK,NRIJK,NGIJK,TIJK,QIJK,FIJK,CXIJK,CYIJK,CZIJK

FORMAT(6I3,2X,6F10.0)

734 IF(1.LE.0) GO TO 16

736 IF(1.GT.1M) GO TO 14

741 IF(1.GT.1M) GO TO 14

744 IF(K.GT.KM) GO TO 14

753 POINTS=POINTS+QIJK

755 TIJK=TIJK+TADD

757 T(I,J,K)=TIJK

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HEAT
757      TG(I,J,K)=TIJK
760      Q(I,J,K)=QIJK
761      F(I,J,K)=FIJK
763      CX(I,J,K)=CXIJK
764      CY(I,J,K)=CYIJK
766      CZ(I,J,K)=CZIJK
767      NC(I,J,K)=NCIJK
771      NR(I,J,K)=NRIJK
772      NG(I,J,K)=NGIJK
774      GO TO 14
C
775      16  WRITE(6,17)
          17  FORMAT(5HODATA/2H T/2H Q/2H F/3H CT/3H RT/3H CX/3H CY/3H CZ)
1001      17J  IF(ITYPE.EQ.1) WRITE(6,170)
          17J  FORMAT(3H NG)
1007      753  WRITE(6,753)POINTS
          753  FORMAT(1H0,F12.5,7H POINTS)
C
1015      DO 18 K=1,KM
1017      WRITE(6,5)(PT(I),I=1,15),IERS,MODIF
1030      WRITE(6,19)K,TIME,ITER
          19  FORMAT(6HOLAYER,I3,8H AT TIME,F10.5,3HSEC,15,4HITER)
1042      20  WRITE(6,20)(I,I=1,IM)
          20  FORMAT(3H0X=,3X,10I10)
1055      DO 18 J=1,JM
1057      WRITE(6,21)J,(T(I,J,K),I=1,IM)
          21  FORMAT(3HOY=,I3,10F10.2)
1073      22  WRITE(6,22)(Q(I,J,K),I=1,IM)
          22  FORMAT(6X,10F10.2)
1106      IF(GSIG.EQ.0.0) GO TO 5433
1107      DO 5432 I=1,IM
1111      IF(F(I,J,K))5431,5432,5432
1117      5431  F(I,J,K)=F(I,J,K)*GSIG
1125      5432  CONTINUE
1130      5433  CONTINUE
1130      WRITE(6,22)(F(I,J,K),I=1,IM)
1143      WRITE(6,23)(NC(I,J,K),I=1,IM)
          23  FORMAT(4X,10I10)
1156      WRITE(6,23)(NR(I,J,K),I=1,IM)
1171      WRITE(6,22)(CX(I,J,K),I=1,IM)
1204      WRITE(6,22)(CY(I,J,K),I=1,IM)
1217      WRITE(6,22)(CZ(I,J,K),I=1,IM)
1232      IF(ITYPE.EQ.1) WRITE(6,23)(NG(I,J,K),I=1,IM)
1247      DO 120 I=1,IM
1251      IF(F(I,J,K))121,121,120
1257      121  IF(NC(I,J,K).LT.1) WRITE(6,150)I,J,K
          150  FORMAT(3HONC,3I5//)
1277      IF(NR(I,J,K).LT.1) WRITE(6,151)I,J,K
          151  FORMAT(3HONR,3I5//)
1317      RKT=RKT+0.0001
1321      120  CONTINUE
1324      DO 24 I=1,IM
C
          C  RECALCULATE CURRENT IN PULSE FROM PULSE-TIME AND MEAN-CURRENT
1343      Q(I,J,K)=Q(I,J,K)*UA*PA*(TOFF+TON)/TON
1344      T(I,J,K)=T(I,J,K)+273-16
1346      24  TG(I,J,K)=T(I,J,K)

