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**RESEARCH AND
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REPORT**

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Metals, Ceramics, and Materials (UC-25)
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UNITED STATES ATOMIC ENERGY COMMISSION

Research and Development Report

A TABULATION OF THE SPECIFIC HEAT
CONTRIBUTION DUE TO THERMAL
EXCITATION (SCHOTTKY EFFECT) OF
THE 4F ELECTRONS OF SOME DI-AND
TRI-VALENT LANTHANIDE ELEMENTS

by

David H. Dennison and Karl A. Gschneidner, Jr.

May, 1965

Ames Laboratory

at

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A TABULATION OF THE SPECIFIC HEAT CONTRIBUTION
DUE TO THERMAL EXCITATION (SCHOTTKY EFFECT)
OF THE 4F ELECTRONS OF SOME DI-AND
TRI-VALENT LANTHANIDE ELEMENTS

David H. Dennison and Karl A. Gschneidner, Jr.

ABSTRACT

The 4f electronic specific heat contribution for divalent samarium and for the trivalent lanthanide elements Ce, Pr, Nd, Pm, Sm, Eu, Tb, Dy, Ho, Er, Tm and Yb has been calculated and is tabulated at 25°K intervals from 50° to 200°K, at 50°K intervals from 200° to 400°K, and at 100°K intervals from 400° to 2000°K.

INTRODUCTION

Today specific heat measurements of many materials are being investigated at high temperatures up to 2000°K. In order to better understand the behavior of solids at high temperatures, the various contributions to the specific heat are being analyzed. For most solids the important contributions are the electronic, lattice, and dilation terms. For the lanthanides, however, there may be two additional important contributions to the specific heat: the magnetic and 4f electronic terms. In this paper we are only concerned with the evaluation of the last term, the specific heat contribution due to the 4f electrons; the symbol C_v^f will designate this contribution.

The C_v^f term is due to a statistical distribution of the 4f electrons over the available energy levels of the ground and excited multiplets as the temperature is increased. This thermal excitation of electrons gives

rise to an excess heat capacity known as the Schottky anomaly¹, and it is given by

$$C_v^f = N \frac{d}{dT} \left[\frac{\sum_J (2J+1) E_J e^{-E_J/kT}}{\sum_J (2J+1) e^{-E_J/kT}} \right] \quad (1)$$

where N is Avogadro's number, J the total angular momentum quantum number, E_J the energy of the Jth level and k Boltzmann's constant.

DISCUSSION

The purpose of this report is to present the tabulated 4f electronic specific heat contribution, C_v^f , for divalent samarium and the trivalent lanthanide elements as a function of temperature up to 2000°K. The C_v^f term was calculated by using Eqn. 1 at 25°K intervals from 50° to 200°K, at 50°K intervals from 200° to 400°K and at 100°K intervals from 400°K to 2000°K.

The 4f electronic energy levels of the ground and excited multiplets, which have energies less than 15,000 cm^{-1} , and their respective term symbols and J values are listed in Table I for divalent samarium and the trivalent lanthanide elements cerium, praseodymium, neodymium, promethium, samarium, europium, terbium, dysprosium, holmium, erbium, thulium and ytterbium. The divalent lanthanides, europium and ytterbium; the trivalent lanthanides, lanthanum, gadolinium and lutetium; and tetravalent cerium do not have any low lying 4f levels ($<15,000 \text{ cm}^{-1}$) and thus are not included in Table I. The energy levels are taken from experimental data obtained from a number of different types of inorganic compounds. The mean value is listed in column four, and the references

from which the data were obtained are given in the last column. The fifth column, $\Delta E_{\max}$, lists the maximum difference between the mean value and the experimental value which is the furthest from the mean.

We have chosen to use the mean experimental values for these energy levels rather than calculated values for the following reasons. Judd² has shown that it is possible to account for the observed energy level separations by allowing a partial breakdown of Russell-Saunders coupling. However, this introduces at least one constant which is quite difficult to evaluate a priori. The choice of this constant in general is dictated by the experimental spectra observed in the compound of interest. Since the "theoretical" values are at least in part based on the experimental values, we believe it is better to be more direct and use the experimental values. Fortunately, the experimental values in most cases agree reasonably well with one another for a given lanthanide ion, even though these data were obtained on several compounds, in which the crystal field environments are quite different. Finally, C_v^f calculated by using either $E_J - |\Delta E_{\max}|$ or $E_J + |\Delta E_{\max}|$ deviates from C_v^f calculated from the mean energy, E_J , by less than 10% in all instances, and since C_v^f usually amounts to less than 20% of the total C_p , the uncertainty in the E_J values amounts to an uncertainty of 2% or less in the calculated C_p value. For divalent samarium and trivalent europium, however, C_v^f may account for about 50% of the total heat capacity at temperatures in the range 100° to 400°K, and thus the uncertainty in the calculated C_p value may be as high as 5%. Thus, it is seen that the total calculated

heat capacity is relatively insensitive to the choice of the value one chooses for E_J .

The results of these computations are listed in Tables II through XIV and are shown in Figs. 1 through 13. In Tables II through XIV, the symbols G, E, T and CM have the following significance: the symbol G is equal to $2J + 1$; E is the energy of a level in units of cm^{-1} ; and T is the absolute temperature at which the $4f$ electronic contribution to the specific heat, CM, has been calculated.

For cerium, holmium and erbium the spread of the C_v^f data is about the width of the line drawn in Figs. 1, 10 and 11. Since we have only one set of experimental energy levels for promethium, divalent samarium, and ytterbium we can only draw a single line for these two lanthanides.

