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SUGGESTIONS FOR A SUB-MICROSECOND SOFT X-RAY SOURCE

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ABSTRACT

It is shown that synchrotron radiation can be considered for a pulsed source of high intensity soft x-ray photons by means of a combination of relativistic electron rings with megagauss magnetic field compression technology. Synchrotron radiation is usually obtained in the visible and ultraviolet spectral regions from laboratory particle accelerators, but our calculations show that the compression of electron rings (with their accompanying magnetic fields) inside imploding metal liners should allow us to make compact synchrotron sources of high intensity. The method has the advantage of being efficient (up to 25% of the energy in a capacitor bank may be convertible to photon output). It can work with existing electron beam machines in the $E = 5\text{--}15$ MeV range, unlike many pellet fusion experiments, which will generally require machines far beyond the present state of the art in energy and energy density. Finally, our proposed method provides a very flexible x-ray source, in that the 5-15 MeV variation in primary electron energy can provide x-ray spectra peaked at energies from about 1-30 keV. The total number of joules radiated depends only on the energy content of the input electron beam, being 20 times the input for efficient compression. The major uncertainty, which needs to be tested experimentally, is the requirement that the energy density of the electron ring be higher than has yet been achieved.

The above possibilities lead us to suggest the program described in this report. We propose to demonstrate feasibility of electron ring formation and compression with an existing 2 MeV, 1.5 kilojoule E-beam machine, plus a capacitor bank liner implosion system. Given the success of this first step, it would be possible to design a facility using a 50 kilojoule E-beam machine, plus auxiliary energy sources for implosion, to provide 100 kJ-500 kJ of kilovolt photon output.

I. INTRODUCTION

The need for generating intense bursts of soft x-ray photons has long been recognized. One well-known source of such x-rays, electron synchrotron radiation, is usually not considered for this application because of its low intensity.¹ In ordinary electron accelerators, such as described in the references, the synchrotron-radiated power comes out over a large area and long periods of time: the machines inherently cannot have a greater radiated power than the input power to their accelerating system. We propose here a technique using imploded-liner megagauss field technology^{2,3} to concentrate relativistic electron rings in a very small volume and cause them to radiate large amounts of energy in a very short time, an idea first discussed by Nowak and Shearer.⁴ (This energy is radiated in a restricted solid angle, thus further increasing the intensity.)

The ability of high current electron rings or "E-layers" to form and survive in a proper plasma for long times (thousands of orbits) has been known for almost 10 years.⁵ A schematic apparatus for forming such layers is shown in Fig. 1: electrons are injected on a spiral path into a magnetic field via a pipe (the "inflector"). They are trapped between regions of strong magnetic field (magnetic mirrors) by means of a dissipative interaction with the background plasma or an appropriate resistive layer near the wall.⁵ If the layer is sufficiently intense and there is a conducting wall around it, the layer will remain centered in the vessel until classical energy loss or scattering causes the individual electrons to be lost, or the layer strength to decay to an unstable level.⁶

The energy of the E-layer electrons can be raised by raising the applied magnetic field B .^{7,8} This can be accomplished either by using a

capacitor bank to provide more ampere-turns in the external coils, or by imploding the metallic liner with explosives, as in Fig. 2, or with a strong magnetic field.^{3,9} As the particle energy increases, proportionally to $B^{1/2}$, the synchrotron radiation rate increases dramatically, being proportional to γ^4/R^2 or B^3 for an adiabatic compression. Here $\gamma = m/m_0 = E/m_0c^2$ is the usual relativistic parameter, assumed much greater than unity. The peak energy of the emitted photon distribution also increases with the compression, being proportional to B^2 for the adiabatic case. Thus, by performing a fast compression of an E-layer, a short pulse of high energy photons is to be expected. A slot is provided in the center of the liner to permit these x-rays to escape: they are concentrated in a narrow angular pancake perpendicular to the axis.^{1,2} The liner-implosion technique has three important advantages: the "wall" stays close to the layer throughout, enhancing stability; the attainment of higher fields is possible; and finally, the efficiency of the process is greater, since one only provides megagauss-level fields in the small final volume.

We turn now to an analysis of the above process. Section II discusses the radiation from an E-layer. We will introduce there the E-beam parameters for the experiment we suggest. Sections III-V provide a theoretical discussion of the formation, stability and compression of the E-layer. Section VI provides a description of a small-scale demonstration of the E-layer trapping principle. Section VII provides a description of the proposed medium scale imploding-liner experiment. Section VIII provides cost estimates and time tables.

II. RADIATION FROM E-LAYERS

The synchrotron radiation spectrum for a single electron in a circular orbit is well known, and is plotted in Fig. 3 in non-dimensional form. The critical photon frequency and energy are given by $\omega_c = 3\gamma^3 (c/a)$, or in cgs units,

$$\text{II.1 (a)} \quad \omega_c = 8.38 \times 10^6 B\gamma^2 \text{ (sec}^{-1}\text{)}$$

$$\text{(b)} \quad \hbar\omega_c = 3.47 \times 10^{-8} B\gamma^2 \text{ (eV)}$$

where B is in gauss, and ω_c is the characteristic upper frequency for the radiation. The corresponding radius of gyration for a single electron is

$$\text{II.2} \quad a = 1.70 \times 10^3 \gamma/B \text{ (cm)}$$

The time scale for the conversion of the kinetic energy of the electron into radiation is of the order of the energy divided by the power and is given by

$$\text{II.3} \quad \frac{\gamma}{d\gamma/dt} \equiv \tau = 5.2 \times 10^8 / \gamma B^2 \text{ (sec)}$$

where the loss rate is

$$\text{II.4} \quad d\gamma/dt = - 1.94 \times 10^{-9} B^2 \gamma^2 \text{ (sec}^{-1}\text{)}$$

Let the magnetic field be changed by a factor λ^2 ; i.e., if the change is induced by an imploding liner, λ would be the ratio of the initial radius to the final radius. Then the above quantities would change by the factors given below:

$$\text{II.5.a} \quad B = B_0 \lambda^2$$

$$\gamma = \gamma_0 \lambda$$

$$a = a_0/\lambda$$

$$\omega_c = \omega_{c0} \lambda^4$$

$$\tau = \tau_0/\lambda^5$$

We envision an axial compression also, but this only alters the motion along the magnetic field, which is assumed small. Note that the layer behaves like a gas with $\gamma_g = 3/2$. We choose as an interesting example the case $\hbar\omega_c = 20$ keV, $\tau = 50$ nsec, $\lambda = 20$, which we shall follow throughout this work as our reference case. The initial and final conditions are:

II.5.b	<u>Initial</u>	<u>Final</u>
Magnetic Field B	$13 \frac{3}{4}$ kG	5.5 MG
Electron Energy E_K (MeV) $\approx \frac{\gamma}{2}$	$7 \frac{3}{4}$ MeV	165 MeV
Orbit Radius a	2.0 cm	0.1 cm
Critical x-ray energy $\hbar\omega_c$	.125 eV	20 keV
Characteristic Time τ	.17 sec	52 nsec

The total energy, W, in the layer depends only on the initial E-layer energy content. We assume a 50 kJ electron beam machine, and 10% - 50% trapping efficiency. This leads to 100-500 kilojoules after compression. We adopt 200 kJ as our reference case (see Section III for more detailed discussion). The final photon spectrum has its peak at about $0.15 \hbar\omega_c$ according to Fig. 3. One might choose other voltages such as the 15 MeV ($\gamma_0 \sim 30$) used in the Aurora machine, where a spectrum with $\hbar\omega_c$ of about 50 keV would be

achieved with $\lambda = 20$; alternatively, one could go to $\lambda = 30$, or complicated orbits (see below). In short, photon spectra peaked in the keV range with sub-microsecond delivery time seem easily accessible.

For strong electron rings the single electron theory described above needs to be modified. A "strong" layer is one in which the layer's diamagnetic current seriously affects the magnetic field. A measure of this is the field reversal parameter $\zeta = \Delta B/B_V$, where ΔB is the jump in field across the layer and B_V is the "vacuum" field present before the layer is introduced. In general one adjusts B_V so that $a_0 = \gamma_0 mc/B_V q$, so that the layer is injected at the equilibrium radius (q is the electron charge). To allow for the possibility of a plasma current in the opposite direction to the layer current, commonly called "current neutralization", we distinguish between $\zeta_L = \mu_0 I_E/LB_0$ as a measure of the ΔB due to the layer alone, and $\zeta = \zeta_{\text{net}} = \mu_0 (I_E - I_P)/LB_0$ based on the total current where B_0 is the average B field. For our example with 10 kJ initial, 200 kJ final, this return current is necessary - see Section V.

If $\zeta_{\text{net}} = 0(1)$, the electron orbits will in general no longer be circular, but will oscillate about the mean radius of the layer with an amplitude of order of but less than the thickness of the layer (see Fig. 4). It is these oscillations which determine the radiation spectrum. They have the effect of hardening the radiation spectrum over that which arises from purely circular motion.

Although ζ_{net} is dependent on the fraction of current neutralization and also time dependent for a less than perfectly conducting plasma, ζ_L is

dependent only on the number of electrons per unit length, N' , and is related to the kinetic energy, W_K , per unit length (L) of the E-layer by

$$\text{II.6} \quad W_K/L = 14.54 \, \epsilon_L \gamma^2 \quad (\text{J/M})$$

where $\gamma \gg 1$. The required ϵ_L is thus given, for a synchrotron source, by the energy content and size of source required. The E-layer will radiate into an angle of $\pm 1/\gamma$ of the plane of the orbit, according to simple theory.² When the final γ is of the order of 100 or more, this is an extremely small angle. The axial betatron oscillations, however, broaden this angle, and involve typically, $v_z = .10c$ at the midplane of the coil, as shown in Fig. 5. The net result is that we should be able to put all the synchrotron radiation from a 1 cm long layer onto a 10 cm wide annular strip at up to 50 cm from the source. Using an electron layer energy of 200 kJ with $\gamma_f = 323$, we find $\epsilon_L = 13.2$ from II.6. This should be easily achieved for $R_{\text{wall}}/R_{\text{layer}} = 1.5$ or less and with partial current neutralization (see Ref. 10, Eq. 33). With the conducting wall at a large radius, (as in a capacitor bank compression near maximum B), the largest possible value of ϵ is 2, so we see an advantage of liner implosions: they permit more joules/unit length of source by letting the wall follow the layer in.

With a final value of $\epsilon_L = 13.2$ and radial compression with $\lambda = 20$, the initial value of $\epsilon_L = 264$. Even with the assumed axial compression of 20:1, the initial value would be back to 13.2. This, of course, requires nearly complete current neutralization. This level of E-layer energy density has not been attained as yet, although there is no known theoretical reason why it cannot be attained. It is equivalent to $v/\gamma = 6.6$ for a propagating beam. We propose to attain it by a combination of

efficient E-layer trapping and axial compression of the layer after it is trapped, plus a large radial pressure gradient set up by the auxiliary Z-pinch shown in Fig. 2a. We emphasize that if these efforts are unsuccessful, one can still get the desired 100 kJ-200 kJ outputs by firing several modules with $\epsilon_L = 2-3$, which is certainly attainable.

One problem which must be faced is that the scaling of γ with radius given by II.5 is not exact as the electrons radiate during the run-in phase of the implosion. For realistic rise time of the magnetic field, it is necessary to consider the time-dependent spectrum of the synchrotron radiation - this can be obtained by solving the following equation:

$$\text{II.7} \quad d\gamma/dt = - c_1 B^2 \gamma^2 + (\gamma/2B) dB/dt$$

Here the second term represents the adiabatic compression and the first term is the synchrotron loss ($c_1 = - 1.9 \times 10^{-9} \text{ sec}^{-1} \text{ gauss}^{-2}$). This equation is valid in the limit $\epsilon_{\text{net}} \ll 1$.

A set of values of the maximum value of ω_c and final τ for a run-in from $R_{Li} = 2.00 \text{ cm}$, $R_{Wi} = 2.83 \text{ cm}$ in times T_0 from 26 microseconds to 1 microsecond is given in Table I. We conclude that run-in velocities of about 1 cm/microsecond are needed to achieve even 25% of the ideal values from II.5. Again, the detailed effect of the betatron orbits, and possible re-absorption of the synchrotron radiation at early stages, needs to be computed. It is to be emphasized, however, that velocities of 1 cm/microsec can be achieved with appropriate liners.