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HEAT

1350	18	CONTINUE
1355		ION=1
1355		TSW=TON
1357	25	ITER=-1
1360		TSUR4=TSUR**4
1362	26	CONTINUE
	C	GAUSS-SIEDEL ITERATION CYCLE STARTS
1362		IF(I TYPE.EQ.0) GO TO 3100
1363		QTG=0.
1364		CK=SUD(CKA,CKB,GT)
1366		GMU=SUD(CMA,CMB,GT)
1371		CR=CMV*CPRES/(82.054*GT)
1375		CP=POL(CPA,CPB,CPC,GT)
1401		CONA=CK*DX*DY*1.62*(1.273*CW*CP/(CK*CD))**0.33333
1413		CONB=3.375E-6*CD**3*CR**2*CB*980.665
1420	3100	CONTINUE
1420		ITER=ITER+1
1422		SUM=0.
1423		DO 30 I=1,IM
1424		DO 30 J=1,JM
1425		DO 30 K=1,KM
1426		DTA=0.
1427		DNA=0.
1427		IF(F(I,J,K))31,32,30
1436	31	DTA=DTA+SIG*F(I,J,K)*(T(I,J,K)**4-TSUR4)
1454		IF(I TYPE)32,32,306
1456	306	IF(NG(I,J,K))32,32,307
1464	307	GMUF=SUD(CMA,CMB,0.5*(GT+T(I,J,K)))
1504		QCG=CONA*(GMU/GMUF)**0.3333
1510		QCG=QCG*(1.+(CONB*(T(I,J,K)-GT)/GMU**2))**0.33333)
	C	LIMIT THE GAS-COOLING TO MAX 1 SURFACE
1520		QCG=QCG*(T(I,J,K)-GT)*F(I,J,K)*0.5
1524		QTG=QTG+QCG
1526		DTA=DTA+QCG
1527	32	L=NC(I,J,K)
1534		CC=VM*POL(AC(L),BC(L),DC(L),T(I,J,K))
1544		IF(CC.LT.0.) CC=0.
1547		DTA=DTA+TG(I,J,K)*CC
1556		IF(ION.EQ.1) DTA=DTA+Q(I,J,K)
1573		DNA=CC
1574		L=NR(I,J,K)
1575		IF(LM.EQ.0) GO TO 74
1577		GO TO(41,42,43),LM
	C	TREAT Z-COORDINATE
1605	43	CC=ZM*CZ(I,J,K)*POL(AZ(L),BZ(L),EZ(L),T(I,J,K))
1624		IF(CC.LT.0.) CC=0.
1627		DNA=DNA+CC
1631		IF(K.EQ.1) GO TO 51
1633		M=NR(I,J,K-1)
1640		IF(M.EQ.0) GO TO 51
1641		CCM=ZM*CZ(I,J,K-1)*POL(AZ(M),BZ(M),EZ(M),T(I,J,K-1))
1657		IF(CCM.LT.0.) CCM=0.
1667		DTA=DTA+CCM*T(I,J,K-1)
1671		DNA=DNA+CCM
1673		GO TO 52
1700	51	DTA=DTA+CC*T(I,J,K)

```

HEAT
1703      DNA=DNA+CC
1704  52    IF(K.EQ.KM) GO TO 53
1706      DTA=DTA+CC*T(I,J,K+1)
1715      GO TO 42
1715  53    DTA=DTA+CC*T(I,J,K)
        C    TREAT Y-COORDINATE
1724  42    CC=YM*CY(I,J,K)*POL(AY(L),BY(L),EY(L),T(I,J,K))
1743      IF(CC.LT.0.) CC=0.
1746      DNA=DNA+CC
1750      IF(J.EQ.1) GO TO 61
1752      M=NR(I,J-1,K)
1756      IF(M.EQ.0) GO TO 61
1760      CCM=YM*CY(I,J-1,K)*POL(AY(M),BY(M),EY(M),T(I,J-1,K))
1775      IF(CCM.LT.0.) CCM=0.
2005      DTA=DTA+CCM*T(I,J-1,K)
2007      DNA=DNA+CCM
2011      GO TO 62
2016  61    DTA=DTA+CC*T(I,J,K)
2021      DNA=DNA+CC
2022  62    IF(J.EQ.JM) GO TO 63
2024      DTA=DTA+CC*T(I,J+1,K)
2033      GO TO 41
2033  63    DTA=DTA+CC*T(I,J,K)
        C    TREAT X-COORDINATE
2042  41    CC=XM*CX(2,4,K)*POL(AX(L),BX(L),EX(L),T(I,J,K))
2056      IF(CC.LT.0.) CC=0.
2061      DNA=DNA+CC
2063      IF(I.EQ.1) GO TO 71
2065      M=NR(I-1,J,K)
2072      IF(M.EQ.0) GO TO 71
2074      CCM=XM*CX(I-1,J,K)*POL(AX(M),BX(M),EX(M),T(I-1,J,K))
2110      IF(CCM.LT.0.) CCM=0.
2120      DTA=DTA+CCM*T(I-1,J,K)
2122      DNA=DNA+CCM
2124      GO TO 72
2131  71    DTA=DTA+CC*T(I,J,K)
2134      DNA=DNA+CC
2135  72    IF(I.EQ.IM) GO TO 73
2137      DTA=DTA+CC*T(I+1,J,K)
2146      GO TO 74
2146  73    DTA=DTA+CC*T(I,J,K)
2155  74    SU=-T(I,J,K)
2162      IF(DNA.EQ.0. .OR. DTA.GT.1.E+100) WRITE(6,75)I,J,K,ITER,DNA,DTA
        75    FORMAT(3H0**,4I5,2E15.4)
2217      T(I,J,K)=DTA/DNA
2221      SU=SU+T(I,J,K)
2223      SUM=SUM+ABS(SU)
2225      IF(T(I,J,K).GT.TMTT) GO TO 400
2235  30    CONTINUE
        C
        C    CALCULATE NEW GAS MEAN-TEMPERATURE
2245      IF(ITYPE.EQ.1) GT=CTC-CTG/(CW*CP)
2252      IF(ITER.GE.1000) GO TO 1001
2255      IF(SUM.GT.RKT) GO TO 26
        C
2260  1001   TIME=TIME+DT