ACKNOWLEDGMENT

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TABLE I. Energy Levels of Divalent Samarium and the Trivalent Lanthanide Elements up to 15,000 cm^{-1} .

Element	Term Symbol	J	Energy (cm^{-1})	$\Delta E_{\text{max.}}$ (cm^{-1})	References
Cerium	2F	5/2	0	-	-
		7/2	2248	8	1, 2, 3
Praseodymium	3H	4	0	-	-
		5	2120	51	4, 5, 6, 7, 8
		6	4237	65	4, 5, 6, 8
	3F	2	4860	90	4, 5, 6, 7, 8
		3	6250	70	4, 5, 6, 8
		4	6694	76	4, 5, 6, 8
	1G	4	6731	47	4, 5, 6, 8
Neodymium	4I	9/2	0	-	-
		11/2	1931	97	9, 10, 11
		13/2	3905	85	9, 10, 11
		15/2	5984	146	11, 12, 13
	4F	3/2	11370	155	9, 10, 11
		5/2	12409	160	9, 10, 11
	2H	9/2	12514	61	9, 10
	4F	7/2	13406	226	9, 10, 11
	4S	3/2	13414	60	9, 10, 11
	4F	9/2	14653	154	9, 10, 11
Promethium	5I	4	0	-	14
		5	1515*	-	14
		6	3166*	-	14
		7	4990	-	14
		8	6670	-	14
	5F	1	12200	-	14
		2	12770	-	14
		3	13720	-	14
	5S	2	14380	-	14
	5F	4	14700	-	14

* Calculated values.

TABLE I. Continued

Element	Term Symbol	J	Energy (cm ⁻¹)	$\Delta E_{\text{max.}}$ (cm ⁻¹)	References
Samarium (+3)	⁶ H	5/2	0	-	-
		7/2	1072	28	15, 16
		9/2	2277	23	15, 16
		11/2	3586	14	15, 16
		13/2	4957	43	13, 15, 16, 17
	⁶ F	1/2	5731	-	16
		3/2	6340	70	13, 16
	⁶ H	15/2	6640	90	13, 16, 17, 18
	⁶ F	5/2	7104	91	13, 16
		7/2	7937	63	13, 16
		9/2	9106	68	13, 16
		11/2	10530	48	13, 16
Samarium (+2)	⁷ F	0	0	-	-
		1	289	-	19
		2	809	-	19
		3	1485	-	19
		4	2259	-	19
		5	3093	-	19
	⁵ D	6	3981*	-	20
		0	14379*	-	20
Europium	⁷ F	0	0	-	-
		1	352	52	13, 15, 18, 21, 22, 23
		2	1064	64	15, 21, 22, 23
		3	1933	78	15, 22, 23
		4	2924	83	15, 22, 23
		5	3949	77	15, 22, 23
		6	5082	60	18, 22, 23
Terbium	⁷ F	6	0	-	-
		5	2075	55	15, 24, 25, 26
		4	3370	102	15, 24, 25, 26
		3	4332	74	15, 24, 25, 26
		2	5010	90	15, 24, 25, 26
		1	5490	110	15, 24, 25, 26
		0	5714	86	15, 24, 25, 26

* Calculated values.

TABLE I. Continued

Element	Term Symbol		Energy (cm^{-1})	ΔE_{max} (cm^{-1})	References
Dysprosium	^6H	15/2	0	-	-
		13/2	3410	100	27, 28, 29
		11/2	5720	120	27, 28, 29
		9/2	7408	74	27, 28
	^6F	11/2	7426	189	27, 28
	^6H	7/2	8932	51	28, 29
	^6F	9/2	9633	284	27, 28, 29
	^6H	5/2	10082	4	28, 29
	^6F	7/2	10941	136	27, 28, 29
		5/2	12321	100	27, 28, 29
		3/2	13169	212	27, 28, 29
		1/2	13957	-	27
Holmium	^5I	8	0	-	-
		7	5068	22	17, 18, 30, 31, 32
		6	8546	101	17, 18, 30, 31, 32
		5	11151	111	18, 30, 31, 32
		4	13268	113	18, 30, 32
Erbium	^4I	15/2	0	-	-
		13/2	6492	64	33, 34, 35, 36, 37
		11/2	10122	69	9, 33, 34, 35, 36, 37, 38
		9/2	12375	136	9, 33, 34, 35, 36, 37, 38
Thulium	^3H	6	0	-	-
		4	5630	180	39, 40, 41
		5	8242	150	4, 39, 40, 41, 42, 43
	^3F	4	12653	153	4, 39, 40, 41, 43
		3	14451	149	4, 39, 40, 41, 43
		2	15011	156	4, 39, 40, 41, 43
Ytterbium	^2F	7/2	0	-	-
		5/2	10300	-	18

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TABLE II. Schottky Specific Heat Contribution From the $4f^1$ Ground Multiplet of Trivalent Cerium

