RELATIONS BETWEEN THE MAJOR SIZE AND COST PARAMETERS OF TOKAMAKS

A. P. Fraas

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CONTENTS

	<u>Page</u>
ABSTRACT	1
ACKNOWLEDGMENT	1
INTRODUCTION	2
DEVELOPMENT OF PARAMETRIC RELATIONS	2
Plasma Power Loss by Conduction	2
Bremsstrahlung Losses	4
Alpha Heating	4
Threshold for Ignition	7
Maximum Magnetic Field	7
Cost of Superconducting Magnet Coils	8
Total Power Output from Plasma	9
Blanket Cost	10
Magnet Shield Cost	10
RESULTS	11
Effects of Torus Size on Magnet Cost for a Feasibility Experiment	11
Plasma Physics Reactor Experiment	14
Cost Effects of Neutral Beam Injection System	14
Effects of Torus Size on Magnet, Blanket, and Shield Costs for Power Reactors	18
Blanket Pumping Power	18
SUMMARY	20
REFERENCES	22

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A. P. Fraas

ABSTRACT

A set of preliminary estimates of the effects of torus size on major design and cost parameters has been made for a series of tokamak reactors with superconducting coils for the toroidal field. The three sets considered include essentially zero power scientific feasibility experiments, low power reactor systems for plasma physics experiments, and full scale power reactors. The prime concern is with the cost of the superconducting magnet coils, but allowances for the blanket and magnet shield systems are also included as well as the system for heating the plasma to the ignition temperature. The prime objective was to show trends for major parameters, not absolute values for costs.

The results of the study indicate that the cost of a superconducting coil system for the toroidal field increases relatively little with torus diameter for a scientific feasibility experiment, is quite insensitive to torus size for a plasma physics test reactor, and actually falls off with increasing size for a full scale power reactor. These effects stem from the reduction in the magnetic field required associated with an increase in torus size together with the increase in the allowable current density in the superconductor material with a reduction in the design value for the magnetic field. The results indicate further that, for a modest increase in the cost of superconducting magnets for the toroidal field, it should be possible to build a feasibility experiment so that it can be converted readily into a plasma physics test reactor.

ACKNOWLEDGMENT

The writer deeply appreciates the help and advice generously given by many colleagues both in evolving the methods used in this study and in reviewing the report as it progressed through several iterations. A particular debt is owed to J. F. Clarke, R. A. Dory, W. F. Gauster, R. Hancox, G. G. Kelley, H. M. Long, M. S. Lubell, D. G. McAlees, O. B. Morgan, H. Postma, D. Steiner, and F. C. Tenney. This study could not have been carried out without their help.

A special debt of appreciation is owed to Bernice Fitzgerald who expertly and cheerfully carried out the calculations and plotted the curves

not once but four times as successive refinements in the analysis made it desirable to repeat the work even though the effects of the changes turned out to be relatively small.

INTRODUCTION

The most important parameters affecting the size and cost of tokamaks are the plasma minor diameter, the aspect ratio, the magnetic field, and the power output. A combination of rational and empirical relations have been presented by Golovin,¹ Yoshikawa,² and Tenney.³ In addition, some important insights to cost effects have been given by Carruthers, Hancox, Mitchell, Gibson, and Bickerton,^{4,5,6} but these have been concerned primarily with minimum capital charges at high power outputs. There has not been much attention given to considerations that now seem to be important for the low power output regime in which feasibility experiments and the early experimental reactors will operate. This study extends Golovin's work in the hope that it will prove helpful in showing some implications of the empirical relations available. It must be emphasized that values attainable at this stage are only rough approximations, but it is believed that the trends given by this analysis are meaningful.

DEVELOPMENT OF PARAMETRIC RELATIONS

Plasma Power Loss by Conduction

Both feasibility and early fusion reactor experiments will probably be conducted with plasma densities and temperatures close to or not much above the threshold for ignition. For these experimental systems a prime consideration will be the total cost of the magnet system rather than the unit cost per kilowatt of power output. Thus, the first step in determining costs is to relate the magnetic field strength required to the torus size for plasma conditions at the threshold for ignition. In this region the principal energy loss mechanisms are conduction and Bremsstrahlung. (Note

that the term "conduction" as used by Golovin and as used here includes ion and electron energy losses to the walls via collision processes, mainly conduction and convection.) Following Golovin's notation except for the use of B rather than H for the magnetic field (see Table 1 for the nomenclature), the conduction loss in eV as given by Golovin¹ in the unnumbered equation following his Eq. 22 is approximately:

$$\text{Conduction power loss} = \frac{\overline{3nT_i}}{\tau} (\text{volume}), \text{ eV/sec} \quad (1)$$