```

HEAT

```

2262      DO 80 K=1,KM
2264      IF(TIME.LT.TMIN)GO TO 2000
2266      WRITE(6,5)(PT(I),I=1,15),IVERS,MODIF
2300      WRITE(6,3)UA,PA,TJN,TOFF,TMLT
2316      IF(OT.LE.O.) WRITE(6,90)K,ITER
2330      IF(OT.GT.O.) WRITE(6,19)K,TIME,ITER
          90      FORMAT(6HOLAYER,I3,12HSTEADY-STATE,I5,4HITER)
2343      2000      CONTINUE
2343      WRITE(6,404)
          404      FORMAT(1H )
2347      IF(ION) 101,101,102
2351      101      WRITE(6,105)
          105      FORMAT(9H+BEAM OFF)
2355      GO TO 103
2356      102      WRITE(6,106)
          106      FORMAT(8H+BEAM ON)
2362      103      CONTINUE
2362      IF(TIME.LT.TMIN)GO TO 2001
2365      WRITE(6,404)
2370      WRITE(6,20)(I,I=1,IM)
2403      2001      CONTINUE
2403      TMN=-1000.
2405      DO 80 J=1,JM
2406      DO 82 I=1,IM
2407      IF(F(I,J,K).GT.O.) GO TO 82
2416      IF(T(I,J,K).LT.TMN) GO TO 82
2427      TMN=T(I,J,K)
2430      IH=I
2430      JH=J
2431      KH=K
2433      82      PR(I)=T(I,J,K)-273.16
2444      IF(TIME.LT.TMIN)GO TO 2002
2447      WRITE(6,21)J,(PR(I),I=1,IM)
2457      2002      CONTINUE
2457      DO 81 I=1,IM
2461      PR(I)=-SIG*F(I,J,K)*T(I,J,K)**4
2474      PR(I)=PR(I)*1000.
2476      IF(F(I,J,K))81,81,83
2503      83      PR(I)=0.
2505      81      TG(I,J,K)=T(I,J,K)
2520      IF(TIME.LT.TMIN) GO TO 80
2522      WRITE(6,22)(PR(I),I=1,IM)
2531      IF(ITYPE) 80,80,315
2533      315      DO 310 I=1,IM
2535      IF(F(I,J,K))311,312,312
2543      311      IF(NG(I,J,K))312,312,313
2551      313      GMUF=SUD(CMA,CMB,0.5*(GT+T(I,J,K)))
2571      QCG=CONA*(GMU/GMUF,**0.33333
2575      QCG=QCG*(1.+(CONB*(T(I,J,K)-GT)/GMU**2)**0.3333)
2605      QCG=QCG*(T(I,J,K)-GT)*F(I,J,K)*0.5
2611      PR(I)=-QCG
2613      GO TO 310
2614      312      PR(I)=0.
2616      310      CONTINUE
2621      WRITE(6,22)(PR(I),I=1,IM)
2627      80      CONTINUE