From Mean Value of E		From Minimum Value of E		From Maximum Value of E	
G	E	G	E	G	E
6.0000	0.00000	6.0000	0.00000	6.0000	0.00000
8.0000	2248.00000	8.0000	2240.00000	8.0000	2256.00000
T	CM	T	CM	T	CM
50.0000	0.00000	50.0000	0.00000	50.0000	0.00000
75.0000	0.00030	75.0000	0.00000	75.0000	0.00000
100.0000	0.00000	100.0000	0.00000	100.0000	0.00000
125.0000	0.00000	125.0000	0.00000	125.0000	0.00000
150.0000	0.00000	150.0000	0.00000	150.0000	0.00000
175.0000	0.00000	175.0000	0.00000	175.0000	0.00000
200.0000	0.00000	200.0000	0.00000	200.0000	0.00000
250.0000	0.00107	250.0000	0.00111	250.0000	0.00102
300.0000	0.00639	300.0000	0.00660	300.0000	0.00620
350.0000	0.02191	350.0000	0.02248	350.0000	0.02136
400.0000	0.05324	400.0000	0.05440	400.0000	0.05210
500.0000	0.17116	500.0000	0.17389	500.0000	0.16847
600.0000	0.34563	600.0000	0.35075	600.0000	0.34255
700.0000	0.54251	700.0000	0.54735	700.0000	0.53769
800.0000	0.72535	800.0000	0.73015	800.0000	0.72055
900.0000	0.87535	900.0000	0.87952	900.0000	0.87117
1000.0000	0.98534	1000.0000	0.98853	1000.0000	0.98213
1100.0000	1.05542	1100.0000	1.05848	1100.0000	1.05432
1200.0000	1.09386	1200.0000	1.09481	1200.0000	1.09286
1300.0000	1.10439	1300.0000	1.10434	1300.0000	1.10440
1400.0000	1.09462	1400.0000	1.09371	1400.0000	1.09549
1500.0000	1.07029	1500.0000	1.06867	1500.0000	1.07187
1600.0000	1.03607	1600.0000	1.03389	1600.0000	1.03821
1700.0000	0.99557	1700.0000	0.99298	1700.0000	0.99814
1800.0000	0.95152	1800.0000	0.94862	1800.0000	0.95440
1900.0000	0.90589	1900.0000	0.90278	1900.0000	0.90898
2000.0000	0.86008	2000.0000	0.85683	2000.0000	0.86331

Fig. 1. Schottky Specific Heat Contribution From the $4f^1$ Ground Multiplet of Trivalent Cerium.

TABLE III. Schottky Specific Heat Contribution From the $4f^2$ Multiplets of Trivalent Praseodymium.

From Mean Value of E		From Minimum Value of E		From Maximum Value of E	
G	E	G	E	G	E
9.0000	0.00000	9.0000	0.00000	9.0000	0.00000
11.0000	2120.00000	11.0000	2069.00000	11.0000	2171.00000
13.0000	4237.00000	13.0000	4172.00000	13.0000	4302.00000
5.0000	4860.00000	5.0000	4770.00000	5.0000	4950.00000
7.0000	6250.00000	7.0000	6180.00000	7.0000	6320.00000
9.0000	6694.00000	9.0000	6618.00000	9.0000	6770.00000
9.0000	6731.00000	9.0000	6684.00000	9.0000	6778.00000
T	CM	T	CM	T	CM
50.0000	0.00000	50.0000	0.00000	50.0000	0.00000
75.0000	0.00000	75.0000	0.00000	75.0000	0.00000
100.0000	0.00000	100.0000	0.00000	100.0000	0.00000
125.0000	0.00000	125.0000	0.00000	125.0000	0.00000
150.0000	0.00000	150.0000	0.00000	150.0000	0.00000
175.0000	0.00000	175.0000	0.00000	175.0000	0.00000
200.0000	0.00000	200.0000	0.00018	200.0000	0.00000
250.0000	0.00181	250.0000	0.00232	250.0000	0.00142
300.0000	0.00963	300.0000	0.01171	300.0000	0.00791
350.0000	0.03023	350.0000	0.03551	350.0000	0.02571
400.0000	0.06876	400.0000	0.07866	400.0000	0.06003
500.0000	0.20375	500.0000	0.22471	500.0000	0.18451
600.0000	0.39563	600.0000	0.42582	600.0000	0.36703
700.0000	0.61350	700.0000	0.64904	700.0000	0.57918
800.0000	0.83439	800.0000	0.87162	800.0000	0.79774
900.0000	1.04665	900.0000	1.08292	900.0000	1.01035
1000.0000	1.24529	1000.0000	1.27886	1000.0000	1.21116
1100.0000	1.42829	1100.0000	1.45803	1100.0000	1.39756
1200.0000	1.59450	1200.0000	1.61965	1200.0000	1.56800
1300.0000	1.74290	1300.0000	1.76299	1300.0000	1.72120
1400.0000	1.87254	1400.0000	1.88727	1400.0000	1.85602
1500.0000	1.98270	1500.0000	1.99194	1500.0000	1.97157
1600.0000	2.07307	1600.0000	2.07684	1600.0000	2.06736
1700.0000	2.14384	1700.0000	2.14230	1700.0000	2.14345
1800.0000	2.19572	1800.0000	2.18914	1800.0000	2.20041
1900.0000	2.22983	1900.0000	2.21859	1900.0000	2.23927
2000.0000	2.24766	2000.0000	2.23219	2000.0000	2.26144

Fig. 2. Schottky Specific Heat Contribution From the $4f^2$ Multiplets of Trivalent Praseodymium.