In his original derivation Golovin based his value for τ on the Bohm time, τ_B , and assumed that $\tau = 300 \tau_B$. In reviewing a preliminary draft of this report, J. F. Clarke recommended that this rough approximation for τ as used by Golovin for substitution in Eq. 1 above be replaced by the more recently derived pseudoclassical expression:

$$\tau_{pc} = \frac{1.7 \times 10^4 a_m^2 e B_w^2 T_e^{1/2}}{\ln \Lambda m_i q^2 A^2 n_e} \quad (2)$$

where: a_m is in meters

T_e is in eV

n_e is number of electrons per cm^3

B_w is in W/m^2

$e/m = 1.7 \times 10^{11}$ coulombs/kg

$\ln \Lambda = 17$

Substituting the above values of $\ln \Lambda$ and e/m in Eq. 2 and changing the units for a and B to be consistent with those used by Golovin (i.e., a in centimeter and B in Gauss) gives:

$$\tau = \frac{170 a^2 B^2 T_e^{1/2}}{q^2 A^2 n_e} \quad (3)$$

Substituting Eq. 3 in Eq. 1 together with the expression for the plasma volume ($2\pi^2 a^2 R$) and the conversion factor from eV to watts, and assuming that the electron and ion temperatures are about the same gives:

$$\begin{aligned} \text{Conduction loss} &= \frac{3n^2 T^{1/2} (2\pi^2 a^2 R) \times 1.6 \times 10^{-19} q^2 A^2}{170 a^2 B^2} \quad (4) \\ &= \frac{2.82 \times 10^{-21} n^2 T^{1/2} q^2 A^2 (2\pi^2 a^2 R)}{a^2 B^2}, \text{ W} \end{aligned}$$

Bremsstrahlung Losses

A second major energy loss from the plasma stems from Bremsstrahlung. Golovin's Eq. 20 gives the Bremsstrahlung loss¹ as:

$$\text{Bremsstrahlung power loss} = 1.1 \times 10^{-32} n^2 T^{1/2} (2\pi^2 a^2 R), \text{ W} \quad (5)$$

Alpha Heating

The alpha heating in the plasma is given in Golovin's Eq. 22 as:

$$\text{Alpha heating power} = (2\pi^2 a^2 R) \frac{n^2}{4} \langle \sigma v \rangle Q_\alpha, \text{ W} \quad (6)$$

However, this expression does not include any allowance for the fact that many of the alpha particles born in the outer region of the plasma will escape because their energy is roughly a hundred times that of the ions in the plasma. Thus, they will not heat the plasma. D. G. McAlees has recently investigated this problem and has developed a method for estimating the fraction of the alphas that will escape from the plasma.⁷ He kindly agreed to carry out a set of calculations for the writer using a typical set of conditions. The results of his calculations for a plasma temperature of 15 keV and an aspect ratio of 4 are plotted as the three points in Fig. 1 for the fraction of the alpha energy that will go into heating the plasma as a function of his parameter P, which is inversely proportional to aB/q .

Fig. 1. Relation between the fraction of the alpha energy that acts to heat the plasma and the parameter P on which the fraction of the alphas lost from the plasma depends for aspect ratio of 4. (Note that McAlees used MKS units, i.e., a in m, B in Tesla, e in Coulombs, m in kg, and V in m/sec. The latter three quantities are for the alpha particles as they are born at 3.5 MeV.)

Also shown is a line through these points defined by an analytical expression that the writer prepared to approximate the values obtained by McAlees. This analytical expression can be used as a factor by which the expression for the alpha heating power of Eq. 6 can be multiplied to allow for the loss of alphas from the plasma.

The calculations used for Fig. 1 were for a plasma centerline temperature and density of $T = 15$ keV and $n = 5 \times 10^{13}$ ions/cm³. Inspection of the equations indicated that the alpha heating loss fraction is insensitive to the plasma temperature and independent of the ion density. To check this McAlees carried out calculations for an additional case for which the torus centerline ion temperature was increased to 25 keV. This yielded values for the alpha heating fraction within less than 1% of those in Fig. 1.

To help put the relations of Fig. 1 in perspective it should be pointed out that the conditions for ignition in a feasibility experiment are likely to be marginal, hence for such a case the region of interest in Fig. 1 will probably be for P between 0.2 and 0.4, i.e., toward the right of Fig. 1. On the other hand, for a full scale power reactor the region of interest will be for values of P less than 0.1, i.e., in the region at the left of Fig. 1. In fact, if damage to the first wall by alpha particles is to be kept to a tolerable level, the incidence of alpha impacts on the wall must be kept very low — of the order of 10^{-4} alphas per fusion reaction, and this is likely to require that the operating regime be at values of P much less than 0.1.