```


HEAT

2634		IF (TIME.GE.TMIN) WRITE(6,404)
2643		IF (ITYPE.EQ.0) GO TO 321
2644		GTT=GT-273.16
2646		WRITE(6,320)GTT
	320	FORMAT(1H+,60X,15H GAS MEAN-TEMP=,F10.2,5HDEG.C)
2654	321	CONTINUE
2654		TMN=TMN-273.16
2656		WRITE(6,2005)TIME, TMN, IH, JH, KH, ITER
	2005	FORMAT(11X,7H T-MAX=,F10.5,F10.2,4I3)
2676		IF (DT.LE.0.) GO TO 1000
2700		IF (TIME.LT.TSW) GO TO 100
2702		IF (ION.EQ.1) TSW=TSW+TOFF
2706		IF (ION.NE.1) TSW=TSW+TON
2711		ION=-ION
2712	100	CONTINUE
2712		IF (TIME.LT.TMAX) GO TO 25
2715		GO TO 1000
	C	AN ELEMENT IS SO HOT THAT IT MELTS AWAY
2715	400	WRITE(6,401)I, J, K, ITER
	401	FORMAT(13H **MELTING AT,4I3,2H**)
2736		F(I, J, K)=1.
2737		T(I, J, K)=273.16
2741		CX(I, J, K)=0.
2741		CY(I, J, K)=0.
2742		CZ(I, J, K)=0.
2743		IF (I.GT.1) CX(I-1, J, K)=0.
2752		IF (J.GT.1) CY(I, J-1, K)=0.
2761		IF (K.GT.1) CZ(I, J, K-1)=0.
2770		GO TO 25
2771		END

APPENDIX 3. Example of the printout from program HEAT v4m1

1/2MIL 1CM DIAH AU-FOIL IN DOUBLE GAUSSIAN AT 1M/UA

HEAT VERS 4 MOD 1

BEAM= 4.000UA, POWER= 2.526E-03MEV/POINT TON= .0050S, TOFF= .0200S, HELTS AT T= 10000.00

DX= .06250CM, DY= .06250CM, DZ= .00127CM, DT= .00100SEC, THAX= .22600SEC, THIN= .00000SEC

TSUR= 20.00DEG

TYPE= 0 VACUUM

RX-FUNC

1	3.29400	-.00057	4183.00
2	1.64700	-.00020	2092.00
3	-0.00000	-0.00000	-0.00

RY-FUNC

1	3.29400	-.00057	4183.00
2	3.29400	-.00057	4183.00
3	1.64700	-.00020	2092.00

RZ-FUNC

1	-0.00000	-0.00000	-0.00
2	-0.00000	-0.00000	-0.00
3	-0.00000	-0.00000	-0.00

C-FUNC

1	2.31000	.00051	-0.00
2	1.15500	.00025	-0.00
3	.57950	.00013	-0.00

9 X 9 X 1 = 2 DIMENSIONAL

DATA

I
Q
F
CT
RT
CX
CY
CZ

90.99750 POINTS

LAYER 1 AT TIME 0.00000SEC -1ITER

X=	1	2	3	4	5	6	7	8	9	
Y= 1	20.00	20.00	20.00	20.00	20.00	20.00	20.00	20.00	20.00	T(°C)
	0.00	.39	.97	1.66	2.80	1.66	.97	.39	.06	P(points)
	1.00	-.02	-.02	-.02	-.02	-.02	-.02	-.02	-.01	F
	2	2	2	2	2	2	2	2	3	Cp-type
	2	2	2	2	2	2	2	2	3	λ-type
	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	C ₁
	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	C ₂
	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	C ₃
Y= 2	20.00	20.00	20.00	20.00	20.00	20.00	20.00	20.00	20.00	
	0.00	.74	1.62	3.22	3.98	3.52	2.28	1.23	.39	
	1.00	-.04	-.04	-.04	-.04	-.04	-.04	-.04	-.02	
	1	1	1	1	1	1	1	1	2	
	1	1	1	1	1	1	1	1	3	
	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	
	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	
	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	
Y= 3	0.00	20.00	20.00	20.00	20.00	20.00	20.00	20.00	20.00	
	0.00	0.00	1.52	2.52	3.32	3.89	3.12	2.28	.97	
	1.00	1.00	-.04	-.04	-.04	-.04	-.04	-.04	-.02	
	0	1	1	1	1	1	1	1	2	
	0	1	1	1	1	1	1	1	3	
	0.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	
	0.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	
	0.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	
Y= 4	0.00	20.00	20.00	20.00	20.00	20.00	20.00	20.00	20.00	
	0.00	0.00	1.00	2.10	3.34	3.95	3.89	3.52	1.66	
	1.00	1.00	-.04	-.04	-.04	-.04	-.04	-.04	-.02	
	0	1	1	1	1	1	1	1	2	
	0	1	1	1	1	1	1	1	3	
	0.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	
	0.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	
	0.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	
Y= 5	0.00	0.00	20.00	20.00	20.00	20.00	20.00	20.00	20.00	
	0.00	0.00	0.00	1.43	2.46	3.34	3.82	3.98	2.00	
	1.00	1.00	1.00	-.04	-.04	-.04	-.04	-.04	-.02	
	0	0	1	1	1	1	1	1	2	
	0	0	1	1	1	1	1	1	3	
	0.00	0.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	
	0.00	0.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	
	0.00	0.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	
Y= 6	0.00	0.00	20.00	20.00	20.00	20.00	20.00	20.00	20.00	
	0.00	0.00	0.00	.74	1.43	2.10	2.82	3.22	1.66	
	1.00	1.00	1.00	-.04	-.04	-.04	-.04	-.04	-.02	
	0	0	1	1	1	1	1	1	2	
	0	0	1	1	1	1	1	1	3	
	0.00	0.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	
	0.00	0.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	
	0.00	0.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	