TABLE IV. Schottky Specific Heat Contribution From the $4f^3$ Multiplets of Trivalent Neodymium

From Mean Value of E		From Minimum Value of E		From Maximum Value of E	
G	E	G	E	G	E
10.0000	0.00000	10.0000	0.00000	10.0000	0.00000
12.0000	1931.00000	12.0000	1834.00000	12.0000	2028.00000
14.0000	3905.00000	14.0000	3820.00000	14.0000	3990.00000
16.0000	5984.00000	16.0000	5838.00000	16.0000	6130.00000
4.0000	11370.00000	4.0000	11215.00000	4.0000	11525.00000
6.0000	12409.00000	6.0000	12249.00000	6.0000	12569.00000
10.0000	12514.00000	10.0000	12453.00000	10.0000	12575.00000
8.0000	13406.00000	8.0000	13180.00000	8.0000	13632.00000
4.0000	13414.00000	4.0000	13354.00000	4.0000	13474.00000
10.0000	14653.00000	10.0000	14499.00000	10.0000	14807.00000
T	CM	T	CM	T	CM
50.0000	0.00000	50.0000	0.00000	50.0000	0.00000
75.0000	0.00000	75.0000	0.00000	75.0000	0.00000
100.0000	0.00000	100.0000	0.00000	100.0000	0.00000
125.0000	0.00000	125.0000	0.00000	125.0000	0.00000
150.0000	0.00000	150.0000	0.00000	150.0000	0.00000
175.0000	0.00000	175.0000	0.00000	175.0000	0.00000
200.0000	0.00043	200.0000	0.00077	200.0000	0.00023
250.0000	0.00439	250.0000	0.00692	250.0000	0.00277
300.0000	0.01342	300.0000	0.02789	300.0000	0.01345
350.0000	0.05354	350.0000	0.07193	350.0000	0.03964
400.0000	0.11086	400.0000	0.14163	400.0000	0.08631
500.0000	0.28516	500.0000	0.34039	500.0000	0.23921
600.0000	0.50762	600.0000	0.57514	600.0000	0.44531
700.0000	0.73613	700.0000	0.80521	700.0000	0.66876
800.0000	0.94902	800.0000	1.01122	800.0000	0.88504
900.0000	1.13685	900.0000	1.18761	900.0000	1.08158
1000.0000	1.29731	1000.0000	1.33501	1000.0000	1.25338
1100.0000	1.43125	1100.0000	1.45598	1100.0000	1.39936
1200.0000	1.54056	1200.0000	1.55341	1200.0000	1.52039
1300.0000	1.62777	1300.0000	1.62986	1300.0000	1.61840
1400.0000	1.69475	1400.0000	1.68773	1400.0000	1.69516
1500.0000	1.74424	1500.0000	1.72925	1500.0000	1.75308
1600.0000	1.77832	1600.0000	1.75675	1600.0000	1.79442
1700.0000	1.79944	1700.0000	1.77244	1700.0000	1.82165
1800.0000	1.80986	1800.0000	1.77844	1800.0000	1.83713
1900.0000	1.81167	1900.0000	1.77674	1900.0000	1.84307
2000.0000	1.80679	2000.0000	1.76914	2000.0000	1.84147

Fig. 3. Schottky Specific Heat Contribution From the $4f^3$ Multiplets of Trivalent Neodymium.

TABLE V. Schottky Specific Heat Contribution From the $4f^4$ Multiplets of Trivalent Promethium.

G	E
9.0000	0.00000
11.0000	1515.00000
13.0000	3166.00000
15.0000	4990.00000
17.0000	6670.00000
3.0000	12200.00000
5.0000	12770.00000
7.0000	13720.00000
5.0000	14380.00000
9.0000	14700.00000
T	CM
50.0000	0.00000
75.0000	0.00000
100.0000	0.00000
125.0000	0.00000
150.0000	0.00025
175.0000	0.00147
200.0000	0.00532
250.0000	0.03014
300.0000	0.08941
350.0000	0.18600
400.0000	0.31085
500.0000	0.59775
600.0000	0.87570
700.0000	1.11403
800.0000	1.30826
900.0000	1.46412
1000.0000	1.58898
1100.0000	1.68906
1200.0000	1.76886
1300.0000	1.83151
1400.0000	1.87919
1500.0000	1.91352
1600.0000	1.93572
1700.0000	1.94722
1800.0000	1.94933
1900.0000	1.94338
2000.0000	1.93064

Fig. 4. Schottky Specific Heat Contribution From the $4f^4$ Multiplets of Trivalent Promethium.

TABLE VI. Schottky Specific Heat Contribution From the $4f^5$ Multiplets of Trivalent Samarium.