Multiplying the right side of Eq. 6 by the equation of Fig. 1 to allow for the alpha particle losses from the plasma yields a revised expression for the alpha heating of the plasma, i.e.,

$$\alpha \text{ Heating} = (2\pi^2 a^2 R) \frac{n^2}{4} \langle \sigma v \rangle Q_{\alpha} e^{-\left(\frac{0.68 \times 10^6 q}{aB} - 0.19\right)}, \quad W \quad (7)$$

(This equation holds only for values of $\frac{0.68 \times 10^6 q}{aB} \geq 0.19$. For smaller values, the alpha heating factor is essentially unity.)

Threshold for Ignition

Equating the alpha heating to the conduction and Bremsstrahlung losses, which defines the threshold for ignition, and dividing through by n^2 times the volume gives:

$$\left\langle \frac{\sigma V}{4} \right\rangle Q_{\alpha} e^{-\left(\frac{0.68 \times 10^6 q}{aB} - 0.19 \right)} = \frac{2.82 \times 10^{-21} T^{1/2} q^2 A^2}{a^2 B^2} + 1.1 \times 10^{-32} T^{1/2} \quad (8)$$

Inasmuch as the objective of this analysis is to relate the torus cost as determined by the torus size and the magnetic field to the plasma parameters, it is especially interesting to note that the approximation of Eq. 8 indicates that the threshold for ignition is independent of the ion density. This characteristic stems from the choice of Eq. 2 for τ . Further, for ion temperatures above 10 keV, calculations made using Eq. 8 indicate that the parameter aB/q does not change much with the ion temperature. Thus, if the values for q and A are fixed, the toroidal field B is inversely proportional to the plasma radius a .

Although Eq. 8 cannot be solved explicitly for B in terms of a , a solution for aB/q can be obtained by iteration using as a starting point the particular solution obtainable for the condition where the alpha heating factor is unity, i.e., a condition for which an explicit solution for aB/q can be obtained. For this condition with $T = 10$ keV and $A = 4$, $aB/q = 1.107 \times 10^6$. An iterative solution of Eq. 8 for the general case with $T = 10$ keV and $A = 4$ yields $aB/q = 0.80 \times 10^6$.

Maximum Magnetic Field

The maximum magnetic field B_m in the conductors in the region close to the polar axis of the torus determines the design and cost of the superconducting magnet coils. For tokamak reactors Golovin's Eq. 12 (Ref. 1) gives B_m as:

$$B_m = \frac{B}{1 - \frac{1}{Ay} - \frac{t_b}{aA}}, \text{ G} \quad (9)$$

where t_b is the thickness of the blanket plus the magnet shield. The value for t_b was taken from Fig. 1 of Ref. 8 which can be represented empirically by:

$$\text{Blanket + shield thickness, } t_b = 190 + 20 \log_{10} \left(\frac{\text{wall heat load}}{0.15} \right), \text{ cm} \quad (10)$$

Cost of Superconducting Magnet Coils

Ideally, the cost of the superconductor should vary directly as the square of the magnetic field, linearly with the major radius, and linearly with the coil radius.⁹ The validity of this can be rationalized readily. For a given torus size and superconductor cross-sectional area, the allowable current density in the conductor will be inversely proportional to the maximum value of the magnetic field. At the same time, the number of ampere turns required will be directly proportional to the magnetic field. Thus, the linear footage of conductor required will increase as the square of the maximum magnetic field. If the maximum magnetic field strength is fixed and the major diameter is increased, the number of coils required and hence the footage and cost of the superconductor will increase linearly with the major diameter. Similarly, for a fixed major diameter the footage of superconductor and hence its cost will be directly proportional to the minor diameter of the torus. The costs of winding the coils should be directly proportional to the footage of superconductor for a given conductor size and shape. The forces acting on the bobbin structure will vary as the square of the magnetic field, the number of coils will increase linearly with the major diameter, and hence the cost of the magnet structure will increase accordingly. As in pressure vessels, the structural weight and cost will increase as the square of the coil diameter.