1/2 MIL ICH DIAM AU-FOIL IN DOUBLE GAUSSIAN AT 1W/UA

HEAT VERS 4 MOD 1

BEAM= 4.030UA, POWER= 2.526E-03MEV/POINT TON= .0050S, TOFF= .0200S, HELTS AT T= 10000.00

LAYER 1 AT TIME .20100SEC 11ITER

BEAM OFF

X=	1	2	3	4	5	6	7	8	9	
Y= 1	20.00 0.00	29.60 .00	41.95 .00	55.33 .01	68.07 .01	78.95 .01	87.17 .01	92.24 .01	93.96 .00	T(°C) P _{radiated} (mW)
Y= 2	20.00 0.00	27.67 .01	40.18 .01	53.68 .01	66.42 .01	77.28 .01	85.47 .01	90.54 .02	92.25 .01	
Y= 3	0.00 0.00	20.00 0.00	35.13 .01	49.03 .01	61.67 .01	72.36 .01	80.45 .01	85.48 .01	87.18 .01	
Y= 4	0.00 0.00	20.00 0.00	29.65 .01	42.38 .01	54.32 .01	64.55 .01	72.38 .01	77.30 .01	78.98 .01	
Y= 5	0.00 0.00	0.00 0.00	20.00 0.00	34.00 .01	44.86 .01	54.32 .01	61.68 .01	66.45 .01	68.10 .01	
Y= 6	0.00 0.00	0.00 0.00	20.00 0.00	27.21 .01	34.01 .01	42.38 .01	49.04 .01	53.69 .01	55.34 .01	
Y= 7	0.00 0.00	0.00 0.00	0.00 0.00	20.00 0.00	20.00 0.00	29.64 .01	35.14 .01	40.18 .01	41.95 .00	
Y= 8	0.00 0.00	0.00 0.00	0.00 0.00	0.00 0.00	0.00 0.00	20.00 0.00	20.00 0.00	27.65 .01	29.58 .00	
Y= 9	0.00 0.00	0.00 0.00	0.00 0.00	0.00 0.00	0.00 0.00	0.00 0.00	0.00 0.00	20.00 0.00	20.00 0.00	
T-MAX=	.20100	93.96	9	1	1	11				

Table I. Approximate grayness factors (g) for some common materials (data from Ref. 7).
O = oxidized, U = unoxidized.

Material	Surface	25° C	100° C	200° C	500° C	600° C	1000° C	1200° C	1500° C
Aluminum	U	0.022	0.028		0.060				
	O			0.11		0.19			
Bismuth	U	0.048	0.061						
Brass	U	0.035	0.035						
	O			0.61		0.59			
Carbon	U	0.81	0.81		0.79				
Chromium	U		0.08						
Cobalt	U				0.13		0.23		
Niobium	U								0.19
Copper	U		0.02						
	O			0.6			0.6		
Gold	U		0.02		0.03				
Iron	U		0.05						
	O		0.74		0.84			0.89	
Lead	U		0.05						
	O			0.63					
Monel	O			0.43		0.43			
Mercury	U	0.10	0.12						
Molybdenum	U						0.13		0.19
Nickel	U	0.045	0.06		0.12		0.19		
	O			0.37				0.85	
Platinum	U	0.037	0.047		0.096		0.152		0.191
Silica	Amorph.						0.80	0.85	
Silver	U		0.02		0.035				
Steel	U		0.08						
	O	0.80		0.79		0.79			
Tantalum	U								0.21
Tin	U	0.043	0.05						
Tungsten	U	0.024	0.032		0.071		0.15		0.23
Zinc	U	0.05							

Table II. Coefficients in the polynomial approximation of the heat conductivity, $\lambda = a + bT + c/T^2$, with T in °K and λ in W/cm degree.