From Mean Value of E		From Minimum Value of E		From Maximum Value of E	
G	E	G	E	G	E
6.0000	0.00000	6.0000	0.00000	6.0000	0.00000
8.0000	1072.00000	8.0000	1044.00000	8.0000	1100.00000
10.0000	2277.00000	10.0000	2254.00000	10.0000	2300.00000
12.0000	3586.00000	12.0000	3572.00000	12.0000	3600.00000
14.0000	4957.00000	14.0000	4914.00000	14.0000	5000.00000
2.0000	5731.00000	2.0000	5731.00000	2.0000	5731.00000
4.0000	6340.00000	4.0000	6270.00000	4.0000	6410.00000
16.0000	6640.00000	16.0000	6550.00000	16.0000	6730.00000
6.0000	7104.00000	6.0000	7013.00000	6.0000	7195.00000
8.0000	7937.00000	8.0000	7874.00000	8.0000	8000.00000
10.0000	9106.00000	10.0000	9038.00000	10.0000	9174.00000
12.0000	10530.00000	12.0000	10482.00000	12.0000	10578.00000
T	CM	T	CM	T	CM
50.0000	0.00000	50.0000	0.00000	50.0000	0.00000
75.0000	0.00000	75.0000	0.00000	75.0000	0.00000
100.0000	0.00000	100.0000	0.00000	100.0000	0.00000
125.0000	0.00176	125.0000	0.00231	125.0000	0.00135
150.0000	0.00958	150.0000	0.01188	150.0000	0.00771
175.0000	0.03056	175.0000	0.03649	175.0000	0.02556
200.0000	0.07037	200.0000	0.08162	200.0000	0.06059
250.0000	0.21086	250.0000	0.23474	250.0000	0.18913
300.0000	0.41026	300.0000	0.44412	300.0000	0.37837
350.0000	0.63177	350.0000	0.67000	350.0000	0.59470
400.0000	0.84759	400.0000	0.88484	400.0000	0.81045
500.0000	1.21534	500.0000	1.24103	500.0000	1.18803
600.0000	1.48900	600.0000	1.49948	600.0000	1.47615
700.0000	1.68987	700.0000	1.68737	700.0000	1.69003
800.0000	1.84041	800.0000	1.82878	800.0000	1.85013
900.0000	1.95783	900.0000	1.94091	900.0000	1.97341
1000.0000	2.05316	1000.0000	2.03408	1000.0000	2.07147
1100.0000	2.13370	1100.0000	2.11470	1100.0000	2.15240
1200.0000	2.20285	1200.0000	2.18535	1200.0000	2.22040
1300.0000	2.26261	1300.0000	2.24735	1300.0000	2.27818
1400.0000	2.31370	1400.0000	2.30089	1400.0000	2.32695
1500.0000	2.35616	1500.0000	2.34568	1500.0000	2.36715
1600.0000	2.38985	1600.0000	2.38136	1600.0000	2.39885
1700.0000	2.41463	1700.0000	2.40769	1700.0000	2.42204
1800.0000	2.43049	1800.0000	2.42461	1800.0000	2.43676
1900.0000	2.43759	1900.0000	2.43233	1900.0000	2.44317
2000.0000	2.43631	2000.0000	2.43125	2000.0000	2.44158

Fig. 5. Schottky Specific Heat Contribution From the $4f^5$ Multiplets of Trivalent Samarium.

TABLE VII. Schottky Specific Heat Contribution From the $4f^6$ Multiplets of Divalent Samarium.

G	E
1.0000	0.00000
3.0000	289.00000
5.0000	809.00000
7.0000	1485.00000
9.0000	2259.00000
11.0000	3093.00000
13.0000	3981.00000
1.0000	14379.00000
T	CM
50.0000	0.10058
75.0000	0.69970
100.0000	1.48115
125.0000	1.99539
150.0000	2.21952
175.0000	2.28414
200.0000	2.28797
250.0000	2.26893
300.0000	2.26765
350.0000	2.28234
400.0000	2.30560
500.0000	2.35928
600.0000	2.40848
700.0000	2.44002
800.0000	2.44421
900.0000	2.41689
1000.0000	2.35956
1100.0000	2.27742
1200.0000	2.17725
1300.0000	2.06570
1400.0000	1.94852
1500.0000	1.83017
1600.0000	1.71390
1700.0000	1.60190
1800.0000	1.49553
1900.0000	1.39557
2000.0000	1.30231

Fig. 6. Schottky Specific Heat Contribution From the $4f^6$ Multiplets of Divalent Samarium.

TABLE VIII. Schottky Specific Heat Contribution From the $4f^6$ Ground Multiplet of Trivalent Europium.

From Mean Value of E		From Minimum Value of E		From Maximum Value of E	
G	E	G	E	G	E
1.0000	0.00000	1.0000	0.00000	1.0000	0.00000
3.0000	352.00000	3.0000	300.00000	3.0000	404.00000
5.0000	1064.00000	5.0000	1000.00000	5.0000	1128.00000
7.0000	1933.00000	7.0000	1855.00000	7.0000	2011.00000
9.0000	2924.00000	9.0000	2841.00000	9.0000	3007.00000
11.0000	3949.00000	11.0000	3872.00000	11.0000	4026.00000
13.0000	5082.00000	13.0000	5022.00000	13.0000	5142.00000
T	CM	T	CM	T	CM
50.0000	0.02437	50.0000	0.07900	50.0000	0.00719
75.0000	0.31501	75.0000	0.61322	75.0000	0.15368
100.0000	0.92997	100.0000	1.37136	100.0000	0.59118
125.0000	1.54380	125.0000	1.88761	125.0000	1.16793
150.0000	1.94262	150.0000	2.08160	150.0000	1.66689
175.0000	2.12515	175.0000	2.08032	175.0000	1.99920
200.0000	2.17368	200.0000	2.00997	200.0000	2.18172
250.0000	2.13252	250.0000	1.88496	250.0000	2.29450
300.0000	2.09349	300.0000	1.85561	300.0000	2.29869
350.0000	2.09569	350.0000	1.88896	350.0000	2.29771
400.0000	2.12584	400.0000	1.94961	400.0000	2.31039
500.0000	2.21520	500.0000	2.08321	500.0000	2.36248
600.0000	2.30579	600.0000	2.20000	600.0000	2.42585
700.0000	2.38367	700.0000	2.29325	700.0000	2.48591
800.0000	2.44407	800.0000	2.36276	800.0000	2.53496
900.0000	2.48348	900.0000	2.40773	900.0000	2.56702
1000.0000	2.49938	1000.0000	2.42735	1000.0000	2.57787
1100.0000	2.49107	1100.0000	2.42195	1100.0000	2.56567
1200.0000	2.45965	1200.0000	2.39344	1200.0000	2.53101
1300.0000	2.40856	1300.0000	2.34494	1300.0000	2.47637
1400.0000	2.34050	1400.0000	2.28025	1400.0000	2.40527
1500.0000	2.26083	1500.0000	2.20335	1500.0000	2.32166
1600.0000	2.17213	1600.0000	2.11793	1600.0000	2.22933
1700.0000	2.07809	1700.0000	2.02725	1700.0000	2.13167
1800.0000	1.98150	1800.0000	1.93400	1800.0000	2.03148
1900.0000	1.88453	1900.0000	1.84030	1900.0000	1.93102
2000.0000	1.78886	2000.0000	1.74777	2000.0000	1.83199

Fig. 7. Schottky Specific Heat Contribution From the $4f^6$ Ground Multiplet of Trivalent Europium.