Lubell has investigated the costs of superconducting magnets that have been built or that have been designed in sufficient detail to make possible a good estimate of their cost and has plotted the costs as functions of a number of parameters including the stored energy. He has concluded that the stored energy gives the best indication of magnet costs, and has fitted analytical expressions to the scatter band of his plotted data. From this work he recommended that the writer use the following relation¹⁰:

$$\text{Cost} = 0.325 E_s^{0.375}, \$ \text{ (for solenoids with } E_s < 2000 \text{ MJ)} \quad (11)$$

$$\text{Cost} = 0.0139 E_s^{0.8}, \$ \text{ (for toroids with } E_s > 2000 \text{ MJ)} \quad (12)$$

The slopes given by these relations are less than indicated by the idealized rationale of the preceding paragraph, the slope being somewhat less for the upper range of stored energy considered, and much less for the lower range. The reduced slopes probably stem from the fact that tooling and set-up costs are nearly fixed irrespective of the number of turns. Thus, the cost is less dependent on magnet size (or stored energy) for the smaller magnets, but approaches the idealized relations indicated by the preceding paragraph for the larger sizes. In view of the convincing case made by Lubell's data, for the purposes of this study the magnet costs were estimated using the relations of Lubell as given in Eqs. 11 and 12. Note that these equations assume the use of Nb-Ti superconductor and that for fields above about 75 kg it would be necessary to employ a more expensive material such as Nb-Sn. Thus, these equations underestimate the cost of the higher field magnets. Fortunately, as will be shown later, the field strengths of interest are largely below 75 kG.

The stored energy was calculated as follows:

$$\text{Stored Energy} = \frac{B_m^2 \pi r^2 a}{10^7} \quad (13)$$

Total Power Output from Plasma

The total power output was taken simply as the plasma volume times the power density as given by Golovin's Eq. 6 (Ref. 1), i.e.,

$$\text{Total power} = P_T = \frac{2\pi^2 a^3 A n^2 \langle \sigma V \rangle Q_T}{4 \times 10^6} \quad (14)$$

The first wall heat loading was taken as the total power output divided by the wall area. This gave the expression

$$\text{Wall heat loading} = P_w = \frac{P_T}{(6.25)^2 a^2 A 10^{-4}}, \text{ MW/m}^2 \quad (15)$$

Blanket Cost

The blanket cost was estimated assuming that the blanket would be fabricated of niobium and graphite and filled with lithium as in Ref. 11, and that its thickness would be 100 cm irrespective of the power density. The unit costs per kilogram for the niobium, graphite, and lithium were taken as \$60, \$2.5, and \$9 respectively. The costs were then ratioed according to the vacuum wall area, i.e., its major and minor diameters, using the weights of the different components of the blanket and magnet shield for the toroidal reactor of Ref. 11 which, for that case, gave a cost of \$15,000,000. This gave the relation

$$\text{Blanket cost} = 15.5 \times 10^6 \frac{r^2 A}{360^2 \times 3}, \$ \quad (16)$$

Magnet Shield Cost

The cost of the magnet shield was obtained in a similar fashion assuming a shield made of graphite or similar material having a density of 1.7 g/cm³ and a cost of \$2.5/kg. This gave the relation

$$\text{Magnet shield cost} = 2.5 \frac{\text{Magnet shield volume} \times 1.7}{1000}, \$ \quad (17)$$

CALCULATIONS

The calculations were carried out for an aspect ratio of 4.0 and $q = 4.0$ for a series of plasma temperatures ranging from 10 to 80 keV to obtain the estimated magnetic field. Table 1 gives the nomenclature, symbols, and units used in the calculations. Table 2 summarizes the results of the iterative calculations for solving Eq. 8 to obtain values for the quantity aB , and indicates the key steps in these calculations. Three or four

iterations usually sufficed. Table 3 summarizes the procedures used in making the successive steps in the main set of calculations and Table 4 presents a typical set of the latter. Note that the maximum magnetic field becomes inordinately high for the smaller plasma minor diameters, particularly for the reactors requiring blanket-plus-magnet shield regions. Because of this, the set of calculations was limited to plasma radii greater than those for 200 kG maximum fields for operation with a blanket and magnet shield. Note that the computational procedure is not difficult, hence additional calculations for other values of the parameters can be made expeditiously.

RESULTS

The most important results of the calculations are plotted in Figs. 2 and 3 for three basic cases, i.e., a simple feasibility experiment with no blanket or shield, a plasma physics reactor experiment with a 60 cm thick water shield, and a full scale power reactor with a complete breeding blanket and magnet shield. Curves are given for plasma temperatures from 10 to 80 keV.