Material	a	b	c	Range (°K)	Max. deviation (%)
Aluminum	2.501	$-4.184 \cdot 10^{-4}$	$1.114 \cdot 10^4$	15-920	+30/-15
Beryllium	1.893	$-9.897 \cdot 10^{-4}$	$1.316 \cdot 10^4$	20-1250	+72/-18
Bismuth	0.03162	$1.091 \cdot 10^{-4}$	$4.308 \cdot 10^2$	70-1070	+22/-26
Cadmium	1.495	$-1.537 \cdot 10^{-3}$	$-2.368 \cdot 10^3$	86-673	+16/-10
Chromium	1.116	$-4.421 \cdot 10^{-4}$	$2.192 \cdot 10^3$	20-1200	+39/-14
Cobalt	2.098	$-4.883 \cdot 10^{-3}$	$1.909 \cdot 10^3$	25-293	+10.7/-6.1
Copper	4.416	$-9.729 \cdot 10^{-4}$	$9.236 \cdot 10^3$	20-1280	+32/-9
Germanium, single cryst.	0.8568	$-1.135 \cdot 10^{-3}$	$7.142 \cdot 10^3$	30-370	+4.1/-5.8
Gold	3.294	$-5.697 \cdot 10^{-4}$	$4.183 \cdot 10^3$	30-1300	+3.5/-3.5
Graphite	0.1405	$4.557 \cdot 10^{-5}$	$2.163 \cdot 10^5$	293-2273	+1.5/-1.6
Iridium	0.8179	$1.232 \cdot 10^{-3}$	$9.477 \cdot 10^3$	24-500	+13/-8
Iron	0.9106	$-6.042 \cdot 10^{-4}$	$3.934 \cdot 10^3$	20-1040	+32/-18
Lead	0.1079	$5.539 \cdot 10^{-5}$	$2.025 \cdot 10^4$	293-1073	+28/-20
Magnesium	1.891	$-8.682 \cdot 10^{-4}$	$4.285 \cdot 10^3$	19-1073	+28/-13
Manganese	-1.143	$9.842 \cdot 10^3$	$1.975 \cdot 10^3$	56-273	---
Mercury	0.02104	$1.221 \cdot 10^{-4}$	$3.360 \cdot 10^3$	83-1070	+53/-21
Molybdenum	1.467	$-3.465 \cdot 10^{-4}$	$2.435 \cdot 10^3$	33-1910	+5.8/-7.6
Nickel	0.5272	$4.078 \cdot 10^{-5}$	$2.401 \cdot 10^3$	33-1620	+38/-24
Niobium	0.4277	$1.347 \cdot 10^{-4}$	$5.718 \cdot 10^2$	30-1900	+10/-6
Palladium	0.1035	$1.837 \cdot 10^{-3}$	$2.390 \cdot 10^3$	30-373	+9.9/-7.1
Platinum	0.6327	$1.307 \cdot 10^{-4}$	$747 \cdot 10^3$	12-2000	+6.0/-4.0
Rhenium	0.4242	$5.748 \cdot 10^{-5}$	$2.34 \cdot 10^3$	30-500	+12/-10
Rhodium	1.903	$-1.110 \cdot 10^{-3}$	$1.879 \cdot 10^3$	10-500	+2.1/-3.4
Selenium, single cryst.	0.02174	$-2.714 \cdot 10^{-7}$	$1.658 \cdot 10^2$	25-400	+13/-14
Silicon, single cryst.	0.9443	$-5.277 \cdot 10^{-4}$	$8.166 \cdot 10^3$	23-1400	+72/-43
Silver	5.334	$-2.454 \cdot 10^{-3}$	$6.717 \cdot 10^3$	15-800	+14/-5.5
Tantalum	0.4709	$6.250 \cdot 10^{-5}$	$1.878 \cdot 10^2$	30-2500	+18/-8
Thorium	0.2298	$2.227 \cdot 10^{-4}$	$-3.261 \cdot 10^2$	300-580	---
Tin	0.2287	$3.867 \cdot 10^{-5}$	$3.478 \cdot 10^4$	293-773	+22/-10
Titanium	0.2934	$-1.245 \cdot 10^{-4}$	$-2.398 \cdot 10^1$	10-800	+22/-17
Tungsten	1.453	$-1.801 \cdot 10^{-4}$	$7.139 \cdot 10^3$	30-3500	+17/-9
Uranium	0.1884	$2.433 \cdot 10^{-4}$	$-3.047 \cdot 10^1$	20-1070	+7.5/-6.2
Vanadium	0.2870	$9.620 \cdot 10^{-5}$	$-9.041 \cdot 10^1$	20-1173	+2.0/-1.3
Zinc	1.256	$-4.509 \cdot 10^{-4}$	$-2.037 \cdot 10^2$	73-673	+0.5/-0.3
Zirconium	0.3256	$-1.364 \cdot 10^{-4}$	$2.143 \cdot 10^2$	36-600	+1.3/-1.9