III. TRAPPING THE E-LAYER

When an electron beam is injected on a spiral path into a magnetic field, as in Fig. 1, there are several means by which the individual

electrons can be trapped between the magnetic mirrors. Efficient trapping is important because the E-beam energy is the most expensive portion of the energy input to the system, even though it is a small fraction ($1/\lambda$) of the total input energy. Conservation of energy and canonical angular momentum would, of course, allow the particles to return to their point of origin and be lost on the injection pipe, in a time-independent field. We therefore must think of dissipative trapping processes or of varying the mirror strength rapidly. One might start with a low mirror value (say 14.3 kilogauss^{*} for the mid plane field in II.5.b) and raise it to a higher value (say 18.3 kilogauss^{**}) in a time comparable with the electron pulse injection time, which is 50-100 nanoseconds for conventional machines. This would require 11-22 kV for a 6 cm coil diameter which might reasonably go with the parameters of II.5.b. With the injected pulse having an axial velocity of .2 c, the resulting layer would be 3 meters long for a 100 ns pulse (folded back on itself in the mirror), and would have to be post-compressed axially to the 2 cm - 4 cm length required for implosion. While such a technique offers a straightforward method of trapping nearly all the electrons produced by the E-beam machine, the difficulties of handling the requisite voltages in the presence of a background plasma, and the difficulties of post-compression, make us view this technique as an adjunct to the dissipative trapping schemes.

Historically, the resistive-layer trapping mechanism, sketched in Fig. 1, was the first to be used.⁵ We will estimate the resistive drag using long-layer theory. In principal one can extract twice the transient mag-

^{*} Corresponding to passing an injected parallel velocity of .2c ($R_m = 1.04$)

^{**} therefore trapping all electrons up to $V_z = .5c$ ($R_m = 1.33$).

netic energy stored between the resistors and the wall as the pulse passes. Fessenden and Stallard have shown that one actually gets about 1/2 the amount based on long-layer theory for realistic pulse shapes.¹² They show that the energy comes out of the axial motion. The spiral motion of the beam is usually washed out in one or two turns by the spread in v_z of the beam electrons,¹³ so the "travelling E-layer" acts like the moving primary of a transformer, with the resistors being the secondary. We will assume the layer length is 20 cm. The ratio of magnetic to kinetic energy for such a layer can be shown to be $(\zeta/4)(1 - R_E^2/R_W^2)$, and some small fraction of this is stored behind the resistors. On the other hand one need only extract a fraction $v_z^2/2c^2$ of the total kinetic energy to bring the layer to rest. Thus a placement of the resistors to intercept about 8% of the layer magnetic energy will allow a $\zeta_{\text{net}} = 1$ layer, injected at $v_z = .2c$ with $R_W = \sqrt{2}R_E$, to stop in a distance equal to its own length, according to ideal theory! The calculations of Fessenden and Stallard suggest that one will need about twice this distance for $\zeta_{\text{net}} = 1$, and this would give a 40 cm stopping length. The case $R_W = \sqrt{2}R_E$ is convenient for injection and stability, and has the property that the transient magnetic energy density is the same inside and outside the layer. Thus to meet the criterion of 8% of the energy being in the ΔR between the resistors and the wall, we need $2\Delta R/R_W = .08$, or $\Delta R = .11$ cm for $R_W = 2.82$ cm, $R_E = 2$ cm from II.5.b. The resistivity of the layer is to be adjusted to give the resistive layer an L/R time comparable to the pulse injection time.¹² Detailed calculations remain to be done, but the dimensions given above, including the e-folding length for stopping, should be very close to the truth, and can in any case be achieved with minor modifications to ΔR .

An even simpler method of trapping the beam has been described by Rostoker and Chu.¹³ They show that in an appropriate weakly collisional plasma the electrons at the front of the e-layer have a reciprocal range, R^{-1} , given by

$$\text{III.1} \quad R^{-1} = \frac{2F_z}{\gamma m V_0^2} = \frac{2n_b \nu_e}{\gamma_0 n_0 V_0} \left(\frac{v_{\perp}^2}{V_0^2} \right)$$

Here ν is the collision frequency, n_b and n_0 are particle densities for the e-layer and plasma respectively, and V_0 is the axial velocity. A discussion of the derivation of III.1 is given in Appendix A. It is shown there that in a reasonable plasma ($n_e = 1.52 * 10^{15}$, $T_e = 8$ eV), the braking length can be made of the order of 40 cm for our sample layer with $\epsilon_L = 13.2$ and $V_0 = .2$ c.

For either the resistor or plasma trapping mechanism, a final magnetic mirror is needed to center the layer in the liner. This is shown in Fig. 2b. Assuming an injection velocity of .2c in the axial direction, and one e-fold down to .0736c, the mirror ratio required to trap the layer is only 1.005! Actually, this means that small field errors will have a serious effect on the layer, so a slow-pulsed field will probably be added to center the layer, as shown in Fig. 2b.

We may conclude, and past experience at LLL and Cornell University bears this out, that 10%-50% (or more) of the injected E-beam electrons will be trapped by a carefully designed system.

IV. STABILITY OF THE LAYER

The stability of electron rings and layers received considerable attention at Livermore during the Astron project.^{5,6,7} There are several

possible instabilities, but at this time all of the observed ones can be stabilized or avoided by well-known means.

The fastest growing and potentially most damaging instabilities are hydromagnetic in nature, such as the so-called "tearing mode". This is a tendency for a layer with an axial perturbation to tear itself up into a series of rings, due to the magnetic self-pinch force. The sequence of events is shown in Figs. 6a, 6b, 6c. The rings tend to coalesce into a new and stable distribution. This process of tearing apart and reassembling was regularly observed in the Livermore Astron experiments - so long as adequate mirrors are provided to contain the individual rings, they will reassemble.¹² The tearing mode is closely related to the filamentation instability for a straight beam,^{18,23} where the reassembly has been observed in computer simulations, and theoretically analyzed.

Another class of instabilities is the set of so-called "precessional modes". The betatron oscillations shown in Fig. 4 become unstable in the presence of resistive walls¹⁴ or plasma with finite resistivity.¹⁵ Fortunately, for an E-layer which is space-charge neutralized, these instabilities are eliminated by the image currents in the walls at high enough E-layer current levels (specifically, $\zeta > f_0/f_m$, as given by Eq. IV.1 of Ref. 6, and subsequent discussion). For our case, this requires $\zeta > .04$, as calculated in Appendix B. This is easily achievable in the initial E-layer.¹⁶ However, as the implosion proceeds, the E-layer length/diameter increases, while ζ decreases, as shown in the next section. For this reason, among others, we choose not to count on wall stabilization, and we elect to put a small toroidal field in the

device by means of a 0.5 mm diameter current-carrying rod down the axis of the device.¹⁷ One then requires

$$\text{IV.1} \quad \Omega_{\theta}/\Omega_z > \eta_M^{1/2}$$

where Ω_{θ} , Ω_z are the cyclotron frequencies based on B_{θ} , B_z respectively, and $\eta_M = (R_M/B_M)(dB_M/dR)$ due to the external mirror field. For the field shape of Fig. 2b, $\eta_M = .0133$ from Appendix B. We thus require $B_{\theta} > .11B_z$ throughout the implosion. This condition is most severe at maximum compression, where $B_{z \text{ max}} = 5.5$ megagauss and $R = 0.1$ cm. The current required is thus at least $I = 2\pi RB_{\theta}/\mu_0 = 302$ kiloamps, which is easily attainable, assuming 6 kV drive voltage, inductance 10^{-7} henry, and rise-time 5 microseconds. The actual current required will be less than this at the end of the implosion due to the intrinsic stabilizing effect of the metal center rod, as explained in Appendix B.