TABLE IX. Schottky Specific Heat Contribution From the $4f^8$ Ground Multiplet of Trivalent Terbium.

From Mean Value of E		From Minimum Value of E		From Maximum Value of E	
G	E	G	E	G	E
13.0000	0.00000	13.0000	0.00000	13.0000	0.00000
11.0000	2075.00000	11.0000	2020.00000	11.0000	2130.00000
9.0000	3370.00000	9.0000	3268.00000	9.0000	3472.00000
7.0000	4332.00000	7.0000	4258.00000	7.0000	4406.00000
5.0000	5010.00000	5.0000	4920.00000	5.0000	5100.00000
3.0000	5490.00000	3.0000	5380.00000	3.0000	5600.00000
1.0000	5714.00000	1.0000	5628.00000	1.0000	5800.00000
T	CM	T	CM	T	CM
50.0000	0.00000	50.0000	0.00000	50.0000	0.00000
75.0000	0.00000	75.0000	0.00000	75.0000	0.00000
100.0000	0.00000	100.0000	0.00000	100.0000	0.00000
125.0000	0.00000	125.0000	0.00000	125.0000	0.00000
150.0000	0.00000	150.0000	0.00000	150.0000	0.00000
175.0000	0.00000	175.0000	0.00000	175.0000	0.00000
200.0000	0.00012	200.0000	0.00017	200.0000	0.00000
250.0000	0.00156	250.0000	0.00203	250.0000	0.00120
300.0000	0.00793	300.0000	0.00978	300.0000	0.00641
350.0000	0.02438	350.0000	0.02903	350.0000	0.02046
400.0000	0.05473	400.0000	0.06342	400.0000	0.04718
500.0000	0.16085	500.0000	0.17971	500.0000	0.14387
600.0000	0.31562	600.0000	0.34433	600.0000	0.28909
700.0000	0.49705	700.0000	0.53311	700.0000	0.46305
800.0000	0.68562	800.0000	0.72576	800.0000	0.64707
900.0000	0.86732	900.0000	0.90828	900.0000	0.82724
1000.0000	1.03302	1000.0000	1.07201	1000.0000	0.99414
1100.0000	1.17735	1100.0000	1.21215	1100.0000	1.14187
1200.0000	1.29769	1200.0000	1.32676	1200.0000	1.26724
1300.0000	1.39348	1300.0000	1.41590	1300.0000	1.36912
1400.0000	1.46561	1400.0000	1.48099	1400.0000	1.44783
1500.0000	1.51594	1500.0000	1.52430	1500.0000	1.50496
1600.0000	1.54685	1600.0000	1.54851	1600.0000	1.54243
1700.0000	1.56097	1700.0000	1.55646	1700.0000	1.56270
1800.0000	1.56095	1800.0000	1.55091	1800.0000	1.56826
1900.0000	1.54930	1900.0000	1.53442	1900.0000	1.56157
2000.0000	1.52830	2000.0000	1.50929	2000.0000	1.54487

Fig. 8. Schottky Specific Heat Contribution From the $4f^8$ Ground Multiplet of Trivalent Terbium.

TABLE X. Schottky Specific Heat Contribution From the $4f^9$ Multiplets of Trivalent Dysprosium.

From Mean Value of E		From Minimum Value of E		From Maximum Value of E	
G	E	G	E	G	E
16.0000	0.00000	16.0000	0.00000	16.0000	0.00000
14.0000	3410.00000	14.0000	3310.00000	14.0000	3510.00000
12.0000	5720.00000	12.0000	5600.00000	12.0000	5840.00000
10.0000	7408.00000	10.0000	7334.00000	10.0000	7482.00000
12.0000	7426.00000	12.0000	7243.00000	12.0000	7609.00000
8.0000	8932.00000	8.0000	8881.00000	8.0000	8983.00000
10.0000	9633.00000	10.0000	9349.00000	10.0000	9917.00000
6.0000	10082.00000	6.0000	10078.00000	6.0000	10086.00000
8.0000	10941.00000	8.0000	10805.00000	8.0000	11077.00000
6.0000	12321.00000	6.0000	12221.00000	6.0000	12421.00000
4.0000	13169.00000	4.0000	12957.00000	4.0000	13381.00000
2.0000	13957.00000	2.0000	13957.00000	2.0000	13957.00000
T	CM	T	CM	T	CM
50.0000	0.00000	50.0000	0.00000	50.0000	0.00000
75.0000	0.00000	75.0000	0.00000	75.0000	0.00000
100.0000	0.00000	100.0000	0.00000	100.0000	0.00000
125.0000	0.00000	125.0000	0.00000	125.0000	0.00000
150.0000	0.00000	150.0000	0.00000	150.0000	0.00000
175.0000	0.00000	175.0000	0.00000	175.0000	0.00000
200.0000	0.00000	200.0000	0.00000	200.0000	0.00000
250.0000	0.00000	250.0000	0.00000	250.0000	0.00000
300.0000	0.00000	300.0000	0.00000	300.0000	0.00000
350.0000	0.00028	350.0000	0.00040	350.0000	0.00020
400.0000	0.00123	400.0000	0.00166	400.0000	0.00091
500.0000	0.00916	500.0000	0.01150	500.0000	0.00728
600.0000	0.03294	600.0000	0.03946	600.0000	0.02744
700.0000	0.07863	700.0000	0.09119	700.0000	0.06777
800.0000	0.14744	800.0000	0.16658	800.0000	0.13029
900.0000	0.23593	900.0000	0.26155	900.0000	0.21254
1000.0000	0.33991	1000.0000	0.37107	1000.0000	0.31089
1100.0000	0.45493	1100.0000	0.49069	1100.0000	0.42126
1200.0000	0.57762	1200.0000	0.61676	1200.0000	0.54020
1300.0000	0.70503	1300.0000	0.74657	1300.0000	0.66491
1400.0000	0.83497	1400.0000	0.87794	1400.0000	0.79309
1500.0000	0.96559	1500.0000	1.00912	1500.0000	0.92281
1600.0000	1.09529	1600.0000	1.13856	1600.0000	1.05241
1700.0000	1.22261	1700.0000	1.26489	1700.0000	1.18036
1800.0000	1.34620	1800.0000	1.38683	1800.0000	1.30527
1900.0000	1.46486	1900.0000	1.50323	1900.0000	1.42586
2000.0000	1.57750	2000.0000	1.61306	2000.0000	1.54098

