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UC-32 – Mathematics and Computers

AN IBM-7090 SUBROUTINE PACKAGE FOR
LAGRANGIAN INTERPOLATION

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operated by

UNION CARBIDE CORPORATION

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ABSTRACT

A FORTRAN subroutine has been written for the purpose of generating a table of data by Lagrangian interpolation in a smaller table of data. There are three options for the interpolation scheme: linear, semilog, and log-log. The number of points to be used in the Lagrangian interpolation is optional. This subroutine provides information for a table lookup, which is usually much less costly than direct computation.

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When a computer code requires that the same type of calculation be performed repeatedly, it might be advantageous to perform the calculation once for the entire range of variables and store the resulting information in the machine in tabular form for future reference. The computing cost of a table lookup is usually much smaller than that of direct computation. Therefore in certain codes, where computational time is at a premium whereas storage area is not, this type of table generation is highly desirable.

A FORTRAN subroutine, LAGRAN, has been written for the purpose of generating a table of data by means of Lagrangian interpolation from a smaller table of data. The interpolation can be one of three types: linear, semilog, or log-log. The number of points to be used in the Lagrangian interpolation is optional. The values in the generated table correspond to values of the independent variable which are evenly spaced either in the variable itself or in its logarithm.

The user must have in storage a data array of functional values in one-to-one correspondence with the values of the independent variable in another data array, which must be arranged in monotonically increasing order.

The call statement for the routine is as follows:

```
CALL LAGRAN(I,NPTS,XCUT,DELX,A,B,NOX,TABLE,NX,X,FX),
```

where

I = type of interpolation: 1, linear; 2, semilog;
3, log-log;

NPTS = number of points to be used in the interpolation;
 $1 \leq \text{NPTS} \leq 10$;

XCUT = a parameter (explained below);

DELX = step-size or spacing of generated table; this quantity
is computed by LAGRAN;

A,B = inclusive lower and upper limits, respectively, of
range of the independent variable in the generated table;

NOX = number of intervals or steps in the generated table of functional values (NOX+1 = number values in table);
 TABLE = the array making up the generated table of functional values;
 NX = number of data points in the array X or FX;
 X = array of the independent variable (monotonically increasing);
 FX = corresponding array of the dependent variable.

In computing a value $F(x_n)$ to be stored in TABLE, the values in the input tables FX and X are selected for use in the interpolation so that, as nearly as possible, an equal number is picked from each side of x_n . If NPTS is odd, more points will be used from one side of x_n than from the other. If $x_n \leq \text{XCUT}$, more values of X larger than x_n will be used. Otherwise, more values of X smaller than x_n will be used. The parameter XCUT may have any value.

The relation of x_n to values in the input table X is established through an index i such that

$$x_i \leq x_n \leq x_{i+1} \quad (i = 1, 2, \dots, \text{NX}-1) ,$$

where x_i and x_{i+1} are adjacent values in the array X. If the above is not satisfied, x_n lies outside the table and

$$i = 1 \quad \text{for } x_n < x_1 ,$$

or

$$i = \text{NX} - 1 \quad \text{for } x_n > x_{\text{NX}} .$$

Some sequence $x_q, x_{q+1}, \dots, x_{q+\text{NPTS}-1}$ contained in the data array X will be used for the interpolation (or extrapolation) to obtain $F(x_n)$.

The index q is determined as follows:

If NPTS is even,

$$q = \min \left\{ (NX + 1 - NPTS), \max \left[1, \left(i - \frac{NPTS}{2} + 1 \right) \right] \right\}.$$

If NPTS is odd and if $x_n \leq XCUT$, then

$$q = \min \left\{ (NX + 1 - NPTS), \max \left[1, \left(i - \frac{NPTS - 1}{2} + 1 \right) \right] \right\}.$$

If NPTS is odd and if $x_n > XCUT$,

$$q = \min \left\{ (NX + 1 - NPTS), \max \left[1, \left(i - \frac{NPTS - 1}{2} \right) \right] \right\}.$$

If $A > B$, the routine interchanges these two quantities and then proceeds with the interpolation.

If $NPTS > NX$, NPTS is set equal to NX .

The quantities to be computed by LAGRAN for the three types of interpolation are determined as follows:

For $I = 1$

$$DELX = (B - A) / NOX,$$

$$x_{n+1} = x_n + DELX,$$

$$F(x_n) = \sum_{k=q}^{q+NPTS-1} \left[F(x_k) \left(\prod_{j \neq k}^{q+NPTS-1} \frac{x_n - x_j}{x_k - x_j} \right) \right].$$

For $I = 2$

$$DELX = (B - A) / NOX,$$

$$x_{n+1} = x_n + \text{DELX},$$

$$F(x_n) = \sum_{k=q}^{q+\text{NPTS}-1} F(x_k)^{c_k},$$

where

$$c_k = \prod_{j \neq k}^{q+\text{NPTS}-1} \frac{x_n - x_j}{x_k - x_j}.$$

For $I = 3$

$$\text{DELX} = \ln(B/A)/\text{NOX},$$

$$x_{n+1} = \ln(x_n) + \text{DELX},$$

$$F(x_n) = \exp \sum_{k=q}^{q+\text{NPTS}-1} \left[\ln F(x_k)^{c_k} \right],$$

where

$$c_k = \prod_{j \neq k}^{q+\text{NPTS}-1} \frac{\ln x_n - \ln x_j}{\ln x_k - \ln x_j}.$$

To select from TABLE the value corresponding to a given x, a FORTRAN statement similar to the following ones is necessary:

For I = 1,2

$K = (x - A)/DELX + 1.49$, where K is the index in TABLE.

For I = 3

$K = \ln(x/A)/DELX + 1.49$.

None of the subroutines should be designated G3R3G3, because LAGRAN has a slave by that name. LAGRAN also calls the routines LOG and EXP.

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