The final class of instabilities is the so-called micro-instabilities such as negative mass¹⁹ and the upper hybrid mode beam-plasma resonances.²⁰ These instabilities quench themselves by non-linear processes after an initial growth phase, are usually absent in the presence of suitably dense plasmas, and in any case involve little E-layer loss.²¹

We can summarize by saying that all known instabilities of the E-layer either have a well-known cure (such as the precessional instabilities), or have non-linear saturation or quenching levels which prevent them from destroying the layer.

V. COMPRESSION OF THE LAYER

It is well known that when a magnetic field is increased in time, the canonical angular momentum of any electron in this field is a constant of

the motion.²² This is true even if the self-fields of the layer are included, as long as they are symmetric.⁷ G. Schmitt has shown that $P_\theta = -qB_0 R^2/2$. This leads to the scaling law $Ba^2 = \text{constant}$, which was used in II.5.a. Further, the work of Ref. 7 can be trivially extended to show that if $\zeta_{\text{net}} \ll 1$, then γ is proportional to $1/a$. Since the layer energy content is proportional to γ and since the energy content of a gas goes like $\frac{1}{a^{2(\gamma_G-1)}}$ where $\gamma_G = C_p/C_v$ is the "adiabatic gamma" in a cylindrical compression, we conclude that the E-layer behaves like a gas with $\gamma_G = 3/2$ in the weak-layer or current neutralized case. We intend to run our experiments nearly or completely current neutralized, so this is an appropriate assumption. The decay of the neutralizing current during the implosion can be made negligible (see below). The centrifugal pressure of the layer has the appropriate behavior ($1/a^3$). In the limit $\zeta_{\text{net}} \gg 1$, the behavior will be quantitatively slightly different, but qualitatively similar; as far as the imploding liner is concerned, the E-layer acts like a gas to be compressed.

The design of the implosion system then hinges on picking the desired compression ratio, λ , and making sure that the stability criteria are met throughout the implosion. Since the E-layer self-field goes up only as $1/a$, while the field it is attempting to reverse goes up as $1/a^2$, the value of ζ_L decreases in a purely radial compression. To achieve a final $\zeta_L = 13.2$ with a reasonable value of λ equal to twenty,²⁴ we would need an initial value of $\zeta_L = 264$. Such a value is impossible¹⁰ without either a strong toroidal field or a complete absence of neutralizing plasma,²⁶ both of which would be difficult to implement in our case due to the problems in injecting large currents. Alternatively, one can have nearly complete electric and magnetic neutralization, so that the

limits of Ref. 10 do not apply. One must supply a plasma pressure gradient to balance the E-layer centrifugal force. We choose the latter path. Following Ref. 10 with appropriate modifications for the current neutralization, we assume the net field reversal must not rise about ± 4 during the ($T_i = 2.7$ microsecond) implosion. Since the plasma return currents are decaying with an L/R time constant toward $\tau = 264$, we must have L/R much greater than $66T_i = 178$ microseconds. Using a layer radius of 2.00 cm, thickness 0.4 cm, we find that a plasma temperature of 100 eV is adequate to ensure this slow decay. The plasma heating by the E-beam should be enough to ensure this.¹³ The plasma density is arbitrary, except that it must exceed the beam density in order to space-charge-neutralize the beam (requiring somewhat above 7.6×10^{13} e⁻/cc for the highest case - the 25 kJ, 20 cm long E-layer shown in Fig. 2). It must also not be dense enough to cut off the x-rays at peak compression, but for a hydrogenic plasma this only requires that the initial density be below $10^{21} - 10^{22}$ e⁻/cc, a limit which is far beyond that set by plasma containment requirements at 100 eV*. Picking $n_e = 15.2 \times 10^{14}$ /cc, the plasma pressure is .4863 atmosphere at 100 eV, which is easily contained by the 13.75 kilogauss field. The plasma confinement will be stable due to the combination of the mirror and toroidal field, plus the residual layer field.

The twenty-fold axial compression is an option, but it is useful to bring all the electrons into the radiating gap, and it reduces the τ problems discussed above. Part will be done by pulsing the axial compression coil and part will be done by shaping the implosion. An overall time-sequence of the magnetic field behavior is shown in Fig. 7. The mirror ratio goes from

*Note: Another limit of about 10^{18} /cc is set by classical scattering during the implosion.

1.33 initially to about 3 at the end. We have assumed, for the purposes of this figure, that the layer and background plasma compress together as a fluid with $\gamma_G = 1.5$. Actually, the background plasma will have a slightly different behavior ($\gamma_G = 1.67$), and we will, of course, incorporate this into future calculations. The radial pressure gradient will be provided by an axial current which rises from 700 KA to 14 MA during the implosion. This technique has been demonstrated in E-beam propagation experiments.³⁰

VI. A SMALL-SCALE LABORATORY DEMONSTRATION EXPERIMENT

The initial effort would be confined to the demonstration of the feasibility of compressing E-layers and the investigation of the scaling necessary to achieve a useful source for simulation purposes. This would include the following.

1. The theoretical and experimental investigation of the formation of intense E-layer $\epsilon_L > 1$ in a geometry suitable for use as a radiation source.
2. The theoretical and experimental investigation of the effects of partial current neutralization on the compression and radiation spectrum of the E-layer.
3. The production of kilojoules of photons with maximum energy in the 0.1 to 1 keV range as a demonstration of the feasibility of the concept.

The fundamental fact confronting any attempt to design a small-scale experiment is the strong γ dependence of emission frequency and radiation rate, shown by II.1 and II.4. It therefore seems to us that we should abandon the hope of producing kilovolt x-rays in the initial, small-

scale experiment, since there is very little question about the theory presented in Section II. Instead we propose to tackle the problems of E-layer formation and compression in relatively small geometries, such as implied by II.5.b. We will try to observe the radiation, but it will be very soft.