Effects of Torus Size on Magnet Cost for a Feasibility Experiment

A striking point is that the cost of the superconducting magnets for a plasma physics experiment for which no blanket or magnet shield region would be employed increases relatively little with an increase in the minor diameter of the torus. This appears to the writer to be an extremely important consideration because it means that for the same cost one can employ a relatively low magnetic field with a large coil diameter in a feasibility experiment, and this will greatly ease the detailed design of the magnet structure from the stress standpoint. The forces acting on individual conductors in the coil will be moderate and hence it will not be difficult to provide the requisite structure for coping with the large lateral forces that will develop in the event that one coil in the system quenches or in the event of other asymmetries in the magnetic field. It will be possible to use NbTi as the superconducting material for these moderate fields, and this should ease the fabrication problems because NbTi is ductile and has been

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Fig. 2. Effects of wall radius and plasma temperature on the toroid centerline and the maximum magnetic fields for the minimum power density for a self-sustained reaction.

Fig. 3. Effects of wall radius and plasma temperature on costs for the minimum power density for a self-sustained reaction.

used in quite a number of magnets, one of them of about the coil size and the field strength required here, i.e., the magnet for the large bubble chamber at the National Accelerator Laboratory, Batavia, Illinois.¹² (This has a bore of 4.8 m and a field of 30 kG.) Further, the large size will be more nearly typical of the size required for a power reactor.

Plasma Physics Reactor Experiment

Figure 3 indicates that there is a relatively small increment in magnet cost in going from a feasibility experiment with no magnet shield to a system designed to include a 60 cm thick magnet shield. This in turn raises a question as to the power outputs to be expected for operation at the minimum power density for a viable self-sustained reaction. Studies by John Clarke have indicated that unless the plasma density is increased to around 5×10^{13} ions/cm³ the reacting plasma will be so sluggish that its dynamic behavior will be difficult to interpret. Figure 4 shows a set of curves for this condition. These curves show that the instantaneous power output will of necessity be large — of the order of at least 30 MW. As shown in a companion report,¹³ this in turn leads to the conclusion that a water shield about 60 cm thick should be included in a feasibility experiment to permit investigation of plasma behavior under self-sustained reaction conditions and yet avoid serious neutron activation of the magnets.

The cost of a magnet shield has not been included in Fig. 3 because it appears to be relatively small. Estimates for an aluminum tank-borated water shield indicate that its cost would run about 5% to 10% of the cost of the magnet coils, i.e., less than the uncertainty in the estimate of the cost of the magnet coils.

Cost Effects of Neutral Beam Injection System

Offsetting the advantages of choosing a large torus size for a feasibility experiment is the fact that the amount of energy that must be put into the plasma to raise its temperature to the ignition point will be directly proportional to the plasma volume. Some indication of the associated cost increment can be obtained by assuming that this energy must be put into the plasma via a neutral beam injection system. Neglecting losses from the plasma and assuming that the energy is injected in 10 sec, the power input to raise the plasma temperature to 15 keV will have to be about 2 MW for an

Fig. 4. Effects of wall radius and plasma temperature on total power and wall heat loading for the minimum power density for a self-sustained reaction.

ideal system having a 3.6 m minor diameter first wall. O. B. Morgan estimates that the cost of the injection system including the power supply to be about $\$1/W$ for a 2 MW injection system.¹⁴ If one assumes that the cost per kilowatt will go down as the design power is increased (as appears to be the case for magnets — see Eqs. 11 and 12), and that this will follow the square root relation that ordinarily applies to variations in unit costs with quantity, one obtains the curve in the lower part of Fig. 5 for the cost of the minimal power neutral beam injection system as a function of the first wall radius. This curve assumes that the injector power input required will be directly proportional to the plasma volume and that there will be no losses.

The combined cost of the magnet system for the toroidal field and the neutral beam injection system for a feasibility experiment with a 60 cm water shield has been plotted in the upper portion of Fig. 5 for two sets of cases. The first set is for the above estimate of the minimum neutral beam power requirement and the magnet costs for plasma temperatures of 10 keV and 20 keV. The second set of curves is for a neutral beam power requirement four times the above minimum to allow for losses and the same magnet costs for plasma temperatures of 10 keV and 20 keV.

In considering the curves in Fig. 5 it seems likely that the higher beam power requirement would be associated with the higher plasma temperature and the lower beam power requirement with the lower plasma temperature, hence the region between the two middle curves seems likely to be representative of the region of principal interest. Note in Fig. 3 that there is little difference in the estimated costs of the magnets for the 20 keV and 40 keV plasma conditions. Note, too, that the region between the two middle curves in the upper portion of Fig. 5 indicates that the combined cost of the magnet and injection systems increases at only a modest rate in going from a first wall radius of 1.25 m to 3 m. Thus, considerations other than the cost of these two major components will probably dominate the choice of torus size.