Table III. Coefficients in the polynomial approximation of the heat capacity, $C_p = a + bT + c/T^2$, with T in °K and C_p in J/ml degree. Values recalculated from Ref. 8.

Material	a	b	c	Range (°K)
Aluminum	2.061	$1.235 \cdot 10^{-3}$	---	298-932
Antimony	1.266	$4.000 \cdot 10^{-4}$	---	298-903
Arsenic, metallic	1.67	$7.09 \cdot 10^{-4}$	---	298-1100
Beryllium	2.902	$2.475 \cdot 10^{-3}$	---	298-1300
Bismuth	0.881	$1.059 \cdot 10^{-3}$	---	298-544
Cadmium	1.707	$9.45 \cdot 10^{-4}$	---	298-594
Calcium	0.8478	$5.50 \cdot 10^{-4}$	---	298-673
Chromium	0.3336	$1.348 \cdot 10^{-3}$	$-5.026 \cdot 10^4$	298-1823
Cobalt	2.949	$2.686 \cdot 10^{-3}$	---	298-718
Copper	3.181	$8.82 \cdot 10^{-4}$	---	298-1357
Germanium, cryst.	1.837	$3.519 \cdot 10^{-4}$	---	298-1213
Gold	2.318	$5.075 \cdot 10^{-4}$	---	298-1336
Graphite	3.219	$8.007 \cdot 10^{-4}$	$-1.649 \cdot 10^5$	298-2300
Iridium	2.702	$6.905 \cdot 10^{-4}$	---	298-1223
Iron	1.985	$4.181 \cdot 10^{-3}$	$2.53 \cdot 10^4$	298-1033
Lead	1.332	$4.354 \cdot 10^{-4}$	---	298-600
Magnesium	1.837	$4.49 \cdot 10^{-4}$	$-2.33 \cdot 10^4$	298-923
Manganese	3.126	$1.854 \cdot 10^{-3}$	$-2.03 \cdot 10^4$	298-1000
Mercury	1.868	---	---	298-634
Molybdenum	2.438	$5.78 \cdot 10^{-4}$	---	298-1800
Nickel	2.576	$4.467 \cdot 10^{-3}$	---	298-633
Niobium	2.183	$3.703 \cdot 10^{-4}$	---	298-1900
Palladium	2.604	$6.194 \cdot 10^{-4}$	---	298-1828
Platinum	2.629	$6.13 \cdot 10^{-4}$	---	298-1800
Rhenium	2.833	---	---	298
Rhodium	2.768	$1.039 \cdot 10^{-3}$	---	298-1900
Selenium, cryst.	1.55	---	---	298
Silicon, cryst.	2.071	$2.016 \cdot 10^{-4}$	$-3.912 \cdot 10^4$	298-1200
Silver	2.073	$8.31 \cdot 10^{-4}$	$1.47 \cdot 10^4$	298-1234
Tantalum	2.234	$2.988 \cdot 10^{-4}$	---	298-1900
Thorium	1.292	$6.182 \cdot 10^{-4}$	$7.05 \cdot 10^3$	298-1500
Tin	1.139	$1.624 \cdot 10^{-3}$	---	298-505
Titanium	2.064	$9.904 \cdot 10^{-4}$	---	298-1150
Tungsten	2.52	$3.34 \cdot 10^{-4}$	---	298-2000
Uranium	1.115	$2.635 \cdot 10^{-4}$	$2.3 \cdot 10^4$	298-935
Vanadium	2.598	$9.622 \cdot 10^{-4}$	---	298-1900
Zinc, metallic	2.445	$1.097 \cdot 10^{-3}$	---	298-692
Zirconium	1.276	$1.240 \cdot 10^{-3}$	$1.678 \cdot 10^4$	298-900

Fig. 1. Pure radiation cooling at different duty cycles [Eq. (39)]; $\frac{1}{4}$ mil ($\sim 1.4 \text{ mg/cm}^2$) carbon foil at 51 W/cm^2 .