We expect to have available a 2 MeV-25 kiloamp E-beam ($\gamma = 5$, 30 ns pulse FWHM). We choose to use larger dimensions than in II.5.b, namely, $R_L = 4.25$ cm, $R_W = 6$ cm, $L_{\text{initial}} = 20$ cm. Equation II.6 tells us that if we trapped all of the one-and-a-half kilojoules of electrons, we would achieve $\epsilon_L = 20$. The field required by the particle energy and radius is 2 kilogauss, requiring 200 joules of initial magnetic energy in the 40 cm long guide field. The amount contained within the liner is 72 joules, and we propose to compress with $\lambda = 20$, leading to a compressed magnetic energy of 28.8 kJ. The E-layer energy will increase by λ to 30 kJ. The plasma energy is small compared to these energies since only 40 eV initial temperature is required. Magnetic liner implosions are calculated to have total efficiencies of 60%-70% for the energy delivered to the volume being compressed (fields plus particles inside). Thus one can take 120 kJ as a moderately conservative estimate of the energy required to drive the magnetic implosion. We find that a one-stage implosion system is adequate to get the velocities needed for this preliminary experiment. A computer calculation of the proposed implosion dynamics starting from radius at which velocity is a maximum is shown in Fig. 8. Here we have assumed $\gamma_G = 1.67$ for all matter inside the liner. The drive-energy requirement is confirmed by this calculation (25% should be added to the calculation for losses in the bank and leads, which were not included).

The required implosion velocity is less for this case than for II.5.b, by roughly the fifth power of the ratio of the γ 's, since synchrotron radiation has much less tendency to rob the layer energy in this case (see Table II). We have thus chosen $U_i = .316$ cm/microsecond. The corresponding ideal parameters initially and at maximum compression are:

VI.1		<u>Initial</u>	<u>Final</u>	
	a	4.27	.21	cm
	E	2.04	50.1	MeV
	B	2×10^3	8×10^5	gauss
	$\hbar\omega_c$	1.7×10^{-3}	2.78×10^2	eV
	τ	26.0	8×10^{-6}	sec

The dwell time of the liner around minimum radius can be estimated as $2.4 (R_{\min}/U_{\text{implosion}}) = 1.7$ microseconds, and it can be made somewhat longer.²⁷ Thus the characteristic 8 microsecond radiation time shown in VI.1 will allow a sizable fraction of the layer energy to be radiated during the liner dwell time. From VI.1 and Fig. 3, the peak photon energy will be at $.15 \hbar\omega_c = 41.5$ eV (about $300 \text{ \AA}$ - in the vacuum UV). These photons should be readily observable in the kilojoule quantities which we expect, although, as stated above, this would not be the primary aim of the first experiment. It is important to keep in an area where synchrotron radiation is significant, even if unobserved, to duplicate the final layer dynamics on a reduced scale.²⁸

A schematic layout of the 2 MeV initial experiment is shown in Fig. 9.

VII. THE MAIN EXPERIMENT

The parameters of the main experiment have been given in II.5.b. As above the energy content of the layer is determined by the desired output. We pick 200 kilojoules as a target number. The magnetic energy content is calculated in Appendix C. The large implosion velocity required leads us to the two-stage magnetic implosion shown in Fig. 2. Thus, to supply up to 200 kilojoules to compress the E-beam, and about 900 kilojoules to compress the magnetic field, at the outer shell we need on the order of 4.4 megajoule to drive the inner shell at 25% overall efficiency. The 14 megamp axial current requires an energy of 1.35 megajoules in a low-inductance bank. We have picked $M_2 = .25 M_1$ to get good velocity multiplication ($V_2 = 1.6 V_0$) while still retaining a sizable mass for the inner liner. The energetic efficiency is $E_2/E_0 = 64\%$ based on momentum transfer alone, and some allowance must be made for magnetic dissipation and liner internal energy losses. To be conservative, one would have to take 5.6 megajoules as the total energy required for the "all-out" 500 kJ yield. We would want to start with the reference yield of 200 kJ of radiation (about 4.4 megajoules in the drive source), and this is what we will base our further calculations and cost estimates on. A 1-D computer calculation of the performance of this implosion system is given in Fig. 10. We are preparing a 2-D computer code which will allow the details of the liner motion around the window guards to be computed. (This code is being prepared for other purposes, and should be available within 6-10 months).

It is entirely possible that chemical explosives may offer the cheapest way to drive the outer shell.⁽³⁾ They typically have overall efficiencies on the order of 10%, however, so the amount of energy

required goes up from 4.4 to 11 MJ (about 6 lbs of TNT equivalent). The problems of shielding the E-beam machine against such a blast would be formidable, but perhaps cheaper than buying a large capacitor bank. The amount of steel for containment is about 2400 lbs. An alternative would be to use inductive energy storage devices or explosive generators located well away from the machine to provide the large energy pulses required. Explosive generators are relatively well developed - however their use entails an expenditure of thousands of dollars per shot, so the breakeven point for a capacitor bank or inductive storage unit is reached in a few hundred shots.

General design sketches of the proposed facility are shown in Fig. 11. Cost estimates are discussed in Section VIII and summarized in Tables IVA, IVB.

VIII. COST ESTIMATES AND TIME SCHEDULES

Cost estimates for the initial experiment are given in Table IIIA: the facility cost would be around \$357,000, assuming installation at LLL in Building 341. This could be cut by \$40K if detailed observation of the UV emission is not required. It could be cut by an additional \$20K if shadow-shielding assumed in the installation estimate can be avoided by beefing up the vault walls during planned FY75 reconstruction. \$297,000 seems like the minimum facility investment required.

Table IIIB shows an estimate of a facility using direct capacitor bank compression of the layer. The costs are very large, due to the large volume which must be filled with magnetic field.

The direct personnel involved on the experiment should be augmented by two physicists doing computational/theoretical work. Again, if this involves sacrificing the detailed UV observations, which I have rated at 1 physicist and 1 technician in Table III, that would be an option. One could thus run the program on a total of 8 committed people and 2 general support allocations. Should the low-level 6 MeV experiments be desired on the Astron accelerator,²⁸ one should budget an additional \$200K operating expenses, and \$200K-\$300K set-up costs.

The saving incurred by not having to make a liner for each shot are not significant, compared with the operating costs of the large capacitor bank.

The requirements for the main facility are shown in Table IV. These are to be regarded as preliminary estimates, but are probably accurate to $\pm 30\%$. The HE-imploded system shown in Table IVB requires the least capital investment, but its operating expenses are considerably higher. Considering the possibility of making a full-scale kilovolt x-ray simulator, we feel that the rewards will justify the admittedly sizable costs.