The curves of Fig. 5 favor the smaller toroids because the time constant of the system increases with size and hence the period of time over which heating takes place can be longer for the larger sizes. On the other hand, losses will also increase with both size and the time spent in heating the plasma to the ignition temperature, hence Fig. 5 is still a fair approximation. The net effect of these factors would reduce the slope of the curves and give substantially less increase in cost with size.

Fig. 5. Effects of wall radius on the estimated cost of neutral beam injectors and on the combined cost of injectors plus the magnet coils for a 60 cm thick magnet shield.

When considering the effects of introducing a 60 cm thick magnet shield in a feasibility experiment to permit short periods of operation at substantial powers to investigate plasma physics problems in this regime, one is struck by the fact that Figs. 2 and 3 show that the minimum cost for the magnet is obtained with magnet coil bores of the order of 4 m.

Effects of Torus Size on Magnet, Blanket, and Shield Costs for Power Reactors

If one considers full-scale power reactors with a breeding blanket and a magnet shield, the effects of torus size on costs are still more pronounced. To help clarify this situation, the minimal cost points defined by Fig. 3 for a full scale power reactor have been plotted to give the solid line in the central portion of Fig. 6 to show the minimum cost of the magnet, blanket, and shield assembly as a function of the thermal power output assuming a maximum permissible wall heat loading of 1 MW/m^2 . Inasmuch as this curve gives a minimum cost for the magnet, blanket, and shield assembly for a power output of about 500 MW, at lower powers it is possible to obtain more favorable costs by holding the torus size constant at the value for 500 MW and allow the wall heat loading to drop. This effect is shown by the dashed line in the right portion of the region. Note the corresponding curves for the first wall radius in the lower portion of Fig. 6. Curves for the corresponding costs in dollars per kilowatt of electrical output are plotted at the top of Fig. 6 assuming an overall thermal efficiency of 50%. These two sets of curves show that the minimum design power output and cost of the magnet-blanket-magnet shield assembly for a demonstration power plant will run roughly 500 MW(t) and \$45,000,000 respectively. The cost and design power output will depend somewhat on the minimum viable power density, but will be insensitive to the aspect ratio. However, the unit capital cost of subsequent commercial power plants will go down fairly rapidly as the output is increased; the curves at the top of Fig. 6 indicate that the capital cost of the magnet-blanket-magnet shield assembly would run about \$85/kW(e) for a 2000 MW(t), or 1000 MW(e) power plant.

Blanket Pumping Power

The pumping power required to circulate lithium through the blanket region is heavily dependent on the magnetic field strength and the first

Fig. 6. Effects of design power output on the cost of the magnet, blanket, and magnet shield assembly for a first wall heat loading of $1 \text{ MW(e)}/\text{m}^2$ and an overall thermal efficiency of 50%. The solid line was drawn for a continuously decreasing minor radius for the first wall whereas the dashed line was drawn for a constant first wall radius for power outputs below that which gave a minimum cost for a $1 \text{ MW(e)}/\text{m}^2$ wall heat loading.

wall heat loading.¹⁵ Inasmuch as the latter two parameters are defined by the above study, it was relatively simple to relate them to the ratio of the pumping power to the heat removed from the blanket. The pumping power-to-heat removal ratio for the lithium blanket system proposed by the writer^{11,15} was determined as delineated in Ref. 15 i.e.,

$$\text{Pumping power-to-heat removal ratio} = 1.21 \times 10^{-18} B^2 a^2 A^2 \quad (18)$$

This yielded a value of 10^{-4} for a wall heat loading of 1 MW/m^2 . This value depends on the wall heat loading and the aspect ratio, but is independent of the plasma radius because the parameter aB is fixed for a given set of conditions establishing the plasma confinement required.

Note that the pumping power-to-heat removal ratio of Eq. 18 includes the estimated electrical and magnetohydrodynamic losses; the value given is much higher than the ideal pumping power-to-heat removal ratio. Note also that this parameter ought not exceed about 1%, or the loss in net power output from the plant is likely to be excessive. This, together with the problems of heat removal from the first wall¹⁶ and radiation damage considerations, strongly favors a first wall heat loading of 1 MW/m^2 or less.

SUMMARY

It is important to maintain perspective when drawing inferences or conclusions from the results of this study. The reader is cautioned to remember that the analysis involves the use of a whole series of approximations and extrapolations, and some of these may be poor. For example, the expression for τ in Eq. 2 may not be very good, or the arbitrary choice of a value of 4 for q may be poor. The cost factors used are believed to be reasonably good approximations, but at this stage it is not possible to say how good the estimate of the magnetic field required may be, and estimates of the power required for a neutral beam injection system and its resulting cost vary by at least a factor of 4. Further, the cost factors considered here have not included the building or the many auxiliaries.