XBL736-3112

Fig. 2. The temperature ratio between Gaussian and uniform beams as a function of the fraction, α , of the beam which is focused on the foil.

Fig. 3. Nomograph for evaluation of foil temperatures at 7.5 MeV/amu incident beam energy and pure radiation cooling. The nomograph is constructed for beryllium, aluminum, and carbon foils, and it consists of four quadrants. In the first quadrant the total dissipated power per current unit is evaluated from foil thickness, ion beam, and material. In the second quadrant the maximal power load per surface and current unit is evaluated from the total power, the foil diameter, and the fraction, α , of a Gaussian beam hitting the foil. The correction for α should be omitted for a uniform beam. In the third quadrant the maximal power per surface unit is evaluated from the beam current (particle current), and in the fourth quadrant the foil temperature is evaluated from the material and duty cycle (in percent).

Fig. 4. Nomograph for the evaluation of foil temperatures at 7.5 MeV/amu incident beam energy, uniform beam shape, and pure conduction cooling. A first-order correction for Gaussian beam shape can be made as follows. First obtain the correction factor ψ/α from Fig. 8, then read off the temperature for uniform beam shape, subtract T_{sur} , multiply the remainder by ψ/α and add T_{sur} again. The nomograph is constructed for beryllium, aluminum, and carbon foils and it consists of four quadrants. In the first quadrant the power per unit thickness and current is evaluated from the foil thickness, ion beam, and foil material. This is converted in the second quadrant to power per unit thickness by multiplication with the beam current. The third quadrant involves mainly a change of scale from logarithmic to linear but it also corrects for the duty cycle. The curves are to be used for foils where the foil diameter (cm) is smaller than $\sqrt{0.05 c \lambda / f \rho C_p}$, where c is the duty cycle (percent), λ is the heat conductivity (W/cm degree) f is the beam pulse frequency (Hz), ρ is the density (g/cm^3) and C_p is the heat capacity ($\text{J}/\text{cm}^3 \text{ degree}$). In the fourth quadrant the maximum temperature is evaluated from the edge temperature and material.

XBL736-3113

Fig. 5. Pure conduction cooling at different duty cycles [Eq. (56)]; $\frac{1}{2}$ -mil, 0.71-mm diameter aluminum foil at 3.18 W/cm^2 .

XBL 736-3114

Fig. 6. Variation of the center temperature with time for $\frac{1}{2}$ -mil circular aluminum foils of 2, 1.4, 1.0, 0.71 and 0.5 cm diameter at a frequency of 50 Hz and a duty cycle of 10%.

XBL736-3115

Fig. 7. The multiplier, ψ , for Gaussian beam shape and conduction-cooled foils as a function of the fraction, α , of the beam focused on the foil [see Eq. (61)].

XBL736-3i16

Fig. 8. The multiplier ψ/α for Gaussian beam shape and conduction-cooled foils as a function of the beam fraction, α , focused on the foil [see Eq. (62)].

XBL736-3117

Fig. 9. $\frac{1}{2}$ -mil, 1-cm-diameter gold foil in a pulsed uniform beam and a total dissipated power of 4W at a frequency of 40Hz and a duty cycle of 20%.

XBL736-3118

Fig. 10. $\frac{1}{2}$ -mil, 1-cm-diameter gold foil in a pulsed Gaussian beam at a total dissipated power of 4W, a frequency of 40Hz and a duty cycle of 20%.

XBL 736-3119

Fig. 11. $\frac{1}{2}$ -mil, 1-cm-diameter gold foil in a pulsed double-Gaussian beam at a total dissipated power of 4W, a frequency of 40Hz and a duty cycle of 20%.

XBL736-3120

Fig. 12. Temperature limits for material as a function of pressure. At high pressure the limit is set by the melting point and at low pressure by the vapor pressure.