We would suggest that work be started in mid-FY75 on installation of the initial experiment, at the level indicated in Table IIIA. We estimate about 1.5 years at this level, followed by about 2 years at the level of Table IVA or IVB, as selected. After that, the facility would presumably go into an operating mode at about 3/4 the operating costs shown in IVA or IVB.

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TABLE I

T_0 (μsec)	V_0 (cm/ μsec)	ω_c (keV)	τ (μsec)	λ
26.3	0.10	0.68	1.40	11.8
15.0	0.18	1.15	0.79	13.1
8.5	0.32	1.93	0.46	14.4
4.8	0.56	3.14	0.25	16.3
2.7	1.00	4.93	0.16	17.6
1.6	1.78	7.33	0.11	18.7
0.9	3.16	10.3	0.08	19.85
0	∞	20.0	0.05	20.0

The above values are for a weak E-Layer with initial conditions given in II.5.b. V_0 is the initial velocity of a liner at radius $\sqrt{2} a$. T_0 is the implosion time.

Table II

<u>T_0 (sec)</u>	<u>V_0 (cm/sec)</u>	<u>ω_c (eV)</u>	<u>τ (sec)</u>
5.80×10^{-3}	1.0×10^3	6.8	3.2×10^{-4}
1.83×10^{-3}	3.16×10^3	2.0×10^1	9.2×10^{-5}
5.80×10^{-4}	1.0×10^4	5.2×10^1	3.1×10^{-5}
1.83×10^{-4}	3.16×10^4	1.2×10^2	1.5×10^{-5}
5.80×10^{-5}	1.0×10^5	1.9×10^2	1.0×10^{-5}
1.83×10^{-5}	3.16×10^5	2.4×10^2	9.0×10^{-6}
5.80×10^{-6}	1.0×10^6	2.7×10^2	8.3×10^{-6}
0	∞	2.9×10^2	8.1×10^{-6}

Similar to Table I except that the above values are for a weak E-layer with initial conditions given in VI.1.

Table IIIA

PHASE I (INITIAL EXPERIMENT): LINER OPTION

Facility Costs:

Use LLL Machine (P/I #316)

Field Coils	\$ 2,000
Implosion Coils	15,000
120 kJ Implosion Bank	120,000
Vacuum UV Diagnostics	40,000
8 Oscilloscopes + Screen Room	85,000 ⁽¹⁾
Machine Shielding + Installation	95,000 ⁽¹⁾
	<u>\$357,000</u>

Personnel Required:

- 1 Project Leader
- 2 Facility Technicians
- 1 Diagnostic Technician
- 1 Experimental Physicist
- 2 Facility Engineers
- 2 Heads Miscellaneous Support
- 3 Theoretical Physicists
- 4 Heads Engineering Design/Fabrication

Estimated Operation Costs:⁽²⁾

500 Liners (4/day * 125 Operating days)	
@ \$200 ea. ⁽³⁾	\$ 100,000
Miscellaneous Expense (Computer, etc.)	125,000
Personnel Costs ⁽⁴⁾	800,000
	<u>\$1,025,000/yr</u>

⁽¹⁾ Two similar installations at LLL (1971-1972) averaged \$130,000 each. Present machine is smaller, but costs have inflated.

⁽²⁾ Per year over 1/2 year set-up + 1-1/2 year operation.

⁽³⁾ Based on assumed mass-production price. Present cost = \$200/liner in quantities of 3-5, but these liners are double-ended.

⁽⁴⁾ Salary, Overhead, burden, stores, minor procurements, travel - \$50K/FTE.

Table IIIB

PHASE I (INITIAL EXPERIMENT): CAPACITOR BANK COMPRESSION

Facility Costs:

Use LLL Machine

Field Coils	\$ 50,000
3.4 MJ Capacitor Bank	1,100,000 ^(1,2)
Vacuum UV Diagnostics	40,000
Oscilloscopes, Screen Room	85,000
Machine Shield + Installation	95,000
	<u>\$1,370,000</u>

Personnel Required:

- 1 Project Leader
- 2 Facility Technicians
- 1 Diagnostic Technician
- 1 Experimental Physicist
- 1 Facility Engineer
- 2 Heads Miscellaneous Support
- 2 Theoretical Physicists

Estimated Operating Costs:

Miscellaneous (Computers, etc.) ⁽³⁾	\$150,000/yr
Personnel Costs ⁽⁴⁾	<u>500,000/yr</u>
	\$650,000/yr

⁽¹⁾ Based on 500 kG, 20 cm diameter, 10 cm long "force-free" coil design.

⁽²⁾ Costs at \$0.33/joule (current LLL large-bank design figure).

⁽³⁾ This approach, of course, uses no liners.

⁽⁴⁾ Salary, overhead, burden, stores, minor procurements, travel - \$50 K/FTE.

Table IVA

PHASE II (MAIN EXPERIMENT): 200 KJ (MAGNETIC IMPLOSION OPTION)

Facility Costs:

50 kJ - 7.5 MeV EB Machine (Installed)	\$ 500,000 ^(1,2)
4400 kJ Implosion Bank	1,500,000 ⁽³⁾
Diagnostic Equipment (Cameras, Scopes)	75,000
Shielding	95,000
Crane (2-ton)	10,000
Vacuum System and Coils	20,000
1.35 MJ Axial Current Bank	445,000
	<u>\$2,645,000</u>

Personnel:

- 1 Project Leader
- 3 Heads Miscellaneous Support
- 1 Facility Engineer
- 2 Facility Technicians
- 1 Diagnostic Technician
- 2 Experimental Physicists
- 2 Theoretical Physicists
- 4 Heads Engineering Design/Fabrication

Operating Costs:

Liners (100/year)	\$ 200,000
Expendables, Computer	150,000/yr
Personnel Costs ⁽⁴⁾	800,000
	<u>\$1,150,000/yr</u>

(1) Based on 1972 costs for a similar installation, * 1.25 inflation.

(2) We could save \$300K if we can use DNA "Facility II" machine, currently housed at P/I.

(3) Based on current LLL large-bank design.

(4) Salary, overhead, burden, stores, minor procurement, travel - \$50K/FTE.