Fig. 9. Schottky Specific Heat Contribution From the $4f^9$ Multiplets of Trivalent Dysprosium.

TABLE XI. Schottky Specific Heat Contribution From the $4f^{10}$ Ground Multiplet of Trivalent Holmium.

From Mean Value of E		From Minimum Value of E		From Maximum Value of E	
G	E	G	E	G	E
17.0000	0.00000	17.0000	0.00000	17.0000	0.00000
15.0000	5068.00000	15.0000	5046.00000	15.0000	5090.00000
13.0000	8546.00000	13.0000	8445.00000	13.0000	8647.00000
11.0000	11151.00000	11.0000	11040.00000	11.0000	11262.00000
9.0000	13268.00000	9.0000	13155.00000	9.0000	13381.00000
T	CM	T	CM	T	CM
50.0000	0.00000	50.0000	0.00000	50.0000	0.00000
75.0000	0.00000	75.0000	0.00000	75.0000	0.00000
100.0000	0.00000	100.0000	0.00000	100.0000	0.00000
125.0000	0.00000	125.0000	0.00000	125.0000	0.00000
150.0000	0.00000	150.0000	0.00000	150.0000	0.00000
175.0000	0.00000	175.0000	0.00000	175.0000	0.00000
200.0000	0.00000	200.0000	0.00000	200.0000	0.00000
250.0000	0.00000	250.0000	0.00000	250.0000	0.00000
300.0000	0.00000	300.0000	0.00000	300.0000	0.00000
350.0000	0.00000	350.0000	0.00000	350.0000	0.00000
400.0000	0.00000	400.0000	0.00000	400.0000	0.00000
500.0000	0.00017	500.0000	0.00018	500.0000	0.00016
600.0000	0.00136	600.0000	0.00142	600.0000	0.00130
700.0000	0.00569	700.0000	0.00590	700.0000	0.00548
800.0000	0.01601	800.0000	0.01651	800.0000	0.01552
900.0000	0.03515	900.0000	0.03613	900.0000	0.03420
1000.0000	0.06442	1000.0000	0.06603	1000.0000	0.06286
1100.0000	0.10430	1100.0000	0.10667	1100.0000	0.10199
1200.0000	0.15396	1200.0000	0.15720	1200.0000	0.15082
1300.0000	0.21221	1300.0000	0.21641	1300.0000	0.20813
1400.0000	0.27732	1400.0000	0.28254	1400.0000	0.27227
1500.0000	0.34768	1500.0000	0.35396	1500.0000	0.34161
1600.0000	0.42171	1600.0000	0.42906	1600.0000	0.41459
1700.0000	0.49798	1700.0000	0.50639	1700.0000	0.48983
1800.0000	0.57526	1800.0000	0.58471	1800.0000	0.56611
1900.0000	0.65251	1900.0000	0.66293	1900.0000	0.64240
2000.0000	0.72883	2000.0000	0.74015	2000.0000	0.71784

Fig. 10. Schottky Specific Heat Contribution From the $4f^{10}$ Ground Multiplet of Trivalent Holmium.

TABLE XII. Schottky Specific Heat Contribution From the $4f^{11}$ Ground Multiplet of Trivalent Erbium.