In spite of the resulting uncertainties in the absolute values for the estimated costs it does appear, however, that the trends indicated by the curves in Figs. 2 to 6 are significant. Some particularly important implications may be summarized as follows:

1. Increasing the size of the torus leads to a relatively small increase in the cost of the superconducting magnet coils required for a feasibility experiment, and actually reduces the cost of the magnet coils for a full scale power plant for the size range of interest.
2. If superconducting coils are used for the toroidal field of a scientific feasibility experiment, provision for a magnet shield can be made for a moderate increase in cost. Such a shield could be installed after the scientific feasibility had been demonstrated, and this would permit extensive tests involving power bursts for hundreds of seconds to investigate the behavior of plasmas in the regimes required for central station power reactors.

REFERENCES

1. I. N. Golovin, Tokamak as a Possible Fusion Reactor - Comparison with Other C.T.R. Devices, pp. 194-221 of Proceedings of the British Nuclear Energy Society Conference on Nuclear Fusion Reactors, held at the U.K.A.E.A. Culham Laboratory, September 17-19, 1969.
2. S. Yoshikawa, Consideration of Power Requirement in Fusion Feasibility Experiment, AEC R&D Report MATT-803, Princeton Plasma Physics Laboratory, June 1971.
3. F. C. Tenney, personal communication, May 1, 1972.
4. R. Carruthers and R. Hancox, The Economics of Toroidal Fusion Reactors, pp. 1109-1117 of Proceedings of the 1971 Intersociety Energy Conversion Engineering Conference, Boston, Massachusetts, August 3-5, 1971.
5. J. T. D. Mitchell and R. Hancox, A Lithium Cooled Toroidal Fusion Reactor, CLM-P 319, U.K.A.E.A. Culham Laboratory, paper presented at the Intersociety Energy Conversion Engineering Conference, San Diego, California, September 25-29, 1972.
6. A. Gibson, R. Hancox, and R. J. Bickerton, Economic Feasibility of Stellarator and Tokamak Fusion Reactors, IAEA-CN-28-K-4, U.K.A.E.A. Culham Laboratory.
7. D. G. McAlees and H. K. Forsen, Thermonuclear Alpha Particle Orbits in a Tokamak, Bulletin of the American Physics Society, Paper 4A13, October 1973.
8. A. P. Fraas, Cryogenic System for the Superconducting Magnets of a Full Scale Thermonuclear Power Plant, USAEC Report ORNL-TM-3097, Oak Ridge National Laboratory, December 1970.
9. M. S. Lubell et al., Engineering Design Studies on the Superconducting Magnet System of a Tokamak Fusion Reactor, CN-28/K-10, Proceedings of the Fourth Conference on Plasma Physics and Controlled Nuclear Fusion Research, Madison, Wisconsin, June 17-23, 1971 (published by I.A.E.A.).
10. M. S. Lubell, personal communication, July 3, 1973.
11. A. P. Fraas, Conceptual Design of the Blanket and Shield Region and Related Systems for a Full-Scale Toroidal Fusion Reactor, USAEC Report ORNL-TM-3096, May 1973.
12. Argonne/Batavia Magnet for the 15 ft Bubble Chamber, CERN Courier, Vol. 12, No. 11, p. 382, November 1972.
13. A. P. Fraas and J. J. Tudor, Mechanical Engineering Design Considerations for a Tokamak Scientific Feasibility Experiment and Plasma Test Reactor, USAEC Report ORNL-TM-4371, November 1973.

14. O. B. Morgan, personal communication, October 16 and 30, 1973.
15. A. P. Fraas, Analysis of a Recirculating Lithium Blanket Designed to Give a Low Magnetohydrodynamic Pumping Power Requirement, USAEC Report ORNL-TM-3756, September 1972.
16. S. J. Leverette, An Investigation of the Temperature Distribution in the Lithium Blanket of a Fusion Reactor, USAEC Report ORNL-TM-3701, May 1972.