Table IVB

PHASE II (MAIN EXPERIMENT): 200 KJ (CHEMICAL IMPLOSION OPTION)

Facility Costs:

500 KJ System

50 kj - 7.5 MeV EB Machine (Installed)	\$ 500,000 ^(1,2)
Crane (5-ton)	25,000
Diagnostic Equipment	75,000
Shielding	95,000
Blast Containment	10,000 ⁽³⁾
Vacuum System	10,000
Axial Current Bank	445,000
	<u>\$1,160,000</u>

Personnel:

- 1 Project Leader
- 2 Heads Miscellaneous Support
- 1 Facility Engineer
- 2 Facility Technicians
- 1 Diagnostic Technician
- 2 Experimental Physicists
- 2 Theoretical Physicists
- 4 Heads Engineering Design/Fabrication
- 3 Heads HE Development

Operating Costs:

Expendables, Computer	\$ 150,000
Personnel Costs ⁽⁶⁾	900,000
Bunker Charges (25 days)	50,000 ⁽⁴⁾
HE Parts (100/yr)	210,000 ⁽⁵⁾
Liners (100/yr)	200,000 ⁽⁵⁾
HE Assembly (100/yr)	140,000
	<u>\$1,650,000/yr</u>

(1) Same basis as Table IVA.

(2) Assumes building available.

(3) Per D. Ravenscroft: 6 lbs HE, 2400 lbs steel, + fast-closing window.

(4) Per R. Anderson: 4 shots/day.

(5) Per J. Scudder: Experience with Roadrunner 4" ID x 18" long systems.

(6) \$50K/FTE, as in previous tables.

TABLE V

T_0 (μsec)	V_0 (cm/ μsec)	ω_c (keV)	τ (μsec)	λ
35.6	0.10	0.72	2.4	8.1
20.0	0.18	1.25	1.2	9.3
11.3	0.32	2.17	0.64	10.5
6.33	0.56	3.72	0.34	11.8
3.56	1.00	6.29	0.18	13.3
2.00	1.78	10.4	0.10	15.0
1.13	3.16	16.8	0.06	16.3
0	∞	100.0	0.014	20.0

Similar to Tables I and II except that the above values are for a weak E-layer with initial conditions given in Sec. VIII.

Figure 1

INJECTION and TRAPPING
OF E-LAYERS

Figure 2a

IMPLODING LINER SYNCHROTRON
EMISSION SOURCE

Figure 2b

MAGNETIC PROFILES FOR Fig.2a

FINAL STATE

Figure 2c

FINAL STATE OF LAYER COMPRESSED
BOTH AXIALLY AND RADIALY.
NOTE RELATIVE ANGULAR SPREAD
OF PHOTON EMISSION FROM CENTER
AND ENDS.

Figure 3

SYNCHROTRON SPECTRUM

Figure 4

RADIAL BETATRON OSCILLATIONS

WE HAVE ASSUMED $\zeta_{\text{net}} > 1$,
SO THE TRAJECTORIES HAVE NEG-
ATIVE CURVATURE INSIDE THE
ORBIT.

Figure 5

EFFECT OF AXIAL BETATRON
OSCILLATIONS ON SYNCHROTRON
EMISSION ANGLE

(a) TEARING

(b) SEPARATED

(c) COALESCED

Figure 6

TEARING INSTABILITY

The effect is due to AXIAL ATTRACTION
between different parts of the LAYER.

Figure 7

TIME VARIATION OF AXIAL MAGNETIC
FIELDS FOR CASE II.5.b (TWO STAGE)

B_D = DRIVE FIELD

B_{12} = FIELD BETWEEN M_1 and M_2

$\bar{B}$ = FIELD INSIDE LINER

Figure 8

R/R_0 vs. T/T_0

FOR "IDEALIZED" IMPLOSION OF LINER
BY MAGNETIC FIELDS

$$\frac{R_{\text{INITIAL}}}{R_{\text{FINAL}}} = 20$$

Figure 9

Figure 10

R/R_0 vs. T/T_0

FOR "IDEALIZED" IMPLOSION OF LINER
BY MAGNETIC FIELDS

$$\frac{R_{\text{INITIAL}}}{R_{\text{FINAL}}} = 20$$

Figure 11

APPENDIX A

We are interested in the retarding forces acting on an e-layer. For a single electron at the front of the e-layer (moving in the positive z-direction) the force is given by

$$A-1 \quad F_z = q \left[\bar{E} + \frac{\bar{V}_\perp \times \bar{B}}{c} \right]_z$$

where the background magnetic field is in the z-direction. If nearly complete current neutralization is assumed we have the following expressions for the return current in the background plasma J_p

$$A-2a \quad J_{p\theta} = - J_{b\theta}$$

$$A-2b \quad J_{pz} = - J_{bz}$$

where the subscript b represents the current in the e-layer. We further assume that

$$A-3 \quad J_{b,p} \sim J_{b,p}(z - V_{||}t)$$

this allows us to replace $\frac{\partial}{\partial t}$ by $-V_{||} \frac{\partial}{\partial z}$ in Maxwell's equations. Utilizing Ohm's law we replace E_z in Eq. A-1 by $J_{||}/\sigma_{||}$, where $J_{||}$ is the return current in the plasma, and $\bar{V}_\perp \times \bar{B}$ by $-V_\perp B_r$. B_r is calculated from Maxwell's equations as follows:

$$A-4 \quad (\bar{\nabla} \times \bar{J}/\sigma)_r = - \frac{1}{c} \frac{\partial B_r}{\partial t} = - \frac{\partial}{\partial z} (J_\perp/\sigma_\perp) = \frac{V_{||}}{c} \frac{\partial B_r}{\partial z}$$

then we have

$$A-5 \quad B_r = - \frac{J_\perp}{\sigma_\perp} \frac{c}{V_{||}}$$

thus Eq. A-1 becomes

$$A-6 \quad F_z = q \left[\frac{J_{||}}{\sigma_{||}} + \frac{J_{\perp}}{\sigma_{\perp}} \frac{V_{\perp}}{V_{||}} \right]$$

this is essentially the Chu-Rostoker result,¹³ with the exception that they derive the expressions for $\sigma_{||}$ and $\sigma_{\perp}$ as well. We assume $\sigma_{||} = 1.97 \sigma_{\perp}$, following Spitzer.²⁹ In terms of the primary layer current, $J_{||}$ and $J_{\perp}$ can be expressed as

$$A-7 \quad J_{||} = - \frac{N'q V_{||}}{2\pi RD}$$

and

$$A-8 \quad J_{\perp} = - \frac{N'q V_{\perp}}{2\pi RD}$$

where R is the radius of the layer, D the thickness, q the charge, and N' the number of charges per unit length. Substituting in Eq. A-6 we have

$$A-9 \quad F_z = - \frac{N'^2 q^2 V_{||}}{2\pi RD} \left[\frac{1}{\sigma_{||}} + \left(\frac{V_{\perp}}{V_{||}} \right)^2 \frac{1}{\sigma_{\perp}} \right]$$

Equation A-9 is valid only if

$$A-10 \quad \frac{4\pi \sigma_{\perp} D^2}{c^2} \gg \frac{D}{V_{||}}$$

where the L.H.S. of Eq. A-10 represents the time for the return current to diffuse a distance D.