From Mean Value of E		From Minimum Value of E		From Maximum Value of E	
G	E	G	E	G	E
16.0000	0.00000	16.0000	0.00000	16.0000	0.00000
14.0000	6492.00000	14.0000	6428.00000	14.0000	6556.00000
12.0000	10122.00000	12.0000	10053.00000	12.0000	10191.00000
10.0000	12375.00000	10.0000	12239.00000	10.0000	12511.00000
T	CM	T	CM	T	CM
50.0000	0.00000	50.0000	0.00000	50.0000	0.00000
75.0000	0.00000	75.0000	0.00000	75.0000	0.00000
100.0000	0.00000	100.0000	0.00000	100.0000	0.00000
125.0000	0.00000	125.0000	0.00000	125.0000	0.00000
150.0000	0.00000	150.0000	0.00000	150.0000	0.00000
175.0000	0.00000	175.0000	0.00000	175.0000	0.00000
200.0000	0.00000	200.0000	0.00000	200.0000	0.00000
250.0000	0.00000	250.0000	0.00000	250.0000	0.00000
300.0000	0.00000	300.0000	0.00000	300.0000	0.00000
350.0000	0.00000	350.0000	0.00000	350.0000	0.00000
400.0000	0.00000	400.0000	0.00000	400.0000	0.00000
500.0000	0.00000	500.0000	0.00000	500.0000	0.00000
600.0000	0.00000	600.0000	0.00000	600.0000	0.00000
700.0000	0.00050	700.0000	0.00055	700.0000	0.00044
800.0000	0.00201	800.0000	0.00221	800.0000	0.00183
900.0000	0.00582	900.0000	0.00632	900.0000	0.00535
1000.0000	0.01345	1000.0000	0.01446	1000.0000	0.01251
1100.0000	0.02616	1100.0000	0.02789	1100.0000	0.02453
1200.0000	0.04507	1200.0000	0.04774	1200.0000	0.04247
1300.0000	0.07069	1300.0000	0.07444	1300.0000	0.06712
1400.0000	0.10319	1400.0000	0.10813	1400.0000	0.09845
1500.0000	0.14234	1500.0000	0.14854	1500.0000	0.13638
1600.0000	0.18767	1600.0000	0.19513	1600.0000	0.18046
1700.0000	0.23848	1700.0000	0.24717	1700.0000	0.23005
1800.0000	0.29398	1800.0000	0.30385	1800.0000	0.28440
1900.0000	0.35334	1900.0000	0.36428	1900.0000	0.34267
2000.0000	0.41567	2000.0000	0.42759	2000.0000	0.40403

Fig. 11. Schottky Specific Heat Contribution From the $4f^{11}$ Ground Multiplet of Trivalent Erbium.

TABLE XIII. Schottky Specific Heat Contribution From the $4f^{12}$ Multiplets of Trivalent Thulium.

From Mean Value of E		From Minimum Value of E		From Maximum Value of E	
G	E	G	E	G	E
13.0000	0.00000	13.0000	0.00000	13.0000	0.00000
9.0000	5630.00000	9.0000	5450.00000	9.0000	5810.00000
11.0000	8242.00000	11.0000	8092.00000	11.0000	8392.00000
9.0000	12653.00000	9.0000	12500.00000	9.0000	12806.00000
7.0000	14451.00000	7.0000	14302.00000	7.0000	14600.00000
5.0000	15011.00000	5.0000	14855.00000	5.0000	15267.00000
T	CM	T	CM	T	CM
50.0000	0.00000	50.0000	0.00000	50.0000	0.00000
75.0000	0.00000	75.0000	0.00000	75.0000	0.00000
100.0000	0.00000	100.0000	0.00000	100.0000	0.00000
125.0000	0.00000	125.0000	0.00000	125.0000	0.00000
150.0000	0.00000	150.0000	0.00000	150.0000	0.00000
175.0000	0.00000	175.0000	0.00000	175.0000	0.00000
200.0000	0.00000	200.0000	0.00000	200.0000	0.00000
250.0000	0.00000	250.0000	0.00000	250.0000	0.00000
300.0000	0.00000	300.0000	0.00000	300.0000	0.00000
350.0000	0.00000	350.0000	0.00000	350.0000	0.00000
400.0000	0.00000	400.0000	0.00000	400.0000	0.00000
500.0000	0.00000	500.0000	0.00000	500.0000	0.00000
600.0000	0.00034	600.0000	0.00049	600.0000	0.00024
700.0000	0.00173	700.0000	0.00235	700.0000	0.00128
800.0000	0.00578	800.0000	0.00748	800.0000	0.00435
900.0000	0.01428	900.0000	0.01783	900.0000	0.01142
1000.0000	0.02903	1000.0000	0.03521	1000.0000	0.02388
1100.0000	0.05125	1100.0000	0.06071	1100.0000	0.04318
1200.0000	0.08157	1200.0000	0.09483	1200.0000	0.07009
1300.0000	0.12017	1300.0000	0.13727	1300.0000	0.10499
1400.0000	0.16644	1400.0000	0.18743	1400.0000	0.14744
1500.0000	0.21966	1500.0000	0.24433	1500.0000	0.19708
1600.0000	0.27875	1600.0000	0.30673	1600.0000	0.25280
1700.0000	0.34266	1700.0000	0.37349	1700.0000	0.31372
1800.0000	0.41027	1800.0000	0.44343	1800.0000	0.37880
1900.0000	0.48054	1900.0000	0.51546	1900.0000	0.44702
2000.0000	0.55246	2000.0000	0.58860	2000.0000	0.51741

Fig. 12. Schottky Specific Heat Contribution From the $4f^{12}$ Multiplets of Trivalent Thulium.

TABLE XIV. Schottky Specific Heat Contribution From the $4f^{13}$ Ground Multiplet of Trivalent Ytterbium.

G	E
8.0000	0.00000
6.0000	10300.00000
T	CM
50.0000	0.00000
75.0000	0.00000
100.0000	0.00000
125.0000	0.00000
150.0000	0.00000
175.0000	0.00000
200.0000	0.00000
250.0000	0.00000
300.0000	0.00000
350.0000	0.00000
400.0000	0.00000
500.0000	0.00000
600.0000	0.00000
700.0000	0.00000
800.0000	0.00000
900.0000	0.00000
1000.0000	0.00012
1100.0000	0.00038
1200.0000	0.00098
1300.0000	0.00217
1400.0000	0.00422
1500.0000	0.00744
1600.0000	0.01213
1700.0000	0.01852
1800.0000	0.02681
1900.0000	0.03711
2000.0000	0.04945

Fig. 13. Schottky Specific Heat Contribution From the $4f^{13}$ Ground Multiplet of Trivalent Ytterbium.