Table 1. Nomenclature

a	Plasma minor radius, cm
A	Aspect ratio
B	Toroidal magnetic field (measured at centerline), Gauss
B_m	Maximum magnetic field in coils, Gauss
n	Ion density, cm^{-3}
P	Power, W
P_T	Total power, W
P_w	Wall heat loading, W/cm^2
q	Stability margin
Q_T	Total energy per fusion reaction, $22.4 \text{ MeV} = 3.6 \times 10^{-12} \text{ J}$
Q_α	Alpha particle energy per fusion reaction, $3.5 \text{ MeV} = 0.562 \times 10^{-12} \text{ J}$
R	Torus major radius, cm
r_c	Magnet coil inner radius, cm
r_w	Wall radius, cm
T	Plasma temperature, eV
T_o	Plasma temperature at the torus centerline, eV
T_e	Electron temperature, eV
T_i	Ion temperature, eV
t_b	Thickness of blanket plus magnet shield, cm
y	Ratio of plasma radius to wall radius, cm
$\langle \sigma v \rangle$	Fusion reaction rate parameter, cm^3/sec
τ	Ion lifetime, sec

Table 2. Results of iterative calculations solving for aB/q as a function of T using Eq. 8 and taking $A = 4$, i.e.,

$$\frac{\langle \sigma v \rangle}{4} Q_\alpha e^{-\left(\frac{0.68 \times 10^6}{aB/q} - 1\right)} = \frac{2.82 \times 10^{-21} T^{.5} A^2}{a^2 R^2 / q^2} + 1.1 \times 10^{-32} T^{.5}$$

① Temperature T (eV)	② Fusion Reaction Rate Parameter $\langle \sigma v \rangle$ (cm ³ /sec) x10 ¹⁶	③ aB/q x10 ⁻⁶	④ $\frac{0.68 \times 10^6}{\textcircled{3}} - 0.19$	⑤ $e^{-\textcircled{4}}$	⑥ $\frac{\textcircled{2} \textcircled{5} \times .14 \times 10^{-12}}{x10^{30}}$	⑦ $\frac{1.1 \times 10^{-32}}{x \sqrt{T}} \times 10^{30}$	⑧ $\frac{(4.5 \times 10^{-20})}{x \sqrt{T} / \textcircled{3}^2} \times 10^{30}$	⑨ $\frac{\textcircled{7} + \textcircled{8}}{= \textcircled{6}} \times 10^{30}$
10,000	1.13	0.8	0.66	0.516	8.16	1.1	7.0469	8.15
20,000	4.31	0.562	1.01996	0.36061	21.76	1.556	20.19	21.75
40,000	7.93	0.5166	1.126299	0.324231	36.00	2.2	33.80	36.00
80,000	8.64	0.5615	1.02104	0.360219	43.572	3.111	40.46	43.57

Table 3. Details of Calculations for Table 4.

Column	Quantity		Operation
(1)	a	Plasma minor radius, cm	Specified
(2)	r_w	Wall radius, cm	$1.25 a$
(3)	T	Plasma temperature, keV	Specified
(4)	aB	Plasma radius x magnetic field, cm·G	From Table 3
(5)	B	Toroidal magnetic field (measured at centerline), G	aB/a
(6)	B_{me}	Max. field in coils - no shield, G	$B/(1 - 1/Ay)$
(7)		Stored energy - no shield, MJ	$B_{me}^2 \pi r_w^2 a \cdot 10^{-7}$
(8)		Magnet cost - no shield, \$	Eq. 12
(9)	$\langle \sigma v \rangle$	Fusion reaction rate parameter, cm^3/sec	Basic data
(10)	P	Power (for $n = 5 \times 10^{13}$), MW	$17.75 \times 10^{16} a^3 \langle \sigma v \rangle$
(11)		Wall surface area (for $A = 4$), m^2	$2\pi^2 a^2 / 10^3$
(12)	P_w	Wall heat loading, MW/m^2	$\frac{\text{Power}}{\text{Wall surface area}}$
(13)	t_b	Shield plus blanket thickness, cm	$190 + 120 \log_{10} (P_w/0.15)$
(14)	B_{ms}	Max. magnet field with blanket and shield, G	$\frac{B}{1 - \frac{1}{Ay} - \frac{t_b}{aA}}$
(15)	r_c	Coil radius with blanket and magnet shield, cm	$r_w + t_b$
(16)		Stored energy with blanket and shield, MJ	$\frac{B_{ms}^2 \pi r_c^2 a}{10^7}$
(17)		Magnet cost with blanket and shield, \$	Eq. 12
(18)		Cost of blanket and shield, \$	Eqs. 16 and 17
(19)		Total cost of blanket, shield, and magnet, \$	(17) + (18)

Table 3 (continued)

Column	Quantity	Operation
(20)	B_{m60} Max. magnetic field with 60 cm shield and no blanket, G	$\frac{B}{1 - \frac{1}{Ay} - \frac{15}{a}}$
(21)	Coil radius with 60 cm water shield, cm	$r_w + 60$
(22)	Stored energy with 60 cm water shield, cm·G	$\frac{B_{m60}^2 \pi (r_w + 60)^2 a}{10^7}$
(23)	Magnet cost with 60 cm shield, \$	Eq. 12

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