Equation A-9 represents the force on an electron at the front of the e-layer. The characteristic stopping distance is computed by setting the

$$RANGE = \frac{\gamma m_0 V_{||}^2}{2F_z}$$

Chu-Rostoker¹³ have evaluated the above in terms of the particle densities and the collision frequencies. Noting that $\sigma_{||} = n_0 e^2 \tau_e / m_e$, we get Eq. III.1 of the text, with $n_b = N' / 2\pi R D$. We note that the right-hand side contains the ratio v_e / n_0 , which is a function of temperature only.²⁹ For our test case ($\gamma_i = 16.1$, $\zeta = 13.2$, $R_i = 2$ cm, $D = .4$ cm, $L = 20$ cm), the temperature of 8.3 eV gives a value of $R = 40$ cm. Condition A-10 is well satisfied: LHS = 448 ns, which is the time for the field to show through. We chose $n_0 = 20$, $n_p = 15.2 * 10^{14}$ /cc as being dense enough to easily neutralize the beam. Then $v_e = 1.03 * 10^9$ /sec. Due to the large heating rate, the initial plasma temperature can be somewhat lower than 8 eV. This effect needs calculational effort.

APPENDIX B

Precessional Stability Calculations

The precessional instability can be suppressed either by the presence of conducting walls (for incomplete current neutralization) or by a toroidal field from the current in a central wire. Since a central wire introduces mechanical complications, we first examine the wall stabilization.

The precession frequency is

$$B-1 \quad \omega_p = \frac{1}{2} \omega_c \left(|\eta_M| - \eta_i \right)$$

Here ω_c is the cyclotron frequency, and the η 's are the logarithmic radial field derivatives, $d \ln B_z / d \ln R$, from the external mirror and the image currents in the wall, respectively. If $\omega_p > 0$, the precession is unstable.⁶

For the mirror field we assume the simple form $B_z = B_0 + 2bz^2 - br^2$, which, with $B_r = -2brz$ yields a curl-free and divergence-free mirror for positive b . From Fig. 2b, we want $B_z/B_0 = 1.333$ at $z = 10$ cm. This yields $b = 1.667 \times 10^{-3} B_0$, and $\eta_M = -2bR^2/B_0$. For $R = 2$ cm from II.5.b, we get

$$B-2 \quad \eta_M = -.01333$$

The image current contribution is evaluated using the notation of Ref. 6.

$$B-3 \quad \eta_i = \frac{(kR)^2 \epsilon(\psi_1 + \psi_2)}{4(1 + k^2/k_0^2)[J_0(k_0 R_0) - J_0(k_0 R_i)]}$$

where $k = 2\pi/L$ for the layer, ζ is defined in Section II, and k_0 sets the radial thickness ($k_0 R_i$, $k_0 R_0$ are adjacent zeroes of the J_1 function). The ψ 's are functions of kR_0 and kR_i . The layer is assumed to have a $J_1(k_0 r)$ current density profile between R_i and R_0 and to be zero elsewhere. R is the radius of peak current density, taken as 2 cm from II.5.b. Picking the fourth zero of J_1' , we get $k_0 R_0 = 11.706$. The rest of the parameters follow:

$$\begin{aligned} \text{B-4} \quad k &= .314159 \text{ cm}^{-1} \\ k_0 &= 5.853 \text{ cm}^{-1} \\ k_0 R_i &= 10.1735 \quad \text{or} \quad R_i = 1.738 \\ k_0 R_0 &= 13.3237 \quad \text{or} \quad R_0 = 2.276 \end{aligned}$$

We choose $R_W = \sqrt{2} R$, and tabulate the quantities needed⁶ in computing ψ_1 and ψ_2 .

$$\begin{aligned} \text{B-5} \quad kR_i &= .5460 \\ kR_0 &= .7150 \\ kR_W &= .8886 \end{aligned}$$

From this we compute $\phi_1 = 3.501$, $\phi_2 = 0$ (notation of Ref. 6). We also need the following quantities:

$$\begin{aligned} \text{B-6} \quad I_1(kR_0) &= .3806 \\ I_1(kR_i) &= .2826 \\ J_0(k_0 R_0) &= +.21836 \\ J_0(k_0 R_i) &= -.24970 \end{aligned}$$

We do not need the K functions since ψ_2 is proportional to ϕ_2 and hence zero.

Evaluating from Ref. 6, Eq. III.24, we get $\psi_1 = 3.501$. Using this in B-3 we find

$$\text{B-7} \quad \eta_1 = .2877\zeta$$

Equating this to η_M from B-2, we find the critical ζ for stability:

$$\text{B-8} \quad \zeta_c = .046$$

This is trivially achieved. Shortening the layer during the axial compression raises the value of ζ_c , however, and one should note that it is the net ζ which counts in stabilizing the precessional mode. In a hot plasma, the net ζ is not easily controlled. For this reason we continue to consider the possibility of having a small center rod in the device, as discussed in the text. This has the additional effect of making $\phi_2 \neq 0$, $\psi_2 \neq 0$, so that additional stabilization is obtained.

APPENDIX C

The magnetic energy content of the system was calculated by considering the self-inductance of two square coils of thickness 6 mm separated by a center-to-center distance of 1.5 cm with a mean radius of 0.5 cm. The current was calculated by assuming a field of 5.5×10^6 gauss at the mid-point of the rings. This procedure yielded a value of 900 kJ. Two other values of interest are the energy stored in a region 1.5 cm long and 0.5 cm in radius which is given by

$$E_{\text{uniform field}} = 142 \text{ kJ} \quad (5.5 \text{ megagauss})$$

and the energy stored in a solenoid 1.5 cm long with radius 0.5 cm.

The latter is given by

$$E_{\text{solenoid}} \cong 474 \text{ kJ} \quad (5.5 \text{ megagauss midplane})$$

The actual choice of the arrangement of the magnetic fields coils will in part be determined by stability conditions.

The magnetic field at either ring is stronger by a factor $.5(a^2 + z^2)^{3/2}/a^3 = 2.9$. This gives a field of 16 megagauss inside either ring, and the nonlinear skin effect permits the field to diffuse about 1 millimeter into the 6 mm thick coil during the 100 ns peak field period.

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